

1. Preliminary Work

- Definition: The pullback m^* (Isomorphism between $K[G \times G]$ and $K[G] \otimes K[G]$)
- Definition: Rational representation (with infinite-dimensional vector spaces)
- Definition: Regular action
- An alternate definition for rational representation for finite vector spaces, and the proof for its equivalence
- Another equivalent definition for a rational representation which immediately follows from the last characterization
- Definition: Invariants and the invariant ring

2. Linearly reductive groups, the Reynolds operator and Hilbert's finiteness theorem

- Definition: Reynolds operator
- Uniqueness of the Reynolds operator
- Definition: linear reductiveness; definition via the Reynolds operator
- Remark about different ways to characterize linear reductiveness
- Definition: A multiplication in $K[G]^*$
- Proposition: The afore mentioned multiplication makes $K[G]^*$ into an associative algebra
- Definition: The action of $K[G]^*$ on V induced by an action of G on V .
- A remark about how we can view the $K[G]^*$ action as an extension of the given one ($\sigma.v = \epsilon_\sigma \cdot v$ where $\epsilon_\sigma \in K[G]^*$ is the evaluation homomorphism).
- Remark: $K[G]^*$ contains the Lie algebra as a subalgebra.
- Proposition: A regular action of G on an affine variety X induces a rational representation $K[X]$ of G
- Proposition: A Reynolds operator on $K[G]$ (action via left-multiplication) yields a Reynolds operator on $K[X]$ for any regular action of G on X
- TODO:
The usual lemmas for Hilbert's finiteness theorem
Hilbert's finiteness theorem

3. Cayley's Ω -Process

- Definition: Cayley's Ω -Process
- A Lemma about a relationship between m^* and Ω

- Lemma and Definition: The constants $c_{p,n} := \Omega^p(\det(Z)^p)$
- Theorem: GL_n is linearly reductive via the Reynolds operator that can be defined by Cayley's Ω -process
- Corollary: SL_n is linearly reductive

4. Examples (known or easy ones)

- The cross ratio
- SAS^T