AC C3

Addition's special cases → carry out from msb
→ zero result
→ negative result.
→ overflow

Overflow: result exceeds capacity

a) for unsigned: X, Y, integers, 3 bit

$X = 5$
$Y = 6$

$$\begin{array}{r} 101 \\ +\ 110 \\ \hline \cancel{1}\,011 = \cancel{3} \end{array}$$

<3> storing capacity

on 4 bit

$$\begin{array}{r} 0101 \\ +\ 0110 \\ \hline 11 = 1011 \end{array}$$

<4> storing capacity

for unsigned, overflow ≡ carry out from msb

b) for signed: X, Y integers, in C2, 4 bit

$X = +5$
$Y = +6$

sign

$$\begin{array}{r} 0101_{C2} \\ +\ 0110_{C2} \\ \hline 1011_{C2} = \cancel{-5} \end{array}$$

sign

$1\,1\,0\,1_{SM} = -5$

$$\begin{array}{r} -5 = 1011_{C2} \\ -6 = 1010_{C2} \\ \hline \cancel{1}\,0101_{C2} = \cancel{+5} \end{array}$$

on 5 bit

sign

$$\begin{array}{r} 00101_{C2} \\ +\ 00110_{C2} \\ \hline 01011_{C2} \end{array}$$

$= +11$

for signed, overflow ≡ adding same sign operand the result has oposite sign

Q:

$$\begin{array}{rl} 1111 & = -1 \\ 0111 & = +7 \\ \hline \cancel{1}\ 0110 & = +6 \end{array}$$

sign $x \neq$ sign y. sign $x = +$; sign $y = -$

$X + Y \equiv$ subtraction

$|X| - |Y| = Z \leq \max\{|X|, |Y|\}$

Overflow for signed.

X, Y – n bits, in C2, integers

$X_{C2} + Y_{C2} = Z_{C2}$.

x_{n-1} sign of X
y_{n-1} — " — Y
z_{n-1} — " — Z

$\text{overflow} \overset{def}{=} V$

inputs			outputs	
x_{n-1}	y_{n-1}	C_{n-1}	z_{n-1}	V
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

Boolean identities:

$I_1: (A \oplus B) \cdot C = A \cdot C \oplus B \cdot C$

$I_2: A + B = A \oplus B \oplus A \cdot B$

$I_2': A \oplus B = (A+B) \oplus A \cdot B$

$\alpha \oplus 0 = \alpha$

$\overline{C_{n-1}} = 1 \oplus C_{n-1}$

$$V = \overline{x_{n-1}}\,\overline{y_{n-1}}\,C_{n-1} + x_{n-1}\,y_{n-1}\,\overline{C_{n-1}} \overset{I_2}{=}$$

$$\overline{x_{n-1}} \cdot \overline{y_{n-1}} \cdot C_{n-1} \oplus x_{n-1} \cdot y_{n-1} \cdot \overline{C_{n-1}} \oplus \underbrace{\cancel{(\overline{x_{n-1}} \cdot \overline{y_{n-1}} \cdot C_{n-1} \cdot x_{n-1} \cdot y_{n-1} \cdot \overline{C_{n-1}})}}_{0} =$$

$$= \overline{x_{n-1}}\,\overline{y_{n-1}} \cdot C_{n-1} \oplus x_{n-1} \cdot y_{n-1} \cdot (1 \oplus C_{n-1}) \overset{I_1}{=}$$

$$\overline{x_{n-1}}\,\overline{y_{n-1}} \cdot C_{n-1} \oplus x_{n-1}\,y_{n-1}\,C_{n-1} \oplus x_{n-1}\,y_{n-1} \overset{I_1}{=}$$

$$(\overline{x_{n-1}}\,\overline{y_{n-1}} \oplus x_{n-1}\,y_{n-1})\,C_{n-1} \oplus x_{n-1}\,y_{n-1} \overset{I_2'}{=}$$

$$(\underbrace{\overline{x_{n-1}}\,\overline{y_{n-1}} + x_{n-1}\,y_{n-1}}_{\overline{x_{n-1} \oplus y_{n-1}} = x_{n-1} \oplus y_{n-1} \oplus 1})\,C_{n-1} \oplus x_{n-1}\,y_{n-1} =$$

