# Punimi Seminarik

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Detyra 1: Duke shfrytezuar induksionin matematik, vertetoni mosbarazimin:

$$n! > n^{\frac{n}{2}} \ (n > 2)$$

Per n=3, vlen :

$$3! > 3^{\frac{1}{2}}$$

Supozojme se vlen per n=k :

$$k! > k^{\frac{k}{2}} \quad (k > 2) \qquad \text{(h.i)}$$

Vertetojme se vlen per n=k+1 :

$$(k+1)! > (k+1)^{\frac{k+1}{2}}$$

Nisemi nga ana e majte :

$$(k+1)! = n! \cdot (k+1) \overset{\textbf{(h.i)}}{>} k^{\frac{k}{2}} \cdot (k+1) > (k+1)^{\frac{k+1}{2}}$$

Sqarim

$$k^{\frac{k}{2}} \cdot (k+1) > (k+1)^{\frac{k+1}{2}} /^2$$

$$k^k \cdot (k+1)^2 > (k+1)^{k+1} / \frac{1}{k^k \cdot (k+1)}$$

$$k+1 > (\frac{k+1}{k})^k$$

$\lim\limits_{k \to \infty} (\frac{k+1}{k})^k = e$ , si dhe $k+1 > 3$

Rjedhimisht

$\left(\frac{k+1}{k}\right) < e < 3$

2) Duke shfrytzuar induksionin matemtik, vertetoni mosbarazimin:

$(2n-1)! < n^{2n-1} \quad (n > 1)$

Per n=2 kemi:

$(2 \cdot 2 - 1)! < 2^{2 \cdot 2 - 1}$

$6 < 8 \quad$ T

Per n=k kemi:

$(2k-1)! < k^{2k-1}$

Per n=k+1 provojme:

$(2(k+1)-1)! < (k+1)^{2(k+1)-1}$

$(2k+2-1)! < (k+1)^{2k+2-1}$

$(2k+2-1)! < (k+1)^{2k-1} \cdot (k+1)^2$

$(2k-1+2)! < (k+1)^{2k-1} \cdot (k+1)^2$

$(2k+1)! < (k+1)^{2k-1} \cdot (k+1)^2$

$(2k+1)2k(2k-1)! < (k+1)^{2k-1} \cdot (k+1)^2$

. (H.I)

$(2k+1) \cdot 2k \cdot k^{2k-1} < (k+1)^{2k-1} \cdot (k+1)^2$

$(2k+1) \cdot 2k \cdot k^{2k-1} < (k+1)^{2k}(k+1)$

$(4k^2+2k)k^{2k-1} < (k+1)^{2k+1}$

$4k^{2k+1} + 2k^{2k} < \binom{2k+1}{0} k^{2k+1} + \binom{2k+1}{1} k^{2k} + \cdots + \binom{2k+1}{2k+1} 1^{2k+1}$

qe duhej vertetuar

4) Funksioni f eshte i perkufizuar ne bashkesine e numrave natyror dhe vlerat e tij jane numra natyror.
Nese $f(n+1) > f(f(n))$ per çdo n, tregoni se $f(n) = n$ per çdo n.

Zgjidhje

Supozojme te kunderten se $f(n) \neq n \Rightarrow f(n) > n$ sepse $f(n) < n$ $\perp$ meqe $f(1) < 1$ nuk mund te jete sepse $f(1) \in N$.
Meqe $f(n) > n \quad \exists a \in N$ qe$f(n) = n + a$. Atehere $f(n+1) = n + 1 + a$.
$f(f(n)) = f(n+a) = n + 2a$
$n + 1 + a > n + 2a \quad \perp$ qe eshte ne kundershtim me $f(n+1) > f(f(n))$.

**5)Te vertetohet mosbarazimi:** $|x_1+x_2+\cdots+x_n| \leqslant \sqrt{n(x_1^2+x_2^2+\cdots+x_n^2)}$
**Per tu liruar nga vlera absolute kemi:**

$$|x_1+x_2+\cdots+x_n| = \sqrt{(x_1+x_2+\cdots+x_n)^2} =$$

$$\sqrt{x_1^2+x_2^2+\cdots+x_n^2+A}$$

$$A = \left[\underbrace{2x_1x_2+2x_1x_3+\cdots+2x_1x_n}_{(n-1)-\text{terma}} + 2x_2x_1+2x_2x_3+\cdots+2x_2x_n+\cdots+2x_nx_1+2x_nx_2+\cdots+2x_nx_{n-1}\right]$$

$$\leqslant \sqrt{x_1^2+x_2^2+\cdots+x_n^2+\left[x_1^2+x_2^2+x_1^2+x_3^2+\cdots+x_n^2+x_{n-1}^2\right]}$$

$$= \sqrt{x_1^2+x_2^2+\cdots+x_n^2+[(n-1)x_1^2+(n-1)x_2^2+\cdots+(n-1)x_n^2]}$$

$$= \sqrt{n(x_1^2+x_2^2+\cdots+x_n^2)}$$

6. Te vertetohet mosbarazimi:

$$\sqrt{\sum_{k=1}^{n}(x_i-y_i)^2} \leq \sqrt{\sum_{k=1}^{n}x_i^2} + \sqrt{\sum_{k=1}^{n}y_i^2}$$

Nisemi nga:

$$\sum_{i=1}^{n}(a_i+b_i)^2 = \sum_{i=1}^{n}(a_i+b_i)(a_i+b_i) =$$

$$\sum_{i=1}^{n}\left[a_i(a_i+b_i)+b_i(a_i+b_i)\right] =$$

$$\sum_{i=1}^{n}a_i(a_i+b_i) + \sum_{i=1}^{n}b_i(a_i+b_i) \leq \text{(Sipas jobarazimit te Holderit)} \sum a_ib_i \leq \sqrt{\sum a_i^2}\cdot\sqrt{\sum b_i^2}$$

Pra:

$$\sqrt{\sum_{i=1}^{n}(a_i+b_i)^2} \leq \sqrt{\sum_{i=1}^{n}a_i^2} \cdot \sqrt{\sum_{i=1}^{n}(a_i+b_i)^2} + \sqrt{\sum_{i=1}^{n}b_i^2} \cdot \sqrt{\sum_{i=1}^{n}(a_i+b_i)^2}$$

Pas pjestimit te jobarazimit te fundit me

$$\sqrt{\sum_{i=1}^{n}(a_i+b_i)^2}$$

fitojme rezultatin e kerkuar:

$$\sqrt{\sum_{i=1}^{n}(a_i+bi)^2} \leq \sqrt{\sum_{i=1}^{n}a_i^2} + \sqrt{\sum_{i=1}^{n}b_i^2}$$

ne te cilin kur marrim per zevendesim

$$a_i = x_i dheb_i = -y_i$$

kemi:

$$\sqrt{\sum_{i=1}^{n}(x_i-y_i)^2} \leq \sqrt{\sum_{i=1}^{n}x_i^2} + \sqrt{\sum_{i=1}^{n}y_i^2}$$

**7) Njehsoni shumën:** $1\cdot 1! + 2\cdot 2! + \cdots + n\cdot n! =$
Sqarim:
**Per n= 1 kemi:**

$$1\cdot 1! = 1 = 2! - 1$$

**Per n= 2 kemi:**

$$1 + 2\cdot 2! = 4 = 3! - 1$$

**Per n= 3 kemi:**

$$1 + 4 + 18 = 23 = 4! - 1$$

$\vdots$

**Per n=n kemi:**

$$1 + 4 + 18 + \cdots + n\cdot n! = (n+1)! - 1$$

$= (n+1)! - 1$

**Detyra 10** Njehsoni shumen: $1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1)$

**Zgjidhje:**

$n(n+1) = n^2 + n$
$S_n = (1^2 + 2^2 + \ldots + n^2) + (1 + 2 + \ldots + n);$
$S_a = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$
$S_b = 1 + 2 + \ldots + n = \frac{n \cdot (n+1)}{2}$
$S_n = S_a + S_b = \frac{n(n+1)(2n+1)}{6} + \frac{(n+1) \cdot n}{2} = \frac{n \cdot (n+1)}{2} \left( \frac{2n-1}{3} + 1 \right)$
$S_n = \frac{n(n+1)}{2} \left( \frac{2(n+2)}{3} \right) = \frac{n(n+1)(n+2)}{3}$

