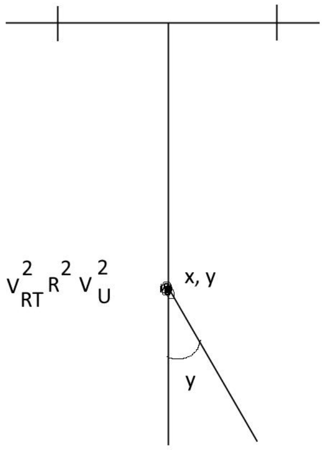
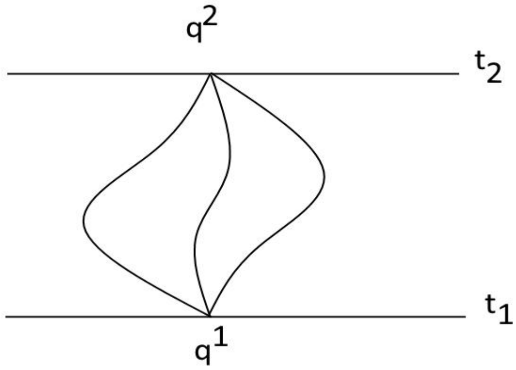


$$\sqrt{x_1^2 + y_1^2} = l$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = l_2$$



q_1(t) | $\frac{dq_1}{dt} = \hat{q}_1(t)$

q_2(t) | $\hat{q}_2(t)$

… | …

q_n(t) | $\hat{q}_n(t)$

$\mathfrak{L}(q_i, \hat{q}_1, t)$ = K - П П = П($q_1, \dots, q_n$) K = K($q_1, \dots, q_n, \hat{q}_1, \dots, \hat{q}_n$)

$$\frac{d}{dt}\frac{d\mathfrak{L}}{dq_i} - \frac{d\mathfrak{L}}{dq_i} = 0 \qquad K = \Sigma_{i,j=1}^{n} q_{ij}(q_1, \dots, q_n)\hat{q}_i, \hat{q}_j$$

$$\Sigma_{i=1}^{n}\frac{dK}{dq_i} * \hat{q}_i = 2K$$

q_1(t) q_2(t) … q_n(t)

$$S = \int_{t1}^{t2} \mathfrak{L}(\bar{q}, \bar{\hat{q}}, t)\, dt$$

$$\delta S = \int_{t1}^{t2} [\mathfrak{L}(\bar{q}, \bar{\hat{q}}, t) + \Sigma_{i=1}^{n}\frac{d\mathfrak{L}}{dq_i} * \delta q_i + \Sigma \frac{d\mathfrak{L}}{d\hat{x}_i}]dt = q^2$$

$$\bar{x}_i = q_i + \delta q_i$$

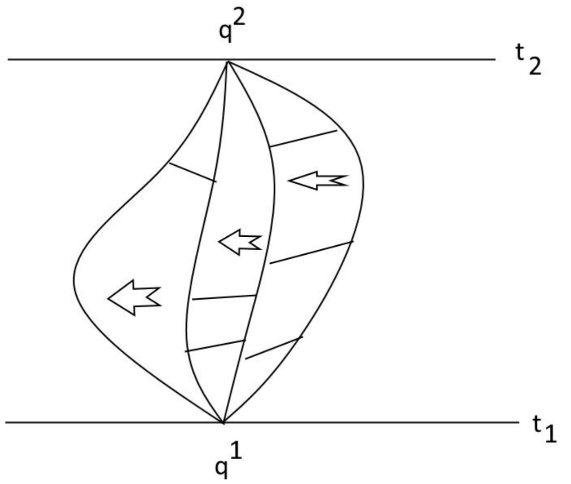
$$\bar{\hat{x}}_i = \hat{q}_i + \delta\hat{q}_i$$

$$(q + \delta q)^2 = q^2 + 2q\delta q + \delta q^2$$

$$2q\delta q + \delta q^2$$

$$= \int_{t1}^{t2} (\Sigma_i \frac{d\mathfrak{L}}{dq_i}\delta q_i - \Sigma_{\delta=1}^{n} \frac{d\mathfrak{L}}{dq_i}\hat{q}_i + \frac{d}{dt}\Sigma_{i=1}^{n}(\frac{d\mathfrak{L}}{d\hat{q}_i}\delta q_i))dt \ =$$

$$= \Sigma_{i=1}^{n}\frac{d\mathfrak{L}}{d\hat{q}_i}\delta q_i|_{t=t_2} - \Sigma_{i=1}^{n}\frac{d\mathfrak{L}}{d\hat{q}_i}\delta q_i|_{t=t_1} + \int_{t1}^{t2}\Sigma_{i=1}^{n}(\frac{d\mathfrak{L}}{dq_i} - \frac{d}{dt}\frac{d\mathfrak{L}}{d\hat{q}_i})\delta q_i dt$$



$$S = \int_{t_1}^{t_2} \mathfrak{L}\mathrm{dt}$$

$$\frac{d}{dt} * \frac{d\mathfrak{L}}{d\hat{q}_i} - \frac{d\mathfrak{L}}{dq_i} = 0$$

Преобразование Лежандра

$$H = \Sigma_{i=1}^{n}(\frac{d\mathfrak{L}}{d\hat{q}_i}\hat{q}_i) - \mathfrak{L}$$

$$\mathfrak{L} = \Sigma\frac{dH}{d\hat{q}_i}\hat{q}_i - H$$

$$\Sigma\frac{dK}{d\hat{q}_i}q_i = 2K$$

Закон сохранения полной энергии

$$\frac{dH}{dt} = \frac{dK}{dt} + \frac{d\Pi}{dt} = 2\frac{dK}{dt} - \frac{d\mathfrak{L}}{dt} =$$

$$\Sigma_{i=1}^{n}\frac{d}{dt}\frac{d\mathfrak{L}}{d\hat{q}_i}\hat{q}_i + \Sigma_{i=1}^{n}\frac{d\mathfrak{L}}{d\hat{q}_i}\hat{\hat{q}} - \Sigma_{i=1}^{n}\frac{d\mathfrak{L}}{dq_i}\hat{q}_i - \Sigma_{i=1}^{n}\frac{d\mathfrak{L}}{d\hat{q}_i}\hat{\hat{q}}_i - \frac{d\mathfrak{L}}{dt} =$$

$$= \Sigma_{i=1}^{n}(\frac{d}{dt}\frac{d\mathfrak{L}}{d\hat{q}_i} - \frac{d\mathfrak{L}}{dq_i})\hat{q}_i - \frac{d\mathfrak{L}}{dt} = - \frac{d\mathfrak{L}}{dt}$$

