

$$1. \quad f = x^2 + xy + 2y^2$$

$$g = x^2 + y^2 = 1$$

$$\nabla f = \langle 2x+1, 4y \rangle$$

$$\nabla g = \langle 2x, 2y \rangle$$

$$2x+1 = \lambda 2x$$

$$4y = \lambda 2y \quad \left\{ \begin{array}{l} \lambda = 2, x = \frac{1}{2}, y = \pm \frac{\sqrt{3}}{2} \\ x = 0, y = 0 \end{array} \right.$$

$$x^2 + y^2 = 1 \quad \left\{ \begin{array}{l} x = \frac{1}{2}, y = \pm \frac{\sqrt{3}}{2} \\ x = 0, y = 0 \end{array} \right. \quad \begin{array}{l} \text{max} \\ \text{min} \end{array}$$

$$2. \quad f = x^2 - xy + y^2$$

$$g = x^2 + y^2 = 1 \quad x, y > 0$$

$$\nabla f = \langle 2x-y, 2y-x \rangle$$

$$\nabla g = \langle 2x, 2y \rangle$$

$$2x-y = \lambda 2x$$

$$2y-x = \lambda 2y \quad \left\{ \begin{array}{l} \lambda = \frac{1}{2}, x = y = \pm \frac{1}{2} \\ x = 0, y = 0 \end{array} \right.$$

$$x^2 + y^2 = 1$$

$$x, y > 0$$

$$\left. \begin{array}{l} (1,0), (0,1) \\ \text{max} \quad \text{max} \end{array} \right. \quad \left. \begin{array}{l} \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \\ \text{min} \end{array} \right.$$