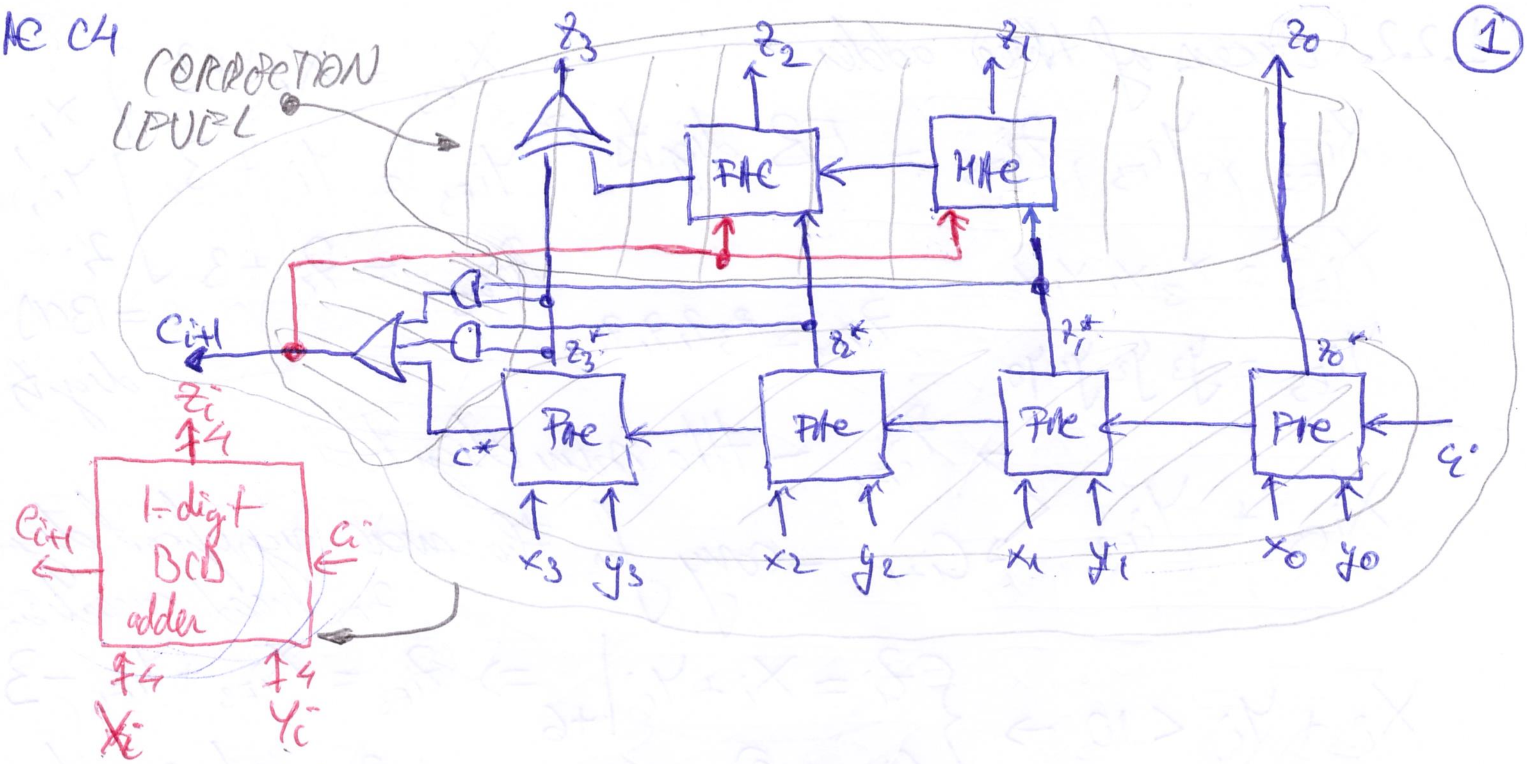
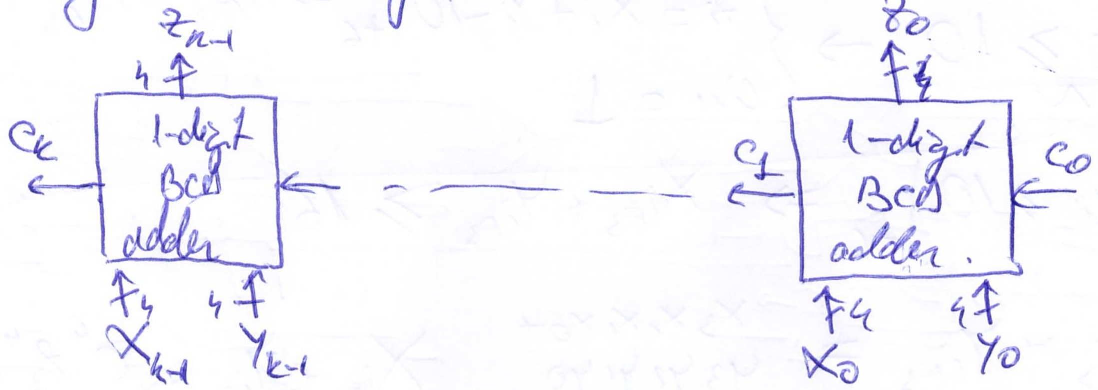


