Problem Set 1.

Problem 1

(a) $\forall w \in B \quad Pr\{w\} \geq 0 \longrightarrow \forall w \in B \quad \frac{Pr\{w\}}{Pr\{B\}} \geq 0$

$\sum_{w \in B} Pr\{w\} = Pr\{B\} \longrightarrow \sum_{w \in B} P_B(w) = 1$

(b)

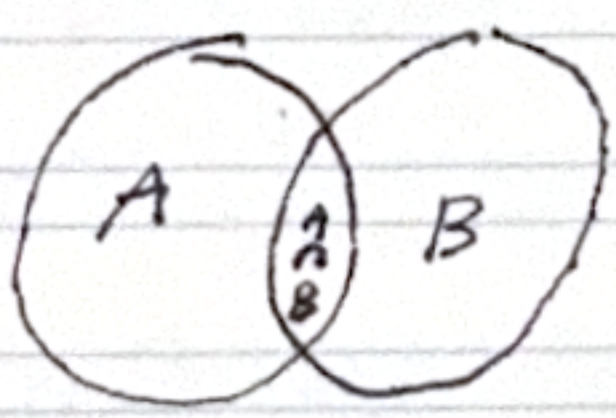
assume A is any Event, $A \subseteq S$.

$$Pr_B\{A\} = \frac{Pr\{A \cap B\}}{Pr\{B\}} = \sum_{w \in A} Pr_B\{w\}$$

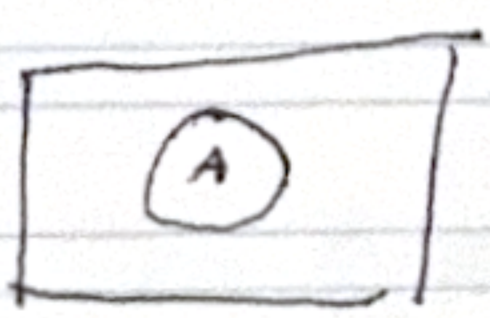
$$= \sum_{w \in A \cap B} \frac{Pr\{w\}}{Pr\{B\}} \checkmark$$

Problem 2

(a)



$Pr\{A\} = Pr\{A - B\} + Pr\{A \cap B\}$



$Pr\{\bar{A}\} = Pr\{S\} - Pr\{A\} = 1 - Pr\{A\}$

$Pr\{A \cup B\} = Pr\{A\} + Pr\{B - A\}$

$= Pr\{A\} + Pr\{B\} - Pr\{A \cap B\}$

$$Pr\{A \cup B\} = Pr\{A\} + Pr\{B\} - Pr\{A \cap B\}$$
$$\leq Pr\{A\} + Pr\{B\}$$

$$Pr\{A\} = Pr\{B\} - Pr\{B - A\}$$
$$= Pr\{B\} - Pr\{B\} + Pr\{A \cap B\}$$
$$= Pr\{A \cap B\} \leq Pr\{B\}$$

(b) Proof. (by Induction)

I.H. $P(n) ::= Pr\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} Pr(A_i)$

