REPORT FOR EE-735 ASSIGNMENT 2

Q1:

A) In this problem we will solve the given PN junction to plot the Charge profile, Electric field and Voltage profile by solving the Gauss Law for the given PN junction diode using numerical integration techniques. We have used the Trapezoidal method to numerically integrate in this problem.

The first thing to find out is the temperature which would be changing according to the last digit of our roll number. My roll number ends with 3 and hence the temperature of operation for my device is T=304.5K.This would automatically change the thermal Voltage V_T . The next step would be to calculate the acceptor doping concentration which can be found from the constant value for V_{bi} (Barrier Potential).

PN Junction Built-in Potential

$$V_{built\text{-}in} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$V_{built\text{-}in}$ ⟶ Built-in potential in volts

N_D ⟶ n type - donor atoms concentration

N_A ⟶ p type - acceptor atoms concentration

n ⟶ concentration of electrons

kT/q ⟶ thermal voltage

T ⟶ temperature in Kelvin

q ⟶ charges in coulombs

Having found the acceptor doping concentration, we will move further to calculate the total depletion length, depletion width on p-side and depletion width on n-side for-

i) Abrupt Junction

Depletion Region

$$W = \sqrt{\frac{2\varepsilon_r \varepsilon_o}{q}\left(\frac{N_a + N_d}{N_a N_d}\right) V_{bi}}$$

- The depletion region is inversely proportional to the doping concentration of the more lightly doped side of a p-n junction

$$W \underset{\sim}{\propto} \sqrt{\frac{1}{N_a}} \text{ if } N_a << N_d \text{ or } W \underset{\sim}{\propto} \sqrt{\frac{1}{N_d}} \text{ if } N_d << N_a$$

From this, the depletion widths on the n-side and p-side of the PN junction can be found easily since the ratio of the widths is inversely proportional to the doping concentrations.

ii)Linearly graded Junction

Linearly graded and abrupt junctions Consider a linearly graded junction in which $N_d - N_a = Bx^m$. If V is the voltage across device, show that the field at the junction E_{max} and the width of the depletion region W are given by,

$$E_{max} = -\frac{eBW^2}{8\varepsilon} \quad \text{and} \quad V_o - V = \frac{eBW^3}{12\varepsilon} \quad .$$

Using one of the above equations and Equation 6.31 to eliminate B, show that

$$W_o^2 = \frac{6\varepsilon V_o}{en_i \exp(eV_o/2kT)}$$

For the linearly graded junction, the depletion widths on the n-side and p-side of the PN junction will be taken as (Total depletion width)/2 as given in the question.

Now we plot the charge profile throughout the depletion layer by applying depletion approximation. We will assume an arbitrary step size according to our tolerance and then initialise the charge profile array across the depletion region depending on whether it is abrupt or linearly graded junction. We can then plot the graph of charge profile.

Then we will use Gauss Law to plot the electric field across the PN junction as follows-

$$E(x) = \int \rho(x)/\varepsilon si \; dx \; ; -xp \leq x \leq xn$$

This integral is performed using the Trapezoidal Numerical Method which is given by-

