Concurrent Signatures

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Algorithms and Computational Complexity
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Outline

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Introduction

What is a Fair Signature Exchange and why important?

Fair Signature Exchange (Example)

- Two parties A(lice) and B(ob)

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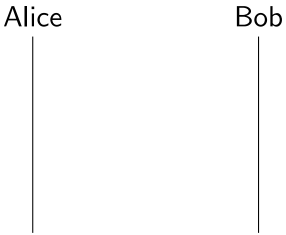

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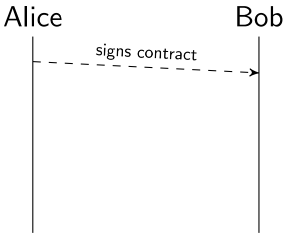

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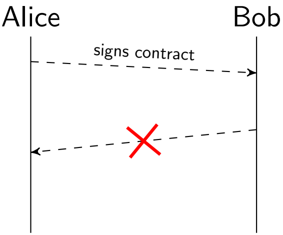

Introduction

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- Is *truly fair* exchange necessary?

Concurrent Approach

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  - denoted as: *keystone*

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Concurrent Scheme

Definition (Concurrent Signature Scheme)

A **concurrent signature scheme** is a digital signature scheme, that holds the following algorithms

- **SETUP**
- **ASIGN**
- **AVERIFY**
- **VERIFY**

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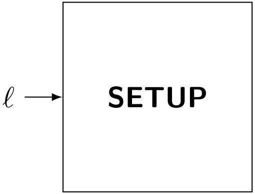
Notation

Let k denote the *keystone*

Algorithms

$\boxed{\textbf{SETUP}}$

Algorithms



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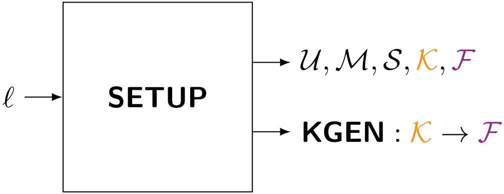
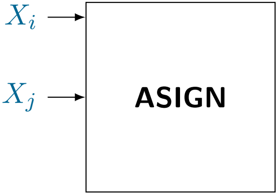

ℓ
SETUP
$\mathcal{U}, \mathcal{M}, \mathcal{S}, \mathcal{K}, \mathcal{F}$
KGEN : $\mathcal{K} \to \mathcal{F}$

Algorithms

ASIGN

Algorithms



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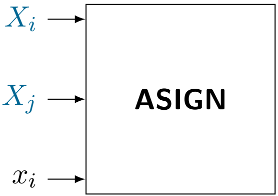

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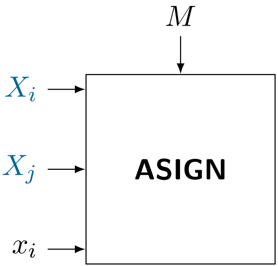

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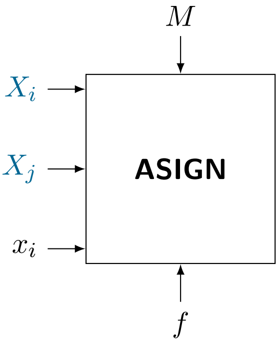

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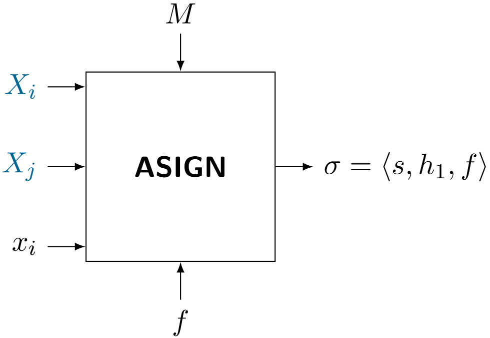
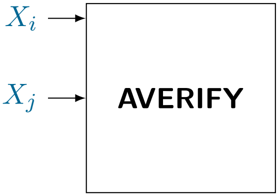


Algorithms

AVERIFY

Algorithms



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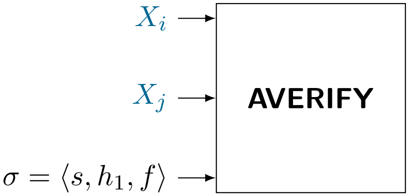

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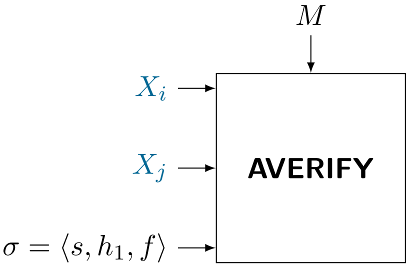

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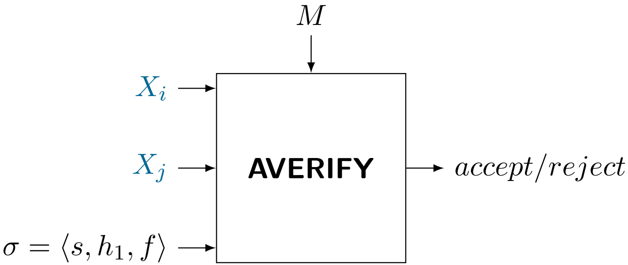

