

Subject: Session 63

Year. Month. Date.

$$1. \frac{M_y}{y} = 6x + 6y = Ny = 6x$$

$$\begin{aligned} f_x &= 3x^2 + 6xy \\ \Rightarrow f &= x^3 + \underline{3x^2} y + g(y) \\ \Rightarrow f_y &= 3x^2 + g'(y) \\ &= 3x^2 + 6y \Rightarrow g'(y) = 6y \Rightarrow g(y) = 3y^2 + C \\ \Rightarrow f &= x^3 + 3x^2 y + 3y^2 + C \end{aligned}$$

$$2. \frac{M_y}{y} = 2xy = Ny = 2xy$$

$$\begin{aligned} f_x &= x + axy^2 \\ \Rightarrow f &= \frac{x^2}{2} + \underline{\frac{ax^2 y^2}{2}} + g(y) \\ \Rightarrow f_y &= 0 + ax^2 y + g'(y) = 3 \\ \Rightarrow & x^2 y + 3y^2 \Rightarrow g'(y) = 3y^2 \Rightarrow g(y) = y^3 + C \\ \Rightarrow f &= \frac{x^2}{2} + \underline{\frac{ax^2 y^2}{2}} + y^3 + C \end{aligned}$$

$$\int_0^{x_1} ax dx + \int_0^{y_1} \left(\frac{x^2}{2} + \underline{\frac{ax^2 y^2}{2}} + y^3 \right) dy = \frac{x_1^2}{2} + ax_1^2 y_1^2 / 2 + y_1^3$$