REPORT FOR EE-735 ASSIGNMENT 1

Q1:

A) The Laplace equation is a partial differential equation describing a scalar function's distribution over space. It is a special case of the Poisson equation where volume charge density in space is zero. To find out the capacitance per unit width of a parallel plate capacitor using Laplace equation, we will use Gauss' Law to estimate the charges on the capacitor plate and then use the formula of C= (Charge per unit width)/Voltage across plates.

To go about implementing this problem in Matlab, we will consider a big box which encloses the capacitor and the voltage at the boundary of the capacitor is 0V. Then we will discretize space in x and y directions(2D) by choosing an appropriate number of points to get proper values according to our error tolerance. To begin the problem, we will initialise the voltage matrix in space according to the dimensions of our plates and where it is situated inside the box. (We consider the plates to be symmetric inside the box.) Then we will compute the voltage derivatives to solve the Laplace equation. The first step in computing the partial double derivative as required in the Laplace equation would be to compute the single derivate in single space. This can be done in two ways – Forward difference and Backward difference method.

Forward Difference:

$$E_{i,j}(x) = -\frac{V_{i+1,j} - V_{i,j}}{\Delta x_{i,j}}$$

Here we will compute the derivative between a point (i,j) and the point in front of it in the X-direction (i+1,j).

Backward difference:

$$E_{i,j}(x) = -\frac{V_{i,j} - V_{i-1,j}}{\Delta x_{i,j}}$$

Here we will compute the derivative between a point (i,j) and the point behind of it in the X-direction (i-1,j).Similarly this will be done in the Y direction also.

Now, to calculate the 2D potential, we will assume the grid spacing is the same in both X and Y directions. This will give us the following expression-

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}, \quad \left(\frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial y} + \frac{\partial^2}{\partial z}\right) V = -\frac{\rho_v}{\epsilon}$$

$$\frac{\left(\frac{V_{i+1,j} - V_{i,j}}{\Delta x_{i,j}}\right) - \left(\frac{V_{i,j} - V_{i-1,j}}{\Delta x_{i,j}}\right)}{\Delta x_{i,j}} + \frac{\left(\frac{V_{i,j+1} - V_{i,j}}{\Delta y_{i,j}}\right) - \left(\frac{V_{i,j} - V_{i,j-1}}{\Delta y_{i,j}}\right)}{\Delta y_{i,j}} = -\frac{\rho_{i,j}(V)}{\epsilon}$$

$$\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta x_{i,j})^2} + \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta y_{i,j})^2} = -\frac{\rho_{i,j}(V_{i,j})}{\epsilon}$$

For zero charge density,

$V_{i,j} = (V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1})/4.$

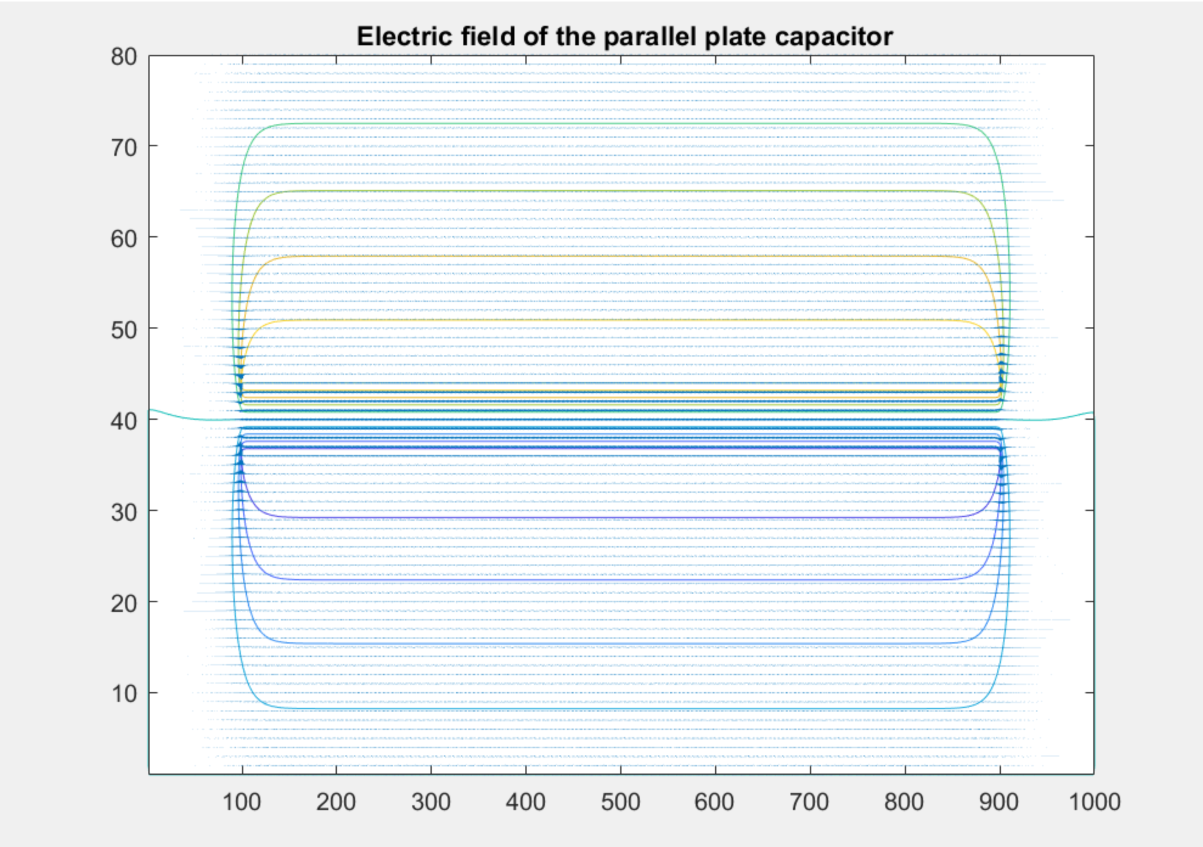
While finding out the potential at all points, care has to be taken that the potential on the plate does not change and this has been included in the code.

