Solving Equations with e and ln x

Session 17

1) $\ln(y+1)+\ln(y-1)=2x+\ln x \Rightarrow e^{\ln(y+1)+\ln(y-1)} = e^{2x+\ln x}$

$(y+1)(y-1)=e^{2x}x \Rightarrow y^2-1=e^{2x}x \Rightarrow y=\pm\sqrt{xe^{2x}+1}$ $\quad y>1$

2) $\log(y+1)=x^2+\log(y-1) \Rightarrow y+1=10^{x^2}(y-1)$

$y+1=10^{x^2}y-10^{x^2} \Rightarrow y(1-10^{x^2})=-1-10^{x^2} \Rightarrow y=\frac{10^{x^2}+1}{10^{x^2}-1}$ $\quad x\neq 1$

3) $2\ln y=\ln(y+1)+x \Rightarrow y^2=(y+1)e^x \Rightarrow y^2-e^x y-e^x=0$

$y=\frac{e^x+\sqrt{e^{2x}+4e^x}}{2}>0$

4) $\frac{e^x+e^{-x}}{e^x-e^{-x}}=y \Rightarrow \frac{u^2+1}{u^2-1}=y \Rightarrow u^2+1=yu^2-y \Rightarrow u^2(1-y)=-(y+1)$

$u^2=-\frac{y+1}{1-y} \Rightarrow u=\sqrt{\frac{y+1}{y-1}} \Rightarrow e^x=\sqrt{\frac{y+1}{y-1}} \Rightarrow x=\ln\left(\sqrt{\frac{y+1}{y-1}}\right)$

$\Rightarrow x=\frac{1}{2}(\ln(y+1)-\ln(y-1)) \quad y>1$

5) $e^x+e^{-x}=y \Rightarrow \frac{u^2+1}{u}=y \Rightarrow u^2-yu+1=0 \Rightarrow u=\frac{y\pm\sqrt{y^2-4}}{2}$

$x=\ln(y\pm\sqrt{y^2-4})-\ln(2)$