Subject: Recitation 16

Year: Month: Date:

2 Triangles

Since $\binom{3}{2}=3$, there are 3 different Es for a pair (E,x). This pair is in fact the pair of a triangle and one of it's edges.

The number of triangles is t and 3 edges each by the def of triangle. So $|C|=3t$.

Also there are $\frac{n(n-1)}{2}$ edges in a graph by the handshaking lemma and λ ~~edges~~ triangles for each. So $|C|=\lambda\frac{n(n-1)}{2}$

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3 counting, counting, canting.

1 $\frac{6!}{3!2!1!}$

2 $|C_1|+|C_2|+|C_3| = 10+20+30 = 60 \to \frac{60!}{1!2!3!}$

3 (Suit Extra, Rank Extra) $= 13\cdot 4 \to \frac{13\cdot 4}{2} = 26$

4

This means either 1 or 2 suits are repeated.

So we have | 1 repeated suit | + | 2 repeated suits |.

for |1 R| we determine the suit and ranks of the repeated card. Then the rest is $\binom{\cancel{4\cdot 52}\ 50}{3}$.

~~Same~~ for |2 R|.
Similiar

$$(4\cdot 13\cdot 12 \cdot \tbinom{50}{3}) + (4\cdot 13\cdot 12\ \cancel{\tbinom{50}{3}} + 3\cdot 13\cdot 12)\binom{48}{1}$$

Subject: Year: Month: Date:

5 $\binom{15}{0} + \binom{15}{1} + \binom{15}{2} + \binom{15}{3}$

6 Since the maximum is 6.3, there are 3 points to be lost in total. Equivalent to a sequence of 0s & 1s of length 5 w/ 2 1s. $\binom{5}{2}$

7 We give ~~every~~ All 20 Pre-frosh in groups of 2. $\binom{13}{3}$

8 $\binom{100}{50\ 50} - \binom{20}{10\ 10}\binom{80}{40\ 40} - \binom{40}{20\ 20}\binom{60}{30\ 30} + \binom{20}{10\ 10}\binom{20}{10\ 10}\binom{60}{30\ 30} = |S_{0-50}| - |S_{0-50} \cap S_{10-50}| - |S_{0-50} \cap S_{10-50}| + |S_{10-50} \cap S_{20-50}|$

9 $\binom{180}{5!^{36}}$

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Subject: Year: Month: Date:

10 we first group the balls in $\binom{10}{1\ 2\ 3\ 4}$ ways. then just permute.

So $\binom{10}{1\ 2\ 3\ 4}4!$

11 Alphabet of three letters. $\binom{64+96+1}{64,96,1}$

4 There's more than one way...

Proof. (by Induction)

I.H. $P(n) ::= \sum_{i=0}^{n} \binom{k+i}{k} = \binom{k+n+1}{k+1}$

B.C. $P(0) ::= 1=1$ ✓

I.S. assume $P(n)$.

$$\sum_{i=0}^{n+1} \binom{k+i}{k} = \binom{k+n+1}{k+1} + \binom{k+n+1}{k}$$

$$= \binom{k+n+1+1}{k+1}$$ ✓ □

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