L0: # Exercise 5 - Selected distributions of a discrete random variable

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L2: # Overview of divisions and their functions

L3: ## Introduction: Probability, Cumulative Probability (Distribution) and Quantile functions

L4: ### Probability function

L5: - starts with the letter \*\* d \*\*: $ p = P (X = x) $: p = d ... (x, ...)

L6: ### Cumulative Probability (Distribution Function)

L7: - starts with the letter \*\* p \*\*: $ p = P (X \ leq x) $: p = p ... (x, ...) - note Cumulative probability is with the alternative definition $ P (X \ leq t) $ - for our distribution function $ F (t) = P (X <t) $: F (t) = p ... (t - 1, ...)

L8: ### Quantile function

L9: - starts with the letter \*\* q \*\*: $ p \ geq P (X \ leq x) $: x = q ... (p, ...) - searches for the smallest $ x $ for which is $ P (X \ leq x) $ greater than $ p $

L10: ## Binomial (Alternative): $ X \ sim Bi (n, π), X \ sim A (π) = Bi (1, π) $

L11: - number of successes in $ n $ Bernoulli attempts (or for one attempt in the case of Alternative) - each attempt has a chance of success $ π $

L12: Probability function P (X = x)

L13: value for which we are looking for a p-st function

L14: selection range

L15: probability of success

L16: this can be used to turn off warnings

L17: turn it on again

L18: draw a probability function

L19: minimum 0, maximum n has a positive probability

L20: Cumulative probability function P (X <= x)

L21: value for which we are looking for a cumulative p-st function

L22: selection range

L23: probability of success

L24: Distribution function F (x) = P (X <x)

L25: value for which we are looking for a cumulative p-st function

L26: selection range

L27: probability of success

L28: or

L29: we draw the distribution function

L30: minimum 0, maximum n has a positive probability

L31: or

L32: minimum 0, maximum n

L33: check correctness at 10

L34: minimum 0, maximum n

L35: find x for given q: q = P (X <= x)

L36: h

L37: selection range

L38: probability of success

L39: Quantile function (inversion of dist. Function): q = F (x) = P (X <x)

L40: probability for which we are looking for a quantile

L41: selection range

L42: probability of success

L43: ## Hypergeometric: $ X \ sim H (N, M, n) $

L44: - number of successes in $ n $ dependent attempts - dependency type:

L45: - $ N $ objects,

L46: - of which $ M $ objects with specified property,

L47: - size selection $ n $

L48: - \*\* we do not return when selecting - the probability of selecting an object with a given property changes with each additional selected object \*\*

L49: - \*\* R function takes as parameters \* hyper (k, M, N - M, n) \*\*

L50: - k is the number of successes for which we calculate the probability,

L51: - M is the number of objects with the specified property,

L52: - NM is the number of objects without the specified property,

L53: - n is the target size of the selection.

L54: Probability function P (X = x)

L55: value for which we are looking for a p-st function

L56: total number of objects

L57: of which with specified property

L58: selection size

L59: draw a probability function

L60: minimum 0, maximum n or M has a positive truth.

L61: Distribution function F (x) = P (X <x)

L62: value for which we are looking for dist. function

L63: total number of objects

L64: of which with specified property

L65: selection size

L66: let's draw the Distribution function

L67: minimum 0, maximum n or M has a positive truth.

L68: Quantile function (inversion of dist. Function): q = P (X <x)

L69: probability for which we are looking for a quantile

L70: total number of objects

L71: of which with specified property

L72: selection size

L73: ## Negative binomial (Geometric): $ X \ sim NB (k, π), X \ sim Ge (π) = NB (1, π) $

L74: - number of attempts up to $ k $. success (inclusive) - each attempt has a chance of success $ π $ - \*\* Negatively binomial NV is defined in Rku as the number of failures before the success \*\*

L75: - therefore we will send x - k as the first parameter

L76: Probability function P (X = x)

L77: number of attempts for which we are looking for truth. fci

L78: required number of successes

L79: truth. individual trials

L80: Note that the first argument must be the number of failures

L81: draw a probability function

L82: minimum k, maximum unlimited

L83: values 0,1,2,3,4 have P (x) = 0

L84: Distribution function F (x) = P (X <x)

L85: number of attempts for which we are looking for truth. fci

L86: required number of successes

L87: truth. individual trials

L88: Note that the first argument must be the number of failures

L89: let's draw the Distribution function

L90: minimum 0, maximum n or M has a positive truth.

