Subject: Midterm Practice Problems Year: Month: Date:

Problem 1

P nor q is equivalent to $\neg P \wedge \neg q$

So we only need to negate the variables

using $P \text{ nor } P \equiv \neg P$ so we have $(P \text{ nor } P) \text{ nor } (q \text{ nor } q)$ $\square$

✓

Problem 2

Proof (by Strong Induction)

I.H. $P(n) ::= a_n \overset{3}{\equiv} 1$

Base case $P(1) ::= 1 \overset{3}{\equiv} 1$ ✓

I.S. assume $P(a)$ for $a \leq n$ to prove $P(n+1)$.

$a_{n+1} \overset{3}{\equiv} a_n + a_{n-1} + a_{n-2} + a_{n-3}$

$\overset{3}{\equiv} 4$

$\equiv 1$ ✓

invoke induction $\square$ ✓

Problem 3

Proof. by Induction

I.H. $P(n) ::=$ after n moves the number of inversions is odd.

B.C. $P(0)$ ✓

I.S. $5n \to 6-n$ remains odd. ✓

invoke induction □

Proof. by ※

0 is even. ※ □

✓

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Problem 4

$-2 \cdot 72 + 17 \cdot 9 = 1$ X

Problem 5

(a) no. odd cycle. ✓

(b) no. odd degree. ✓

(c) 10. bottom right to bottom left. X
-2

(d) 5. all the left + center going to right.
+4 X

Problem 6.

1. no. 

$A \to B \wedge B \to C$ but $\neg A \to C$ ✓

2. no. $A \to B \longrightarrow \neg B \to A$ ✓

3. yes. $\neg$symmetric $\therefore$ the reflexivity doesnt occur so it doesnt matter. ✓

4. no. no cycles. ✓

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Problem 7

Proof by Induction.

I.H. $P(n) ::=$ n-node outer planar graph is 3 colorable.

B.C. $P(1)$ ✓

I.S. ~~n~~-node O.P. G has a node v
n+1

degree ≤ 2. remove to obtain

n-node G'. $P(n)$. add v. Color w/ 2. ✓

□ ✓

Problem 8.

$$f(n) + \int_1^n f(i)\,di \leq \sum_{i=1}^{n} f(i) \leq f(1) + \int_1^n f(i)\,di$$

$$\frac{1}{n^3} + \left(-\frac{1}{2n^2} + \frac{1}{2}\right) \leq \sum_{i=1}^{n} f(i) \leq 1 + \left(-\frac{1}{2n^2} + \frac{1}{2}\right)$$

$$1 - \frac{1}{n^3} \leq 1$$ ✓ ✓

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Problem 9

(a) O, Ω, θ ✓

(b) Ω, ω ✓

(c) $\sim, \theta, O, \Omega$ X O, o

(d) $\sim, \theta, O, \Omega$ ✓

(e) θ, O, Ω ✓