IEC6 Optimal length of RCA segments
- segments RCA : b biti
- leftmost & rightmost segments have no skip logic
- operands on n bits, $n = k * b$, $k \in N$

$$D^{z/cout}_{CSkA-n} = 2b\,d + 2\left(\frac{n}{b} - 2\right)\cdot d + 2\,b\cdot d \qquad 2d \quad +2d.$$

$$= \left(\frac{2n}{b} + 4b - 4\right) d.$$

$$b_{opt} = \frac{d\,D^{z/cout}_{CSkA-n}}{d\,b} = 0 \Rightarrow -\frac{2n}{b^2} + 4 = 0 \Rightarrow b_{opt} = \frac{\sqrt{2n}}{2}$$

$$D^{z/cout}_{CSkA-n\,opt.} = 4(\sqrt{2n} - 1)d.$$

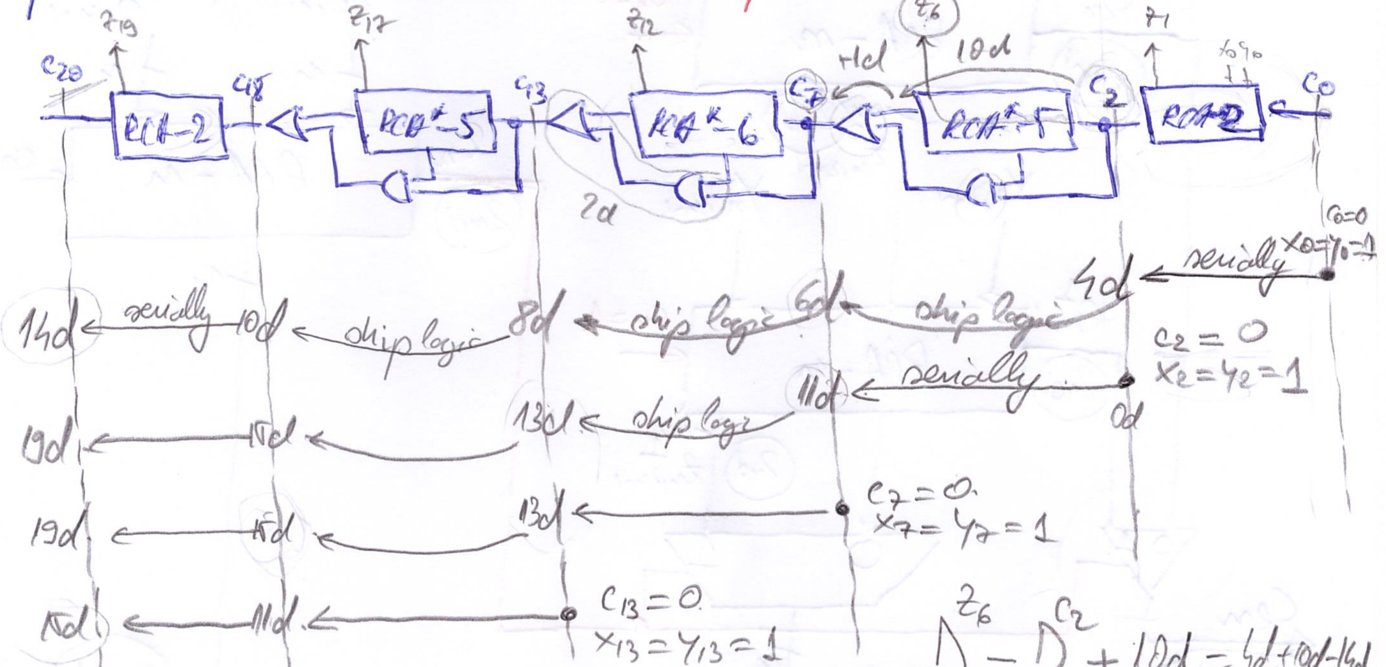
$n = 32 \rightarrow b_{opt} = 4$

$D^{z/cout}_{CSkA-32\,opt} = 28d.$!!!

$D^{z/cout}_{RCA-32} = 64d.$!!!