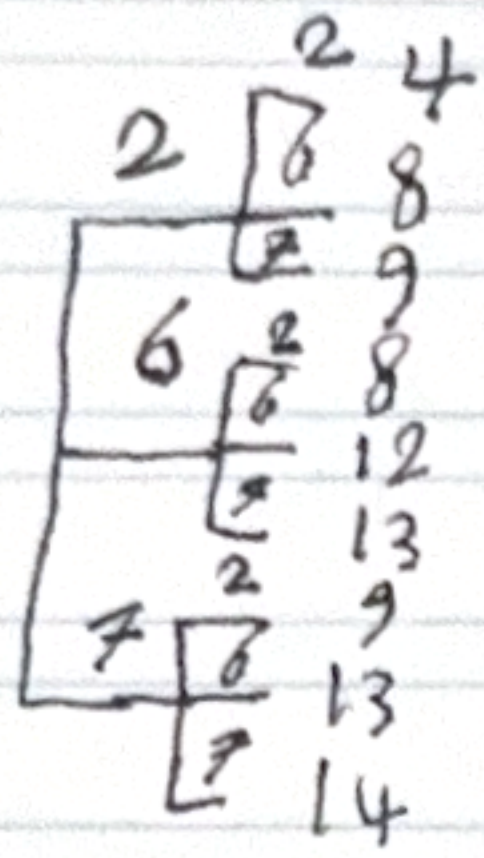
B.C. $P(1) ::= Pr(A_1) = Pr(A_1)$ ✓

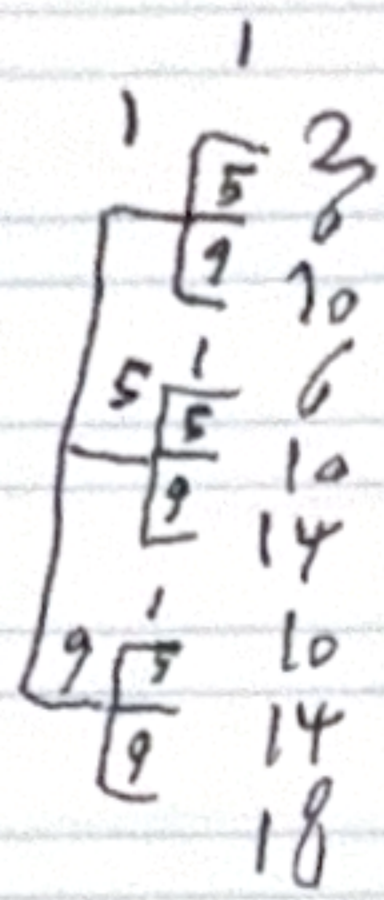
I.S. assume $P(n)$.

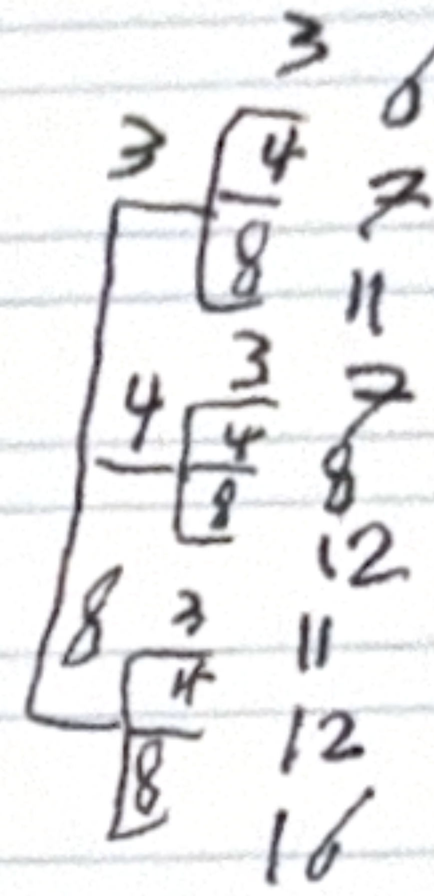
$$Pr(A_1 \cup \dots A_{n+1}) = Pr(A_1 \cup \dots A_n)$$
$$+ Pr(A_{n+1})$$
$$- Pr(A \cup \dots A_n \cap A_{n+1})$$
$$\leq \sum_{i=1}^{n} Pr(A_i)$$ ✓

□

Problem 3

A 

B 

C 

$Pr(C > A) = \frac{1}{81}(1+1+5+1+5+5+5+5+9) = \frac{37}{81}$

$Pr(C = A) = \frac{1}{81}(1+1+2) = \frac{4}{81}$

$\longrightarrow$ $A > C$ is more likely.

