Subject: Final Exam

Year: Month: Date:

Problem 1

a) $P = \langle 1, 1, 2 \rangle$

$Q = \langle 0, 3, -1 \rangle$

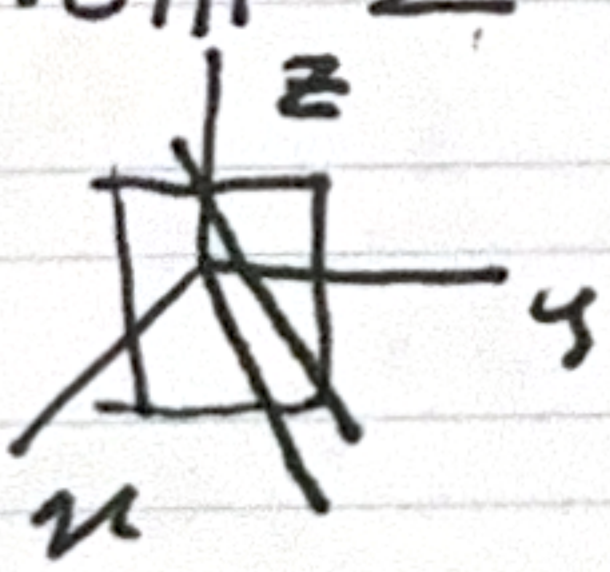
$R = \langle -1, 1, 2 \rangle$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ -2 & 0 & 0 \end{vmatrix} = 0i + 6j + 4k$$

$\Rightarrow 6y + 4z = 14$

b) $\dfrac{\langle 2, 1, 1 \rangle}{\sqrt{6}} \cdot \dfrac{\langle -1, 2, -3 \rangle}{\sqrt{14}} = \dfrac{-3}{\sqrt{6} \cdot \sqrt{14}}$

Problem 2

a) 

$r = \langle 0 + t, 0 + 2t, 1 \rangle$

b) $x + 2y = 0$

c) P on L $\Rightarrow$ $P = (t, 2t, 1)$

$P^* = \langle -t, 2t, 1 \rangle$

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Problem 3

a) $\begin{vmatrix} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & a \end{vmatrix} = (a+1) + 0 + 3(-1)$

$\xRightarrow{a=2} 3 - 3 = 0$

b) $\begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ -2 & 1 & -1 \end{vmatrix} = -3i - 5j + k$

$\Rightarrow t \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix}$

c) $-(a+3) = -4 \Rightarrow p = -4$

Problem 4

a) $\hat{T} = \frac{1}{\sqrt{2}} \langle -\sin(e^t), \cos(e^t), 1 \rangle$

b) $\hat{T}' = \frac{1}{\sqrt{2}} \langle -\cos(e^t), -\sin(e^t), 0 \rangle$

Problem 5

a) $F_x = \frac{xz}{(x^2+y)^{\frac{1}{2}}}$

$F_y = \frac{z}{2(x^2+y)^{1/2}} + \frac{2}{z}$

$F_z = (x^2+y)^{1/2} - \frac{2y}{z^2}$

$\Rightarrow \nabla F(1,3,2) = \langle 1, \frac{3}{2}, \frac{1}{2} \rangle$

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b) $|F_y| = 3/2 \Rightarrow \frac{3}{2}(0.1) = 0.15$

c) $\frac{df}{ds}\Big|_{P_0,u}$ $\Delta S \Rightarrow$ $0.1 = \frac{1}{6}\Delta S \Rightarrow \Delta S = 0.6$

Problem 6

a) $f_x = 1 - \frac{2}{x^2 y} = 0$

$f_y = 4 - \frac{2}{x y^2} = 0$

$\Big\} \; y = \frac{1}{2}, x = 2$

b) $f_{xx} = \frac{4}{x^2 y}$ $\quad AC - B^2 = 12 > 0$

$f_{yy} = \frac{4}{x y^3}$ $\quad A > 0$

$f_{xy} = \frac{2}{x^2 y^2}$ $\quad \rightarrow (2, \frac{1}{2})$ min

Problem 7

$$2(x - x_0) = \lambda A$$

$$2(y - y_0) = \lambda B$$

$$2(z - z_0) = \lambda C$$

$$Ax + By + Cz = D$$

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Problem 8

$$\frac{\partial F}{\partial \phi} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial \phi} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial \phi}$$

