① IZRAČUNAJ LIMITE

a) $$\lim_{(x,y)\to(5,1)} \frac{xy}{x+y} = \frac{5}{6}$$ ZA ZVEZNO FUNKCIJO KAR VSTAVIMO x,y.

b) $$\lim_{(x,y)\to(1,1)} \frac{2x^2-xy-y^2}{x^2-y^2} = \lim_{(x,y)\to(1,1)} \frac{(2x+y)(x-y)}{(x+y)(x-y)} = \lim_{(x,y)\to(1,1)} \frac{2x+y}{x+y} = \frac{3}{2}$$

② ZA NASLEDNJE $f(x,y)$ IZRAČUNAJ $\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y)$, $\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y)$, $\lim\limits_{(x,y)\to(0,0)} f(x,y)$

a) $$f(x,y) = \frac{x^2-y^2+2x^3+3y^3}{x^2+y^2}$$

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{x\to 0} \frac{x^2+2x^3}{x^2} = 1, \quad \lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0} \frac{-y^2+3y^3}{y^2} = -1$$

$\lim\limits_{(x,y)\to(0,0)} f(x,y)$ NE OBSTAJA (ČE BI OBSTAJALA, BI BILA VREDNOST NEODVISNA OD POTI DO $(0,0)$)

b) $$f(x,y) = \frac{x^2y}{(x^2+y)^2}$$

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y) = 0 \qquad \lim_{y\to 0}\lim_{x\to 0} f(x,y) = 0$$

$\lim\limits_{(x,y)\to(0,0)} f(x,y) = ?$ NIČ SE NE POKRAJŠA, POLARNE KOORDINATE NE DELUJEJO... DAJMO PROBAT POKAZAT, DA NE OBSTAJA, S TEM DA NAJDEMO POT DO $(0,0)$ KI DA DRUGAČEN REZULTAT.

$(0,0)$ SE PRIBLIŽUJEMO PO POTI $y=x$: $$\lim_{x\to 0} f(x,x) = \lim_{x\to 0} \frac{x^3}{(x^2+x)^2} = 0$$

$y=x^2$:
$$\lim_{x\to 0} f(x,x^2) = \lim_{x\to 0} \frac{x^4}{4x^4} = \frac{1}{4} \Rightarrow \lim_{(x,y)\to(0,0)} f(x,y)$$ NE OBSTAJA

1. KOLOKVIJ 2020 1. NALOGA

$$f(x,y) = 1 + e^{\sqrt{1-x^2-y^2}}$$

a) DOLOČI D_f, Z_f

$D_f: 1-x^2-y^2 \geq 0 \Rightarrow x^2+y^2 \leq 1$

$D_f = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$

Z_f: V D_f $\sqrt{1-x^2-y^2}$ ZAVZAME VREDNOSTI IZ $[0,1]$

$\sqrt{1-x^2-y^2} = 0 \Rightarrow f(x,y) = 2$

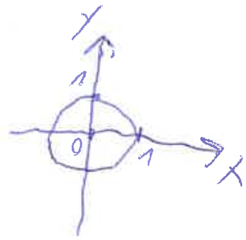
$\sqrt{1-x^2-y^2} = 1 \Rightarrow f(x,y) = 1+e$

$Z_f = [2, 1+e]$

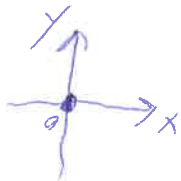
b) NARIŠI NIVOJNICE f ZA VREDNOSTI $2, 1+e, 1+\sqrt{e}$

NIVOJNICA: $f(x,y) = c$

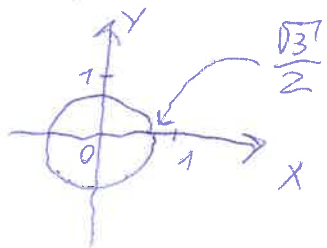
$c=2$: $2 = f(x,y) = 1+e^{\sqrt{1-x^2-y^2}} \Rightarrow \sqrt{1-x^2-y^2} = 0 \Rightarrow x^2+y^2 = 1$



