

Hoare triples exercises

6 Settembre 2018

$$\{N > 0\}^P C \{x = 3(N-1)\}^Q$$

$x := 0; i := 1;$ C_1
 while $i < N$ do
 C $x := 3+x$
 $i := i+1$ W
 endwhile

$$N = 4$$

$$x = 0 \quad 3 \quad 6 \quad 9$$

$$i = 1 \quad 2 \quad 3 \quad 4$$

$$Inv: x = 3(i-1) \wedge i \leq N$$

$$(A) \{P\} C_1 \{Inv\}$$

$$\vdash (ass.) \{0 = 3(1-1) \wedge 1 \leq N\} C_1 \{Inv\}$$

$$(calcolo) \equiv \{N \geq 1\} C_1 \{Inv\} \quad (a \geq b \leftrightarrow a > b-1)$$

$$\equiv \{N > 0\} C_1 \{Inv\}$$

$$(B) \{Inv \wedge \neg E\} W \{Q\} \quad (Inv \wedge \neg E \rightarrow Q)$$

$$x = 3(i-1) \wedge i \leq N \wedge i \geq N$$

$$\equiv x = 3(i-1) \wedge i = N$$

$$\equiv x = 3(N-1)$$

$$(C) \{Inv \wedge E\}^W C \{Inv\}$$

$$\vdash (ass.) \{3+x = 3(i+1-1) \wedge i+1 \leq N\} W \{Inv\}$$

$$\equiv \{x = 3(i)-3 \wedge i \leq N-1\} W \{Inv\}$$

$$\equiv \{x = 3(i-1) \wedge i < N\} W \{Inv\} \quad (a < b \leftrightarrow a \leq b-1)$$

$$(D) Inv \rightarrow t \geq 0 \quad t: N-i$$

$$x = 3(i-1) \wedge i \leq N$$

$$\rightarrow i \leq N \quad (a \wedge b \rightarrow a)$$

$$\equiv N \geq i \equiv N-i \geq 0 \equiv t \geq 0$$

$$(E) \{Inv \wedge E \wedge t=V\} W \{t < V\}$$

$$\vdash (ass.) \{N-i-1 < V\} W \{t < V\}$$

$$\{N-i \leq V\} W \{t < V\}, \quad Inv \wedge E \wedge t=V$$

$$Inv \wedge E \wedge N-i=V \rightarrow N-i \leq V$$

$$\vdash (pre) \{Inv \wedge E \wedge t=V\} W \{t < V\}$$