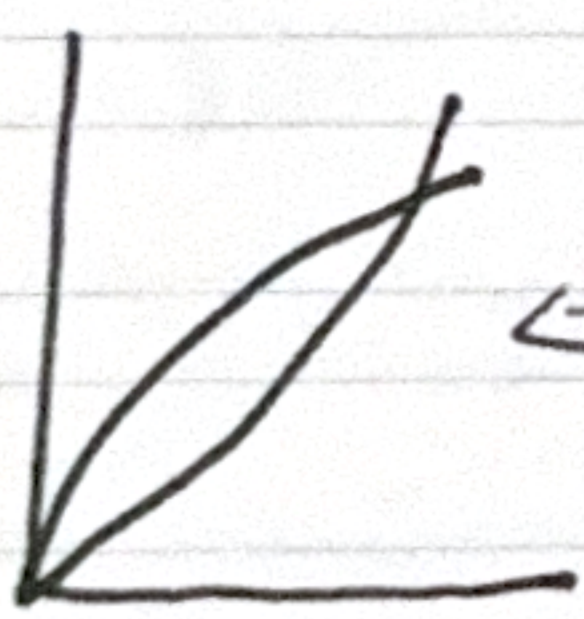
$$\frac{\partial x}{\partial \phi} = 1$$

$$\frac{\partial y}{\partial \phi} = -1$$

$$\frac{\partial z}{\partial \phi} = -\sqrt{2}$$

b) Not a gradient!

Problem 9



a)

b) $\int_0^4 \int_{y^2/8}^{\sqrt{y}} f(x,y)\,dx\,dy$

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Problem 10

$$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} u^{-2/3}v^{-1/3}/3 & -u^{1/3}v^{-4/3}/3 \\ u^{-2/3}v^{2/3}/3 & 2u^{1/3}v^{-1/3}/3 \end{vmatrix}$$

$$= \frac{1}{3}u^{-1/3}v^{-2/3} \Rightarrow \int_1^2 \int_4^9 f(u^{1/3}v^{-1/3}, u^{1/3}v^{2/3})$$

$$\left(\frac{1}{3}u^{-1/3}v^{-2/3}\right) du\, dv$$

Problem 11

a)

Positive

b)

$$\left.\begin{aligned} \int_{C_1} &= 1 \\ \int_{C_2} &= 0 \\ \int_{C_3} &= -\frac{1}{2} \end{aligned}\right\} \frac{1}{2}$$

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c) $\operatorname{div} F = 1 \Rightarrow \text{area } R \Rightarrow \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

Problem 12

a) $\int_0^{2\pi} \int_0^1 \int_{2r}^2 z \, dz \, r \, dr \, d\theta = \pi$

b) $\bar{z} = \frac{1}{m} \iiint_G z \cdot dv = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \int_{2r}^2 z^2 \, dz \, r \, dr \, d\theta$

c) $\frac{1}{\pi} \int_0^{2\pi} \int_0^{\tan^{-1}(1/2)} \int_0^{2\sec\phi} (\rho \cos\phi)^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