Once the potential is found, we will use the gradient function to calculate the 2D electric field. To estimate the charge per unit width on the plates, we will use Gauss’ Law. We assume the Electric field of a parallel plate capacitor to be

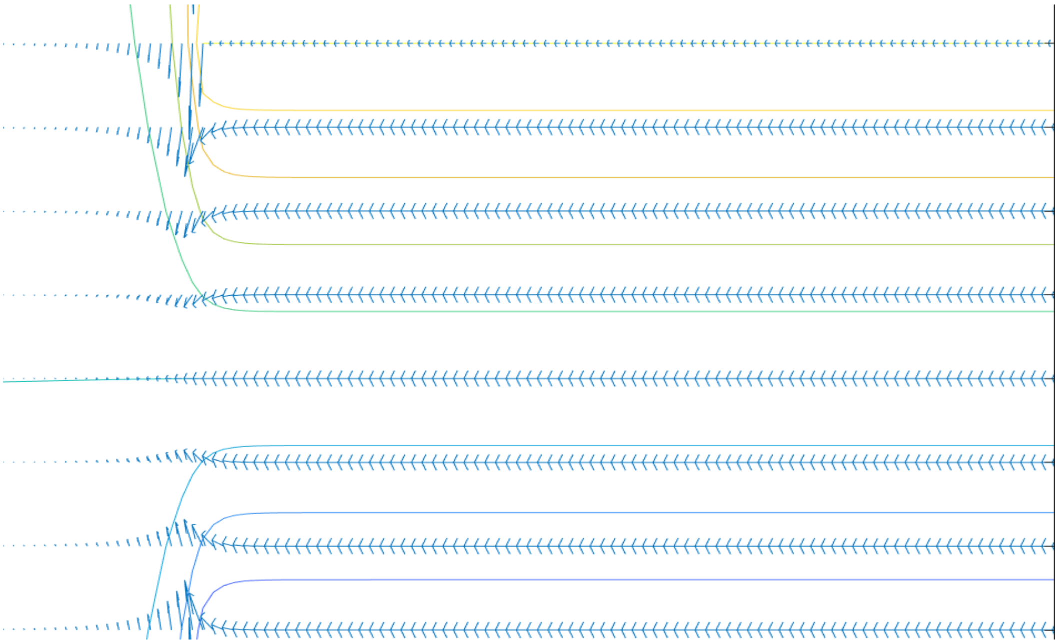
E= (charge per unit area)/Epsilon. From here we can find the charge per unit length by summing (Epsilon*E*dx) over the entire plate. Here ‘E’ implies the total magnitude of the electric field. For proper calculation of charge, the thickness of the plate has been considered in the code. It has also been commented in the appropriate place. Finally we will compute the capacitance per unit width from charge and voltage on the plates.

Plots have been done for the 2D electric field, 2D potential and Equipotential surfaces.

c)2D Electric field -



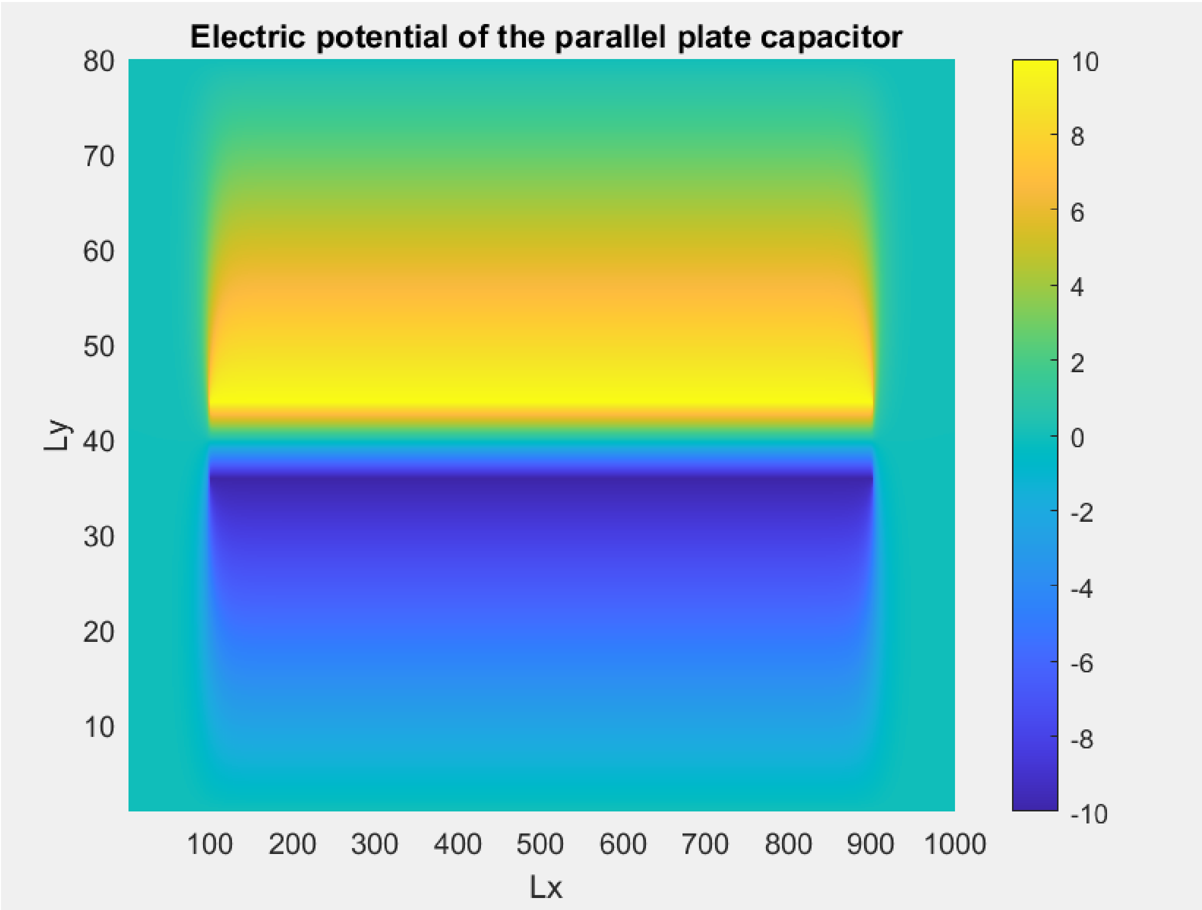
Due to dimensions of the capacitor it is hard to notice the exact field lines however if we zoom in we can see the field lines exiting the positive plate and entering the negative plate including some fringing of fields around the edges.



