Machine Learning

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1. Introduction
2. Logistic Regression
3. Evaluation

What is Classification?

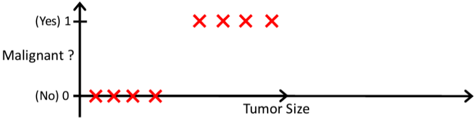
Classification is a supervised learning task where the goal is to predict the category or class label of a given input based on its features. It involves learning a decision boundary that separates different classes in the data.

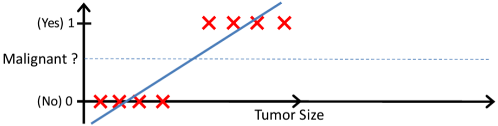
Binary Classification Examples

- **Medical Diagnosis:** Predicting whether a tumor is *benign* or *malignant*.
- **Spam Detection:** Classifying emails as *spam* or *not spam*.
- **Loan Approval:** Predicting whether a loan application is *approved* or *rejected*.
- **Weather Prediction:** Classifying if it will *rain* or *not rain* tomorrow.

Classification Problem

Can we solve this problem by Linear Regression?



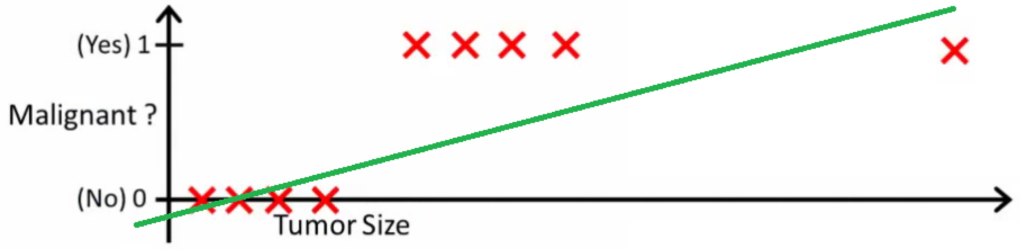


We could fit a straight line and define a threshold at 0.5:

- If $h_\theta(x) \geq 0.5$, predict $y = 1$
- If $h_\theta(x) < 0.5$, predict $y = 0$

Classification Problem

- What about now? (By adding a new data point)



- **Classification:** $y = 0$ or $y = 1$
- $h_\theta(x)$ can be > 1 or < 0
- Another drawback of using linear regression for this problem

What we need:

1. Introduction
2. **Logistic Regression**
3. Evaluation

Introduction

Problem: Distinguish if a person is *overweight* or *not overweight* based on features like age, gender, height, weight, and BMI.

Age	Gender	Height (cm)	Weight (kg)	BMI	Overweight
25	Male	175	80	25.3	0
30	Female	160	60	22.5	0
...	...	...	...	...	...
35	Male	180	90	27.3	1

Notation:

- Features of a sample: **vector** x
- Label: y

Logistic Regression: We try to find a function $\sigma(w^T x)$ that predicts the posterior probabilities $P(y = 1 \mid x)$.

Introduction (cont.)

- $\sigma(w^T x)$: probability that $y = 1$ given x (parameterized by $\mathbf{w}$)

$$P(y = 1|x, \mathbf{w}) = \sigma(\mathbf{w}^T x)$$
$$P(y = 0|x, \mathbf{w}) = 1 - \sigma(\mathbf{w}^T x)$$

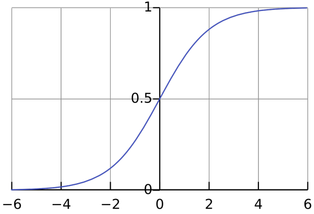
- We need to look for a function which gives us an output in the range [0, 1]. (like a probability).
- Let's denote this function with $\sigma(.)$ and call it the **activation function**.

Introduction (cont.)

