Discretisation of 2D domains, bounded by NURBS curves

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- Multipatch NURBS are also common.

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- Requirement 1: Algorithm takes N or h as input.
- Requirement 2: Quasiuniformness, quasirandomness.

inRS

- inRS[1] algorithm for calculating indicator function of Ω.

[1] Sommariva, Vianello inRS: Implementing the indicator function of NURBS-shaped planar domains.

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- For each I_k, calculate the roots of the derivatives of the appropriate rational function.
- For each $I_{k,j}$ find $a_{k,j} = \min_{I_{k,j}} \alpha_k(\xi)$
- Cover $\partial\Omega$ by monotone boxes $\mathcal{B}_{k,j} = [a_{k,j}, b_{k,j}] \times [c_{k,j}, d_{k,j}]$.

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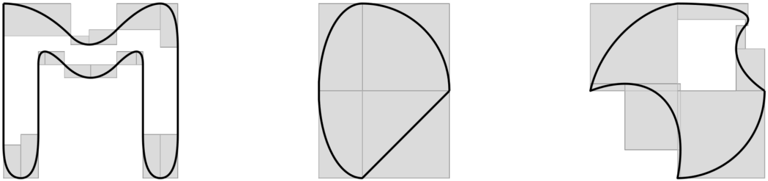
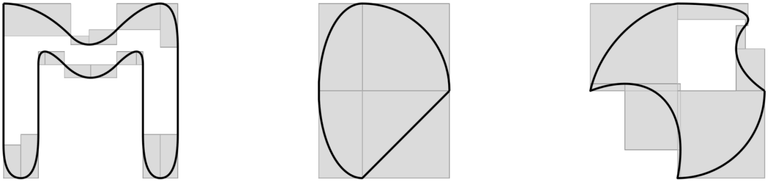


- If (x_0, y_0) outside global bounding box $\rightarrow$ also outside Ω.



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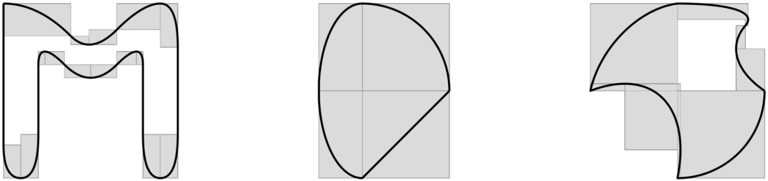
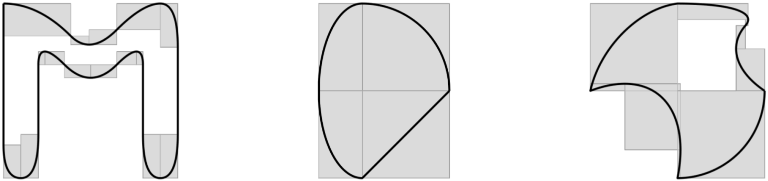


- If (x_0, y_0) outside global bounding box $\rightarrow$ also outside Ω.
- If (x_0, y_0) outside monotone boxes - polygon case.
- If inside one of the monotone boxes - polynomial solve.



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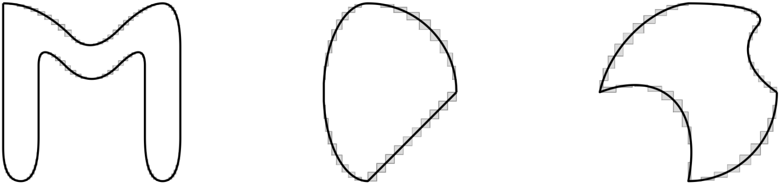
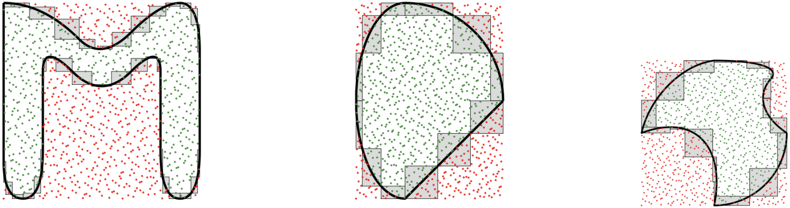
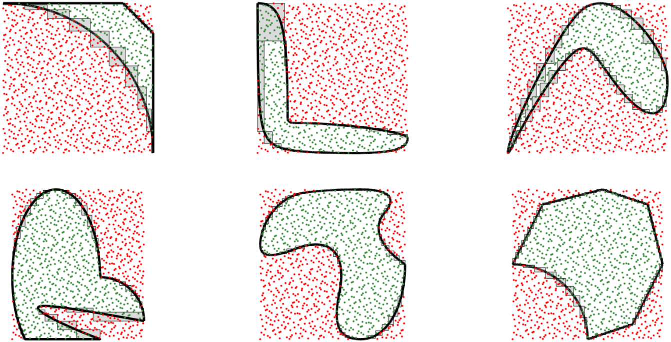