$$\frac{dH}{dt} = - \frac{d\mathfrak{L}}{dt}$$

$$\lambda(q_i, \hat{q}_i) = \mathfrak{L}(q_i, \hat{q}_i)$$

$\bar{t} = t + \alpha$

—-----------------------

$a_1 * a_2 \rightarrow a_3$

1) Существование нейтрального элемента

$l_{\Lambda} * a = a$

$a * l_{\Pi} = a$

$l_{\Lambda} = l_{\Lambda} l_{\Pi} = l_{\Pi}$

2) Ассоциативность

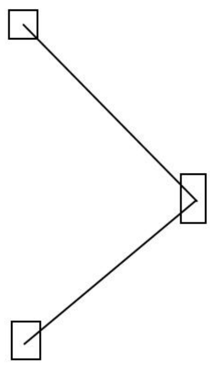
$(a_1 a_2) a_3 = a_1 (a_2 a_3)$

3) Существование обратного элемента

$a_1 a_1^{-1} = e$

$a^{-1} a = e$

Группа
преобразований



Непрерывная группа
S — числовой параметр

$\bar{q} = \phi(q_0, t_0, S)$

$t_s = \xi(q_0, t_0, S)$

$q_{s1} = \phi(q_0, t, S_1)$

$q_{s,s1} = \phi(q_s, t_{s1}, S_2)$

$q_{s3} = \phi(q_s, t_s, S_2)$

$\phi(q_0, t_0, 0) = q_0$

$\xi(q_0, t_0, 0) = t_0$

$\overline{q_s} = \phi(q_0, t_0, S)$

S = 0

$t_s = \xi(q_0, t_0, S)$

$q_s = \phi(q_0, t_0, 0) + \frac{\partial\phi}{\partial S}(q_0, t_0) \bullet S$

$q_s = q_0 + \psi(q_0, t_0)S$

$t_s = t_0 + \xi(q_0, t_0, S) \bullet$ S

Теорема Нетер

$$I = \xi L - \sum_{i=1}^{n}(\phi_i - \xi\widehat{q_i})\frac{\partial L}{\partial\widehat{q_i}}$$

$$\int_{t1}^{t1,\delta t} Z\alpha t$$

$$L(\overline{q_0}, \widehat{q_0} t_0)\delta t_0 = L((\overline{q_s}, \widehat{q_s} t_s)\delta t_s$$

Проверим, что является законом сохранения

$$\delta t_s = t_s(q_0, \widehat{q_0}t_0 + \delta t_0) - t_s(q_0, \widehat{q_0 t_0}) =$$

$$= t_0 + \delta t + \xi(q_0 t_0 + \delta t)S - t_0 - \xi(q_0\delta_0)S =$$

$$(1 + \frac{\partial\xi}{\partial t}S)\sigma T = \frac{\partial\xi}{\partial t}\delta t S$$

$$L(\overline{q_0}, \widehat{q_0}t_0)\delta t = L(\overline{q_s}, \widehat{q_s}t_s)\delta t_s = L(\overline{q_0} + \psi(q_0 t_0)S, \frac{dq_0 + \psi(q_0 t_0)S}{dt_0}, t_0 +$$

$$+ \xi(q_0 t_0)S)(1 + \frac{\partial\xi}{\partial t}S)\ (=)$$

$$q_s = q_0 + \psi(q_0 t_0)\mathrm{S}$$
$$t_s = t_0 + \xi(q_0 t_0)S$$

$$\frac{dt(t_s)}{dt_s} = \frac{\partial f(t_0 \text{ от } \xi(q_0 t_0)S}{\partial Z}$$
$$t_0 = t_s - \xi(q_0 t_0)S$$
$$t_0 = t_s - \xi(q_0 t_s)S$$

$$(=)\ L(q_0 + \psi S \frac{d}{dt_0} (\overline{q_0} + \overline{\psi} S)(1 - \frac{\partial \xi}{\partial t} S)$$

$$L(\overline{q_0})\delta t_0 = L(q_0, \widehat{q_0}, t_0) + \left\{ \sum_{i=1}^{n} \frac{\partial L}{\partial q_i}\psi_i + \sum_{i=1}^{n} \frac{\partial Z}{\partial q_i} (\frac{d\psi_i}{dt} - Z_i \frac{d\xi}{dt}) + \frac{\partial L}{\partial t}\xi \right\} S = i$$
$$\left(\widehat{q_0} + \frac{d\psi}{dt} S\right)\left(1 - \frac{d\xi}{dt} S\right)$$
$$\widehat{q_0} + \frac{d\psi}{dt}$$
$$\sum_{i=1}^{n} \frac{\partial L}{\partial q_i}\psi_i + \frac{\partial L}{\partial \widehat{q_i}} (\frac{d\psi_i}{dt} - \widehat{q_i} \frac{d\xi}{dt}) + \xi \frac{\partial Z}{\partial t} + L \frac{\partial \psi}{\partial t} = 0$$
$$\frac{\partial L}{\partial t} = \frac{\partial \xi}{\partial t} L + \xi(\frac{\partial Z}{\partial t} + \sum_{i=1}^{n} \frac{\partial Z}{\partial \partial} \widehat{q_i} + \sum_{i=1}^{n} \frac{\partial Z}{\partial \widehat{q_i}} \widehat{\widehat{q_i}}) +$$
$$+ \sum_{i=1}^{n} (\psi_i - \xi \widehat{q_i}) \frac{\partial Z}{\partial \widehat{q_i}} + \frac{\partial L}{\partial q_i} (\frac{\partial \psi_i}{\partial t} - \frac{\partial \xi}{\partial t} y_i)$$ — Сокращается. Ч.Т.Д.
$$q_s = q_0 + \psi(q_0(t_0), t_0)\mathrm{S}$$
$$q_s = t_0 + \xi(q_0(t_0), t_2)S$$

Для систем электродинамических уравнений — $\psi(x, y, z, t)$

$$\overline{A} = (A_x(x, y, z, t),\ A_y(x, y, z, t), A_z(x, y, z, t))$$
$$L = L_{\text{частицы(ее кинетическая энергия)}} + L_{\text{частицы+поля}} + L_{\text{поля(его энергетическая характеристика)}}$$

Как измерить элементарное поле?

$$L = \frac{mv^2}{2} + \frac{e}{c}(A, U) - e\varphi$$

$$v\frac{\partial L}{\partial V} - L = v(mv + \frac{e}{c}A) - \frac{mv^2}{2} - \frac{e}{c}(A, U) + l\varphi = \frac{mv^2}{2} + e\varphi$$

$$S = \int_{t_1}^{t_2}(\frac{mv^2}{2} + \frac{e}{c}(A, \varphi) - e\varphi)dt$$

Минковского пространство

$$- x_1y_3 + x_2y_2 + x_3y_3 + x_4y_4$$

$$- x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2} + \ ...$$

$$\delta S = \int_{t_1}^{t_2} mv + \delta v + \frac{e}{c} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z)dt\ (=)$$

$$\delta(Av) = \delta(A_xV_x + A_yv_y + A_zv_z) = A_xV_x\delta + A_yv_y\delta + A_zv_z\delta +$$

$$+ v_x(\frac{\partial A_x}{\partial x}\delta x + \frac{\partial Ay}{\partial y}\delta y + \frac{\partial A_z}{\partial z}\delta z) + v_y(\frac{\partial A_y}{\partial x}\delta x + \frac{\partial Ay}{\partial y}\delta y + \frac{\partial A_z}{\partial z}\delta z) +$$

$$+ v_z(\frac{\partial A_z}{\partial x}\delta x + \frac{\partial Az}{\partial y}\delta y + \frac{\partial A_z}{\partial z}\delta z)$$

$$(=) \int_{t_1}^{t_2}\{- \left(\frac{dmv_x}{dt}\delta x + \frac{dmv_y}{dt}\delta y\frac{dmv_z}{dt}\delta z + \frac{e}{c}\left(\frac{dA_x}{dt}\delta x + \frac{dAy}{dt}\delta y + \frac{dA_z}{dt}\right)\right)$$

$$- \frac{dmv}{dt} - \frac{e}{c}\left(\frac{\partial A_x}{\partial t} + \frac{\partial Ax}{\partial x}v_x + \frac{\partial Ax}{\partial y}v_y + \frac{A_z}{\partial z}v_z\right) +$$

$$+ \frac{e}{c}\left(\frac{\partial A_y}{\partial x}v_x + \frac{\partial Ay}{\partial x}dy + \frac{\partial Az}{\partial x}v_y\right) = 0 - e\frac{\partial\varphi}{\partial x}$$

$$\frac{dmv}{dt} + \frac{e}{c}\left(\frac{dA_x}{\partial z} + v_y\left\{\frac{\partial A_x}{\partial y} - \frac{\partial Ay}{\partial x}\right\} + v_x\left\{\frac{\partial A_x}{\partial z} - \frac{\partial At}{\partial x}\right\}\right) + \frac{e\partial\varphi}{\partial x} = 0$$

