Master Ingénierie des Systèmes Complexes
- Parcours Robotique et Objets Connectés -
- Marine and Maritime Intelligent Robotics study track -

Automatique non-linéaire
Advanced Control for Autonomous Vehicles
(Assignements list)

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A1 - Draw a representation of the vector field below, defined over $\mathbb{R}^2$.

Let $X(t) = \begin{pmatrix} x \\ y \end{pmatrix}$ denote the state variable. The considered vector field is : $f(X(t)) = \begin{pmatrix} -\frac{-6x}{(1+x^2)^2} + 2y \\ -\frac{2x+2y}{(1+x^2)^2} \end{pmatrix}$.

It is up to you to decide the domain [1] over which the vector field is drawn. The point is to be able to display interesting/useful information about the vector field.

A2 - Draw a representation of the Lotka-Volterra vector field below, defined over $[0,a]^2$ where $a > 0$ is a constant that allows you to properly draw the trajectories.

Let $X(t) = \begin{pmatrix} x \\ y \end{pmatrix}$ denote the state variable. The considered vector field is : $f(X(t)) = \begin{pmatrix} x(1-y) \\ y(x-1) \end{pmatrix}$.

A3 - Check that

$$x(t) = c\sin(t)e^{-t} \quad \text{for} \quad c \in \mathbb{R}$$

is a family of solutions to the differential equation $\dot{x}(t) = (cotan(t) - 1)\, x(t)$ with $x(0) = 0$ (where $cotan$ is the cotangent function).

Note : this is an exemple of a system that doesn't comply with the hypothesis asked in the existence theorem given in the course (Picard–Lindelöf theorem, Picard's existence theorem, Cauchy–Lipschitz theorem or existence and uniqueness theorems). There exist other existence theorems that guarantee existence by not uniqueness of the solution (i.e. Peano Theorem & Caratheodory theorem). The above equation illustrates Caratheodory Theorem.

A4 - Pick an initial point for the differential system in **A1** and simulate, using the order 4 Runge-Kutta [2] algorithm a trajectory that starts from this point. Draw the simulated (2D) trajectory over the vector field drawn in **A1**.

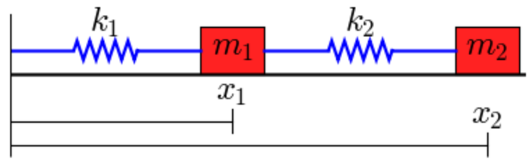
A5 - Same question as in **A4** but with the Lotka-Volterra system given in **A2**.

A6 - This one concerns the differential equation in A3.

Show that the point $x = 0$ in not stable following the definition of Lyapunov stability given in the course. (This is somewhat counter intuitive since $X(t) \to 0$ as time goes to infinity).

1. A square of the form $[a,b] \times [c,d]$
2. In addition to this one, you can also consider a build-in fonction in Matlab and/or Python.

A7 - We consider the two masses with springs problem, where x_1 and x_2 represents the linear position of the center of mass of the first, and second mass, respectively.



The length of the springs at rest is denoted l_0.
The two masses have the same weight $m_1 = m_2 = m$.
The springs have the same stiffness constant, denoted $k_1 = k2 = k$.
Modelling is performed with the help of the Euler-Lagrange formalism.
Kinetic energy $\quad E_c = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2)$.
Potential energy $\quad E_p = \frac{k}{2}(x_1 - l_0)^2 + \frac{k}{2}(x_2 - x_1 - l_0)^2$.
The Lagrangian is $L = E_c - E_p$.
The Euler-Lagrange equations are

$$\begin{aligned} \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_1} - \frac{\partial L}{\partial x_1} &= 0 \\ \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_2} - \frac{\partial L}{\partial x_2} &= u(t) \end{aligned}$$

where $u(t)$ is the control variable (pushing and/or pulling the second mass).

A7 - q1 - Determine the equations of motion for this system and write them in the state space representation — i.e. $\dot{X} = f(X)$ where $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}$.

A7 - q2 - Consider the equilibrium point $\begin{pmatrix} l_0 \\ 2l_0 \\ 0 \\ 0 \end{pmatrix}$ and use the Jurdjevic-Queen technique in order to design a feedback that stabilises the system toward this equilibrium point —*The total energy $E_c + E_p$ is a good candidate for the V function.*

A7 - q3 - Perform a simulation of the looped system.