**Detyra 11** Njehsoni shumen: $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + n(n+1)(n+2)$

**Zgjidhje:**

$n(n+1)(n+2) = n^3 + 3n^2 + 2n$

$S_n = (1^3 + 2^3 + \ldots + n^3) + 3(1^2 + 2^2 + \ldots + n^2) + 2(1 + 2 + \ldots + n);$

$S_a = 1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

$S_b = 3(1^2 + 2^2 + \ldots + n^2) = \frac{3n(n+1)(2n+1)}{6} = \frac{n(n+1)(2n+1)}{2}$

$S_c = 2(1 + 2 + \ldots + n) = \frac{2n(n+1)}{2} = n(n+1)$

$S_n = S_a + S_b + S_c = \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1)$

$S_n = \frac{n^2(n+1)^2 + 2n(n+1)(2n+1) + 4n(n+1)}{4}$

$S_n = \frac{[n(n+1)][n(n+1) + 2(2n+1) + 4]}{4}$

$S_n = \frac{n(n+1)(n^2+5n+6)}{4}$

$S_n = \frac{n(n+1)(n+2)(n+3)}{4}$

**13)Tregoni se çdo nenbashkesi M $\subset$ R e cila eshte e kufizuar nga siper ka supremum ne R**

Zgjidhje:

*Meqe M eshte e kufizuar nga siper rrjedh qe* $\exists M^* \in R$ *i tille qe* $m \leq M^*$ $\forall m \in M$.

*Dhe kushti tjeter i suprimumit eshte qe* $(\forall \varepsilon > 0)$ $(\exists m \in M)$ *ashtu qe* $M^* - \varepsilon < m < M^*$ *eshte e vertet sepse ne te kunderten* $M^* - \varepsilon > m > M^*$

$\rightarrow\leftarrow$ *Kontradiksion*

**14)Tregoni se çdo nenbashkesi M $\subset$ R e cila eshte e kufizuar nga poshte ka infimum ne R**

**Zgjidhje:**

*Meqe M eshte e kufizuar nga poshte rrjedh qe* $\exists M^* \in R$ *i tille qe* $M^* \leq m$, $\forall m \in M$

*Dhe nga kushti tjeter i infimumit eshte qe* $(\forall \varepsilon > 0)$ $(\exists m \in M)$ *ashtu qe* $m < M^* + \varepsilon$ *eshte i vertet sepse ne te kunderten* $m > M^* + \varepsilon$

$\rightarrow\leftarrow$ *Kontradiksion*

**Detyra 15** Tregoni se per cdo numer natyror $n, n^3 + 2n$ plotepjestohet me 3.

**Zgjidhje:**

Per n=1
$n^3 + 2n = 3$ plotepjestohet me 3
Per n=2
$n^3 + 2n = 12$ plotepjestohet me 3
Supozojme se vlen per n=k;
$k^3 + 2k$ plotepjestohet me 3 => $k^3 + 2k = 3e \ldots$ (hi)
Tregojme per n=k+1;
$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2 = k^3 + 2k + 3(k^2 + 3k + 1) \stackrel{hi}{=} 3e + 3(k^2 + k + 1)$ plotepjestohet me 3.

**17)Te vertetohet mosbarazimi:**
$2^n > n^2$ per $n \geq 5$

Per $n \geq 5$ kemi:
$2^5 > 5^2$
$35 > 25$ T

Per $n = k$ kemi
$2^k > k^2 \ldots$ (H.I)

Provojme per n=k+1
$2^{k+1} > (k+1)^2$
$2 \cdot 2^k > k^2 + 2k + 1$
$. \vdots$
$(H.I)$
$2 \cdot k^2 > k^2 + 2k + 1$
$k^2 - 2k + 1 > 0$
$(k-1)^2 > 0$

**18) Tregoni se vlen barazimi:**
$arcctg3 + arcctg5 + \cdots + arcctg(2n+1) = arctg2 + arctg\frac{3}{2} + \cdots + arctg\frac{n+1}{n} - narctg1$

**Zgjidhje:**

**Per n=1 kemi:**

$arcctg3 = arctg2 - 1arctg1$

$arcct3 = arctg\frac{2-1}{1+2\cdot 1}$

$arcctg3 = arctg\frac{1}{3}$ E sakte

**Supozojme per n:**

$arcctg3 + arcctg5 + \cdots + arcctg(2n+1) = arctg2 + arctg\frac{3}{2} + \cdots + arctg\frac{n+1}{n} - narctg1 \cdots h.i.$

**Vertetojme per n+1:**

$arcctg3 + arcctg5 + \cdots + arcctg(2n+1) + arcctg(2n+3) = arctg2 + arctg\frac{3}{2} + \cdots + arctg\frac{n+1}{n} + arctg\frac{n+2}{n+1} - narctg1 - (n+1)arctg1 + narctg1$

$\Rightarrow$ Nga supozimi

$arcctg(2n+3) = arctg\frac{n+2}{n+1} - arctg1$

$arcctg(2n+3) = arctg\frac{\frac{n+2}{n+1}-1}{1-\frac{n+2}{n+1}-1}$

$arcctg(2n+3) = arctg\frac{\frac{n+2-n-1}{n+1}}{\frac{n+1+n+2}{n+1}}$

$arcctg(2n+3) = arctg\frac{1}{2n+3}$ Gje qe duhej vertetuar

**Detyra 20.** Tregoni se:

$(\arctan x)^{(n)} = \frac{(n-1)!}{\sqrt{(1+x^2)^n}} \cdot sin(n(\arctan x + \frac{\pi}{2}))$

**Zgjidhje:**

1. Per n=1:

$(\arctan x)' = \frac{(1-1)!}{\sqrt{1+x^2}} \cdot sin(1(\arctan x + \frac{\pi}{2}))$

$\frac{1}{1+x^2} = \frac{1}{\sqrt{1+x^2}} \cdot cos \arctan x$

Sqarim:

$\tan^2 x = \frac{sin^2 x}{cos^2 x}$

$cos^2 x tg^2 x = sin^2 x$

$cos^2 x tg^2 x = 1 - cos^2 x$

$cos^2 x(tg^2 x + 1) = 1$

$cos^2 x = \frac{1}{tg^2 x+1}$

$cos x = \frac{1}{\sqrt{tg^2 x+1}}$

$\frac{1}{1+x^2} = \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{tg^2 arctgx+1}}$

$\frac{1}{1+x^2} = \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}}$

$\frac{1}{1+x^2} = \frac{1}{1+x^2}(T)$

2. Supozojm se vlen per n=k:

$(\arctan x)^{(k)} = \frac{(k-1)!}{\sqrt{(1+x^2)^k}} \cdot sin(k(\arctan x + \frac{\pi}{2})) \cdots\cdots (hi)$

3. Duhet treguar per n=k+1:

$(\arctan x)^{(k+1)} = \frac{(k)!}{\sqrt{(1+x^2)^{k+1}}} \cdot sin(k+1(\arctan x + \frac{\pi}{2}))$
Nisemi nga ana e majte:

$(\arctan x)^{(k+1)} = \left[\frac{(k-1)!}{\sqrt{(1+x^2)^k}} \cdot sin(k(\arctan x + \frac{\pi}{2}))\right]' =$

$= \frac{-(k-1)!\left[\sqrt{(1+x^2)^k}\right]'}{\sqrt{(1+x^2)^{2k}}} sin \overbrace{\left[k(arctgx + \frac{\pi}{2}\right]}^{\alpha} + \frac{(n-1)!}{\sqrt{(1+x^2)^n}} \cdot cos\left[n(arctgx + \frac{\pi}{2}\right] \frac{n}{1+x^2} =$

