Subject: Problem Set 6

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2ま-1 $g = x+2y+3z=18$

(a) $f = xyz$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 1, 2, 3 \rangle$

$$\left.\begin{aligned} yz &= \lambda 1 \\ xz &= \lambda 2 \\ xy &= \lambda 3 \\ x+2y+3z &= 18 \end{aligned}\right\} \quad x = 2y = 3z = 6$$

(b) $f = xyz$

$g = x^2+2y^2+4z^2=12$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 2x, 4y, 8z \rangle$

$$\left.\begin{aligned} yz &= \lambda 2x \\ xz &= \lambda 4y \\ xy &= \lambda 8z \\ x^2+2y^2+4z^2 &= 12 \end{aligned}\right\} \quad x^2 = 2y^2 = 4z^2 = 4$$

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2J_3 $f = 2yz + 4xz + 3xy$

$g = xyz = 1$

$\nabla f = \langle 4z+3y, 2z+3x, 2y+4x \rangle$

$\nabla g = \langle yz, xz, xy \rangle$

$$\left.\begin{array}{l} 4z+3y = \lambda yz \\ 2z+3x = \lambda xz \\ 2y+4x = \lambda xy \\ xyz = 1 \end{array}\right\} x:y:z = 2:4:3$$

2J_1

a) $w = (z-y)^2 + y^2 + z^2 = z + z^2 \rightarrow \left(\frac{\partial w}{\partial y}\right)_z = 0$

b) $\left(\frac{\partial w}{\partial z}\right)_y = 1 + 2z$

2J_2

a) $0 = 2x\left(\frac{\partial x}{\partial y}\right)_z + 2y \rightarrow \left(\frac{\partial x}{\partial y}\right)_z = -\frac{y}{x}$

$\left(\frac{\partial w}{\partial y}\right) = 2x\left(\frac{\partial x}{\partial y}\right)_z + 2y = 2x\left(\frac{-y}{x}\right) + 2y = 0$

$1 = 2x\left(\frac{\partial x}{\partial z}\right)_y \rightarrow \left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2x}$

$\left(\frac{\partial w}{\partial z}\right)_y = 2x\left(\frac{\partial x}{\partial z}\right)_y + 2z = 1 + 2z$

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b) $dw = 2x\,dx + 2y\,dy + 2z\,dz$

$dz = 2x\,dx + 2y\,dy \longrightarrow dw = 0\,dy + (1+2z)\,dz$

2 ـ 3

a) $\left(\frac{\partial w}{\partial t}\right)_{x,z} = x^2 z - z^2$

b) $\left(\frac{\partial w}{\partial z}\right)_{x,y} = -2t$

2 ـ 4

b) $dw = 2x^2 y\,dx + (x^2 z - z^2)\,dt$

$+ (x^2 t - 2zt)\,dz = x^2 z - z^2$

2 ـ 5

a) $\left(\frac{\partial S}{\partial P}\right)_V = S_P + S_T \left(\frac{\partial T}{\partial P}\right)_V$

$= S_P + S_T \cdot \frac{RV}{nR}$

b) $\left(\frac{\partial S}{\partial V}\right)_P = S_V + S_T \left(\frac{\partial T}{\partial V}\right)_P$

$= S_V + S_T \cdot \frac{P}{nR}$

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2.J_7

$$df\Big|_p = 2dx + dy - 3dz$$

$$dz = 2x\,dx + dy$$

$$2dx + dz - 2x\,dx - 3dz$$

$$\Rightarrow df\Big|_p = \sout{2dx + dy - 6x\,dx - 3dy}$$

$$\nabla g\Big|_{(1,1)} = \langle 0, -2 \rangle$$

Part II

Problem 1 Mathlet Not working

Problem 2

(a) $$f = I_1^2 R_1 + I_2^2 R_2$$

$$\left.\begin{aligned} I_1 + I_2 &= I \\ 2 I_1 R_1 &= \lambda \\ 2 I_2 R_2 &= \lambda \end{aligned}\right\} \quad I_1 = \frac{R_2}{R_1+R_2} I, \quad I_2 = \frac{R_1}{R_1+R_2} I$$

b) $$f = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

$$\left.\begin{aligned} I_1 + I_2 + I_3 &= I \\ 2 I_1 R_1 &= \lambda \\ 2 I_2 R_2 &= \lambda \\ 2 I_3 R_3 &= \lambda \end{aligned}\right\} \quad I_1 = \frac{R_2 R_3}{D} I, \quad I_2 = \frac{R_1 R_3}{D} I, \quad I_3 = \frac{R_1 R_2}{D} I, \quad D = R_1R_2 + R_2R_3 + R_3R_1$$

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Problem 3

a) $f = \frac{1}{2}xy = \frac{1}{2}x^2\tan\theta$

$\Rightarrow \left(\frac{\partial w}{\partial x}\right)_\theta = x\tan\theta \;,\; \left(\frac{\partial w}{\partial \theta}\right)_x = \frac{1}{2}x^2\sec^2\theta$

b) $\left(\frac{\partial w}{\partial x}\right)_\theta = \frac{y}{2} + \frac{x}{2}\cdot\tan\theta = y$

$\left(\frac{\partial w}{\partial \theta}\right)_x = \underbrace{\frac{y}{2}}_{w_x}\cdot 0 + \underbrace{\frac{x^2}{2}\cdot\sec^2\theta}_{w_y}$ $\left(\frac{\partial x}{\partial \theta}\right)_x$ $\left(\frac{\partial y}{\partial \theta}\right)_x$

c) $df = \frac{y}{2}dx + \frac{x}{2}dy \qquad dy = \tan\theta\, dx + x\sec^2 d\theta$

$\Rightarrow \left(\frac{df}{d\theta}\right)_x = 0 + 0 + \frac{x^2}{2}\sec^2$

d) $\frac{\Delta y}{\Delta x}$ and $\frac{1}{2}x^2\sec^2\theta$