Addition of 2 k-digit BCD numbers: K cells:



Exemplu:

$$\begin{array}{r} 683\ + \\ 197 \\ \hline \underline{8}\,\underline{8}\,0 \end{array}$$

$$\begin{array}{r} 1 \\ 0110\ + \\ \underline{0001} \\ \hline 1000 \end{array} \qquad \begin{array}{r} 1 \\ 1000\ + \\ \underline{1001} \\ \hline 10010\ + \ \geq 16 \\ \underline{0110} \\ 1000 \end{array} \qquad \begin{array}{r} 0011\ + \\ \underline{0111} \\ \hline 1010\ + \ \geq 10 \\ \underline{0110} \\ 0000 \end{array}$$

$$\underbrace{1000}_{8} \qquad \underbrace{1000}_{8} \qquad \underbrace{0000}_{0}$$

1.2.2. Excess of three adder.

$X_{iE3}, Y_{iE3}, Z_{iE3}$ – E3 digits $\rightarrow$

$$\left.\begin{aligned} X_{iE3} &= X_i + 3 \\ Y_{iE3} &= Y_i + 3 \\ Z_{iE3} &= Z_i + 3 \end{aligned}\right\} \begin{aligned} &X_i, \\ &Y_i, \\ &Z_i \end{aligned}$$

X_i, Y_i, Z_i = BCD digits.

$$X_{iE3} = x_3 x_2 x_1 x_0$$

$$Y_{iE3} = y_3 y_2 y_1 y_0$$

$$Z_{iE3} = z_3 z_2 z_1 z_0$$

$$X_{iE3} + Y_{iE3} \begin{cases} Z_{iE3} & \text{– the sum digit} \\ C_{i+1} & \text{– carry for the most significant digit} \end{cases}$$

Z_{iE3} need correction.

$$X_i + Y_i < 10 \rightarrow \begin{cases} Z_i = X_i + Y_i \,\big|_{+6} \\ C_{i+1} = 0 \end{cases} \Rightarrow Z_{iE3} = X_{iE3} + Y_{iE3} - 3$$

Z_{iE3} need correction

$$\underline{X_i + Y_i \geq 10} \rightarrow \begin{cases} Z_i = X_i + Y_i - 10 \,\big|_{+6} \\ C_{i+1} = 1 \end{cases} \Rightarrow Z_{iE3} = X_{iE3} + Y_{iE3} - 13$$

$$X_i + Y_i \geq 10 \,\big|_{+6} \Rightarrow \underline{X_{iE3} + Y_{iE3} \geq 16}$$

$$X_{iE3} + Y_{iE3} = \begin{array}{r} x_3 x_2 x_1 x_0 + \\ \underline{y_3 y_2 y_1 y_0} \\ C^4 z_3^4 z_2^4 z_1^4 z_0^4 \end{array}$$