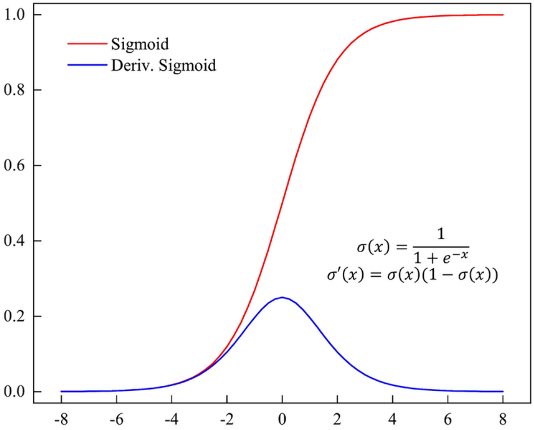
- Sigmoid (logistic) function.

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

- A good candidate for activation function.
- It gives us a number between 0 and 1 **smoothly**.
- It is also **differentiable**



Sigmoid function & its derivative



Introduction (cont.)

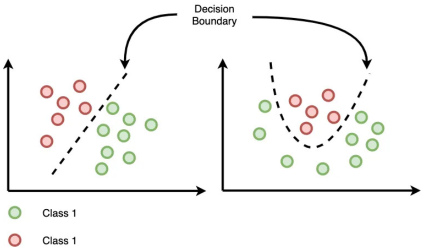
- The sigmoid function takes a number as input but we have:

$$x = [x_0 = 1, x_1, \ldots, x_d]$$
$$w = [w_0, w_1, \ldots, w_d]$$

- So we can use the **dot product** of x and w.
- We have $0 \leq \sigma(\mathbf{w}^T x) \leq 1$. which is the estimated probability of $y = 1$ on input x.
- An Example : A basketball game (Win, Lose)
  - $\sigma(\mathbf{w}^T x) = 0.7$
  - In other terms 70 percent chance of winning the game.

Decision surface

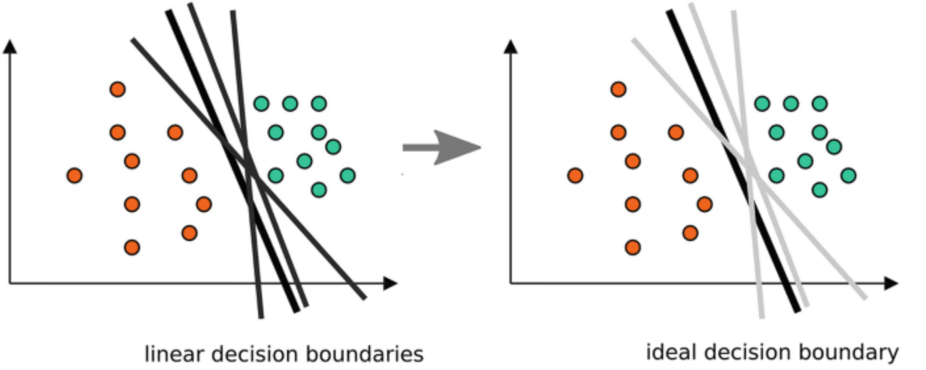
- Decision surface or decision boundary is the region of a problem space in which the output label of a classifier is ambiguous. (could be linear or non-linear)
- In binary classification it is where the probability of a sample belonging to each $y = 0$ and $y = 1$ is equal.



- Decision boundary hyperplane always has **one less dimension** than the feature space.

Decision surface (cont.)

- An example of linear decision boundaries:



Decision surface (cont.)

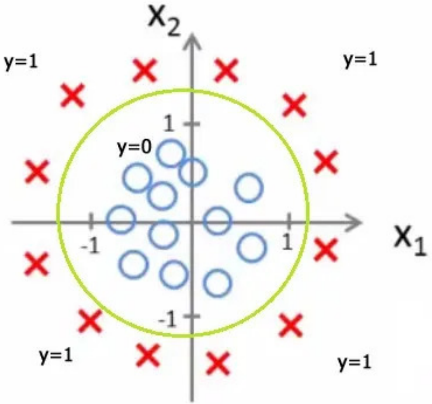
- Back to our logistic regression problem.
- Decision surface $\sigma(\mathbf{w}^T x)$ = **constant**.

