Subject: Problem Set 9
Year. Month. Date. ( )

4D-1 c)

$$\int_0^1 xy\,dx + y^2\,dy = \int_0^1 x^3\,dx + 2x^5\,dx$$

$$= \frac{x^4}{4} + \frac{x^6}{3}\Big|_0^1 = \frac{7}{12}$$

$$\int_1^0 xy\,dx + y^2\,dy = \int_1^0 x^2\,dx + x^2\,dx$$

$$= \frac{2}{3}x^3\Big|_1^0 = -\frac{2}{3}$$

$$\frac{7}{12} - \frac{2}{3} = -\frac{1}{12}$$

$$\int_0^1\int_{x^2}^{x} -x\,dy\,dx = -\frac{1}{12}$$

4D-2

$$\oint_C 4x^3y\,dx + x^4\,dy = \iint_R 4x^3 - 4x^3 = 0$$

4D-3

$$\frac{1}{2}\int_0^{2\pi} 3\sin^4\cos^2 + 3\sin^2\cos^4 = \frac{3\pi}{8}$$

PAPCO

Subject:
Year. Month. Date. ( )

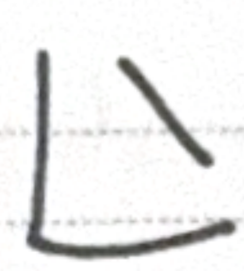
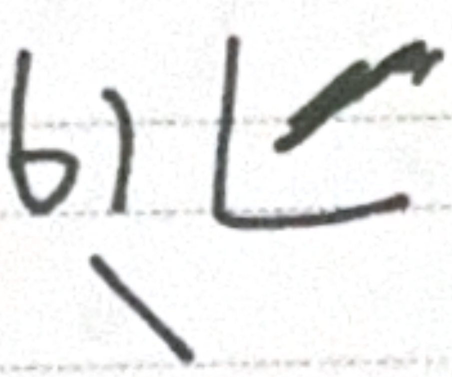
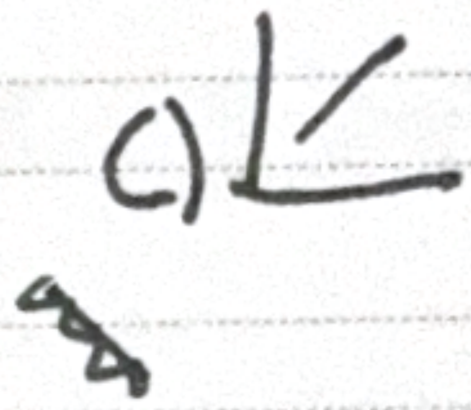
4D_4

