Subject: Problem Set 5

Year: Month: Date:

Problem 1

(a) each Component is connected and acyclic.

the node adj to the deleted node had either been a leaf or an internal node.

(b) Proof by strong Induction.

I.H. $P(L)$ :: a SB tree with L leaves has $2L-1$ nodes.

B.C. $P(1)$ ✓

I.S. Consider SBT with $L+1$ leaves.

delete the root. by (a) we have 2 SBTs. with leaves $L_1, L_2 < L+1$. by I.H. we have $2L_1 - 1 + 2L_2 - 1 + 1$ nodes which is $N_1 + N_2 + 2$ nodes.

Problem 2

(a) lemma: we can 2-color a 2D grid like a chess Board. So it's bipartite.

Corollary: $N \times M$ when $N \wedge M$ are odd is an odd cycle, So it's not bipartite.

Subject: Year: Month: Date:

(b) Proof. (by Induction)

I.H. $P(n) ::=$ n is even $\longrightarrow \exists$ ham cycle

B.C. $P(2)$, outer edges. ✓

I.S. Consider the $((n-1,0),(n-1,1))$ edge. remove it. add $((n-1,0),(n,0))$, $((n,0),(n+1,0))$, $((n+1,i),(n+1,i+1))$ for $0 \le i < m$, $((n,i),(n,i+1))$ for $1 \le i < m$ and $((n,m-1),(n+1,m-1))$.

2. Induction is based on even.

3. yes. No.

Problem 3.

(a) "by induction, each of the two pieces are connected"

(b) 

(c) Proof. (by $\divideontimes$)

assume mangled but not connected.

split into 2 sets. only 1 ~~has~~ could have more than $\lceil \frac{n}{2} \rceil$ nodes $\longrightarrow \divideontimes$

Subject: Year: Month: Date:

Problem 4

(a) Proof (by Induction)

I.H. $P(n) ::=$ n-node tree has $n-1$ edges.

B.C. $P(1)$ ✓

I.S. assume $n+1$-node tree. remove a leaf. $P(n)$.

add back. $n-1+1$ edges. ✓

Proof (by ※)

assume $n-1$ node connected graph but not tree.

$\rightarrow \exists$ cycle. remove edges until no cycle.

then we have tree with $< n-1$ edges

※

(b) Proof (by Induction)

I.H. $P(e) ::$ $e \geq n+1$ ^ connected $\rightarrow \exists$ SPT

B.C. $e = n+1$ ✓

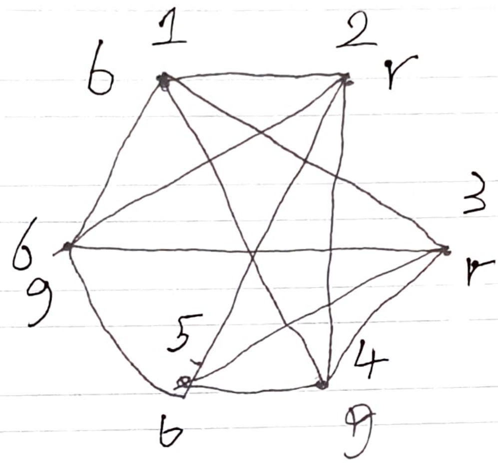
I.S. $e+1 \geq n+1$. then we know if we remove

an edge, by $P(e)$ $\exists$ SPT.

Subject: Year: Month: Date:

Problem 5

(a)



(b) 2. every 2 nodes that are not connected directly are connecte by 2 edges.

(c) 1-6-3-4-5-2-1. uses every node.

(d) can't be <3 because of k_3

Problem 6

(a) v needs even degree to be the first and last.

(b) W has 1 node that goes to it last +2k for every other time.