

Unit 2. Applications
of Differentiation

Problem Set 3

Σ A. APProximation

$$1) \sqrt{a+b\kappa} = \sqrt{a} \left(1 + \frac{b}{a}\kappa\right)^{\frac{1}{2}} \approx \sqrt{a} \left(1 + \frac{b}{2a}\kappa\right)$$

$$3) \frac{(1+\kappa)^{3/2}}{1+2\kappa} \approx (1 + \frac{3}{2}\kappa)(1 - 2\kappa) \approx 1 - \frac{\kappa}{2}$$

$$6) \tan \theta = \frac{\sin \theta}{\cos \theta} \approx \frac{\theta}{\cancel{a}} \left(1 - \frac{\theta^2}{2}\right)^{-1} \approx \theta \left(1 + \frac{\theta^2}{2}\right) \approx \theta$$

$$11) PV^K = C \Rightarrow P = CV^{-K} \Rightarrow P = C(V + \Delta V)^{-K} = CV^{-K} \left(1 + \frac{\Delta V}{V}\right)^{-K} \\ \approx CV^{-K} \left(1 + -K \frac{\Delta V}{V} + \frac{K(K+1)}{2} \Delta V^2\right)$$

$$12a) \frac{e^\kappa}{1-\kappa} \approx \left(1 + \kappa + \frac{\kappa^2}{2}\right) \left(1 - \kappa\right)^{-1} \approx \left(1 + \kappa + \frac{\kappa^2}{2}\right) \left(1 + \kappa + \frac{\kappa^2}{2}\right) \approx 1 + 2\kappa \\ + \frac{5}{2}\kappa^2$$

$$12d) \ln(\cos \kappa) \approx \ln\left(1 - \frac{\kappa^2}{2}\right) \approx -\frac{\kappa^2}{2}$$

$$12e) n \ln(n) \approx (h+1) \ln(1+h) \approx (h+1) \left(h - \frac{1}{2}h^2\right) \approx (n-1) + \frac{(n-1)^2}{2}$$

2B. Curve Sketching

2a) $y = x^3 - 3x + 1$ $y' = 3x^2 - 3$ $y'' = 6x$
 $y'' = 0 \Rightarrow x = 0$

2c) $y = \frac{x}{x+4}$ $y' = \frac{1(x+4) - (x+4)x}{(x+4)^2} = \frac{4}{(x+4)^2}$