Legend:

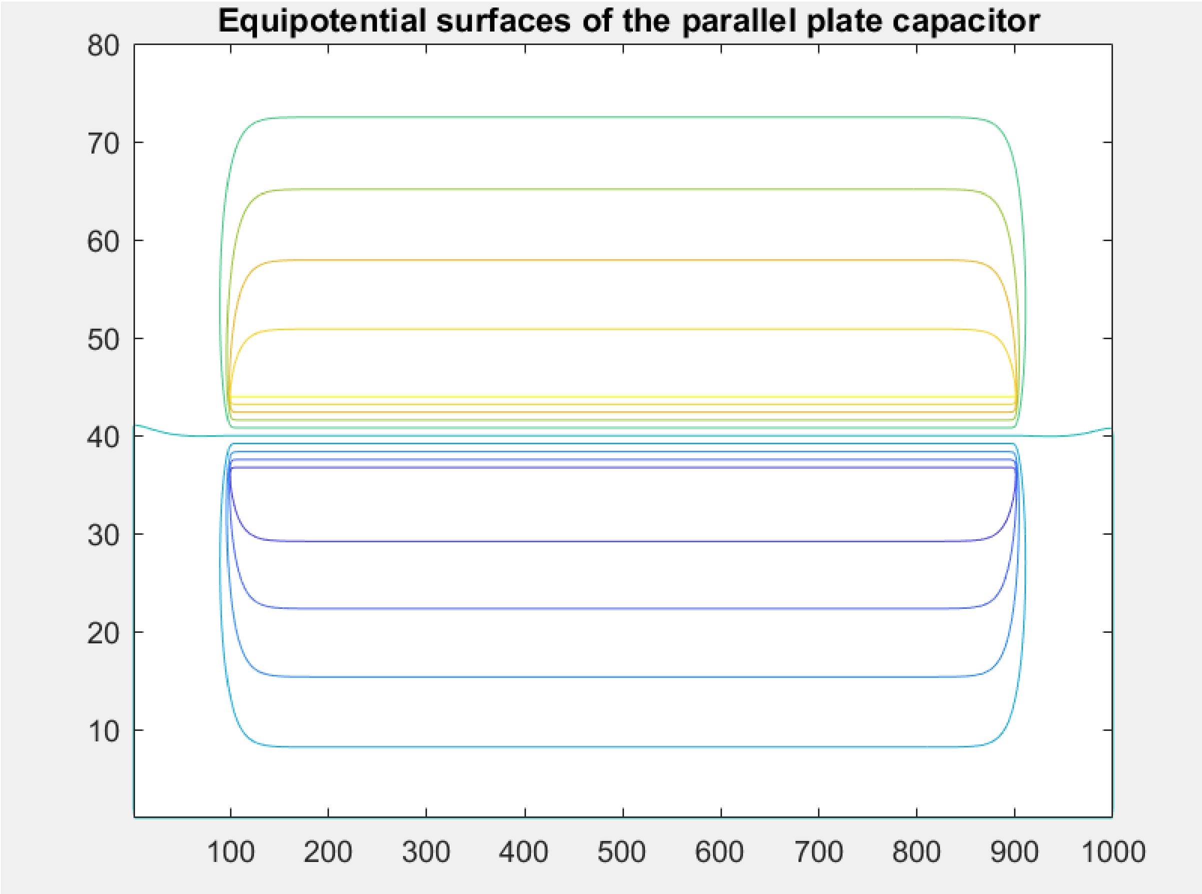
Electric field lines

b)2D potential-



The potential shows what we expect. The potential slowly fades away as we move away from the plates on both sides eventually falling to 0V at the box boundary.

