Subject: Problem Set 2 Year: Month: Date:

Problem 1

lemma: if sorted ~~subset~~ ~~set~~ index → ※

(a) 1: $a_1 < a_2 < a_3$ ※

2: $a_1 < a_3 < a_2$ ※

if any

3: $a_3 < a_1 < a_2$

1: $a_1 < a_3 < a_2 < a_4$ ※

2: $a_1 < a_3 < a_4 < a_2$ ※

3: $a_1 < a_4 < a_3 < a_2$ ※

4: $a_4 < a_1 < a_3 < a_{\cancel{4}\,2}$ ※

(b) 1: $a_4 < a_3 < a_1 < a_2$ ※

2: $a_3 < a_1 < a_2 < a_4$ ※

3: if any $a_3 < a_1, a_4 < a_2$

(c) 1: $a_3 < a_1 < a_4 < a_2$

2: $a_3 < a_4 < a_1 < a_2$

1: 3, 4, 5

2: 5, 4, 2

1: $a_3 < a_1 < a_4 < a_5$ ※

2: $a_5 > a_4 > a_2$ ※

(d) with $a_1 < a_2$ All cases → ※

symmetry.

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Problem 2.

P(by WOP)

$P(n)$: $\sum_{i=0}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

$C = \{ n \mid P(n) \text{ is false} \}$

$\exists c$ by WOP $c = \min(C)$

$\sum_{i=0}^{c-1} i^3 = \left(\frac{(c-1)(c)}{2}\right)^2$

$\left(\frac{(c-1)(c)}{2}\right)^2 + c^3 = \sum_{i=0}^{c} i^3$

$\rightarrow$ $P(c) = T = F$ B

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Problem 3

I.H. $P(t) ::=$ After t mins

perimeter stays

the same.

Base case: $P(0)$ ✓

I.S. an square is infected

when two neighbors are,

1: $P_{n+1} = P$

2: $P_{n+1} = P + 2 - 2$ ✓

theorem. the perimeter never reaches $4n$

Problem ~~3~~ 4. "implies that"

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Problem 5.

P(by ~~strong~~ structural Induction.)

I.H. $P(n) ::= G_n = 3^n - 2^n$

B.C. $P(0)$ ✓ $P(1)$ ✓

I.S. $5(3^n - 2^n) - 6(3^{n-1} - 2^{n-1}) = 3^{n+1} - 2^{n+1} = G_{n+1}$ ✓

(Constructive case)

Problem 6.

(a) $i \longrightarrow i+1 \Longrightarrow$ no

$\updownarrow$

$i-1$

(b) $\left.\begin{array}{l}(i, i+1), (i, i+2), (i, i+3) \\ \text{or} \\ (i, i-1), (i, i-2), (i, i-3)\end{array}\right\}$ 3 Pairs

(c) none, no Pairs per (a).

(d) $+\cancel{2}1, -1, +3, -3 \rightarrow$ changes. (induction)

(e) lemma, blank switches ~~each~~ rows odd row with i.

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(f) odd moves $\rightarrow$ even Parity

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Problem 7.

I.H. $P(n) ::= \forall n \in \mathbb{N} \quad B \geq Z$

B.C. $P(0)$ ✓

I.S. $Z_{n+1} = \left\lfloor \frac{B_n - Z_n}{2} \right\rfloor \qquad B_{n+1} = \left\lfloor \frac{B_n - Z_n}{2} \right\rfloor \times 2$