$\frac{-(n-1)! \cdot \frac{n}{2}[1+x^2]^{\frac{n}{2}-1} 2x}{(1+x^2)^n} \cdot sinx + \frac{n!}{(1+x^2)^{\frac{n}{2}+1}} \cdot cos\alpha =$

$\frac{n!}{\sqrt{(1+x^2)^{n+2}}} \cdot cos\alpha - \frac{n!x}{(1+x^2)^{n-\frac{n}{2}+1}} \cdot sin\alpha =$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} \left( \frac{1}{\sqrt{1+x^2}} \cdot cos\alpha - \frac{x}{\sqrt{1+x^2}} \cdot sin\alpha \right)$

Sqarim:

$cosx = \frac{1}{\sqrt{1+tg^2x}}; sinx = \frac{tgx}{\sqrt{1+tg^2x}}$

$\frac{1}{\sqrt{1+x^2}} = cos(arctgx)$

$\frac{x}{\sqrt{1+x^2}} = sin(arctgx)$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} (cosarctgx \cdot cos\alpha - sinarctgx \cdot sinx) =$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} [cos(arctgx + \alpha)] =$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} [sin(\frac{\pi}{2} + arctgx + \alpha)] =$

$\boxed{\alpha = n(arctgx + \frac{\pi}{2})}$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} [sin(\frac{\pi}{2} + arctgx + arctgx + n\frac{\pi}{2})] =$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} [sin((n+1)\frac{\pi}{2} + (n+1)arctgx)] =$

$\frac{n!}{\sqrt{(1+x^2)^{n+1}}} [(n+1)(arctgx + \frac{\pi}{2})] \cdots (T)$

**21)Tregoni se:** $(e^x sinx)^{(n)} = 2^{\frac{n}{2}} e^x sin\left(x + \frac{n\pi}{4}\right)$

**Zgjidhje:**

**Per n=1 kemi:**

$(e^x sinx)^{(')} = \sqrt{2} e^x sin\left(x + \frac{\pi}{4}\right)$

$e^x sinx + e^x cosx = \sqrt{2} e^x sin\left(x + \frac{\pi}{4}\right)$

$e^x \cdot \left(sinx + sin\left(\frac{\pi}{2} - x\right)\right) = \sqrt{2} e^x sin\left(x + \frac{\pi}{4}\right)$

$e^x \cdot 2sin\frac{\pi}{4} \cdot cos\frac{x-\frac{\pi}{2}+x}{2} = \sqrt{2} e^x sin\left(x + \frac{\pi}{4}\right)$

$e^x \cdot \sqrt{2} \cdot cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}e^x sin\left(x + \frac{\pi}{4}\right)$

$e^x\sqrt{2}sin\left(x + \frac{\pi}{4}\right) = \sqrt{2}e^x sin\left(x + \frac{\pi}{4}\right)$
E sakte

**Supozojme per n:**

$(e^x sinx)^{(n)} = 2^{\frac{n}{2}} e^x sin\left(x + \frac{n\pi}{4}\right)$

**Vertetojme per n+1:**

$(e^x sinx)^{(n+1)} = 2^{\frac{n+1}{2}} e^x sin\left(x + \frac{(n+1)\pi}{4}\right)$

$(e^x sinx)^{(n+1)} = \left[2^{\frac{n}{2}} e^x sin\left(x + \frac{n\pi}{4}\right)\right]' =$

$2^{\frac{n}{2}}\left[e^x sin\left(x + \frac{n\pi}{4}\right) + e^x cos\left(x + \frac{n\pi}{4}\right)\right] =$

$2^{\frac{n}{2}} e^x\left[sin\left(x + \frac{n\pi}{4}\right) + sin\left(\frac{\pi}{2} + x + \frac{n\pi}{4}\right)\right] =$

$2^{\frac{n}{2}} e^x\left[2sin\frac{2x + \frac{2n\pi}{4} + \frac{\pi}{2}}{2} cos\frac{\pi}{4}\right] =$

$2^{\frac{n}{2}+1} e^x \frac{\sqrt{2}}{2} sin\left(x + \frac{(n+1)\pi}{4}\right) =$

$2^{\frac{n}{2}+1-1+\frac{1}{2}} e^x sin\left(x + \frac{(n+1)\pi}{4}\right) =$

$2^{\frac{n+1}{2}} e^x sin\left(x + \frac{(n+1)\pi}{4}\right)$ Qe duhej vertetuar

**Detyra22** Le të jetë dhënë vargu $(a_n)_{n\geq 1}$ ashtu që
$\sum\limits_{k=1}^{n} a_k = \frac{3n^2+9n}{2}, \forall n \geq 1.$
Njehësoni
$\lim\limits_{n\to\infty} \frac{1}{na_n} \sum\limits_{k=1}^{n} a_k.$

**Zgjidhje:**

$a_n = \sum\limits_{k=1}^{n} a_k - \sum\limits_{k=1}^{n-1} a_k = \frac{3n^2+9n}{2} - \frac{3(n-1)^2+9(n-1)}{2} =$
$= \frac{3n^2+9n-3n^2+6n-3-9n+9}{2} = \frac{6n+6}{2} = 3n + 3$

$\lim\limits_{n\to\infty} \frac{1}{n\cdot a_n} \sum\limits_{k=1}^{n} a_k = \lim\limits_{n\to\infty} \frac{1}{n\cdot 3(n+1)} \cdot \frac{3n^2+9n}{2} = \lim\limits_{n\to\infty} \frac{3n^2+9n}{6n^2+6n} = \frac{3}{6} = \frac{1}{2}$

23) Le te jete dhene vargu $(a_n)$ $n \geq 1$ ashtu qe $a_1 = a_2 = 0$ dhe $a_{n+1} = \frac{1}{3}(a_n + a_{n-1}^2 + b)$,ku $0 \leq b < 1$. Tregoni se vargu i dhene eshte konvergjent dhe njehsoni

$$\lim_{n\to\infty} a_n$$

.

Zgjidhje

Qe te tregojme qe vargu eshte konvergjent mjaftone te tregojme se ka monotoni dhe eshte i kufizuar

$a = \frac{1}{3}(a + a^2 + b)$
$3a = (a + a^2 + b)$
$a^2 - 2a + b = 0$
$a_{1\backslash 2} = \frac{2\pm\sqrt{1-4b}}{2}$
$a_{1\backslash 2} = 1 \pm \sqrt{1-b}$

- Monotonia
$a_3 = \frac{b}{3}$
$a_4 = \frac{1}{3}(\frac{b}{3} + b) = \frac{1}{3}(\frac{b+3b}{3}) = \frac{4b}{9} \Rightarrow a_n > a_3$
Me ane te induksionit matematike:
$a_{n+1} > a_n$

$a_{n+1} = \frac{1}{3}(a_n + a_{n+1}^2 + b) > \overbrace{\frac{1}{3}(a_{n-1} + a_{n-2}^2 + b)}^{a_n} = a_n$

- Kufizueshmeria

$a_1 < 1 + \sqrt{1-b}$
$a_2 < 1 + \sqrt{1-b}$
$a_3 \neq \frac{b}{3} < 1 + \sqrt{1-b}$
Supozojme se $a_n < 1 + \sqrt{1-b}$ - (i kufizueshem nga larte $< 1 + \sqrt{1-b}$)
Duhet vertetuar:
$a_{n+1} < 1 + \sqrt{1-b}$
$a_{n+1} = \frac{1}{3}(a_n+a_{n-1}^2+b) < \frac{1}{3}(a_{n-1}+a_{n-1}^2+b) < \frac{1}{3}(1+\sqrt{1-b}+(1+\sqrt{1-b})^2+b) = \frac{1}{3}(1 + \sqrt{1-b} + 1 + 2\sqrt{1-b} + 1 - b + b) = \frac{1}{3}(3 + 3\sqrt{1-b}) = 1 + \sqrt{1-b}$

$$\lim_{n\to\infty} a_n = 1 + \sqrt{1-b}$$

**24)Njehsoni:**

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k!\cdot k}{(n+1)!}$$

**Zgjidhje**

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k!\cdot k}{(n+1)!} = \lim_{n\to\infty} \frac{(n+1)!-1}{(n+1)!} = \lim_{n\to\infty} \left(1 - \frac{1}{(n+1)!}\right) = 1-0 = 1$$