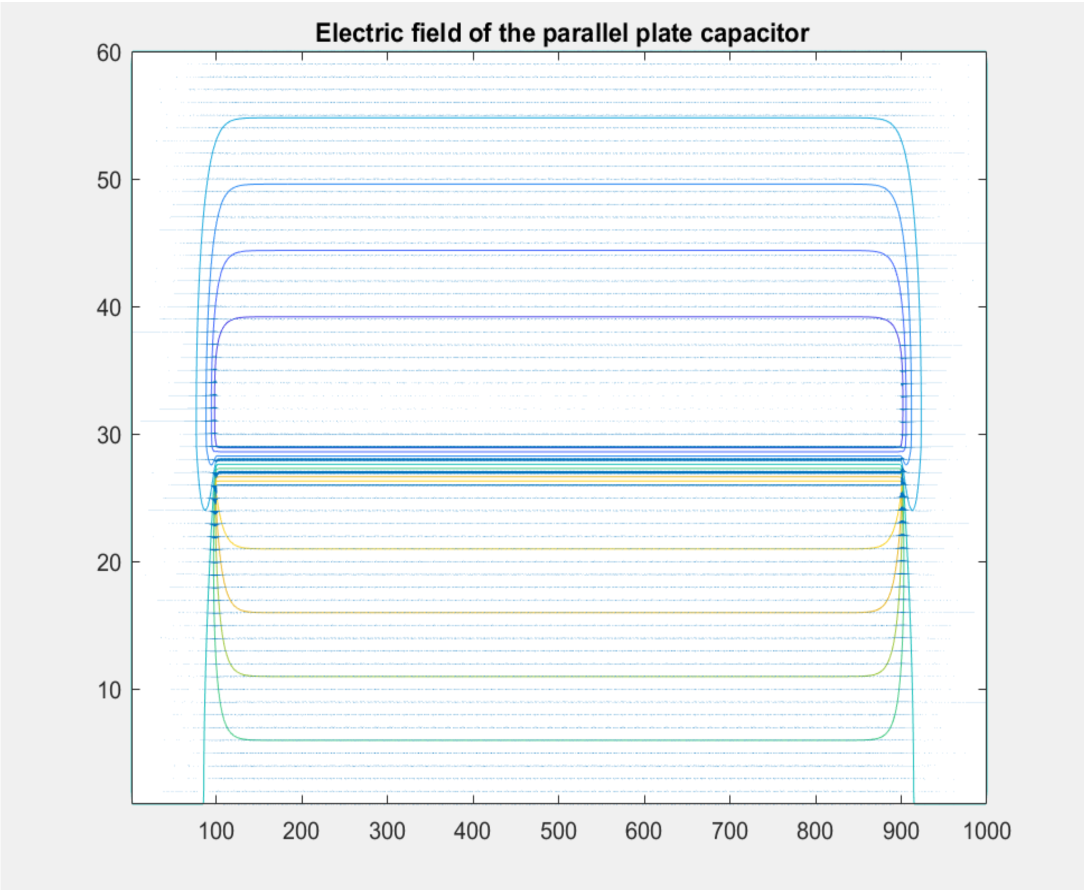
Equipotential surfaces-

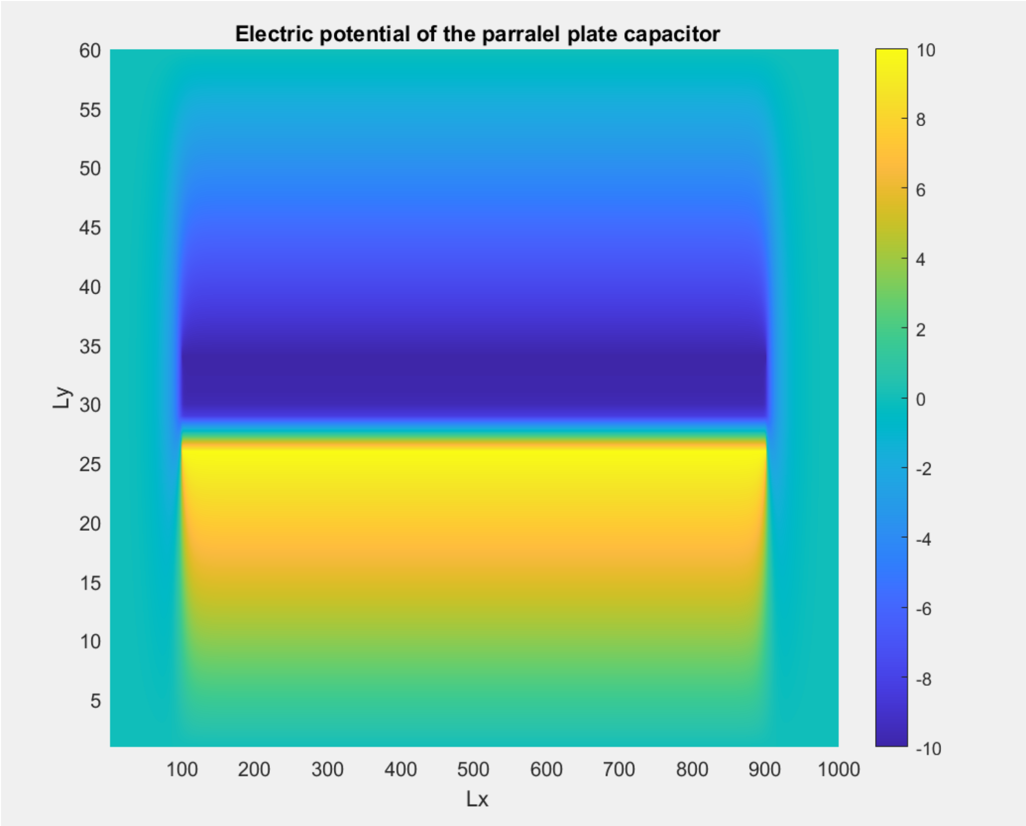


The equipotential surfaces are exactly parallel to the field lines. This also indicates what we expected. The equipotential surfaces also show the fringing field around the plate edges.

d)The theoretical value of the capacitance turns out to be 8.84e-10 F/m. The simulated value turns out to be 13.866e-10 F/m. The excess value in the capacitance accurately explains the assumptions that we made for calculating the electric field of the parallel plate capacitor. We assumed the plates to be infinite with no fringing at the edges. If we have two parallel plates forming a capacitor, the electric field does not end abruptly at the edge of the plates. There is some field outside that plates that curves from one to the other. While summing the electric field on the plate, this causes the real capacitance to be larger than what we would calculate using the ideal formula. We also have more electric field because of the fringe fields.

e)




