

P1.

Proof (by str Ind.)

$$\text{I.H. } P(m \times n) \text{ ~~is~~ } ::= \text{ ^{number} ~~of~~ splits for } m \times n \\ = mn - 1$$

$$\text{B.C. } P(1 \times 1) = 1 - 1 = 0 \quad \checkmark$$

I.S. assume $P(1) \dots P(n)$ (for str Ind.)

We break the piece

into p & q s.t. $p + q = mn$

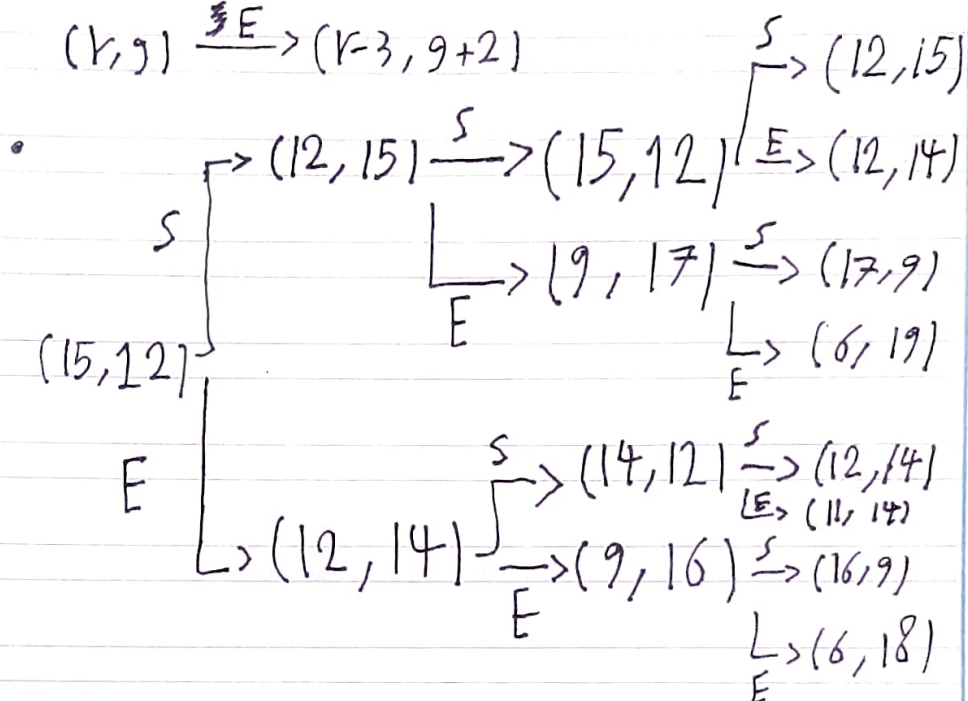
then ~~of~~ ^{number} splits $= p - 1 + q - 1 + 1 \quad \checkmark$

P2.

- Pairs like (r, g)

- $(r, g) \xrightarrow{SE} (g, r)$

- $(r, g) \xrightarrow{NE} (r-3, g+2)$



Theorem 1.

Lemma 1. $P(n) := r - g \equiv 3 \pmod{5}$

or $r - g \equiv 2$

$\& n$ holds after n steps

I.H. $P(n)$

(invariance)

B.C. $P(1) = 15 - 12 = 3 \checkmark$

I.S. after n steps

$S \rightarrow r_{n+1} - g_{n+1} = -(r_n - g_n)$

$\& r_{n+1} - g_{n+1} = -(r_n - g_n)$

or $-2 \equiv 3 \pmod{5} \quad = -3 \equiv 2 \pmod{5} \checkmark$

by Lemma 1 you ~~can't~~ ^{won't}

reach $r - g \equiv 0 \pmod{5}$

$E \rightarrow r - 3 - g - 2 = r - g - 5 \equiv 3 \pmod{5} \checkmark$

or $r - g - 5 \equiv 2 \pmod{5}$

Theorem 2.

lemma 1. $\exists h \forall$. $P(h) ::= \text{after } h \text{ steps } r+g \leq 27$

B.C. $P(0) \checkmark$

I.S. $r+g \leq 27 \begin{cases} \xrightarrow{S} g+r \leq 27 \checkmark \\ \xrightarrow{} r+g-3+1 \leq 27 \checkmark \end{cases}$

by lemma 1 you can't have

more than $\underbrace{27}_{\text{greens}}$ or $\underbrace{27-9}_{\text{reds}}$.



~~unique. (not considering swaps as unique)~~

$$28 + 27 + \dots + 1 = 406$$

Theorem 3. $25 \text{ E} + 27 \text{ G} + 1 = 52$

$$\neq 108 \leftarrow$$