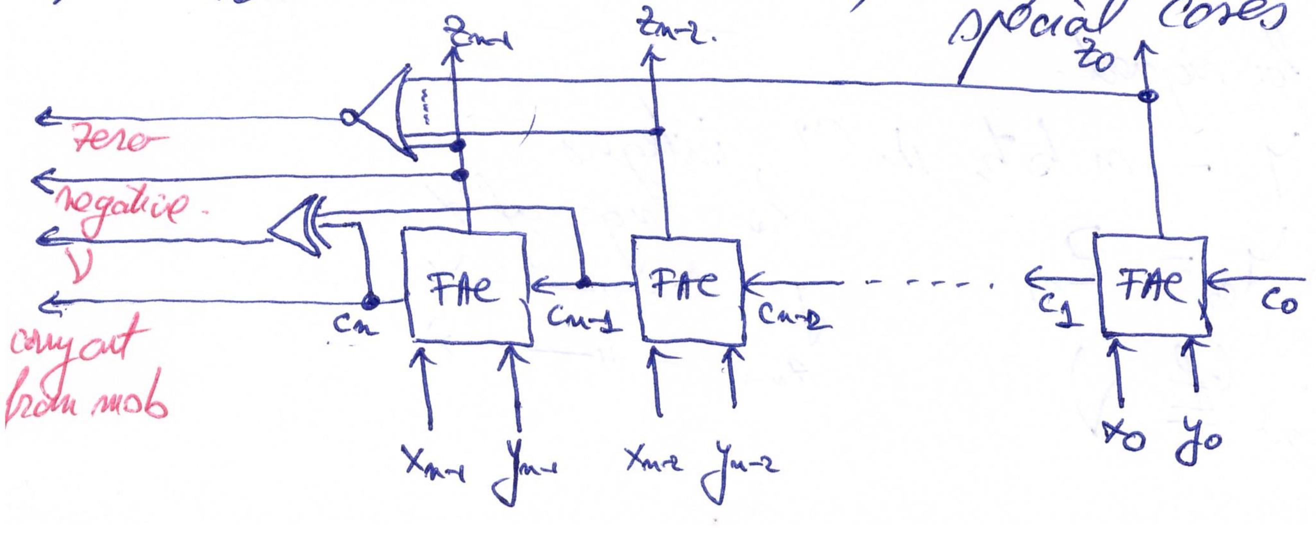
$$(x_{n-1} \oplus y_{n-1} \oplus 1)\,C_{n-1} \oplus x_{n-1}\,y_{n-1} \overset{I_1}{=}$$

$$\underbrace{x_{n-1}C_{n-1} \oplus y_{n-1}C_{n-1} \oplus x_{n-1}\,y_{n-1}}\oplus C_{n-1} \overset{I_2'}{=}$$

$$(\underbrace{x_{n-1}C_{n-1} + y_{n-1}C_{n-1} + x_{n-1}\,y_{n-1}}_{C_n}) \oplus C_{n-1}$$

