Subject: Problem Set 7 Year: Month: Date:

Problem 1

$$\sum_{i=0}^{n} x^i = \frac{1-x^n}{1-x}$$

derive both sides

$$\sum_{i=0}^{n} i x^{i-1} = \frac{(1-x)(-nx^{n-1}) - (1-x^n)(-1)}{(1-x)^2}$$

$$= \frac{(1-x)(-nx^{n-1}) + 1 - x^n}{(1-x)^2}$$

$$= \frac{-nx^{n-1} - nx^n + 1 - x^n}{(1-x)^2}$$

multiply by x then

$$= \frac{-x^n(n+1) - x^{n-1} n + 1}{(1-x)^2}$$

derive both sides again

$$\sum_{i=0}^{n} i x^i = \frac{-x^{n+1}(n+1) - x^n n + 1}{(1-x)^2}$$

$$\sum_{i=0}^{n} i^2 x^{i-1} = \frac{(-(n+1)^2 x^n - n^2 x^{n-1})(x^2+1-2x) - (2x-2)(-x^{n+1}(n+1) - x^n n + 1)}{(1-x)^4}$$

$$= \frac{1 + x - (n+1)^2 x^n + (2n^2+2n-1)x^{n+1} - n^2 x^{n+2}}{(1-x)^3}$$

multiply by x →

$$\sum_{i=0}^{n} i^2 x^i = \frac{x(1+x-(n+1)^2 x^n + (2n^2+2n-1)x^{n+1} - n^2 x^{n+2})}{(1-x)^3}$$

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Problem 2

(a) $\prod_{k=1}^{n} 2^{2k-1} = 2^{\sum_{k=1}^{n} 2k-1} = 2^{n^2}$

(b) $\sum_{i=0}^{n} \cancel{3} \sum_{j=0}^{m} 3^{i+j} = \sum_{i=0}^{n} 3^i \cdot \sum_{j=0}^{m} 3^j = \left(\frac{1-3^n}{1-3}\right)\left(\frac{1-3^m}{1-3}\right)$

$= \frac{(3^{n+1}-1)(3^{m+1}-1)}{4}$

(c) $\sum_{i=1}^{n}\sum_{j=1}^{n}(i+j) = \sum_{i=1}^{n} i \sum_{j=1}^{n}\left(1+\frac{j}{i}\right)$

$= \sum_{i=1}^{n} i\cdot\left(n+\sum_{j=1}^{n}\frac{j}{i}\right) = \sum_{i=1}^{n} i\cdot\left(n+\frac{1}{i}\sum_{j=1}^{n} j\right)$

$= \sum_{i=1}^{n} i\cdot\left(n+\frac{n(n+1)}{2i}\right) = \sum_{i=1}^{n} ni + \frac{(n(n+1))}{2}$

$= \frac{n^2(n+1)}{2} + \sum_{i=1}^{n} ni = \frac{n^2(n+1)}{2} + \frac{n^2(n+1)}{2} = n^2(n+1)$

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(d) $\prod_{i=1}^{n} \prod_{j=1}^{n} 2^{i} \cdot 3^{j} = \prod_{i=1}^{n} 2^{\frac{n(n+1)}{2} i} \prod_{j=1}^{n} 3^{j}$

$= \prod_{i=1}^{n} 2^{\frac{n(n+1)}{2} i} \, 3^{\frac{n(n+1)}{2}} = 3^{\frac{n(n+1)}{2}} \prod_{i=1}^{n} 2^{\frac{n(n+1)}{2} i}$

$= 3^{\frac{n(n+1)}{2}} \, 2^{\frac{n^2(n+1)^2}{4}}$

Problem 3

(a)

$$f(1) + f(2) + \int_2^{\infty} + f(\infty) \leq F \leq$$

$$f(1) + f(2) + \int_2^{\infty} + f(2)$$

$$\frac{1}{9} + \frac{1}{25} + \frac{1}{180} + 0 \leq \sum_{i=1}^{\infty} \frac{1}{(2i+1)^2} \leq \frac{1}{9} + \frac{1}{25} + \frac{1}{180} + \frac{1}{25}$$

(b) first one holds indefinitely

Second one holds upto a Point.

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Problem 4

(a) $\frac{1}{100!}$

(b) $\frac{H_n}{100}$

(c) $\frac{\ln}{100} \geq 1 \rightarrow \ln \geq 100 \rightarrow e^{100}$ seconds

Problem 5

(a) Ω, O, Θ

(b) o, O

(c) $o, O, \Theta,$

(d) $\cancel{\omega, \Omega, o, O, \Theta, \infty}$

(e) ω, Ω

Subject:

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Problem 6

(a) $\lim\limits_{n \to \infty} \frac{n!}{(n+1)!} = \frac{1}{n+1} = 0 \to 0, 0$ ✓

(b) $\lim\limits_{n \to \infty} \frac{\sqrt{2\pi n}\,(\frac{n}{e})^n}{(\frac{n}{\cancel{2}3})^{n+e}} = \lim\limits_{n \to \infty} \frac{3^e \sqrt{2\pi \cancel{n}}}{n e^{-\frac{1}{2}}} (\frac{3}{e})^n = \infty$ ✓

(c) $\lim\limits_{n \to \infty} \frac{\sqrt{2\pi n}\,(\frac{n}{e})^n}{2^n} = \lim\limits_{n \to \infty} (\frac{n}{2e})^n \sqrt{2\pi n} = \infty$ ✓