Electric potential of the parralel plate capacitor
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-8
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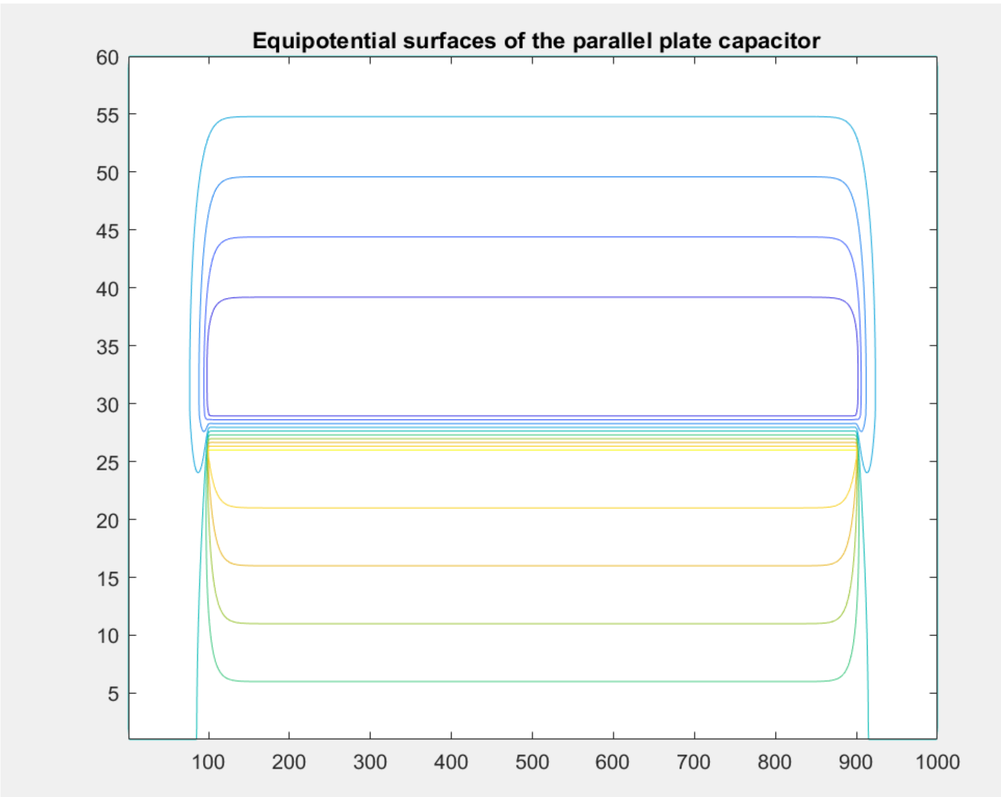
Equipotential surfaces of the parallel plate capacitor
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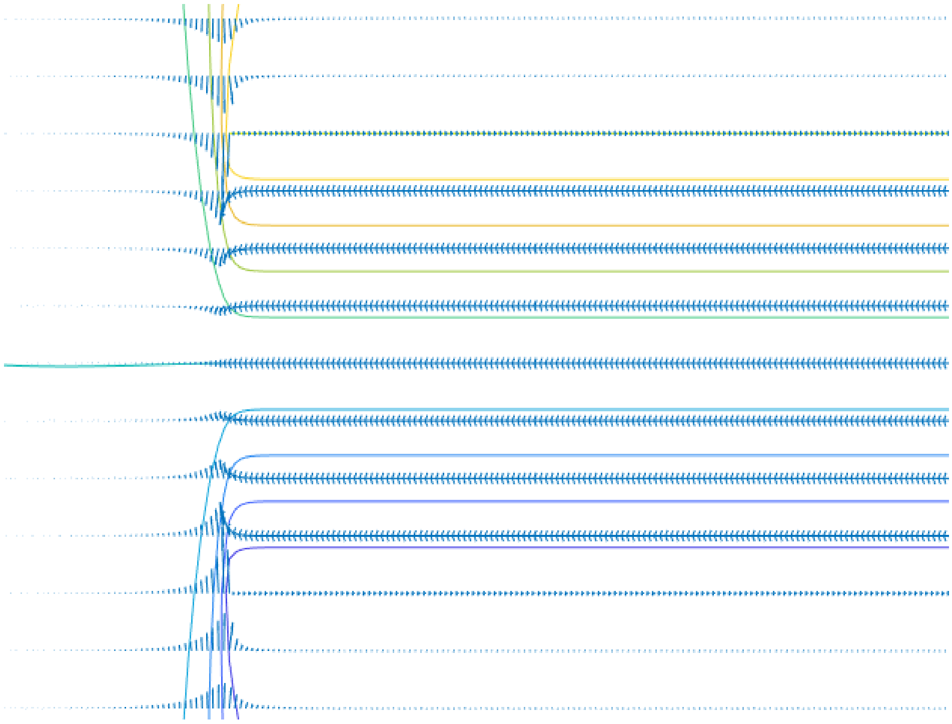
For the second part, we have to additionally consider the boundary conditions at the dielectric interface.

$$\epsilon_{i-1}\frac{V_i - V_{i-1}}{\Delta x_{i-1}} = \epsilon_i \frac{V_{i+1} - V_i}{\Delta x_i}$$

Arranging this we find that the voltage at any point (i , j) is given by

Vt(i,j) = (Vt(i-1,j)+5*Vt(i+1,j))/6

We find that the nature of the electric field, potential and equipotential are exactly the same except for the fact that the electric field inside the dielectric is much stronger. This is denoted by the higher density of field lines in the dielectric region. Also the electric potential fades slower in the dielectric region.

