2010 Final

Problem 1

(a) Proof (by Induction)

I.H. $P(n) :: T_n \overset{2}{\equiv} 1 \quad \forall n \geq 0$ ✓

B.C. $P(0)$ ✓

I.S. $T_{n-1} \overset{2}{\equiv} 1$

$T_{n} \overset{2}{\equiv} 1$

$T_{n} + 2T_{n-1} \overset{2}{\equiv} 3 \overset{2}{\equiv} 1 \equiv T_{n+1}$ ✓

□

(b) Proof (by Induction)

I.H. $P(n) :: \overset{gcd}{(T_{n+1}, T_{n})} = 1$ ✓

B.C. $P(1).\ \overset{gcd}{(1, 1)} = 1$

I.S. $\gcd(T_{n+1}, T_n) = 1$

$\longrightarrow \gcd(T_{n+1} + 2T_n + 2T_n, T_{n+1}) = 1$ □

Problem 2

Hom Sol: $\alpha^2 - 11\alpha + 30 = 0$

$$\alpha = \frac{+11 \pm \sqrt{11^2 - 120}}{2} = \frac{11 \pm 1}{2} = 5, 6$$

$$\alpha = A(5)^n + B(6)^n$$

Bdry Cnds: $x_0 = 4 = \cancel{5}A + \cancel{6}B$

$x_1 = 23 = 5A + 6B$

$A = 1$

$B = 3$

Gen Sol: $5^n + 3 \cdot 6^n$

Problem 3

(a) (Suit, R_1, R_2, R_3, R_4, Suit, Rank, Suit, Rank,

Suit, Rank) $\quad 4 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 3 \cdot 13 \cdot 2 \cdot 13 \cdot 1 \cdot 13$

(b) (suit, $R_1, R_2 \cdots R_7$, $\cancel{S}_2$ $C_1 \cdots C_7$)

$$4 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \binom{39}{7}$$

Problem 4

merge 2s. 12 chickens. 12 zeroes. 2 ones.

$$\binom{14}{2}$$

Problem 5

each person can have ≤ 6 biscuits.

there are 7 people.

