

1. $f = x^2 + x + 2y^2$

$g = x^2 + y^2 = 1$

$\nabla f = \langle 2x+1, 4y \rangle$

$\nabla g = \langle 2x, 2y \rangle$

$2x+1 = \lambda 2x$

$4y = \lambda 2y$

$x^2 + y^2 = 1$

$\left. \begin{array}{l} 2x+1 = \lambda 2x \\ 4y = \lambda 2y \\ x^2 + y^2 = 1 \end{array} \right\} \lambda = 2, x = \frac{1}{2}, y = \pm \frac{\sqrt{3}}{2}$

$\left(\frac{1}{2}, \pm \frac{\sqrt{3}}{2} \right), (1, 0), (-1, 0)$
 $\begin{array}{cc} \text{max} & \text{min} \end{array}$

2. $f = x^2 - xy + y^2$

$g = x^2 + y^2 = 1, x, y \geq 0$

$\nabla f = \langle 2x-y, 2y-x \rangle$

$\nabla g = \langle 2x, 2y \rangle$

$2x-y = \lambda 2x$

$2y-x = \lambda 2y$

$x^2 + y^2 = 1$

$x, y \geq 0$

$\left. \begin{array}{l} 2x-y = \lambda 2x \\ 2y-x = \lambda 2y \\ x^2 + y^2 = 1 \\ x, y \geq 0 \end{array} \right\} \lambda = \frac{1}{2}, x, y = \pm \frac{\sqrt{1}}{2}$

$(1, 0), (0, 1), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$
 $\begin{array}{ccc} \text{max} & \text{max} & \text{min} \end{array}$