

## Problem 1

 $P \equiv q$  $A \quad B \quad C$  $A \vee B$  $A \vee B \rightarrow C$  $T \quad T \quad T$  $T \quad T$  $T \quad F \quad T$  $T \quad T$  $F \quad T \quad T$  $T \quad T$  $F \quad F \quad T$  $F \quad F$  $T \quad T \quad F$  $T \quad T$  $T \quad F \quad F$  $T \quad T$  $F \quad T \quad F$  $T \quad T$  $F \quad F \quad F$  $F \quad T$

## Problem 2 (Chd..)

A	B	C	$\neg C \Rightarrow \neg A$	$\neg C \Rightarrow \neg B$	?
T	T	T	T	T	T
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F
T	T	F	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

Tables Show  $P \equiv ?$  ✓

$$\frac{97}{120}$$

## Problem 2

Proof. by Strong Ind.

よ. H.  $P(n) := G_n \setminus 3^h$

B.C. P(0) 153<sup>0</sup> ✓

J.S. Gains 3<sup>n-1</sup>

$$3G_{n-2} \leq 3+3^{n-2} + 3^{n-1}$$

$$3G_{n-3} \leq 3 \cdot 3^{n-3} \leq 3$$

$$G_{n+1} = G_{n-1} + 3G_{n-2} + 3G_{n-3} \leq 3^{\frac{n-2}{2}} \leq 3^{\frac{n-1}{2}} \leq 2\sqrt{3}^{\frac{n}{2}}$$

invoke 569 □

int.

97 81%  
120

## Problem 3

(a) Proof, by Ind



T.H. P(h): # of shapes decreases by 1 on each h.

B.C. ✓

T.S. either case ✓

We need  $n-1$  moves to turn  $n$  shapes to 1. ✓~~-F~~(b) \* circles odd  $\leftrightarrow$  when 1 circleRemains its pair with a square to make  
circle and by Ind so on...  $\leftrightarrow$  winner

## Problem 4

$$(a) 4 \times 113 - 41 \times 11 = 1 \rightarrow |13 - 4| = 72 \checkmark$$

$$\begin{aligned} (b) & \quad 112 \parallel \quad 112 \overset{100}{\cancel{\times}} \cancel{12} \cancel{+} \cancel{32} \parallel \quad \begin{matrix} 100 \\ 112 \parallel \\ \equiv 11 \cdot 11 \end{matrix} \quad \begin{matrix} 100 \\ 112 \parallel \\ \equiv 11 \cdot 11 \end{matrix} \quad \begin{matrix} 100 \\ 112 \parallel \\ \equiv 11 \cdot 11 \end{matrix} \\ & \quad \equiv 11^2 \quad B - 4 \end{aligned}$$

$$11 \overset{11^2}{\equiv} 1$$

$$11 \overset{11^3}{\equiv} 8$$

$$11^4 \overset{11^3}{\equiv} 8^2 \equiv 64$$

$$11^8 \overset{11^3}{\equiv} 64^2 \overset{11^3}{\equiv} 856 \equiv 65$$

$$11^{11} \overset{11^3}{\equiv} 65 \cdot 8 \cdot 11 \equiv 5 \cdot 6 \cdot 11 \equiv 88 \checkmark$$

Subject:

Year:

Month:

Date:

Problem 5

-#2

(a) ~~52 43 31 20 19~~

DCBAFE 5

(b) DBAFEGCD ✓

(c) A B R

B G

C B ✓

D R

E R

F G

(d) No. C has odd degree. ✓

(e) DC ✓

C B

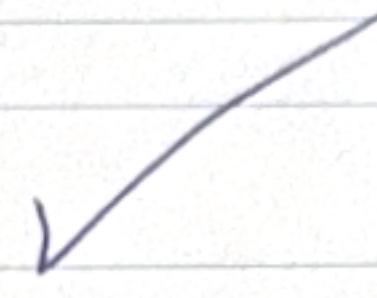
A B

E F

E C

## Problem 6

Proof by Induction.

J.o.H.  $P(m) := m\text{-edge } G \text{ has } C_m \geq m - h + k$ B.C.  $P(0) := 0 \geq 0 - l + 1 \checkmark$ I.S.  $P(m) \checkmark \rightarrow C_{m+1} \geq (m+1) - h + k$ Case 1: We remove edge  $e$  from cycle~~the removes~~  $\rightarrow C_{m+1} \geq m - h + k$  $\rightarrow \text{add back} \rightarrow C_{m+1} \geq m + 1 - h + k \checkmark$ Case 2: Cycles are untouched.  $\rightarrow C_{m+1} = C_m$  $\geq m - h + k \checkmark$ ~~- 545~~Invoke Ind.  $\square$

Problem 7

$$f(\infty) + \int_{i=1}^{\infty} f(i) di \leq \sum_{i=1}^{\infty} f(i) \leq \int_{i=1}^{\infty} f(i) di + f(1)$$

✓

Problem 8

$$(a) \lim_{n \rightarrow \infty} \frac{n^{\ln n}}{n^n} = \infty \rightarrow \times$$

✓

$$(b) \lim_{n \rightarrow \infty} \frac{n^{1/\ln n}}{n^n} = \frac{1}{1^n} \rightarrow \times$$

✓

$$(c) \lim_{n \rightarrow \infty} \frac{n^{n+1}}{n^n} = \infty \rightarrow \checkmark$$

✓

$$(d) \lim_{n \rightarrow \infty} \frac{n^{n+\frac{1}{2}}}{n^n} = \infty \rightarrow \times \quad \times - \cancel{3}$$