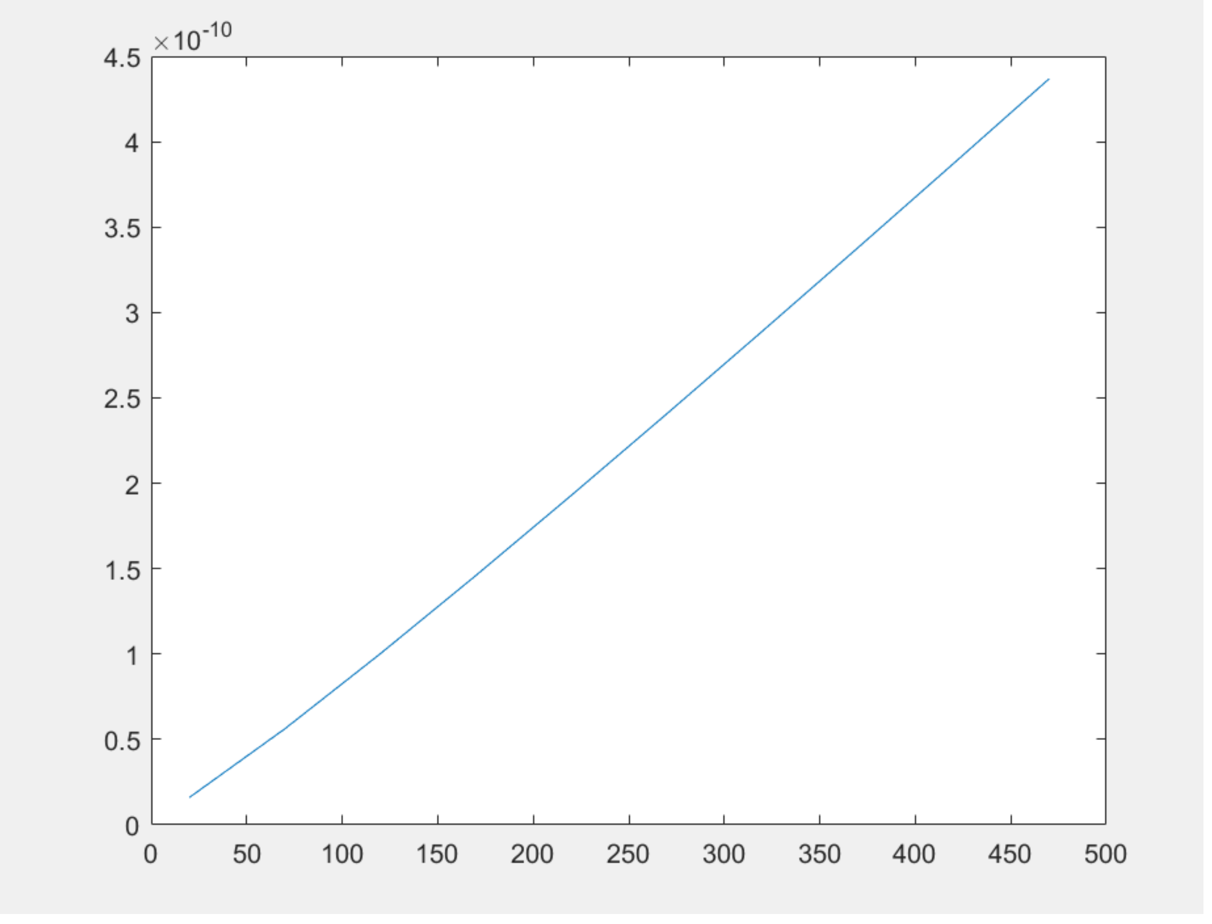


The theoretical value of the capacitance for this case is 14.75e-10 F/m and the practical value comes out to be 21.72e-10. The excess value in the capacitance accurately explains the assumptions that we made for calculating the electric field of the parallel plate capacitor. We assumed the plates to be infinite with no fringing at the edges. If we have two parallel plates forming a capacitor, the electric field does not end abruptly at the edge of the plates. There is some field outside that plates that curves from one to the other. This causes the real capacitance to be larger than what we would calculate using the ideal formula.

We also have more electric field because of the fringe fields. There is also some fringing at the dielectric interface. This increases the capacitance further.

Q3:

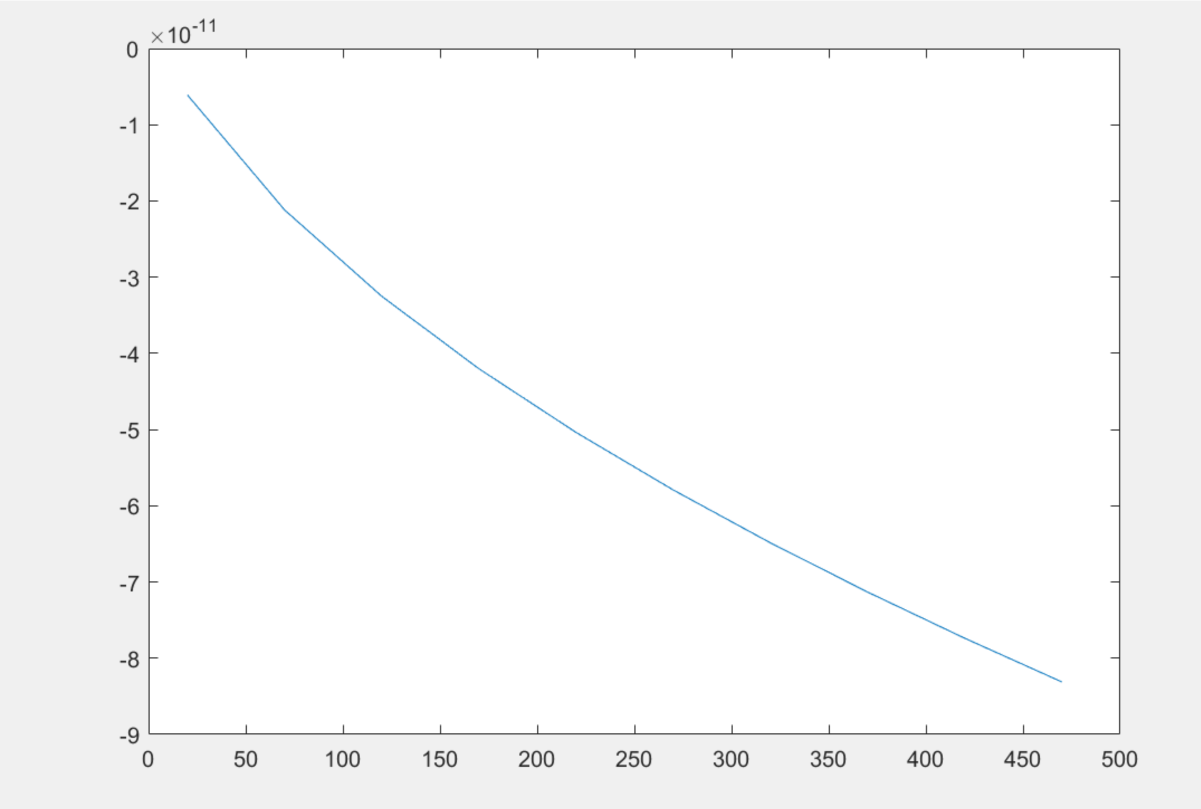
a)



As we vary L from 20nm to 1500nm, we see that the capacitance increases almost linearly as we would expect from the ideal capacitor formula.

