Subject: Midterm Practice Problems Year: Month: Date:

Problem 1

P nor q is equivalent to $\neg P \land \neg q$

So we only need to negate the variables

using P nor $P \equiv \neg P$ so we have $(P \text{ nor } P) \text{ nor } (q \text{ nor } q)$ $\square$

Problem 2

Proof (by Strong Induction)

I.H. $P(n) ::= a_n \equiv_3 1$

Base case $P(1) ::= 1 \equiv_3 1$

I.S. assume $P(a)$ for $a \leq n$ to prove $P(n+1)$.

$$a_{n+1} \equiv_3 a_n + a_{n-1} + a_{n-2} + a_{n-3}$$

$$\equiv_3 4$$

$$\equiv 1$$

invoke induction $\square$

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Problem 3

Proof. by Induction

I. H. $P(n) ::=$ after n moves the number of ~~[illegible]~~ inversions is odd.

B.C. $P(0)$ ✓

I.S. $x \to 6 - x$ remains odd. ✓

invoke induction □

Proof. by ※

0 is even. ※ □ ✓

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Problem 4

$-2\cdot 72+17\cdot 9=1$ X

Problem 5

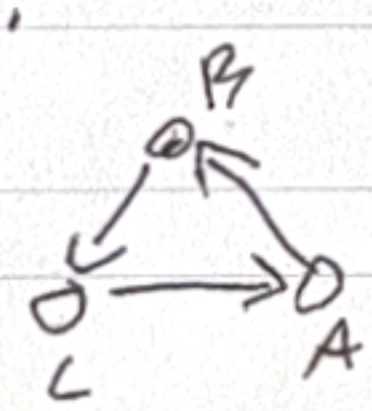
(a) no. odd cycle. ✓

(b) no. odd degree. ✓

(c) 1. bottom right to bottom left. X
-2

(d) 5. all the left + center going to right.
+4 X

Problem 6.

1. no.  A->B ∧ B->C but ¬ A->C ✓

2. no. A->B ⟶ ¬B->A ✓

3. yes. ¬symmetric ∴ the reflexivity doesnt occur so it doesnt matter. ✓

4. no. no cycles. ✓

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Problem 7

Proof by Induction.

I.H. $P(n) ::=$ n-node outer planar graph is 3 colorable.

B.C. $P(1)$ ✓

I.S. ~~n~~ $n+1$-node O.P. G has a node v degree ≤ 2. remove to obtain n-node G'. $P(n)$. add v. color w/ 2. ✓

□ ✓

Problem 8.

$$f(n) + \int_1^n f(i)\,di \leq \sum_{i=1}^{n} f(i) \leq f(1) + \int_1^n f(i)\,di$$

$$\frac{1}{n^3} + \left(-\frac{1}{2n^2} + \frac{1}{2}\right) \leq \sum_{i=1}^{n} f(i) \leq 1 + \left(-\frac{1}{2n^2} + \frac{1}{2}\right)$$

$$1 - \frac{1}{n^3} \leq 1$$ ✓ ✓

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Problem 9

(a) O, Ω, θ ✓

(b) Ω, ω ✓

(c) $\sim, \theta, O, \Omega$ X O, o

(d) $\sim, \theta, O, \Omega$ ✓

(e) θ, O, Ω ✓