Variable-sized RCA segments
- operands on 20 bits

critical path ??



$$D^{z_6}_{max} = D^{c_2}_{max} + 10d = 4d + 10d = 14d$$

$D^{cout}_{CSkA} = 19d$

$D^{z}_{CSkA} = 23d.$

$$D^{z_{12}}_{max} = D^{c_7}_{max} + \underbrace{2\cdot 6d}_{RCA^*-6} = 11d + 12d = 23d$$

$$D^{z_{17}}_{max} = D^{c_{13}}_{max} + 2\cdot 5d = 13d + 10d = 23d$$

$$D^{z_{19}}_{max} = D^{c_{18}}_{max} + 2\cdot 2d = 15d + 4d = 19d$$

Multilevel CSkA.

1.4.3. Carry Select Adder (CSeA)

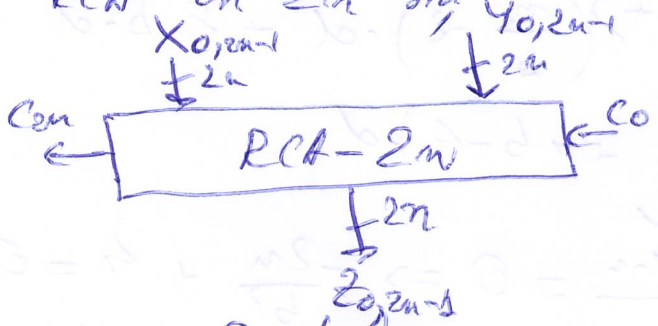
- principle of sum conditioned by carry

$$c_i \begin{cases} 0 \\ 1 \end{cases} \quad \text{calculate } z_i,\ c_{i+1} \text{ in 2 variants} \begin{cases} \text{if } c_i == 0 \\ \text{if } c_i == 1 \end{cases}$$

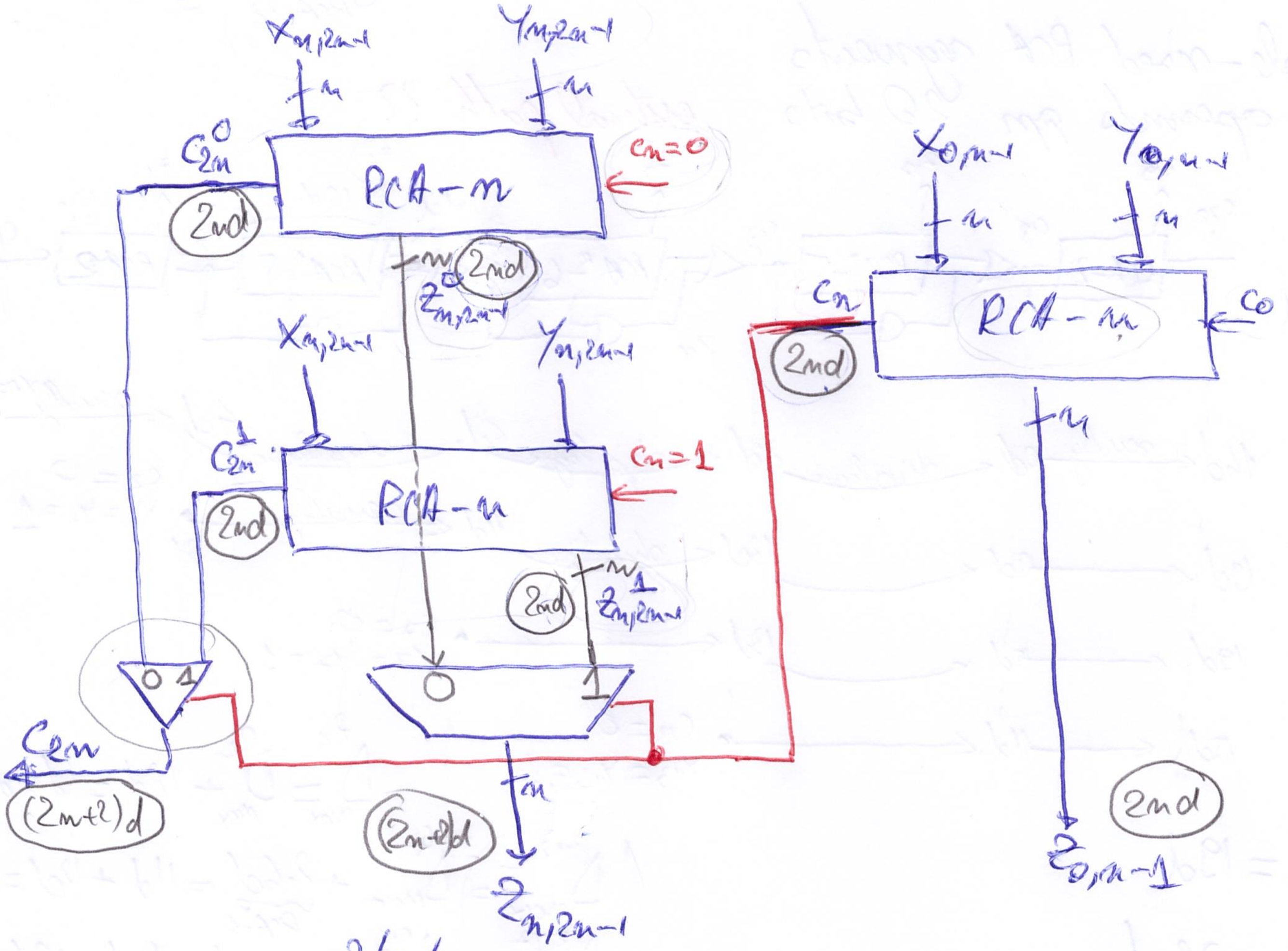
$$\Rightarrow \begin{cases} (z_i^0,\ c_{i+1}^0) - c_i == 0 \\ (z_i^1,\ c_{i+1}^1) - c_i == 1 \end{cases}$$