(binary adder)

$$X_{iE3} + Y_{iE3} = C^4 z_3^4 z_2^4 z_1^4 z_0^4$$

$$X_{iE3} + Y_{iE3} \geq 16 \equiv C^4 == 1$$

Subtracting 3 on 4 bits $\equiv$ adding 13 on 4 bits.

$$(a + b - 3) \bmod 16 = (a + b - 3 + 16) \bmod 16$$
$$= (a + b + 13) \bmod 16$$

Subtracting 13 on 4 bits $\equiv$ adding 3 on 4 bits

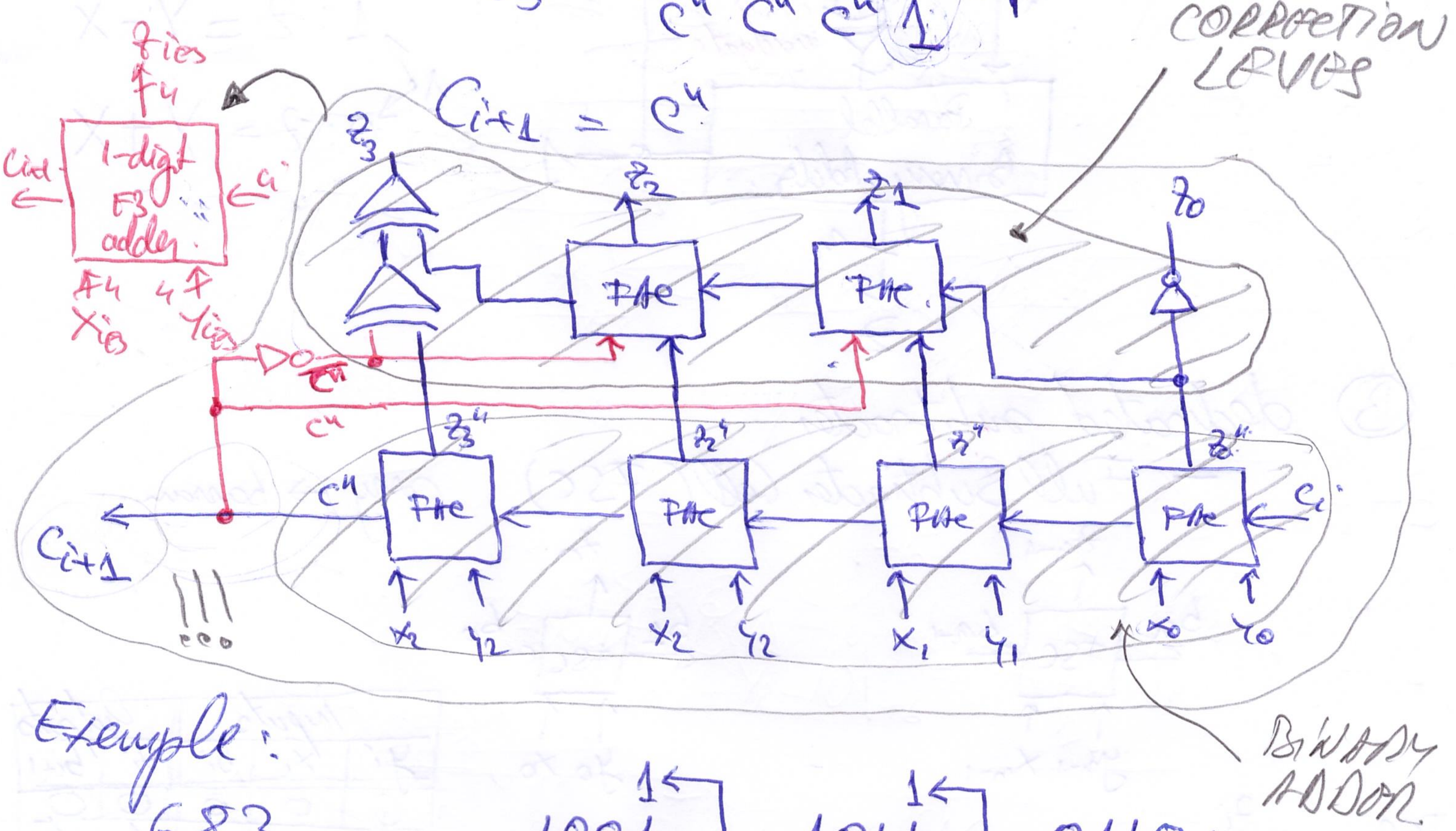
Z_{iE3}'s correction depend:

$$C^4 \begin{cases} 1 \;(X_i + Y_i \geq 10) \Rightarrow \begin{cases} Z_{iE3} = z_3^4 z_2^4 z_1^4 z_0^4 + 0011 \\ C_{i+1} = 1 \end{cases} \\[2ex] 0 \;(X_i + Y_i < 10) \Rightarrow \begin{cases} Z_{iE3} = z_3^4 z_2^4 z_1^4 z_0^4 \; 1101 \\ C_{i+1} = 0 \end{cases} \end{cases}$$

Correction stage:

$$z_{iE3} = \frac{z_3^4}{c^4}\,\frac{z_2^4}{c^4}\,\frac{z_1^4}{c^4}\,\frac{z_0^4}{1} +$$

$$C_{i+1} = c^4$$



Example:

683		1001	1011	0110
197		0100	1100	1010
880		0 1110	1 1000	1 0000
		1101	0011	0011
	*	1011	1011	0011
		8	8	0

E3 Advantages:
- C_{i+1} faster generation
  → faster addition
- can use binary adders
  → has access to intermediate carries

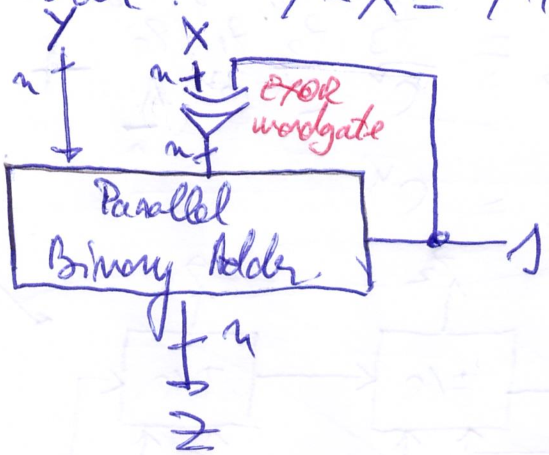
1.3. Subtracters based on Serial Propagation of Carry / Borrow

X = subtrahend $\qquad X = x_{n-1} x_{n-2} \ldots x_1 x_0$

Y = subtracter $\qquad Y = y_{n-1} y_{n-2} \ldots y_1 y_0$

$Z = Y - X$

Ⓐ use binary adder $Y - X = Y + (-X)$

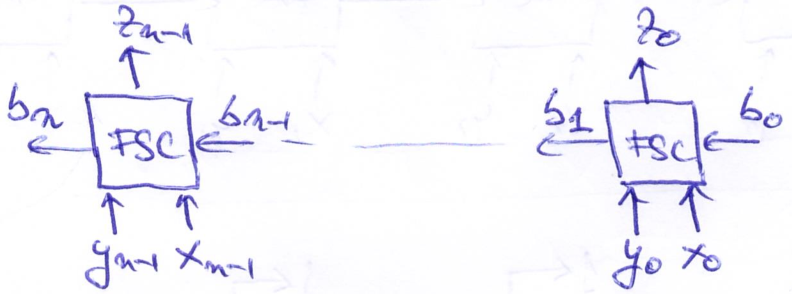


$1 : Z = Y - X$

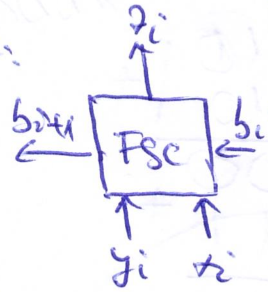
$0 : Z = Y + X$

Ⓑ dedicated subtracter.