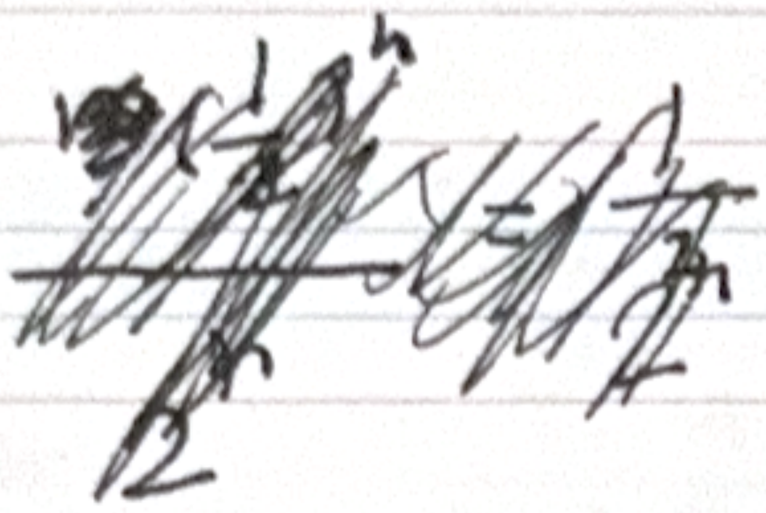
pigeon hole. $\square$

Problem 6

m boys. w girls. $m+w=n$

i from boys. $j=k-i$ from girls. □

Problem 7

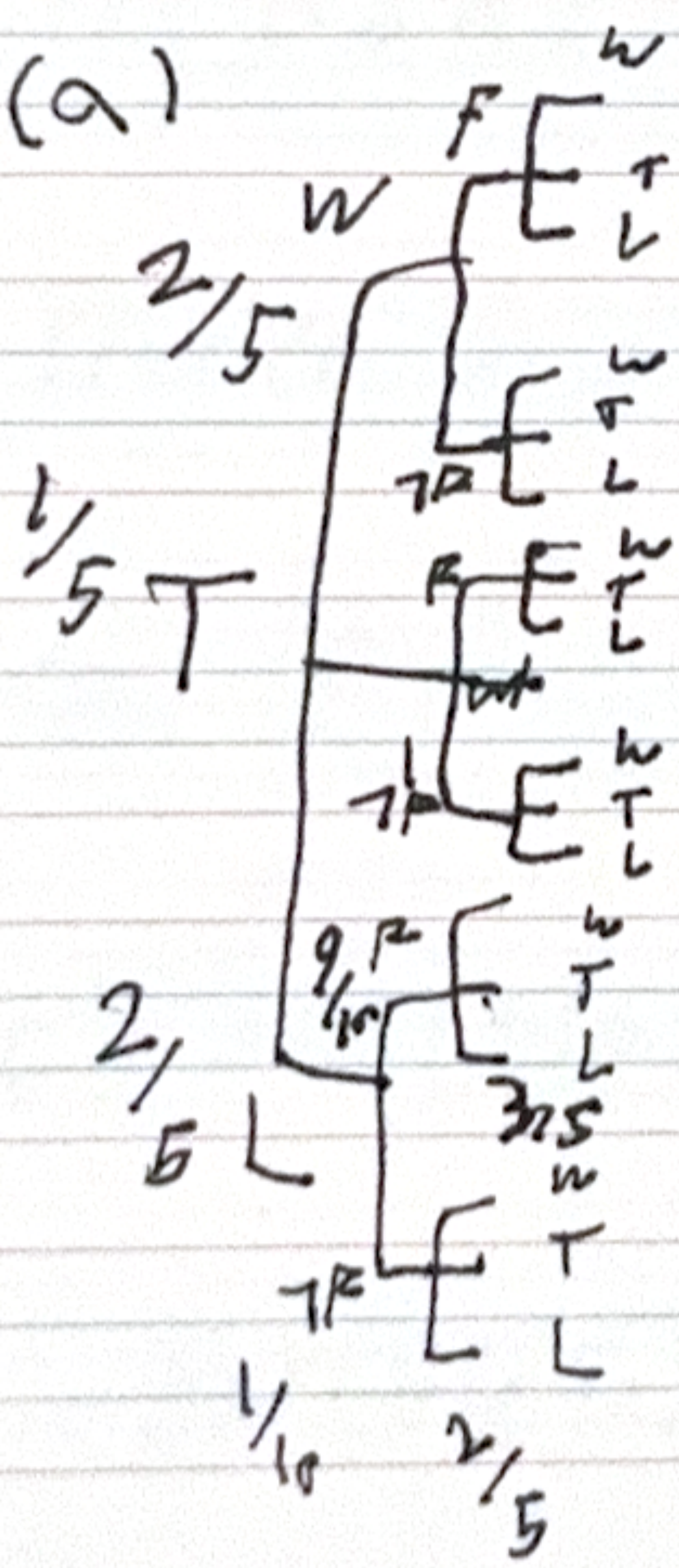


$\frac{1}{2^{n-1}}$ first one whatever.

then. sides must agree.

Problem 8

(a)



✓ □ ○ a

✓ $\frac{2}{5}\cdot\frac{2}{5}$ b

✓ □ ○ c

✓ $\frac{1}{5}\cdot\frac{2}{5}$ d

✓ □ $\frac{2}{5}\cdot\frac{9}{10}\cdot\frac{3}{5}$ e

✓ $\frac{2}{5}\cdot\frac{1}{10}\cdot$ f

$\frac{e}{a+b+c+d+e+f}$

$\frac{2}{5}\cdot\frac{9}{10}\cdot\frac{3}{5}$

(b) $\frac{2}{5}\cdot\frac{1}{10}\cdot\frac{2}{5}$

$+\ \frac{2}{5}\cdot\frac{9}{10}\cdot\frac{3}{5}$

Problem 9

(a) $\frac{1}{10} \cdot \frac{1}{5} \cdot \frac{1}{3} \cdot \frac{1}{2}$

(b) $\leq \frac{1}{\cancel{4}50}$

(c) $\leq \frac{1}{10}$

Problem 10

$$\sum_{i=1}^{n} \frac{i}{n} = \frac{1}{n}\sum_{i=1}^{n} i = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

Problem 11

$$Pr(R \geq x) = Pr(R^3 \geq x^3) \leq \frac{15}{x^3}$$

Problem 12

(a) 7

~~1225~~

2.5 1.5 .5

(b) $Var(Die) = \big((1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2$

$+ (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2\big)\frac{1}{6} =$

.5 1.5 2.5

$Var(Die) \cdot 2$

(C) ~~[illegible]~~

(d) $p = \frac{3}{36} + \frac{6}{36} \rightarrow \frac{1}{p}$

(e) $Ex = \frac{2}{6} \cdot 10 = \frac{20}{6}$

$Pr(R \geq \frac{20}{6})$

Problem 13

$p\, x_{n+2} + (1-p)\, x_{n-1}$