2008 Final

Problem 1

(a) $Ex_{TF} = 36 \cdot \frac{1}{2} = 18$

(b) $Ex_{ind} = \frac{1}{3} \cdot \frac{3}{5} \cdot 15 + \frac{2}{3} \cdot \frac{15}{2} = 8$

$Ex_{Gia} = 49 - 16 = 33$

$Ex_{Test} = 18 + 8 + 33 = 59$

(b) $Var_{TF} = \frac{36}{4} = 9$

(c) $Var(ind \mid marten) = (0-8)^2 \cdot \frac{2}{5} + (15-8)^2 \cdot \frac{3}{5}$

$= Ex(ind^2 \mid marten) - Ex(ind \mid marten)^2$

$= \frac{3}{5} \cdot 15^2 \; - 9^2 = 54$

(d) The tests ≥ 0

(e) $Pr(R \geq 24) \leq Pr(R \geq 3 \cdot 8) \leq \frac{1}{8^2} \leq \frac{1}{64}$

Problem 2

(a) $Pr(L|C) = \dfrac{Pr(C|L)\,Pr(L)}{Pr(C)} = \dfrac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{2}{3}} = \frac{1}{4}$

(b) $\overset{7}{Ex(W)} = Ex(W|C)\,\overbrace{Pr(C)}^{2/3} + \overset{1}{Ex(W|\bar{C}\cap L)}\,\overbrace{Pr(\bar{C}\cap L)}^{1/6}$
$\underbrace{+Ex(W|\bar{C}\cap\bar{L})\,Pr(W|\bar{C}\cap\bar{L})}_{0}$

$\rightarrow Ex(W|C) = \dfrac{7-\frac{1}{6}}{2/3} = \frac{41}{4}$

Problem 3

(a)
(Rank of ¬A, Suit of ¬A)

$\rightarrow \cancel{\dfrac{12\cdot 4\cdot 48}{52\cdot 51\cdot 50\cdot 49}}$ $\qquad \dfrac{48}{\binom{52}{5}}$

(b) (Rank of ¬included, Suit of ¬included,
Rank of 4)

$1-\dfrac{\overset{3}{\cancel{13}}\cdot 4\cdot 12}{\binom{52}{5}}$

(c) $1\Big/\dfrac{1-624}{\binom{52}{5}}$

(d) $\dfrac{4\cdot 13\cdot 12\cdot 11\cdot 40\cdot 39\overset{3}{!}\cdot 4!}{52!}$

Problem 4

(a) $\frac{6}{9} \cdot 3 = 2 = Ex(T)$

$Var(T) = 3 \cdot \frac{2}{3}(1 - \frac{2}{3}) = \frac{2}{3}$

(b) $\frac{2}{3}$. Linearity of Ex does not depend on indep.

Problem 5

(a) $\dfrac{n^2}{\binom{n}{2} - 2n}$

(b) $\left(1 - \dfrac{n}{\binom{2^n}{} - 2n}\right)^K$

Problem 6

$Pr\{T \geq 30e\} \leq e^{-(e-e+1)\cdot 30} = e^{-30}$

Problem 7

(a) $S_n = S_{n-1} + H_{n-1} + C_{n-1}$

$H_n = S_{n-1} + C_{n-1}$

$C_n = H_{n-1}$

(b) proof (by ind)

I.H. $P(n)$: $T_n = 2^n$ ✓

B.C $T_0 = 1 = 2^0$ ✓

I.S. $T_{n+1} = 2S_n + 2H_n + 2C_n$

$= 2T_n$

$= 2^{n+1}$ ✓

$\square$

(c)

$T_{n-1} = H_{n-1} + S_{n-1} + C_{n-1}$

$T_{n-1} - H_{n-1} = S_{n-1} + C_{n-1} = H_n$

(d) $H_n = 2^{n-1} - H_{n-1}$

2008 Final (cntd..)

Problem 7 (cntd..)

(d) $H_n = 2^{n-1} - H_{n-1}$

Hom Sol: $\alpha + 1 = 0 \longrightarrow \alpha = -1$

Part Gen Sol: $a2^n + a2^{n-1} = \frac{1}{2}(2^n)$

$$a2^n + \frac{a}{2}2^n = \frac{1}{2}2^n$$

$$a + \frac{a}{2} = \frac{1}{2}$$

$$a = \frac{1}{3}$$

Gen Sol: $H_n = A(-1)^n + \frac{1}{3}2^n$

Bdry cnds: $H_0 = 0 \rightarrow A = -\frac{1}{3} \rightarrow$

$$\rightarrow H_n = (-\tfrac{1}{3})(-1)^n + \tfrac{1}{3}2^n$$

Problem 8

(a) No (b) Yes (c) ~~Yes~~ No (d) Yes

(e) ~~No~~ Yes (f) Yes

Problem 9

Since the graph is finite, by WOP, consider the longest path. Consider the last node.

v_n if $v_n \to v_{m>n}$ then the path is longer.

$\to v_n \to v_{i<n}$ therefor $v_i \cdots v_n \cdots v_i$

Problem 18

Proof (by invariance)

I.H. P(n) ::= # of inversions is odd.

B.C. 1 ✓

I.S. if we have x inversions we have x or

$6-x$. ✓ □