unit2. Applications of Differentiation — Problem Set 5

2G. Mean-Value Theorem

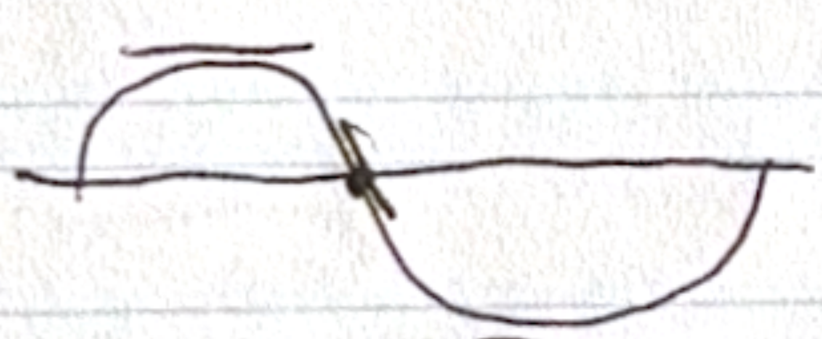
1b) $(\ln x)'_c = \ln 2 \Rightarrow \frac{1}{c} = \ln 2 \Rightarrow c = \frac{1}{\ln 2}$

2b) $(\sqrt{1+x})' = \frac{1}{2\sqrt{1+x}} \Rightarrow \sqrt{1+x} = 1 + \frac{1}{2\sqrt{1+c}}\,x < 1 + \frac{x}{2}$

5a) on $[a,b]$ $\bar{v}$ is 0 => by mvt $f'(c_1) = 0$

" $[b,c]$ $\bar{v}$ " 0 => " " $f'(c_2) = 0$

on $[c_1, c_2]$ $\bar{a}$ " 0 => " " $f''(p) = 0$

5b) 

6) $f(b) = f(a) + \overset{+}{f'(c)}\,\overset{+}{(b-a)} \Rightarrow f(b) > f(a)$

$'' \;=\; '' \quad \underset{0}{''} \quad = f(b) = f(a)$

Unit 3. Integration

3A. Differentials, indefinite integration

1 f) $d(e^{3x}\sin x) = (e^{3x}\sin x + e^{3x}\cos x)\,dx$

1 e) $dy = \left(1 - \frac{1}{\sqrt{x}}\right)dx$

2 a) $\frac{2}{5}x^5 + x^3 + \frac{1}{2}x^2 + 8x + C$

2 c) $\frac{2}{27}(8+9x)^{3/2} + C$

2 e) $\frac{-\sqrt{8-2x^2}}{2} + C$

2 i) $\frac{1}{3}\ln 3x+2 + C$

2 k) $x - \ln x+5 + C$

3 a) $-\frac{1}{5}\cos 5x + C$

3 c) $-\frac{1}{3}\cos^3 + C$

3e) $5\tan(x/5)+C$

3g) $\int u^8\,du = \frac{1}{9}u^9 = \frac{1}{9}\sec^9 + C$

3F. Differential equations:

Separation of variables

1c) $\frac{dy}{dx} = 3/\sqrt{y} \Rightarrow dy\sqrt{y} = 3dx \Rightarrow \frac{2}{2}y^{\frac{3}{2}} = 3x + C$

1d) $\frac{dy}{dx} = xy^2 \Rightarrow \frac{dy}{y^2} = x\,dx \Rightarrow -y^{-1} = \frac{1}{2}x^2$

2a) $\frac{dy}{dx} = 4xy \Rightarrow \frac{dy}{y} = 4x\,dx \Rightarrow \ln y = 2x^2 + C \Rightarrow C = \ln 3 - 2$

$y_{(3)} = a e^{2x^2} = 3e^{16}$

2c) $\frac{dy}{dx} = \frac{x^2}{y} \Rightarrow dy\,y = dx\,x^2 \Rightarrow \frac{1}{2}y^2 = \frac{1}{3}x^3 + 50 \Rightarrow y(5) = \sqrt{550/3}$

2e) $dy\,e^{-y} = dx \Rightarrow -e^{-y} = x + C \Rightarrow y = -\ln(4-x) \Rightarrow x<4$

4b) $dT\left(\frac{1}{(T_e - T)}\right) = dt\,k \Rightarrow \ln|T - T_e| = -kt + C$

$T - T_e = \pm e^C e^{-kt} = Ae^{-kt} \Rightarrow T = (T - T_0)e^{-kt} + T_e$

4c) $t \to \infty \Rightarrow T \to T_c$

8b) $y' = \frac{-y}{x} \Rightarrow \frac{dy}{dx} = \frac{-y}{x} \Rightarrow \frac{dy}{y} = \frac{-dx}{x} \Rightarrow \ln|y| = -\ln|x| + C_1$

$\Rightarrow |y| = \frac{C}{|x|}$