Subject: PSet 12

Year: Month: Date:

6D-1 a) $\vec{F} = \langle t^2, t^3, -t \rangle$

$$dr = \langle 1, 2t, 3t^2 \rangle$$

$$\int_0^1 t^2 + 2t^4 - 3t^3 \, dt$$

$$= \frac{t^3}{3} + \frac{2}{5}t^5 - \frac{3}{4}t^4 \Big|_0^1$$

$$= \frac{1}{3} + \frac{2}{5} - \frac{3}{4} = -\frac{1}{60}$$

6D-2

Observe the field is radially outwards.

Always $\perp \hat{t}$. $F \cdot \hat{t} \cdot ds = 0$

6D-4 a) $\vec{F} = \langle 2x, 2y, 2z \rangle$

b) (i) $d\vec{r} = \langle -\sin t, \cos t, 1 \rangle$

$$\int_0^{2n\pi} 2\cos t(-\sin t) + 2\sin t(\cos t) + 2t \, dt = (2n\pi)^2$$

(ii) $r(t) = \langle 1, 0, t \rangle$

$$\int F \cdot dr(t) = \int_0^{2n\pi} 2t \, dt = (2n\pi)^2$$

(iii)

$$\int_C F \cdot dr = f(1,0,2n\pi) - f(1,0,0)$$

$$= (2n\pi)^2$$

6D_5 $1-(-1)=2$

$(1,1,-\pi/2),\ (1,1,\pi/2)$

6E-3 a) (ii) $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+z & x^2 & x \end{vmatrix} = 0$

b) (ii) $f_x = 2xy + z$

$\Rightarrow f = x^2y + zx + g(y,z)$

$\Rightarrow f_y = x^2 + g_y(y,z) = x^2 \Rightarrow g_y(y,z) = 0$

$\Rightarrow g(y,z) = c + h(z) \Rightarrow f = x^2y + zx$
$+ h(z) \Rightarrow f_z = x + h'(z) = x \Rightarrow h'(z) = 0$

$\Rightarrow h(z) = c \Rightarrow f = x^2y + zx$

6E_5 $N_z = P_y \Rightarrow 2xz + ay = bxz + 2y$

$M_z = P_x = 2yz = byz \Rightarrow a = b = 2$

$f_x = yz^2 \Rightarrow f = xyz^2 + g(y,z);\ f_y = xz^2 + g_y$
$= xz^2 + 2yz \Rightarrow g_y = 2yz \Rightarrow g = y^2z + h(z)$

$f_z = 2xyz + y^2 + h'(z) = 2xyz + y^2 \Rightarrow h = c$

$\Rightarrow f = xyz^2 + y^2z + c$

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6F_1 b) $\oint_C y\,dx = \int_0^{2\pi} -\sin^2 t\,dt = -\pi$

$$-\iint_S (x+y+z)\,dS = -\pi$$

6F_2

$$\oint_C y\,dx + z\,dy + x\,dz = -3\pi$$

$$-\iint_{S'} 3\,dA = -3\pi$$

6F_5 a) $\operatorname{curl} F = -2x\mathbf{j} + 2\mathbf{k}$

$$\iint \operatorname{curl} F \cdot n\,dS = 2\pi a^2$$

b) $\iint_S \operatorname{curl} F . dS = \oint_0^{2\pi} (a^2 \sin\theta^2 + a^2\cos\theta^2)\,d\theta = 2\pi a^2$

Part II

Problem 1

(a) $\operatorname{curl} F = 0$

(b) F is conservative $\Rightarrow \oint_C F.dr = 0$

(c) No. ~~l_θ~~

(d) $\oint_{C_2} F.dr = 2\pi$

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Problem 2

a) $\mathrm{curl}\, G = 0$

b) Yes. Avoid origin.

c) $\mathbb{R}^3 - \{0\}$ is simply connected.

Problem 3

a) $\nabla \cdot F = 0$

b) $\nabla \cdot F = 0 \xrightarrow[\text{Theorem}]{\text{Divergence}}$ any closed surface flux is 0.

Problem 4

a) $\nabla \times F = 2\langle \cos t, \sin t, 0 \rangle$

b) $|\nabla \times F| = 2 \longrightarrow W_{max} = 1$

c) $ht \cdot v = 0$

d) pure flow.

e) Rotation in a plane; while the plane rotates.

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