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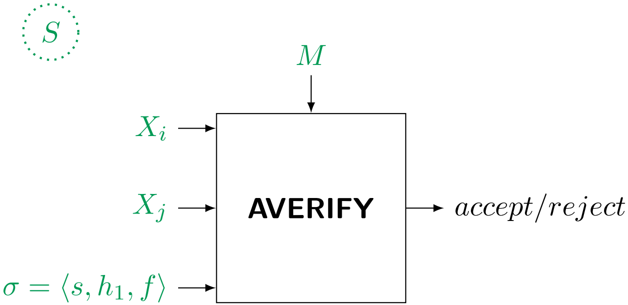

Algorithms

VERIFY

Algorithms

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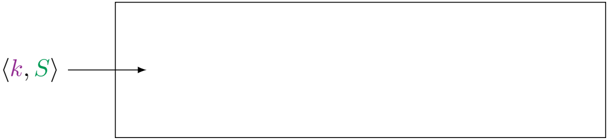
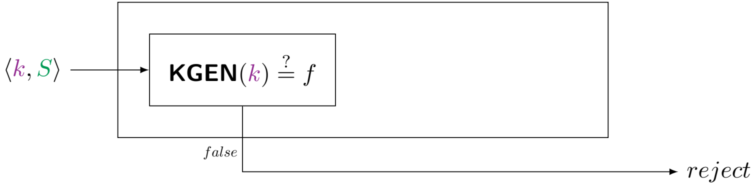


VERIFY

$\langle k, S \rangle \longrightarrow \boxed{\textbf{KGEN}(k) \stackrel{?}{=} f}$

VERIFY



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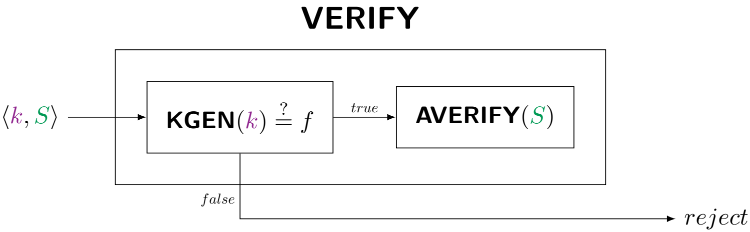
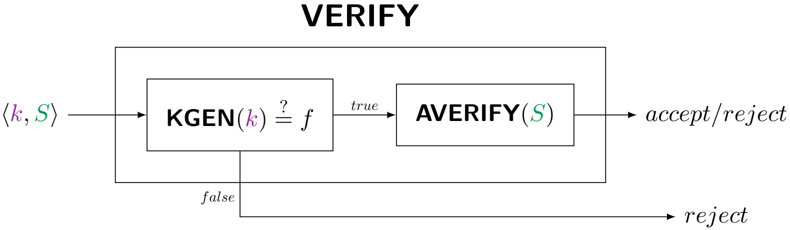

VERIFY
$\langle k, S \rangle$
$\textbf{KGEN}(k) \stackrel{?}{=} f$
$true$
$\textbf{AVERIFY}(S)$
$false$
$reject$

Algorithms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Concurrent Signature Protocol

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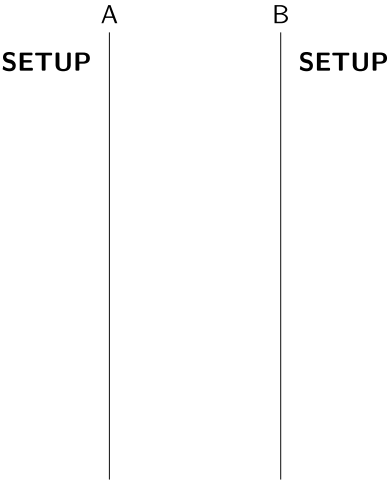
- both parties *ambiguous* until k released

Concurrent Signature Protocol



Concurrent Signature Protocol

A	B

SETUP | **SETUP**

$k \in \mathcal{K},\ f = \textbf{KGEN}(k)$

Concurrent Signature Protocol

A	B
SETUP	**SETUP**
$k \in \mathcal{K}$, $f = \textbf{KGEN}(k)$	
$\textbf{ASIGN}(X_A, X_B, x_A, f, M_A)$	

Concurrent Signature Protocol

A B

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$= \langle s_A, h_A, f \rangle =: \sigma_A$

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$A \xrightarrow{\sigma_A} B$