L91: Quantile function (inversion of dist. Function): q = P (X <x)

L92: truth. for quantile

L93: required number of successes

L94: true individual trials

L95: ## Poisson: $ X \ sim Po (λt) $

L96: - number of events in the Poisson process in a closed area (in time, on area, in volume) - with occurrence density $ λ $ - in time / area / volume of size $ t $

L97: Probability function P (X = x)

L98: number of attempts for which we are looking for truth. fci

L99: occurrence density

L100: true individual trials

L101: draw a probability function

L102: minimum 0, maximum unlimited

L103: Distribution function F (x) = P (X <x)

L104: number of attempts for which we are looking for truth. fci

L105: density of occurrence

L106: true individual trials

L107: let's draw the Distribution function

L108: minimum 0, maximum n or M has a positive truth.

L109: Quantile function (inversion of dist. Function): q = P (X <x)

L110: true for quantile

L111: density of occurrence

L112: true individual trials

L113: # Examples

L114: ## Example 1.

L115: Bridge is played with 52 bridge cards, which are dealt among 4 players. There are always 2 players playing together. When dealing (13 cards) you received 2 aces. What is the probability that your partner will have the remaining two aces?

L116: X ... number of aces among 13 cards

L117: X ~ H (N = 39, M = 2, n = 13)

L118: P (X = 2)

L119: 52-13

L120: calculation

L121: which is dhyper (2,2,37,13)

L122: graph of probability function

L123: all possible implementations of NV X.

L124: values of the probability function for x

L125: ## Example 2.

L126: Experiments have shown that a radioactive substance emits within 7.5 s an average of 3.87 α-particles. Determine the probability that this substance will emit at least one α-particle in 1 second.

L127: X ... number of radiated alpha particles during 1 s

L128: X ~ Po (lt = 3.87 / 7.5)

L129: frequency of occurrence

L130: in 1 second

L131: Poisson distribution parameter

L132: P (X> = 1) = P (X> 0) = 1 - P (X <= 0)

L133: graph of probability function

L134: theoretically up to an infinite number of particles can be emitted,

L135: from a certain value the probability is negligible

L136: values of the probability function for x

L137: ## Example 3.

L138: A friend sends you to the cellar to bring 4 bottled beers - two dozen and two twelve. You don't know where to light it, so you take 4 bottles blindly from the bass. How likely were you to comply if you knew that there were a total of 10 tens and 6 twelve in the base?

L139: X ... number of 10 ° beers among 4 selected

L140: X ~ H (N = 16, M = 10, n = 4)

L141: P (X = 2)

L142: probability function graph

L143: all possible implementations of NV X.

L144: values of the probability function for x

L145: ## Example 4.

L146: On average, there are 15 certain microorganisms in one milliliter of a perfectly mixed solution. Determine the probability that there will be less than 5 of these micro-organisms in a test tube if a sample of 1/2 milliliter is randomly selected.

L147: X ... number of microorganisms in 0.5 ml of solution

L148: X ~ Po (lt = 15/2)

L149: Poisson distribution parameter

L150: P (X <5) = P (X <= 4)

L151: or

L152: graph of probability function

L153: theoretically there can be up to an infinite number of microorganisms in solution,

L154: from a certain value the probability is negligible

L155: values of the probability function for x

L156: ## Example 5.

L157: Pour 15 coins on the table. What is the probability that the number of coins lying face up is from 8 to 15?

L158: X ... number of coins that fall face up out of a total of 15 coins

L159: X ~ Bi (n = 15, p = 0.5)

L160: P (8 <= X <= 15) = P (X <= 15) - P (X <8) = P (X <= 15) - P (X <= 7)

L161: otherwise: P (8 <= X <= 15) = P (X> 7) = 1-P (X <= 7)

L162: graph of probability function

L163: all possible implementations of NV X

L164: values of the probability function for x

L165: ## Example 6.