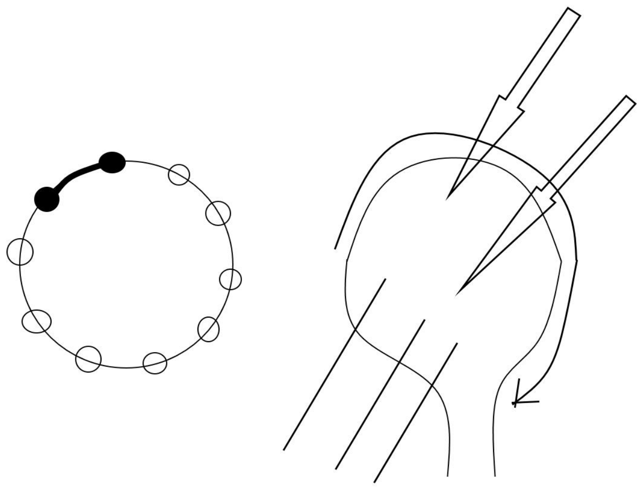
$$cot \bullet v \bullet dS = \oint vde$$

i	j	k
$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
A_x	A_y	A_z

$$R_x = \frac{\partial A_z}{\partial y} - \frac{\partial Ay}{\partial z}$$
$$R_y = \frac{\partial A_x}{\partial z} - \frac{\partial Az}{\partial y}$$
$$R_z = \frac{\partial A_y}{\partial x} - \frac{\partial Ax}{\partial y}$$
$$v_y R_z - v_z R_y$$

$$\frac{dmv_y}{dt} + \frac{e}{c}\left(\frac{dA_x}{\partial z} + v_y\left\{- \frac{\partial A_x}{\partial y} + \frac{\partial Ay}{\partial x}\right\} + v_x\left\{\frac{\partial A_x}{\partial z} - \frac{\partial At}{\partial x}\right\}\right) + \frac{e\partial\varphi}{\partial x} = 0$$

$$\frac{dmV_y}{dt} = - \frac{e}{c}[V, rot\ A]_y - e\frac{\partial\varphi}{\partial y} - l\frac{\partial A_y}{\partial t}$$
$$\frac{dmV_x}{dt} = + \frac{e}{c}[V, rot\ A]_x - e\frac{\partial\varphi}{\partial x} - \frac{l}{c}\frac{\partial A_x}{\partial t}$$
$$\frac{dmV_z}{dt} = + \frac{l}{c}[V, rot\ A] - e\frac{\partial\varphi}{\partial y} - \frac{l}{c}\frac{\partial A_z}{\partial t}$$
$$\frac{d}{dt}mV = \frac{l}{c}[V,\ rot\ A] - e(\frac{1}{c}\frac{\partial A}{\partial t} + grad\ \varphi)$$
$$H = rot\ A$$
$$E = - \frac{1}{c}\frac{\partial A}{\partial t} - grad\ \varphi$$
$$\frac{dmV}{dt} = \frac{l}{c}[V,\ H] + lE$$
$$d{:}VH = 0 \Leftarrow rot\ graf\ A = 0$$
$$rot\ E = -\frac{1}{c} * \frac{\partial}{\partial t} rot\ A$$
$$\frac{1}{c}\frac{\partial H}{\partial t} = - rot\ E$$



$\int\limits_S (H, n) dS$

$H \bullet S$ - меняется угол поворота

Калибровочная инвариантность

$A^{/} = A + grad\,\psi$

$\varphi^{/} = \varphi - \frac{1}{c}\,\frac{\partial\psi}{\partial t}$

$-\frac{1}{c}\frac{\partial A^{/}}{\partial t} - grad\varphi^{/} = -\frac{1}{c}\frac{\partial A}{\partial t} - \frac{1}{c}\frac{\partial}{\partial t}grad\psi - grad\varphi + \frac{1}{c}\,\frac{\partial}{\partial t}\,grad\,\psi$

$\psi = C\int\limits_0^t \varphi dt$

$\frac{d}{dt}\,\frac{mV^2}{2} = l(E, V)$

$E^2 = (E, E)$

$H^2 = (H, H)$

$\mathcal{L}_{\text{поля}} = \frac{1}{8\pi}[E^2 - H^2]$

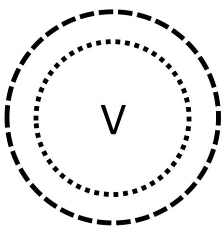
ρ − плотность зарядов

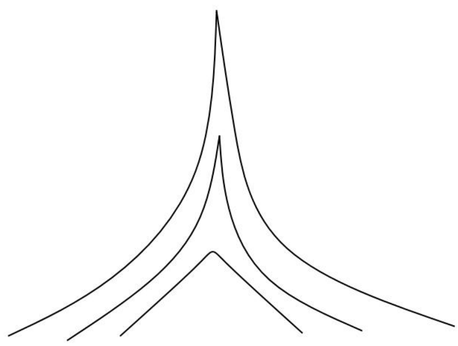
j − плотность тока

$Q = \int\limits_V \rho\, dV$

$Q = E\, e_i$

$\rho = \Sigma\delta(\bar{x} - \bar{x}_1) l_i$





$$Q = \int\limits_V \rho \, dV$$

$$\bar{j} = \Sigma\delta(\bar{x} - \bar{x}_j) l_i \bar{V}_i$$

$$S = \int\limits_{t1}^{t2} \mathfrak{L}\, dt = \int\limits_{t1}^{t2} \Sigma \frac{mV_i^2}{2} + \frac{1}{c}\Sigma\, l_i (A\, V_i) - \Sigma\, l_i \varphi + \int\limits_V \frac{1}{\partial \bar{u}} (E^2 - H^2) dV) dt =$$

$$\ominus \int_{t_1}^{t_2} \sum \frac{mV_i^2}{2} dt \int_{t_1}^{t_2} \int_V \frac{1}{c}(A, j) - \rho\varphi + \frac{1}{\partial \bar{u}} \left(\left(\frac{1}{c}\frac{\partial A}{\partial t} + grad\varphi, \frac{1}{c}\frac{\partial A}{\partial t} + grad\phi \right) + (rotA, rotA) \right)$$

$$\dots$$

$$dS = \int_{t_1}^{t_2} \int_V \left(\frac{1}{c}(dA, j) - \frac{2}{c}\frac{\partial}{\partial x}\frac{\partial A_x}{\partial t} + \frac{\partial}{\partial y}\frac{\partial A_y}{\partial t} + \frac{\partial}{\partial z}\frac{\partial A_z}{\partial t} \right) d\varphi$$

$$Q = \int_V \rho dV$$

$$-\rho + \frac{1}{4\pi}\left(div \left(-\frac{1}{e}\frac{\partial A}{\partial t} - grad\varphi \right) \right) = 0$$

$$divE = 4\pi\rho$$

$$\left(dA, \left[\frac{1}{c} j + \frac{1}{4\pi} \left(\frac{-\pi A}{c^2 \partial t^2} - \frac{1}{c}\frac{\partial}{\partial t} grad\varphi \right) \right] (rotA, rotB) \right) =$$

