Subject: Problem Set 5

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2C-1 a) $dw = w_x dx + w_y dy + w_z dz$

$= \frac{yz}{xyz} \cdot dx + \frac{1}{y} \cdot dy + \frac{1}{2} dz$

b) $dz = z_x dx + z_y dy$

$= \frac{2y\,dx - 2x\,dy}{(x+y)^2}$

2C-2 $dV = yz\,dx + xz\,dy + xy\,dz$

$= 50\,dx + 20\,dy + 350\,dz$

$\Rightarrow 5 + 20 + 10 = 35$

2C-5 a) $-w^{-2}dw = -t^{-2}dt - u^{-2}du - v^{-2}dv$

$dw = w^2\left(\frac{dt}{t^2} + \frac{du}{u^2} + \frac{dv}{v^2}\right)$

b) $2u\,du + 4v\,dv + 6w\,dw = 0 \Rightarrow dw = -\frac{u\,du + 2v\,dv}{3w}$

2E-1 a)

c) $dw/dt = yz\,dx/dt + xz\,dy/dt + xy\,dz/dt$

$= t^5 + 2t^5 + 3t^5 = 6t^5$

$xyz = t^6 \Rightarrow 6t^5$

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2E-2 b) $\frac{dw}{dt} = \frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dy}{dt}$

$= -y\sin t + x\cos t$

$= -\sin^2 t + \cos^2 t \longrightarrow t = \frac{\pi}{4} + \frac{\pi}{2}k$

c) $df/dt = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t}$

$= 1\cdot 1 + (-1)\cdot\frac{2}{3} + 2\cdot 3$

$= 5$

2E-8 a) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial x} = f'(u)\cdot(-yx^{-2})$

$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial y} = f'(u)\cdot x^{-1}$

$\Rightarrow x\cdot f'\cdot(-yx^{-2}) + y\cdot f'\cdot(x^{-1}) = 0$

2D_1

a) $\nabla f = \langle 3x^2, 6y^2\rangle$

$\nabla f|_P = \langle 3, 6\rangle$

$\nabla f|_P \cdot \langle 1/\sqrt{2}, -1/\sqrt{2}\rangle$

$= 3/\sqrt{2} - 6/\sqrt{2} = -3/\sqrt{2}$

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e) $\nabla f = 2(4+2v+3w)(i+2j+3k)$

$\nabla f|_P = \langle 4, 8, 12 \rangle$

$\nabla f|_P \cdot \frac{u}{|u|} = \frac{4}{3}(i+2j+3k) \cdot \frac{-2i+2j-k}{3} = -\frac{4}{3}$

2D-2

b) (i) $\nabla w = \langle y+z, x+z, x+y \rangle$

$\nabla w|_P = \langle +1, 3, 0 \rangle \Rightarrow \max \frac{df}{ds}\Big|_u = \sqrt{10}$

$\min = -\sqrt{10}$

(ii) $\langle 1, 3, 0 \rangle$

(iii) $\langle -3, 1, 0 \rangle$

2D-3

a) $\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

$\nabla f \cdot \langle x, y, z \rangle \Big|_P$

$= \langle 4, 12, 36 \rangle \cdot \langle 3, 2, 1 \rangle = 12+24+36$

$= 72$

$\Rightarrow 4x+12y+36z = 72$ is the Tangent surface

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2D-8 $P = 30 + (x+1)(y+2)e^z$, $P|_0 = 32$

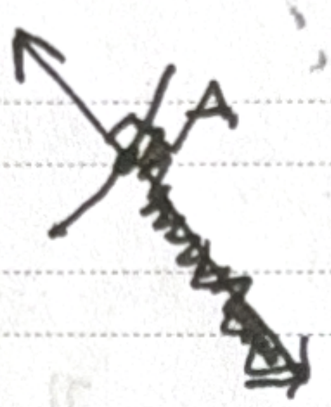
$\nabla P = \langle (y+2)e^z, (x+1)e^z, (x+1)(y+2)e^z \rangle$

$\nabla P|_0 = \langle 2, 1, 2 \rangle$

$|\nabla P|_0| = 3 \Rightarrow -3 \cdot \Delta s = -.9 \Rightarrow \Delta s = .3$

