

## Linear Recurrences

1. Hom Sol: eq:  $\alpha^2 - \alpha - 1 = 0$

$$\text{Root: } \alpha = s\left(\frac{1+\sqrt{5}}{2}\right) + t\left(\frac{1-\sqrt{5}}{2}\right)$$

Part Sol:  $C = C + C + 1$

$$C = -1$$

$$\text{Gen Sol: } T(n) = s\left(\frac{1+\sqrt{5}}{2}\right)^n + t\left(\frac{1-\sqrt{5}}{2}\right)^n + \underset{-1}{\tilde{C}}$$

$$\text{Bdry Cnd: } T(0) = 0 = s + t + \underset{-1}{\tilde{C}} \rightarrow s = 1 - t$$

$$T(1) = 1 = s\left(\frac{1+\sqrt{5}}{2}\right) + t\left(\frac{1-\sqrt{5}}{2}\right) + \underset{-1}{\tilde{C}}$$

$$\rightarrow 2 = s\left(\frac{1+\sqrt{5}}{2}\right) + (1-s)\left(\frac{1-\sqrt{5}}{2}\right)$$

$$\rightarrow T(n) = \left(\frac{5+3\sqrt{5}}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n$$

$$+ \left(\frac{5-3\sqrt{5}}{10}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$$

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Subject:

Year:

Month:

Date:

2. Hom Sol: eq:  $\alpha^2 - 6\alpha + 9 = 0$

$$(\alpha - 3)^2 = 0$$

Roots:  $\alpha = 3, 3$

Geh Sol:  $T(n) = A \cdot 3^n + Bn \cdot 3^n$

$$\rightarrow T(n) = A \cdot 3^n + Bn \cdot 3^n$$

Boundary Chds:  $T(0) = 0 = A + B$

$$\rightarrow A = -B$$

$$T(1) = 1 = A \cdot 3^0 - B \cdot 1 \cdot 3^0$$

$$\rightarrow 1 = 3A - B \cdot A$$

$$\rightarrow A = \frac{1}{3}$$

Comp Sol:  $T(n) = 3^{n-1} - \frac{1}{3} \cdot 3^{n-1}$

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## Trimerge Sort

$$1. h-1 = 2$$

$$2. h-1$$

$$3. T_h = 3T\left(\frac{n}{3}\right) + h-1$$

$$4. P=1$$

$$5. \text{Yes. } C=1$$

$$6. T_h = \Theta\left(n\left(1 + \int_1^{\frac{n}{3}} \frac{h-1}{u^2} du\right)\right)$$

$$= \Theta\left(n + h \ln n + \frac{1}{h} - h\right)$$

$$= \Theta(n \ln(h))$$