Note: Due to very high simulation times, the graph only shows L variation upto 500nm but the code has been written such that it can be run upto 1500nm.

b)



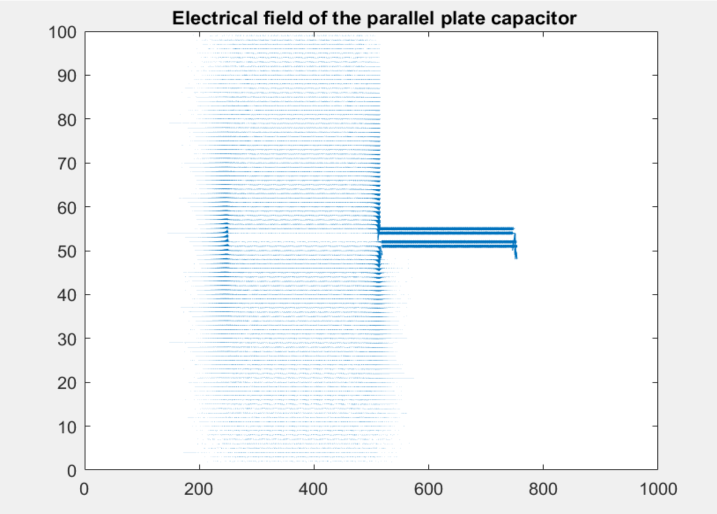
The plot of Cparasitic vs L is shown above. The plot validates our assumptions that as we increase L, our assumption of electric field of the infinite parallel plate capacitor becomes more and more accurate since the parasitic capacitance decreases. If we were to further increase L , we would find that parasitic capacitance would decrease further and eventually fall to zero. Thus the theoretical and practical values of capacitances would be almost same.

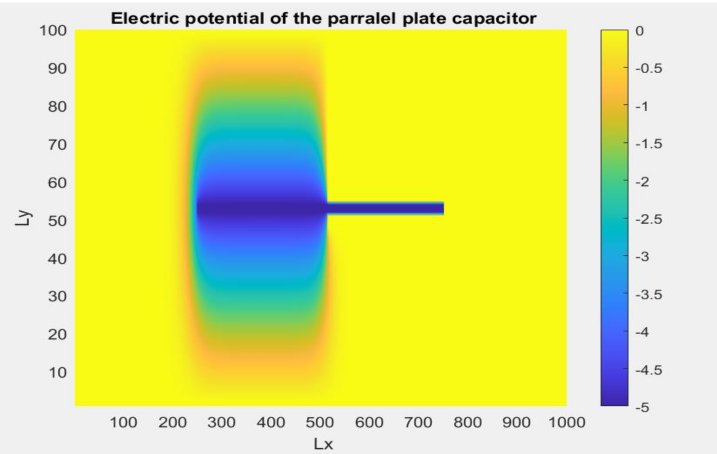
Q2:

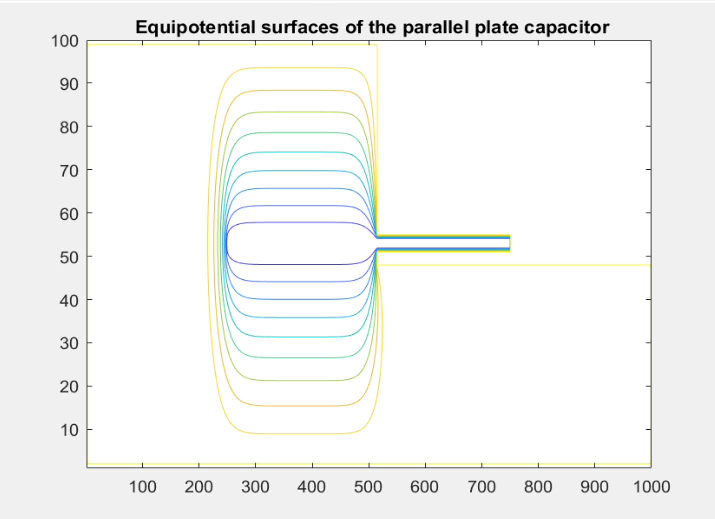
In this problem we proceed exactly as the first problem except the initialising the conditions of the parallel plates differently. After solving the laplace equation and applying necessary boundary conditions, the plots obtained are as follows-

