Subject: Practice Final

Year: Month: Date:

Problem 1

a) $PQ = \langle 0, 1, 1 \rangle$, $PR = \langle -3, 1, 3 \rangle$

$$\begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ -3 & 1 & 3 \end{vmatrix} = 2i - 3j + 3k$$

b) $2x - 3y + 3z = -4$

Problem 2

a) $$\begin{vmatrix} 1 & 0 & c \\ 2 & c & 1 \\ 1 & -1 & 2 \end{vmatrix} = (2c + 1) \times 1 + 0 + c(-2 - c)$$

$= 2c + 1 - 2c - c^2 = 1 - c^2$

$\Rightarrow c \neq \pm 1$

b) $c = 2 \Rightarrow det = -3$; $-1 \times \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = 1$

$\Rightarrow X = \frac{-1}{3}$

Problem 3

$OQ + QP = \alpha \langle \cos\theta, \sin\theta \rangle + a\theta \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$\Rightarrow x = \alpha \cos\theta + a\theta \frac{\sqrt{2}}{2}$

$y = a \sin\theta + a\theta \frac{\sqrt{2}}{2}$

Subject:

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Problem 4

a) $\frac{dr}{dt} = \langle -3\sin t, 5\cos t, -4\sin t \rangle$

$$\left|\frac{dr}{dt}\right| = \sqrt{9\sin^2 + 25\cos^2 + 16\sin^2} = 5$$

b) $x = 0 \Rightarrow 3\cos t = 0 \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}$

Problem 5

a) $\nabla w = \langle 2xy - y^3, x^2 - 3xy^2 \rangle$

$\nabla w_p = \langle 3, -2 \rangle$

$\frac{dw}{ds}\Big|_A = \langle 3, -2 \rangle \cdot \langle 3, 4 \rangle \times \frac{1}{5} = \frac{1}{5}$

b) 0.002

Problem 6

a) $\nabla w = \langle 2x, 4y, 4z \rangle$

$\nabla w_p = \langle 2, 4, 4 \rangle$

$x + 2y + 2z = 5$

b) $\langle 1,2,2 \rangle \cdot \langle 0,0,1 \rangle = 2 = |p||q|\cos\theta$

$\Rightarrow \cos\theta = \frac{2}{\sqrt{9} \cdot 1} = \frac{2}{3} \Rightarrow \cos^{-1}\frac{2}{3}$

Subject:

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Problem 7

$$2x = \lambda 2$$

$$2y = \lambda$$

$$2z = -\lambda$$

$$\underline{2x+y-z=6}$$

$$\lambda = 2 \Rightarrow (2, 1, -1)$$

Problem 8

a) $0 = dg = 2\,dx - dy - dz$

$\overset{dy=0}{\Rightarrow} \left(\frac{dz}{dx}\right)_y = 2$

b) $dw = dx + dy + 2\,dz$

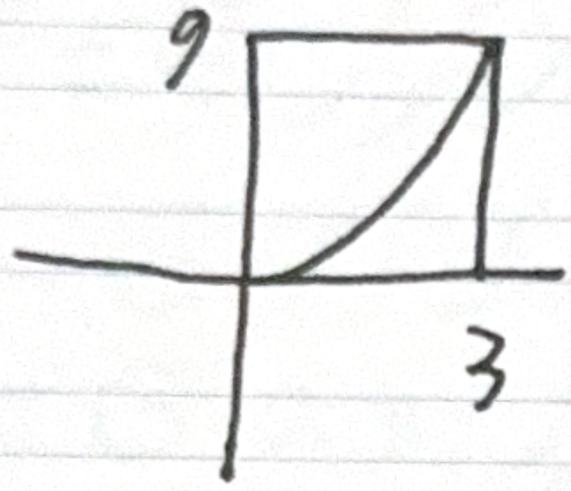
$$\Rightarrow \left(\frac{dw}{dx}\right)_y = \left(\frac{dx}{dx}\right)_y + \left(\frac{dy}{dx}\right)_y + 2\left(\frac{dz}{dx}\right)_y$$

$$\Rightarrow \left(\frac{dw}{dx}\right)_y = 1 + 0 + 2 \cdot 2 = 5$$

Subject:

Year: Month: Date:

Problem 9



$$\int_0^3 \int_{x^3}^{9} x e^{-y^2}\, dy\, dx$$

$$= \int_0^9 \int_0^{\sqrt{y}} x e^{-y^2}\, dx\, dy$$

Inner: $\frac{\sqrt{y}}{2} e^{-y^2}$

outer: $\frac{1}{4}\left[1 - e^{-81}\right]$

Problem 10

$$8 \int_0^{\frac{\pi}{4}} \int_0^{2\cos\theta} r^2 \cdot r\, dr\, d\theta$$

Problem 11

a) $\oint_C -Q\, dx + P\, dy$

b) $\iint_R (a+b)\, dx\, dy = \text{area} \Rightarrow a+b=1$

Subject:

Year: Month: Date:

Problem 12

$$F = G\iiint \frac{\cos\phi}{\rho^2}\,\delta\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= G\int\int\int \rho\cos^2\phi\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= G\cdot 2\pi\cdot\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\cos^2\phi\sin\phi\,d\phi\int_0^1 \rho\,d\rho$$

$$= \frac{\pi G}{3}$$

Problem 13

$$r = \langle 1+t, 1+3t, 1+7t\rangle$$

$$\int_0^1 2t\,dt - 4.7t\,dt = -13$$

Problem 14

a) $\left.\begin{aligned} 2ay &= 2y \\ 2y &= 2by \end{aligned}\right\}\ a,b=1$

b) $f = xy^2 + y^2z + \frac{z^3}{3}$

c) $xy^2 + y^2z + \frac{z^3}{3} = C$

Subject:

Year: Month: Date:

Problem 15

$$\iiint_V 3\,dV = 3\,\text{Volume } V$$

$$\text{Volume } V = \int_0^{2\pi}\int_0^1 (1-r^2)\,r\,dr\,d\theta = \frac{\pi}{2}$$

$$\Rightarrow 3\frac{\pi}{2}$$

Problem 16

$$n\,dS = \langle 2x, 2y, 1\rangle\,dx\,dy$$

$$F\cdot n\cdot dS = (2x^2+2y^2+z)\,dx\,dy = (x^2+y^2+1)\,dx\,dy$$

$$\iint x^2+y^2+1\,dx\,dy = \int_0^{2\pi}\int_0^1 r^2+1\,r\,dr\,d\theta = 3\frac{\pi}{2}$$

Problem 17

$$\nabla\cdot F\cdot h = 0 \Rightarrow 0$$

Subject:

Year: Month: Date:

Problem 18

a) $\int_0^\infty e^{-x^2} dx \int_0^\infty e^{-y^2} dy = I \cdot I$

b) $\int_0^{\frac{\pi}{2}} \int_0^\infty e^{-r^2} \cdot r \, dr \, d\theta = \frac{\pi}{2} \cdot \left[\frac{e^{-r^2}}{-2}\right]_0^\infty$

$= \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4} = I^2 = \frac{\sqrt{\pi}}{2}$