$c=1+e$: $1+e = f(x,y) \Rightarrow \sqrt{1-x^2-y^2} = 1 \Rightarrow (x,y) = (0,0)$

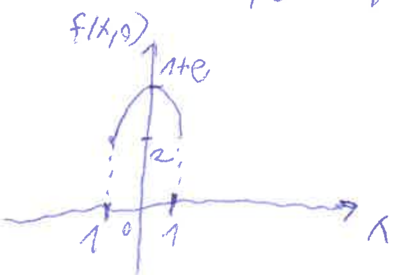


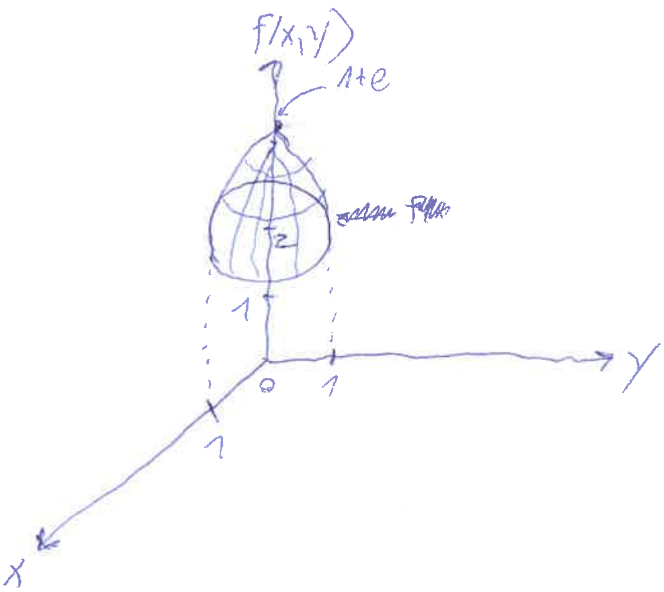
$c = 1+\sqrt{e}$: $1+\sqrt{e} = f(x,y) \Rightarrow \sqrt{1-x^2-y^2} = \frac{1}{2} \Rightarrow x^2+y^2 = \frac{3}{4}$



c) SKICIRAJ GRAF f

PREREZ $f(x,0) = 1 + e^{\sqrt{1-x^2}}$





d) $z=f(x,y)$, $x=s$, $y=st^2$. IZRAČUNAJTE $\frac{\partial z}{\partial t}, \frac{\partial z}{\partial s}$ ZA $t=1, s=\frac{1}{2}$

$$z=f(x(s,t),y(s,t)) \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\underbrace{\frac{\partial x}{\partial t}}_{=0} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} =$$

$$= e^{\sqrt{1-x^2-y^2}} \cdot \frac{(-y)}{\sqrt{1-x^2-y^2}} \cdot 2st = e^{\sqrt{1/2}} \cdot \frac{(-1/2)}{\sqrt{1/2}} \cdot 1 =$$

$t=1, s=\frac{1}{2} \Rightarrow x=\frac{1}{2}, y=\frac{1}{2}$

$$= -\frac{\sqrt{2}}{2} e^{\sqrt{1/2}} = \boxed{-\frac{\sqrt{2}}{2} e^{\sqrt{2}/2}}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = -\frac{e^{\sqrt{1-x^2-y^2}}}{\sqrt{1-x^2-y^2}}\left(x\cdot 1 + y\cdot t^2\right) = -e^{\sqrt{2}/2}\sqrt{2}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$= \boxed{-\sqrt{2}\, e^{\sqrt{2}/2}}$$

