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the center of an structure is obtained through averaging the centers of it's sub parts.

the center of a single L:

$$\left(\frac{1}{2}+\frac{n}{2}\right)/2 = \frac{n+1}{4} = c_1$$

the center of h Ls:

$$c_h = c_1 + (c_1+1) + (c_1+2) + \dots + (c_1+(h-1)) / h$$

$$= \left(h c_1 + \frac{h(h-1)}{2}\right)/h = c_1 + \frac{h-1}{2}$$

every c_i for $i \neq 1$ should be $c_i \leq \underbrace{n-1}_{\text{length of base}}$
(base starts at -1)

for c_k it should be:

(we shift frame) $n-1 \leq c_k \leq n$

$\longrightarrow (n+1)/4 + (k-1)/2 \leq n - 1/2$

$n-1 \leq (n+1)4 + (k-1)/2 \leq n$

$\longrightarrow \frac{3n-3}{2} \leq k \leq \frac{3n-1}{2}$

for odd n we have 2 choices

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- $$\sum_{k=1}^{n} H_k = \sum_{k=1}^{n} \sum_{j=1}^{k} 1/j$$

- $$\sum_{k=1}^{n} H_k$$ is similiar to $$\int_1^n \ln = n \ln n - n + 1$$

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k \ j	1	2	3	4	...	n
1	1					
2	1	1/2				
3	1	1/2	1/3			
4	1	1/2	1/3	1/4		
	...					
n	1	1/2	1/3	1/4	...	1/n

- $$\sum_{j=1}^{n} \sum_{k=j}^{n} 1/j$$

- $$\sum_{j=1}^{n} 1/j \sum_{k=j}^{n} 1 = \sum_{j=1}^{n} 1/j\,(n-j+1)$$

$$= \sum_{j=1}^{n} \frac{n-j+1}{j} = \sum_{j=1}^{n} \frac{n+1}{j} - \sum_{j=1}^{n} 1$$

$$= (n+1) \sum_{j=1}^{n} 1/j - n = (n+1) H_n - n$$