SLAM and MOT

Xuesong LI

1 Notation

$Robotstate: X_r = \begin{bmatrix} x_r \\ y_r \\ \alpha_r \end{bmatrix}$

$Landmarkstate: X_{lm} = \begin{bmatrix} x_{lm} \\ y_{lm} \end{bmatrix}$ (To simplify equation, just use one landmark)

$Movingobjectstate: X_{mot} = \begin{bmatrix} x_{mot} \\ y_{mot} \\ \alpha_{mot} \\ v_{mot} \end{bmatrix}$ (To simplify equation, just use one moving object)

2 Processing model

Motion model is $X(k+1) = F(X(k), u(k))$

$$\begin{bmatrix} x_r \\ y_r \\ \alpha_r \\ x_{lm} \\ y_{lm} \\ x_{mot} \\ y_{mot} \\ \alpha_{mot} \\ v_{mot} \end{bmatrix}(k+1) = \begin{bmatrix} x_r \\ y_r \\ \alpha_r \\ x_{lm} \\ y_{lm} \\ x_{mot} \\ y_{mot} \\ \alpha_{mot} \\ v_{mot} \end{bmatrix}(k) + dt * \begin{bmatrix} v_r(k) * cos(\alpha_r(k) + g(k)) \\ v_r(k) * sin(\alpha_r(k) + g(k)) \\ v_r(k) * sin(g(k))/WB \\ 0 \\ 0 \\ v_{mot}(k) * cos(\alpha_{mot}(k)) \\ v_{mot}(k) * sin(\alpha_{mot}(k)) \\ 0 \\ 0 \end{bmatrix}$$

$g(k)$ is steering angle, $v_r(k)$ is robot speed, WB is the vehicle wheel-base. Constant velocity and orientation assumption is made for moving objects.

Jacobian of states in process model is as following.

$$J = \begin{bmatrix} 1 & 0 & -v_r(k)*dt*sin(\alpha_r(k)+g(k)) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & v_r(k)*dt*cos(\alpha_r(k)+g(k)) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -v_{mot}(k)*dt*sin(\alpha_{mot}(k)) & dt*cos(\alpha_{mot}(k)) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & v_{mot}(k)*dt*cos(\alpha_{mot}(k)) & dt*sin(\alpha_{mot}(k)) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Jacobian of inputs in process model is as following.

$$J_u = \begin{bmatrix} dt*cos(\alpha_r(k)+g(k)) & -v_r(k)*dt*sin(\alpha_r(k)+g(k)) \\ dt*sin(\alpha_r(k)+g(k)) & v_r(k)*dt*cos(\alpha_r(k)+g(k)) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Prediction step:

$\hat{X}(k+1) = F(\hat{X}(k), u(k))$

$P(k+1|k) = J*P(K|K)*J^T + J_u*Q_u*J_u^T + Q_{mot}$

Q_{mot} is noise in motion model of moving object.

$$Q_{mot} \text{ is } \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_v \end{bmatrix}$$

3 Observation Model

Observation model is $Y(K) = h(X(K))$, the observation is range and bearing. We assume that the landmark and moving object are observed simultaneously. (r_{lm}, b_{lm}) is the observation from landmark, and (r_{mot}, b_{mot}) is the observation from moving object.

$$Y(k) = \begin{bmatrix} r_{lm} \\ b_{lm} \\ r_{mot} \\ b_{mot} \end{bmatrix}(k) = \begin{bmatrix} \sqrt{(x_{lm}(k)-x_r(k))^2+(y_{lm}(k)-y_r(k))^2} \\ atan2(y_{lm}(k)-y_r(k), x_{lm}(k)-x_r(k)) - \alpha_r(k) \\ \sqrt{(x_{mot}(k)-x_r(k))^2+(y_{mot}(k)-y_r(k))^2} \\ atan2(y_{mot}(k)-y_r(k), x_{mot}(k)-x_r(k)) - \alpha_r(k) \end{bmatrix}$$

Jacobian of observation model is H as following.

