

Problems for Recitation 23

1 Getting Dressed

$$(a) E_X(X) = 0.2 + 0.1 + 0.1 + 0.3 = 0.7$$

$$(b) \overset{Pr}{E_X(X \geq 1)} \leq \frac{E_X(X)}{1} = 0.7$$

$$(c) \overset{Pr}{E_X(X \geq 3)} \leq \frac{E_X(X)}{3} = 0.23$$

$$(d) E_X(X \geq 2) = E_X(|X - E_X(X)| \geq 1.3)$$

$$\begin{aligned} \text{Var}(X) &= E_X(X^2) - (E_X(X))^2 \\ &= (0.2 \cdot 1^2 + 0.1 \cdot 2^2 + 0.1 \cdot 3^2 + 0.3 \cdot 4^2) - (0.7)^2 \\ &= 0.55 \end{aligned}$$

$$\sigma(X) = \sqrt{0.55} \approx 0.7416$$

$$\overset{Pr}{E_X(|X - E_X(X)| \geq \sigma(X))} \leq \frac{1}{1.7^2} \approx 0.33$$

$$(e) E_X(X \geq 1) \geq 1 - e^{-E_X(X)} \approx 0.5$$

$$(g) \quad \Pr \{n \geq 4/3 E_x(n)\} \leq e^{-\left(4/3 \ln(4/3) - 4/3 + 1\right) E_x(n)} \approx 0.14$$

$$(h) \quad \Pr \{n \geq 3 E_x(n)\} \leq e^{-\left(3 \ln(3) - 3 + 1\right) 36} \approx 5.49 \times 10^{-22}$$

2 A Financial Crisis

$$1. \Pr(X \geq 2.1 E_X(X)) \leq e^{-(2.1 \ln(2.1) - 1.1)} 50 \approx e^{-0.802} \approx 0.$$

None!!

$$2. \Pr(X \geq 2.0 E_X(X)) \leq \frac{1}{2.0} = 5\%. \text{ as good as any conditional.}$$

$$3. ~~50\%~~ 50\%$$

$$4. 100 E_X(X) = 5000$$

$$5. \Pr(X \geq 9/5 5000) \leq e^{-(1.8 \ln(1.8) - 1.8 + 1)} 5000 \approx 0$$

$$\Pr(X \geq 7/5 5000) \leq e^{-(1.4 \ln(1.4) - 1.4 + 1)} 5000 \approx 0$$

$$6. \Pr(X \geq 9/5 5000) \leq \frac{1}{9/5} = \frac{5}{9} = 55\%$$

$$\Pr(X \geq 7/5 5000) \leq \frac{1}{7/5} = \frac{5}{7} = 71\%$$