Subject: Problem Set 4

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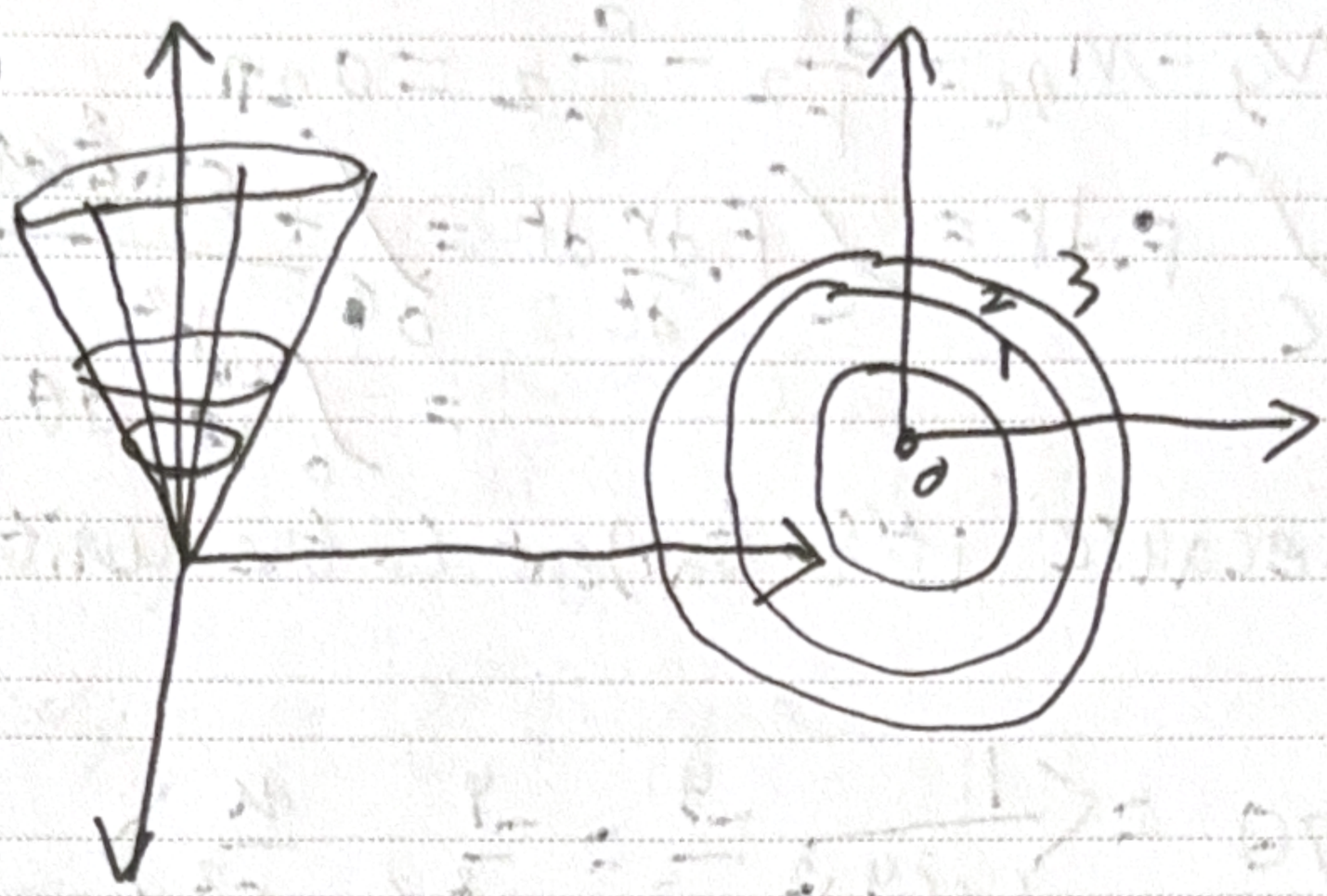
2A_1 c)



