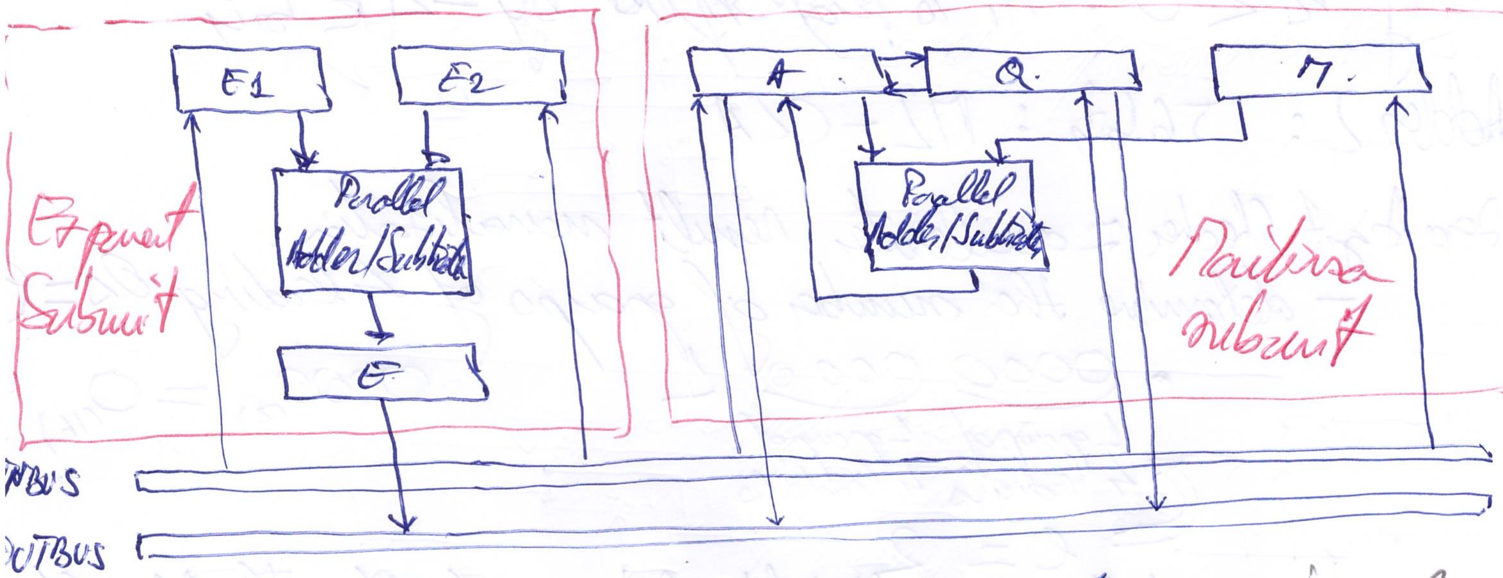
ACC9

2.1. 2 types of architectures
- loosely coupled: shared buses
- tightly coupled: direct connections

Loosely coupled:



Tightly coupled: direct connections → faster operation

IBM f.p. format, S360 Model 91

2 f.p. units
- +, − more complex
- *, /

operands on 32 or 64 bits

IBM f.p. format

Radix = 16 (=2^4)

