Subject: Midterm

Year: Month: Date:

Problem 1

$P \equiv q$

A	B	C	$A \vee B$	$A \vee B \rightarrow C$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	F
T	T	F	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Subject: Year: Month: Date:

Problem 2 (Cntd..)

A	B	C	$\neg C \Rightarrow \neg A$	$\neg C \Rightarrow \neg B$	q
T	T	T	T	T	T
T	F	T	T	F	T
F	T	T	F	T	T
F	F	T	F	F	F
T	T	F	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

Tables show $P \equiv q$ ✓

$\frac{97}{120}$

Subject: Year: Month: Date:

Problem 2

Proof. by Strong Ind.

I. H. $P(n) ::= G_n \leq 3^n$

B. C. $P(0)$ $1 \leq 3^0$ ✓

I. S. $G_{n-1} \leq 3^{n-1}$

$3G_{n-2} \leq 3 \cdot 3^{n-2} \leq 3^{n-1}$

$3G_{n-3} \leq 3 \cdot 3^{n-3} \leq 3^{n-2}$

$G_{n+1} = G_{n-1} + 3G_{n-2} + 3G_{n-3} \leq 3^{n-2} 8 \leq 3^{n-1} 2 \leq 3^n$ ✓

invoke stg ind. □

✓

97/120 81%

Subject: Year: Month: Date:

Problem 3

(a) Proof. by ind ✓

I.H. $P(n)$ ::= # of Shapes decreases by 1 on move n.

B.C. ✓

I.S. either case ✓

□

we need n-1 moves to turn n shapes to 1. ✓

-7

(b) # circles odd ⟷ when 1 circle remains its paird with a square to make circle and by ind so on... ⟷ winner

Subject: Year: Month: Date:

Problem 4

(a) $4 \times 113 - 41 \times 11 = 1 \longrightarrow 113 - 41 = 72$ ✓

(b) $11^{11211} \equiv 11^{11200} \cdot 11^{11} \equiv (11^{112})^{100} \cdot 11^{11} \equiv (11^{\varphi(113)})^{100} \cdot 11^{11}$

$\equiv 11^{11}$

B −4

$11 \overset{113}{\equiv} 11$

$11^{2} \overset{113}{\equiv} 8$

$11^{4} \overset{113}{\equiv} 8^{2} \equiv 64$

$11^{8} \overset{113}{\equiv} 64^{2} \overset{113}{\equiv} 856 \equiv 65$

$11^{11} \overset{113}{\equiv} 65 \cdot 8 \cdot 11 \equiv 5 \cdot 60 \equiv 88$ ✓

Subject: Year: Month: Date:

Problem 5

−2

(a) ~~[illegible]~~ D C B A F E 5

(b) D B A F E C D

(c) A ~~G~~ R

B G

C B

D R

E R

F G

(d) no. C has odd degree.

(e) DC

CB

AB

EF

EC

Subject:

Year: Month: Date:

Problem 6

Proof. by Induction.

I.H. $P(m) ::=$ m-edge G has $C_m \geq m-n+k$

B.C. $P(0) ::= 0 \geq 0-1+1$

I.S. $P(m+1) \rightarrow C_{m+1} \geq (m+1)-n+k$

Case 1: we remove edge e. $\in$ a cycle

removes. $\rightarrow C_m \geq m-n+k$

$\rightarrow$ add back $\rightarrow C_m+1 \geq m+1-n+k$

Case 2: cycles are untouched. $\rightarrow C_{m+1} = C_m$

$\geq m-n+k$

-5 5

invoke Ind. $\square$

Subject:

Year: Month: Date:

Problem 7

$$\underbrace{0}_{f(\infty)} + \int_{i=1}^{\infty} f(i)\,di \leq \sum_{i=1}^{\infty} f(i) \leq \int_{i=1}^{\infty} f(i)\,di + \underbrace{f(1)}_{1}$$ ✓

Problem 8

(a) $\lim\limits_{n\to\infty} \dfrac{n \ln n}{n} = \infty \to$ ✗ ✓

(b) $\lim\limits_{n\to\infty} \dfrac{n/100}{n} = \dfrac{1}{100} \to$ ✗ ✓

(c) $\lim\limits_{n\to\infty} \dfrac{n^{n+1}}{n^{n}} = \infty \to$ ✓ ✓

(d) $\lim\limits_{n\to\infty} \dfrac{n^{n+\frac{1}{2}}}{n} = \infty \to$ ✗ ✗ -3