– Full Subtracter Cell (FSC) carry → borrow



FSC:



$y_i - x_i - b_i \begin{cases} z_i \\ b_{i+1} \end{cases}$

Inputs			Outputs	
y_i	x_i	b_i	z_i	b_{i+1}
0	0	0	0	0
1 0	0	1	1	1
0	1	0	1	1
1 0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1 0	1	1	1	1

$1 - 1 - 1 \equiv 0 - 0 - 1$

z_i:

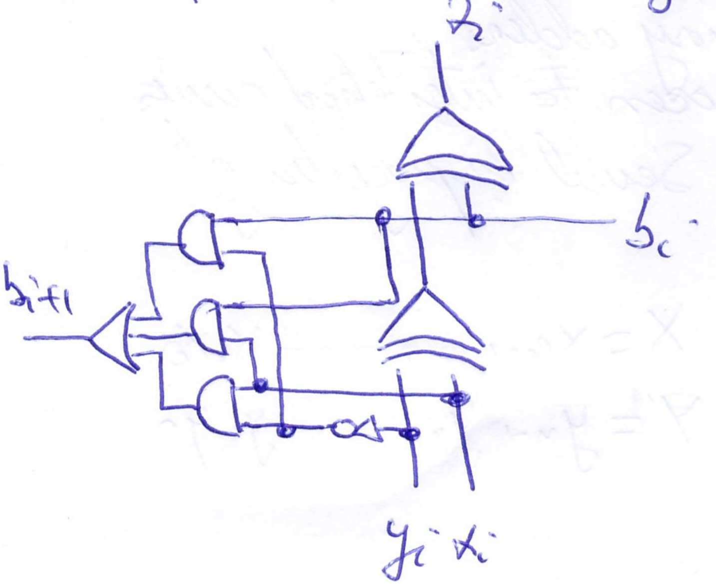
y_i \ $x_i b_i$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$z_i = x_i \oplus y_i \oplus b_i$

b_{i+1}:

y_i \ $x_i b_i$	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$b_{i+1} = \overline{y_i}\, b_i + \overline{y_i}\, x_i + b_i x_i$



© BCD subtraction

Let $X^{(k)}, Y^{(k)}$ – BCD numbers, k-digits

$$Z^{(k)} = Y^{(k)} - X^{(k)}$$

$$Y^{(k)} = Y_{k-1} Y_{k-2} \,\_\_\, Y_0$$

$$X^{(k)} = X_{k-1} X_{k-2} \,\_\_\, X_0$$

Let 9's complement of a BCD digit X_i

$$\overline{X_i^*} = 9 - X_i$$

⇒ 9's complement of a BCD number on k digits, $X^{(k)}$

$$\overline{X^{*}}^{(k)} = \overline{X_{k-1}^*}\,\overline{X_{k-2}^*} \,\_\_\, \overline{X_0^*} = \overbrace{\underbrace{9\;9\,\_\_\,9}_{10^k-1}}^{<k> \text{ digits}} - \underbrace{X_{k-1} X_{k-2} \,\_\_\, X_0}_{X^{(k)}}$$

$$= 10^k - 1 - X^{(k)}$$

$$\overline{X^{*}}^{(k)} = 10^k - 1 - X^{(k)}$$

$$Z^{(k)} = \left(Y^{(k)} - X^{(k)}\right) \bmod 10^k =$$

$$= \left(Y^{(k)} + \underbrace{10^k - 1 - X^{(k)}}_{\overline{X^{*}}^{(k)}} + 1\right) \bmod 10^k$$

$$= \left(Y^{(k)} + \overline{X^{*}}^{(k)} + 1\right) \bmod 10^k$$

9's Complementing of a BCD digit.

$$X_i = x_3x_2x_1x_0 \rightarrow \overline{X_i^*} = x_3^*x_2^*x_1^*x_0^*$$

$$\overline{X_i^*} = 9 - X_i$$

Inputs				Output			
x_3	x_2	x_1	x_0	x_3^*	x_2^*	x_1^*	x_0^*
0	0	0	0	1	0	0	1
0	0	0	1	1	0	0	0
0	0	1	0	0	1	1	1
1	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0