Case1: -5V,d2=30nm

The Numerical Value of Capacitance is 2.779806e-11

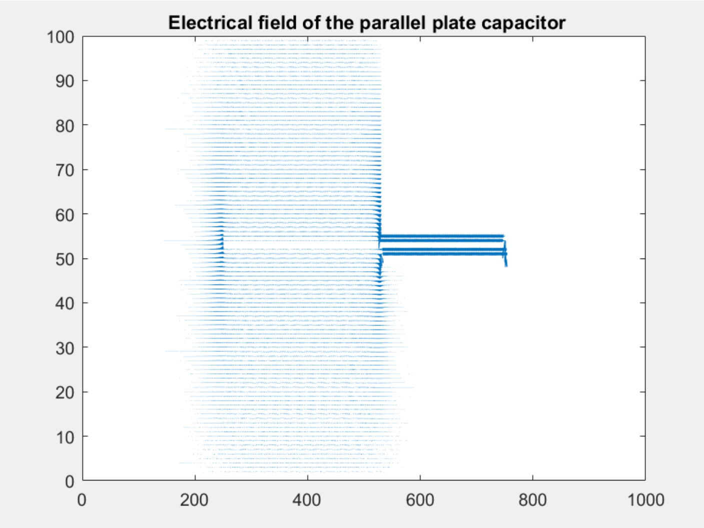


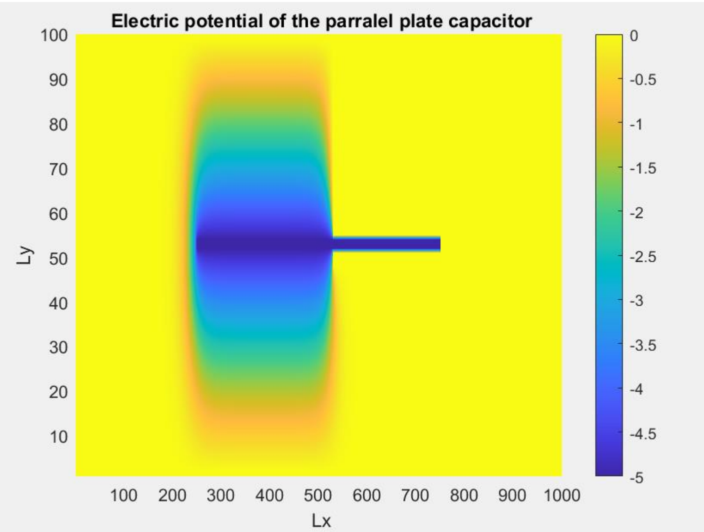


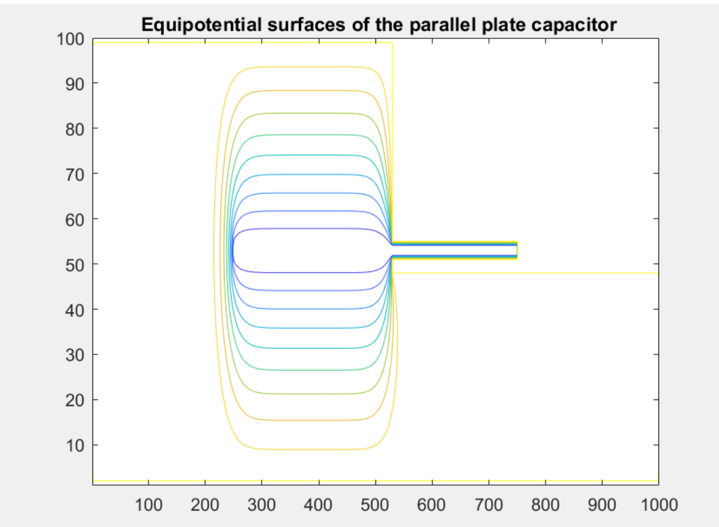


Case2: -5V,d2=60nm

The Numerical Value of Capacitance is 2.596763e-11

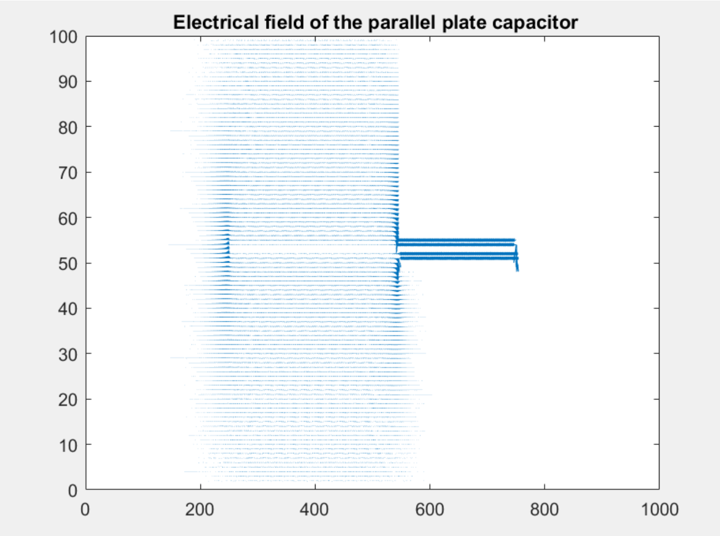


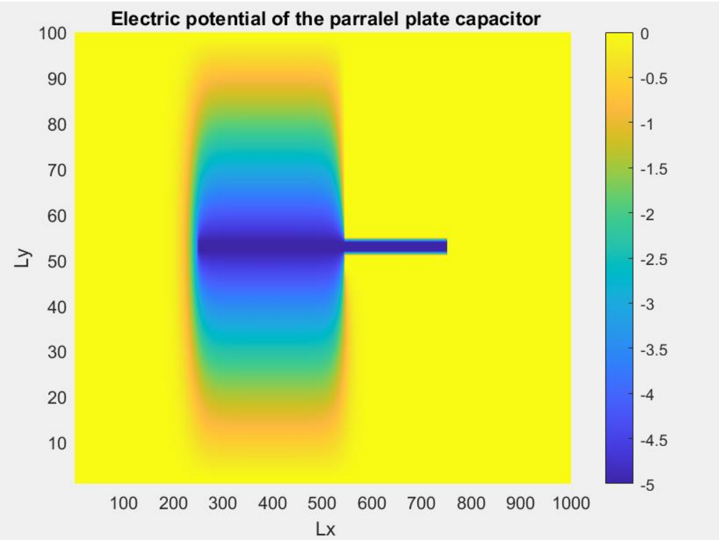


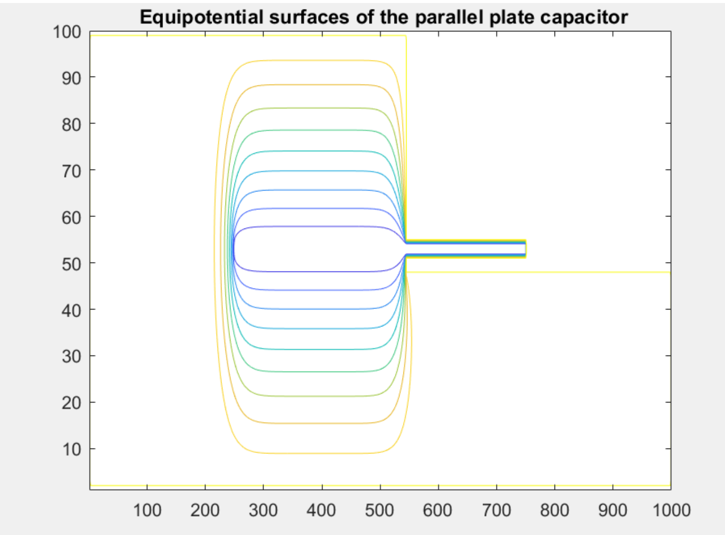


Case3: -5V,d2=90nm

The Numerical Value of Capacitance is 2.427246e-11







The capacitance value for subsequent cases are –

Case4: 0V,d2=30nm ;Capacitance is 2.508358e-12

Case5: 0V,d2=60nm ;Capacitance is 2.492505e-12

Case6: 0V,d2=90nm;Capacitance is 2.475907e-12

Case7: 5V,d2=30nm;Capacitance is 2.917664e-11

Case8: 5V,d2=60nm Capacitance is 2.734620e-11

Case9: 5V,d2=90nm Capacitance is 2.565104e-11

Photos of plots are provided in a separate folder.

d)According to different cases in this problem, the capacitance values will change since their configuration will also change. The two capacitors will hence-

1) Converge to a single capacitance when V3=-5V

2)Be in series when V3=0V

3) Be in parallel when V3=5V