

## Recitation 22

### 1 Properties of Variance

$$\begin{aligned}(a) \text{Var}_R &= E_x((R - E_x(R))^2) \\&= E_x(R^2 - 2RE_x(R) + E_x(R)^2) \\&= E_x(R^2) + E_x^2(R) - 2E_x(R)E_x(R) \\&= E_x(R^2) + E_x^2(R) - 2E_x^2(R) \\&= E_x(R^2) - E_x^2(R)\end{aligned}$$

$$\begin{aligned}(b) \text{Var}[aR+b] &= E_x((aR+b)^2) - E_x^2(aR+b) \\&= E_x(a^2R^2) + 2E_x(abR) + E_x(b^2) - E_x^2(aR+b) \\&= a^2E_x(R^2) + 2abE_x(R) + b^2 - (E_x(R+b))^2 \\&= a^2E_x(R^2) + 2abE_x(R) + b^2 - a^2E_x^2(R) - 2abE_x(R) - b^2 \\&= a^2(E_x(R^2) - E_x^2(R)) = a^2 \text{Var}(R)\end{aligned}$$

$$\begin{aligned}(c) E_x((R_1+R_2)^2) - E_x^2(R_1+R_2) \\&= E_x(R_1^2) + E_x(R_1)E_x(R_2) + E_x(R_2^2) - (\underbrace{E_x(R_1) + E_x(R_2)}_{E_x(R)})^2 \\&\quad E_x^2(R_1) + 2E_x(R_1)E_x(R_2) + E_x^2(R_2) \\&= E_x(R_1^2) - E_x^2(R_1) + E_x(R_2^2) - E_x^2(R_2)\end{aligned}$$

$$(d) R_1 = R_2 \rightarrow \text{Var}(2R_1) = 4\text{Var}(R_1) \neq 2\text{Var}(R_1)$$

$$(e) E_x(\sum_{i=1}^n I_i) = \sum_{i=1}^n E_x(I_i) = nP(1-P) = Var$$

$$\sqrt{n P(1-P)} = SD$$

$$(f) Var(T) = E_x(T^2) - E_x^2(T)$$

$$E_x(T_j^2) = \sum_{\substack{n \in [0, 1] \\ R(T_j)}} n^2 \Pr(n) \leq \sum_{n \in R(T_j)} n \Pr(n) = E_x(T_j)$$

$$\rightarrow E_x(T^2) - E_x^2(T) \leq E_x(T) - E_x(T_j)^2 \leq E_x(T)$$