Problems For Recitation 23

1 Getting Dressed

(a) $Ex(X) = 0.2 + 0.1 + 0.1 + 0.3$

$= 0.7$

(b) $Pr_x(x \geq 1) \leq \frac{Ex(x)}{1} = 0.7$

(c) $Pr_x(x \geq 3) \leq \frac{Ex(x)}{3} = 0.23$

(d) $Pr_x(x \geq 2) = Ex(|x - Ex(x)| \geq 1.3)$

$Var(x) = 0.2(1-0.7)^2 + 0.1(2-0.7)^2 + 0.1(3-0.7)^2 + 0.3(0-0.7)^2$

$= 0.018 + 0.169 + 0.529 + 0.147$

$= 0.863$

$\sigma(x) = \sqrt{0.863} \approx 0.9374$

$Pr_x(|x - Ex(x)| \geq 1.7\sigma(x)) \leq \frac{1}{1.7^2} \approx 0.3460$

(e) $Pr_x(x \geq 1) \geq 1 - e^{-Ex(x)} \approx 0.5$

(g) $\Pr\{x \geq 4/3\, E_x(x)\} \leq e^{-(4/3 \ln(4/3) - 4/3 + 1)\, E_x(x)} \approx 0.14$

(h) $\Pr\{x \geq 3\, E_x(x)\} \leq e^{-(3\ln(3) - 3 + 1)\, 36} \approx 5.49 \times 10^{-21}$

2 A Financial Crisis

1. $Pr(X \geq 2.1 Ex(x)) \leq e^{-(2.1 \ln(2.1) - 1.1)5} \approx e^{-8.02} \approx 0.$

None!!

2. $Pr(x \geq 20 Ex(x)) \leq \frac{1}{20} = 5\%$. as good as any conditional.

3. ~~$$~~ 50%

4. $100 Ex(X) = 5000$

5. $Pr(X \geq 9/5\, 5000) \leq e^{-(1.8 \ln(1.8) - 1.8 + 1)5000} \approx 0$

   $Pr(X \geq 7/5\, 5000) \leq e^{-(1.4 \ln(1.4) - 1.4 + 1)5000} \approx 0$

6. $Pr(x \geq 9/5\, 5000) \leq \frac{1}{9/5} = 5/9 = 55\%$

   $Pr(x \geq 7/5\, 5000) \leq 1/7/5 = 5/7 = 71\%$