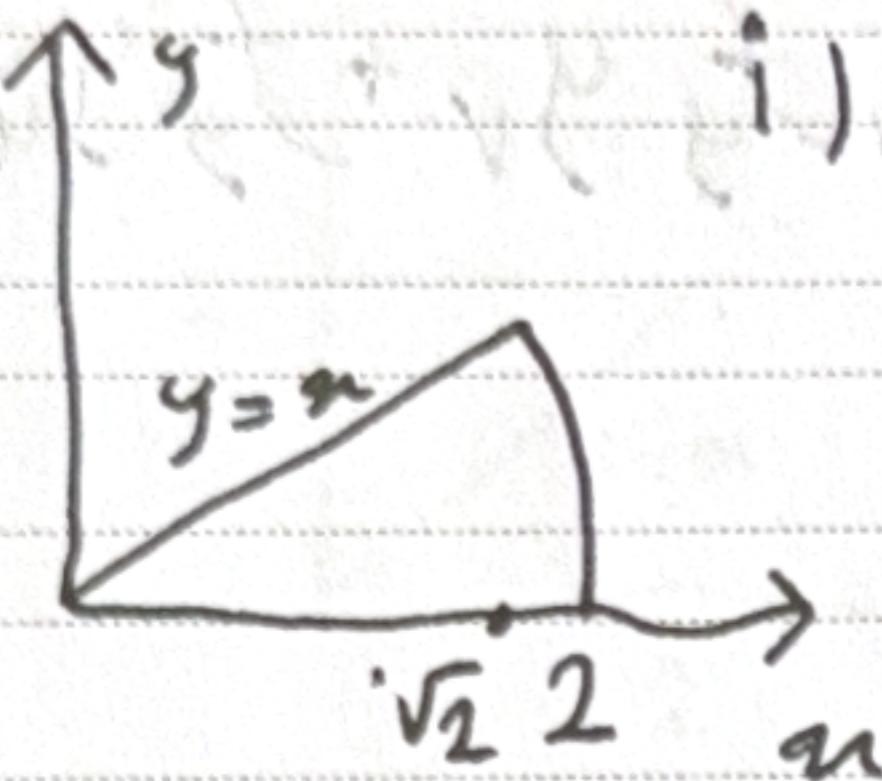


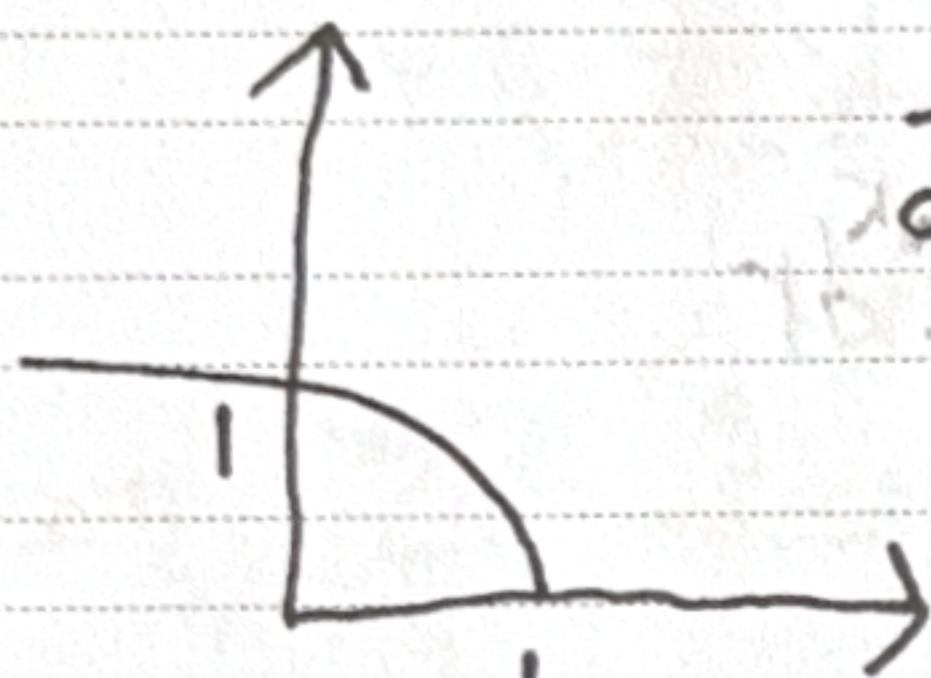
3A-2 c)



$$\text{i) } \int_0^{\sqrt{2}} \int_0^{\sqrt{2}-\sqrt{4-x^2}} dy dx + \int_0^{\sqrt{2}} \int_{\sqrt{4-y^2}}^{\sqrt{2}-\sqrt{4-x^2}} dy dx$$

$$\text{ii) } \int_0^{\sqrt{2}} \int_0^{\sqrt{1-y^2}} dy dx$$

3A-3 b)



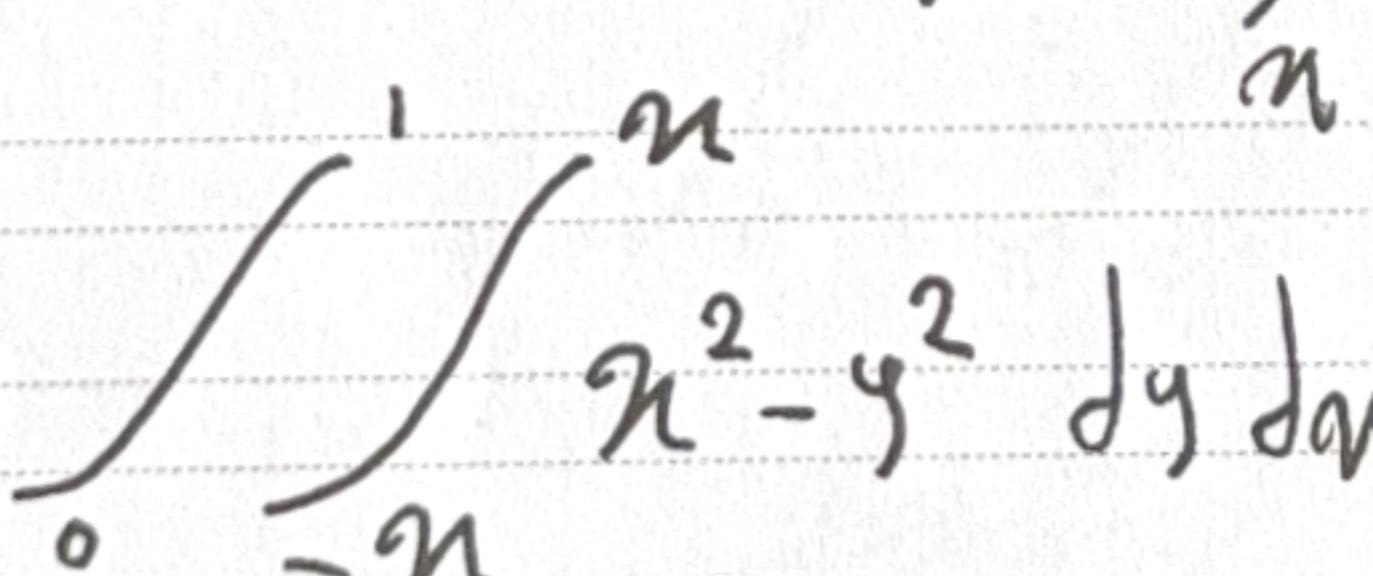
$$\int_0^1 \int_0^{1-y^2} (2x+y^2) dy dx$$

