Subject: Problem Set 6

Year: Month: Date:

Problem 1

(a) R, $n \mid (x-x)$. S, $n \mid (x-y) \to n \mid (y-x)$

T, $n \mid x-y \wedge n \mid y-z \to n \mid x-y+y-z$

(b) asymmetric.

(c) irreflexive.

(d) R, Path of 0 by convention. S, undirected graph.

T, join the $x-y$ with $y-z$.

Problem 2

(a) not injective, $f(0)=f(\pi)$. Surjective,

continuous and unbounded. ✓

(b) injective. Surjective like (a). bijective. ✓

(c) not surjective because incontinuous. injective

because of ~~graph~~, strictly increasing.

(d) just surj. mapping that many primes -2.

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Problem 3

(a) R, $i \leq j$ iff $a_i \leq a_j$ ✓

AS, $i \leq j \wedge j \leq i$ iff $a_i = a_j \wedge i = j$ ✓

T, $i \leq j$, $j \leq k \to i \leq k$

$a_i \leq a_j$, $a_j \leq a_k \to a_i \leq a_k$ ✓

(b) by Dilworth's lemma

Case 1. $\exists$ a chain of length $n \leq$ with the above relation. $\longrightarrow$ non-dec sub.

Case 2. length of the largest chain is $c \leq n-1 \longrightarrow$ we have c antichains one of size

$$\geq \frac{(n-1)(m-1)+1}{c} \geq \frac{(n-1)(m-1)+1}{(n-1)} = m - 1 + \frac{1}{(n-1)} \geq m$$ ✓

(c) 3,4,1,2,

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Problem 4

d SS ~~*~~S C LMC CML

$L_N + 4$ 2×2 $3NL_N + 3N$ 1 $2L_{N+3}$ $\sqrt{N}$ ~~+~~

Problem 5

(a) upper-Bound on $\min\{2^i, 2^{n-i}\} = 2^{n/2} = \sqrt{n}$

(b) all ~~nodes~~ inputs from $Z\underbrace{000}_{n/2}$ to $\underbrace{000}_{n/2}Z'$

go through first node of $1_{n/2} \cdot 2^{n/2} Z_{s.}^{n/2}$