$$\left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \right]$$

$$- \left(\frac{\partial^2 A_z}{\partial y \partial y} - \frac{\partial^2 A_y}{\partial y \partial z} \right) B_z - \left(\frac{\partial^2 A_z}{\partial y \partial z} - \frac{\partial^2 A_y}{\partial t^2} \right) B_y - \left(\frac{\partial^2 A_y}{\partial z^2} - \frac{\partial^2 A_z}{\partial y \partial z} \right) B_x$$

$$- \left(\frac{\partial^2 A_y}{\partial x^2} - \frac{\partial^2 A_x}{\partial x \partial y} \right) B_z - \left(\frac{\partial^2 A_y}{\partial x \partial y} - \frac{\partial^2 A_z}{\partial y \partial z} \right) B_y - \left(\frac{\partial^2 A_x}{\partial x \partial t} - \frac{\partial^2 A_y}{\partial y \partial z} \right) B_x$$

$$- \frac{\partial^2 A_y}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_x}{\partial z^2} + \frac{\partial^2 A_z}{\partial x \partial z}$$

$$\frac{1}{c}j + \frac{1}{4\pi e}\left(\frac{\partial}{\partial t}\underbrace{\left[-\frac{1}{c}\frac{\partial A}{\partial t} - grad\varphi\right]}_{E}\right) + rot\,\underbrace{rotA}_{H} = 0$$

$$divE = 4\pi\rho$$

$$\frac{\partial\rho}{\partial z} + div\,\rho V = 0$$

$$4\pi\frac{\partial\rho}{\partial t} = div\frac{\partial E}{\partial t} = div(e \cdot rot(H) - 4\pi j) = -4\pi divj$$

$$\frac{\partial\rho}{\partial t} + div\,j = 0$$

$$\boxed{\frac{1}{c}\frac{\partial E}{\partial t} = rot(H) - \frac{4\pi}{e}j}$$

$$\frac{1}{c}\frac{\partial H}{\partial t} = -rot(E)$$

$$\int\frac{1}{\delta\pi}(E^2 + H^2)dV = \oint\frac{c}{4\pi}\left[H \times E\right]dS + \int_V jdV$$

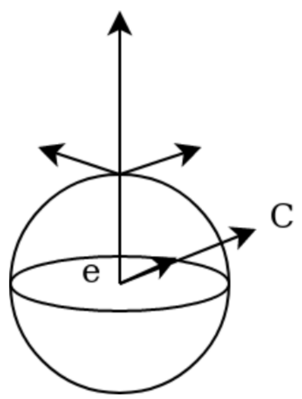
$$\frac{d}{dt}\sum\frac{mV_i^2}{2} = \sum eEV_j$$

$$\int_V\frac{1}{\delta\pi}\left(E^2 + H^2\right)dV + \frac{\sum m_iV_i^2}{2} = \oint\frac{c}{4\pi}\left[H \times E\right]dS$$

$$E = \frac{e}{c} - grad(\varphi) = -grad(\varphi)$$

$$divE = 4\pi\rho$$

$$div\,grad\varphi = 4\pi e\delta(x, y, z)$$



$$E = \vec{n}|E|$$

$$\int_{\psi_R} div\, grad\varphi dV^2 = \int_{\psi_R} 4\pi e\delta(x,y,z)dV$$

$$\oint (E,n)dS = -4\pi e \qquad \oint_{S_R} (grad\,\phi \cdot n)dS = 4\pi e$$

$$|E| \oint dS = 4\pi e \qquad |E| = \frac{4\pi e}{4\pi R^2} = \frac{e}{R^2}$$

$$A' = A + grad(f) \qquad \varphi' = \varphi - \frac{1}{c}\frac{\partial f}{\partial t}$$

$$E' = E \qquad H' = H \qquad rotA' = rotA$$

$$E = -\frac{1}{c}\frac{\partial A}{\partial t} - grad\varphi$$

$$\begin{aligned} E' &= -\frac{1}{c}\frac{\partial A'}{\partial t} - grad\varphi' \\ &= -\frac{1}{c}\frac{\partial A}{\partial t} - \frac{1}{c}\frac{\partial}{\partial t}grad f - grad\varphi' \\ &= -\frac{1}{c}\frac{\partial A}{\partial t} - grad\varphi - \frac{1}{c}\frac{\partial}{\partial t}grad f \\ &= -\frac{1}{c}\frac{\partial A}{\partial t} - grad\varphi = -\frac{1}{c}\frac{\partial A}{\partial t} + E \end{aligned}$$

$$f = c\int_0^t \varphi dt + B(x,y,z)$$

$$\varphi - \frac{1}{c}\frac{\partial f}{\partial t} = 0$$

$$t = 0$$

$$divA' = divA + c \cdot grad \int_0^t \varphi dt + div\, grad\, B$$

$$div\, grad\, B = \mathcal{F}$$

$$divA|_{t=0} = 0$$

Используем уравнение Максвелла
$H = rot A$

$$\frac{\partial E}{\partial t} = rot H \qquad div E = 0 \qquad div A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\Delta A = grad\, div\, A - rot\, rot A$$

$$\Delta A = \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \\ \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \\ \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix} grad A = \begin{pmatrix} \frac{\partial}{\partial x} div A \\ \frac{\partial}{\partial y} div A \\ \frac{\partial}{\partial z} div A \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial x \partial y} + \frac{\partial^2 A_z}{\partial x \partial z} \\ \frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial y \partial z} \\ \frac{\partial^2 A_x}{\partial x \partial z} + \frac{\partial^2 A_y}{\partial y \partial z} + \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{\partial A_y}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2} + \frac{\partial^2 A_z}{\partial x \partial z} \\ \frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial y \partial z} \\ \frac{\partial^2 A_x}{\partial x \partial z} + \frac{\partial^2 A_y}{\partial y \partial z} + \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix}$$

$$E = -\left(\frac{1}{c}\frac{\partial A}{\partial t}\right)$$

$$-\frac{1}{c}\frac{\partial A}{\partial t^2} = rot\, rot A - grad\, div A$$

$$\frac{\partial^2 A}{\partial t^2} = c^2(grad\, div - rot\, rot) = c^2 \Delta A$$

$\frac{\partial A^2}{\partial t^2} = c^2 \Delta A$ $\qquad$ $\frac{\partial^2 E}{\partial t^2} = \Delta E$

$\frac{\partial^2 A}{\partial t^2} = -\ c^2 rot\ A$ $\qquad$ $\frac{\partial^2 rot\ A}{\partial t^2} = -\ c^2 rot\, rot\ (rot\ A)$

$div\ E\ =\ 0$ $\qquad$ $div\ H\ =\ 0$

$div\ A\ =\ 0$ $\qquad$ $\varphi\ =\ 0$

$E\ =-\ \frac{1}{c}\frac{\partial A}{\partial z}$

$g(x, t)$ $\qquad$ $div\ A\ \equiv\ 0$

$\frac{\partial}{\partial y}\ \equiv\ 0$ $\qquad$ $\frac{\partial}{\partial z}\ \equiv\ 0$

$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\ =\ 0$

$\frac{\partial E_x}{\partial x} = 0$ $\qquad$ $\frac{\partial H_x}{\partial x} = 0$

$E_x = 0$ $\qquad$ $H_x = 0$

$g(x, t)$ $\qquad$ $div\, A \equiv 0$

$\frac{\partial}{\partial y} \equiv 0$ $\qquad$ $\frac{\partial}{\partial z} \equiv 0$

$\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial E_y}{\partial x^2}$ $\qquad$ $\frac{\partial^2 E_z}{\partial t^2} = c^2 \frac{\partial^2 E_x}{\partial x^2}$