**Sqarim:**

$$\left[\sum_{k=1}^{n} \frac{k!\cdot k}{(n+1)!} = \frac{1\cdot 1!}{(n+1)!} + \frac{2\cdot 2!}{(n+1)!} + \frac{3\cdot 3!}{(n+1)!} + \cdots + \frac{n\cdot n!}{(n+1)!} = \frac{1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \cdots + n\cdot n!}{(n+1)!}\right]$$

Per n=1 kemi:

$$1\cdot 1! = 1$$

Per n=2 kemi:

$$1 + 2\cdot 2! = 1 + 4 = 5$$

Per n=3 kemi:

$$1 + 2\cdot 2! + 3\cdot 3! = 5 + 18 = 23$$

Per n=4 kemi:

$$1 + 2\cdot 2! + 3\cdot 3! + 4\cdot 4! = 119$$

$\vdots$

Per n=n kemi:

$$1 + 2\cdot 2! + \cdots + n\cdot n! = (n+1)! - 1$$

textbf*Detyra 25:* Njehsoni

$$\lim_{n\to\infty} \left(\frac{3^{3n}(n!)^3}{(3n)!}\right)^{\frac{1}{n}}$$

**Duke perdorur Teoremen e dyte te Koshit mbi limitet**

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

**kemi:**

$$\lim_{n\to\infty} \left(\frac{3^{3n}(n!)^3}{(3n)!}\right)^{\frac{1}{n}} = \lim_{n\to\infty} 27\sqrt[n]{\frac{(n!)^3}{(3n)!}} = 27\lim_{n\to\infty} \sqrt[n]{\frac{(n!)^3}{(3n)!}} \overset{(*)}{=} 27\lim_{n\to\infty} \frac{(n+1)!^3(3n)!}{(n!)^3(3n+3)!} =$$

$$= 27\lim_{n\to\infty} \frac{(n+1)^3(n!)^3(3n)!}{(n!)^3(3n+3)(3n+2)(3n+1)(3n)!} = 27\lim_{n\to\infty} \frac{n^3+3n^2+3n+1}{27m^3+54n^2+33n+6} = 27\frac{1}{27} = 1$$

**27)Njehsoni:**

$$\lim_{x\to\ 0} \frac{ln(1+x+x^2+\cdots+x^n)}{nx}$$

Zgjidhje:

$$\lim_{x\to\ 0} \frac{ln(1+x+x^2+\cdots+x^n)}{nx} =$$

$$\lim_{x\to 0} \frac{ln\left(\frac{1-x^n}{x-1}\right)}{nx} =$$

Sqarim

$$1+x+x^2+\cdots+x^n = \text{Varg gjeometrikë}$$

$$s_n = \frac{1-x^n}{x-1}$$

Sipas rregulles se Lopitalit kemi:

$$\lim_{x\to 0} \frac{\left(ln\left(\frac{1-x^n}{x-1}\right)\right)'}{(nx)'} =$$

$$\lim_{x\to 0} \frac{\frac{x-1}{1-x^n}\left(\frac{(-nx^{n-1}-(1-x^n)}{(x-1)^2}\right)}{n} =$$

$$\lim_{x\to\ 0} \frac{\frac{x-1}{1-x^n}\left(\frac{-nx^{n-1}-1+x^n}{(x-1)^2}\right)}{n} =$$

$$\lim_{x\to\ 0}\frac{\frac{x-1}{1-x^n}\left(\frac{-nx^{n-1}-1+x^n}{(x-1)^2}\right)}{n}=$$

$$\frac{\frac{-1}{1}\cdot\frac{-n0^{n-1}-1+0^n}{(-1)^2}}{n}=\frac{1}{n}$$

**28)Te caktohet parametri a ashtu qe:**

$$\lim_{x\to 0}\frac{(1-cosax)}{x^2}=\lim_{x\to 0}\frac{sinx}{\pi-x}$$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-cosax}{x^2}=\lim_{x\to 0}\frac{2sin^2\frac{ax}{2}}{x^2}=$$

$$\lim_{x\to 0}\frac{2sin^2\frac{ax}{2}}{\left(\frac{ax}{2}\right)^2\cdot\frac{4}{a^2}}=1\cdot\lim_{x\to\ 0}\frac{1}{2}\frac{4}{a^2}=\frac{a^2}{2}$$

$$\Rightarrow\frac{a^2}{2}=\lim_{x\to\ 0}\frac{sinx}{\pi-x}$$

$$\frac{a^2}{2}=\frac{0}{\pi}$$

$$a^2=0$$

$$a=0$$

**Detyra 29** Nese a,b,c $\in R$, njehsoni: $\lim\limits_{x\to\infty}(a\sqrt{x+1}+b\sqrt{x+2}+c\sqrt{x+3})$;

**Zgjidhje:**

Sqarim: Me që $\lim\limits_{x\to\infty}(\sqrt{x+1})=\lim\limits_{x\to\infty}(\sqrt{x+2})=\lim\limits_{x\to\infty}(\sqrt{x+3})$

$$Ateher:\lim_{x\to\infty}(a+b+c)\cdot\sqrt{x+1}=\begin{cases}-\infty, & a+b+c<0\\ 0, & a+b+c=0\\ +\infty, & a+b+c>0\end{cases}$$

**Ne rastin e pergjithshem kemi :**

$$\lim_{x\to\infty}(a\sqrt{x+1}+b\sqrt{x+2}+c\sqrt{x+3})\,[x+2=t;x+1=t-1;x+3=t+1;x\to\infty;t\to\infty]$$

$$\lim_{t\to\infty}(a\sqrt{t-1}+b\sqrt{t}+c\sqrt{t+1})\left[t=tg^2x;t\to\infty;x\to\frac{\pi}{2}\right]$$

$$\lim_{x\to\{\frac{\pi}{2}}\left(a\sqrt{tg^2x-1}+btgx+c\sqrt{tg^2x+1}\right)=$$

$$\lim_{x\to\{\frac{\pi}{2}}\left(a\frac{\sqrt{sin^2x-cos^2x}}{cosx}+b\frac{sinx}{cosx}+c\frac{1}{cosx}\right)=$$

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{a\sqrt{sin^2x-cos^2x}+b\cdot sinx+c}{cosx}\right)=$$

$$\frac{a+b+c}{0}=\infty$$

**Detyra 30** Njehsoni:

$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}$

**Zgjidhje:**

Nga teorema mbi tre limitet:

$$\left.\begin{array}{l}\frac{1}{\sqrt{n^2+n}}=\frac{1}{\sqrt{n^2+k}}<\frac{n}{\sqrt{n^2+1}}\\ \frac{1}{\sqrt{n^2+n}}<\frac{2}{\sqrt{n^2+k}}<\frac{n}{\sqrt{n^2+1}}\\ \dots\dots\dots\dots\dots\dots\dots\dots\\ \frac{1}{\sqrt{n^2+n}}<\frac{n}{\sqrt{n^2+k}}=\frac{n}{\sqrt{n^2+1}}\end{array}\right\}(+)$$

$\frac{n}{\sqrt{n^2+n}}\leq\sum\limits_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}\leq\frac{n}{\sqrt{n^2+1}}$

$\lim\limits_{n\to\infty}\frac{n}{\sqrt{n^2+n}}\leq\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}\leq\lim\limits_{n\to\infty}\frac{n}{\sqrt{n^2+1}}$

$1\leq\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}\leq 1 => \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}=1$