Problem 13

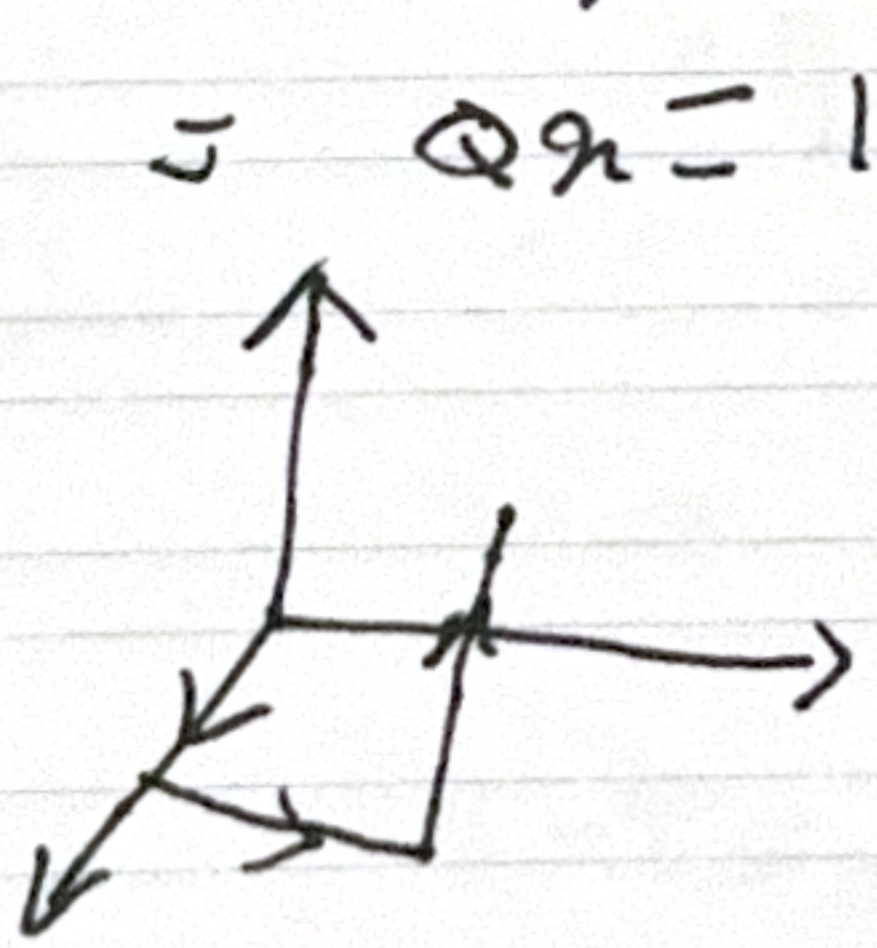
a) $P_z = R_x = y^2$

$Q_z = R_y = -1 + 2xy$

$P_y = Q_x = 1 + 2yz$

$\int_0^{x_1} + \int_0^{y_1} + \int_0^{z_1}$

b)



$= xy - yz + xy^2z + C$

c) $-10 + 3 = -7$

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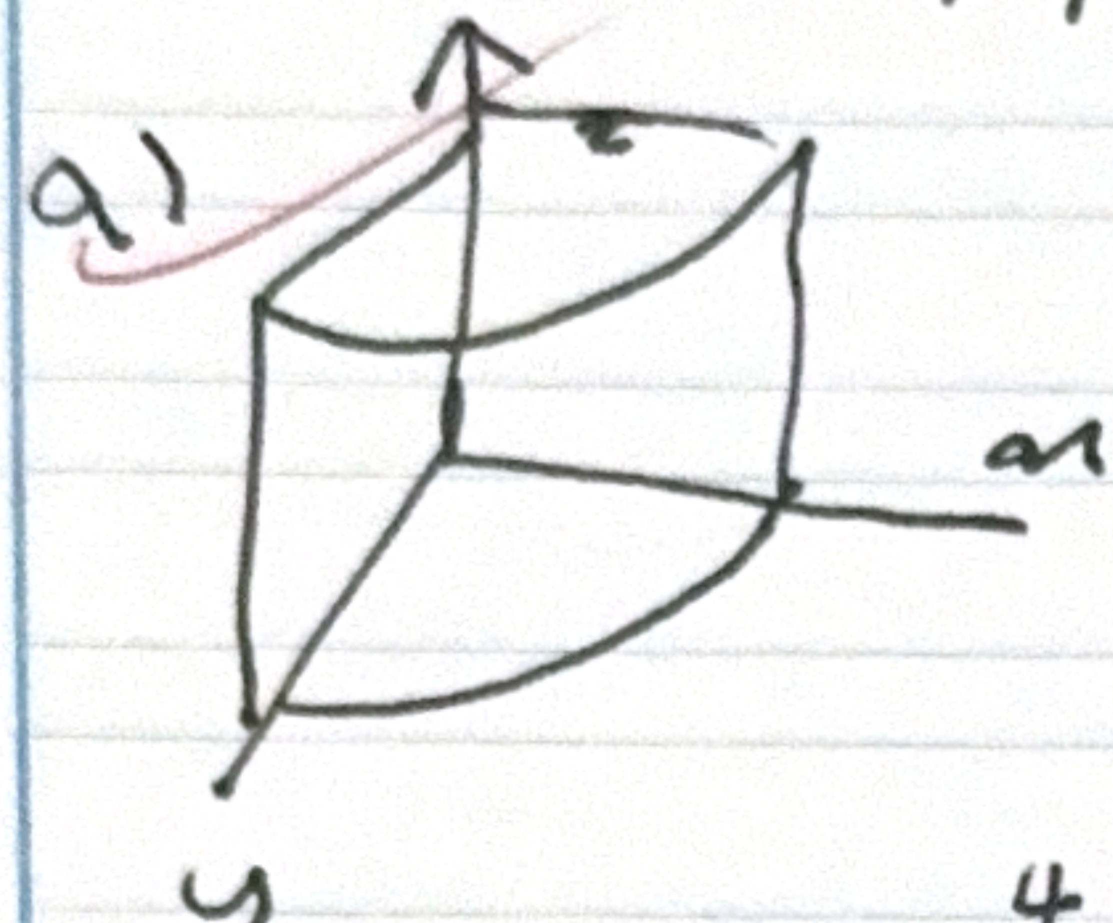
Problem 14

