# Linear Recurrences

1. Hom Sol: eq: $\alpha^2 - \alpha - 1 = 0$

$$\text{root}: \alpha = S\left(\frac{1+\sqrt{5}}{2}\right) + t\left(\frac{1-\sqrt{5}}{2}\right)$$

Part Sol: $c = c + c + 1$

$$c = -1$$

Gen Sol: $T(n) = S\left(\frac{1+\sqrt{5}}{2}\right)^n + t\left(\frac{1-\sqrt{5}}{2}\right)^n + c^{-1}$

Bdry Cnd: $T(0) = 0 = S + t + c^{-1} \rightarrow S = 1 - t$

$$T(1) = 1 = S\left(\frac{1+\sqrt{5}}{2}\right) + t\left(\frac{1-\sqrt{5}}{2}\right) + c_{-1}$$

$$\rightarrow 2 = S\left(\frac{1+\sqrt{5}}{2}\right) + (1-S)\left(\frac{1-\sqrt{5}}{2}\right)$$

$$\rightarrow T(n) = \left(\frac{5+3\sqrt{5}}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n$$

$$+ \left(\frac{5-3\sqrt{5}}{10}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$-1$$

2. Hom Sol: eq: $x^2 - 6x + 9 = 0$

$$(x-3)^2 = 0$$

roots: $x = 3, 3$

Gen Sol: $\overset{T(n)=}{A\,3^n} + Bn\,3^n$

$\longrightarrow T(n) = A\,3^n + Bn\,3^n$

Bdry Chds: $T(0) = 0 = A + B$

$\longrightarrow A = -B$

$T(1) = 1 = A3^o - An3^o$

$\longrightarrow 1 = 3A - 3.0.A$

$\longrightarrow A = \dfrac{1}{3}$

Comp Sol: $T(n) = 3^{n-1} - A3^{n-1}$

# Trimerge Sort

1. $h - 1 = 2$

2. $h - 1$

3. $T_h = 3T(n/3) + h - 1$

4. $P = 1$

5. Yes. $C = 1$

6. $T_n = \theta\left(n\left(1 + \int_1^{nu} \frac{h-1}{u^2}\right)\right)$

$$= \theta\left(n + h\ln n + \frac{1}{n} - h\right)$$

$$= \theta(n \ln(n))$$