$\xi = x + ct$ $\qquad$ $x = \frac{\xi+\eta}{2}$

$\eta = x - ct$ $\qquad$ $t = \frac{1}{c}\frac{\xi-\eta}{2}$

$A_\xi(x, t) = \overline{A}(\frac{\xi+\eta}{2}, \frac{1}{c}\frac{\xi-\eta}{2}) = \overline{A}(\frac{\xi(x,t)+\eta(x,t)}{2}, \frac{1}{c}\frac{\xi(x,t)-\eta(x,t)}{2})$

$$\frac{\partial A_x}{\partial x} = \frac{\partial A}{\partial \xi} * \frac{\partial \xi}{\partial t} + \frac{\partial A_\xi \partial \eta}{\partial \eta \partial t} = (\frac{\partial A_\xi}{\partial \xi} - \frac{\partial A_\xi}{\partial \eta})c = c^2(\frac{\partial A_\xi}{\partial \xi^2} + 2\frac{\partial A_\xi}{\partial \xi \partial \eta} + \frac{\partial^2 A_\eta}{\partial \eta^2})$$

$c^2(\frac{\partial^2 A_\xi}{\partial \xi^2} - \frac{\partial^2 A_\xi}{\partial \xi \partial \eta})$

$\frac{\partial A_y}{\partial x^2} = c^2 \frac{\partial^2 A_\xi}{\partial x^2}$

$= c^2(\frac{\partial^2 A_\eta}{\partial \xi \partial \eta} + \frac{\partial^2 A_y}{\partial \eta^2})$

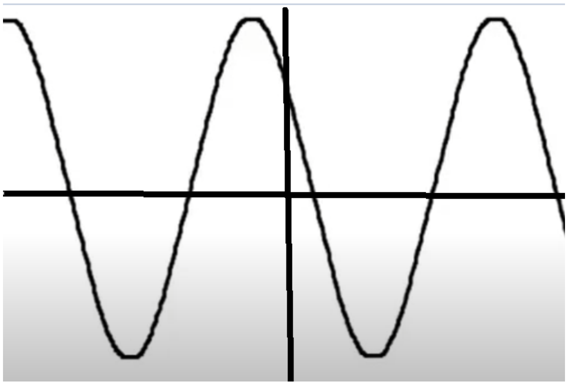
$\frac{\partial A_\xi}{\partial t} = \frac{\partial A_\xi}{\partial \xi} * \frac{\partial \xi}{\partial t} + \frac{\partial A_\xi}{\partial \eta} * \frac{\partial \eta}{\partial t} = \frac{\partial A_\xi}{\partial t}$

$A_y = f(\xi) + g(\eta) = f(x + ct) + \xi(x - ct)$

x_1 $\quad$ t_1 $\qquad$ $x_1 - ct_1 = x_2 = x - ct_2$

x_2 $\quad$ t_2 $\qquad$ $x_1 = x_2 - ct$

График сдвинулся на ct



$A_y = f_y(x - ct)$

$E_y = -\frac{1}{c}\frac{\partial A_y}{\partial t} = -\frac{1}{c}f'_y(-c) = f'_y$

$A_z = f_z(x - ct)$ $\qquad\qquad\qquad\qquad$ $E_z = f'_z$

$S = \frac{c}{8\pi}[E \times H] = \frac{c}{8\pi}(E_\xi^2 + E_z^2) = \frac{c}{8\pi}[E^2]$

$E_y = f'_y(x - ct) = A_y cos(\omega t + \alpha_y) = A_y cos(\omega t - kx + \beta_\xi)$

$E_z = f'_z(x - ct) = A_z cos(\omega t + \alpha_z) = A_z cos(\omega z - kx - \beta_z)$

$\alpha_y = kx + \beta_y$

$\alpha_z = kx + \beta_z$

$\omega t + \frac{\omega}{c}x + \alpha_y$

Если возьмем плоскость

$E_y = A_y cos(\omega t - kx + \alpha)$

$E_z = A_z sin(\omega t + kx + \alpha)$

$y' = ycos(\varphi) - zsin(\varphi)$

$z' = ysin(\varphi) + zcos(\varphi)$

$E_y = A_y cos(\omega t - kx + \alpha)$

$E_z = A_z sin(\omega t + kx + \alpha)$

$\frac{E_\xi^2}{A_\xi^2} + \frac{E_z^2}{A_z^2} = 1$

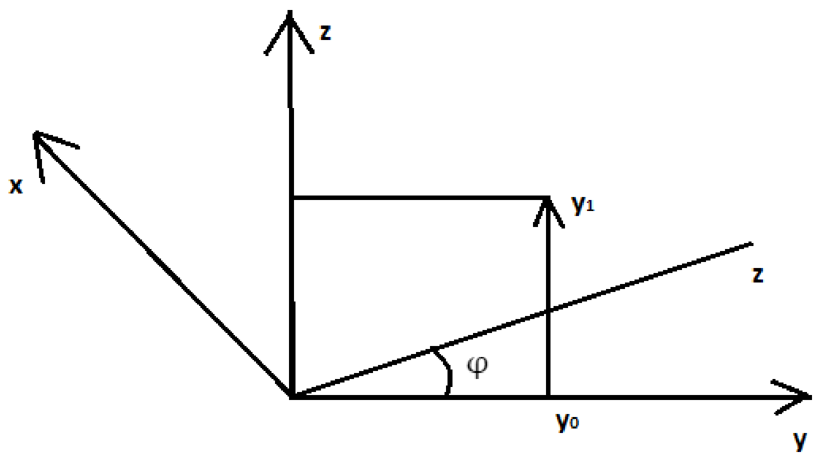


график выше против часовой стрелки

$y' = y\,cos(\varphi) - z\,sin(\varphi)$
$z' = y\,sin(\varphi) + z\,cos(\varphi)$ - по часовой

$z' = z\,cos(\varphi) - y\,cos(\varphi)$
$y' = \frac{y}{cos(\varphi)} + (z - \frac{y\,sin(\varphi)}{cos(\varphi)})\,sin(\varphi) = y(\frac{1}{cos(\varphi)} - \frac{sin^2(\varphi)}{cos(\varphi)} + z\,sin(\varphi)) =$
$= y\,cos(\varphi) + z\,sin(\varphi)$

$E_y' = E_y\,cos(\varphi) - E_z\,sin(\varphi)$
$E_z' = E_y\,sin(\varphi) + E_z\,cos(\varphi)$

$E_y' = A_y'cos(\omega t - kx + \alpha) = A_y'cos(\Omega\,cos\alpha - sin\Omega\,sin\alpha)$
$E_z' = A_z'sin(\omega t - kx + \alpha) = A_z(sin\Omega\,cos\alpha + cos\Omega\,sin\alpha)$

$E_y' = E_y\,cos(\varphi) - E_z\,sin(\varphi) = A_y\,(cos\Omega\,cos\beta_y - sin\Omega\,sin\beta_y)\,cos\varphi -$
$- A_z(cos\Omega\,cos\beta_z - sin\Omega\,sin\beta_z)\,sin\varphi$
$E_z' = E_y\,sin\varphi + E_z\,cos\varphi$

$A_y'\,cos\alpha = A_y cos\beta_y\,cos\varphi - A_z cos\beta_z sin\varphi$
$A_y'\,sin\alpha = A_y sin\beta_y\,cos\varphi - A_z sin\beta_z sin\varphi$

$E_y' = E_y\,cos(\varphi) - E_z\,sin(\varphi) = A_y\,(cos\Omega\,cos\beta_y - sin\Omega\,sin\beta_y)\,cos\varphi -$
$- A_z(cos\Omega\,cos\beta_z - sin\Omega\,sin\beta_z)\,sin\varphi$
$E_z' = E_y\,sin(\varphi) + E_z\,cos(\varphi) = A_y\,(cos\Omega\,cos\beta_y - sin\Omega\,sin\beta_y)\,sin\varphi -$
$+ A_z(cos\Omega\,cos\beta_z - sin\Omega\,sin\beta_z)\,cos\varphi$

