Problem Set 12

Problem 1

(a) $2N \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} = N$

$(2N - N) \cdot \frac{1}{2} + (0 - N) \cdot \frac{1}{2} = N^2$

(b) $R = R_1 + \cdots + R_{\cancel{N}}$

$Ex(R) = Ex(R_1) + \cdots + Ex(R_N)$

$= N$

$Var(R) = Var(R_1) + \cdots + Var(R_N)$

$= N$

(c) Second strategy is less Risky.

(d) $3.5 = Ex$

$(1 - 3.5)^2 \frac{1}{6} + (2 - 3.5)^2 \frac{1}{6} + (3 - 3.5)^2 \frac{1}{6}$

$+ (4 - 3.5)^2 \frac{1}{6} + (5 - 3.5)^2 \frac{1}{6} + (6 - 3.5)^2 \frac{1}{6}$

(e) $Ex = \cancel{(1 + 3.5)} (1 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3) \frac{1}{6}$

$Var = (1^3 - \mu)^2 \frac{1}{6} + (2^3 - \mu)^2 \frac{1}{6} + (3^3 - 4)^2 \frac{1}{6}$

$+ \cdots + (6 - \mu)^2 \frac{1}{6}$

Problem 2

(a) $R = R_1 + R_2 + \cdots + R_7$

$Ex(R) = \frac{7}{8} \cdot 7$

(b) Sometimes $Ex(R)$ has to $^{15}R \to {}^{3}R = 7$

Problem 3

(a) $1 \cdot 2/3 + 2 \cdot 1/3 = 4/3$

(b) $\frac{1}{1/6} = 6 \to 5 = 6-1$

(c) $(3.5)^2$

(d) $\frac{1}{2} \cdot 4/3 + 1/3 \cdot 5 + 1/6 \cdot 3.5^2$

Problem 4

$Pr\{J \geq t_n\} \leq 0.05 \longleftrightarrow CDF(t_n) \geq 0.95$

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Problem 5

(a) $10 \cdot \left(2 \cdot \frac{3}{4} + 0 \cdot \frac{1}{4}\right) + 4 \cdot (3 + 7) + \frac{12}{6} + \frac{18}{6}$

(b) $3.5^2 + \frac{4}{10}\, 40 + \frac{3}{10}\, 50 + \frac{3}{10}\, 60$

(c) $\frac{4}{7} \cdot (a) + \frac{\cancel{3}\,2}{7} \cdot (b) + \frac{1}{7} \cdot 84$

Problem 6

(a) $Pr(x_i = 1) = \left(1 - \frac{1}{n}\right)^n$

(b) $n\left(1 - \frac{1}{n}\right)^n \sim n/e$

(c) $Pr(E_{k\ \text{specific}} \cup E_{k\ \text{diff}} \cup \cdots E_{k\ \text{diff}})$

$\leq \binom{n}{k}\left(\frac{1}{n}\right)^k$

(d) $Pr(R \geq k) \leq n(c) \leq n\binom{n}{k}\left(\frac{1}{n}\right)^k \leq \frac{n}{k!}$

(e) $Pr(R \geq k) \leq \frac{n}{k!} \sim \frac{n}{\sqrt{2\pi k}\,(k/e)^k} \leq \frac{n}{(k/e)^k} = \frac{e^{k + \ln n}}{e^{k \ln k}} \xrightarrow[n \to \infty]{} 0$