Zlepki nad triangulacijami, DN 4

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1 Četrta naloga

Naloga 4 [28. 3. – 3. 4.].
Naj bodo $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2 \in \mathbb{R}^2$ točke, ki določajo baricentrično ogrodje t za $\mathbb{R}^2$. Za vsak $i \in \{0,1,2\}$ naj bodo $\alpha_i, \alpha_{i,1}, \alpha_{i,2} \in \mathbb{R}$ parametri, ki določajo trojico koeficientov polinoma $P \in \mathbb{P}_3^2$ v Bernstein–Bézierjevi reprezentaciji glede na t tako, da velja

$$P(\boldsymbol{p}_i) = \alpha_i, \quad \frac{\partial}{\partial_1} P(\boldsymbol{p}_i) = \alpha_{i,1}, \quad \frac{\partial}{\partial_2} P(\boldsymbol{p}_i) = \alpha_{i,2}.$$

Preostali koeficient $b_{1,1,1}\langle t\rangle$ polinoma P naj bo podan z

$$\begin{aligned} b_{1,1,1}\langle t\rangle \;\; = \;\; & \frac{1}{3}\left(\frac{3}{4}b_{2,1,0}\langle t\rangle + \frac{3}{4}b_{2,0,1}\langle t\rangle - \frac{1}{2}b_{3,0,0}\langle t\rangle\right) + \\ & \frac{1}{3}\left(\frac{3}{4}b_{0,2,1}\langle t\rangle + \frac{3}{4}b_{1,2,0}\langle t\rangle - \frac{1}{2}b_{0,3,0}\langle t\rangle\right) + \\ & \frac{1}{3}\left(\frac{3}{4}b_{1,0,2}\langle t\rangle + \frac{3}{4}b_{0,1,2}\langle t\rangle - \frac{1}{2}b_{0,0,3}\langle t\rangle\right). \end{aligned}$$

Sestavite metodo, ki sprejme tabelo velikosti 3×2 z ogrodjem t in tabelo velikosti 3×3 s parametri $\alpha_i, \alpha_{i,1}, \alpha_{i,2}$, $i = 0,1,2$, vrne pa koeficiente $b_{\boldsymbol{d}}\langle t\rangle$, $\boldsymbol{d} \in \mathbb{D}_3^2$, polinoma P.

Izpeljimo formulo za algoritem. Ključne so enačbe:

$$P(\mathbf{p}) = \mathcal{B}[P](\mathbf{p} : d) \tag{1}$$
$$\mathcal{B}[P](t : \mathbf{d}) = b_{\mathbf{d}}\langle t\rangle \tag{2}$$
$$D_{\mathbf{v}} f(\mathbf{p}) = \nabla^T f(\mathbf{p}) \cdot \mathbf{v} \tag{3}$$
$$D_{\mathbf{v}} P(\mathbf{p}) = d(\mathcal{B}[P](\mathbf{q}+\mathbf{v}, \mathbf{p} : d-1) - \mathcal{B}[P](\mathbf{q}, \mathbf{p} : d-1)) \tag{4}$$
$$\tag{5}$$

Računajmo. Pri nas je $d = 3$. Iz vrednosti polinoma dobimo

$$P(\mathbf{p}_0) = \mathcal{B}[P](\mathbf{p}_0 : 3) = b_{3,0,0}\langle t\rangle \tag{6}$$
$$P(\mathbf{p}_1) = \mathcal{B}[P](\mathbf{p}_1 : 3) = b_{0,3,0}\langle t\rangle \tag{7}$$
$$P(\mathbf{p}_2) = \mathcal{B}[P](\mathbf{p}_2 : 3) = b_{0,0,3}\langle t\rangle \tag{8}$$

Najprej odvodi v točki $\mathbf{p}_0$.

$$D_{\mathbf{p}_1-\mathbf{p}_0}(P(\mathbf{p}_0)) = 3(\mathcal{B}[P](\mathbf{p}_1, \mathbf{p}_0 : 2) - \mathcal{B}[P](\mathbf{p}_0 : 3)) = 3(b_{2,1,0}\langle t\rangle - b_{3,0,0}\langle t\rangle) \tag{9}$$

$$D_{\mathbf{p}_2-\mathbf{p}_0}(P(\mathbf{p}_0)) = 3(\mathcal{B}[P](\mathbf{p}_2, \mathbf{p}_0 : 2) - \mathcal{B}[P](\mathbf{p}_0 : 3)) = 3(b_{2,0,1}\langle t\rangle - b_{3,0,0}\langle t\rangle) \tag{10}$$

V točki $\mathbf{p}_1$:

$$D_{\mathbf{p}_0-\mathbf{p}_1}(P(\mathbf{p}_1)) = 3(\mathcal{B}[P](\mathbf{p}_0, \mathbf{p}_1 : 2) - \mathcal{B}[P](\mathbf{p}_1 : 3)) = 3(b_{1,2,0}\langle t\rangle - b_{0,3,0}\langle t\rangle) \tag{11}$$

$$D_{\mathbf{p}_2-\mathbf{p}_1}(P(\mathbf{p}_1)) = 3(\mathcal{B}[P](\mathbf{p}_2, \mathbf{p}_1 : 2) - \mathcal{B}[P](\mathbf{p}_1 : 3)) = 3(b_{0,2,1}\langle t\rangle - b_{0,3,0}\langle t\rangle) \tag{12}$$

V točki $\mathbf{p}_2$:

$$D_{\mathbf{p}_0-\mathbf{p}_2}(P(\mathbf{p}_2)) = 3(\mathcal{B}[P](\mathbf{p}_0, \mathbf{p}_2 : 2) - \mathcal{B}[P](\mathbf{p}_2 : 3)) = 3(b_{1,0,2}\langle t\rangle - b_{0,0,3}\langle t\rangle) \tag{13}$$

$$D_{\mathbf{p}_1-\mathbf{p}_2}(P(\mathbf{p}_2)) = 3(\mathcal{B}[P](\mathbf{p}_1, \mathbf{p}_2 : 2) - \mathcal{B}[P](\mathbf{p}_2 : 3)) = 3(b_{0,1,2}\langle t\rangle - b_{0,0,3}\langle t\rangle) \tag{14}$$