$$\text{Inner: } y + \frac{y^3}{3} - y^2$$

$$\text{Outer: } \left[\frac{y^2}{2} + \frac{2y^5}{5} \right]_0^1 - \left[y - \frac{1}{3} y^3 \right]_0^1$$

$$= \frac{2}{40}^{2/3}$$

3A-4 c)



$$\text{Inner: } n^2 y - \frac{y^3}{3}]_0^n = \frac{4}{3} n^3$$

$$\text{Outer: } \frac{1}{3} n^4]_0^1 = \frac{1}{3} n^4$$

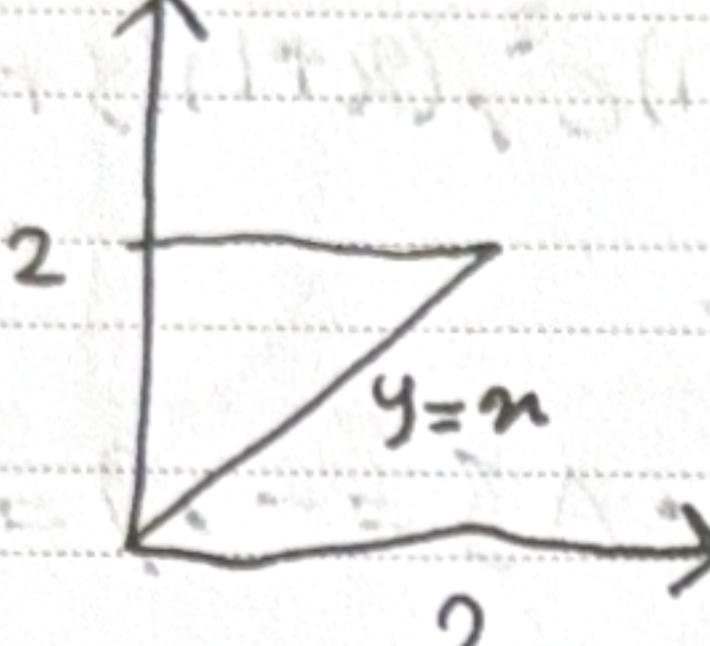
$$\int_0^1 \int_{-n}^n n^2 - y^2 dy dx$$

Subject:

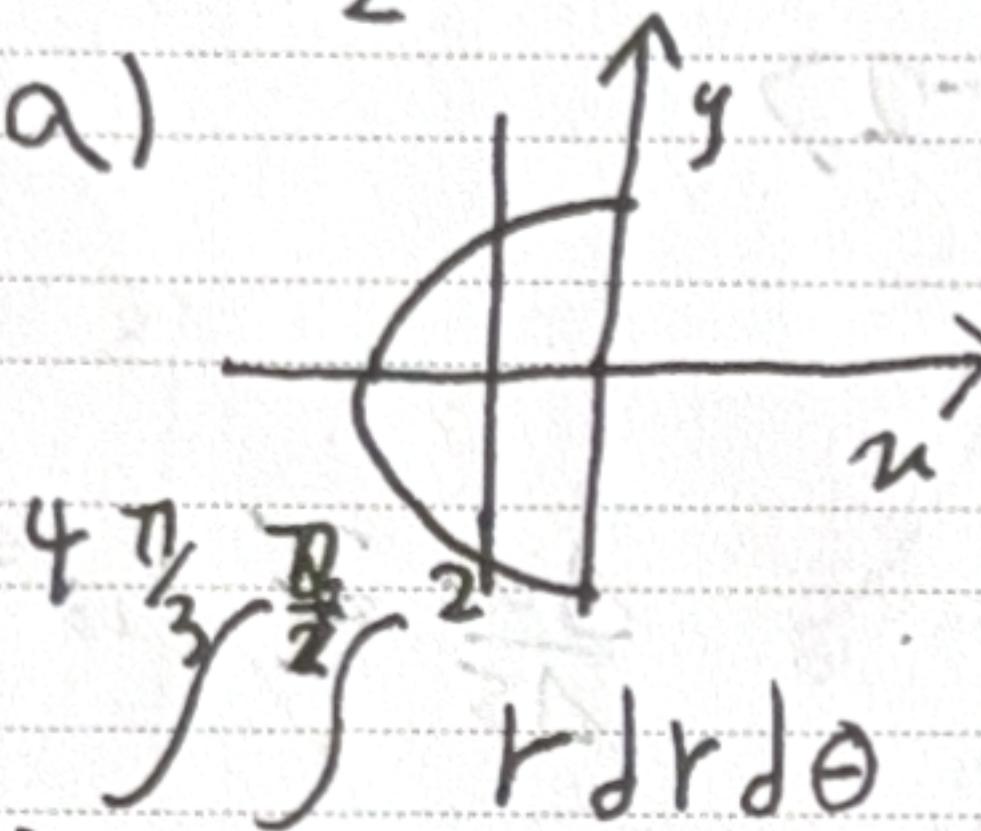
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$$3A-5 \text{ a) } \int_0^2 \int_{y^2}^2 e^{-y^2} dy dx$$