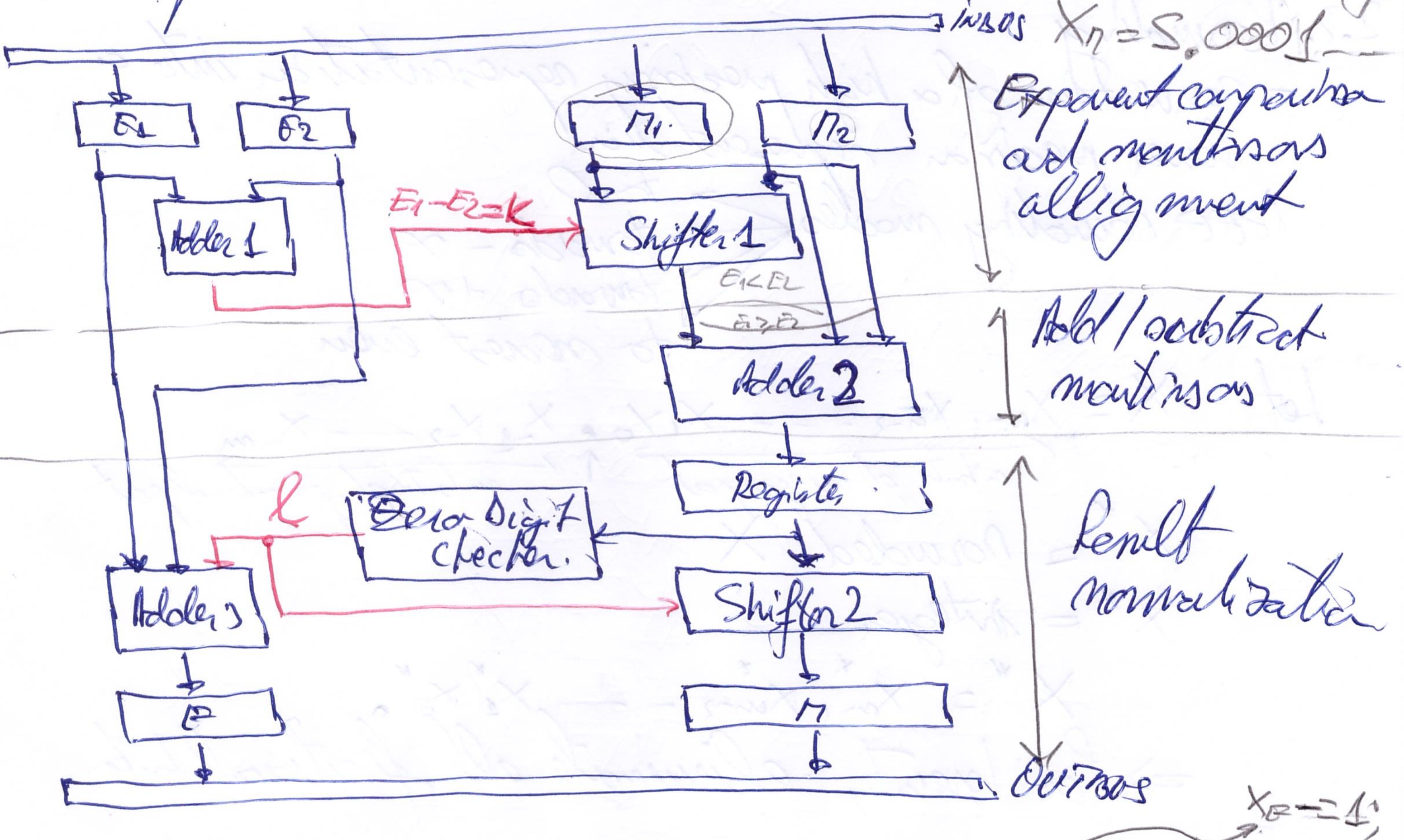
X_E - 7 bits

X_M = 24/56

$X = (-1)^{S_M} * 16^{X_E - 64} * X_M$

X_M - normalized at most 3 leading 0

$X_M = S.0001\_\_$



$X_E = 1$;

IBM: 0.0000 110...0

Radix is 16 !!!
Shifter 1 = accelerates mantissa allignment
if $k \geq 0$: M_2 is RightShifted by $4k$ bits.
if $k < 0$: M_1 is RightShifted by $-4k$ bits.

Adder 2 : 56 bits : ML-CLA.