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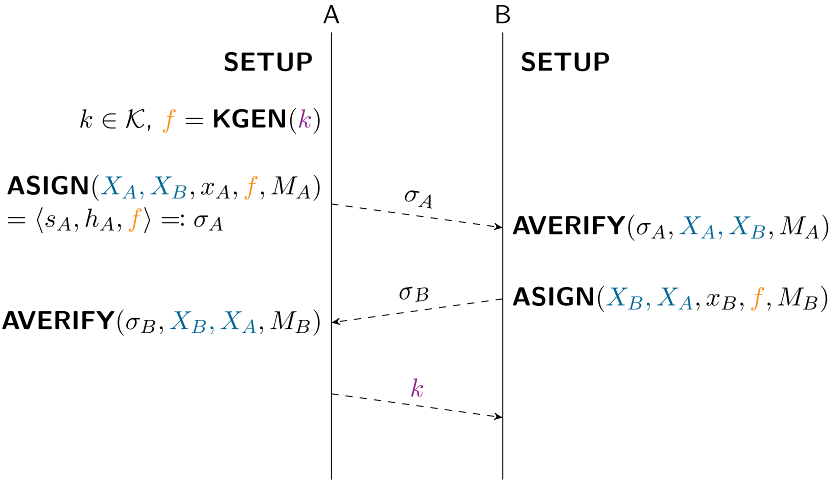

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- Fairness
- Unforgeability
- Ambiguity

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  - **AVerify / Verify...**

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  - **Private Key Extraction. . .**

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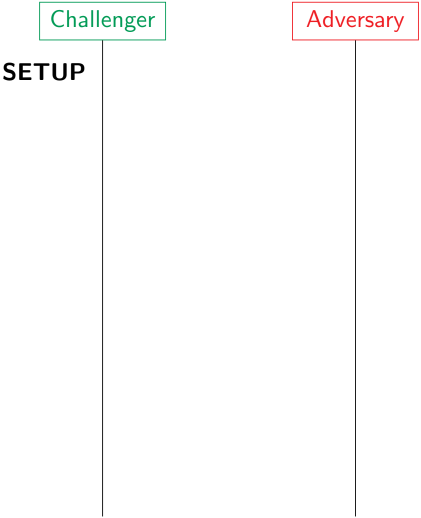
  2. E creates additional valid signature S' with same keystone-fix
     - but only one of both is accepted by **VERIFY**
     - i.e. E creates signature with same keystone but is not bound by it

Ambiguity



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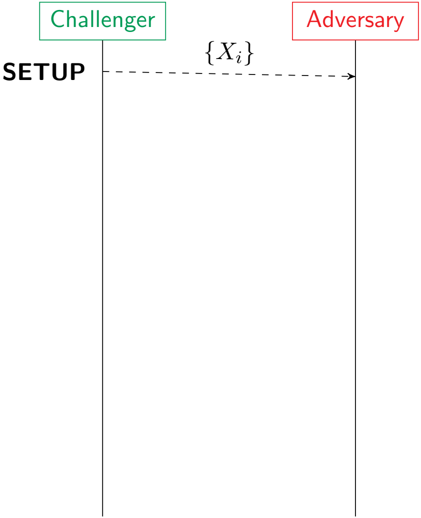

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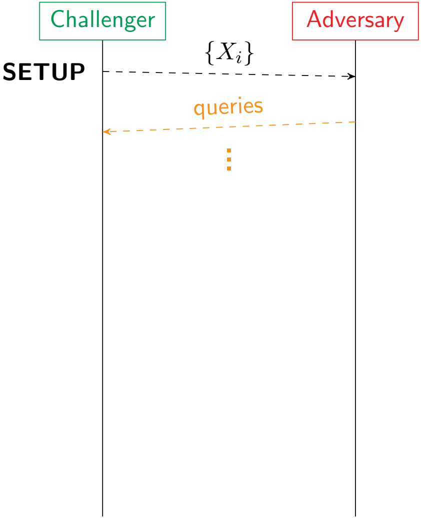

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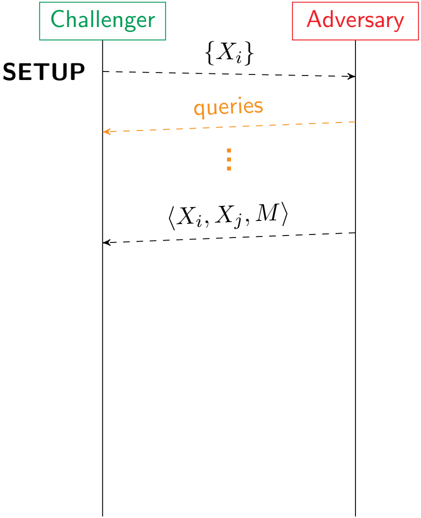
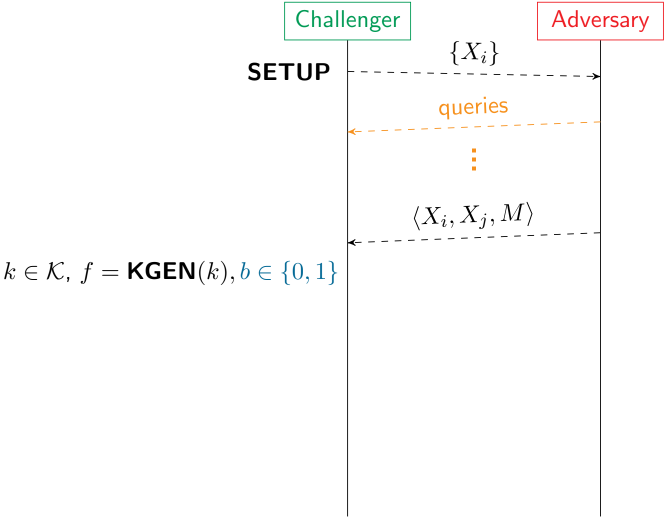