L166: The probability that we will call the studio of the radio station that has just announced a telephone competition is 0.08. What is the probability that we will appeal on the 4th attempt at the most?

L167: X ... number of attempts before we call the radio studio

L168: X ~ NB (k = 1, p = 0.08) or G (0.08)

L169: P (X <= 4)

L170: probability function graph

L171: theoretically we can make infinitely many attempts,

L172: from a certain value the probability is negligible

L173: values of the probability function for x

L174: ## Example 7.

L175: The factory produces 10% of defective parts per day. What is the probability that if we remove thirty components from the daily production, at least two will be defective?

L176: X ... number of defective parts out of 30 selected

L177: X ~ Bi (n = 30, p = 0.1)

L178: P (X> = 2) = 1 - P (X <2) = 1 - P (X <= 1)

L179: or P (X> = 2) all except 0 and 1

L180: graph of probability function

L181: all possible implementations of NV X

L182: probability function values for x

L183: ## Example 8.

L184: There are 200 parts in stock. 10% of them are defective. What is the probability that if we remove thirty parts from the warehouse, at least two will be defective?

L185: X ... number of defective parts out of 30 selected from 200

L186: X ~ H (N = 200, M = 20, n = 30)

L187: P (X> = 2) = 1 - P (X <2) = 1 - P (X <= 1)

L188: graph of probability function

L189: all possible implementations of NV X

L190: probability function values for x

L191: ## Example 9.

L192: A company found that some illegal software was installed on 33% of computers. Determine the probability and distribution function of the number of computers with illegal software among the three computers inspected.

L193: X ... number of computers with illegal software out of 3 checked

L194: X ~ Bi (n = 3, p = 0.33)

L195: probabilistic function

L196: all possible implementations of NV X.

L197: values of the probability function for x

L198: rounding probabilities to 3 des. places

L199: completion of the last value by 1

L200: Create a table of probability functions

L201: graph of probability function

L202: distribution function

L203: simplified distribution function listing

L204: ## Example 10.

L205: Sports is a lottery game in which the bettor bets six numbers out of forty-nine, which he expects to fall in a future draw. To participate in the game, it is necessary to choose at least one combination of 6 numbers (always 6 numbers per column) and use these crosses to mark these numbers in the columns on Sazka as in the columns, starting with the first column. The bettor wins if he guesses at least three numbers from the drawn six numbers. What is the probability that in order for the bettor to win, he will have to fill in:

L206: First the probability that we get in one column

L207: Y ... number of guessed numbers in 6 drawn from 49

L208: Y ~ H (N = 49, M = 6, n = 6)

L209: P-st guess at least 3 numbers in one column

L210: P (Y> = 3) = 1 - P (Y <3) = 1 - P (Y <= 2)

L211: ### a)

L212: just three columns,

L213: X… the number of columns the bettor will have to fill in order to win

L214: X ~ NB (k = 1, p = pp)

L215: a) P (X = 3)

L216: ### b)

L217: at least 5 columns,

L218: b) P (X> = 5) = 1 - P (X <5) = 1 - P (X <= 4)

L219: ### c)

L220: less than 10 columns,

L221: c) P (X <10) = P (X <= 9)

L222:

L223: more than 5 and at most 10 columns?

L224: P (5 <X <= 10) = P (X <= 10) - P (X <= 5)

L225: or P (X <11) - P (X <6)

L226: ## Example 11.

L227: The probability of throwing 6 on a 6-wall cube is 1/6. We roll until we roll six times 10 times.

L228: ### a)

L229: What is the mean value of the number of throws.

L230: X… rolls the dice before we roll 10 sixes

L231: X ~ NB (k = 10, p = 1/6)

L232: ### b)

L233: How many throws do we have to count on if we want the probability of throwing 10 sixes to be at least 70%.

L234: P (X <= k)> = 0.7