$$= \int_0^2 \int_{-y}^y e^{-y^2} dy dx = \frac{1}{2} (1 - e^{-4})$$



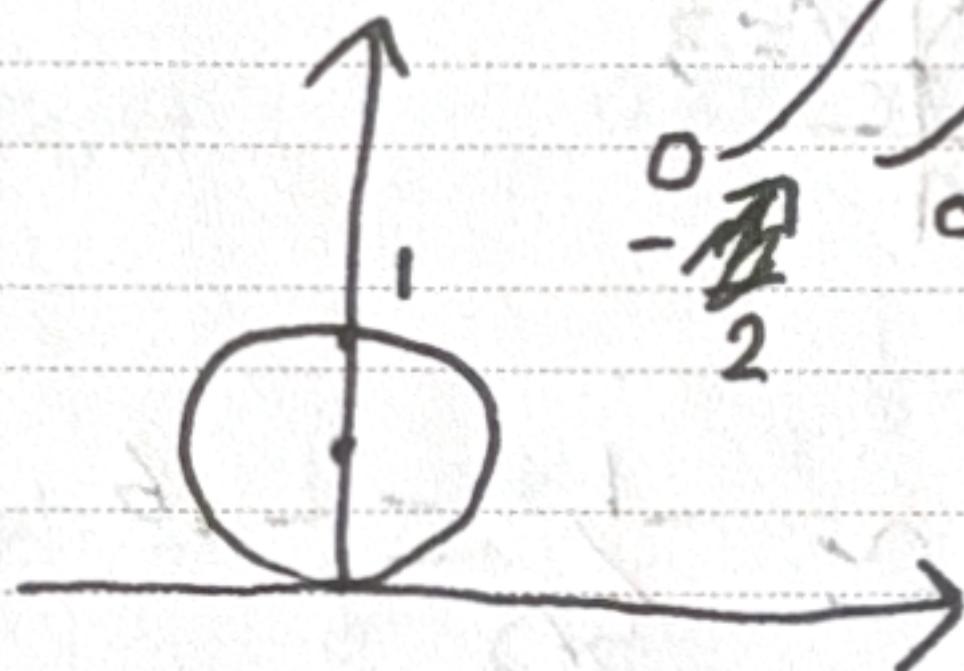
$$3B-1 \text{ a)}$$



$$2 \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^R r dr d\theta$$

$$\frac{\pi}{2} \sin \theta$$

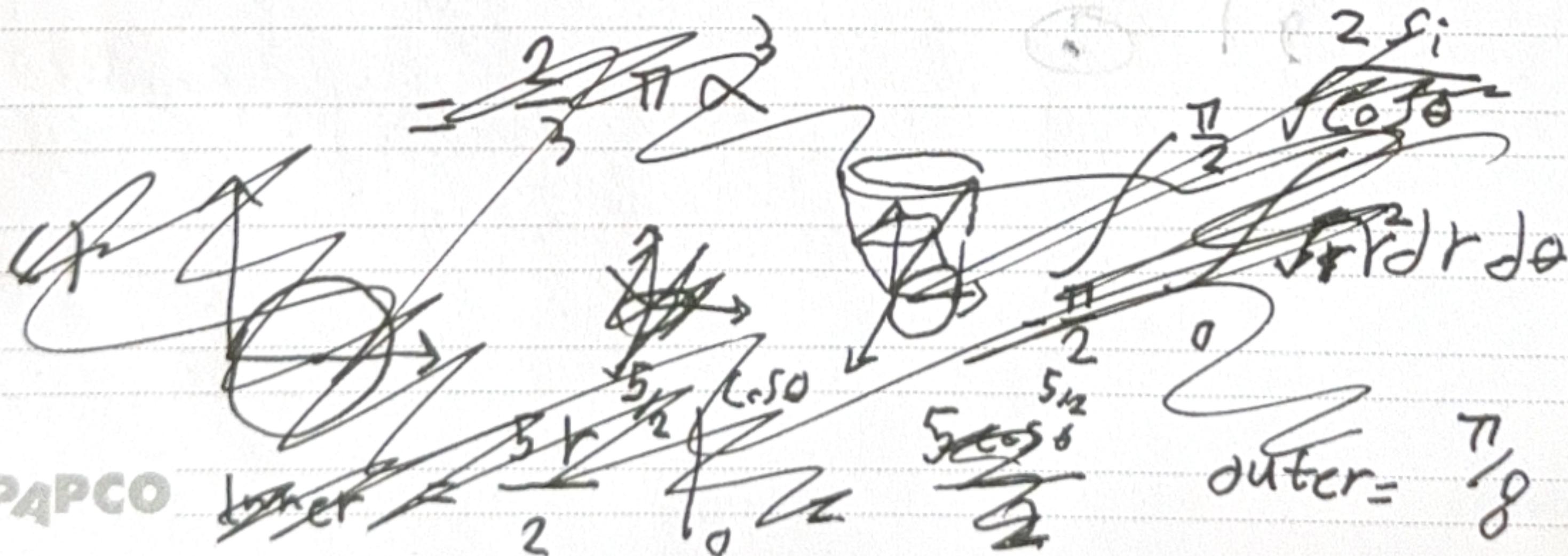
$$3B-2 \text{ d)}$$



$$\int_0^{\frac{\pi}{2}} \sqrt{1-r^2} = \frac{\pi}{2} - 1$$

$$3B-3 \text{ a)}$$

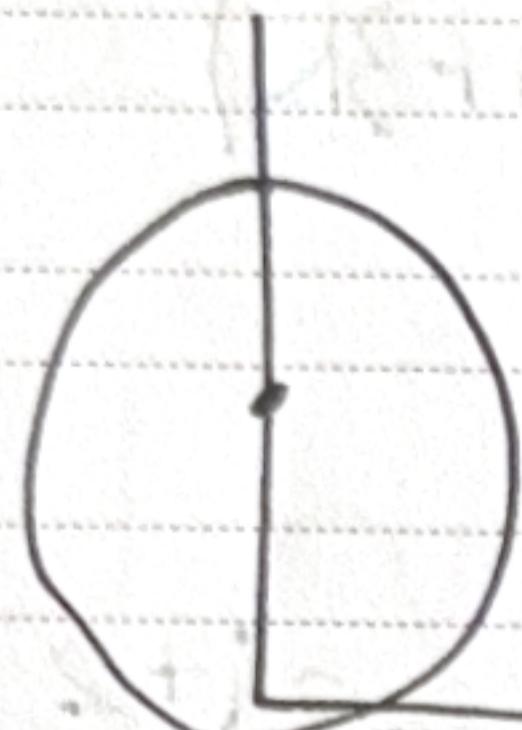
$$\int_0^{2\pi} \int_0^a \sqrt{a^2 - r^2} r dr d\theta$$



