200~~6~~4 Final

Problem 1

Proof (by ind)

I.H. $P(n) :: \gcd(F_n, F_{n+1}) = 1 \quad \forall n \geq 0$

B.C. $\gcd(0, 1) = 1 \quad P(0)$ ✓

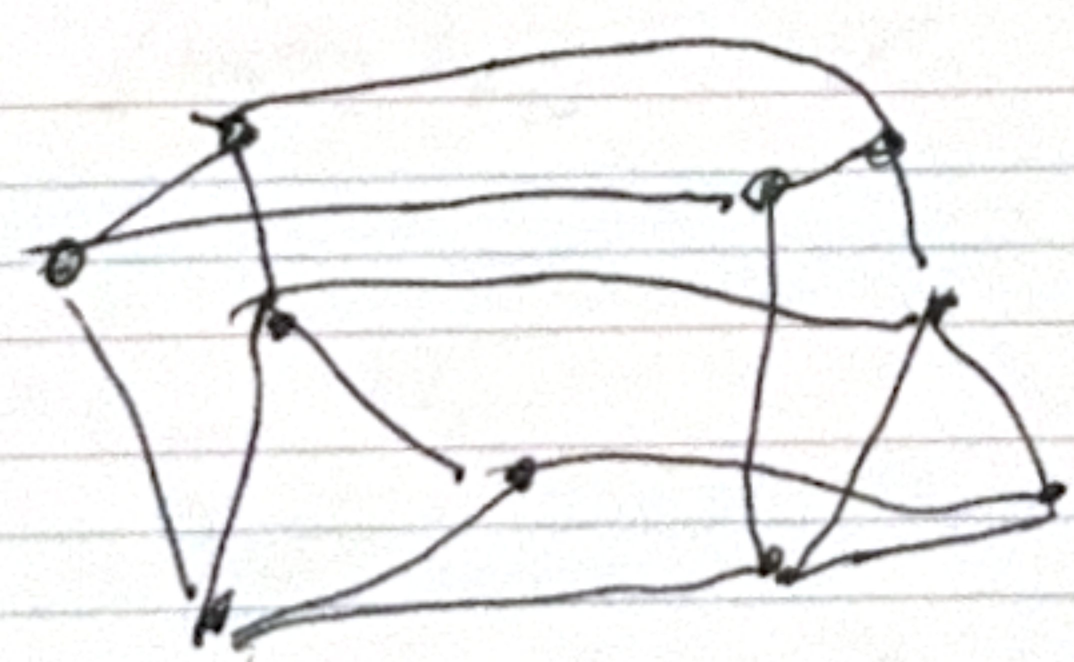
I.S. $\gcd(F_n, F_{n+1}) = \gcd(F_n, \gcd(F_n, F_{n-1}))$

$= \gcd(F_n, 1) = 1$ ✓

□

Problem 2

(a)



(b) Proof (by ind)