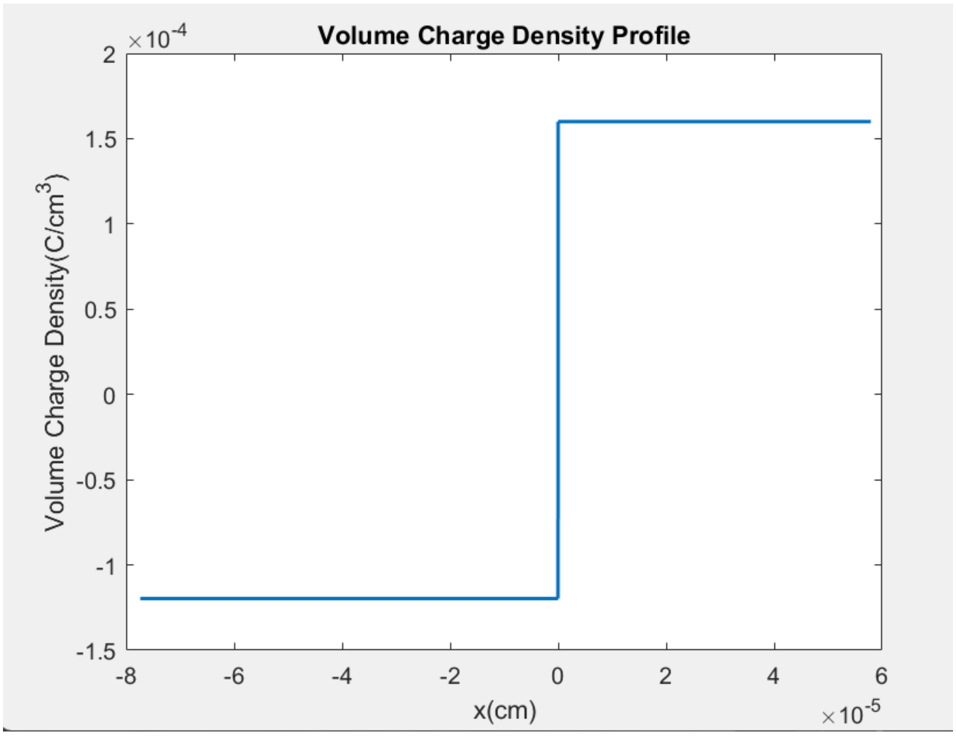
$$\int_a^b f(x)\, dx \;\; \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

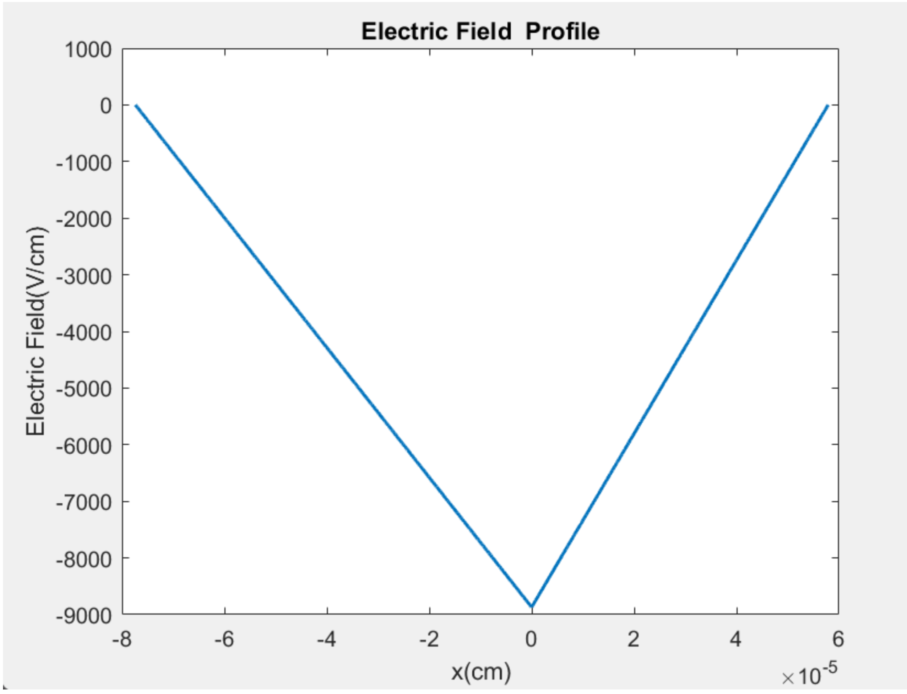
Having found the Electric field , we can now find the potential profile across the PN junction by integrating the electric field array using the Trapezoidal Numerical Method. The Plots obtained are given below.