ZeroDigit Checker = accelerates result normalization.
- determine the number of groups of 4 leading 0s $= \ell$

$$.\underbrace{0000}_{\text{1 group of 4 leading 0s}}\ \underbrace{0000}_{\text{1 group of 4 leading 0s}}1 \qquad 0000_{(2)} = 0_{(16)}$$

$\ell = 2$

→ Shifter 2 : LeftShift, for normalization, the Result by 4ℓ bits

→ Adder 3 : subtract ℓ from result's exponent.

Adder 3 : selects either E1, or E2, depending on sign of $E_1 - E_2$

2.2. Rounding

- Conversion of a high precision representation into a lower precision representation

IEEE rounding modes
→ to 0
→ towards $-\infty$
→ towards $+\infty$
→ to nearest even

Let $X = \underbrace{x_{n-1} x_{n-2} \dots x_1 x_0}_{\text{n bits of integer part}} . \underbrace{x_{-1} x_{-2} \dots x_{-m}}_{\text{m bits of fractional part}}$

$X^* =$ rounded X

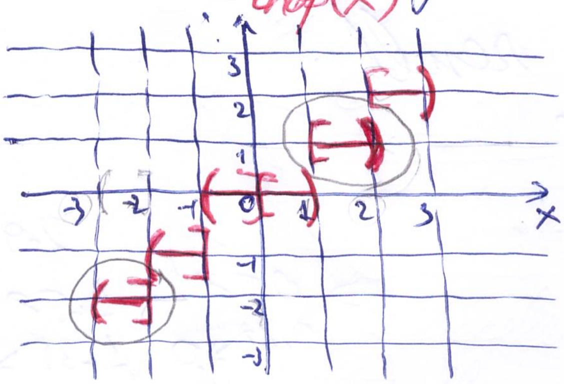
$X^* =$ integer

$X^* = x^*_{n-1} x^*_{n-2} \dots x^*_1 x^*_0 .$

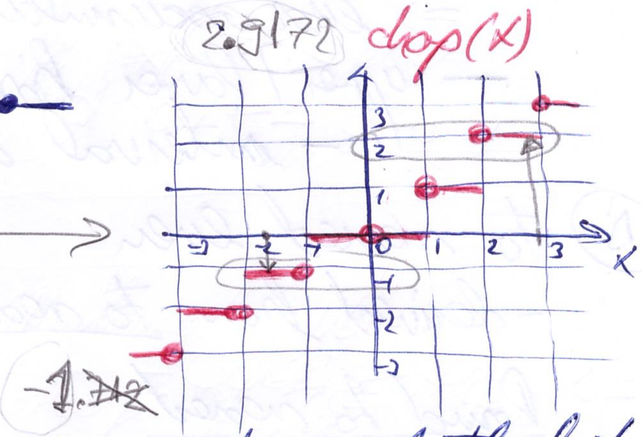
⇒ for brevity eliminate all fractional bits

(A) to 0 (inwards rounding)

X^* is the largest integer, for which $|X^*| \leq |X|$



$[-) \equiv \bullet\!-$

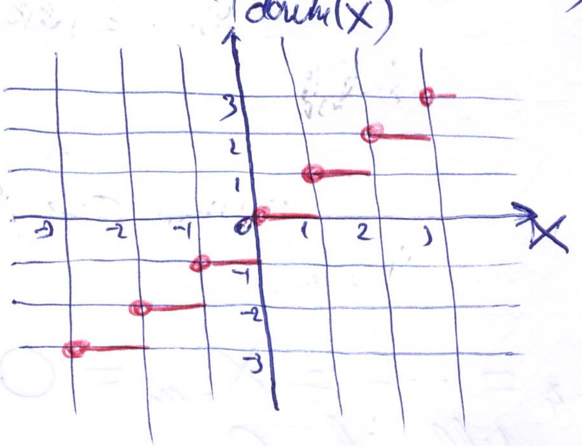


if X in SM. inwards rounding ≡ truncate the fractional bits

(B) towards $-\infty$ (downwards rounding)