- select the correct one AFTER c_i's computation

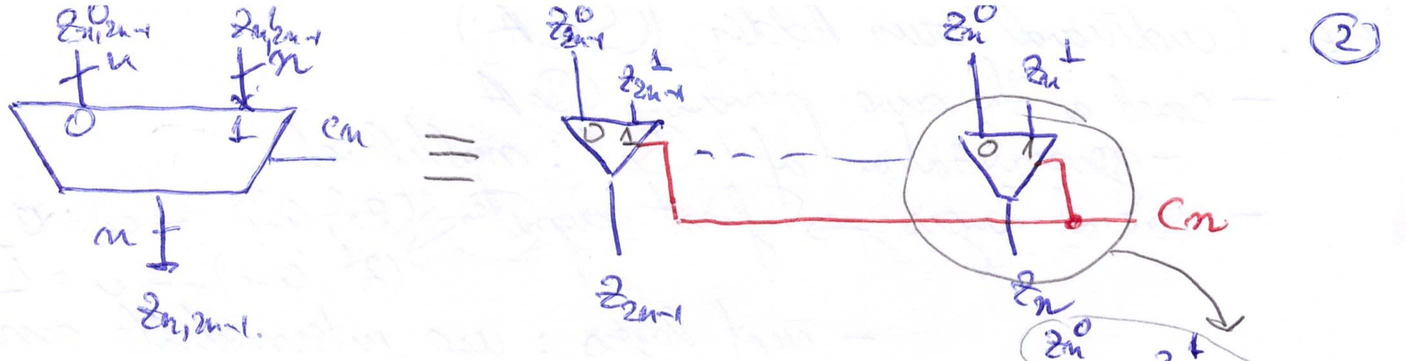
Consider an RCA on 2n bits,

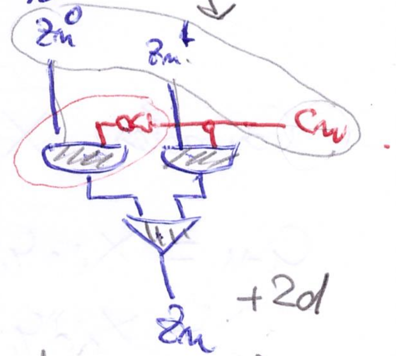


- split the adder in 2 halves
  - duplicate the most signif. half $\begin{cases} c_n = 0 \\ c_n = 1 \end{cases}$



$$\Delta^{z/cout}_{CSeA-2n} = (2n+2)d.$$





<u>C_{2n}</u> optimise for area

Inputs			Outputs
C_n	C^0_{2n}	C^1_{2n}	C_{2n}
0	0	0	0
0	0	1	0
0	1	0	d
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	d
1	1	1	1

$C^0_{2n} = 1$

$C^1_{2n} = 0$?!?