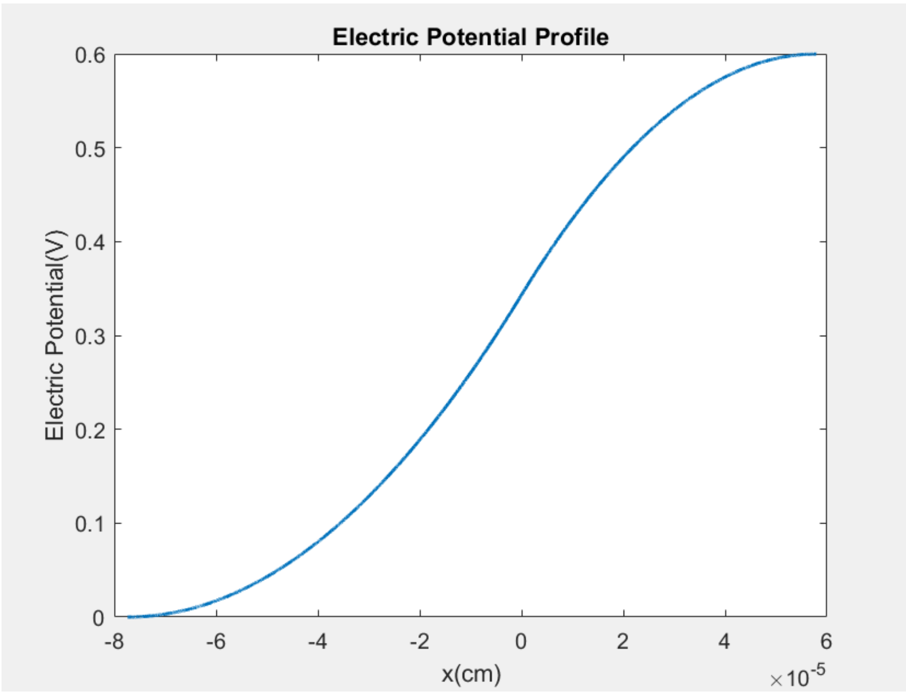
Comparing with the inbuilt ‘trapez’ function, we find that the results are extremely close which means that our algorithm using trapezoidal integration is correct.

Decreasing the step size gives us more accurate field and potential values at points inside the depletion region. But it also increases the run time. We stick to a grid size of 0.0001 for optimal results.

