1 Properties of Variance

(a)
$$Var_{(R)} = Ex((R - Ex(R)^2)$$
$$= Ex(R^2 - 2R Ex(R) + E(R)^2)$$
$$= Ex(R^2) + E^2_{(R)} \overset{Ex}{-} 2R Ex(R))$$
$$= Ex(R^2) + Ex^2(R) - 2E(R Ex(R))$$
$$= Ex(R^2) + Ex^2(R) - 2Ex^2(R)$$
$$= Ex(R^2) - Ex^2(R)$$

(b)
$$Var[aR+b] = Ex((aR+b)^2) - Ex^2(aR+b)$$
$$= Ex(a^2R^2) + 2Ex(abR) + Ex(b^2) - Ex^2(aR+b)$$
$$= a^2 Ex(R^2) + 2ab Ex(R) + b^2 - (Ex(aR+b))^2$$
$$= a^2 Ex(R^2) + 2ab Ex(R) + b^2 - a^2 E^{2R}_{x(m)} - 2ab Ex(R) - b^2$$
$$= a^2(Ex(R^2) - E^2(R)) = a^2 Var(R)$$

(c)
$$Ex((R_1 + R_2)^2) - Ex^2(R_1 + R_2)$$

$$= Ex(R_1^2) + 2 Ex(R_1) Ex(R_2) + Ex(R_2^2) - \underbrace{(Ex(R_1) + Ex(R_2))}_{\parallel}^2$$
$$\qquad\qquad\qquad Ex^2(R_1) + 2 Ex(R_1) Ex(R_2) + Ex^2(R_2)$$

$$= Ex(R_1^2) - Ex^2(R_1) + Ex(R_2^2) - Ex^2(R_2)$$

(d) $R_1 = R_2 \rightarrow Var(2R_1) = 4 Var(R_1) \neq 2 Var(R_1)$

(e) $E_x\left(\sum\limits_{i=1}^{n} I_i\right) = \sum\limits_{i=1}^{n} E_x(I_i) = \dfrac{n\,P(1-P)}{\sqrt{n\,P(1-P)}} = \dfrac{Var}{= SD}$

(f) $Var(T) = E_x(T^2) - E_x^2(T)$

$E_x(T_j^3) = \sum\limits_{\substack{x \in \{0,1\} \\ R(Tj)}} x^2 \, Pr(x) \leq \sum\limits_{x \in R(Tj)} x \, Pr(x) = E_x(Tj)$

$\longrightarrow E_x(T^2) - E_x^2(T) \leq E_x(T) - E_{x(x)}^{T^2} \leq E_x(T)$