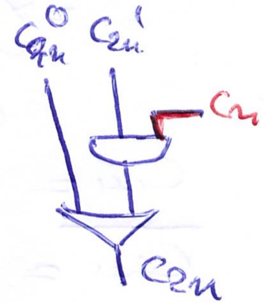
$\{C^0_{2n}, z^0_{n,2n-1}\} = (X_{n,2n-1} + Y_{n,2n-1} + 0)$

$\{C^1_{2n}, z^1_{n,2n-1}\} = (X_{n,2n-1} + Y_{n,2n-1} + 1)$

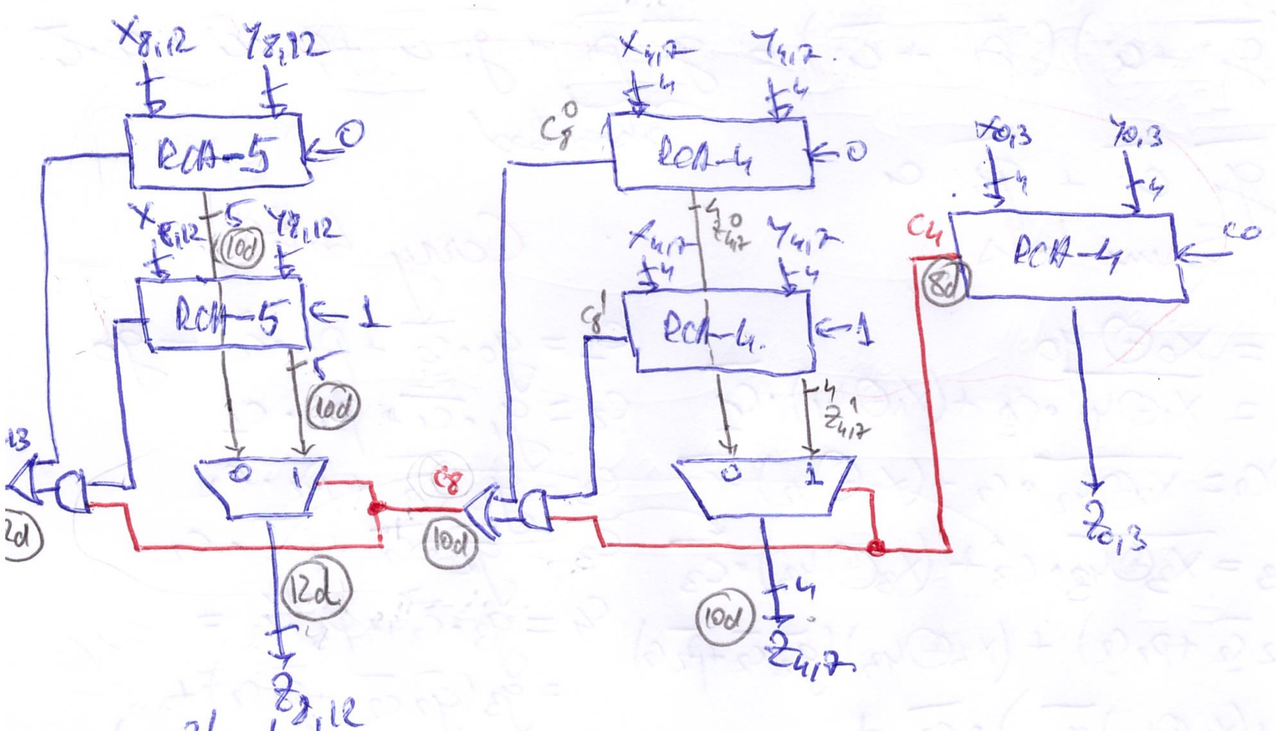
$C^0_{2n} > C^1_{2n}$

C_n \ $C^0_{2n}C^1_{2n}$	00	01	11	10
0	0	0	1	d
1	0	1	1	d

$C_{2n} = C^0_{2n} + C^1_{2n} \cdot C_n$



Variable-sized RCA segments



$D^{\text{3l-colut}}_{CSeA} = 12d.$

Multilevel CSeA.