X^* is the largest integer, for which $X^* \leq X$

- for positive X, towards $-\infty$ ≡ ~~towa~~ 0

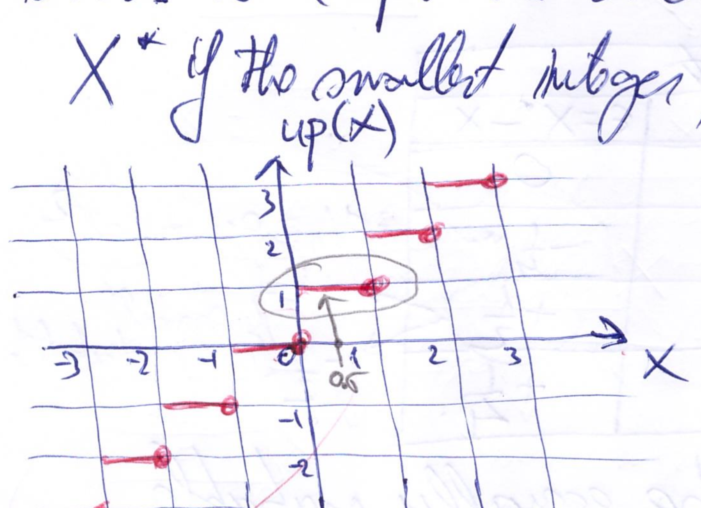


- if X in C2, downwards rounding ≡ truncate the fractional bits
- if X in SM:
  - if $X \geq 0$: truncate the fractional bits
  - if $X < 0$:

$$X^* = \begin{cases} x_{n-1}x_{n-2}\ldots x_1x_0 - 1, & \text{if } .x_{-1}x_{-2}\ldots x_{-m} \neq 0 \\ x_{n-1}x_{n-2}\ldots x_1x_0, & \text{if } .x_{-1}x_{-2}\ldots x_{-m} = 0 \end{cases}$$

(C) towards $+\infty$ (upwards rounding)

X^* is the smallest integer, for which $X^* \geq X$



- for negatives upwards rounding ≡ inwards rounding

upwards & downwards rounding:
- errors in the same direction
  $\Rightarrow$ error accumulation
- upper / lower bounds for result
  - interval arithmetic.

error $\varepsilon = X^* - X$.

① to nearest even
- derived from to nearest.

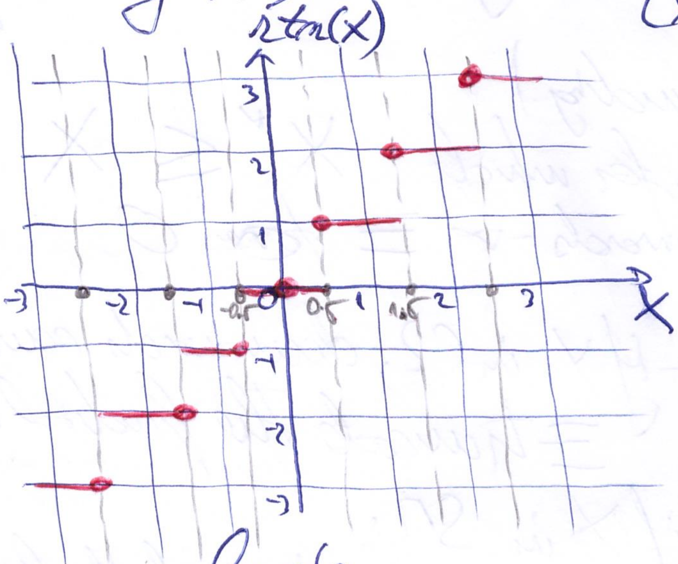
$2.9 \nearrow 3$

$\rightarrow$ $0.5 \nearrow 1$ $1.5 \nearrow 2$

$0.45 \searrow 0$ $1.35 \searrow 1$

Round to nearest

if $X \geq 0$ $X^* = \begin{cases} x_{n-1}x_{n-2}\dots x_1x_0. , & \text{if } .x_{-1}x_{-2}\dots x_{-m} < \frac{1}{2} \\ x_{n-1}x_{n-2}\dots x_1x_0. + 1 , & \text{if } .x_{-1}x_{-2}\dots x_{-m} \geq \frac{1}{2} \end{cases}$



! Similarly defined for negatives

Error analysis.
- for brevity, $x_{-3} = x_{-4} = \dots = x_{-m} = 0$
  only x_{-1} and x_{-2} can differ from 0
  = only 2 bits of fractional part.