2A_2

b) $z = x/y$ $z_x = 1/y$ $z_y = -x/y^2$

e) $z = x \ln(2x+y)$ $z_x = \ln(2x+y) + \frac{2x}{2x+y}$

$z_y = \frac{x}{2x+1}$

2A_3

b) $f = \frac{x}{x+y}$ $f_x = \frac{y}{(x+y)^2}$

$f_{xy} = \frac{x-y}{(x+y)^3}$ $f_y = \frac{-x}{(x+y)^2}$

$f_{yx} = \frac{-(y-x)}{(x+y)^3}$

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2A-5

a) $w_x = ae^{ax}\sin ay$

$w_{xx} = a^2 e^{ax}\sin ay$

$w_y = e^{ax} a\cos ay$

$w_{yy} = e^{ax}(-\sin ay)$

→ ✓

2B-1 b) $w = \frac{y^2}{x}$

$w_x = \frac{-y^2}{x^2} = 0$

$w_y = \frac{0 - 2y}{x^2}$

$w = 4^{2x^2} - 4(x-1) + 4(y-2)$

→

2B-6

$V = \pi r^2 h$

$V_r = 2\pi rh$

$V_h = \pi r^2$

$\Delta V \approx (2\pi rh)\Big|_0 \Delta r + (\pi r^2)\Big|_0 \Delta h$

$= (12\pi)\Delta r + 4\pi \Delta h$

$= 16\pi\epsilon < .1$

$\rightarrow \epsilon < \frac{1}{160\pi} < .002$

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2B_9

a) $\Delta W = W_x|_0 \Delta x + W_y|_0 \Delta y$

$= 2x(y+1)|_0 \Delta x + x^2|_0 \Delta y$

$= 2\Delta x + \Delta y$

$\longrightarrow \Delta x$ is more sensitive

b) $\Delta W = 2\Delta x + \Delta y = 0$

$\longrightarrow \frac{\Delta y}{\Delta x} = -2$

2F_1 (a) $z^2 = \frac{1}{xy}$

$\Leftrightarrow D = x^2 + y^2 + \frac{1}{xy}$

$D_x = 2x - \frac{1}{x^2 y} = 0$

$D_y = 2y - \frac{1}{y^2 x} = 0$

$\Leftrightarrow x = \pm y$

$\Leftrightarrow x^4 = \frac{1}{2}$

$\Leftrightarrow (\frac{1}{2}^{1/4}, \frac{1}{2}^{1/4}, 2^{1/4})$

2F_2 $A = 3xy + \frac{4}{y} + \frac{2}{x}$

$A_x = 3y - \frac{2}{x^2}$ $A_y = 3x - \frac{4}{y^2}$

$\longrightarrow y = 2x$

$\longrightarrow 3x^3 = 1 \longrightarrow x = \frac{1}{3}^{1/3}$

$\longrightarrow x:y:z = 1:2:\frac{3}{2}$

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2G-1 c) $\sum x_i^2 a + \sum x_i b = \sum x_i y_i$

$\sum x_i a + n b = \sum y_i$

14 6

$\Leftrightarrow \frac{14}{3} a + \frac{6}{3} b = 13$

$6a + 3b = 6$

$\Leftrightarrow a = 1/2$

$b = 1$

2G-4

$$D = \sum_{i=0}^{n} (z_i - (a + bx_i + cy_i))^2$$

$$D_a = \sum_{i=0}^{n} 2(z_i - (a + bx_i + cy_i))$$

$$D_b = \sum_{i=0}^{n} 2x_i (z_i - (a + bx_i + cy_i))$$

$$D_c = \sum_{i=0}^{n} 2y_i (z_i - (a + bx_i + cy_i))$$

$\Leftrightarrow$

$$na + \sum x_i b + \sum y_i c = \sum z_i$$

$$\sum x_i a + \sum x_i^2 b + \sum x_i y_i c = \sum x_i z_i$$

$$\sum y_i a + \sum x_i y_i b + \sum y_i^2 c = \sum y_i z_i$$

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2H-1 c) $z = 2x^4 + y^2 - xy + 1$

$z_x = 8x^3 + 0 - y + 0$

$z_y = 0 + 2y - x + 0$

$\Leftrightarrow \quad 8x^3 - y = 0$

$-x + 2y = 0$

$\Leftrightarrow (0,0), (\frac{1}{2}, \frac{1}{4}), (-\frac{1}{2}, -\frac{1}{4})$

$z_{xx} = 24x^2$

$z_{xy} = -1$

$z_{yy} = 2$

$\Leftrightarrow (0,0): AC - B^2 = -1 < 0$ Saddle

$(\frac{1}{2}, \frac{1}{4}): AC - B^2 = 12^2 > 0, A > 0$ local min

$(-\frac{1}{2}, -\frac{1}{4}): AC - B^2 = 12^2 > 0, A > 0$ local max

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2H-3 $f_x = 2x + 0 + 2 + 0 - 0$

$f_y = 0 + 2y + 0 + 4 - 0$

$\overset{-2}{\Leftrightarrow} (\overset{-1}{a}, \overset{-2}{-4/2}) \Leftrightarrow$ Not in R!