$$\begin{bmatrix} \frac{-dx_{lm}(k)}{\sqrt{dx_{lm}(k)^2+dy_{lm}(k)^2}} & \frac{-dy_{lm}(k)}{\sqrt{dx_{lm}(k)^2+dy_{lm}(k)^2}} & 0 & \frac{dx_{lm}(k)}{\sqrt{dx_{lm}(k)^2+dy_{lm}(k)^2}} & \frac{dy_{lm}(k)}{\sqrt{dx_{lm}(k)^2+dy_{lm}(k)^2}} & 0 & 0 & 0 & 0 \\ \frac{dy_{lm}(k)}{dx_{lm}(k)^2+dy_{lm}(k)^2} & \frac{-dx_{lm}(k)}{dx_{lm}(k)^2+dy_{lm}(k)^2} & -1 & \frac{-dy_{lm}(k)}{dx_{lm}(k)^2+dy_{lm}(k)^2} & \frac{dx_{lm}(k)}{dx_{lm}(k)^2+dy_{lm}(k)^2} & 0 & 0 & 0 & 0 \\ \frac{-dx_{mot}(k)}{\sqrt{dx_{mot}(k)^2+dy_{mot}(k)^2}} & \frac{-dy_{mot}(k)}{\sqrt{dx_{mot}(k)^2+dy_{mot}(k)^2}} & 0 & 0 & 0 & \frac{dx_{mot}(k)}{\sqrt{dx_{mot}(k)^2+dy_{mot}(k)^2}} & \frac{dy_{mot}(k)}{\sqrt{dx_{mot}(k)^2+dy_{mot}(k)^2}} & 0 & 0 \\ \frac{dy_{mot}(k)}{dx_{mot}(k)^2+dy_{mot}(k)^2} & \frac{-dx_{mot}(k)}{dx_{mot}(k)^2+dy_{mot}(k)^2} & -1 & 0 & 0 & \frac{-dy_{mot}(k)}{dx_{mot}(k)^2+dy_{mot}(k)^2} & \frac{dx_{mot}(k)}{dx_{mot}(k)^2+dy_{mot}(k)^2} & 0 & 0 \end{bmatrix}$$

where
$dx_{lm}(k) = x_{lm}(k) - x_r(k)$,
$dy_{lm}(k) = y_{lm}(k) - y_r(k)$,
$dx_{mot}(k) = x_{mot}(k) - x_r(k)$,
$dy_{mot}(k) = y_{mot}(k) - y_r(k)$.

Updating step:
$P(k+1|k) = (P(k+1|k) + P(k+1|k)^T) * 0.5$,
$S = H * P(k+1|k) * H^T + R$,
$K = P(k+1|k) * H^T * (S)^{-1}$,
$X(k+1|k+1) = X(k+1|k) + K * v$,
$P(k+1|k+1) = P(k+1|k) - K * S * K^T$.

4 Inialization

4.1 New landmark

When another new landmark is observed, the state vector will increase two new elements and its corresponding covaraince will also be added.

New landmark is $X_{lm1} = \begin{bmatrix} x_{lm1} \\ y_{lm1} \end{bmatrix}$.

Its new observation is $z_{lm1} = \begin{bmatrix} r_{lm1} \\ b_{lm1} \end{bmatrix}$.

Asuming the mapping function from X to X_{lm1} is $X_{lm1} = f(X, z_{lm1})$

$$\begin{bmatrix} x_{lm1} \\ y_{lm1} \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \end{bmatrix} + \begin{bmatrix} cos(b_{lm1}+\alpha_r) * r_{lm1} \\ sin(b_{lm1}+\alpha_r) * r_{lm1} \end{bmatrix}.$$

The jacobian matrix from X to X_{lm1} is

$$G_v = \begin{bmatrix} 1 & 0 & -sin(b_{lm1}+\alpha_r) * r_{lm1} & 0 & 0 & 0 & 0 \\ 0 & 1 & cos(b_{lm1}+\alpha_r) * r_{lm1} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$G_r = \begin{bmatrix} 1 & 0 & -sin(b_{lm1}+\alpha_r)*r_{lm1} \\ 0 & 1 & cos(b_{lm1}+\alpha_r)*r_{lm1} \end{bmatrix}$

The jacobian matrix from z_{lm1} to X_{lm1} is

$$G_u = \begin{bmatrix} cos(b_{lm1}+\alpha_r) & -sin(b_{lm1}+\alpha_r)*r_{lm1} \\ sin(b_{lm1}+\alpha_r) & cos(b_{lm1}+\alpha_r)*r_{lm1} \end{bmatrix}$$

The new state vector is $\left[x_r, y_r, \alpha_r, x_{lm}, y_{lm}, x_{lm1}, y_{lm1}, x_{mot}, y_{mot}, \alpha_{mot}, v_{mot}\right]^T$,

And augmented covariance matrix $P = \begin{bmatrix} P_{rr} & P_{rlm} & P_{rlm1} & P_{rmot} \\ P_{lmr} & P_{lmlm} & P_{lmlm1} & P_{lmmot} \\ P_{lm1r} & P_{lm1lm} & P_{lm1lm1} & P_{lm1mot} \\ P_{motr} & P_{motlm} & P_{motlm} & P_{motmot} \end{bmatrix}$

$P_{lm1lm1} = G_v * P * G_v^T + G_u * R * G_u^T$, where R is sensor noise.