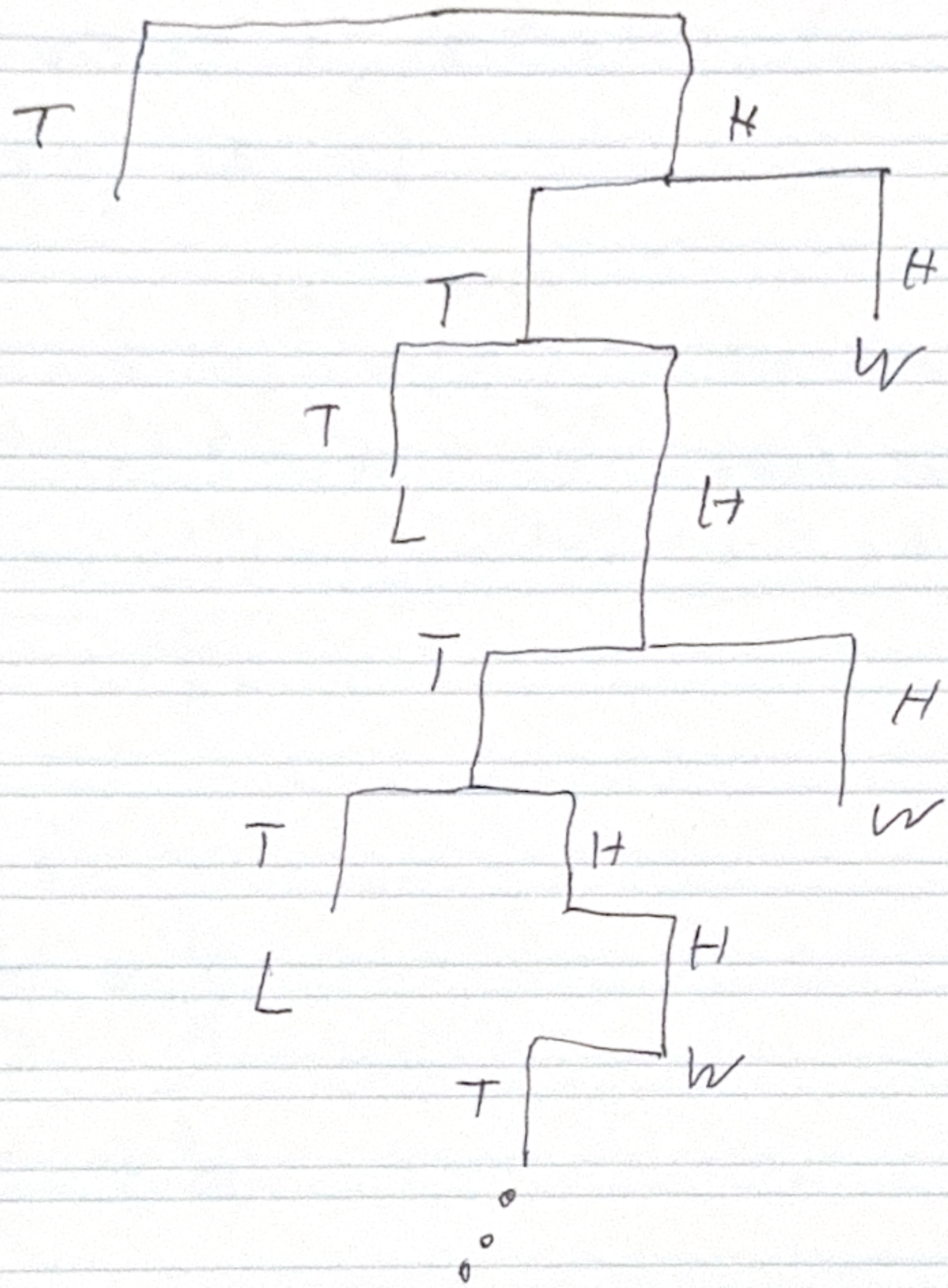
$Pr(C > B) = \frac{1}{81}(1+3+6+6+6+6+6+6+8) = \frac{48}{81}$

$\longrightarrow$ $C > B$ is more likely

$Pr(B > A) = \frac{1}{81}(0+1+5+1+5+8+5+8+9) = \frac{41}{81}$

$\longrightarrow$ $B > A$ is more likely.

Problem 4



$$Pr(W)_{\text{single Branch}} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^{\infty}} = \frac{1}{4}\,\frac{1}{1-\frac{1}{4}} = \frac{1}{3}$$

$$Pr(w)_{\text{overAll}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{3\,2^{\infty}} = \frac{1}{3}\,\frac{1}{1-\frac{1}{2}} = \frac{2}{3}$$

Suit of 5

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Problem 5

Suit of 4 Suit of 1

/ /

(a) ((Suit of 3, Suit of 2, (R_{31}, R_{32}, R_{33})

$, (R_{21}, R_{22}))$

$$4 \times 3 \times \binom{13}{3} \times \binom{13}{2}$$ uniform space

$$+ \ 4 \times \binom{13}{4} + 3\binom{13}{1} + 4 \cdot \binom{13}{5}$$ each hand $1/\binom{52}{5}$