$A_y'\,cos\alpha = A_y cos\beta_y cos\varphi - A_z cos\beta_z sin\varphi$
$A_y'\,sin\alpha = A_y sin\beta_y cos\varphi - A_z sin\beta_z sin\varphi$

$A_z'\,cos\alpha =- [A_y sin\beta_y sin\varphi + A_z sin\beta_z cos\varphi]$
$A_z'\,sin\alpha = [A_y cos\beta_y sin\varphi + A_z cos\beta_z cos\varphi]$

$$\frac{A_y \cos\beta_y \cos\varphi - A_z \cos\beta_z \sin\varphi}{A_y \sin\beta_y \cos\varphi - A_z \sin\beta_z \sin\varphi} = -\frac{A_y \sin\beta_y \sin\varphi + A_z \sin\beta_z \cos\varphi}{A_y \cos\beta_y \sin\varphi + A_z \cos\beta_z \cos\varphi}$$

$$A_y^2 \cos\beta_y^2 \cos\varphi \sin\varphi - A_y A_z \cos\beta_z \cos\beta_y \sin^2\varphi - A_z^2 \cos^2\beta_y \cos\varphi \sin\varphi$$

$$[A_y^2 \cos\beta_y^2 - A_z^2 \cos^2\beta_y] \frac{\sin(2\varphi)}{2} + A_y A_z \cos\beta_z \cos\beta_y \cos(2\varphi)$$

$$-A_y^2 \sin^2\beta_y \sin\varphi \cos\varphi - A_y A_z \sin\beta_y \sin\beta_z \cos^2\varphi + A_y A_z \sin\beta_y \sin^2\varphi +$$

$$+ A_z^2 \sin^2\beta_z \sin\beta_y$$

$$tg(2\varphi) = \frac{2A_y A_z(\cos\beta_y \cos\beta_z + \sin\beta_y \sin\beta_z)}{(A_z - A_y)(A_z + A_y)}$$

$$tg(\varphi) = 1$$

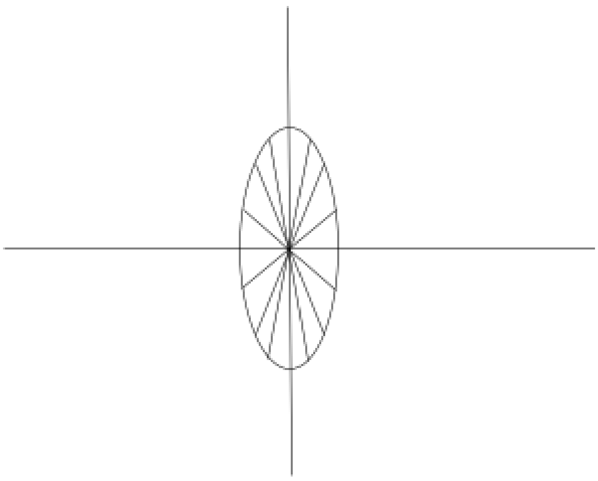
$$\frac{2\sin\varphi\cos\varphi}{\cos^2\varphi - 1} = Q$$

$$2\sin^2\varphi\cos^2\varphi = Q(\cos^2\varphi - 1)^2$$

$$2(1 - \cos^2\varphi)\cos^2\varphi = Q(\cos^2\varphi - 1)^2$$

$$E_y = A'_y \cos(\Omega + \alpha)$$

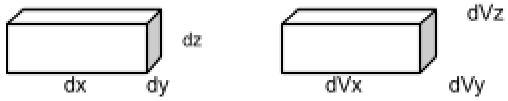
$$E_z = A'_z \sin(\Omega + \alpha)$$



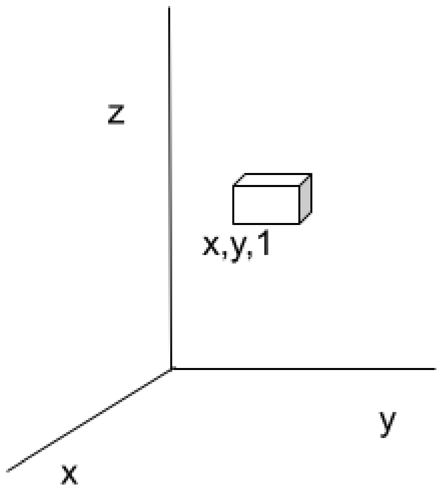
$$\Omega = \quad - \alpha$$

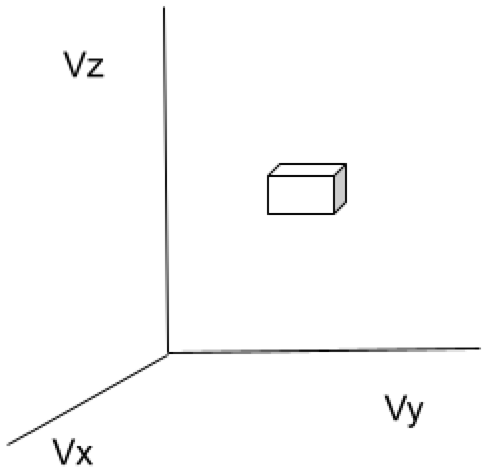
$$A'_z = 0$$

$$f(x , y, z, t, v_x, v_y, v_z)$$



n dx dy dz





$x < x_m < x + dx$ $\qquad$ $v_x < v_{x_m} < v_x + dv_x$

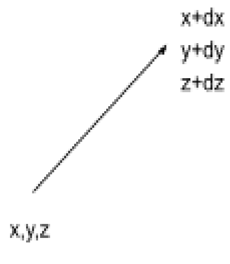
$y < y_m < y + dy$ $\qquad$ $v_y < v_{y_m} < v_y + dv_y$

$z < z_m < z + dz$ $\qquad$ $v_z < v_{z_m} < v_z + dv_z$

$$\rho = m \int_{-\infty}^{+\infty} f(x,\ y,\ z, v_x, v_y, v_z)\cdot dv_x v_y v_z$$

$$E = \frac{mv^2}{2} \int_{-\infty}^{+\infty} f(x,\ y,\ z, v_x, v_y, v_z)\cdot dv_x v_y v_z$$

$$P = \int_{-\infty}^{+\infty} mvf(dv_x dv_y dv_z)$$



$dx = v_x dt$

$dy = v_y dt$

$dz = v_z dt$

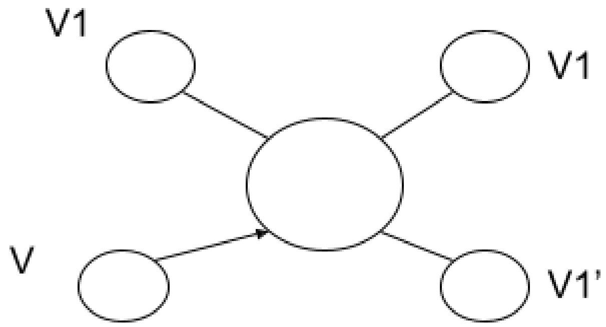
$v_x + dv_x$ $\qquad$ $dv_x = a_x dx = \frac{F_x}{m} dt$