	Inputs		Output	
	x_{-1}	x_{-2}	X^*	$\varepsilon = X^* - X$
$.00_{(2)} = .0_{(10)}$	0	0	$x_{n-1}x_{n-2}\dots x_1x_0.$	0
$.01_{(2)} = .25_{(10)}$	0	1	$x_{n-1}x_{n-2}\dots x_1x_0.$	$-\frac{1}{4}$
$.10_{(2)} = .5_{(10)}$	1	0	$x_{n-1}x_{n-2}\dots x_1x_0. + 1$	$+\frac{1}{2}$
$.11_{(2)} = .75_{(10)}$	1	1	$x_{n-1}x_{n-2}\dots x_1x_0. + 1$	$+\frac{1}{4}$

$.01_{(2)} = 0 \cdot 2^{-1} + 1 \cdot 2^{-2}$

$\left.\begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix}\right\}$ equal probability

Consider all 4 cases to be equally probable

$$\varepsilon_{mean} = \frac{0 - \frac{1}{4} + \frac{1}{2} + \frac{1}{4}}{4} = \frac{1}{8}$$

if $.10_{(2)}$ is more probable $\varepsilon_{mean} > \frac{1}{8}$

Solution: round the case $.10_{(2)}$ $(.5_{(10)})$, with equal probability (3)
upwards, and
downwards.

$\Rightarrow$ round to nearest even: inspect x_0 of X

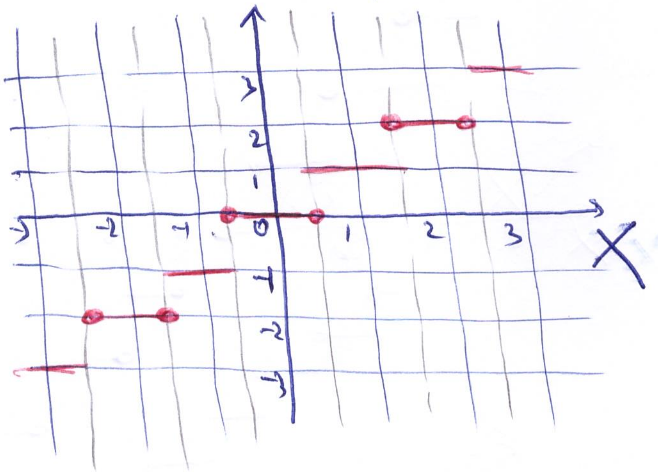
$x_0 = 1$ / X is odd $X.5 \nearrow X+1$

round to nearest even
$x_0 = 0$ X is even $X.5 \searrow X.$

if $X \geq 0$

$$X^* = \begin{cases} x_{n-1}x_{n-2}\dots x_1x_0., & \text{if } .x_{-1}x_{-2}\dots x_{-n} < \frac{1}{2}, \text{ or} \\ & \text{if } .x_{-1}x_{-2}\dots x_{-n} = \frac{1}{2} \text{ and } x_0 = 0 \\ x_{n-1}x_{n-2}\dots x_1x_0.+1, & \text{if } .x_{-1}x_{-2}\dots x_{-n} > \frac{1}{2}, \text{ or} \\ & \text{if } .x_{-1}x_{-2}\dots x_{-n} = \frac{1}{2} \text{ and } x_0 = 1 \end{cases}$$

rtne(X)



$0.5 \to 0$
$-0.5 \to 0$
$1.5 \to 2$
$-1.5 \to -2$