Subject: Problem Set 1

Year: Month: Date:

P1

(a) $\exists p \in \mathcal{N} . S(P) \wedge A(P).$

(b) $\forall p \in \mathcal{N} . S(P) \wedge T(P) \longrightarrow A(P).$

(c) $\cancel{\forall p \in \mathcal{N}} \neg(\exists p \in \mathcal{N} . T(P) \wedge \neg A(P))$

(d) $\exists P_1, P_2, P_3 . T(P_1) \wedge T(P_2) \wedge T(P_3) \wedge \neg S(P_1) \wedge \neg S(P_2) \wedge \neg S(P_3)$

P2

(a) T Valid

(b) F

P	Q	R	b(P,Q,R)
T	T	F	F

inValid

P3 (a)

(i) $\neg$(A nand B)

(ii) $\neg(\neg(\neg A$ nand $\neg B)) = \neg A$ nand $\neg B$

(iii) A nand $\neg$ B

(b) A nand A

(c) A nand $\neg A = T$ $\neg$(A nand $\neg$A) = F

P4 12

6 > 6

3 > 3 ①

1 1

Subject: Year: Month: Date:

P5

$$R(r^{1/5}) \longrightarrow R(r)$$

$$r^{1/5} = \frac{a}{b} \longrightarrow r = \frac{a^5}{b^5} \qquad \exists a, b \in \mathbb{Z}$$

P6

$$\underline{E(z) \longrightarrow \exists E(x, y, z)}$$

Case 1: 2 odd, 1 Even

$$(2i+1)^2 + (2j+1)^2 + (2k)^2$$

$$= 2^2 i^2 + 4i + 1 + 2^2 j^2 + 4j + 1 + 4k^2$$

$$= 4m + 2 \overset{4}{\equiv} 2 \ \times$$

Case 2: 2 Even 1 odd

$$(2i)^2 + (2j)^2 + (2k+1)^2$$

$$= 4i^2 + 4j^2 + 4k^2 + 4k + 1 \overset{4}{\equiv} 1 \ \times$$

Case 3: 3 odd

$$(2i+1)^2 + (2j+1)^2 + (2k+1)^2$$

$$= 4i^2 + 4i + 1 + 4j^2 + 4j + 1 + 4k^2 + 4k + 1$$

$$\overset{4}{\equiv} 3 \ \times \qquad 4l$$

Case 4: $4i^2 + 4j^2 + 4k^2 = 4l^2$ ✓