- Halton nodes + rejection sampling





DIVG

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[2]Slak, Kosec: On generation of node distributions for meshless PDE discretizations

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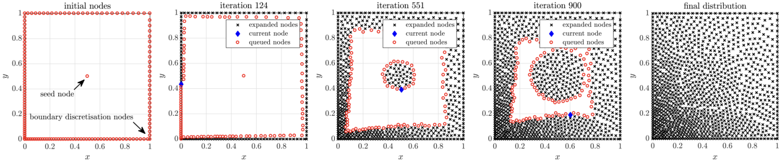

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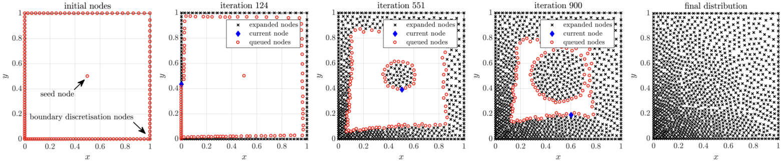

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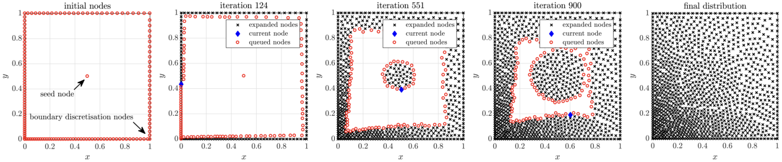

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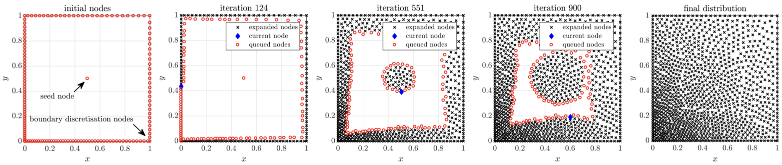

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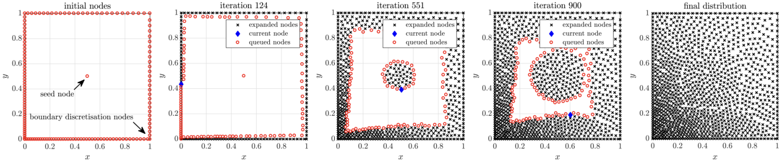
- Algorithm is efficient with the help of a kd-tree.



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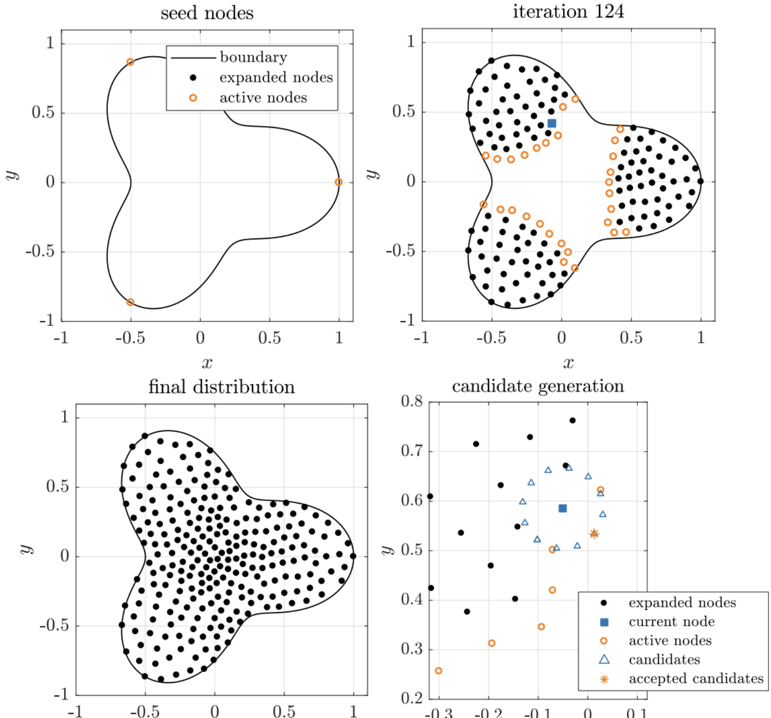

seed nodes
boundary
expanded nodes
active nodes
iteration 124
final distribution
candidate generation
expanded nodes
current node
active nodes
candidates
accepted candidates
x
y

- sDIVG[3] idea - use DIVG in the parameter space.