Challenger
Adversary
SETUP
$\{X_i\}$
queries
$\langle X_i, X_j, M \rangle$

Ambiguity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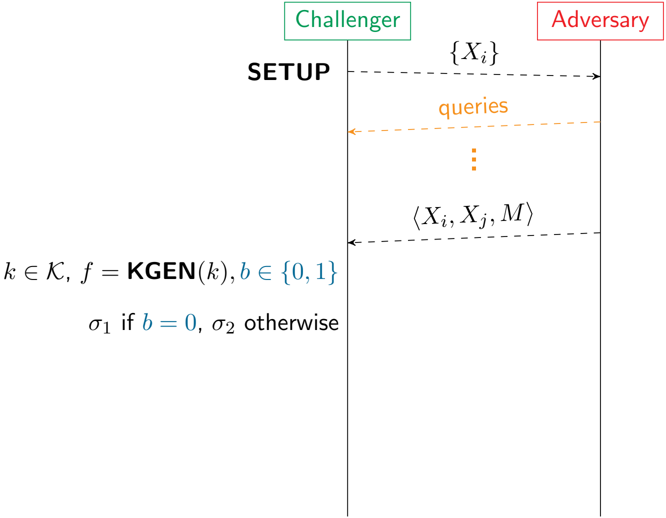
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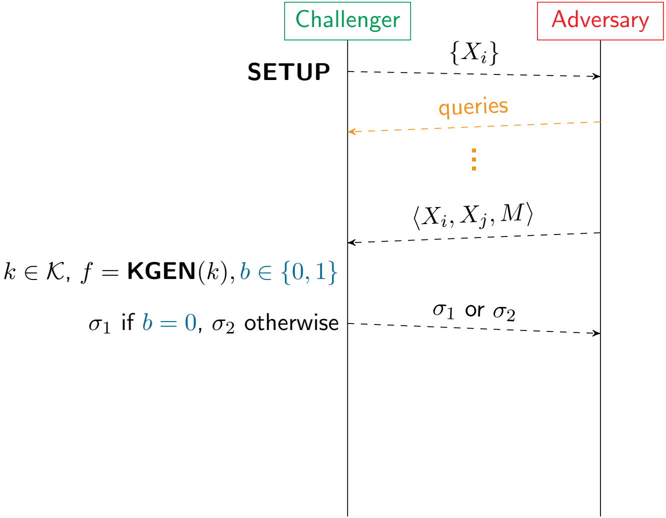
σ_1 if $b = 0$, σ_2 otherwise

Ambiguity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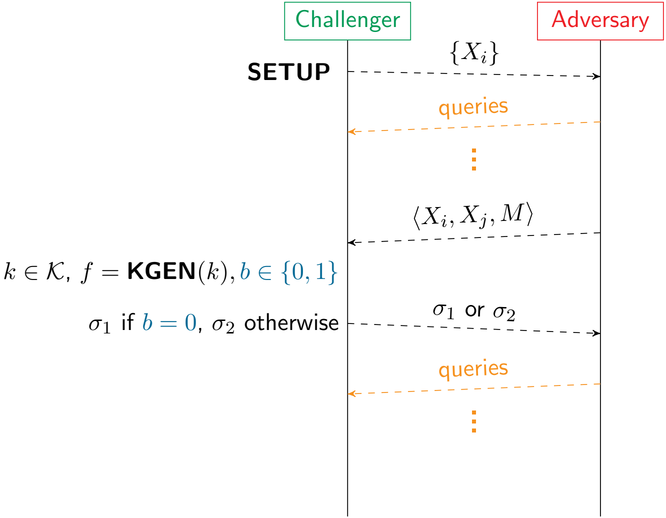
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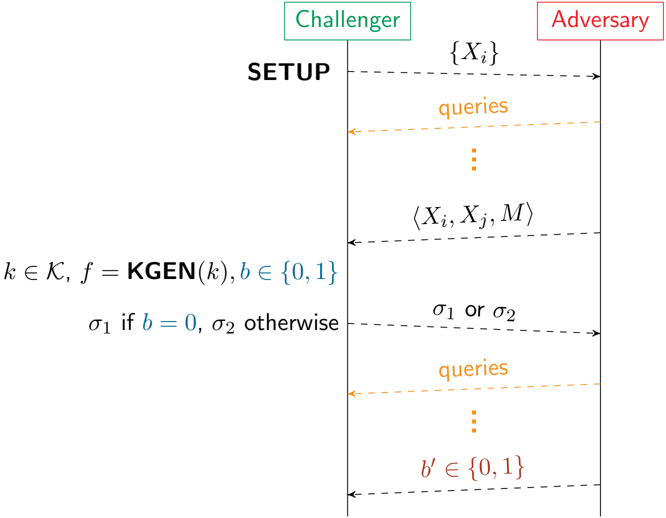

