

Subject: Problem Set 8

Year:

Month:

Date:

Problem 1

$$(a) \Theta(n^3(1 + \int_1^n \frac{u}{u^4} du)) = \Theta(n^3 + \frac{n^3}{-2n^2}) = \Theta(n^3)$$

$$(b) \Theta(n^{-3}(1 + \int_1^n \frac{u}{u^{-3}+1} du)) = \Theta(n^{-3} + n) = \Theta(n)$$

$$(c) P < 1 \rightarrow \Theta(n)$$

$$(d) \text{XXXXXXXXXX} P = \frac{1}{2} \rightarrow \Theta(n^{\frac{1}{2}}(1 + \int_1^n \frac{n^{\frac{1}{2}}}{n^{\frac{3}{2}}} du)) = \Theta(n^{\frac{1}{2}} \ln(n))$$

$$(e) P = \frac{1}{2} \rightarrow \Theta(n^{\frac{1}{2}}(1 + \int_1^n \frac{1}{u^{\frac{3}{2}}} du)) = \Theta(n^{\frac{1}{2}})$$

Problem 2

$$(a) \pm. H.$$

$$(b) T(n/2) \leq C n^{\frac{1}{2}}$$

$$\rightarrow 4 T(n/2) \leq \frac{n}{2} C$$

$$\rightarrow T(n) \leq \frac{n}{2} C$$

$$\rightarrow T(n) \leq C(n)$$

$$(c) P = 2 \rightarrow \Theta(n^2(1 + \int_1^n \frac{n}{n^3} du)) = \Theta(n^2)$$

$$(d) P < 1 \rightarrow \frac{a}{b^P} < 1 \rightarrow a < b^P \rightarrow a < b$$

Problem 3

Proof. (by Induction)

$$\text{I.H. } P(n) ::= A_n \leq \sqrt{2} + \frac{1}{2^n} \quad \forall n \geq 0$$

$$\text{B.C. } P(0) ::= 2 \leq \sqrt{2} + 1 \quad \checkmark$$

I.S. Failed!

Proof. (by dir arg)

Failed!

I.S.

$$\text{lemma: } \sqrt{2} + \frac{1}{2^n} \geq \sqrt{2}$$

$$\begin{aligned} \rightarrow A_{n+1} &= A_n / 2 + \frac{1}{A_n} \\ &\leq \left(\sqrt{2} + \frac{1}{2^n} \right) / 2 + \frac{1}{\sqrt{2}} \\ &= \sqrt{2} + \frac{1}{2^{n+1}} \quad \checkmark \quad \square \end{aligned}$$

Problem 4

$$(a) \text{ Hom Sol: eq: } x^3 - 4x^2 - x - 6 = 0$$

$$\text{roots: } x = -1, 2, 3$$

$$\text{Gen Sol: } T_n = A(-1)^n + B(2)^n + C(3)^n$$

$$\text{Bdry Cnds: } T_0 = 3 = A + B + C$$

$$T_1 = 4 = -A + 2B + 3C$$

$$T_2 = 14 = A + 4B + 9C$$

$$\text{Comp Sol: } A = 1, B = 1, C = 1$$

$$(b) \text{ Hom Sol: eq: } x^2 + x - 2 = 0$$

$$\text{roots: } x = 1, -2$$

Part

$$\text{Gen Sol: } f(n) = an^2 + bn + c$$

$$\rightarrow a = 1/6 \quad b = 7/18 \quad c = 0$$

$$\text{Gen Sol: } A(-1)^n + B2^n + \frac{1}{6}n^2 + \frac{7}{18}n$$

$$\text{Bdry Cnds: } 5 = A + B$$

$$-4/9 = -A + B + 1/6 + 7/18$$

$$\rightarrow A = 3 \quad B = 2 \rightarrow T_n = 3 + 2(-2)^n + \frac{1}{6}n^2 + \frac{7}{18}n$$