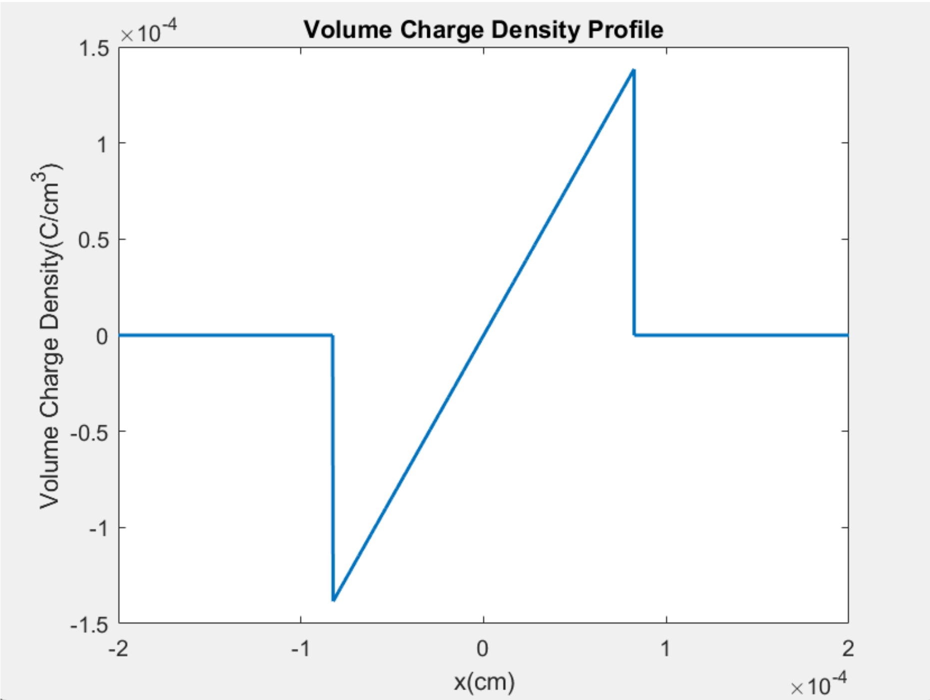
Abrupt Junction

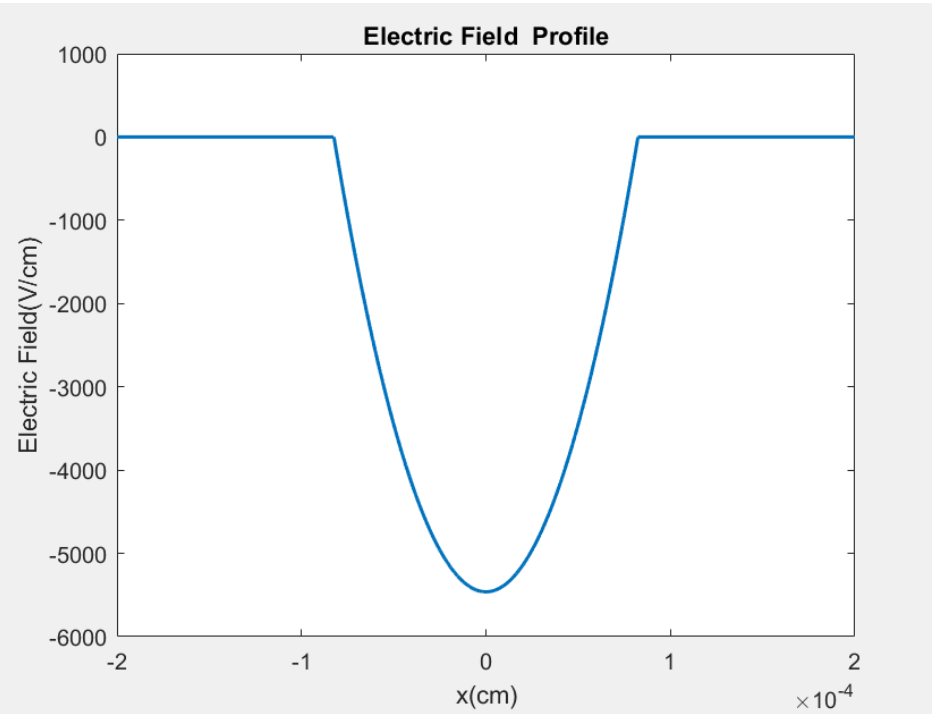


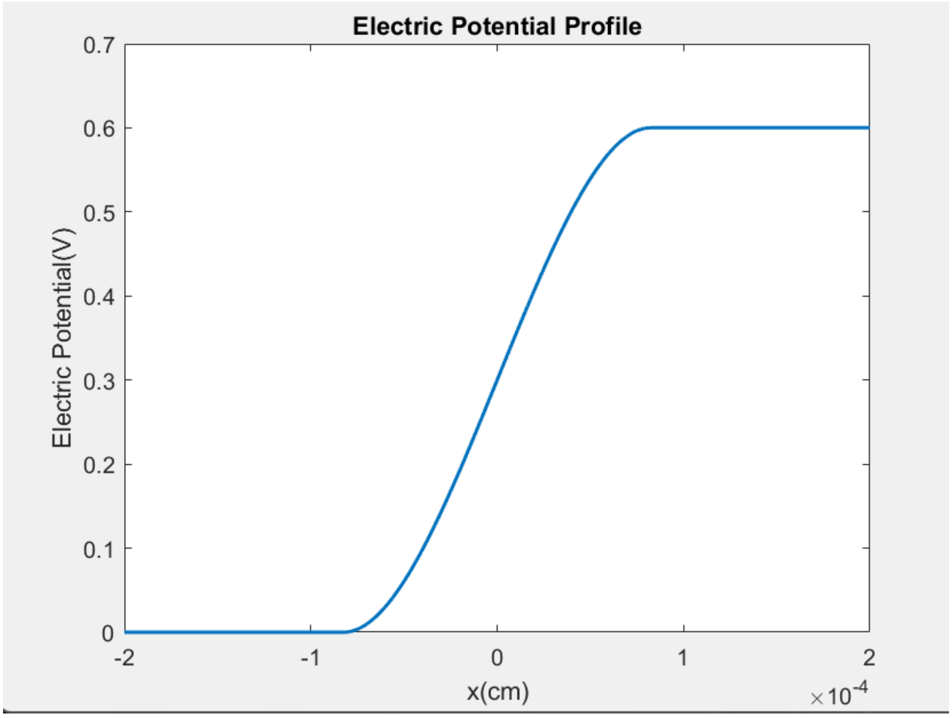

Electric Field Profile
1000
0
-1000
-2000
-3000
-4000
-5000
-6000
-7000
-8000
-9000
Electric Field(V/cm)
-8
-6
-4
-2
0
2
4
6
x(cm)
×10^-5