$v_y + dv_y$ $\qquad$ $dv_y = \frac{F_y}{m} dt$

$v_z + dv_z$ $\qquad$ $dv_z = \frac{F_z}{m} dt$

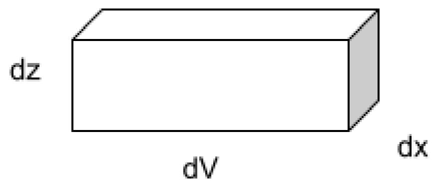
$$f(x\ ,\ y,\ z,\ t, v_x,\ v_y,\ v_z) = f(x + v_x dt\ , y + v_y dt, z + v_z dt, t + dt, v_x + \frac{F_x}{m} dt\ ,$$
$$v_y + \frac{F_y}{m} dt, v_z + \frac{F_z}{m} dt) + \frac{\partial f}{\partial x} v_x dt + \frac{\partial f}{\partial y} v_y dt + \frac{\partial f}{\partial z} v_z dt +$$
$$\frac{\partial f}{\partial t} t dt + \frac{\partial f}{\partial v_x} \frac{F_x}{m} dt + \frac{\partial f}{\partial v_y} \frac{F_y}{m} dt + \frac{\partial f}{\partial v_z} \frac{F_z}{m} dt + O(t)$$

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} + \frac{F_x}{m} \frac{\partial f}{\partial v_x} + \frac{F_y}{m} \frac{\partial f}{\partial v_y} + \frac{F_z}{m} \frac{\partial f}{\partial v_z} = I$$



$$-\int_{-\infty}^{+\infty} \int \int \omega(v_1, v'\ , v'_1, v\ ) \qquad f(v)\ \ f(v'\ )\, dv'\, dv_1\, dv'_2$$

$$\omega(v_1, v'\ , v'_1, v\ ) \ \ f(v_1)\ f(v'_1)$$



$$-\int \int_{-\infty}^{+\infty} \int \omega(v_1, v' , v'_1, v) \quad f(v_1) \; f(v'_1)\, dv'\, dv_1\, dv'_z$$

$$\omega(v_1, v' , v'_1, v) = \omega(v_2, v' , v_1, v)$$

$$\omega(v_2, v' , v'_2, v) = \omega(v_2, v' , v_2, v)$$

Больцмана:

$$\frac{\partial f}{\partial t} + \overline{v}\frac{\partial f}{\partial \overline{x}} + \frac{\overline{F}}{m}\frac{\partial f}{\partial \overline{v}} = \int \int_{-\infty}^{+\infty} \int \omega(v_1, v' , v'_1, v)(f(v_1) \bullet f(v'_2)) -$$

$$- f(v) \; f(v')\, dv' dv_1 dv'_1$$

$$dv' = dv'_x dv'_y dv'_z$$

$$I_{\varphi}(v) - \int_{-\infty}^{+\infty} I(v)\varphi(v)dv = \int \int_{-\infty}^{+\infty} \int \omega(v_1, v' , v'_1, v)\bullet$$

$$\bullet(f(v_1) \bullet f(v'_1) - f(v) \; f(v'))\varphi(v)dv\, dv'\, dv_1 dv'_1$$

$$\int_a^b f(x, y)$$

$$J_{\varphi} = \int_{-\infty}^{+\infty} I(v)\varphi(v)dv =$$

$$\int \int_{-\infty}^{+\infty} \int \omega(v_1, v' , v'_1, v)\bullet(f(v_1) \bullet f(v'_1) - f(v) \; f(v'))\varphi(v)dv\, dv'\, dv_1 dv'_1 =$$

$$= \int \int \int_{-\infty}^{+\infty} \int \omega(v_1, v' , v'_1, v)(f(v_1) \bullet f(v'_1) - f(v) \; f(v'))$$

$$\varphi(v')dv'\, dv'\, dv_1 dv'_1 = \int \int \int_{-\infty}^{+\infty} \int \omega(v_1, v' , v'_1, v)(f(v_1) \bullet f(v'_1) -$$

$$- f(v) \; f(v'))\varphi(v'_1)v_1 dv'\, dv_1 dv'_1.$$

$$J = \int \int \int_{-\infty}^{0} \int \omega(v_1, v' , v'_1, v)(f(v_1) \bullet f(v'_1) - f(v) \; f(v'))\bullet$$

$$\bullet\varphi(v')dv\, dv'\, dv_1 dv'_1$$

$$J_{\varphi} =- \int\int\int_{-\infty}^{+\infty}\int \omega(v_1, v' , v'_1, v)\bullet(f(v_1) \bullet f(v'_1) - f(v) \ f(v'))\bullet$$

$$\bullet\varphi(v_1)dv\, dv'\, dv_1 dv'_1$$

$$J_{\varphi} =- \int\int\int_{-\infty}^{+\infty}\int \omega(v_1, v' , v'_1, v)\bullet(f(v_1) \bullet f(v'_1) - f(v) \ f(v'))\bullet$$

$$\bullet\varphi(v'_1)dv\, dv'\, dv_1 dv'_1$$

$$J_{\varphi} = \frac{1}{4}\int\int\int_{-\infty}^{+\infty}\int \omega(v_1, v' , v'_1, v)\bullet(f(v_1) \bullet f(v'_1) - f(v) \ f(v'))\bullet$$

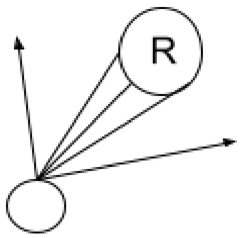
$$\bullet(f(v) \ + \varphi(v') - \varphi(v_1) - \varphi(v'_1))dv\, dv'\, dv_1 dv'_1$$

N = 10

$$H(t) \ = \int\int f\bullet \ln f \, d\overline{x} d\overline{v}$$

$$\frac{\partial H}{\partial t}\leq 0$$

Модель твердых шаров:



$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial t}{\partial v} = J \mid + \ ln f$$

$$\frac{\partial f}{\partial t}(1 \ + \ ln f) \ = \frac{\partial}{\partial t}(f ln f) \ - \frac{\partial f}{\partial t} ln f \ + \frac{\partial f}{\partial t}$$

$$\frac{\partial f \ ln f}{\partial t} \ + \ v\frac{\partial f \ ln f}{\partial x} + \frac{F}{m}\frac{\partial f \ ln f}{\partial v} = J(1 \ + \ ln f)$$

$$\frac{\partial H}{\partial t} + \int_{-\infty}^{+\infty} v$$

$$\int_{-\infty}^{+\infty}\frac{\partial f \ ln f}{\partial x} dx dv \ \ + \int_{-\infty}^{+\infty}(\frac{F}{m}\int_{-\infty}^{+\infty}\frac{\partial f \ ln f}{\partial v}) dx \ = \int J(1 \ + \ ln) dv dx \ = J_{1 + ln f}$$

$H(t) = \int\int f \bullet \ln f \, d\bar{x} d\bar{v}$ $\qquad \frac{\partial H}{\partial t} \leq 0$

$$= \int\limits_{-\infty}^{+\infty} I((1+\ln f(v)) + (1+\ln f(v')) - (1+\ln f(v_1)) - (1+\ln f(v_1')))$$
$$\cdot dv dv' dv_1 dv_1'$$

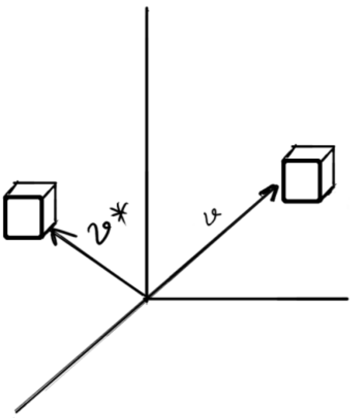
$$= \int\limits_{-\infty}^{+\infty} \left[\iiiint\limits_{-\infty}^{+\infty} w(v, v', v_1, v_1')(f(v)f(v_1') - f(v)f(v')) \cdot \ln \frac{f(v)f(v')}{f(v_1)f(v_1')} \cdot dv \cdot dv_1 \cdot dv' \cdot dv_1' \right] dx$$