[3] Duh, Kosec, Slak: Fast variable density node generation on parametric surfaces with application to mesh-free methods

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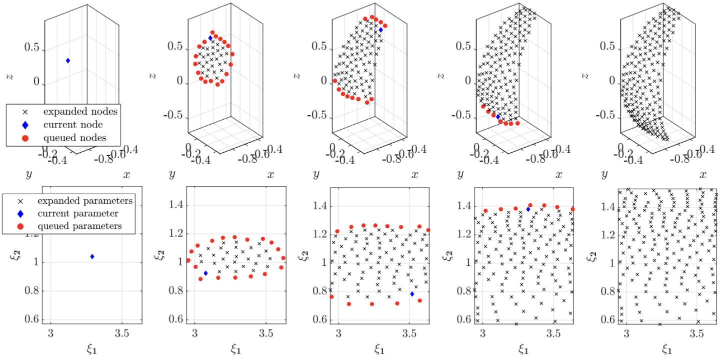

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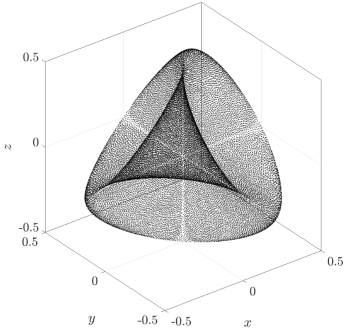
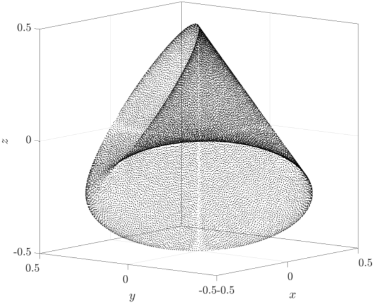
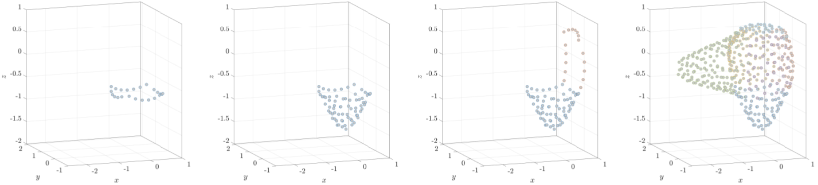

0.5
0
-0.5
z
0.5
0
-0.5
y
-0.5
0
0.5
x


0.5
0
-0.5
z
0.5
0
-0.5
y
-0.5
0
0.5
x

- NURBS-DIVG[4]

[4] Duh, Shankar, Kosec: Discretization of non-uniform rational B-spline (NURBS) models for meshless isogeometric analysis

- NURBS-DIVG[4]
- Multipatch - Discretise each patch seperately with sDIVG.



[4] Duh, Shankar, Kosec: Discretization of non-uniform rational B-spline (NURBS) models for meshless isogeometric analysis

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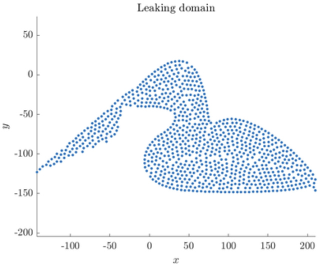
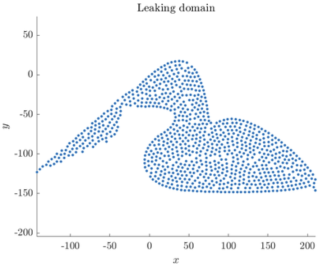
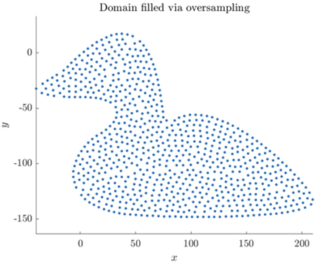
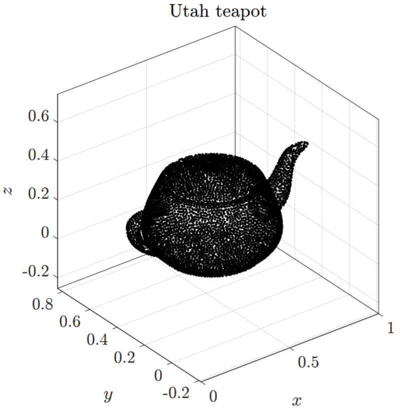


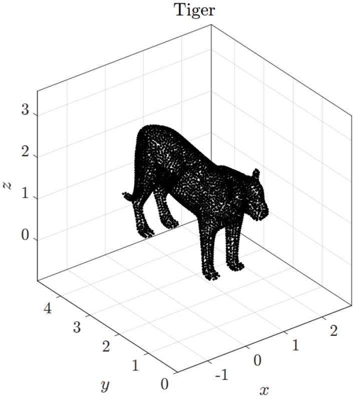
- Interior check - $(\mathbf{p} - \mathbf{x}) \cdot \mathbf{n} > 0$
- Improve accuracy with supersampling.