$$V = C_n \oplus C_{n-1}$$

RCA, n bits with addition's special cases



Addition with constant
- only odd constants

$Y = \text{constant}$ $\quad Y = y_{m-1} y_{m-2} \dots y_0$

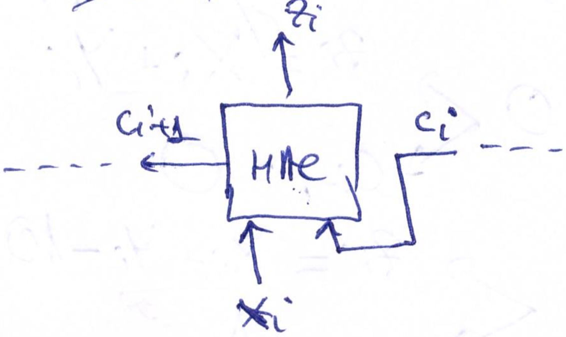
X $\quad X = x_{m-1} x_{m-2} \dots x_0$

$\quad z = z_{m-1} z_{m-2} \dots z_0$

$z = X + Y$

if $y_i == 0$

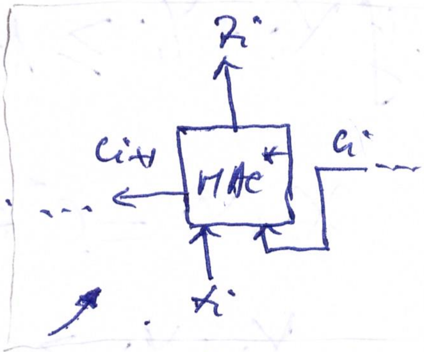
$$\begin{cases} z_i = x_i \oplus 0 \oplus c_i = x_i \oplus c_i \\ c_{i+1} = \cancel{x_i \cdot 0} + \cancel{0 \cdot c_i} + x_i \cdot c_i = x_i \cdot c_i \end{cases} \Big\} HAC$$



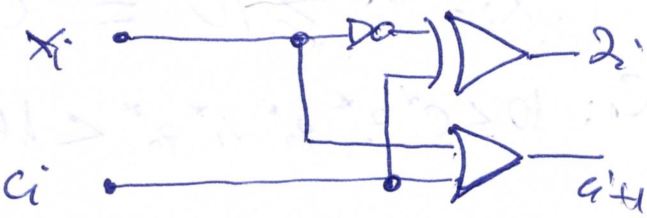
if $y_i == 1$

$$\begin{cases} z_i = x_i \oplus 1 \oplus c_i = \overline{x_i} \oplus c_i \\ c_{i+1} = x_i \cdot 1 + 1 \cdot c_i + x_i c_i = x_i + \underbrace{c_i + x_i c_i} \end{cases}$$

$$= x_i \underbrace{(1 + c_i)}_{1} + c_i = x_i + c_i$$



$$\begin{cases} z_i = \overline{x_i} \oplus c_i \\ c_{i+1} = x_i + c_i \end{cases} \Big\} HAC^*$$

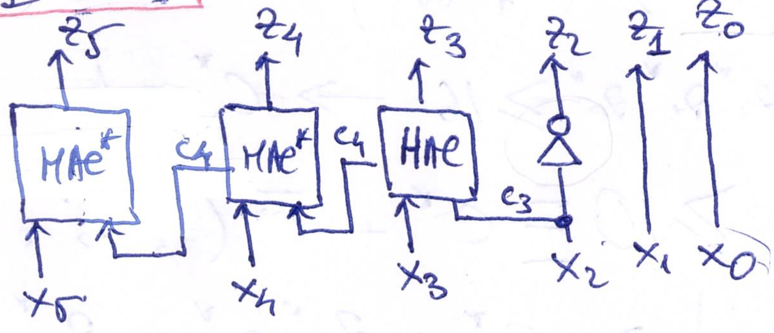


Example: X, Y 6-bit

$Y = y_5 y_4 y_3 y_2 y_1 y_0$ $\quad X = x_5 x_4 x_3 x_2 x_1 x_0$ $\quad z = X + Y$

$= 1\ 1\ 0\ 1\ 0\ 0$

x_3	$x_1 x_0$
1	00



$z_2 = x_2 + y_2,\ c_2 = 0$

$z_2 = x_2 \oplus y_2 \oplus c_2 =$

$= x_2 \oplus 1 \oplus 0 =$

$z_2 = \overline{x_2}$

$c_3 = x_2 \cdot y_2 + x_2 c_2 + y_2 \cdot c_2$

$= x_2 \cdot 1 + x_2 \cdot 0 + 1 \cdot 0$

$c_3 = x_2$

1.2. Decimal Adders based on Serial Carry Propagation

1.2.1. BCD adder.

BCD
E3

Let X_i, Y_i, Z_i – BCD 8421 digits.

