

Hoare triples exercises

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$$\{y=K \wedge K>0\}^P \quad C \quad \{x=K^2\}^Q$$

$z:=0; x:=0$

while $(y>0)$ do

$x:=2z+x+1$

$z:=z+1$

$y:=y-1$

endwhile

C_1

W

C

$$\text{Inv: } x=z^2 \wedge z=K-y \wedge y \geq 0$$

Correttezza

(A) $P \rightarrow \text{Inv}, \{P\} C_1 \{ \text{Inv} \}$

(B) $\text{Inv} \wedge \neg E \rightarrow Q, \{ \text{Inv} \wedge \neg E \} W \{ Q \}$

(C) $\{ \text{Inv} \wedge E \} C \{ \text{Inv} \}$

(D) $\text{Inv} \rightarrow t \geq 0$

(E) $\{ \text{Inv} \wedge E \wedge t=V \} C \{ t < V \}$

con C corpo dell'iterazione ($W \equiv C$ in questo esercizio).

(A)

#2

$$\{P\} C_1 \{ \text{Inv} \}$$

$$\{P\} C_1 \{ x=z^2 \wedge z=K-y \wedge y \geq 0 \}, x=z^2 \wedge z=K-y \wedge y \geq 0 \rightarrow x=z^2 \quad (a \wedge b \wedge c \rightarrow a)$$

$$\vdash (\text{post}) \{P\} C_1 \{ x=z^2 \} \star$$

$$\vdash (\text{ass.}) \{ \text{True} \} C_1 \{ x=z^2 \}, \text{True} \rightarrow P$$

$$\vdash (\text{pre}) \{P\} C_1 \{ x=z^2 \}$$

quindi \star è stato derivato. ✓

(B) $\{ \text{Inv} \wedge \neg E \} W \{ Q \}$

$$x=z^2 \wedge z=K-y \wedge (y \geq 0) \wedge y \leq 0$$

$$\equiv x=z^2 \wedge z=K$$

$$\equiv x=K^2 \quad \text{quindi} \quad \text{Inv} \wedge \neg E \rightarrow x=K^2 \quad \checkmark$$

$$(C) \{Inv \wedge E\} \underset{(w)}{C} \{Inv\}$$

$$\vdash (ass.) \{2z+x+1=(z+1)^2 \wedge z+1=K-(y-1) \wedge y-1 \geq 0\} W \{Inv\}$$

$$2z+x+1=z^2+2z+1 \wedge z+1=K-y+1 \wedge y \geq 1$$

$$(calcolo) \{x=z^2 \wedge z=K-y \wedge y \geq 1\} W \{Inv\}$$

La correttezza parziale è dimostrata da (A), (B) e (C).

$$(D) Inv \rightarrow t \geq 0$$

$$t \cdot y$$

$$\dots \wedge y \geq 0 \rightarrow y \geq 0 \quad \checkmark$$

$$(E) \{Inv \wedge E \wedge y=V\} \underset{(w)}{C} \{y < V\}$$

$$\vdash (ass.) \{y-1 < V\} W \{y < V\}$$

$$x=z^2 \wedge z=K-y \wedge y \geq 0 \wedge y > 0 \wedge y=V \equiv Inv \wedge E \wedge y=V$$

$$\rightarrow y=V \wedge y > 0$$

$$\rightarrow y-1 < V$$

$$\vdash (pre) \{Inv \wedge E \wedge y=V\} W \{t < V\}$$

La correttezza totale è dimostrata dalla correttezza parziale e (D), (E).