Subject: Recitataion 3

Year: Month: Date:

P1.

Proof (by Stg Ind.)

I.H. $P(m \times n)$ ::= ~~#~~ number of splits for m by n

$= mn - 1$

B.C. $P(1 \times 1) = 1 - 1 = 0$ ✓

I.S. assume $P(1) \ldots P(n)$ (for Stg Ind).

we break the piece

into p & q s.t. $p + q = mn$

then ~~#~~ number of spits $= p - 1 + q - 1 + 1$ ✓

P2.

- Pairs like (r,g)
- $(r,g) \xrightarrow{S} (g,r)$

  $(r,g) \xrightarrow{E} (r-3, g+2)$
- $(15,12)$
  - $\xrightarrow{S} (12,15)$
    - $\xrightarrow{S} (15,12)$
      - $\xrightarrow{S} (12,15)$
      - $\xrightarrow{E} (12,14)$
    - $\xrightarrow{E} (9,17)$
      - $\xrightarrow{S} (17,9)$
      - $\xrightarrow{E} (6,19)$
  - $\xrightarrow{E} (12,14)$
    - $\xrightarrow{S} (14,12)$
      - $\xrightarrow{S} (12,14)$
      - $\xrightarrow{E} (11,14)$
    - $\xrightarrow{E} (9,16)$
      - $\xrightarrow{S} (16,9)$
      - $\xrightarrow{E} (6,18)$

Theorem 1.

Lemma 1. $P(n) := r-g \equiv 3 \pmod 5$ or $r-g \equiv 2$ holds after n steps

(invariance)

I.H. $P(n)$

B.C. $P(1)$ $15-12=3$ ✓

I.S. after n steps

$\xrightarrow{S}$ $r_{n+1}-g_{n+1} = -(r_n-g_n)$ $= -3 \equiv 2$ ✓ or $-2 \equiv 3$

$\xrightarrow{E}$ $r-3-g-2 = r-g-5 \equiv 3$ ✓ or $r-g-5 \equiv 2$

by Lemma 1 you won't reach $r-g \equiv 0$ $\square$

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Theorem 2.

lemma 1. Inv. $P(n) ::=$ after n steps $r+g \le 27$

B.C. $P(0)$ ✓

I.S. $r+g \le 27$ $\xrightarrow{S} g+r \le 27$ ✓

$\longrightarrow r+g-3+2 \le 27$ ✓

by lemma 1 you cant have

more than $\underbrace{27 r}_{\text{greens}}$ $\underbrace{\text{or } 27-g}_{\text{reds}}$.

~~Unique. (not considering~~ swaps as unique)

$28+27+\dots+2 = 406$

3

3

$+$

27

$+$

27

$=$

Theorem 3. $25\,E + 2\cancel{7}6\,S + 1 = 52$

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