Subject: Problem Set 5 Year: Month: Date:

Problem 1

(a) each component is connected and acyclic.

the node adj to the deleted node had either been a leaf or an internal node.

(b) Proof by strong Induction.

I.H. $P(L)$ :: a SB tree with L leaves has $2L-1$ nodes.

B.C. $P(1)$ ✓

I.S. Consider SBT with $L+1$ leaves.

delete the root. by (a) we have 2 SBTs with leaves $L_1, L_2 < L+1$. by I.H. we have $2L_1 - 1 + 2L_2 - 1 + 1$ nodes which is $N_1 + N_2 + 2$ nodes.

Problem 2

(a) lemma: we can 2-color a 2D grid like a chess Board. So it's bipartite.

Corollary: $N \times M$ when $N \wedge M$ are odd is an odd cycle, so it's not bipartite.

(b) Proof. (by Induction)

I. H. $P(n) ::=$ n is even $\longrightarrow \exists$ ham cycle

B. C. $P(2)$, outer edges. ✓

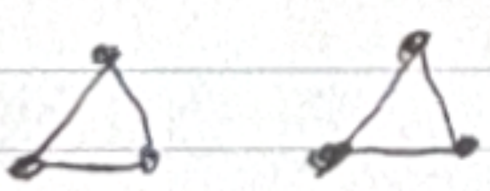
I. S. Consider the $((n-1,0),(n-1,1))$ edge. remove it. add $((n-1,0),(n\cancel{-1},0))$, $((n,0),(n+1,0))$, $((n+1,i),(n+1,i+1))$ for $0 \le i < m$, $((n,i),(n,i+1))$ for $1 \le i < m$ and $((n,m-1),(n+1,m-1))$.

2. Induction is based on even.

3. yes. No.

Problem 3.

(a) "by induction, each of the two pieces are connected"

(b) 

(c) Proof. (by ※)

assume mangled but not connected.

split into 2 sets. only 1 ~~has~~ could have more than $\lceil \frac{n}{2} \rceil$ P_g nodes $\longrightarrow$ ※

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Problem 4

(a) Proof (by Induction)

I.H. $P(n) ::=$ n-node tree has $n-1$ edges.

B.C. $P(1)$ ✓

I.S. assume $n+1$-node tree. remove a leaf. $P(n)$.

add back. $n-1+1$ edges. ✓

Proof (by ※)

assume $n-1$ node connected graph but not tree.

$\rightarrow \exists$ cycle. remove edges until no cycle.

then we have tree with $<n-1$ edges

※

(b) Proof (by Induction)

I.H. $P(e) ::\ e \geq n+1 \wedge$ Connected $\rightarrow \exists$ SPT

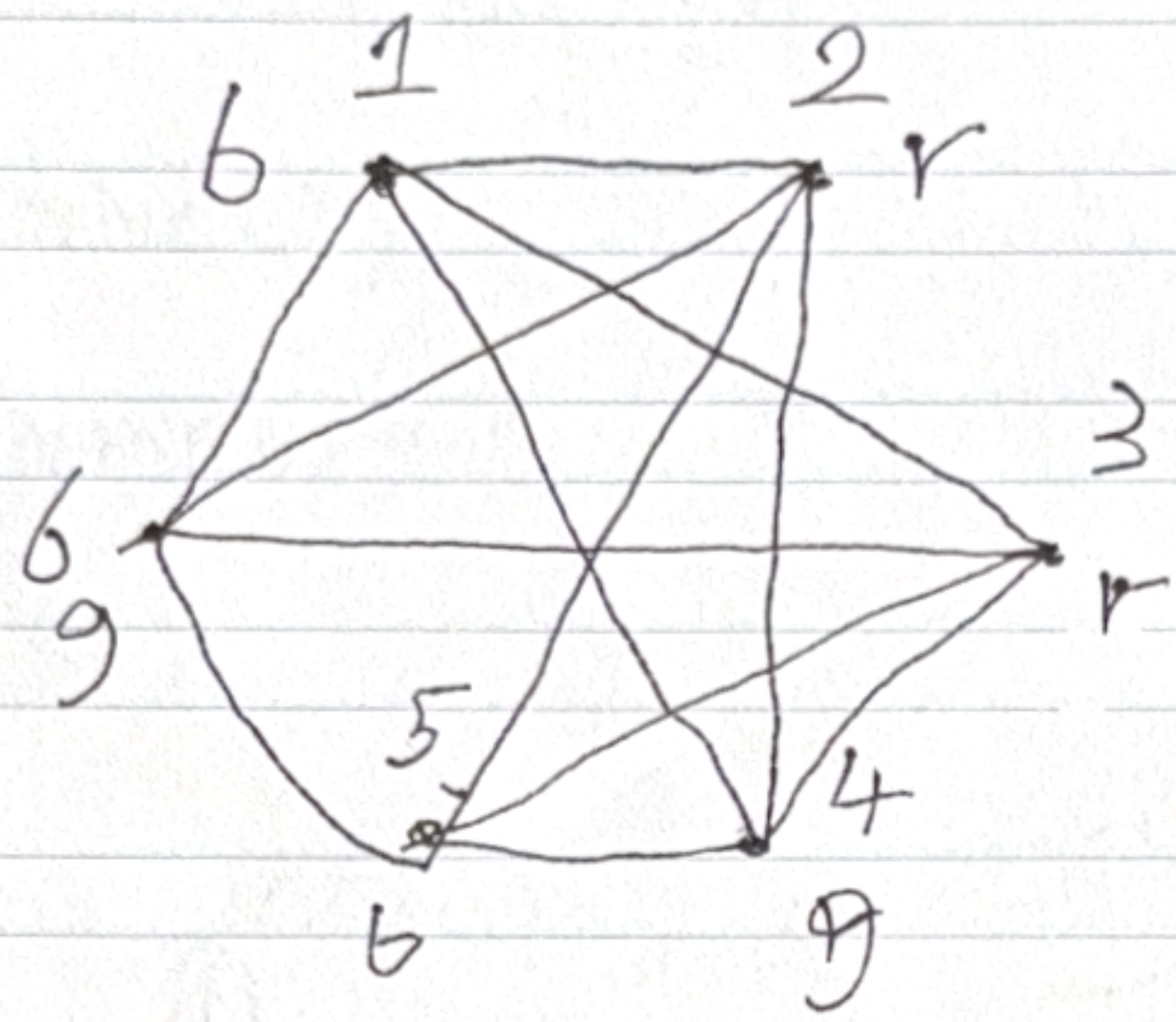
B.C. $e = n+1$ ✓

I.S. $e+1 \geq n+1$. then we know if we remove

an edge, by $P(e)$ $\exists$ SPT.

Problem 5

(a)



(b) 2. every 2 nodes that are not connected directly are connecte by 2 edges.

(c) 1-6-3-4-5-2-1. uses every node.

(d) can't be <3 because of K_3

Problem 6

(a) v needs even degree to be the first and last.

(b) W has 1 node that goes to it last +2k for every other time.