4.4. Conditional Sum Adder (CSuA)

- based on the same principle CSeA
  - generalization of CSeA: multilevel
- several layers → first computes → (z_i^0, c_{i+1}^0) − $c_i = 0$
  → (z_i^1, c_{i+1}^1) − $c_i = 1$
  - next layers: use intermediate carries to select bits of sum

Consider X, Y − 4 bits $c_0 = 0$

$$c_{i+1} = x_i \cdot y_i + x_i \cdot c_i + y_i \cdot c_i = x_i \cdot y_i (\overbrace{c_i + \overline{c_i}}^{1}) + x_i \cdot c_i + y_i \cdot c_i$$

$$= x_i \cdot y_i \cdot \overline{c_i} + \underbrace{x_i \cdot y_i \cdot c_i + x_i \cdot c_i} + y_i \cdot c_i =$$

$$= x_i \cdot y_i \cdot \overline{c_i} + x_i \cdot c_i \underbrace{(y_i + 1)}_{1} + y_i \cdot c_i =$$

$$= x_i \cdot y_i \cdot \overline{c_i} + \underbrace{x_i \cdot c_i + y_i \cdot c_i} =$$

$$= \underbrace{x_i \cdot y_i}_{g_i} \cdot \overline{c_i} + \underbrace{(x_i + y_i)}_{p_i} \cdot c_i$$

$$a \oplus b = \overline{a} \cdot b + a \cdot \overline{b}$$

$$c_{i+1} = g_i \cdot \overline{c_i} + p_i \cdot c_i$$

$$\overline{c_{i+1}} = \overline{g_i \cdot \overline{c_i} + p_i \cdot c_i} = \overline{g_i \overline{c_i}} \cdot \overline{p_i \cdot c_i} =$$

$$= (\overline{g_i} + c_i)(\overline{p_i} + \overline{c_i}) = \underbrace{\overline{g_i}\,\overline{p_i}}_{\text{absorbed}} + \overline{g_i}\,\overline{c_i} + \overline{p_i} \cdot c_i + \cancel{c_i \overline{c_i}}$$

$$\overline{c_{i+1}} = \overline{g_i}\,\overline{c_i} + \overline{p_i}\, c_i$$

Sum bits

$$z_0 = x_0 \oplus y_0 \oplus \overset{=0}{c_0} = x_0 \oplus y_0$$

$$z_1 = (x_1 \oplus y_1) \oplus c_1 = \overline{x_1 \oplus y_1} \cdot c_1 + (x_1 \oplus y_1) \cdot \overline{c_1}$$

$$z_2 = (x_2 \oplus y_2) \oplus c_2 = \overline{x_2 \oplus y_2} \cdot c_2 + (x_2 \oplus y_2) \cdot \overline{c_2}$$

$$z_3 = x_3 \oplus y_3 \oplus c_3 = \overline{x_3 \oplus y_3} \cdot c_3 + (x_3 \oplus y_3) \cdot \overline{c_3}$$

$$= \overline{(x_3 \oplus y_3)}(g_2 \overline{c_2} + p_2 c_2) + (x_3 \oplus y_3)(\overline{g_2}\,\overline{c_2} + \overline{p_2} c_2)$$

$$= (\overline{x_3 \oplus y_3} \cdot g_2 + (x_3 \oplus y_3)\overline{g_2}) \cdot \overline{c_2} +$$

$$(\overline{x_3 \oplus y_3} \cdot p_2 + (x_3 \oplus y_3)\overline{p_2}) \cdot c_2$$

Carry bits

$$c_1 = g_0 \cdot \overline{c_0} + p_0 \cdot \overset{=0}{c_0} = g_0$$

$$c_2 = g_1 \cdot \overset{=1}{\overline{c_1}} + p_1 \cdot c_1$$

$$c_3 = g_2 \cdot \overline{c_2} + p_2 \cdot c_2$$

$$\overline{c_3} = \overline{g_2} \cdot \overline{c_2} + \overline{p_2} \cdot c_2$$