**Detyra 31.**Njehsoni:

$\lim\limits_{n\to\infty}(e^x+\sin x)^{\frac{1}{x}}$

**Zgjidhje:**

$\lim\limits_{n\to\infty}(e^x+\sin x)^{\frac{1}{x}} = \lim\limits_{n\to\infty}(e^x)^{\frac{1}{x}}(1+\frac{\sin x}{e^x})^{\frac{1}{x}} =$
$e\lim\limits_{x\to 0}(1+\frac{\sin x}{e^x})^{\frac{e^x}{\sin x}\cdot\frac{\sin x}{x\cdot e^x}} =$
$= e\lim\limits_{x\to 0}\left[(1+\frac{\sin x}{e^x})^{\frac{e^x}{\sin x}}\right]^{\frac{\sin x}{x}\cdot\frac{1}{e^x}} = e\cdot e = e^2$

33) Njehsoni

$$\lim_{x\to 0}\frac{tgx-x}{x^2} = \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}-x}{x^2} = \lim_{x\to 0}\frac{\frac{\sin x - x\cos x}{\cos x}}{x^2} = \lim_{x\to 0}\frac{\sin x - x\cos x}{x^2\cos x} = \lim_{x\to 0}\frac{\sin x}{x^2\cos x} - \lim_{x\to 0}\frac{x\cos x}{x^2\cos x} =$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{x\cos x} - \lim_{x\to 0}\frac{1}{x} = 1\cdot\lim_{x\to 0}\frac{1}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x} - \lim_{x\to 0}\frac{1}{x} = \lim_{x\to 0}(1-1)\cdot\lim_{x\to 0}\frac{1}{x} = 0\cdot\lim_{x\to 0}\frac{1}{x} = 0$$

34. Njehsoni:

$$\lim_{x\to 0}\frac{tgx-arctgx}{x^2} = (MeLopital) = \lim_{x\to 0}\frac{(tgx-arctgx)'}{(x^2)'} = \lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}}{2x} =$$

$$\lim_{x\to 0}\frac{\left(\frac{(1+x^2)-\cos^2 x}{\cos^2 x(1+x^2)}\right)'}{(2x)'} = \lim_{x\to 0}\frac{\frac{(2x-2\sin x\cos x)\cos^2 x(1+x^2)-2\sin x\cos x(2x)(1+x^2-\cos^2 x)}{\cos^4 x(1+x^2)^2}}{2}$$
$=$

$$\lim_{x\to 0}\frac{(2x-sin2x)\cos^2 x(1+x^2)-sin2x(1+x^2-\cos^2 x)}{2\cos^4 x} = \frac{(0-0)1(1+0)-0(1+0-1)}{2\cdot 1(1+0)^2} = \frac{0}{2} = 0.$$

37 - Te shqyrtohet dhe te paraqitet grafikisht funksioni:

f(x)=$\frac{x^2-2x+1}{x^2+1}$

1)**Domena**

$x^2+1 \neq 0$
$x^2 \neq 1$
$x \neq \pm\sqrt{-1}$

$D_f = \{-\infty, \infty\}$

2) **Zerot e funksionit**

$x=0 \quad y=\frac{0-0+1}{0+1}=1$
$A(0,1)$

$y=0 \quad \frac{x^2-2x+1}{x^2+1}=0$
$x_{1/2}=\frac{2\pm\sqrt{4-4}}{2}=\frac{2\pm0}{2}=1$
$B(1,0)$

3) **Simetria**

$f(x)=\frac{(-x)^2-2(-x)+1}{(-x)^2+1}=\frac{x^2+2x+1}{x^2+1} \Rightarrow asimetrike$

4) **Asimptotat**

$$\lim_{x\to\infty}\frac{x^2-2x+1}{x^2+1}=2$$

- asimptote horizontale ne y=2.
Pasi qe nuk ka keputje domena funksioni nuk ka asimptote vertikale.
Pasi qe ka asimptote horizontale rrjedhimisht nuk ka asimptote te pjerte.

6) **Monotonia dhe vlerat ekstreme**

$f^{"}(x)=\frac{x^2-2x+1}{x^2+1}=\frac{(2x-2)(x^2+1)-2x(x^2-2x+1)}{(x^2+1)^2}=\frac{2x^3-2x^2+2x-2-2x^3+4x^2-2x}{(x^2+1)^2}=$
$\frac{2x^2-2}{(x^2+1)^2}=\frac{2(x^2-1)}{(x^2+1)^2}$

$2(x^2-1)=0$
$x^2-1=0 \Rightarrow x=\pm 1$

| x | $(-\infty,-1)$ | (-1,1) | $(1,\infty)$ |
|---|---|---|---|
| f(x) | + | - | + |
| $f^{'}$ (x) | $\nearrow$ | $\searrow$ | $\nearrow$ |

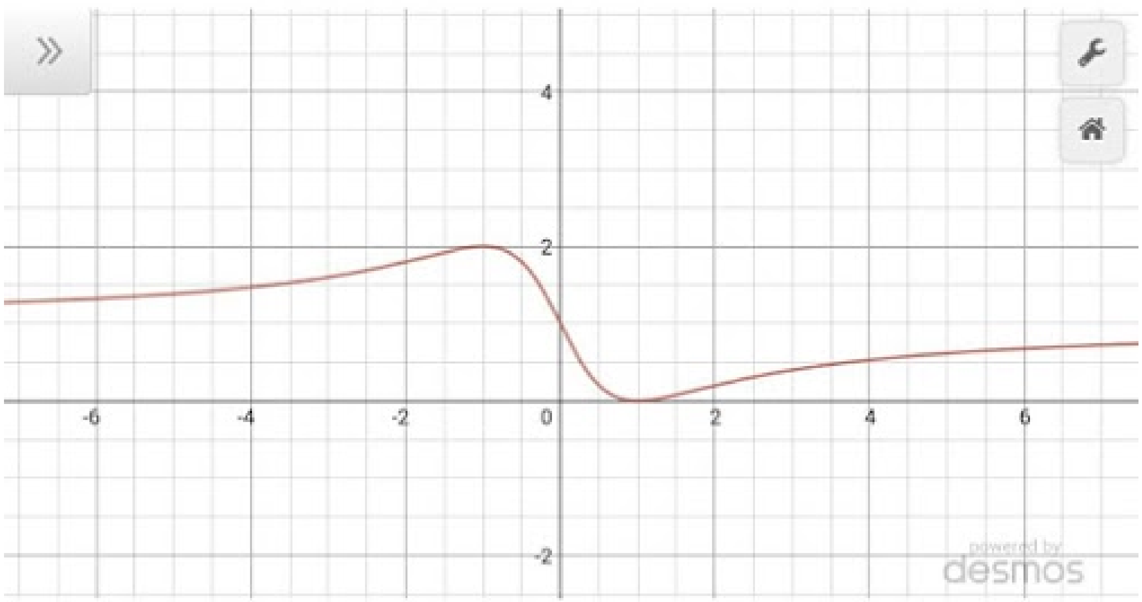
7) **Konkaviteti dhe konveksiteti**

$f^{'}(x)=\frac{2x^2-2}{x^4+2x^2+1}$
$f^{"}(x)=\frac{4x(x^4+2x^2+1)-(4x^3+4x)(2x^2-2)}{(x^2+1)^4}=\frac{4x^5+8x^3+4x-8x^5+8x^3-8x^3+8x}{(x^2+1)^4}=$
$\frac{-4x^5+8x^3+12x}{(x^2+1)^4}=\frac{-4x(x^4-2x^2+3)}{(x^2+1)^4}$

$-4x(x^2-1)^2=0$
$x=0 \quad dhe \quad x^2-1=0 \Rightarrow x=\pm 1$

| x | $(-\infty,0)$ | $(0,\infty)$ |
|---|---|---|
| $f^{'}$(x)- | + | |
| $f^{"}(x)$ | $\cap$ | $\cup$ |