$x\,g(t) = t$

$y(t) = -t$

$g(t) = t^2 + t^2 + 2at - 4t - 1$

$g_t = 4t \overset{!}{=} 2$

$\Leftrightarrow (1/2, -1/2)$ min in Boundry

2H-4

a) $f = xy - x - y + 2$

$f_x = y - 1 \qquad f_{xx} = 0 \qquad f_{xy} = 1$

$f_y = x - 1 \qquad f_{yy} = 0$

$\Leftrightarrow (1,1) \Leftrightarrow AC - B^2 = -1 < 0$ Saddle

Boundries:

$-x + 2$

$-y + 2$ } No minimum

$\lim_{x \to \infty} f(x,x) = \infty$ } $\max \infty$

b) $x = 2 \Rightarrow 2y - 2 - y + 2 = y$

$y = 2 \Rightarrow 2x - x - 2 + 2 = x$

$\Rightarrow \underbrace{(0,0)}_{\max 2}, \underbrace{(0,2)}_{\text{Saddle}}, \underbrace{(2,0)}_{\text{Saddle}}, \underbrace{(2,2)}_{\max = 2}$

c) (1,1) Saddle Point

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2.H-6 a) $z = 4 - x - y$

$$z_{xx} f = xy + \frac{1}{4}(4-x-y)^2$$

$$f_x = y - \frac{1}{2}(4-x-y) = 0$$

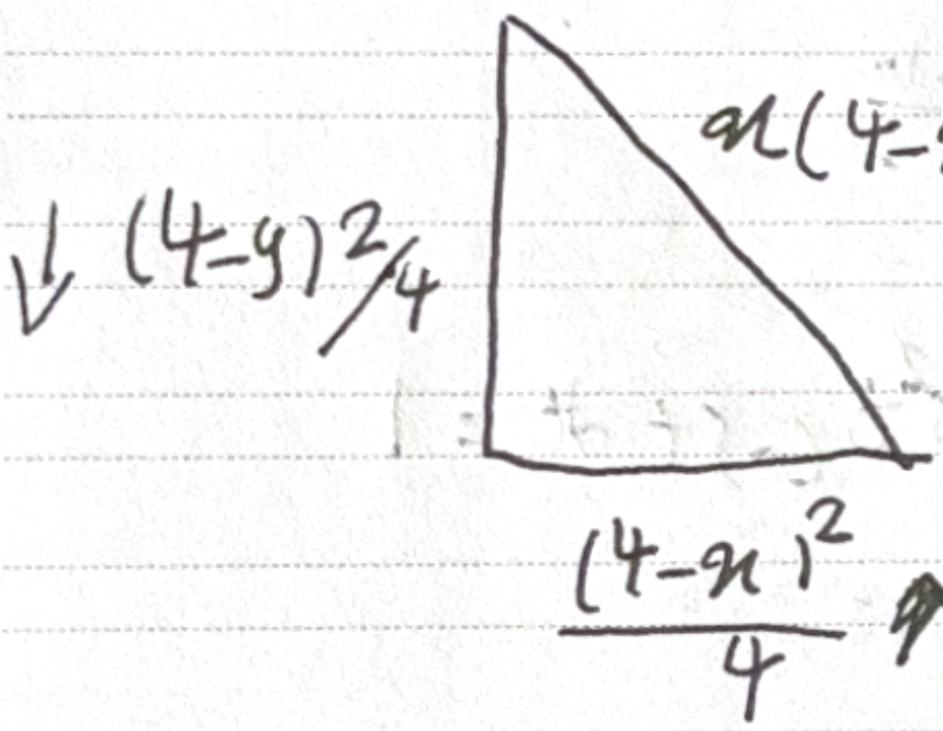
$$f_y = x - \frac{1}{2}(4-x-y) = 0$$

<a> (1,1)