Electric Potential Profile
0.6
0.5
0.4
0.3
0.2
0.1
0
Electric Potential(V)
-8
-6
-4
-2
0
2
4
6
x(cm)
×10^-5

Linear Junction







B) By replacing the continuous partial derivatives by finite-difference Equation as shown below, ∇2V(i) = (V(i−1) − 2V (i) + V(i+1))/h2. solve the Poisson equation with depletion approximation. For this calculation first compute the L and U matrices numerically. Use these matrices to solve the system of linear equations [A]n×n [V]n×1 = [b]n×1 using LU decomposition method (Do not use inbuilt LU command) and compare graphically the result with inbuilt MATLAB operator A\b.

Simulation Approach :

In this question we will solve the possions equation for finding Voltage across the PN junction . which is shown below for 2D

$$\emptyset_{i-1} - 2\emptyset_i + \emptyset_{i+1} = (\Delta x)^2 \frac{\rho_i}{\epsilon}$$

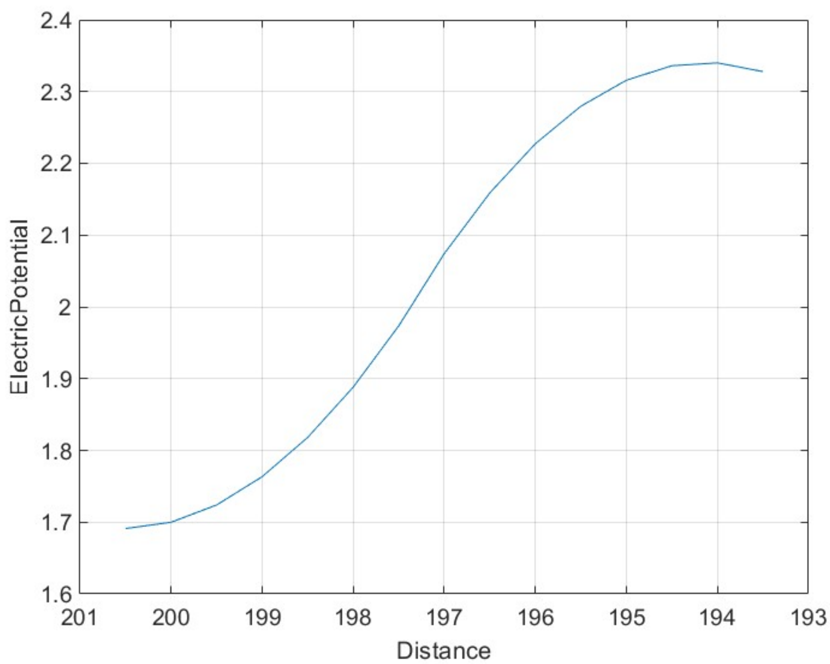
Matrix form of 2D Poissions equation in Presence of Charge

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \emptyset_3 \\ \emptyset_4 \\ \emptyset_5 \\ \emptyset_6 \\ \vdots \\ \emptyset_{n-1} \\ \emptyset_n \end{bmatrix} = \frac{(\Delta x)^2}{\epsilon} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \vdots \\ \rho_{n-1} \\ \rho_n \end{bmatrix}$$