8) **Grafiku i funksionit**




**Detyra 38.** Te shqyrtohet dhe te paraqitet grafikisht funksioni:
$f(x) = \frac{2x}{x^2+1}$

**Zgjidhje:**

**1.** Domena:
$x \in (-\infty, +\infty)$

**2.**Zerot:
$\frac{2x}{x^{+}1} = 0 => x = 0$

**3.**Simetria:
$f(-x) = \frac{2(-x)}{(-x)^2+1} = -\frac{2x}{x^2+1} = -f(x)tek$

**4.**Shenja:

| x | -∞ 0 | 0 ∞ |
|---|---|---|
| f(x) | - | + |

**5.**Asimptotat:
1(A.H)
$\lim_{x\to\infty} \frac{2x}{x^2+1} = 0$
Ka asimptot horizontale y=0.
2(A.V)
Nuk ka asimptote vertikale
3(A.P)
Nuk ka asimpotote te pjerret

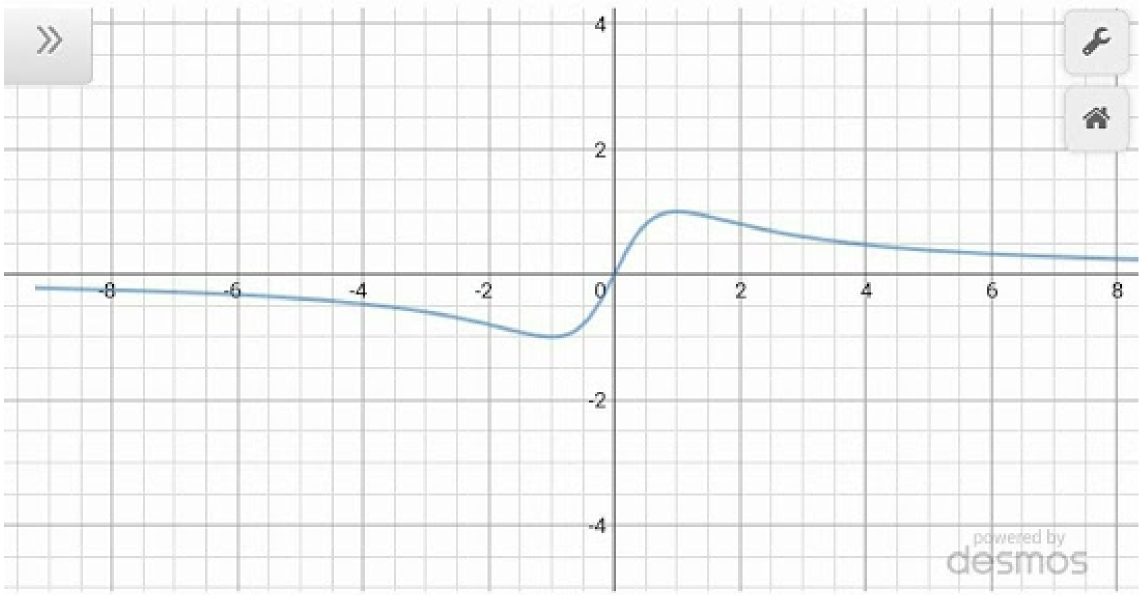
**6.**Monotonia $f'(x) = \left(\frac{2x}{(x^2+1)}\right)' = \frac{2(x^2+1)-2x(2x)}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2} =>$
$-2x^2 + 2 = 0 => x_{1/2} = \pm 1$

| | -∞ , -1 | -1, 1 | 1, + ∞ |
|---|---|---|---|
| f'(x) | - | + | - |
| f(x) | ↘ | ↗ | ↘ |

**7.**Konkaviteti dhe konveksiteti:
$f''(x) = \left(\left(\frac{2x}{x^2+1}\right)'\right)' = \left(\frac{2-2x^2}{(x^2+1)^2}\right)' = \left(\frac{2(1-x^2)}{(x^2+1)^2}\right)' = \frac{2(1-x^2)'(x^2+1)^2-2(1-x^2)(x^2+1)^2}{(x^2+1)^4} =$
$\frac{-4x(x^2+1)^2-8x(1-x^2)(x^2+1)}{(x^2+1)^4} = \frac{4x^3-12x}{(x^2+1)^3} => x = 0, x_{1/2} = \pm\sqrt{3}$

| | -∞ $-\sqrt{3}$ | $-\sqrt{3}$ 0 | 0 $\sqrt{3}$ | $\sqrt{3}$ $+\infty$ |
|---|---|---|---|---|
| f''(x) | + | + | - | - |
| f(x) | $\bigcup$ | $\bigcup$ | $\bigcap$ | $\bigcap$ |

**Grafiku**




**Detyra 39.** Te shqyrtohet dhe te paraqitet grafikisht funksioni:

$f(x) = \frac{4x}{4-x^2}$

**Zgjidhje:**

**1.**Domena:

$x \in (-\infty,-2)\bigcup(-2,0)\bigcup(0,2)\bigcup(2,\infty)$

**2.**Perioda:
Nuk ka periode.

**3.**Asimtotat:

$\lim\limits_{x\to\infty} \frac{4x}{4-x^2} = 0 => y = 0$ eshte asimtote horizontale
$\lim\limits_{x\to 0^+} = \lim\limits_{\varepsilon\to 0} \frac{4\varepsilon}{4-\varepsilon^2} = 0$ pika x=0 nuk eshte A.V
$\lim\limits_{x\to 2^+} \frac{4x}{4-x^2} = \lim\limits_{\varepsilon\to 0} \frac{4(2+\varepsilon)}{4-(2+\varepsilon)^2} = -\infty$
$\lim\limits_{x\to 2^-} \frac{4x}{4-x^2} = \lim\limits_{\varepsilon\to 0} \frac{4(2-\varepsilon)}{4-(2-\varepsilon)^2} = \infty$ pika x=2 eshte A.V
$\lim\limits_{x\to -2^+} \frac{4x}{4-x^2} = \lim\limits_{\varepsilon\to 0} \frac{4(-2+\varepsilon)}{4-(-2+\varepsilon)^2} = -\infty$
$\lim\limits_{x\to -2^-} \frac{4x}{4-x^2} = \lim\limits_{\varepsilon\to 0} \frac{4(-2-\varepsilon)}{4-(-2-\varepsilon)^2} = \infty$ pika x=-2 eshte A.V

**4.**Monotonia:
$\left(\frac{4x}{4-x^2}\right)' = \frac{4(4-x^2)-4x(-2x)}{(4-x^2)^2} = \frac{16+4x^2}{(4-x^2)^2} => x = -2, x = 2$

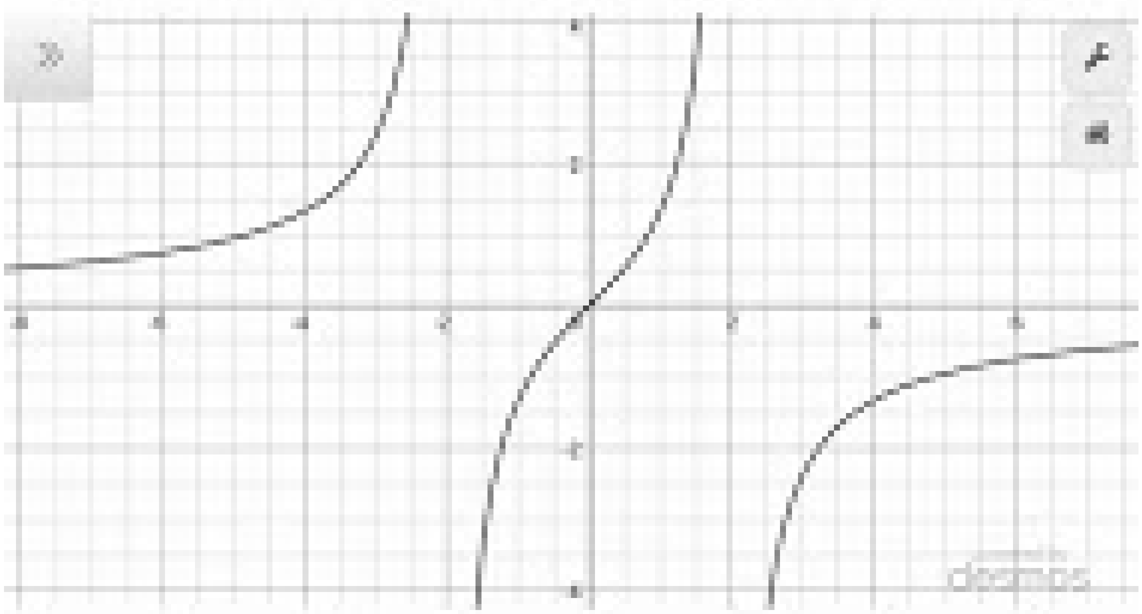
| | -∞,-2 | -2, 0 | 0, 2 | 2, +∞ |
|---|---|---|---|---|
| f'(x) | + | - | + | - |
| f(x) | rrites | zvg | rrites | zvg |

