Subject: Problem Set 4

Year: Month: Date:

Problem 1

Proof (by Induction)

I.H. $P(n) :: G' = (V, M_1 \cup M_2)$, n-node G' is bipartite.

B.C. $P(1)$ ✓

I.S. assume $(n+1)$ node graph with edges $= M_1 \cup M_2$, remove $(n+1)$st node and it's edges. apply $P(n)$. add $(n+1)$st.

Case 1. no edges, add to L or R.

Case 2. 1 edge, add to opposite.

Case 3. 2 edges,

Case 3.1 both edges go to one set, ✓

Case 3.2 ~~to~~ edges go to vertices on different sides with diff components, divide each to L and R and put $L' = L_1 \cup R_2$ $R' = L_2 \cup R_1$. now same side, add to other.

Case 3.3 different sides, same component.

※. odd cycle —> both edges from one M.

Problem 2.

(a) each edge contributes to 2 degrees.

(b) 111·17 ~~27~~

27

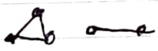
(c) $\frac{n \cdot (n-1)}{2}$

Problem 3

(a) 1, 3, 4

(b) degree 4: a ≠ b, a = c, a ≠ d, b ≠ d, cycle ≠ degree 4, one to one -> degree 4

Problem 4 ~~bin~~

(a) 

(b) "Removing V reduces ..."

Problem 5

lemma, ∀girl rated worst by at ~~least~~ most (n-~~1~~2) boys

Proof by ※, if ∀girl rated worst by at least

(n-1) boys —> ∃ n(n-1) boys or more ※

now if ∃ girl with worst boy ∧ ∃ boy with worst girl —> rogue.

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Problem 8

(a) Color line (a) with $\{a,b\}$ and line (b) with $\{c,d\}$ now color all with $\{a,b\} \times \{c,d\}$

(b) all even numbers are adj to odd and vice versa.