2010 Final

Problem 1

(a) Proof (by induction)

I.H. $P(n) :: T_n \overset{2}{\equiv} 1 \quad \forall n \geq 0$ ✓

B.C. $P(0)$ ✓

I.S. $T_{n-1} \overset{2}{\equiv} 1$

$T_n \overset{2}{\equiv} 1$

$T_n + 2T_{n-1} \overset{2}{\equiv} 3 \overset{2}{\equiv} 1 \equiv T_{n+1}$ ✓

□

(b) Proof (by induction)

I.H. $P(n) :: \gcd(T_n, T_{n+1}) = 1$ ✓

B.C. $P(1)$. $\gcd(1,1) = 1$

I.S. $\gcd(T_n + T_{n+1}, T_n) = 1$

$\rightarrow \gcd(T_{n+1} + 2T_n + 2T_n, T_{n+1}) = 1$ □

Problem 2

Hom Sol: $\alpha^2 - 11\alpha + 30 = 0$

$$\alpha = \frac{+11 \pm \sqrt{11^2 - 120}}{2} = \frac{11 \pm 1}{2} = 5, 6$$

$$\alpha = A(5)^n + B(6)^n$$

Bdry Cnds: $a_0 = 4 = A + B$

$$a_1 = 23 = 5A + 6B$$

$$A = 1$$

$$B = 3$$

Gen Sol: $5^n + 3 \cdot 6^n$

Problem 3

(a) (Suit, R_1, R_2, R_3, R_4, Suit, Rank, Suit, Rank, Suit, Rank) $4 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 3 \cdot 3 \cdot 2 \cdot 13 \cdot 1 \cdot 13$

(b) (suit, $R_1, R_2 \cdots R_7$, $C_1 \cdots C_7$)

$4 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \binom{39}{7}$

Problem 4

merge 2s. 12 chickens. 12 zeroes. 2 ones.

$\binom{14}{2}$

Problem 5

each person can have ≤ 6 biscuits.

there are 7 people.

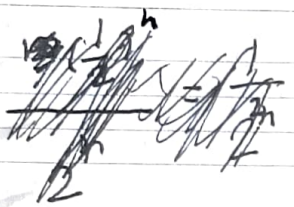
Pigeon hole. □

Problem 6

m boys. w girls. $m+w=n$

i from boys. $j=k-i$ from girls. □

Problem 7

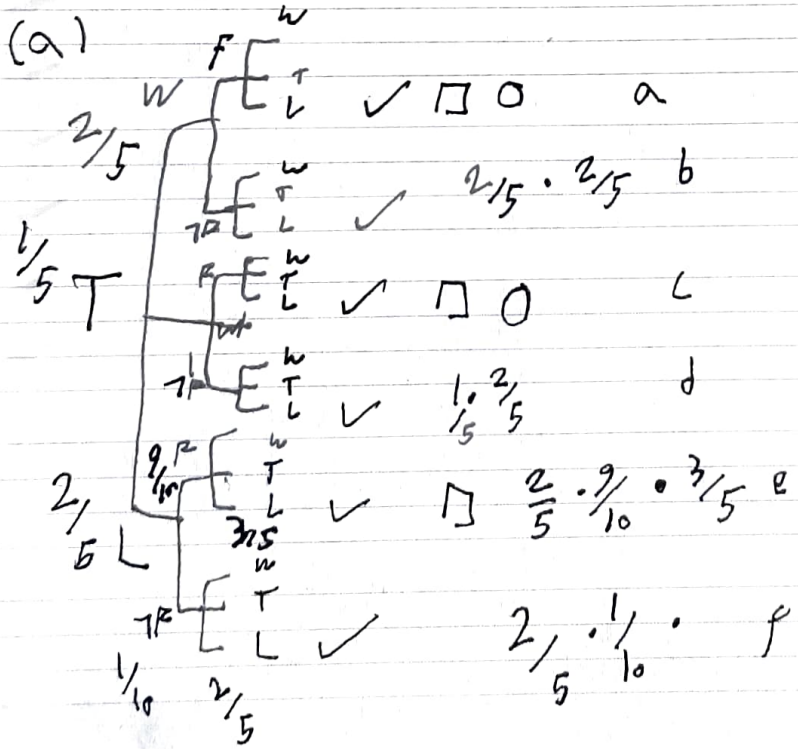


$\frac{1}{2^{n-1}}$ first one whatever.

then. sides must agree.

Problem 8

(a)



$$\frac{e}{\frac{a+b+c}{+d+e+f}}$$

$$2/5 \cdot 9/10 \cdot 3/5$$

(b) $2/5 \cdot 1/10 \cdot 2/5 + 2/5 \cdot 9/10 \cdot 3/5$

Problem 9

(a) $\frac{1}{10} \cdot \frac{1}{5} \cdot \frac{1}{3} \cdot \frac{1}{2}$

(b) $\leq \frac{1}{450}$

(c) $\leq \frac{1}{10}$

Problem 10

$$\sum_{i=1}^{n} \frac{i}{n} = \frac{1}{n} \sum_{i=1}^{n} i = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

Problem 11

$$Pr(R \geq a) = Pr(R^3 \geq a^3) \leq \frac{15}{a^3}$$

Problem 12

(a) 7

(b) $Var(Die) = ((1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2) \frac{1}{6} =$

$2.5 \quad 1.5 \quad .5 \quad .5 \quad 1.5 \quad 2.5$

$Var(Die) \cdot 2$

(c) [illegible]

(d) $p = \frac{3}{36} + \frac{6}{36} \rightarrow \frac{1}{p}$

(e) $Ex = \frac{2}{6} \cdot 10 = \frac{20}{6}$

$Pr(R \geq \frac{20}{6})$

Problem 13

$p\, x_{n+2} + (1-p)\, x_{n-1}$