$$y'' = \frac{0 - 8}{(x+4)^3} \quad y'' = 0 \Rightarrow x = \text{No Answer}$$

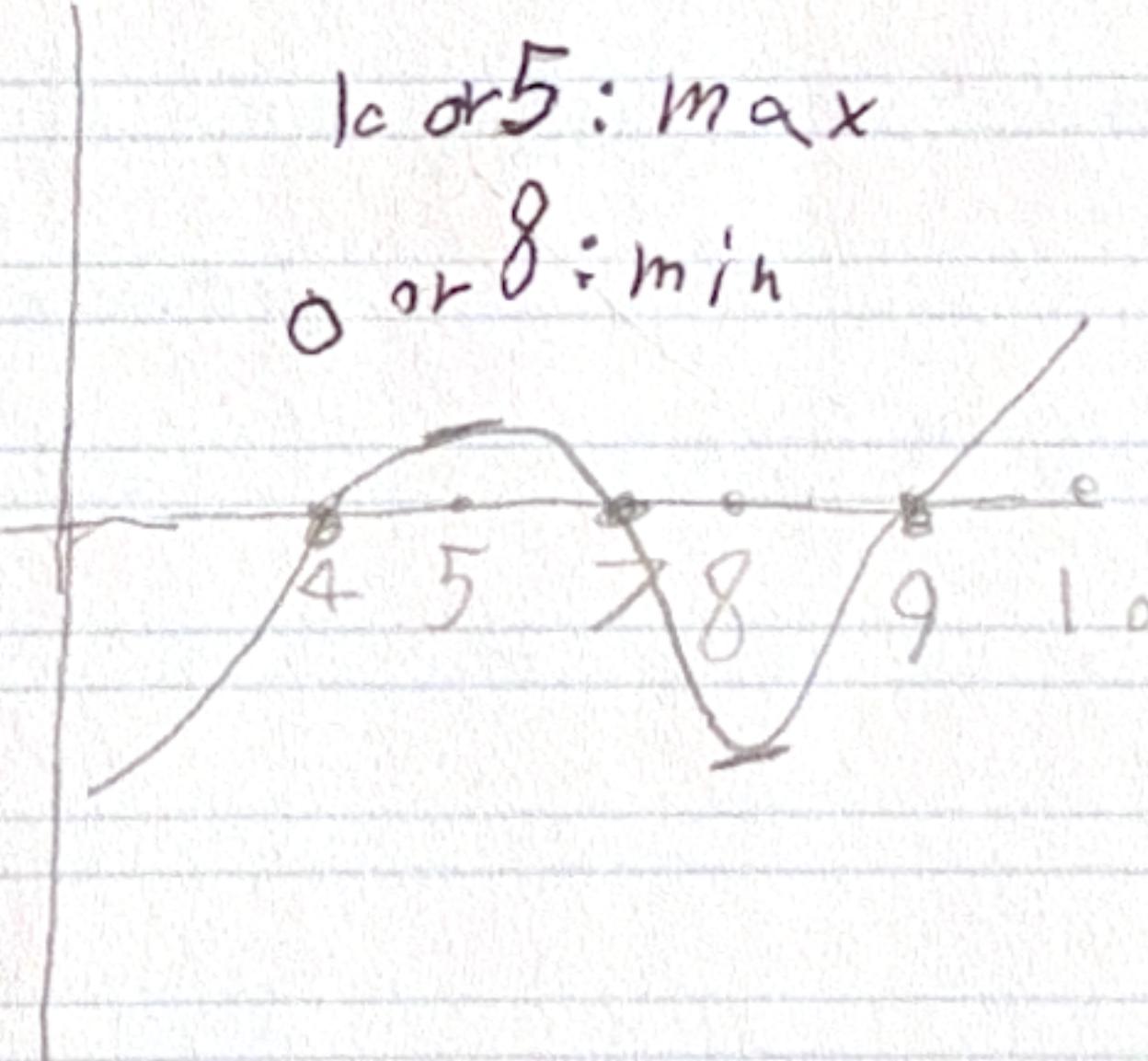
2h) $y = e^{-x^2}$ $y' = e^{-x^2}(-2x)$ $y'' = e^{-x^2}(-2x)^2 + -2e^{-x^2}$

$$y'' = 0 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

4)