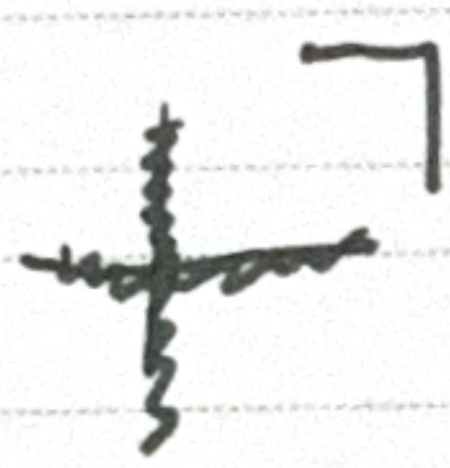
$$\iint_R 3x^2+3y^2 \, dA \geq 0$$

4E-1 a) 0

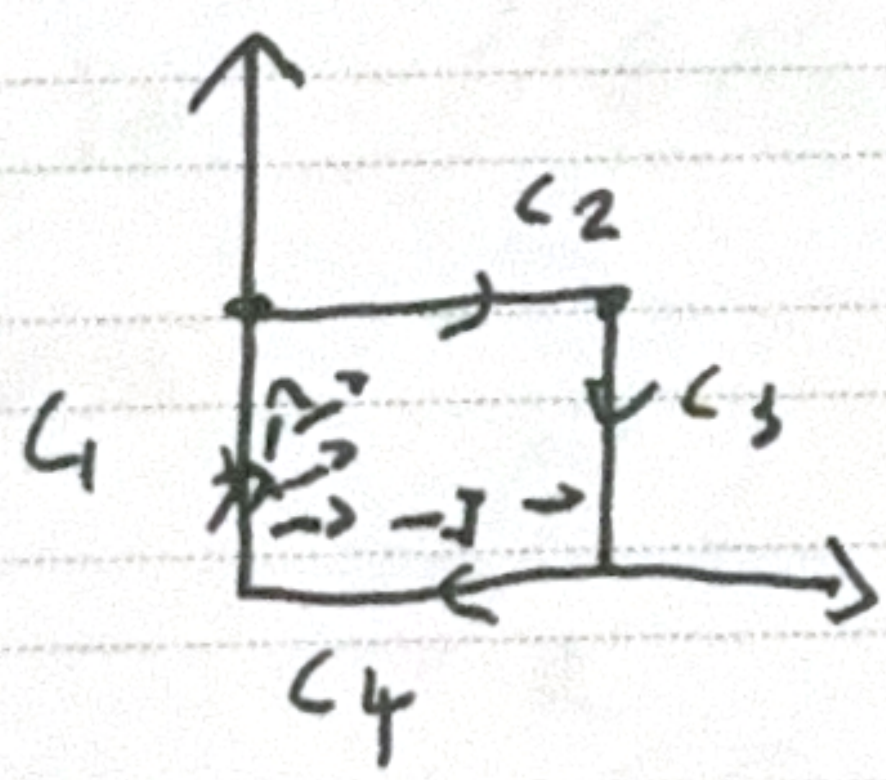
b) 0

c) $F \cdot n = -x \Rightarrow \int_0^1 -x \, dx = \frac{1}{2}$

4E-2 a)  b)  c) 

d)  e) $\pm\sqrt{2}$

4E-4




$$\int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$$

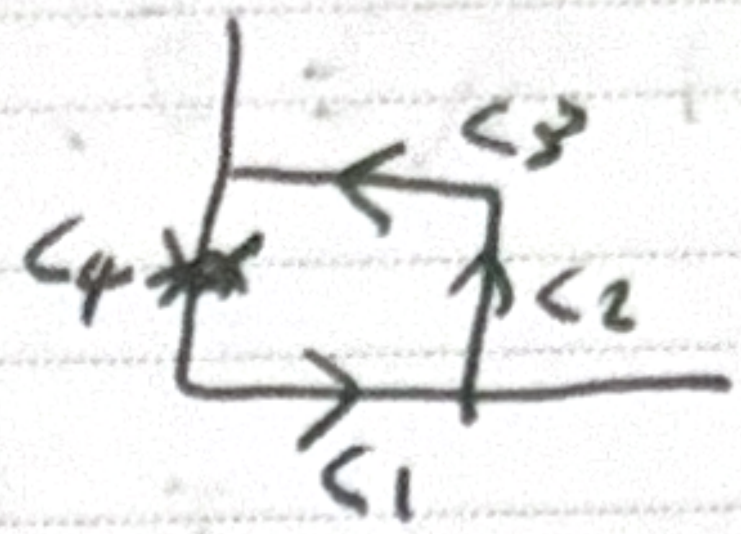
$$= \int x\,dy - y\,dx = \int -y\,dx + x\,dy = 0 + \int_0^1 -1 + \int_1^0 1 + 0$$

$$= -2$$

4E-5 a) $\oint_C F \cdot n \, ds = \int_0^{2\pi} a^{m+1} d\theta = 2\pi a^{m+1}$

b) $-1$

# 4F_4




$$\int_C -xy\,dx + x^2\,dy$$

$$= \int_{C1} + \int_{C2} + \int_{C3} + \int_{C4}$$

$$= 0 + \int_0^1 1 + \int_1^0 -x + \int 0 = \cancel{2}\ \frac{3}{2}$$

$$\iint_R 3x\,dA = \int_0^1\int_0^1 3x\,dy\,dx = \frac{3}{2}$$

## Part II

Problem 1

a) $\iint_R N_x - M_y \, dA$

$$= \iint_R 6 - 3x^2 - 3y^2 + 6\,dy\,dx$$

$$= \iint_R 12 - 3(x^2+y^2)$$

$$= \iint_R 12 - 3\cancel{4}(x^2+y^2)$$

$\therefore x^2+y^2 \le 4 \Rightarrow$ radius 2 circle

counterclockwise

Subject:
Year. Month. Date. ( )

b) $\iint_R (12-3x^2-3y^2)\,dx\,dy = \int_0^{2\pi}\int_0^{2} (12-3r^2)\,r\,dr\,d\theta$

