1F. Chain rule, implicit differentiation

3) $y = x^{1/n} \Rightarrow y^n = x^1 \Rightarrow n y^{n-1} y' = 1 \Rightarrow y' = \frac{y^{1-n}}{n} = \frac{1}{n} x^{\frac{1}{n}-1}$

5) $\sin x + \sin y = 1/2 \Rightarrow \cos x + y' \cos y = 0 \Rightarrow \cos x = 0$

$\Rightarrow x = \frac{\pi}{2} \pm k\pi \Rightarrow \sin x = \pm 1 \Rightarrow \sin y = \pm 1 + \frac{1}{2} \Rightarrow \sin x = -1$

$\Rightarrow \sin y = -\frac{1}{2} \Rightarrow y = \frac{-\pi}{6} + 2n\pi, 7\frac{\pi}{6} + 2n\pi, x = \frac{\pi}{2} + 2k\pi$

8a) $V = 1/3 \pi r^2 h \Rightarrow 0 = \frac{\pi}{3}(2rr'h + r^2) \Rightarrow r' = \frac{-r^2}{2rh} = \frac{-r}{2h}$

8c) $c^2 = a^2 + b^2 - 2ab\cos\theta \Rightarrow 0 = 2aa' + 2b - 2\cos\theta(a'b + a)$

$\Rightarrow a' = \frac{a\cos\theta - b}{a - b\cos\theta}$

1G. Higher derivatives

4) $-(x+1)^{-2}, 2(x+1)^{-3}, -6(x+1)^{-4} \Rightarrow (-1)^n n!(x+1)^{-n-1}$

5b) $(x^p(1+x)^q)^{(q+p)} = \binom{q+p}{p} x^{p(p)} (x+1)^{q(q)} = (q+p)!$

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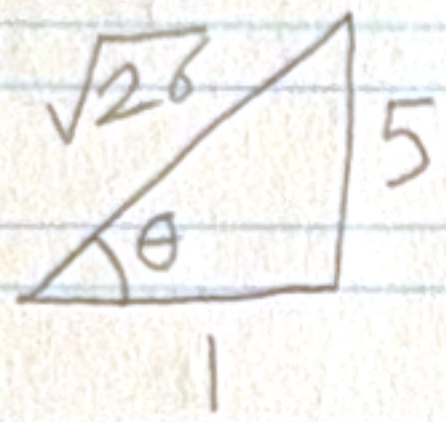
5A. Inverse trigonometric functions; Hyperbolic functions

1a) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

1b) $\sin^{-1}(\sqrt{3}/2) = \pi/3$

1c) 

$\Rightarrow \sin^{+1}(\theta) = \frac{5}{\sqrt{26}}$, $\cos(\theta) = \frac{1}{\sqrt{26}}$, $\sec(\theta) = \sqrt{26}$

3f) $\sin^{-1'}(a/x) = \frac{1}{\sqrt{1-(a/x)^2}} \cdot \frac{-a}{x^2} = \frac{-a}{x\sqrt{x^2-a^2}}$

3g) $y = x/\sqrt{1-x^2}$, $dy/dx = (1-x^2)^{-3/2}$, $1+y^2 = 1/(1-x^2)$

$\Rightarrow \frac{d}{dx}\tan^{-1}(y) = \frac{1}{\sqrt{1-x^2}}$

3h) $\sin^{-1}\sqrt{1-x} \Rightarrow y = \sqrt{1-x} \Rightarrow \frac{1 \cdot -\frac{1}{2}(1-x)^{-1/2}}{\sqrt{1-(1-x)}} = \frac{1 \cdot -\frac{1}{2}(1-x)^{-1/2}}{\sqrt{x}} = \frac{-1}{2\sqrt{x}\sqrt{1-x}}$

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1H. Exponentials and Logarithms: Algebra

1a) $e^{-kt} = 1/2 \Rightarrow -kt = -\ln 2 \Rightarrow t = \frac{\ln 2}{k}$

1b) $y_1 = y_0 e^{-kt_1} \Rightarrow y_0 e^{-k(t_1+\lambda)} = y_0 e^{-kt_1} e^{k\lambda} = y_1 \cdot 1/2$