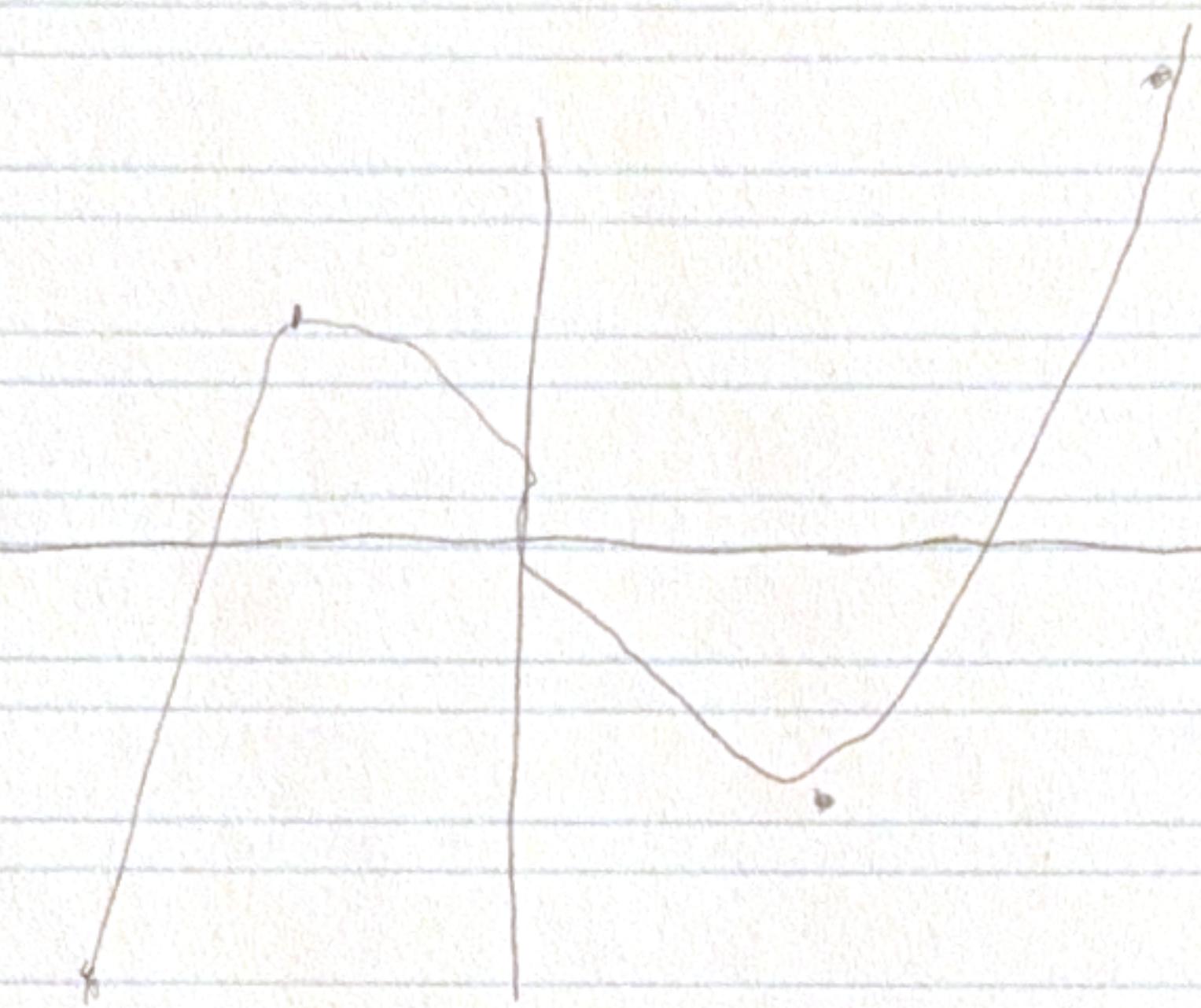
1 or 5: max

0 or 8: min



$$\begin{aligned}
 6a) \quad & a n^3 + b n^2 + c n + d \Rightarrow 3 a n^2 + 2 b n + c \\
 \Rightarrow & 3a - 2b + c = 0 \text{ and } 3a + 2b + c = 0 \\
 \Rightarrow & -4b = 0 \Rightarrow b = 0 \Rightarrow 3a = -c \\
 \text{and } & -1 < n < 1 \Rightarrow f(n) \xrightarrow{n \rightarrow 0} n^3 - 3n
 \end{aligned}$$

6b)



$$7a) \quad f'(a) = \lim_{\Delta n \rightarrow 0} \frac{\Delta y}{\Delta n}$$

$$y \text{ increasing} \Rightarrow \left\{ \begin{array}{l} \Delta y > 0 \Rightarrow \Delta n > 0 \\ \Delta y < 0 \Rightarrow \Delta n < 0 \end{array} \right\} \frac{\Delta y}{\Delta n} > 0$$

$$\Rightarrow \lim_{\Delta n \rightarrow 0} \frac{\Delta y}{\Delta n} \geq 0$$

7b) $\frac{\Delta y}{\Delta n} > 0 \neq \lim_{\Delta n \rightarrow 0} \frac{\Delta y}{\Delta n} > 0$

Counter: $n^3 f'(n^*) \neq 0$