Subject: Problem Set 4

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Problem 1

Proof (by Induction)

I.H. $P(n) :: G' = (V, M_1 \cup M_2)$, n-node G' is bipartite.

B.C. $P(1)$ ✓

I.S. assume $(n+1)$ node graph with edges $= M_1 \cup M_2$, remove $(n+1)$st node and it's edges. apply $P(n)$. add $(n+1)$st.

Case 1. no edges, add to L or R.

Case 2. 1 edge, add to opposite.

Case 3. 2 edges,

Case 3.1 both edges go to one set, ✓

Case 3.2 ~~to~~ edges go to vertices on different sides with diff components, divide each to L and R and put

$L' = L_1 \cup R_2$ $R' = L_2 \cup R_1$. now same side, add to other.

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Case 3.3 different sides, same component.

※. odd cycle → both edges from one M.

Problem 2.

(a) each edge contributes to 2 degrees.

(b) $\frac{2\nmid}{111 \cdot 17}$ ※

(c) $\frac{n \cdot (n-1)}{2}$

Problem 3

(a) 1, 3, 4

(b) $\overset{\text{degree }4}{a \neq b}$, $a = c$, $a \neq d$, $b \neq d$, one to one → degree 4 (cycle ≠ degree 4)

Problem 4

(a)

(b) "Removing V reduces ..."

Problem 5

lemma, ∀girl rated worst by at ~~least~~ most $(n-\cancel{1})^2$ boys

Proof by ※, if ∀girl rated worst by at least

$(n-1)$ boys → ∃ $n(n-1)$ boys or more ※

now if ∃ girl with worst boy ∧ ∃ boy′ with worst girl → rogue.

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Problem ۵

(a) Color line (a) with $\{a,b\}$ and line (b) with $\{c,d\}$ now color all with $\{a,b\} \times \{c,d\}$

(b) all even numbers are adj to odd and vice versa.

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