Subject:

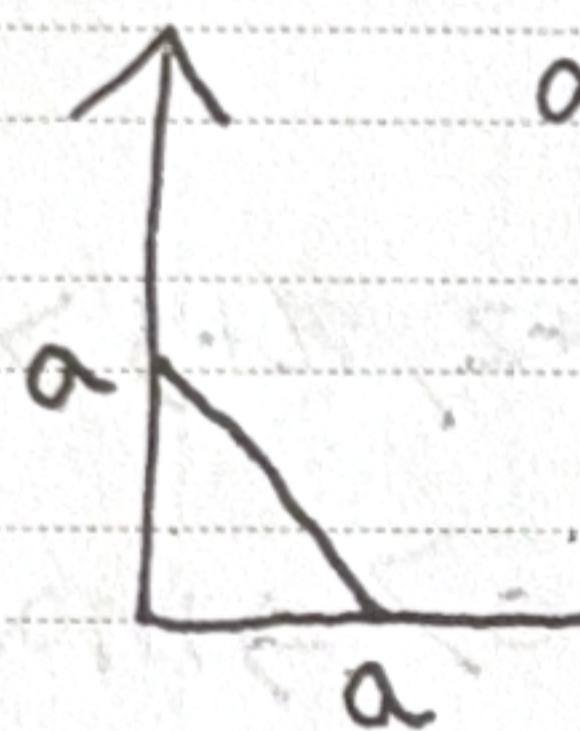
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(1)



$$2 \int_0^{\pi} r^2 \sin \theta \, r dr d\theta = \frac{32}{9}$$

3C-1



a) a-a-n

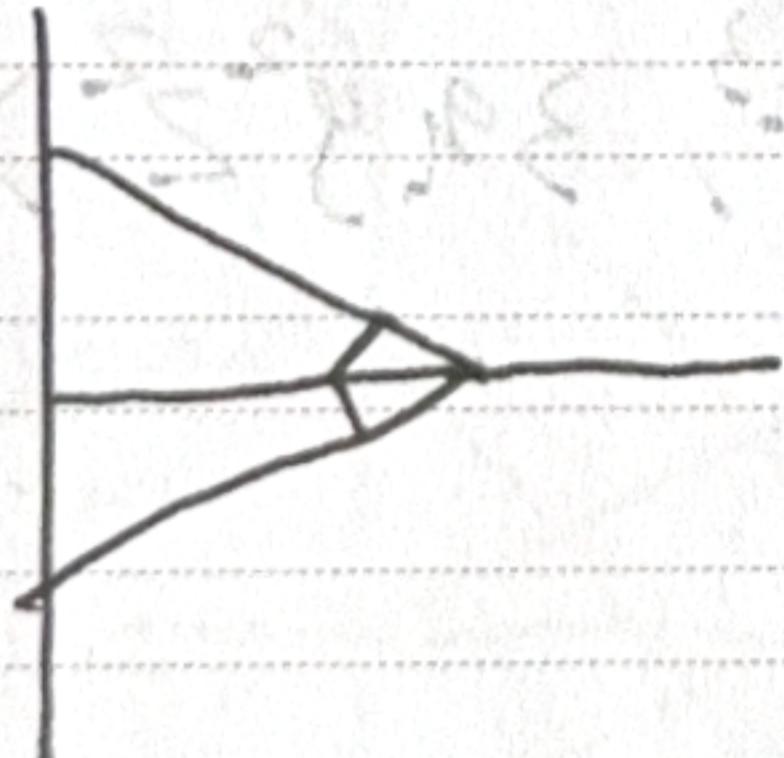
$$\int_0^a \int_0^{a-a} n^2 dy dx \\ \text{inner} = y n^2 \Big|_0^{a-a} \\ = n^2 a - n^3$$

$$b) \int_0^R \int_R^a n^2 dA$$

$$\text{outer} = \frac{n^3}{3} a - \frac{n^4}{4} \Big|_0^a \\ = \frac{n^4}{3} - \frac{n^4}{4} = \frac{n^4}{12}$$

$$+ \int_R^a y^2 dA = \frac{1}{6} a^4$$

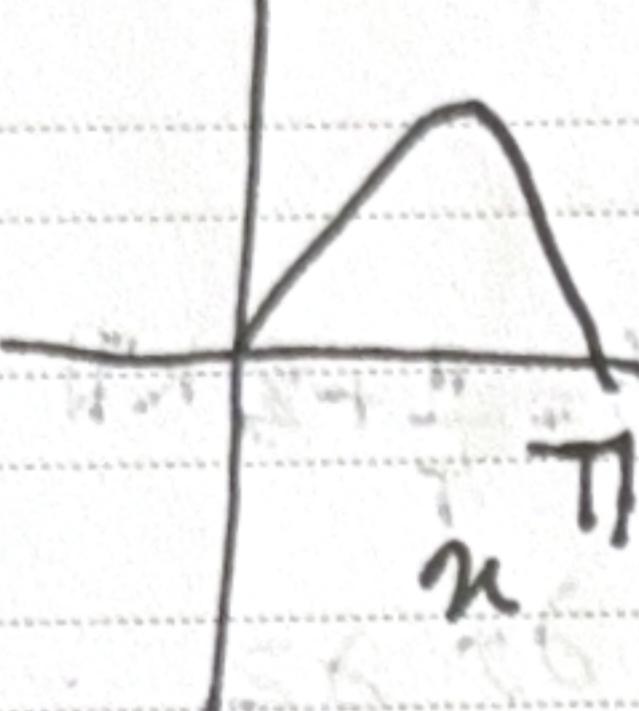
(1)



$$\frac{a^4}{24}$$

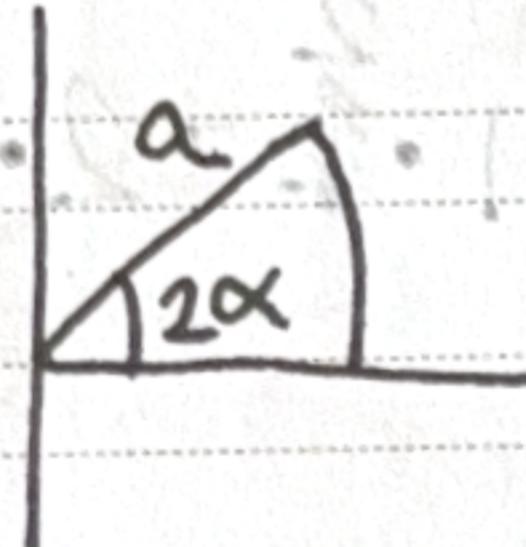
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3C-2 a) $M = \int_0^{\pi} \sin \theta d\theta$



$$= \int_0^{\pi} \sin \theta d\theta = 2$$

$$\bar{y}_{\text{ave}} = \frac{1}{M} \int_0^{\pi} y_{\theta} d\theta = \frac{\pi}{8}$$

