

$$1. M_y = 6x + 4y = N_x = 6x$$

$$f_x = 3x^2 + 6xy$$

$$\Rightarrow f = x^3 + 3x^2y + g(y)$$

$$\Rightarrow f_y = 3x^2 + g'(y)$$

$$= 3x^2 + 6y \Rightarrow g'(y) = 6y \Rightarrow g(y) = 3y^2 + C$$

$$\Rightarrow f = x^3 + 3x^2y + 3y^2 + C$$

$$2. M_y = 2xy = N_x = 2xy$$

$$f_x = x + xy^2$$

$$\Rightarrow f = \frac{x^2}{2} + \frac{x^2y^2}{2} + g(y)$$

$$\Rightarrow f_y = 0 + x^2y + g'(y)$$

$$= x^2y + 3y^2 \Rightarrow g'(y) = 3y^2 \Rightarrow g(y) = y^3 + C$$

$$\Rightarrow f = \frac{x^2}{2} + \frac{x^2y^2}{2} + y^3 + C$$

$$\int_0^{x_1} x dx + \int_0^{y_1} x_1^2 y + 3y^2 dy = \frac{x_1^2}{2} + \frac{x_1^2 y_1^2}{2} + y_1^3$$