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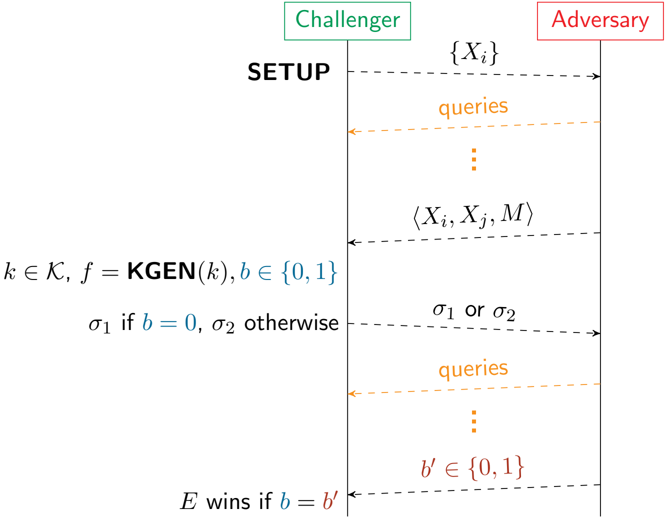

Challenger
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σ_1 or σ_2
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queries
$b' \in \{0,1\}$
E wins if $b = b'$

Unforgeability

- consider previous adversary-challenger game

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  1. No **ASIGN** query with our parameters, and no **Private Key Extraction** query for X_c or X_d
  2. No **ASIGN** query with $\langle X_c, X_i, f, M \rangle$, no **Private Key Extraction** query for X_c

Outline

Discrete Logarithmic Problem

Definition (discrete logarithm)

- Group $\mathbb{G}$ given
- b^k always defined in $\mathbb{G}$ through $b^x = \underbrace{b \cdot b \cdots b}_{x \text{ times}}$
- the **discrete logarithm** is an integer x, such that $b^x = a$

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Definition (group generator)

Let $\mathbb{G}$ be a cyclic group of order p. Then $g \in \mathbb{G}$ is generator of $\mathbb{G}$ if

$$\mathbb{G} = \{g^i \bmod p \mid i \in \mathbb{N}\}$$

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  - public keys $X_i = g^{x_i} \bmod p$ published

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- **AVERIFY** : $\langle\langle s, h_1, f\rangle, X_i, X_j, M\rangle$
  - returns: $h_1 + f \stackrel{?}{=} H_2(g^s {X_i}^{h_1} {X_j}^f \bmod p \parallel M) \bmod q$

Lemmas

Lemma (Fairness)

The concurrent signature scheme in our example is fair in the random oracle model

Lemma (Ambiguity)

The concurrent signature scheme in our example is ambiguous in the random oracle model

Lemma (Unforgeability)

The concurrent signature scheme in our example is existentially *unforgeable* under a *chosen message attack* in the random oracle model, assuming the hardness of the discrete logarithm problem

Random Oracle

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An **oracle** (machine) is an abstract machine that takes an input and generates a solution for it without knowing its inner workings.

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Definition (Random Oracle)

A **random oracle** is a oracle, which fullfills the following properties

- returns each *unique* request with a truly random value (chosen from output domain)
- repeated requests (always) return the same response

Definitions

Definition (Chosen Message Attack)

A chosen message attack allows the adversary to query the signature query with messages of her choice.

Definition (Negligible Function)

A negligible function, is a function negl such that

$$\mathrm{negl}(n) < \frac{1}{n^c}$$

for all sufficiently large n

Forking Lemma

applies to signature schemes with signature form $\langle r_1, h, r_2 \rangle$ on message M

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Definition

If an algorithm E creates a signature $\langle r_1, h, r_2 \rangle$ from public data with non-negligible probability, there exists algorithm A which controls E and forces it to rerun to create a second valid signature $\langle r_1, h', r_2' \rangle$.