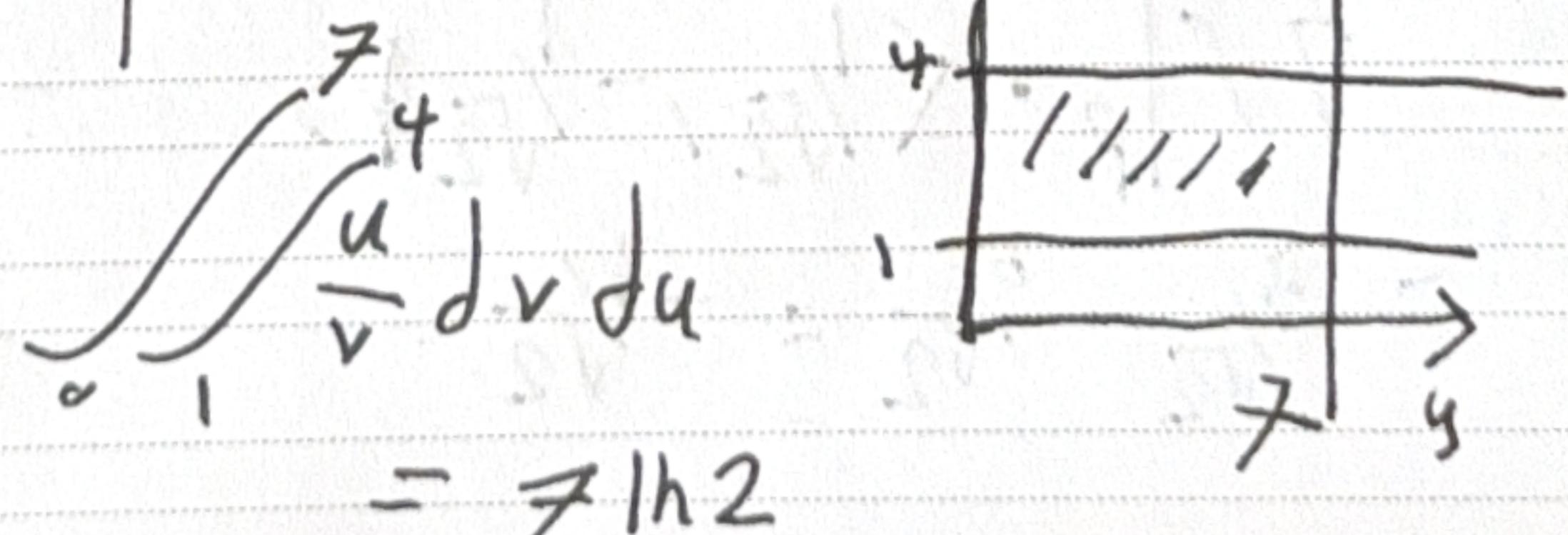
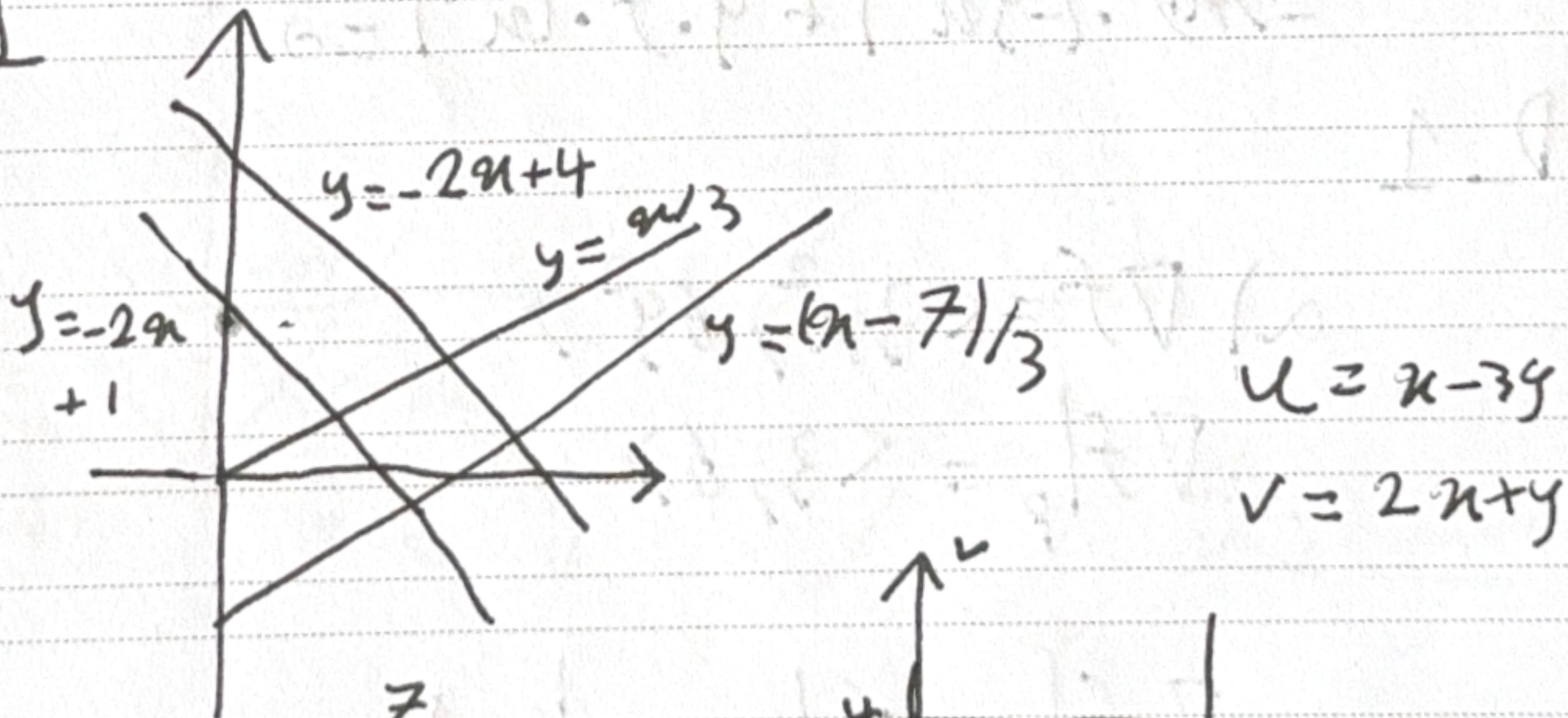
3C-4  $\int_0^{2\alpha} \int_0^a r dr d\theta = M$

