

Subject: Practice Exam 4

Year:

Month:

Date:

Problem 1

a) $M_{xy} = N_x = ze^y$

P $M_z = P_x = ye^x$

$N_z = P_y = e^x + 2y \quad \checkmark$

b) $f_{yz} = e^y y z \Rightarrow f = e^y y z + g(y, z)$

$$\Rightarrow f_y = e^y z + g_y(y, z) = e^y z + 2yz$$

$$\Rightarrow g_y(y, z) = e^y 2yz \Rightarrow g(y, z) = y^2 z$$

$$+ h(z) \Rightarrow f = e^y y z + y^2 z + h(z) \Rightarrow$$

$$f_z = e^y y + y^2 + h'(z) \Rightarrow h'(z) = 1 \Rightarrow h(z) = z$$

$$\Rightarrow f = e^y y z + y^2 z + z$$

c)

$$M_y = 1 = N_{yz} = 1$$

$$M_z = 0 = P_{yz} = 0$$

$$N_z = 0 \neq P_y = 1$$

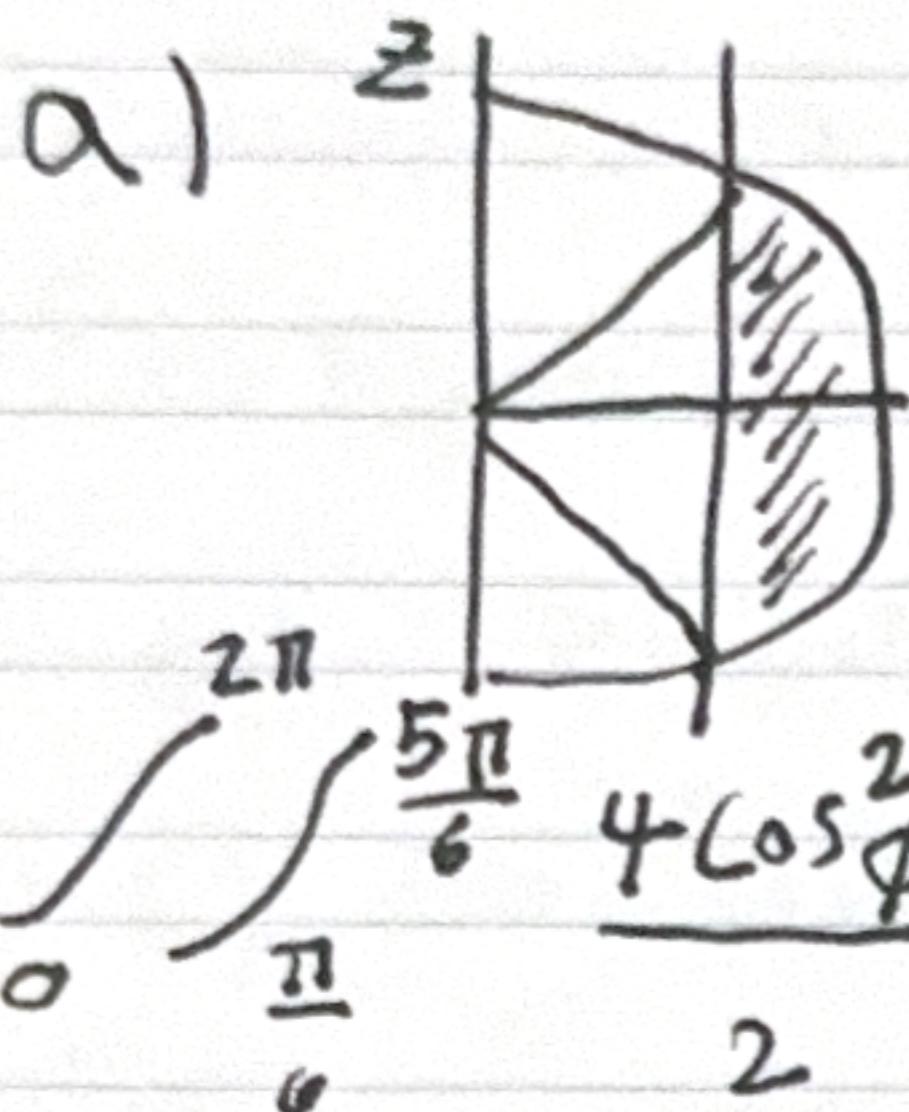
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Problem 2



$$h = \frac{1}{2} \langle u, g, z \rangle$$
$$F \cdot h = \frac{z^2}{2}$$

$$\int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{4 \cos^2 \phi}{2} 4 \sin \phi d\phi d\theta = 4\sqrt{3} \pi$$

b) $h = \pm \langle u, g, 0 \rangle \Rightarrow F \cdot h = 0$

c) $\operatorname{div} F = 1 \Rightarrow V = \iiint_R 1 dV = 4\sqrt{3} \pi$

Problem 3 $r(t) = \langle \cos t, \sin t, 1 \rangle$

$$\int_0^{2\pi} (-\cos t \sin t + \sin t \cos t) dt = 0$$

b) $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos t & \sin t & 1 \end{vmatrix} = 1i + uj + ok$

c) $\iint_S (\nabla \times F) \cdot h dS = \iint_S \frac{\vec{u+uy}}{\sqrt{2}} dS$

Problem 4

$$\operatorname{div} \mathbf{F} = 0 \Rightarrow \iiint \operatorname{div} \mathbf{F} dV = 0$$

Problem 5

a) $z^2 > 0$

b) $z = \rho \cos \phi \Rightarrow \rho \cos \phi = \rho^4 \Rightarrow \cos \phi = \rho^3$

c)

2π $\frac{\pi}{2}$ $(\cos \phi)^{\frac{1}{3}}$
 $\rho^2 \sin \phi d\rho d\phi d\theta$

Problem 6

$$d\vec{s} = h ds = \langle -f_x, -f_y, 1 \rangle dx dy$$

$$= \langle -y, -x, 1 \rangle$$

$$\iint_{x^2+y^2<1} (-y^2-x^2+xy) dx dy = \int_0^{2\pi} \int_0^1$$

$$(-r^2 + r^2 \cos \theta \sin \theta) r dr d\theta = -\frac{\pi}{2}$$