3. NALOGA $\quad f(x,y) = y^{14}e^{x} + \sqrt{x}\sin(\sqrt{x}y)$

a) DOLOČI D_f $\qquad D_f = \{(x,y)\in\mathbb{R}^2 \mid x\geq 0\}$

b) RAZVIJ $f(x,y)$ V TAYLORJEVO VRSTO OKOLI $(x,y)=(0,0)$

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!} \qquad \sin(x) = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!}$$

$$\Rightarrow f(x,y) = y^{14}\sum_{k=0}^{\infty}\left[\frac{x^k}{k!}\right] + \sqrt{x}\sum_{k=0}^{\infty}\left[(-1)^k\frac{(\sqrt{x}y)^{2k+1}}{(2k+1)!}\right] =$$

$$= \sum_{k=0}^{\infty}\left(\frac{x^k y^{14}}{k!} + (-1)^k\frac{x^{k+1}y^{2k+1}}{(2k+1)!}\right)$$

c) IZRAČUNAJTE $\frac{\partial^{14}f}{\partial x^5\partial y^9}(0,0)$ IN $\frac{\partial^{14}f}{\partial x^7\partial y^7}(0,0)$

$$\frac{\partial^{14}f}{\partial x^5\partial y^9}(0,0) = 5!\cdot 9!\cdot(\text{KOEF. PRED } x^5y^9) = 5!\cdot 9!\cdot\frac{1}{9!} = 5!$$

$$\frac{\partial^{14}f}{\partial x^7\partial y^7}(0,0) = 0$$, KER ČLENA x^7y^7 NI V RAZVOJU

d) S POMOČJO LINEARNEGA PRIBLIŽKA OCENI VREDNOST $f(-0.5, 0.1)$

LINEARNIH ČLENOV V RAZVOJU NI, TOREJ JE REZULTAT 0.

1. KOLOKVIJ 2019 1. NALOGA

$$f(x,y) = \sqrt{x}\,\sqrt[5]{1+y^2}$$

a) RAZVIJ f V TAYLORJEVO VRSTO OKOLI $(1,0)$.

$$f(x,y) = (1+x-1)^{1/2}\cdot(1+y^2)^{1/5} = \left[\sum_{k=0}^{\infty}\binom{1/2}{k}(x-1)^k\right]\cdot\left[\sum_{k=0}^{\infty}\binom{1/5}{k}y^{2k}\right]$$

b) S POMOČJO TAYLORJEVEGA POLINOMA DRUGE STOPNJE PRIBLIŽNO IZRAČUNAJ $f(1.01,-0.5)$

$$f(x,y) = \left(1+\frac{1}{2}(x-1)+\left(-\frac{1}{8}\right)(x-1)^2+\text{višji členi}\right)\cdot\left(1+\frac{1}{5}y^2+\text{višji čl.}\right)$$

$$\binom{1/2}{2} = \frac{1/2\cdot(1/2-1)}{2!} = -\frac{1}{8}$$

$$= 1+\frac{y^2}{5}+\frac{x-1}{2}-\frac{(x-1)^2}{8}+\text{višji členi} =$$

$$= 1+\underbrace{\frac{25\cdot 10^{-2}}{5}}_{0.05}+\underbrace{\frac{0.01}{2}}_{0.005}-\underbrace{\frac{10^{-4}}{8}}_{0.0000125}+\text{višji členi} =$$

$$= 1.055 - 0.0000125 + \text{višji} =$$

$$= \boxed{1.054875 + \text{višji čl.}}$$

c) IZRAČUNAJ $\frac{\partial^{2020} f}{\partial x^{2019}\partial y}(1,0)$ IN $\frac{\partial^{2020} F}{\partial x^{2018}\partial y^2}(1,0)$

$$\frac{\partial^{2020} f}{\partial x^{2019}\partial y}(1,0) = 0$$ KER ČLENA $(x-1)^{2019}y$ NI V RAZVOJU

$$\frac{\partial^{2020} f}{\partial x^{2018}\partial y^2}(1,0) = 2018!\cdot 2!\cdot\binom{1/2}{2018}\cdot\frac{1}{5} = \boxed{\frac{2}{5}\cdot\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)\cdot\ldots\cdot\left(\frac{1}{2}-2017\right)}$$