**5.**Simetria:
$f(-x) = \frac{-4x}{4-x^2} = -f(x)$ eshte tek.

**6.**Sjellja:
$\lim\limits_{x\to -\infty} \frac{4x}{4-x^2} = 0$
$\lim\limits_{x\to 0} \frac{4x}{4-x^2} = 0$
$\lim\limits_{x\to -2} \frac{4x}{4-x^2} = -\infty$
$\lim\limits_{x\to 2} \frac{4x}{4-x^2} = \infty$
$\lim\limits_{x\to \infty} \frac{4x}{4-x^2} = 0$
**7.**Konkaviteti dhe konveksiteti
$f''(x) = \left(\left(\frac{4x}{4-x^2}\right)'\right)' = \frac{8x(4-x^2)^2+4x(16x-4x^2+8x^2)(4-x^2)}{(4-x^2)^4} = \frac{8x(4-x^2)+4x(16+4x^2)}{(4-x^2)^3} =$
$\frac{8x(x^2+12)}{(4-x^2)^3} => x = 0, x = 2, x = -2$

| | -∞,-2 | -2, 0 | 0, 2 | 2, +∞ |
|---|---|---|---|---|
| f''(x) | - | - | + | + |
| f(x) | $\bigcap$ | $\bigcap$ | $\bigcup$ | $\bigcup$ |

**Grafiku :**



**Detyra 41.**Te shqyrtohet dhe te paraqitet grafikisht funksioni:

$f(x) = xe^x$

**Zgjidhje:**

**1.**Domena:

$x \in (-\infty, +\infty)$

**2.**Perioda:
Nuk ka periode.

**3.**Asimptotat:
$\lim_{x\to\infty} xe^x = \infty$ nuk ka asimptote horizontale

$\lim_{x\to-\infty} xe^x = 0$

$\lim_{x\to\infty} xe^x = \infty$

Boshti $O_y$ eshte asimptota vertikale

**4.**Sjellja:
$\lim_{x\to-\infty} xe^x = 0$
$\lim_{x\to\infty} xe^x = \infty$

**5.**Zerot:
$xe^x = 0 => x = 0$

**6.**Monotonia:
$f'(x) = (xe^x)' = e^x \cdot xe^x = e^x(1+x) => x = -1$

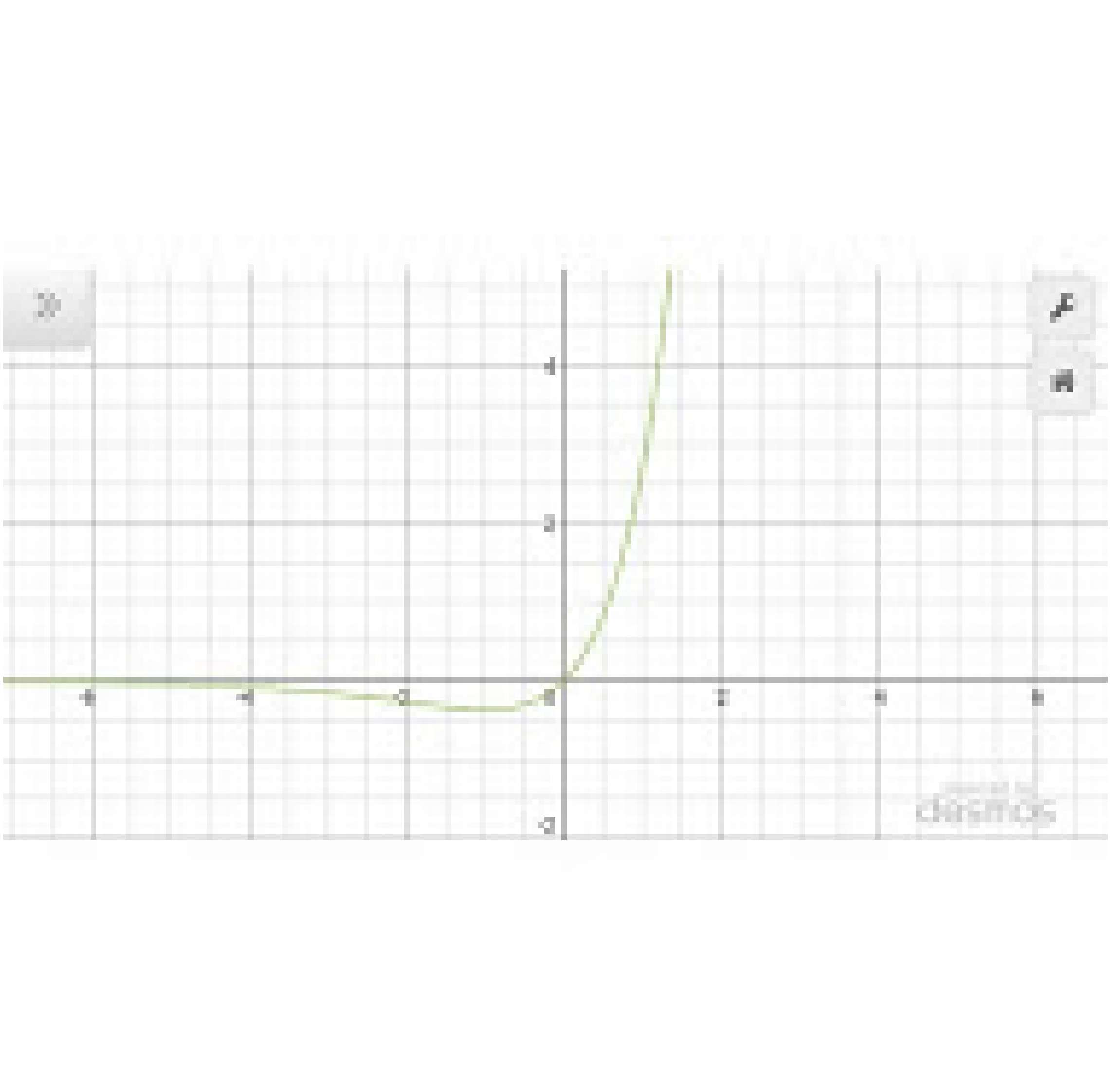
| | $-\infty$, -1 | -1, $+\infty$ |
|---|---|---|
| f'(x) | - | + |
| f(x) | zvg | rrites |

**7.**Konkaviteti dhe konveksiteti:

$f''(x) = ((xe^x)')' = (e^x + xe^x)' = e^x x + 2e^x = e^x(x+2) => x = -2$

| | $-\infty$, -2 | -2, $+\infty$ |
|---|---|---|
| f'(x) | - | + |
| f(x) | $\cap$ | $\cup$ |

**Grafiku:**



**Detyra 42.** Te shqyrtohet dhe te paraqitet grafikisht funksioni:

$f(x) = x^2 e^{-x}$

**Zgjidhje:**

**1.**Domena:

$x \in (-\infty, +\infty)$

**2.**Perioda:
Nuk ka periode.

**3.**Asimtotat:

$\lim\limits_{x\to\infty} x^2 e^{-x} = 0 => y = 0$ eshte asimtota horizontale
$\lim\limits_{x\to-\infty} x^2 e^{-x} = \infty$
$\lim\limits_{x\to\infty} x^2 e^{-x} = 0$