Unforgeability - Proof

Proof Idea:

1. Assumption: hardness of the discrete logarithm

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2. Proof by contraposition: Scheme is forgeable with an non-negligible probability
3. Create system, that uses the adversary to solve the discrete logarithm problem

Proof - Setup

- need signature format $\langle r_1, h, r_2 \rangle$

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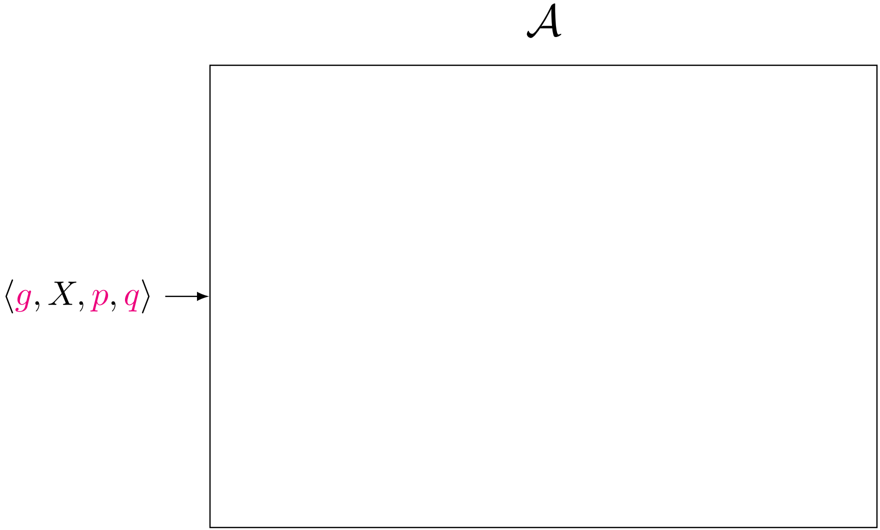
- $\mathcal{A}$ is an algorithm that solves the discrete logarithm problem
  - simulates random oracles and challenger C
  - input: $\langle g, X, p, q \rangle$
  - goal: find $x \in \mathbb{Z}_q$ s.t. $g^x = X \bmod p$

Proof - Simulation

$\mathcal{A}$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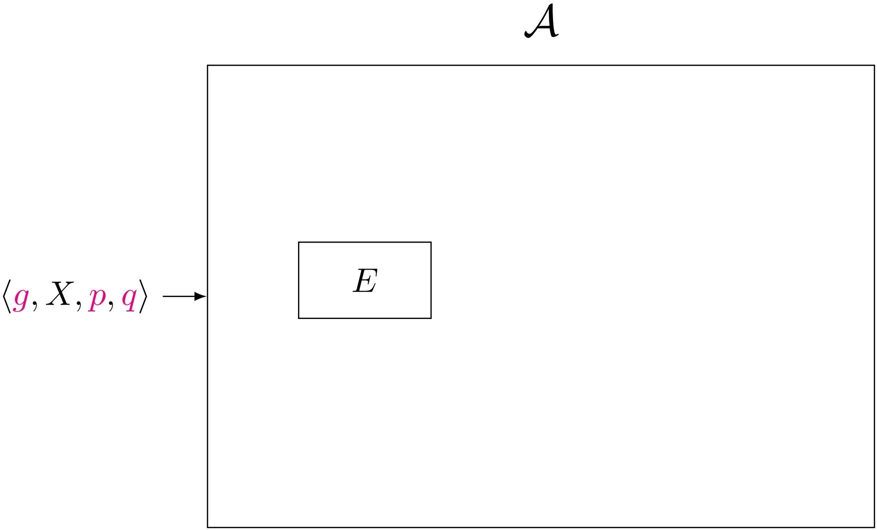

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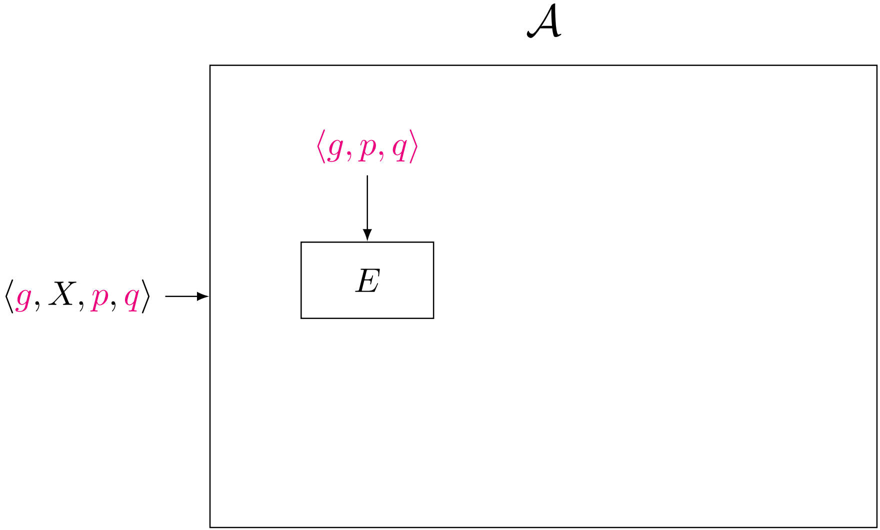