2) $[H^+]_{Dil} = \frac{1}{2}[H^+]_{org} \Rightarrow PH_{Dil} = PH_{org} + \log 2$

3a) $y^2 - 1 = e^{2x} x \Rightarrow y = \sqrt{e^{2x} x + 1}$

5b) $u^2 - yu + 1 = 0 \Rightarrow x = \ln\left(\frac{y \pm \sqrt{y^2-4}}{2}\right)$

1I. Exponentials and Logarithms: Calculus

1c) $-2x \cdot e^{-x^2}$

1d) $\ln x + 1 - 1 = \ln x$

1e) $\frac{2x}{x^2} = \frac{2}{x}$

1f) $\frac{2}{x} \ln x$

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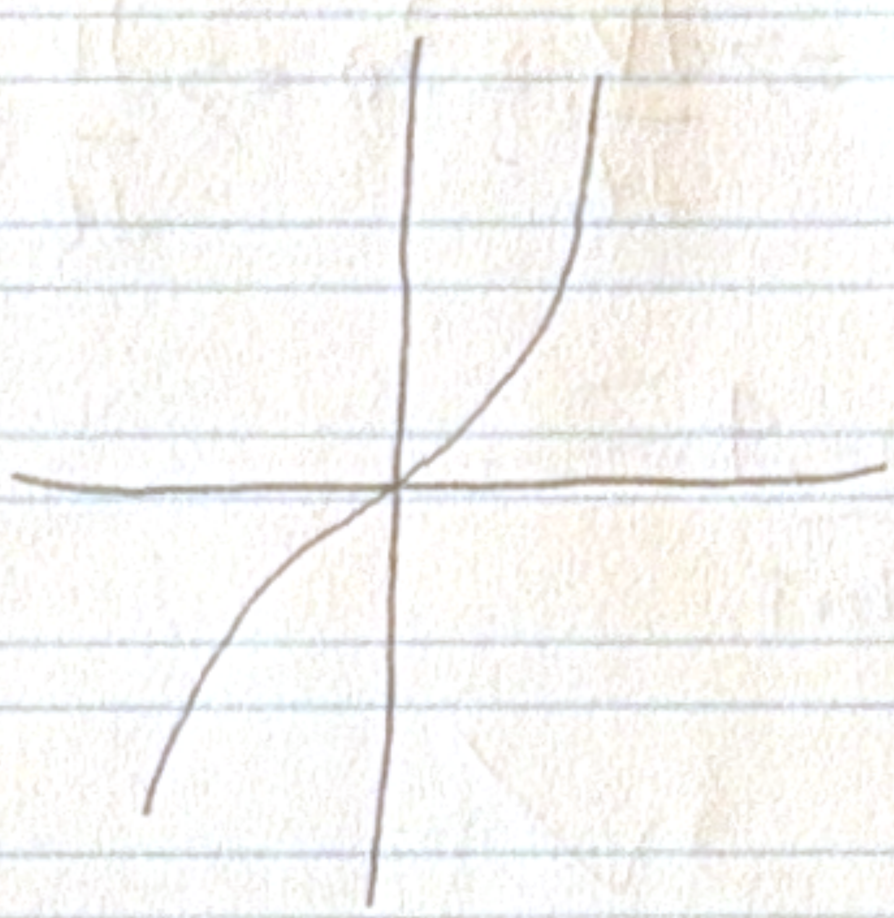
1m) $\frac{-2e^{x}}{(1+e^{x})^2}$

4a) $\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^{3n} = e^3$

5A. Inverse trigonometric functions; Hyperbolic functions

5a) no critical point, point of inflection $x=0$, is odd



5b) $y = \sinh^{-1} x \iff x = \sinh y$ Domain $= \mathbb{R}$

5c) $1 = \cosh y \, y' \Rightarrow y' = \frac{1}{\cosh y} = \frac{1}{\sqrt{\sinh^2 y + 1}} = \frac{1}{\sqrt{x^2+1}}$