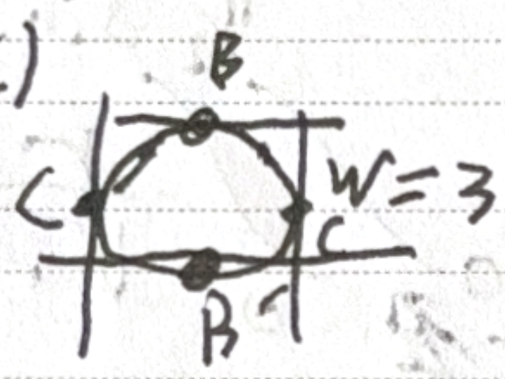
$\Rightarrow \frac{1}{10}\langle -2, -1, -2 \rangle$

2D-9 a)



$\frac{\Delta w}{\Delta s} \approx 1/.5 = 2$

b) c)



d) $\frac{\Delta w}{\Delta s}\Big|_i \approx \frac{-1}{5/3} = -.6$ $\frac{\Delta w}{\Delta s}\Big|_j \approx \frac{-1}{1} = -1$

e) $\frac{\Delta w}{\Delta s} \approx 1/.5 = 2$

f) $\frac{\Delta w}{\Delta s} \approx \frac{-1}{5/4} = -.8$

g)

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2.E_7:

$$\nabla f = \langle f_x x_u y_u + f_y y_u, f_y y_v + f_y y_u \rangle$$

$$= \langle f_x, f_y \rangle \begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}$$

Part 二

Problem 1

a) $dR = \frac{k \cdot 1}{r^4} dw + (-5^4) \frac{w \cdot k}{r^5} dr$

b) $\frac{dR}{R} = -4\frac{dr}{r} + \frac{dw}{w}$

c) r Contributes more

Opposite sign: r has negative factor.

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Problem 2

$$\frac{Df}{Dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

$$+\frac{\partial f}{\partial t}\frac{dt}{dt} = V \cdot \nabla f + \frac{\partial f}{\partial t}$$

Problem 3

a) $P = P(t) \Rightarrow \nabla P = \langle P_x, P_y, P_z \rangle = 0$

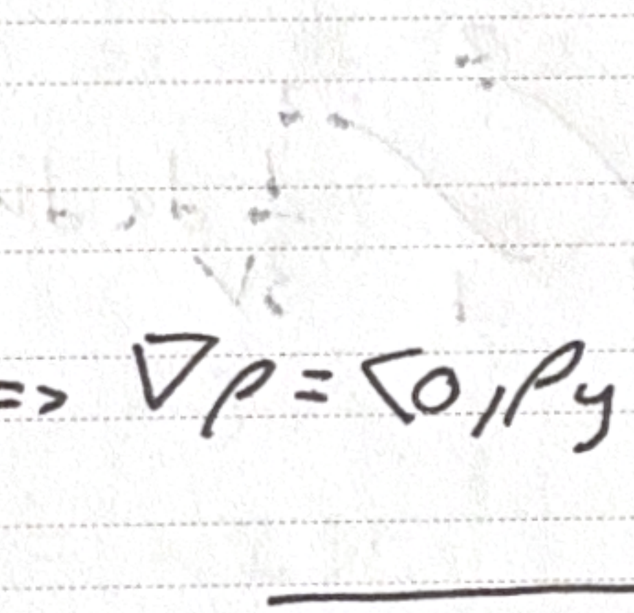
$$\Rightarrow \frac{DP}{Dt} = 0 \Leftrightarrow \frac{dP}{dt} = 0$$

b) $V \cdot \nabla p = 0$

c) $P = P(y) \Rightarrow \nabla p = \langle 0, P_y \rangle$

Problem 4

a)



b) $\nabla f = \langle -1, -4 \rangle$

c) $r(t) = -t\,i - 4t\,j$

$\Rightarrow x = 4/17,\ y = 16/17$

d) $\langle -1, -4 \rangle \cdot \frac{\langle -2, -1 \rangle}{\sqrt{5}} = 6/\sqrt{5}$