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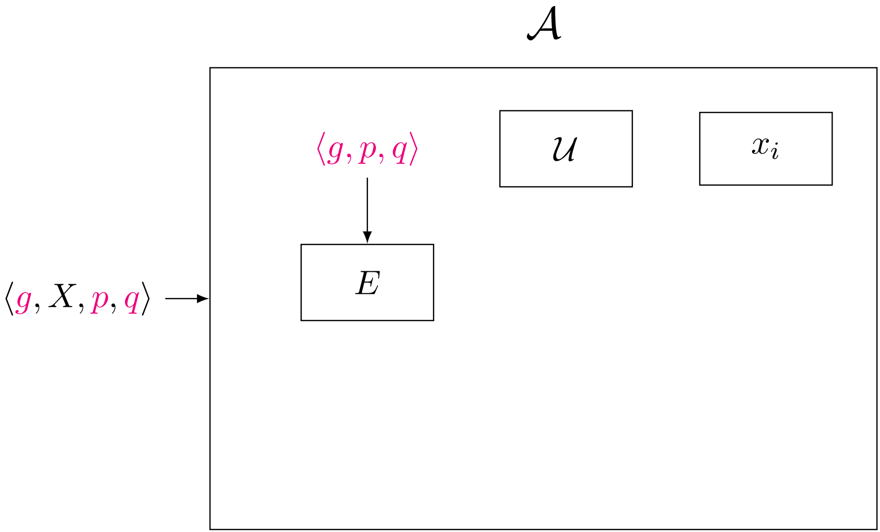

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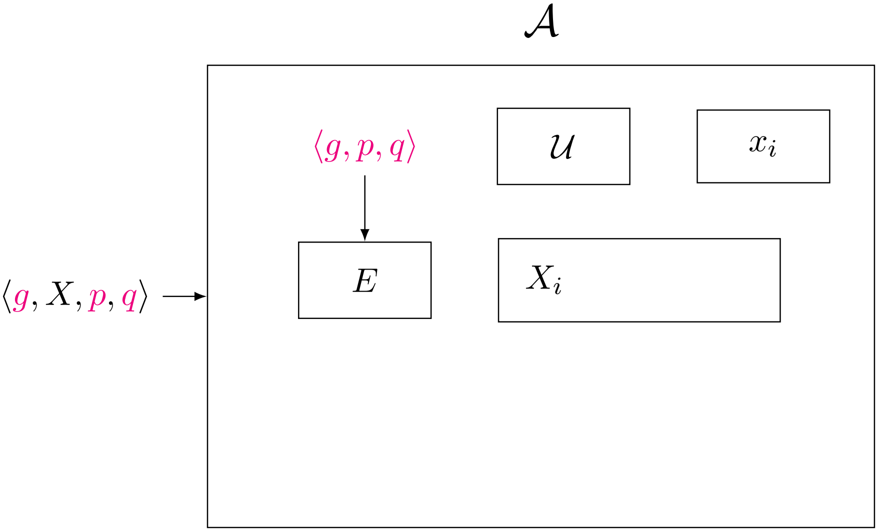

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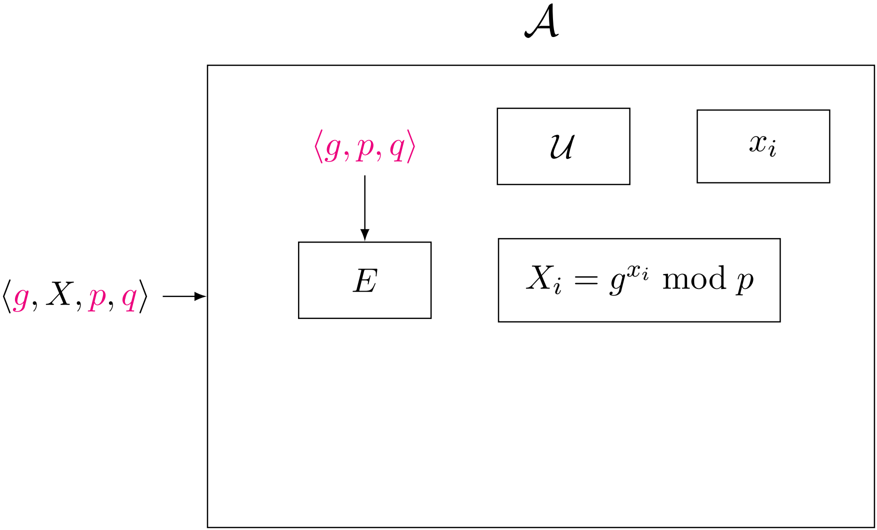

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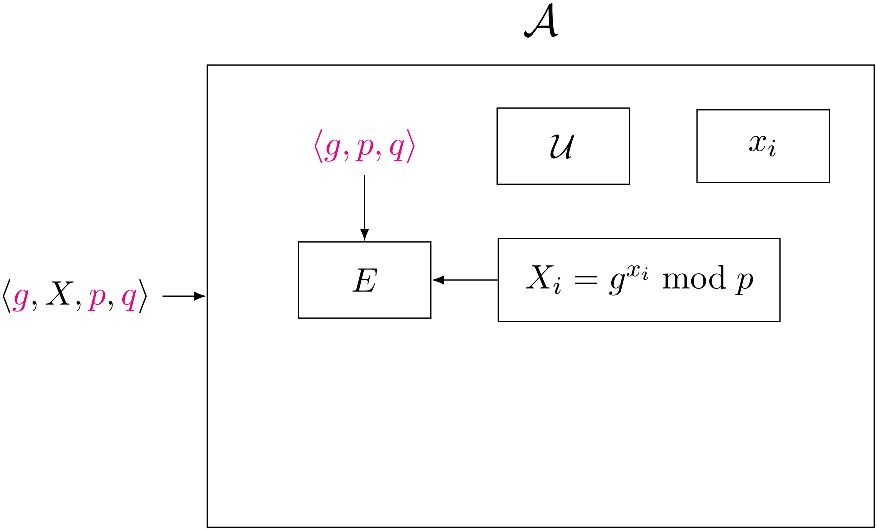

