Subject: Recitation 2

Year: Month: Date:

Problem 1

(a) Proof. (by WOP & Contradiction)

$$P(n) ::= 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

$$C = \{ n \mid P(n) \text{ is False} \}$$

by WOP $\exists c = \min(C)$

therefor $\sum_{i=0}^{c-1} r^i = \frac{1 - r^c}{1 - r}$

but $\frac{1 - r^c}{1 - r} + r^c = \frac{1 - r^{c+1}}{1 - r}$

So it must be $P(c) = T = F$

※ $\square$

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(b) Proof. (by induction)

I.H. $P(n) ::= \sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$

Base case: $P(0) = 1 = 1$ ✓

Inductive Step:

assume $P(n)$ for induction to prove $P(n+1)$.

$\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$ the $\sum^{n}$ sum of up to n

$\frac{1-r^{n+1}}{1-r} + r^{n+1} = \frac{1-r^{n+2}}{1-r} \rightarrow P(n+1)$ is true ✓

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Problem 2.

lemma 1. Purple People see 1 less Purple than red People.

$P<n$: All P are gone.

$P=n$: All P are leaving.

$P>n$: All P are here.

$P<1$: Vacant.

$P=1$: the single P sees all r and leaves.

$P>1$: they stay.

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$P < h+1$: Case 1. $P = h$ therefore all left. yesterday

Case 2. $P < h$ they left way before.

$P = h+1$: $P = h+1 > h$. they stood yesterday. leave today.

$P > h+1$: $P > h+1 > h$. they stay.