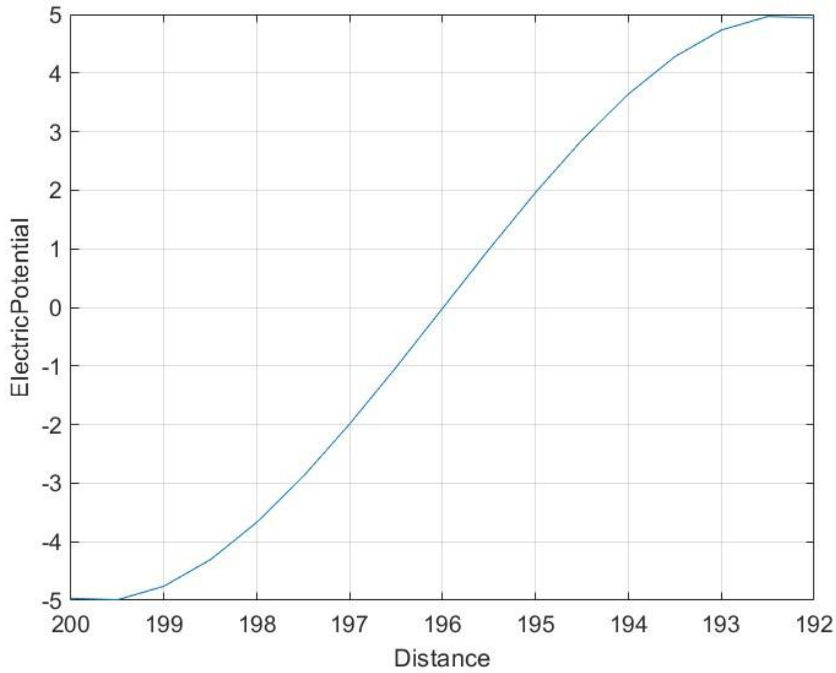
This matrix can be represented as AX-B=0. This family of linear equations can be solved Conveniently by using numerical methods like Gauss Seidel and LU Decomposition . So in this question we are using LU Decomposition. In which we have LU = A and Ld = B combining both we get LUX – Ld =0 . We find d form Ld=B which is forward substitution and then used d to find X from UX=d backward substitution .

Simulation Results :

STEP GRADED JUNCTION



LINEARLY GRADED JUNCTION



A. Use Gauss-Seidel method to solve the linear equations [A]n×n [V]n×1 = [b]n×1 and compare the result graphically with result got from part B.

Simulation Approach :

In this question we will solve the possions equation for finding Voltage across the PN junction . which is shown below for 2D

$$\emptyset_{i-1} - 2\emptyset_i + \emptyset_{i+1} = (\Delta x)^2 \frac{\rho_i}{\epsilon}$$

Matrix form of 2D Poissions equation in Presence of Charge

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \emptyset_3 \\ \emptyset_4 \\ \emptyset_5 \\ \emptyset_6 \\ \vdots \\ \emptyset_{n-1} \\ \emptyset_n \end{bmatrix} = \frac{(\Delta x)^2}{\epsilon} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \vdots \\ \rho_{n-1} \\ \rho_n \end{bmatrix}$$

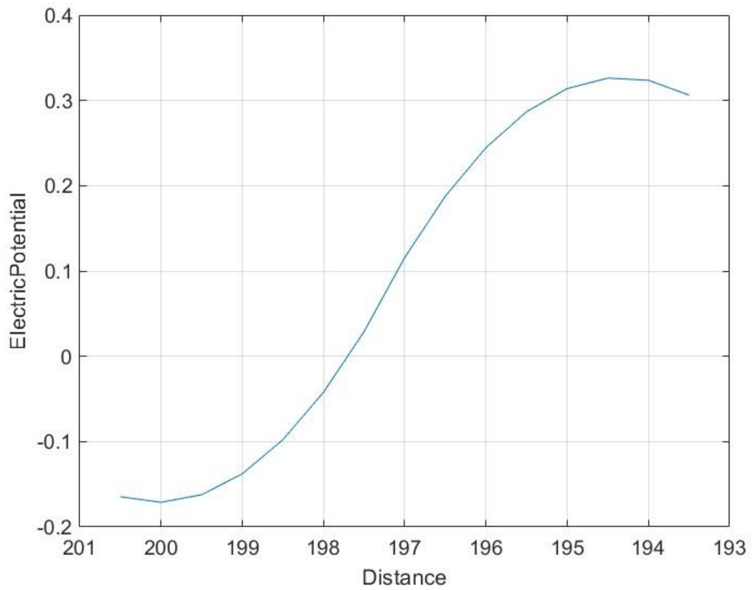
This matrix can be represented as AX-B=0. This family of linear equations can be solved Conveniently by using numerical methods like Gauss Seidel and LU Decomposition . So in this question we are using Gauss Seidel . Main Equation of Gauss Seidel is given as

$$\mathbf{x}_{n+1} = -(\mathbf{D}+\mathbf{L})^{-1}\mathbf{U}\mathbf{x}_n + (\mathbf{D}+\mathbf{L})^{-1}\mathbf{b}$$