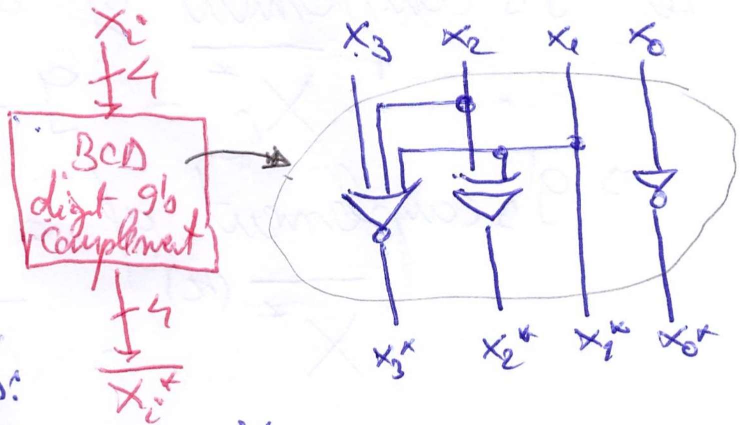
! don't care. $x_3x_2x_1x_0 \in [10;15]$

$$x_3^* = \overline{x_3 + x_2 + x_1}$$

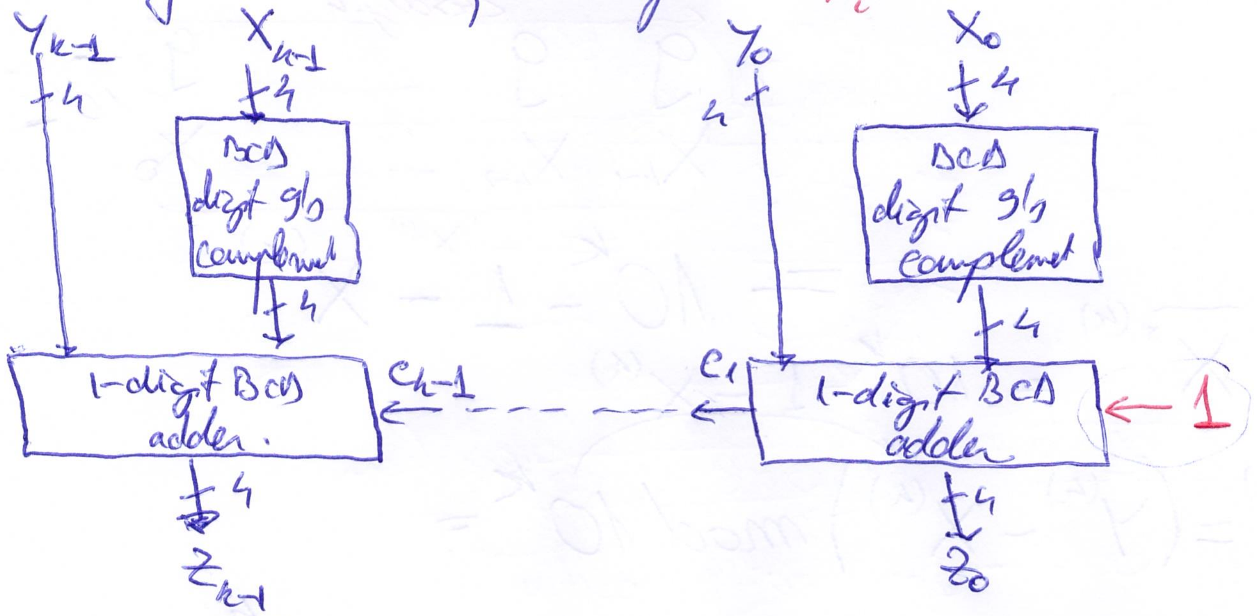
$$x_2^* = x_2 \oplus x_1$$

$$x_1^* = x_1$$

$$x_0^* = \overline{x_0}$$



Subtracting 2 BCD nos. on k digits:



1.4. Parallel Computation of Sum