Robotics Modelling
Modeling and control of underwater vehicle: Sparus

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Abstract

This document allows to model ”Sparus”, the last autonomous underwater vehicle of IQUA Robotics (Girona). This report is designed for students, so they can swiftly understand theory and codes that have been written. All the work done here can be adapted to an other shape if future work needs it.

Contents

1 Introduction

1.1 Main caracteristic of the Sparus

The Sparus AUV is represented by the figure Fig.1. It allows to define the main axes and the position of the gravity center.

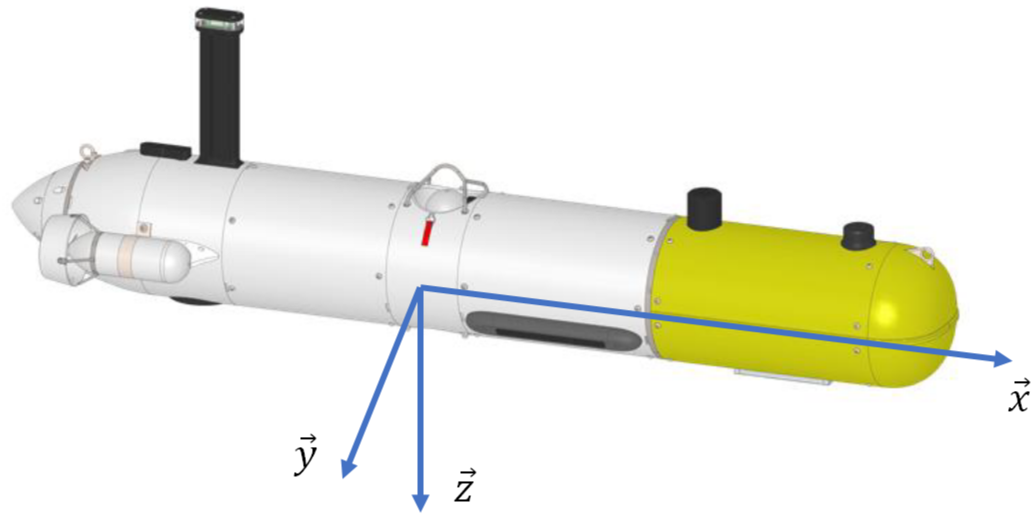

Figure 1: The Sparus in 3D

Therefore we consider the origin of the base "body" in the gravity center of the Sparus. It is placed at the x-position of the central thruster and at middle of the cylinder.

$$\overrightarrow{r}_g = {}_b\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{1}$$

Moreover, the coordinates of the buoyancy center are:

$$\overrightarrow{r}_b = {}_b\begin{bmatrix}0\\0\\-0.02\ m\end{bmatrix} \tag{2}$$

1.2 Notations

For your study, the following scalar notations has to be used :

ρ_w	water mass density	$1000\ kg.m^{-3}$
R	Radius	$0.115\ m$
L	Length	$1.6\ m$
m	Mass	$52\ Kg$
m_b	Buoyancy mass	$52,1\ Kg$
g	Earth gravity	$9.81\ m.s^{-2}$
P	Weight	$P = m.g$

And the following vector and matrix notations has to be defined :

G^b	Gravity vector
B^b	Buoyancy vector
U^b	Thrust mapped vector
K^b	Friction matrix
E^b	Mapping matrix
$M_G^b = M_B^b + M_A^b$	Generalized mass matrix
M_B^b	Body mass matrix
M_A^b	Added mass matrix
$C_G^b = C_B^b + C_A^b$	Generalised Coriolis matrix
v	Velocity vector in Body frame
η	Position vector in Earth frame
J_θ	Generalised rotation matrix

1.3 Sensor and Thruster positions

Moreover, the values of the sensors and thrusters positions (represented on the figure Fig.2) are :

	Coordinates in basse b	Values in m
Motor 1	$\overrightarrow{r}_{t1}^b = (d_{1x}, d_{1y}, d_{1z})^T$	$(0, 0, 0.08)$
Motor 2	$\overrightarrow{r}_{t2}^b = (d_{2x}, d_{2y}, d_{2z})^T$	$(-0.59, 0.17, 0)$
Motor 3	$\overrightarrow{r}_{t3}^b = (d_{3x}, d_{3y}, d_{3z})^T$	$(-0.59, -0.17, 0)$
DVL	$\overrightarrow{r}_{t3}^b = (d_{dx}, d_{dy}, d_{dz})^T$	$(-0.4145, 0, 0.11)$
IMU and Depth	$\overrightarrow{r}_{t3}^b = (d_{ix}, d_{iy}, d_{iz})^T$	$(0.364, -0.021, -0.085)$
USBL	$\overrightarrow{r}_{t3}^b = (d_{ux}, d_{uy}, d_{uz})^T$	$(0.44, 0, -0.14)$

Figure 2: The plan projections of Sparus

2 Dynamic Modelling

The objective of this section is to help you to developp the dynamic model of the Sparus.