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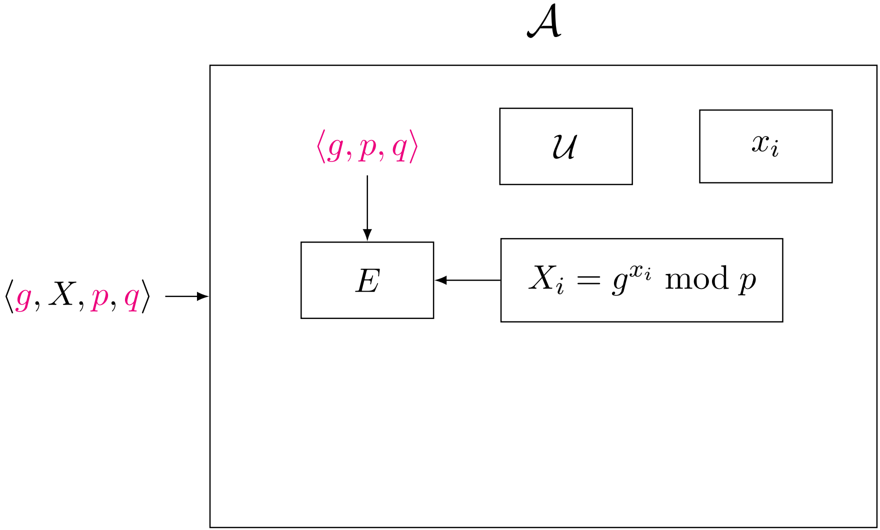

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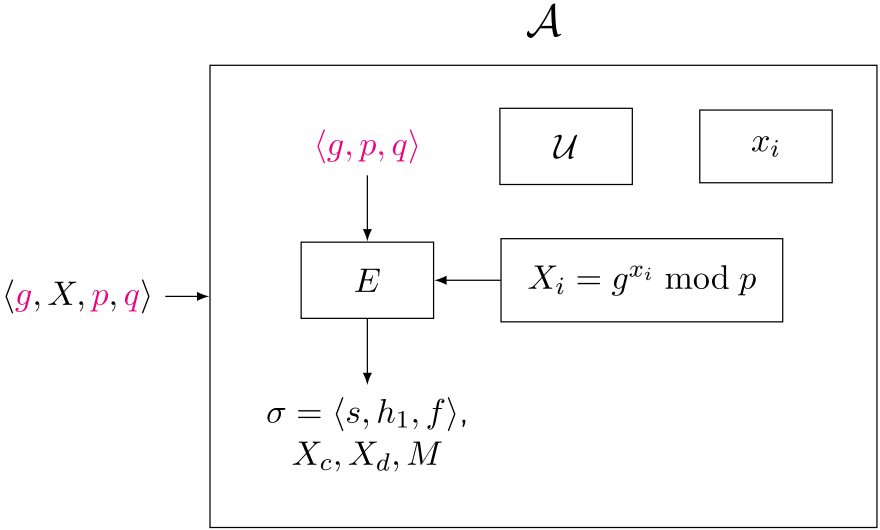

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Proof - Simulation

with second case we have equation

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Proof - Simulation

$$g^{s} X^{h_1} {X_d}^{f} = g^{s'} X^{h_1'} {X_d}^{f}$$
$$s + x h_1 + f = s' + x h_1' + f$$

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- solved the discrete logarithm problem with non-negligible probability
- Henceforth, $\mathcal{A}$ solves the discrete logarithm problem in case 1

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Case 2: $h' = h$, output of signature query $\langle X_{c'}, X_{d'}, f', M' \rangle$

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- due to $h = h'$ we have

$$h = H_2(g^s X^{h_1} X_d^f \parallel M) = H_2(g^{s'} X_{c'}^{h'_1} X_{d'}^{f'} \parallel M') = h'$$

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- when $X_{c'}, X_{d'} \neq X$, or $X_{c'}, X_{d'} = X$ but exponents are different

$$g^s X^{h_1} X_d^f = X_{c'}^{h_1'} X_{d'}^{f'}$$

  is easily solveable for x

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- $\mathcal{A}$ can solve the discrete logarithm in case 2
- Hence, $\mathcal{A}$ can solve the discrete logarithm $\lightning$

Outline

Conclusion

- sign entities without a significant disadvantage
- ambiguous until all parties are committed
- still not *truly fair*