$$\sigma(\mathbf{w}^T x) = \frac{1}{1 + e^{-(\mathbf{w}^T x)}} = 0.5$$

- Decision surfaces are **linear functions** of x
  - if $\sigma(\mathbf{w}^T x) \geq 0.5$ then $\hat{y} = 1$, else $\hat{y} = 0$
  - Equivalently, if $\mathbf{w}^T x + w_0 \geq 0.5$ then decide $\hat{y} = 1$, else $\hat{y} = 0$

$\hat{y}$ is the predicted label

Non-Linear Boundary Example



Predict $y = 1$ if $-1 + x_12 + x_22 \geq 0$

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Quote of the Day

It doesn't matter how beautiful your theory is ... If it doesn't agree with experiment, it's wrong.

Richard Feynman

Confusion Matrix and Accuracy

Confusion Matrix:

	Predicted Positive	**Predicted Negative**
Actual Positive	True Positive (TP)	False Negative (FN)
Actual Negative	False Positive (FP)	True Negative (TN)

Accuracy:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

Example: Cancer Detection

- **Scenario:** Predicting whether a tumor is malignant (1) or benign (0).
- **Confusion Matrix:**

	Predicted Malignant	Predicted Benign
Actual Malignant	50 (TP)	10 (FN)
Actual Benign	5 (FP)	85 (TN)

Calculations:

- **Accuracy:** $\frac{50+85}{50+85+10+5} = 0.935\ (93.5\%)$.
- **Precision:** $\frac{50}{50+5} = 0.909\ (90.9\%)$.
- **Recall:** $\frac{50}{50+10} = 0.833\ (83.3\%)$.

Precision and Recall

Precision:

$$\text{Precision} = \frac{TP}{TP + FP}$$

Focus: How many positive predictions are actually correct.

Recall (Sensitivity):

$$\text{Recall} = \frac{TP}{TP + FN}$$

Focus: How many actual positives are correctly identified.

F1 Score

F1 Score:

$$F1 = 2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$$

Use: Balances Precision and Recall, especially in imbalanced datasets.

ROC Curve and AUC

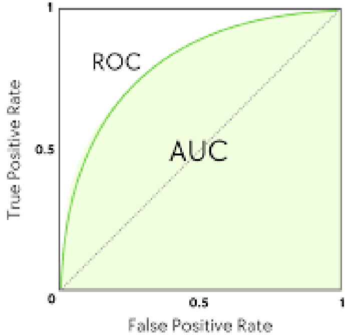
ROC Curve:

- Plots True Positive Rate (TPR) vs. False Positive Rate (FPR).
- Evaluates classifier performance at various thresholds.

AUC:

- Measures the overall ability of the model to distinguish between classes.
- Range: 0.5 (No discrimination) to 1.0 (Perfect discrimination).

ROC-AUC



Log-Loss (Logarithmic Loss)

Formula:

$$\text{Log-Loss} = -\frac{1}{n}\sum_{i=1}^{n}\left[y^{(i)}\log(p^{(i)}) + (1-y^{(i)})\log(1-p^{(i)})\right]$$

Use: Penalizes incorrect predictions with confidence.

Specificity (True Negative Rate)

Formula:

$$\text{Specificity} = \frac{TN}{TN + FP}$$

Focus: How many negatives are correctly identified.

Summary of Evaluation Metrics

Metric	Formula	Use Case
Accuracy	$\frac{TP+TN}{TP+TN+FP+FN}$	Overall correctness
Precision	$\frac{TP}{TP+FP}$	Positive prediction quality
Recall	$\frac{TP}{TP+FN}$	Positive detection rate
F1 Score	$2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision}+\text{Recall}}$	Balancing Precision and Recall
Specificity	$\frac{TN}{TN+FP}$	Negative detection rate
Log-Loss	$-\frac{1}{n}\sum \log$	Probabilistic predictions

For more information and code check the related notebook

End of Classification