Nuk ka asimptota vertikale

**4.**Sjellja me skaje te domenit:

$\lim\limits_{n\to\infty} x^2 e^{-x} = 0$

$\lim\limits_{n\to-\infty} x^2 e^{-x} = \infty$

**5.**Zerot:

$x^2 e^{-x} = 0 => x = 0$

**6.**Monotonia:

$\left(\frac{x^2}{e^x}\right)' = \frac{2xe^x - x^2e^x}{e^{2x}} = \frac{e^x(2x-x^2)}{e^{2x}} => x = 0$ dhe $x = 2$

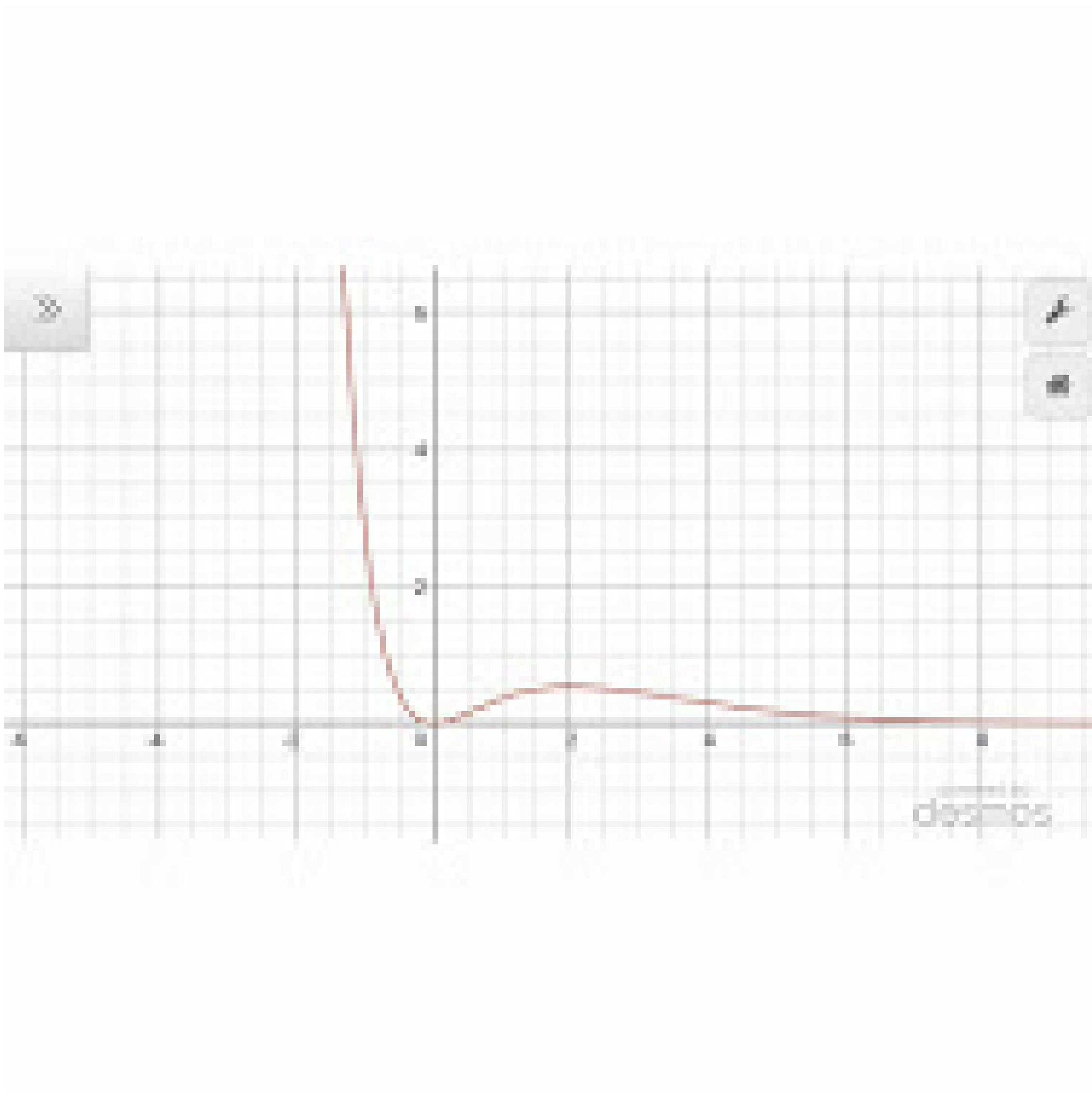
| | -∞, 0 | 0, 2 | 2, +∞ |
|---|---|---|---|
| f'(x) | - | + | - |
| f(x) | zvg | rrites | zvg |

**7.**Konkaviteti dhe konveksiteti:

$f"(x) = \left((x^2e^{-x})'\right)' = \left(\frac{2xe^x - x^2e^x}{e^{2x}}\right)' = \frac{e^x(x^2-4x+2)}{e^{2x}}$
$x_1 = 2 + \sqrt{2} = 3.41$
$x_2 = 2 - \sqrt{2} = 0.6$

| | $-\infty$, 0.6 | 0.6, 3.4 | 3.4, +∞ |
|---|---|---|---|
| f"(x) | + | - | + |
| f(x) | $\bigcup$ | $\bigcap$ | $\bigcup$ |

**Grafiku :**



**Detyra 44.** Te shqyrtohet dhe te paraqitet grafikisht funksioni:

$f(x) = \frac{lnx}{x^2}$

**Zgjidhje:**

**1.** Domena:

$x > 0 \wedge x \neq 0$
$x \in (0, +\infty)$

**2.**Zerot:

$\frac{lnx}{x^2} = 0, x = 1$

**3.**Simetria:
$f(-x) = \frac{ln(-x)}{(-x)^2} = \frac{lnx}{x} simetrike$

**4.**Shenja:
Gjithmon pozitive

**5.**Asimptotat:

1(A.V.)
$\lim\limits_{x \to 0} \frac{lnx}{x^2} = \lim\limits_{x \to 0} \frac{(lnx)'}{(x^2)'} = \lim\limits_{x \to 0} \frac{\frac{1}{x}}{2x} = \infty$
Asimptota vertikale x=0

2(A.H.)
$\lim\limits_{x \to 0} \frac{lnx}{x^2} = ln \lim\limits_{x \to 0} \frac{x}{x^2} = ln \lim\limits_{x \to 0} \frac{1}{x} = 0$

Asimptota horizontale y=0

3(A.P.)
Nuk ka asimptot te pjerrt pasi qe ka horizontale

**6.**Monotonia:

$\left(\frac{lnx}{x^2}\right)' = \frac{\frac{1}{x}x^2 - lnx2x}{x^4} = \frac{x(1-2lnx)}{x^4} => 1 - 2lnx = 0; lnx = \frac{1}{2}$
$\sqrt{e} = x = 1.64$

| | 0, 1.64 | 1.64, $+\infty$ |
|---|---|---|
| f'(x) | + | - |
| f(x) | $\nearrow$ | $\searrow$ |

**7.**Konkaviteti:
$f''(x) = \left(\left(\frac{lnx}{x^2}\right)'\right)' = \left(\frac{1-2lnx}{x^3}\right)' = \frac{(1-2lnx)'x^3-(1-2lnx)3x^2}{x^6} = \frac{-2\frac{1}{x}x^3-(1-2lnx)3x^2}{x^6} =$
$\frac{x^2(-2-3(1-2lnx))}{x^6} = \frac{-5+6lnx}{x^3} =>$
$-5 + 6lnx = 0$
$lnx = \frac{5}{6}$
$e^{\frac{5}{6}} = x = 2.3$

| | 0, 2.3 | 2.3, $+\infty$ |
|---|---|---|
| f''(x) | - | + |
| f(x) | $\cap$ | $\cup$ |

**Grafiku**