$$= \int_0^{2\alpha} \frac{a^2}{2} d\theta = \alpha a^2$$

$$\bar{r} = \frac{1}{M} \int_0^{2\alpha} \int_0^a r \cos \theta r dr d\theta$$

$$= \frac{2}{3} a \cdot \frac{\sin \alpha}{\alpha}$$

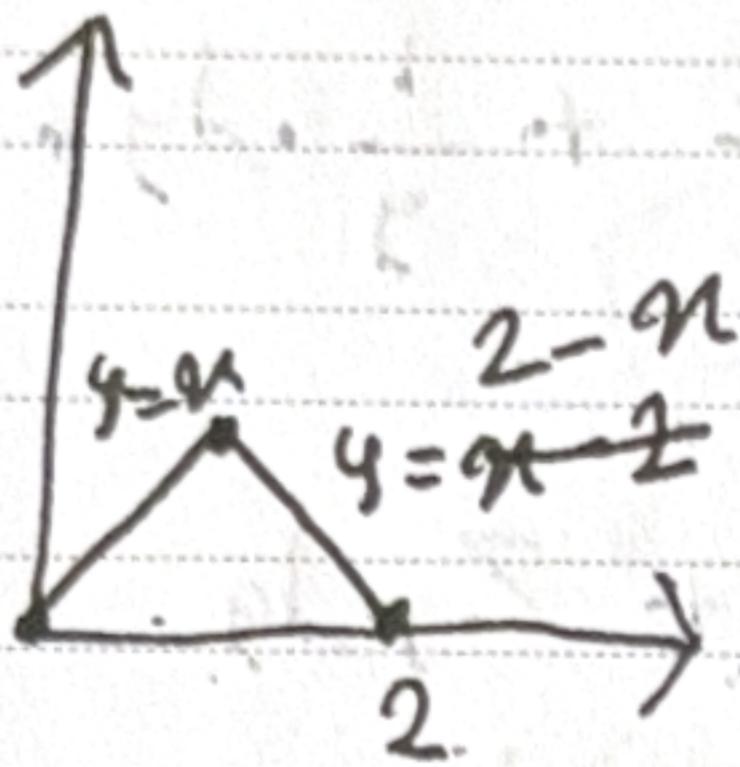
3D-1



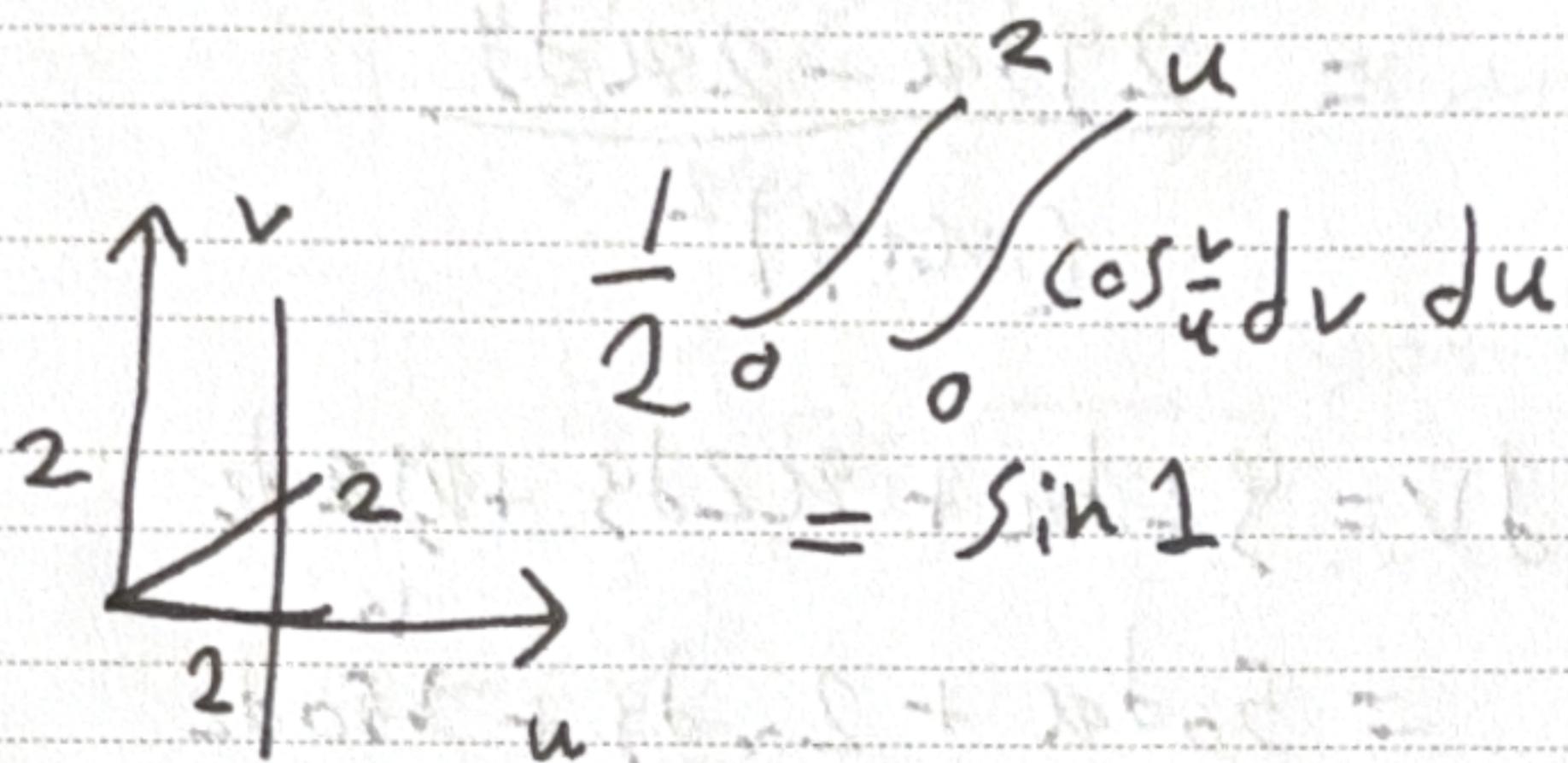
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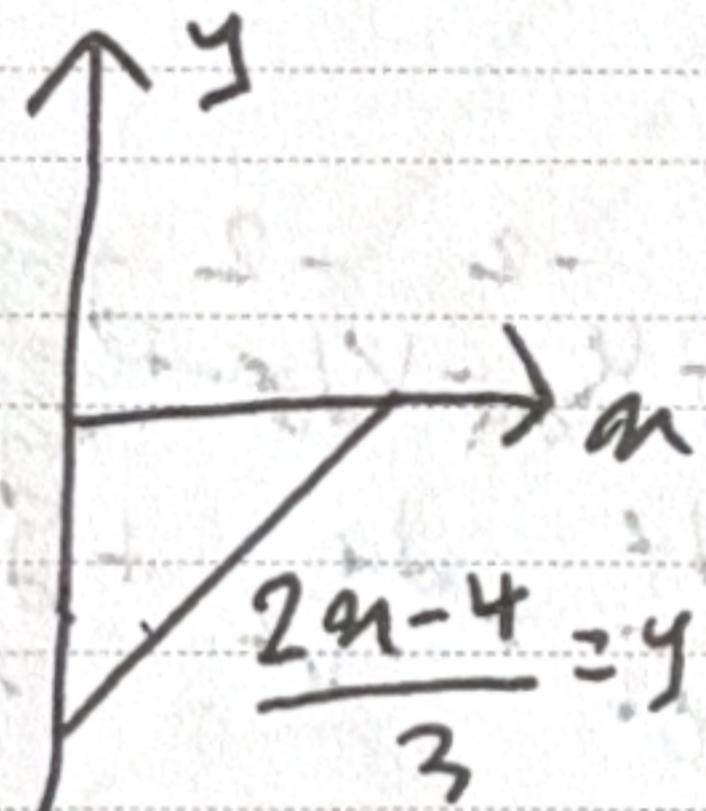
3D-2