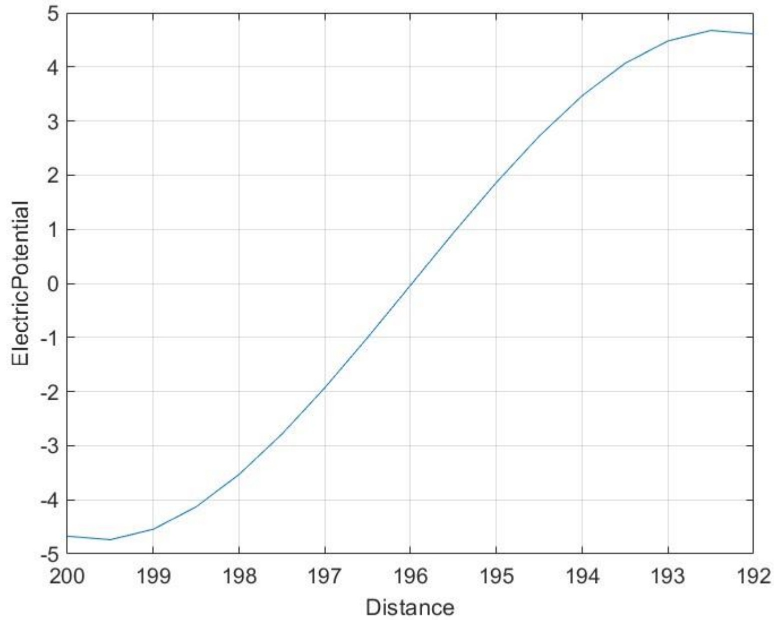
Where Xn+1 is new value of x, D is a Diagonal matrix of A, L is the lower triangular matrix of A, U is the upper triangular matrix of A . Now I solved iteratively of Step graded and linearly graded junctions obtained in question 1 .

Simulation Results :

STEP GRADED JUNCTION



LINEARLY GRADED JUNCTION



Observations : So Using having been used Two numerical methods Gauss Seidel and LU decomposition and having been found there results it seems that both of them converge to same result but gauss seidel being some what slow for higher iterations because it has to compute inverse of the matrices many times but code for gauss seidel is compact and clean . finally two methods are giving same results.

Q2:

In this problem, we have to solve the charge neutrality equation using Newton Raphson method for finding the fermi energy E_F of a compensated semiconductor (Si) bar with doping concentrations $N_D = 1e15\ cm{-}3\ and\ N_A = 1e17\ cm{-}3$.

To solve this problem using the charge neutrality equation, we have to make certain assumptions. They are stated below and also inside the code.

- We have considered the Si bar to be doped with Aluminium(acceptor ionization energy level in silicon is 67meV above E_V) as P type dopant and Arsenic(donor ionization energy level in silicon is 54meV below E_C) as N type dopant
- Silicon band gap is assumed to be 1.12eV
- At room temperature, we consider thermal voltage to be 25.8mV
- The Effective Density of States in the Conduction Band (NC) per cm^-3 is 3e19
- The Effective Density of States in the Valence Band (NV) per cm^-3 is 1e19
- The uppermost level of valence band E_V is assumed to be at 0 eV
- The intrinsic fermi level is considered to be at the centre of E_C and E_V

We perform the Newton Raphson method using the charge neutrality equation and consider the Fermi level to be at the acceptor energy level initially. After about 2000 iterations, increasing the number of iterations does not change the Fermi level which implies that we have arrived at our final result. The fermi level of the compensated semiconductor is 0.127536 eV above the Valence Band.

The plotted Energy Band diagram comes as follows-