

Unit 3. Integration

Problem Set 6

3B. Definite Integrals

$$2a) \sum_{i=1}^6 (2i+1)(-1)^{2i}$$

$$2b) \sum_{i=1}^n \frac{1}{i^2}$$

$$3b) \text{left: } (-1)^2 + 0^2 + 1^2 + 2^2 \quad \text{Right: } 0^2 + 1^2 + 2^2 + 3^2 \\ \text{upper: } (-1)^2 + 1^2 + 2^2 + 3^2 \quad \text{lower: } 0^2 + 0^2 + 1^2 + 2^2$$

$$4a) ((1^2 + \dots + b^2) - (0^2 + \dots + (b-1)^2)) \frac{b}{n} = \frac{b^3}{n}; \text{ yes.}$$

$$5) \sum_{i=1}^n \sin(bi) \Delta i = \int_0^1 \sin(bu) du = -\frac{1}{b} \cos(bu) \Big|_0^1 = (1 - \cos b)/b$$

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Unit 4. Applications of Integration

4f. other Applications

$$1) \int_1^{100} n K \frac{\pi}{4} dn$$

Unit 3. Integration

3C. Fundamental theorem of Calculus

$$1) \int_3^6 \frac{1}{\sqrt{u-2}} du = \int_1^4 \frac{1}{u^{\frac{1}{2}}} du = 2u^{\frac{1}{2}} \Big|_1^4 = 2$$

$$2a) \int_0^2 \sqrt{3u+5} du = \int_5^{11} u^{\frac{1}{2}} \cdot \frac{1}{3} du = \frac{2}{3} \cdot \frac{1}{3} u^{\frac{3}{2}} \Big|_5^{11} = \frac{\sqrt{11^3} - \sqrt{5^3}}{3} \cdot \frac{2}{3}$$

$$3a) \int_1^2 \frac{m du}{m^2 + 1} = \int_2^5 \frac{1}{2} \frac{du}{u} = \frac{1}{2} \ln(u) \Big|_2^5 = \frac{1}{2} \ln\left(\frac{5}{2}\right)$$

$$5a) \int_0^\pi \sin u du = -\cos u \Big|_0^\pi = 1 - (-1) = 2$$

3E. Change of Variables;

Estimating Integrals

$$6b) \sin < 1 \Rightarrow \sin^2 < \sin \Rightarrow \int_0^\pi \sin^2 < 2$$

$$6c) \sqrt{u^2 + 1} > \sqrt{u^2} \Rightarrow \int_{10}^{20} \sqrt{u^2 + 1} > \int_{10}^{20} \sqrt{u^2} = \int_{10}^{20} \sqrt{u^2 + 1} > 15.$$

Unit

4f. Other Applications

$$2) R = \int_0^1 r q \lambda_0 e^{-k(\frac{t}{r})} dt = \frac{r q \lambda_0}{k} (1 - e^{-b_0 k})$$