I.H. P(n):: G_n is bipartite $\forall n \geq 1$ ✓

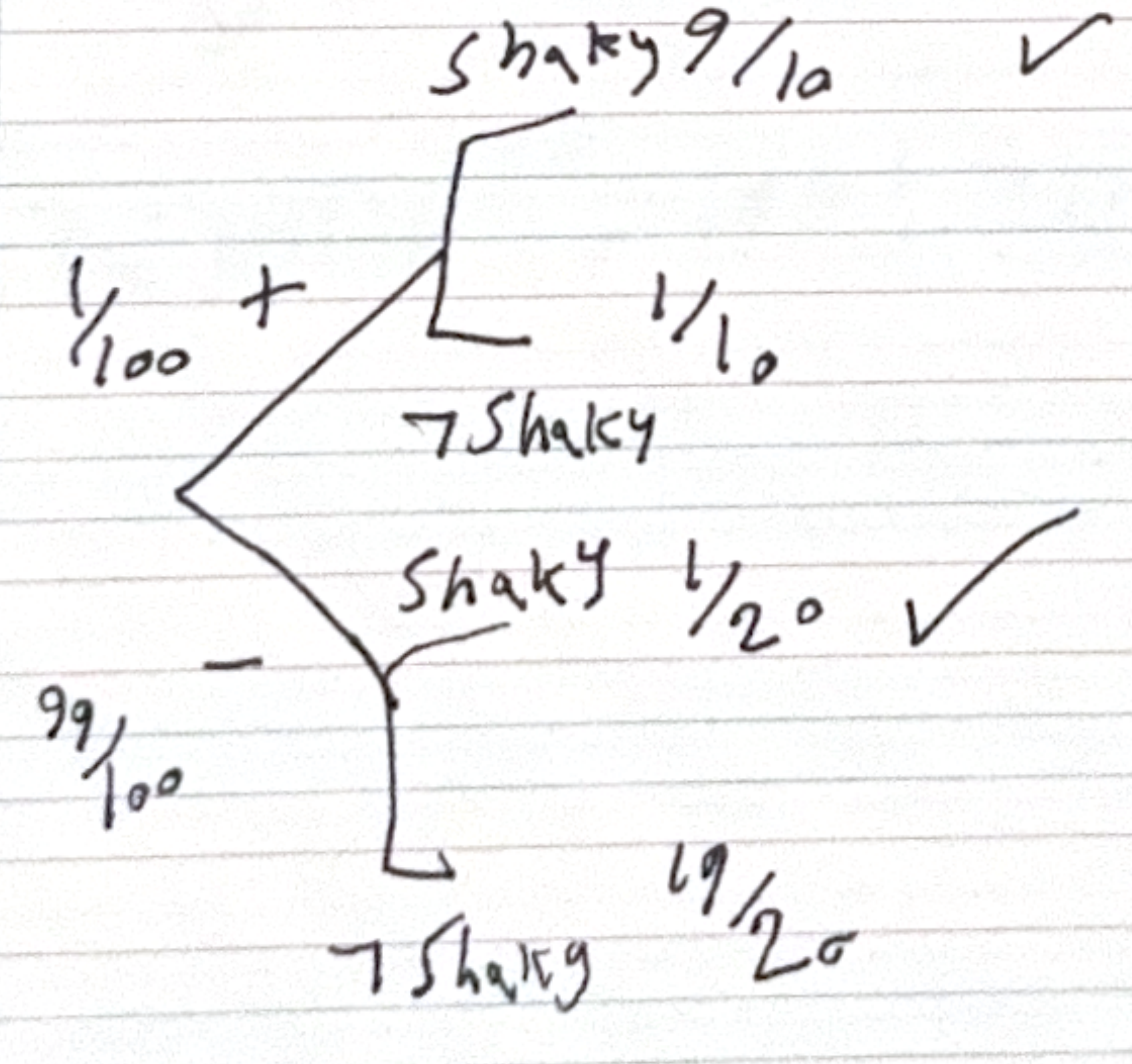
B.C. G_1 ✓

I.S. WLOG G_n only has edges $\{L\} - \{R\}$. G_{n+1} has $\underbrace{\{L\}}_{G_n} - \underbrace{\{R\}}_{G_n}$

and $\underbrace{\{L\}}_{G_n} - \underbrace{\{L\}}_{G_n'}$ so we partition. ✓

□

Problem 3



$$\frac{9/1000}{\frac{9}{1000} + \frac{99}{2000}}$$

(a) $$\frac{6\cdot 5\cdot 4\cdot 3\cdot 2}{6^5}$$

(b) $$\frac{\binom{5}{2}\cdot 6\cdot\binom{5}{3}}{6^5}$$

(c) $$\frac{\binom{5}{2}\cdot\binom{6}{2}\cdot\binom{3}{2}\cdot 5\cdot 4}{6^5}$$

problem 5

(a) $\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4}$

(b) $5\cdot 4^{n-1}$

(c) $|A| \geq 4^{32} > 1\cancel{9}6! > |B|$ Pigeonhole.

Problem 6

(a) $Ex(B) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \Rightarrow 4$

(b) $\frac{1}{\frac{1}{6}} - 1 = 5$

(c) $3.5 \cdot 3.5 = 12.25$

(d) $\frac{1}{12} \cdot \frac{4}{\cancel{}} + \frac{1}{3} \cdot 5 + \frac{1}{6} \cdot 12.25$

Problem 7

$\frac{6}{6} = 1$

Problem 8

(a) $\frac{300}{3} + \frac{200}{4} + \frac{400}{4} + \frac{250}{5} + \frac{2 \cdot 500}{5} = 500$

(b) $Pr(X \geq 1500) \leq \frac{500}{1500} = \frac{1}{3}$

(c) $Pr(X \leq (1-\delta\, Ex(X))) \leq e^{-\delta^2 Ex(X)/2}$

$= e^{-10}$