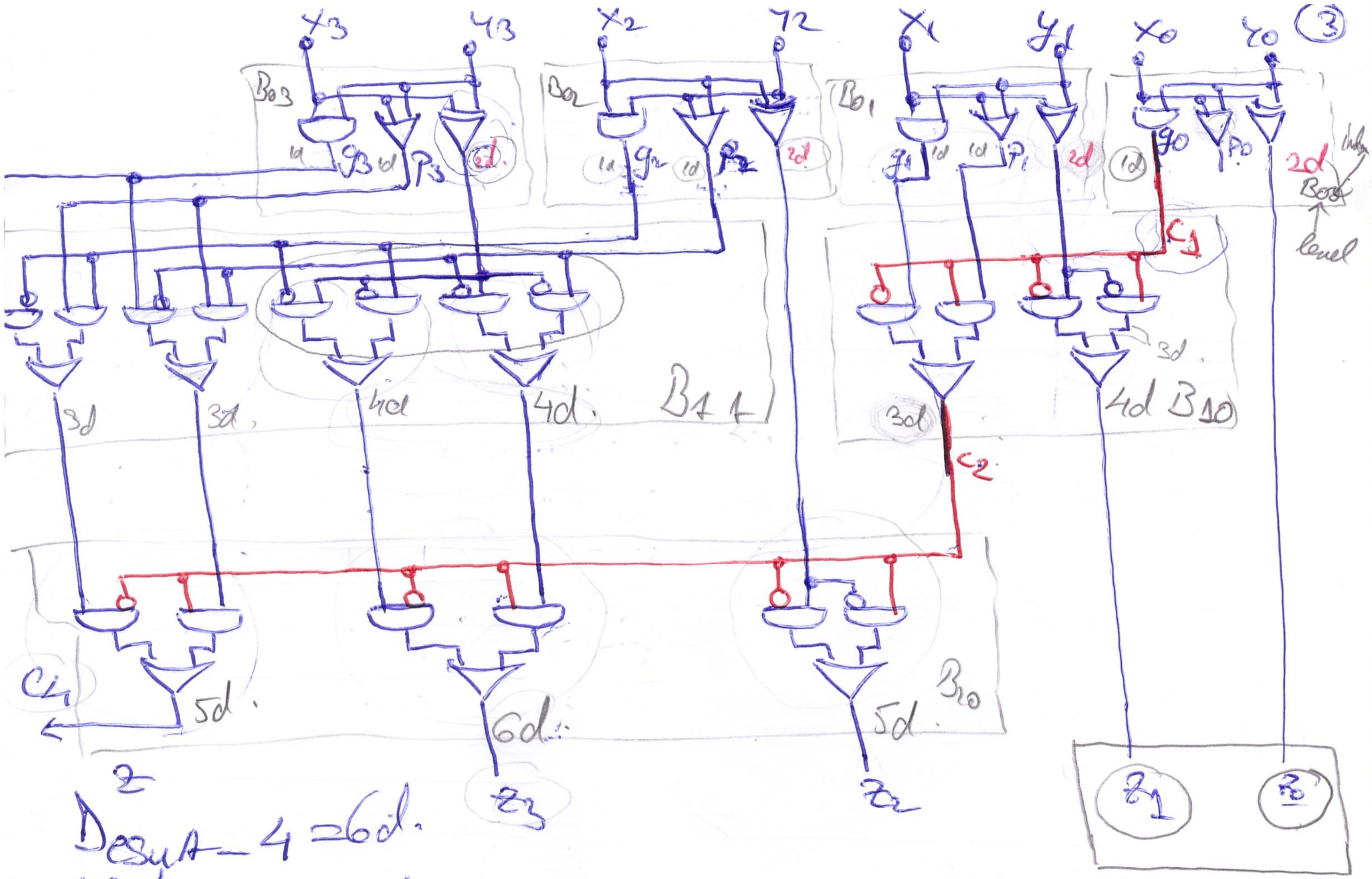
$$c_4 = g_3 \cdot \overline{c_3} + p_3 \cdot c_3 =$$

$$= g_3(\overline{g_2}\,\overline{c_2} + \overline{p_2} c_2) +$$

$$p_3(g_2 \overline{c_2} + p_2 c_2) =$$

$$= (g_3 \overline{g_2} + p_3 g_2)\overline{c_2} +$$

$$(g_3 \overline{p_2} + p_3 p_2) c_2$$



$D_{CSuA-4} = 6d.$

$D^{cout}_{CSuA-4} = 5d.$