$= 24\pi$

Problem 2

a) $\iint_R \frac{\partial \rho}{\partial t}\,dA + \iint_R \mathrm{div}(F)\,dA = 0$

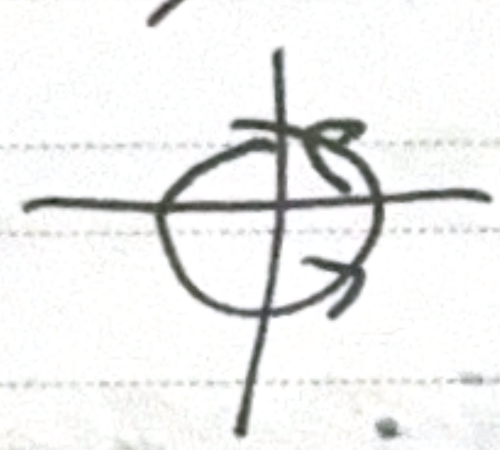
b) $\mathrm{div}(gG) = (g_x M + g_y N) + (g M_x + g N_y)$

$= G \cdot \nabla g + g\,\mathrm{div}(G)$

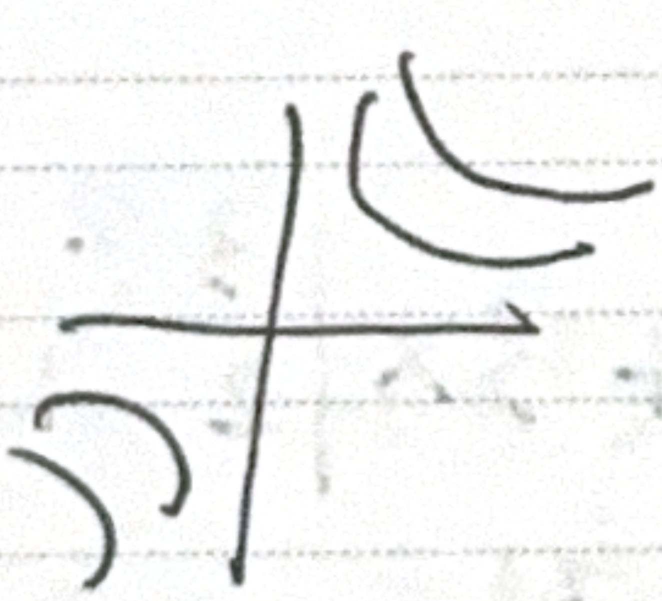
c) $\frac{\partial \rho}{\partial t} + \mathrm{div}(F) = \frac{\partial \rho}{\partial t} + v \cdot \nabla \rho + \rho\,\mathrm{div}(v) = \frac{D\rho}{Dt} + \rho\,\mathrm{div}(v)$

Problem 3

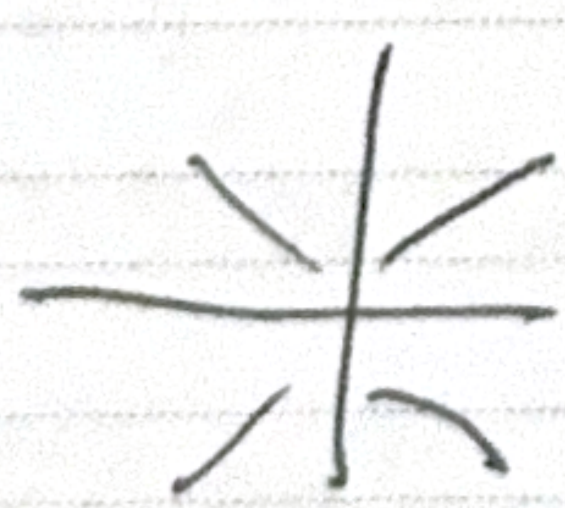
(i)



(ii)



(iii)



PAPCO