(b) $$\frac{4 \times 3 \times \binom{13}{3} \times \binom{13}{2} + 4 \cdot \binom{13}{4} + 3 \cdot 13 + 4\binom{13}{5}}{\binom{52}{5}}$$

Problem 6

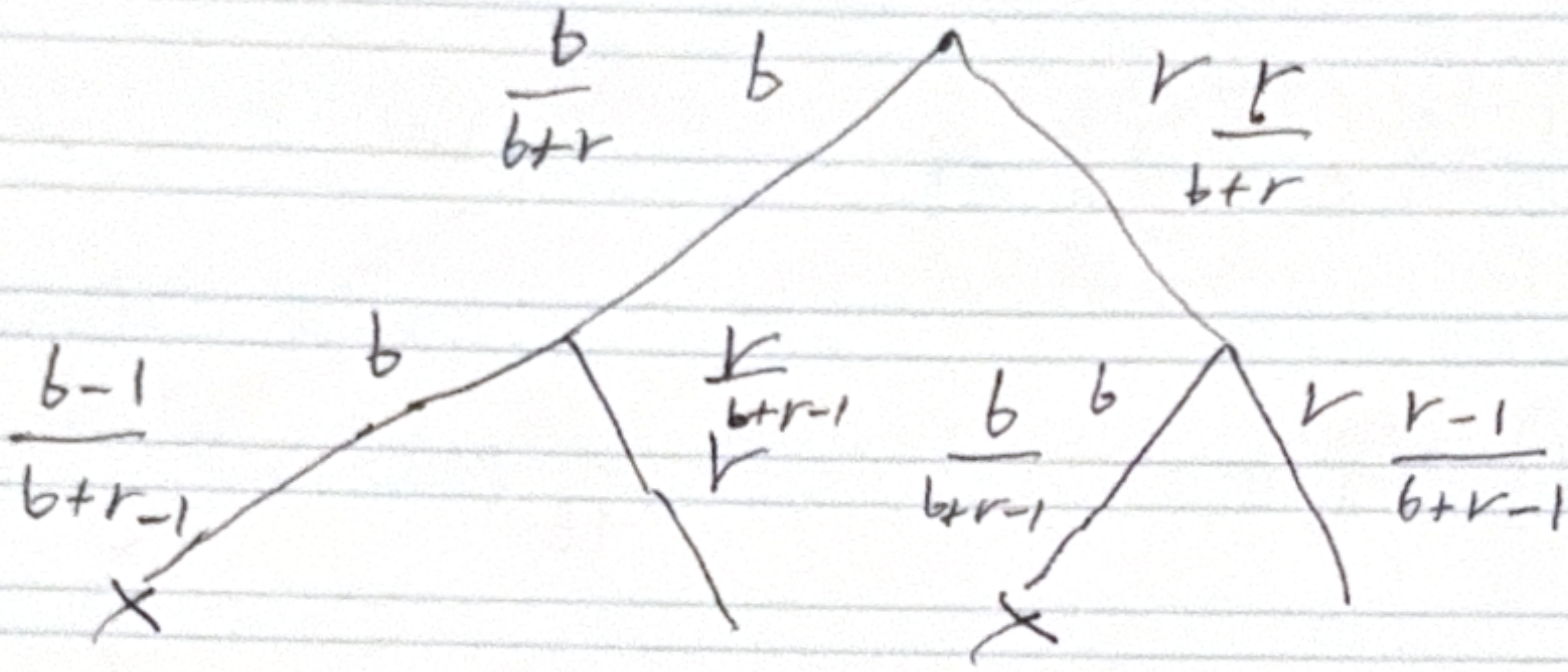
(a) $\frac{26}{52} = 1/2$

(b) $\frac{26}{51} > 1/2$

(c) b winning cards left

r+b total cards

(d) $\neg\exists$. ~~If~~ ~~we~~ w/out seeing the first card. this one and the next have the same chances.



$$\left(\frac{b}{b+r}\right)\left(\frac{b-1}{b+r-1}\right)+\left(\frac{r}{b+r}\right)\left(\frac{b}{b+r-1}\right)=b/_{b+r}$$

$$= S_{b,r}$$