Applying Nash Equilibrium to Rock, Paper, Scissors, Lizard, and Spock

	Rock	Paper	Scissors	Spock	Lizard
Rock	0,0	-1,1	1,-1	-1,1	1,-1
Paper	1,-1	0,0	-1,1	1,-1	1,-1
Scissors	-1,1	1,-1	0,0	-1,1	1,-1
Spock	1,-1	-1,1	1,-1	0,0	-1,1
Lizard	-1,1	1,-1	-1,1	1,-1	0,0

 σ_R = Probability of player 1 playing Rock

 E_R = Expected outcome of player 2 while playing Rock

$$\sigma_R + \sigma_P + \sigma_{SC} + \sigma_{SP} + \sigma_L = 1$$

$$\sigma_{I} = 1 - (\sigma_{R} + \sigma_{P} + \sigma_{SC} + \sigma_{SP})$$

$$\begin{split} \mathsf{E}_{\mathsf{R}} &= (\,\textbf{0} \,\,^*\,\sigma_{\mathsf{R}}) + (\,\textbf{-1} \,\,^*\,\sigma_{\mathsf{P}}) + (\,\textbf{1} \,\,^*\,\sigma_{\mathsf{SC}}) \, + (\,\textbf{-1} \,\,^*\,\sigma_{\mathsf{SP}}\,) + (\,\textbf{1} \,\,^*\,\sigma_{\mathsf{L}}) \\ &= -1\sigma_{\mathsf{P}} + \,\,\sigma_{\mathsf{SC}} - \sigma_{\mathsf{SP}} + 1 - \sigma_{\mathsf{R}} - \sigma_{\mathsf{P}} - \,\,\sigma_{\mathsf{SC}} - \sigma_{\mathsf{SP}} \\ &= -\sigma_{\mathsf{R}} \,\, - 2\sigma_{\mathsf{P}} - 2\,\,\sigma_{\mathsf{SP}} + 1 \end{split}$$

$$\begin{split} E_{P} &= (\,\mathbf{1} * \sigma_{R}) + (\,\mathbf{0} * \sigma_{P}) + (\,\mathbf{-1} * \sigma_{SC}) \, + (\,\mathbf{1} * \sigma_{SP}\,) + (\,\mathbf{-1} * \sigma_{L}\,) \\ &= \sigma_{R} - \sigma_{SC} + \sigma_{SP} - (\,\mathbf{1} - \sigma_{R} - \sigma_{P} - \sigma_{SC} - \sigma_{SP}\,) \\ &= \sigma_{R} - \sigma_{SC} + \sigma_{SP} - 1 + \sigma_{R} + \sigma_{P} + \sigma_{SC} + \sigma_{SP} \\ &= 2\sigma_{R} + 2\sigma_{SP} + \sigma_{P} - 1 \end{split}$$

$$\begin{split} E_{\text{SC}} &= (-1 * \sigma_{\text{R}}) + (1 * \sigma_{\text{P}}) + (0 * \sigma_{\text{SC}}) + (-1 * \sigma_{\text{SP}}) + (1 * \sigma_{\text{L}}) \\ &= -\sigma_{\text{R}} + \sigma_{\text{P}} - \sigma_{\text{SP}} + (1 - (\sigma_{\text{R}} + \sigma_{\text{P}} + \sigma_{\text{SC}} + \sigma_{\text{SP}})) \\ &= -\sigma_{\text{R}} + \sigma_{\text{P}} - \sigma_{\text{SP}} + 1 - \sigma_{\text{R}} - \sigma_{\text{P}} - \sigma_{\text{SC}} - \sigma_{\text{SP}} \\ &= -\sigma_{\text{R}} + \sigma_{\text{P}} - \sigma_{\text{SP}} + 1 - \sigma_{\text{R}} - \sigma_{\text{P}} - \sigma_{\text{SC}} - \sigma_{\text{SP}} \\ &= -2\sigma_{\text{R}} - \sigma_{\text{SC}} - 2\sigma_{\text{SP}} + 1 \end{split}$$

$$\begin{aligned} \mathsf{E}_{\mathsf{SP}} &= (\mathbf{1} * \sigma_{\mathsf{R}}) + (-\mathbf{1} * \sigma_{\mathsf{P}}) + (\mathbf{1} * \sigma_{\mathsf{SC}}) + (\mathbf{0} * \sigma_{\mathsf{SP}}) + (-\mathbf{1} * \sigma_{\mathsf{L}}) \\ &= \sigma_{\mathsf{R}} - \sigma_{\mathsf{P}} + \sigma_{\mathsf{SC}} - (\mathbf{1} - (\sigma_{\mathsf{R}} + \sigma_{\mathsf{P}} + \sigma_{\mathsf{SC}} + \sigma_{\mathsf{SP}})) \\ &= \sigma_{\mathsf{R}} - \sigma_{\mathsf{P}} + \sigma_{\mathsf{SC}} - \mathbf{1} + \sigma_{\mathsf{R}} + \sigma_{\mathsf{P}} + \sigma_{\mathsf{SC}} + \sigma_{\mathsf{SP}} \\ &= 2 \, \sigma_{\mathsf{R}} + 2 \, \sigma_{\mathsf{SC}} + \sigma_{\mathsf{SP}} - 1 \end{aligned}$$

$$E_{L} = (-1 * \sigma_{R}) + (-1 * \sigma_{P}) + (1 * \sigma_{SC}) + (1 * \sigma_{SP}) + (0 * \sigma_{L})$$

$$= -\sigma_{R} - \sigma_{P} + \sigma_{SC} - \sigma_{SP}$$

$$E_R = E_P = E_{SC} = E_{SP} = E_I$$

Using these equations, we will eventually reach that the Nash Equilibrium for the game Rock, Paper, Scissors, Spock and Lizard is:

For player 1,
$$\sigma_R$$
 = ½, σ_P = ½, σ_{SC} = ½, σ_{SP} = ½, and σ_L = ½ and similarly

For player 2,
$$\sigma_R$$
 = ½, σ_P = ½, σ_{SC} = ½, σ_{SP} = ½, and σ_L = ½