2.1 Kinematics

First, we need to describe two frames:

- Earth inertial frame (E-frame)
- Body-fixed frame (B-frame)

Then we will define the relationship between those two frames using the Euler angle theory in order to be able to describe a velocity belonging to the B-frame in the E-frame. We don't speak about a relationship between positions in those two frames because position in body-fixed frame is physically irrelevant and has no meaning. So, the equation is:

$$\dot{\overrightarrow{\eta}} = J_\theta . \overrightarrow{\upsilon} \tag{3}$$

With,

$$\dot{\overrightarrow{\eta}} = \begin{pmatrix} \overrightarrow{v^e} \\ \overrightarrow{\omega^e} \end{pmatrix} = \begin{pmatrix} \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{pmatrix}^T \tag{4}$$

$$\dot{\overrightarrow{\upsilon}} = \begin{pmatrix} \overrightarrow{v^b} \\ \overrightarrow{\omega^b} \end{pmatrix} = \begin{pmatrix} \dot{u} & \dot{v} & \dot{w} & \dot{p} & \dot{q} & \dot{r} \end{pmatrix}^T \tag{5}$$

Where $\dot{\overrightarrow{\eta}}$ the generalised velocity is vector in E-frame and it is composed of Nessie linear velocity vector $\overrightarrow{v^e}$ and Nessie angular velocity vector $\overrightarrow{\omega^e}$.

And $\overrightarrow{\upsilon}$ is the generalised velocity is vector in B-frame and it is composed of Nessie linear velocity vector $\overrightarrow{v^b}$ and Nessie angular velocity vector $\overrightarrow{\omega^b}$.

In addition, the generalized rotation and transfer matrix J_θ is composed of 4 sub-matrixes according to equation:

$$J_\theta = \begin{bmatrix} R_\theta & 0_{3*3} \\ 0_{3*3} & T_\theta \end{bmatrix} \tag{6}$$

Where the rotation R_θ and the transfer T_θ matrices are defined according in the lesson.

2.2 Dynamics

The dynamics is described in the B-frame :

$$M_G^b \dot{\upsilon} + C_G^b \upsilon = G^b - B^b + K^b + U^b \tag{7}$$

where :

- M_G^b is the general mass matrix of the body
- C_G^b is the general body Coriolis matrix.
- $G^b + B^b$: Gravity and Buoyancy
- $K^b = \sum K_i^b$: Sum of all body drag forces
- U^b: Thruster

2.2.1 Gravity and Buoyancy

The gravity G^b and buoyancy B^b vector can be defined as follow:

$$G^b = \begin{pmatrix} \overrightarrow{F}_g^b \\ \overrightarrow{r}_g \wedge \overrightarrow{F}_g^b \end{pmatrix} = \begin{pmatrix} R_\theta^{-1} \overrightarrow{F}_g^e \\ \overrightarrow{r}_g \wedge R_\theta^{-1} \overrightarrow{F}_g^e \end{pmatrix} \tag{8}$$

and

$$B^b = \begin{pmatrix} \overrightarrow{F}_b^b \\ \overrightarrow{r}_b \wedge \overrightarrow{F}_b^b \end{pmatrix} = \begin{pmatrix} R_\theta^{-1} \overrightarrow{F}_b^e \\ \overrightarrow{r}_b \wedge R_\theta^{-1} \overrightarrow{F}_b^e \end{pmatrix} \tag{9}$$

where $\overrightarrow{F}_g^e$ and $\overrightarrow{F}_b^e$ are respectively the force vectors of the gravity and buoyancy expressed in the earth frame.

2.2.2 Thruster

The Thruster forces and moments can be represented by the following equation:

$$U^b = E^b F_T^b = E_{6*3}^b \begin{bmatrix} F_{T1}^b \\ F_{T2}^b \\ F_{T3}^b \end{bmatrix} \tag{10}$$

Where F_{T1}^b is the force of thruster (or motor) 1 in Body-fixed frame. We have the same definition for the six forces.

E^b has to be determined thanks to the position of the thruster (see section 1.3). For exemple, the moment produced by the thruster 1 is $\overrightarrow{r}_{t1}^b \wedge \overrightarrow{F_{T1}^b}$.

After adjusting the model behaviour with experimental data, we have chosen the following first order transfer function :

$$G(s) = \frac{Kt}{\tau s + 1} \tag{11}$$

Consequently, the input is a percentage of the maximal force ($F_{max} = Kt$ at 100%).

The values of the gain and constant times are defined below:

	Forward	rearward		
Kt_1	$55\ N$	$28.5\ N$	τ_1	$0.4\ s$
Kt_2	$71.5\ N$	$30\ N$	τ_2	$0.8\ s$
Kt_3	$71.5\ N$	$30\ N$	τ_3	$0.8\ s$

3 Simulation of an experimentation

3.1 Scenario

The objective is to command the Sparus AUV with the following scene:

a. At beginning, the AUV is on the water,

b. Move in a straight heave motion until a 5 meter depth.

c. Move forward at constant speed: $1\ m.s^{-1}$. The roll, pitch and yaw motions have to be stabilized around $0\,\mathrm{deg}$.

d. Stop when it arrives at the location $(20\ m, 0, 5\ m)^T$

3.2 Available sensors

The AUV is set-up with a 3D accelerometer, a Doppler radar and an USBL. You are not able to measure the thruster forces.

3.3 Work to do

You have to send me back your work by email which contains a report (max 15 pages) and your code.

You have to realize in order of importance before the deadline (to be determined):

a. Estimate the global mass matrices at the gravity center

b. Estimate all drag matrices at the gravity

c. Complete the simulator with some simple experiments to validate it.

d. Highlight the importance of the different coefficients in the added mass and friction matrices. Conclude what can be neglected.

e. Define controllable degrees of freedom

f. (bonus) Develop and Validate a simple commands law for scenario.