$$\begin{aligned} u + v &= u + v \\ u - v &= u - v \\ &= -2 \end{aligned}$$

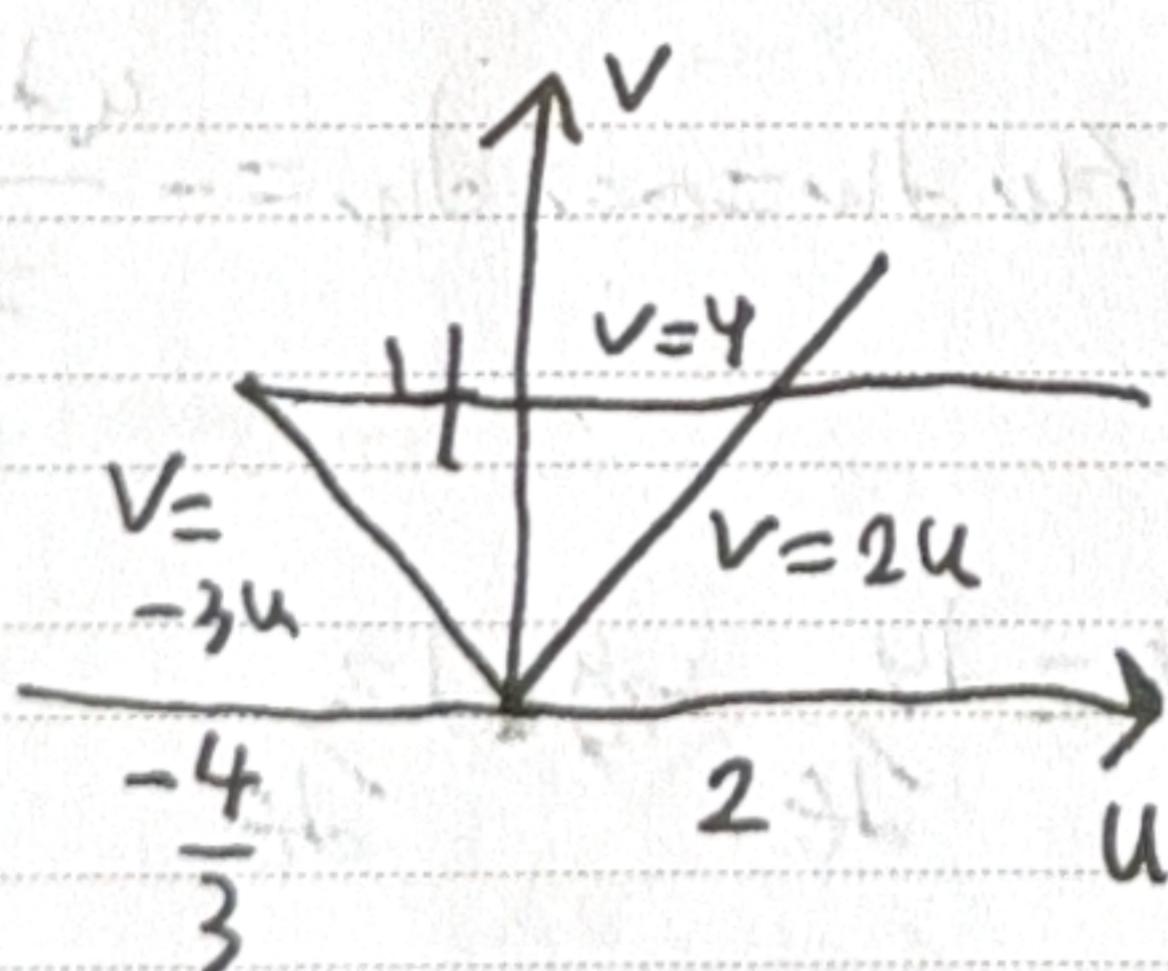


3D-4



$$u = u + v$$

$$v = 2u - 3y$$



$$\int \int dz du$$

$$\int_0^4 \int_{-\frac{v}{3}}^{2u} \frac{1}{2} v^2 u^2 \frac{du dv}{5}$$

$$= \frac{4^6}{6 \cdot 15} \left(\frac{1}{8} + \frac{1}{27} \right)$$

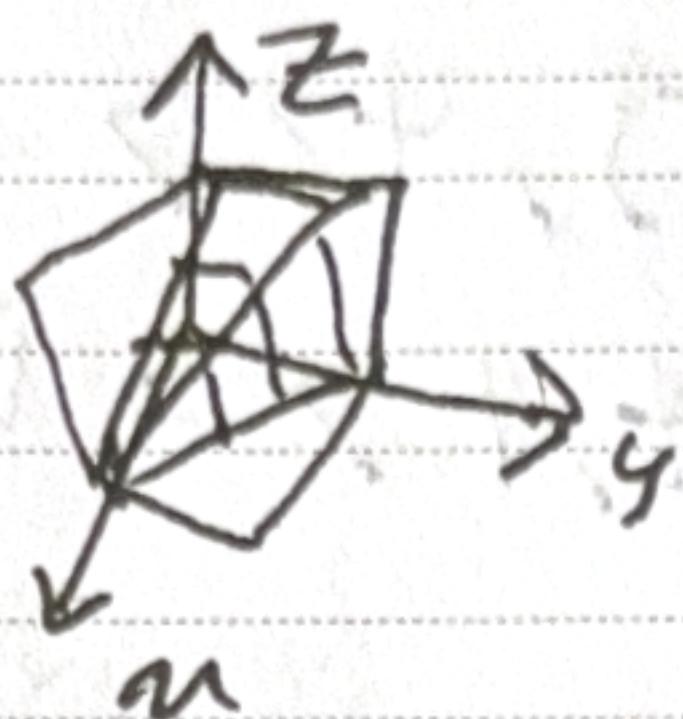
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Part 2

Problem 1 a)

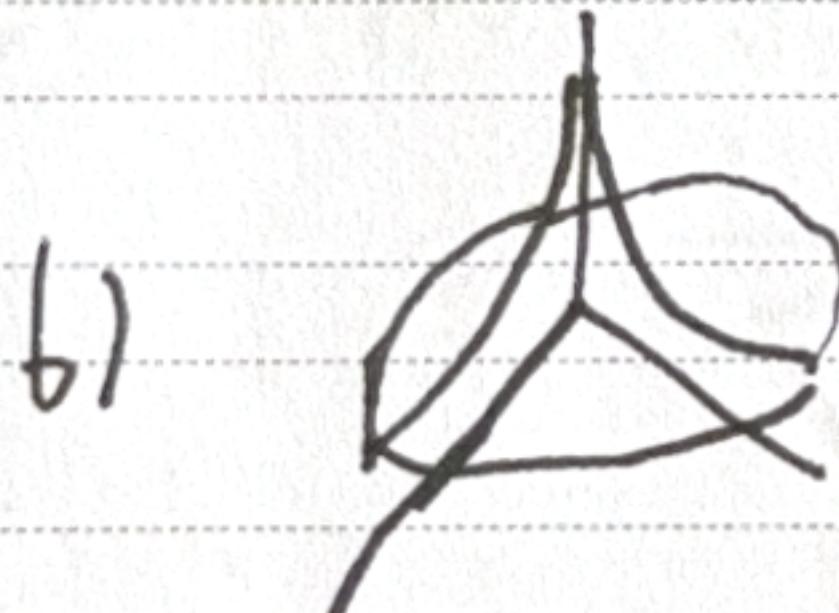


$$b) \int_0^4 \int_0^{4-n} \sqrt{4-n} dy dn$$

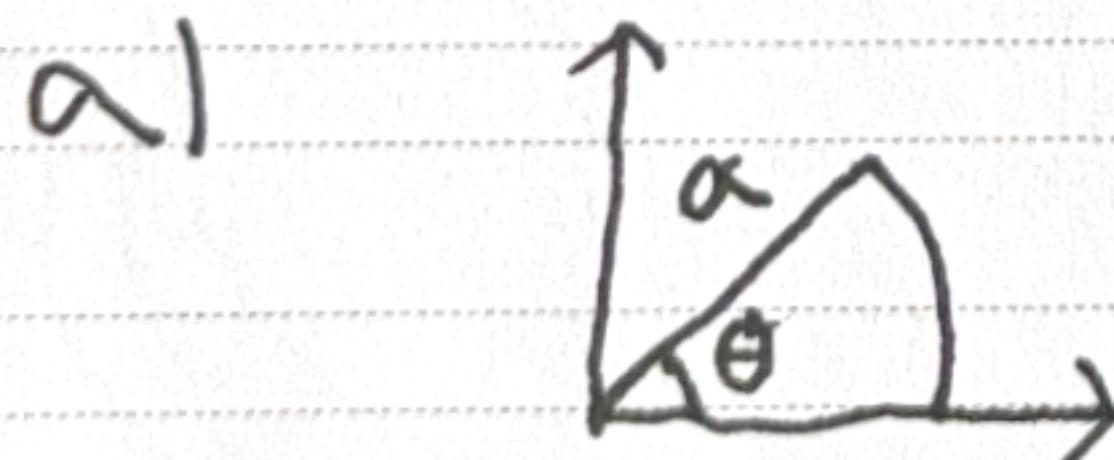
$$\begin{aligned} &= \int_0^4 \left[y\sqrt{4-n} \right]_{y=0}^{y=4-n} dn \\ &= \frac{64}{5} \end{aligned}$$

Problem 2

$$a) \int_0^{2\pi} \int_0^a f(r) r dr d\theta$$
$$= \int_0^a 2\pi r f(r) dr$$



Problem 3



$$\begin{aligned} \bar{r} &= \frac{1}{\pi} \int_{-\theta}^{\theta} \int_0^a r \cos \theta r dr d\theta \\ &= \left(\frac{2 \sin \theta}{3\theta} \right) a \end{aligned}$$

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b)

$$\bar{g}_{\Delta} = \left(\frac{2}{3}\right) a$$

c) the triangle is closer.

$$\frac{\sin \theta}{\theta} < 1 \rightarrow f_s \theta < \frac{2}{3} = f_D$$

Problem 4

ii, i) A: $\begin{vmatrix} 1+t & 0 \\ 0 & 1+t \end{vmatrix} = -(1+t)^2$

B: $\begin{vmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{vmatrix} = -1$

C: $\begin{vmatrix} 1+t & 0 \\ 0 & \frac{1}{1+t} \end{vmatrix} = -1$

(iii)

Problem 5 i)