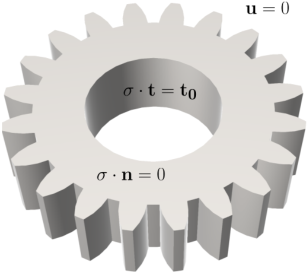


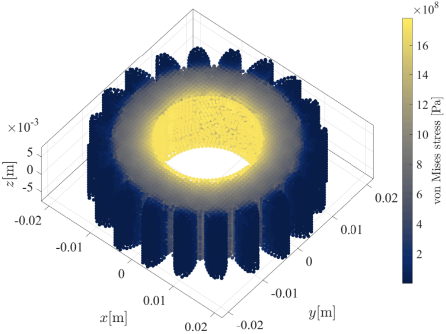


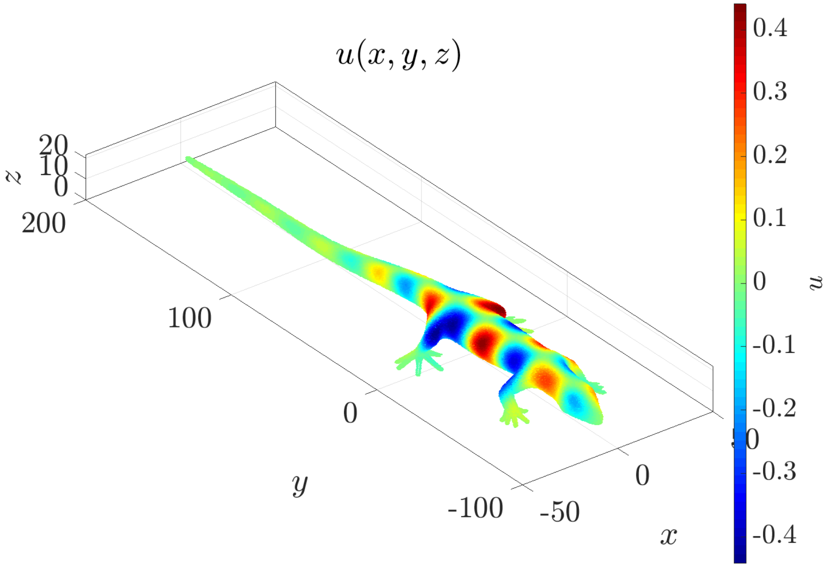
Utah teapot
z
0.6
0.4
0.2
0
-0.2
y
0.8
0.6
0.4
0.2
0
-0.2
x
0
0.5
1



Tiger
z
3
2
1
0
y
4
3
2
1
0
x
-1
0
1
2

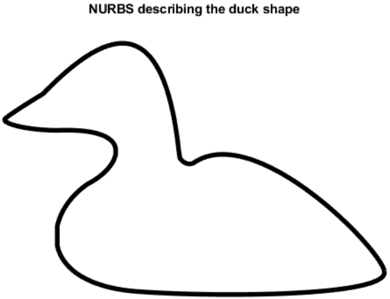

$\mathbf{u} = 0$
$\sigma \cdot \mathbf{t} = \mathbf{t_0}$
$\sigma \cdot \mathbf{n} = 0$

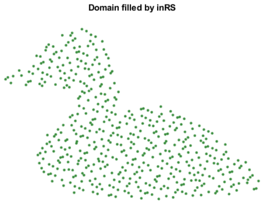

$\times 10^{-3}$
5
0
-5
z[m]
-0.02
-0.01
0
0.01
0.02
x[m]
-0.02
-0.01
0
0.01
0.02
y[m]
$\times 10^{8}$
16
14
12
10
8
6
4
2
von Mises stress [Pa]

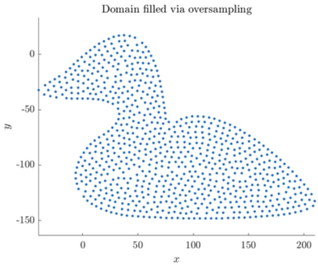


$u(x, y, z)$
z
20
10
0
200
100
0
-100
y
-50
0
50
x
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
u

Comparison


NURBS describing the duck shape


Domain filled by inRS


Domain filled via oversampling
0
-50
-100
-150
y
0
50
100
150
200
x