Boundaries: $x + y \leq 4$

$x \geq 0$

$y \geq 0$



$\Rightarrow f_{(0,0)} = 4$ max

$f_{(4,0)} = 0$ min

$f_{(0,4)} = 0$ min

$f_{(2,2)} = 4$ max

$f_{(1,1)} = 2$ saddle

b) $f_{xx} = 1/2$

$f_{xy} = 3/2$

$f_{yy} = 1/2$

$AC - B^2 = -2 < 0$ Saddle

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Part II

Problem 1

(a) Mathlet not found

Problem 2

(a) $x^2 - y^2 = 2 + (x-y)^2$

$\Leftrightarrow \quad x = y + \frac{1}{y}$

$\Leftrightarrow \quad x = t + t^{-1}$

$y = t$

$z = 2 + t^{-2}$

(b) $f_x = 2x$
$f_y = -2y$
forward
} $\langle 4, -2, -1 \rangle = n$

$f_{2x} = 2(x-y)$
$f_{2y} = -2(x-y)$
forward
} $\langle 2, -2, -1 \rangle$

$\cos\theta = \frac{8+4+1}{\sqrt{21}\sqrt{9}}$

(c) $r' = \langle 1 - t^{-2}, 1, -2t^{-3} \rangle$

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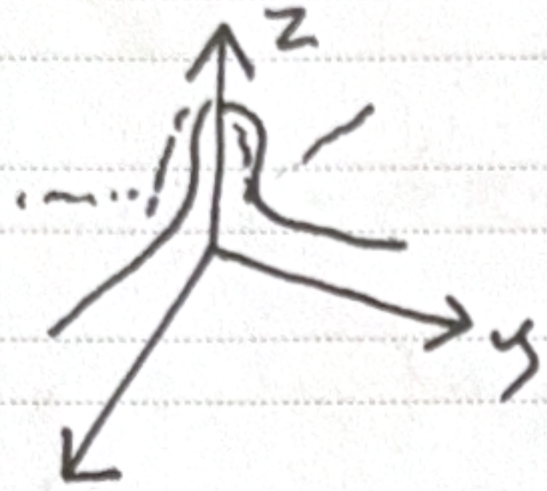
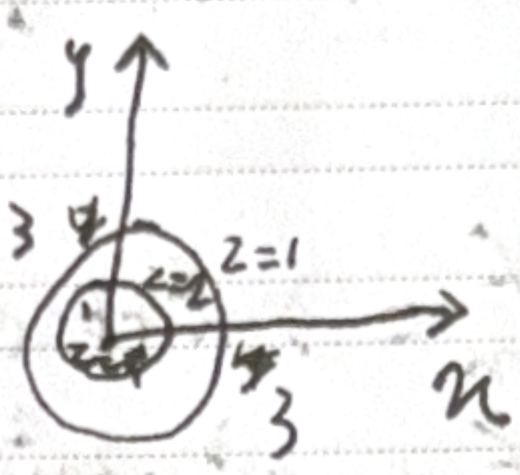
Problem 3

a)





b) $x = t$, $y = 3/2 - t^2$, $z = \frac{4}{1+t^2+(1.5-t^2)^2}$

c) ✓

d) $z_t = 4t(t^2-1) \Rightarrow (0, 3/2, 16/13)$, min

$(\pm 1, 1/2, 16/9)$ max

e) $D^2 = t^2 + (3/2 - t^2)^2 \Rightarrow (0, 3/2, 16/13)$ min

$(\pm 1, 1/2, 16/9)$ max

Problem 4

$z^2 = 27 - x^2 - y^2$

$f = x^3 + y^3 + (27 - x^2 - y^2)^{3/2}$

$f_x = 3x^2 + 0 + \frac{3}{2}(27 - x^2 - y^2)^{1/2}(-2x)$

$f_y = 0 + 3y^2 + \frac{3}{2}(27 - x^2 - y^2)^{1/2}(-2y)$

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$\Leftrightarrow$ $(0,0), (0,\sqrt{27/2}), (\sqrt{27/2},0), (3,3)$

$$f_{xx} = 6x - 3(27-x^2-y^2)^{1/2} + 3x^2(27-x^2-y^2)^{1/2}$$

$$f_{xy} = 3xy(27-x^2-y^2)^{-1/2}$$

$$f_{yy} = 6y - 3(27-x^2-y^2)^{1/2} + 3y^2(27-x^2-y^2)^{-1/2}$$

$AC-B^2>0, A<0 \rightarrow (0,0)$ max

$AC-B^2<0 \rightarrow (0,3\sqrt{3/2}), (3\sqrt{3/2},0)$ Saddle

$AC-B^2>0, A>0 \rightarrow (3,3)$ min

Boundaries: $x^2+y^2=27$

$\Rightarrow x = 3\sqrt{3}\cos t$

$y = 3\sqrt{3}\sin t$

$f_{(x,y)} = 27(\cos^3 t + \sin^3 t)$

$\Rightarrow f_t = 0 \Rightarrow t = 0, \frac{\pi}{4}, \frac{\pi}{2} \ldots$

$0, \frac{\pi}{2}$ max $\Rightarrow S = 81\sqrt{3}$

$\frac{\pi}{4}, \frac{3\pi}{4}$ min

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Problem 5

(a) $f = \cos\alpha \cos\beta \cos(\alpha+\beta)$

$$f_\alpha = -\sin\alpha \cos\beta \cos(\alpha+\beta) - \cos\alpha \cos\beta \sin(\alpha+\beta)$$

$$f_\beta = -\cos\alpha \sin\beta \cos(\alpha+\beta) - \cos\alpha \cos\beta \sin(\alpha+\beta)$$

$\Leftrightarrow$ $(\pi/3, \pi/3)$

(b) $f_{(\pi/3, \pi/3)} = -\frac{1}{8}$