

P1

(a)  $\exists p \in \mathcal{K}. S(p) \wedge A(p).$

(b)  $\forall p \in \mathcal{K}. S(p) \wedge T(p) \rightarrow A(p).$

(c)  ~~$\forall p \in \mathcal{K}. \neg(\exists p \in \mathcal{K}. T(p) \wedge \neg A(p))$~~

(d)  $\exists p_1, p_2, p_3. T(p_1) \wedge T(p_2) \wedge T(p_3) \wedge \neg S(p_1) \wedge \neg S(p_2) \wedge \neg S(p_3)$

P2

(a) T

Valid

(b) F

P	Q	R	b(P, Q, R)
T	T	F	F

Invalid

P3(a)

(i)  $\neg(A \text{ hand } B)$

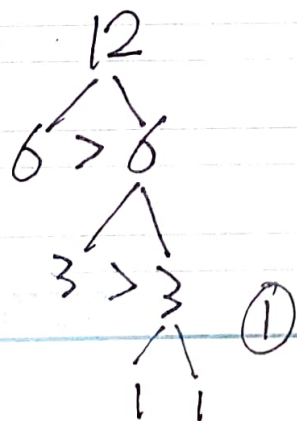
(ii)  $\neg(\neg(\neg A \text{ hand } \neg B)) = \neg A \text{ hand } \neg B$

(iii)  $A \text{ hand } \neg B$

(b)  $A \text{ hand } A$

(c)  $A \text{ hand } \neg A = T$        $\neg(A \text{ hand } \neg A) = F$

P4



P5

$$R(r^{1/5}) \rightarrow R(r)$$

$$r^{1/5} = \frac{a}{b} \rightarrow r = \frac{a^5}{b^5} \quad \exists a, b \in \mathbb{Z}$$

P6

$$E(z) \rightarrow E(n, y, z)$$

Case 1: 2 odd, 1 Even

$$\begin{aligned} (2i+1)^2 + (2j+1)^2 + (2k)^2 \\ = 2^2 i^2 + 2^4 i + 1 + 2^2 j^2 + 2^4 j + 1 + 4k^2 \\ \equiv 4m + 2 \equiv 2 \pmod{4} \end{aligned}$$

Case 2: 2 Even 1 odd

$$\begin{aligned} (2i)^2 + (2j)^2 + (2k+1)^2 \\ = 4i^2 + 4j^2 + 4k^2 + 4k + 1 \equiv 1 \pmod{4} \end{aligned}$$

Case 3: 3 odd

$$\begin{aligned} (2i+1)^2 + (2j+1)^2 + (2k+1)^2 \\ = 4i^2 + 4i + 1 + 4j^2 + 4j + 1 + 4k^2 + 4k + 1 \\ \equiv 3 \pmod{4} \end{aligned}$$

$$\text{Case 4: } 4i^2 + 4j^2 + 4k^2 = 4l^2 \quad \checkmark$$