$\underline{f(v_1)f(v_1') > f(v)f(v')}$

$$\frac{dH}{dt} = \int\limits_{-\infty}^{+\infty} v \int\limits_{-\infty}^{+\infty} \frac{\partial f \ln f}{\partial x} \cdot dx \cdot dv + \int\limits_{-\infty}^{+\infty} \left(\frac{F}{m} \int\limits_{-\infty}^{\infty} \frac{\partial f \ln f}{\partial v} \cdot dv \right) dx =$$
$$= \int\limits_{-\infty}^{+\infty} I(1+\ln f) dv dx$$

$$f(v_x, v_y, v_z) = f(\sqrt{v_x^2 + v_y^2 + v_z^2})$$

$ndxdydz$



$dxdydz \quad N \quad =$

$\varphi(v_x)$

$v_x \quad v_x + dv_x$

$\varphi(v_y)$

$v_y \quad v_y + dv_y$

$\varphi(v_z)$

$v_z \quad v_z + dv_z$

$$\frac{\varphi(v_x)dv_x}{N} \; \frac{\varphi(v_y)dv_y}{N} \; \frac{\varphi(v_z)dv_z}{N}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\frac{f(v)}{N} = \frac{\varphi(v_x) \cdot \varphi(v_y) \cdot \varphi(v_z)}{N^3}(dxdydz)^3$$

$$f(v) = \frac{(dv_x dv_y dv_z)^3}{N^2}\varphi(v_x)\varphi(v_y)\varphi(v_z)$$

$$\frac{df}{dv_x} = n^2\varphi'(v_x)\varphi(v_y)\varphi(v_z)$$

$$\frac{\partial f'}{\partial v} \cdot \frac{v_x}{v} = n^2\varphi'(v_x)\varphi(v_y)\varphi(v_z) \cdot \frac{1}{dv_x}\blacksquare$$

$$\frac{\partial \ln f}{\partial v} \cdot \frac{1}{v} = \frac{\varphi'(v_y)}{v_y\varphi(v_y)}$$

$$\frac{\varphi'(v_x)}{v_x\varphi(v_x)} = -\alpha$$
$$(\ln\varphi(v_x))' = -\alpha v_x$$
$$\ln\varphi = -\alpha\frac{v_x^2}{2} + \beta$$
$$\varphi = \upsilon e^{-\frac{\alpha v_x^2}{2}}$$
$$\int\limits_{-\infty}^{+\infty}\varphi(v)dv = n\cdot dxdydz = N$$
$$\upsilon\int\limits_{-\infty}^{+\infty}e^{-\frac{\alpha v^2}{2}}dv = n$$
$$\int\limits_{-\infty}^{+\infty}e^{-x^2}dx = \sqrt{\pi}\ \text{(из мат. анализа)}$$
$$\sqrt{\frac{2}{\alpha}}v' = v$$
$$\upsilon\int\limits_{-\infty}^{+\infty}e^{-v^2}\sqrt{\frac{2}{\alpha}}dv' = n$$
$$\upsilon\sqrt{\frac{2}{\alpha}}\cdot\int\limits_{-\infty}^{+\infty}e^{-v'^2}dv' = n$$
$\upsilon =$ не успел. см. видео

$$\varphi(v) = \frac{n\sqrt{\alpha}}{\sqrt{2\pi}}e^{-\frac{\alpha^2v^2}{2}}$$

$$\left[\frac{mv^2}{2} \cdot \varphi(v) \cdot dv\right] dxdydz$$

$\dfrac{3}{2}kT \rightarrow$ сколько кин. эн. в 1м градуса с учетом $\dfrac{3}{2}$

$$\left[\int_{-\infty}^{+\infty} \frac{mv^2}{2} \cdot \varphi(v) \cdot dv\right] dxdydz = \left(\frac{3kT}{2}\right) ndxdydz$$

$$f(v_x, v_y, v_z) f(\sqrt{v_x^2 + v_y^2 + v_z^2})$$

$$f(\sqrt{v_x^2 + v_y^2 + v_z^2}) = \frac{n\alpha\sqrt{\alpha}}{2\pi\sqrt{2\pi}} e^{-\frac{\alpha(v_x^2+v_y^2+v_z^2)}{2}}$$

$$\iiint_{-\infty}^{+\infty} \frac{n\alpha\sqrt{\alpha}}{2\pi\sqrt{2\pi}} (v_x^2 + v_y^2 + v_z^2) e^{-\frac{\alpha(v_x^2+v_y^2+v_z^2)}{2}} dxdydz = \frac{3kT}{m}$$

$$\frac{n\alpha\sqrt{\alpha}}{2\pi\sqrt{2\pi}} \int_0^{\infty} ve^{-\frac{\alpha v^2}{2}}$$

$$dxdydz = R^2 \cdot sin\theta$$

$$\frac{n\alpha\sqrt{\alpha}}{2\pi\sqrt{2\pi}}\int\limits_0^{\pi}\int\limits_0^{2\pi}\int\limits_0^{\infty} v^2 e^{-\frac{\alpha v^2}{2}} v^2 dv d\theta d\varphi = \frac{2\alpha\sqrt{\alpha}}{\sqrt{2\pi}}\int\limits_0^{\infty} v^4 e^{-\frac{\alpha v^2}{2}} dv = \frac{3kT}{m}$$

Замена: $\sqrt{\frac{2}{\alpha}}v' = v$

$$\frac{8n\sqrt{2}}{\alpha\sqrt{2}} = \frac{8n}{\sqrt{\pi}}$$
$$\frac{2n\alpha\sqrt{\alpha}}{\sqrt{2\pi}} \cdot \sqrt{\frac{2}{\alpha}} \cdot \frac{4}{\alpha^2}$$
$$\sqrt{\frac{2}{\alpha}}v' = v$$

тут много пропущенного

$$\frac{2n}{\alpha\sqrt{\pi}}\int\limits_0^{\infty} e^{-v^2} dv = \frac{n}{\alpha} = \frac{kT}{m}$$

$$\alpha = \frac{nm}{kT}$$

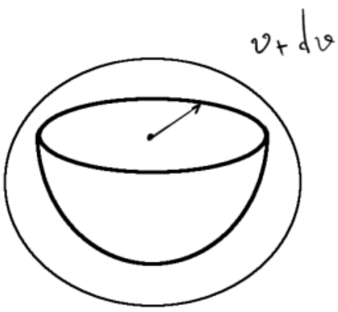
$$\frac{mv^2}{2}\iiint\left[\frac{2\alpha\sqrt{\alpha}}{\sqrt{2\pi}} e^{-\frac{\alpha v^2}{2}} dv_x dv_y dv_z\right] dx dy dz = \frac{3}{2}kT$$

- $\alpha = \frac{m}{kT}$

$\ldots$

$$f(v_x, v_y, v_z) = \frac{nm^{3/2}}{(kT)^{3/2}(2\pi)^{3/2}} e^{-\frac{mV^2}{2kT}}$$

$$f(v_x, v_y, v_z) = \frac{nm^{3/2}}{(2k\pi T)^{3/2}} e^{-\frac{m}{2kT}(v_x - v_{x_1})^2 + (v_y - v_{y_1})^2 + (v_z - v_{z_1})^2}$$



$$f(v) = \frac{\sqrt{2\pi} m^{3/2}}{(kT)^{3/2}} v^2 e^{-\frac{mV^2}{2kT}}$$