5B. Integration by direct substitution

9) $\int e^x (1+e^x)^{-1/3} dx = (1+e^x)^{2/3} \cdot \frac{3}{2} + C$

11) $\int \sec^2 9x \, dx = \frac{\tan 9x}{9} + C$

13) $\int \frac{x^2 dx}{1+x^6} = \int \frac{du}{3(1+u^2)} = \frac{\tan^{-1}(x^3)}{3} + C$

16) $\int_{-1}^{1} \frac{\tan^{-1} x \, dx}{1+x^2} = \frac{u^2}{2}\Big|_{-\pi/4}^{\pi/4} = 0$

5C. Trigonometric integrals

5) $\int \sin^3 x \cos^2 x \, dx = \int (1-\cos^2 x)\cos^2 x \sin x \, dx = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

7) $\int \sin^2(4x)\cos^2(4x)\, dx = \int \frac{(1-\cos 16x)\, dx}{8} = \frac{1}{8} - \frac{\sin 16x}{128} + C$

9) $\int \sin^3 x \sec^2 x \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \sin x \, dx = \cos x + \sec x + C$

11) $\int \sin x \cos 2x \, dx = \int \sin x (2\cos^2 x - 1)\, dx = \cos x - \frac{2}{3}\cos^3 x + C$

5D. Integration by inverse substitution

1) $\int \frac{dx}{(a^2-x^2)^{\frac{3}{2}}} = \frac{1}{a^2}\int \sec^2\theta\, d\theta = \frac{1}{a^2}\tan(\theta)+C = \frac{x}{a^2\sqrt{a^2-x^2}}+C$

2) $\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = a^3\int \sin^3\theta\, d\theta = a^3\left(-\cos\theta + \frac{1}{3}\cos^3(\theta)\right)+C$

3) $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = \int \frac{\tan^2\theta\, d\theta}{\sec\theta} = \int(\sec\theta-\cos\theta)\,d\theta$

$= \ln(\sec\theta+\tan\theta)-\sin\theta+C$

10) $\int \frac{dx}{(x^2+4x+13)^{3/2}} = \int \frac{dx}{(x+2)^2+3^2} = \frac{1}{9}\sin\theta+C$