$X_i = x_3x_2x_1x_0 \quad Y_i = y_3y_2y_1y_0 \quad Z_i = z_3z_2z_1z_0$

$X_i + Y_i < \begin{matrix} z_i = \text{sum digit} \\ c_{i+1} = \text{carry out for next digit.} \end{matrix}$

if $\cancel{X_i} + Y_i < 10 < \begin{matrix} Z_i = X_i + Y_i \\ c_{i+1} = 0 \end{matrix}$

if $X_i + Y_i \geq 10 < \begin{matrix} Z_i = X_i + Y_i - 10 \\ c_{i+1} = 1 \end{matrix}$

Z_i's CORRECTION

5 – 0101
3 – 0011
$\overline{1000} = 8$

7 0111
8 1000
$\overline{15}$ $\overline{1111} = 15$
1 5 Z_i

$X_i + Y_i = C^* z_3^* z_2^* z_1^* z_0^*$

$X_i + Y_i \geq 10 \equiv C^* z_3^* z_2^* z_1^* z_0^* \geq 10$ $\begin{cases} 10 \leq C^* z_3^* z_2^* z_1^* z_0^* < 16 & C1 \\ C^* z_3^* z_2^* z_1^* z_0^* \geq 16 & C2 \end{cases}$

$C_1: 10 \leq C^* z_3^* z_2^* z_1^* z_0^* < 16$ $\begin{cases} C^* = 0 \ (\ldots < 16) \\ z_3^* z_2^* z_1^* z_0^* \geq 10 \end{cases}$

$z_3^*z_2^*$ \ $z_1^*z_0^*$	00	01	11	10
00				
01				
11	1	1	1	1
10			1	1

$z_3^* z_2^* z_1^* z_0^* \geq 10$ dacă $z_3^* z_2^* + z_3^* z_1^* = 1$

$C_1: \overline{C^*}(z_3^* z_2^* + z_3^* z_1^*)$

$C_2: C^* z_3^* z_2^* z_1^* z_0^* \geq 16 \rightarrow C^* = 1$

$X_i + Y_i \geq 10 \equiv C^* + \overline{C^*}(z_3^* z_2^* + z_3^* z_1^*)$

$a + \bar{a}b = a + b$

$\hookrightarrow C^* + z_3^* z_2^* + z_3^* z_1^*$

$(a+b) \bmod c = (a+b+c) \bmod c$

How to subtract $10_{(10)}$ from $X_i + Y_i$

$(X_i + Y_i - 10) \bmod 2^4 = (X_i + Y_i - 10) \bmod 16$

$= (X_i + Y_i + 16 - 10) \bmod 16 = (X_i + Y_i + 6) \bmod 16$

subtracting 10 from $x_i + y_i \equiv$ adding 6 to $x_i + y_i$

Ex:

$$\begin{array}{r} 15 - \\ 10 \\ \hline 5 \end{array} = \begin{array}{r} 1111 - \\ 1010 \\ \hline 0101 \end{array} \qquad \begin{array}{r} 15 + \\ 6 \end{array} = \begin{array}{r} 1111 + \\ 0110 \\ \hline \cancel{1}\,0101 \end{array} = 5$$

$$\begin{array}{r} 18 - \\ 10 \\ \hline 8 \end{array} = \begin{array}{r} 10010 - \\ 1010 \\ \hline 1000 \end{array} \qquad \begin{array}{r} 18 + \\ 6 \end{array} = \begin{array}{r} \cancel{1}\,0010 + \\ 0110 \\ \hline 1000 \end{array} = 8$$

z_i's correction depends on:

$$c^* + z_3^* z_2^* + z_3^* z_1^* \begin{cases} 1 \; (x_i + y_i \geq 10) \Rightarrow \begin{cases} z_i = z_3^* z_2^* z_1^* z_0^* + 0110 \; (+6) \\ c_{i+1} = 1 \end{cases} \\ 0 \; (x_i + y_i < 10) \Rightarrow \begin{cases} z_i = z_3^* z_2^* z_1^* z_0^* + 0000 \; (+0) \\ c_{i+1} = 0 \end{cases} \end{cases}$$

Correction stage for z_i

$$z_i = z_3^* z_2^* z_1^* z_0^* + 0\, c_{i+1}\, c_{i+1}\, 0$$

and $c_{i+1} = c^* + z_3^* z_2^* + z_3^* z_1^*$