Problem Set 11

Problem 1

(a)

T	O
GGO	GGO
GGy	GGy
GBO	
BGy	

(b) $Pr(T|O) = \frac{2/8}{4/8} = 1/2$

(c) $O \not\perp G$

Problem 2

(a)

4/48 S

3/48

1/48 Rs

8/48 OG

6/48 S OG

2/48 Rs OG

12/48

9/48

3/48 S Rs

(b) $Pr\{S\} = 13/48$

(c) $\frac{10/48}{1 - 6/48} = 5/21$

(d) $\frac{6/48}{13/48} = 6/13$

O R

Problem 3

(a) $\frac{N!}{(N-m)!\,N^K}\,(h-m)^{K-m} = \frac{N!}{(N-m)!\,N^m}\left(1-\left(\frac{m}{K}\right)^{K-m}\right)$

$= e^{-\frac{m^2}{2N}-\frac{m}{h}(K-m)}$

(b) $-\frac{m^2}{2N}-\frac{mK-m^2}{N} = -\ln(2)$

$-m^2-2mK+2m^2 = -2N\ln(2)$

$m^2-2mK+2N\ln(2)=0$

$m = \frac{2K-\sqrt{4K^2-8N\ln 2}}{2}$

$= K-K\left(1-\frac{2N\ln 2}{2K^2}\right)^2$

$= \frac{N\ln 2}{K}$

Problem 4

(a) $\Pr\{w\} = \Pr\{TH\}\cdot\sum_{i=0}^{N}\Pr\{Tie\}^i =$

$\Pr\{HT\}\cdot$ // // $=$

$\Pr\{L\}$

(b) $(p^2+(1-p)^2)^N$

Problem 5

(a) $P(A|B) \overset{=}{\neq} 0 \neq P(A)$

$P(B|A) = 0 \neq P(B)$

(b) $P(A \cap \bar{B}) = Pr(A) - Pr(A)\,Pr(B)$

$= Pr(A)(1-P(B))$

(c)

$A ::=$ 1st flip is heads

$B ::=$ 2nd flip is heads

$C ::=$ odd heads

$$\frac{Pr(A \cap (B \cup C))}{Pr(B \cup C)} = \frac{Pr(HH, HT)}{Pr(HH, TH, HT)} = 2/3 \neq 1/2 = Pr(A)$$

(d) $Pr(A \cup B|C) = Pr(A|C) + Pr(B|C) - Pr(A \cap B|C)$

$= Pr(A) + Pr(B) - Pr(A \cap B)$

$= Pr(A \cup B)$

Problem 6

(a) mut Indep

(b) $\frac{1}{15,000,000}$. No.

(c) $\frac{1}{5000}$

(d) The researcher didn't do $Pr(C \mid A \cap B)$.
meaning B & C may not be indep given A.