a)



b) $\hat{n} = \frac{1}{2}\langle x, y, 0\rangle$

$F = \langle x, 0, 0\rangle$

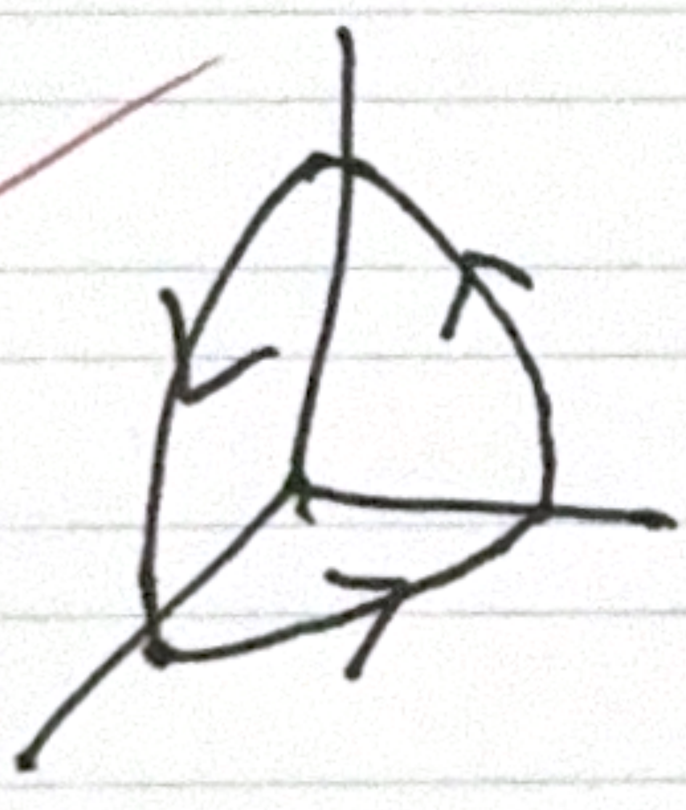
$\Rightarrow 4\int_0^4 dz \int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta = 16\frac{\pi}{4}$

c) $\operatorname{div} F = 1 \Rightarrow 4\pi$

d) zero

Problem 15

a)



$4\int_0^{\frac{\pi}{2}}\int_0^2 (r^2-2)\, r \, dr \, d\theta$

$= 0$

b) $\int_{C_1+C_2+C_3} = 4-4=0$