$$[P_{rlm1} P_{lmlm1} P_{motlm1}] = G_v * \begin{bmatrix} P_{rr} & P_{rlm} & P_{rmot} \\ P_{lmr} & P_{lmlm} & P_{lmmot} \\ P_{motr} & P_{motlm} & P_{motmot} \end{bmatrix} =$$

$$\begin{bmatrix} G_r & 0 & 0 \end{bmatrix} * \begin{bmatrix} P_{rr} & P_{rlm} & P_{rmot} \\ P_{lmr} & P_{lmlm} & P_{lmmot} \\ P_{motr} & P_{motlm} & P_{motmot} \end{bmatrix}$$

$P_{lm1r} = P^T_{rlm1} = E[(X_{lm1} - \hat{X}_{lm1})(X_r - \hat{X}_r)^T] = G_r * P_{rr}$
$P_{lm1lm} = P^T_{lmlm1} = E[(X_{lm} - \hat{X}_{lm})(X_{lm1} - \hat{X}_{lm1})^T] = G_r * P_{rlm}$
$P_{lm1mot} = P^T_{motlm1} = E[(X_{mot} - \hat{X}_{mot})(X_{lm1} - \hat{X}_{lm1})^T] = G_r * P_{rmot}$

4.2 New moving object

When another new moving object is observed, the state vector will increase foure new elements and its corresponding covaraince will also be added.

New moving object is $X_{mot1} = \begin{bmatrix} x_{mot1} \\ y_{mot1} \\ \alpha_{mot1} \\ v_{mot1} \end{bmatrix}$.

Its new observation is $z_{mot1} = \begin{bmatrix} r_{mot1} \\ b_{mot1} \end{bmatrix}$.

Asuming the mapping function from X to X_{lm1} is $X_{mot1} = f(X, z_{mot1})$

$$\begin{bmatrix} x_{mot1} \\ y_{mot1} \\ \alpha_{mot1} \\ v_{mot1} \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} cos(b_{mot1}+\alpha_r)*r_{mot1} \\ sin(b_{mot1}+\alpha_r)*r_{mot1} \\ -(\alpha_r + b_{mot1}) \\ 0 \end{bmatrix}.$$

The jacobian matrix from X to X_{lm1} is

$$G_{mv} = \begin{bmatrix} 1 & 0 & -sin(b_{mot1}+\alpha_r)*r_{mot1} & 0 & 0 & 0 & 0 \\ 0 & 1 & cos(b_{mot1}+\alpha_r)*r_{mot1} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_{mr} = \begin{bmatrix} 1 & 0 & -sin(b_{mot1}+\alpha_r)*r_{mot1} \\ 0 & 1 & cos(b_{mot1}+\alpha_r)*r_{mot1} \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

The jacobian matrix from z_{mot1} to X_{mot1} is

$$G_{mu} = \begin{bmatrix} cos(b_{mot1}+\alpha_r) & -sin(b_{mot1}+\alpha_r)*r_{mot1} \\ sin(b_{mot1}+\alpha_r) & cos(b_{mot1}+\alpha_r)*r_{mot1} \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

And augmented covariance matrix

$$P = \begin{bmatrix} P_{rr} & P_{rlm} & P_{rmot} & P_{rmot1} \\ P_{lmr} & P_{lmlm} & P_{lmmot} & P_{lmmot1} \\ P_{motr} & P_{motlm} & P_{motmot} & P_{motmot1} \\ P_{mot1r} & P_{mot1lm} & P_{mot1mot} & P_{mot1mot1} \end{bmatrix}$$

$P_{mot1mot1} = G_{mr}*P*G_{mr}^T + G_{mu}*R*G_{mu}^T$, where R is sensor noise.

Similarly,

$$[P_{rmot1}P_{lmmot1}P_{motmot1}] = G_v * \begin{bmatrix} P_{rr} & P_{rlm} & P_{rmot} \\ P_{lmr} & P_{lmlm} & P_{lmmot} \\ P_{motr} & P_{motlm} & P_{motmot} \end{bmatrix} =$$

$$\begin{bmatrix} G_{mr} & 0 & 0 \end{bmatrix} * \begin{bmatrix} P_{rr} & P_{rlm} & P_{rmot} \\ P_{lmr} & P_{lmlm} & P_{lmmot} \\ P_{motr} & P_{motlm} & P_{motmot} \end{bmatrix}$$

$P_{rmot1} = P_{mot1r}^T = E[(X_{mot1} - \hat{X}_{mot1})(X_r - \hat{X}_r)^T] = G_{mr} * P_{rr}$

$P_{lmmot1} = P_{mot1lm}^T = E[(X_{mot1} - \hat{X}_{mot1})(X_{lm} - \hat{X}_{lm})^T] = G_{mr} * P_{rlm}$

$P_{mot1mot} = P_{mot1mot}^T = E[(X_{mot1} - \hat{X}_{mot1})(X_{mot} - \hat{X}_{mot})^T] = G_{mr} * P_{rmot}$