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From cellular automata to urban models: new principles for model development and implementation

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Abstract. Integration with geographic information systems (GIS) has helped move cellular automata (CA)-based urban and regional models from the realm of instructive metaphors to that of potentially useful qualitative forecasting tools. Such models can now be fully interactive for exploratory purposes and they can be based on actual data. New problems, however, arise as the formal integrity of the original CA framework is lost through successive relaxations of the assumptions and as the resulting complicated models become increasingly difficult to implement and understand. In this paper I propose that the theoretical problem can find a satisfactory answer in the notion of *proximal space* and the practical one can be handled successfully within the formalism of *geo-algebra,* a mathematical expression of proximal space which builds a bridge between GIS data and operations, on the one hand, and traditional robust classes of urban and regional models, on the other.

Introduction

> "Everything that happens in our world resembles a vast game in which nothing is determined in advance but the rules, and only the rules are open to objective understanding" (Eigen and Winkler, 1981, page xi).

It is no coincidence that the first widely popular cellular automaton was called the 'Game of Life'. But if the world is like a game and cellular automata (CA) are a kind of game, then CA are (perhaps) like the world, in that simple well-understood interaction rules can determine the most complex, unpredictable, forms of evolution. As Wittgenstein (1953) has shown, the notion of game is an extremely rich and profound one: it is no wonder it has served as a powerful metaphor for all kinds of complex interactive situations. Being an inherently spatial form of game, CA have become a popular metaphor for a range of geographic and urban or regional processes. Yet, after over a decade of marveling at the suggestive power of such constructs, the time is ripe to move ahead towards a more practically useful class of models.

Between the evocative metaphor and the rigorous prediction lies the region of qualitative forecasts. We have learned by now that, in all but the most trivial cases, to hope for good predictive models in the realm of social phenomena is futile. The inability to describe, let alone fully explain and predict, urban development is as true of CA as of any other kind of model we already have or may still develop in the future. It is not just that we are not yet smart enough or knowledgeable enough today, but may be so tomorrow: there are also laws of complexity at work (sensitivity to initial conditions, uncomputability, NP-completeness, and so on), which virtually guarantee that the detailed future trajectory of urban and regional systems will remain forever intractable. This is even without taking into account the effects of deliberate planning intervention, the purpose of which is to counteract and change 'natural' development tendencies.

But right there between prediction and game playing, between science and science fiction, lies the realm of sharpened intuition, informed speculation, and educated guess. It is here, in the area of serious qualitative forecasts, that the impact of

CA-based urban and regional models can be most significant. I believe there are two major requirements for this to happen: interactivity and realism. Things have been moving in that direction for some time, especially through the integration of CA models with geographic information systems (GIS). This development has already largely taken care of the interactivity requirement and has helped improve realism to a considerable degree. Can we do any better? What problems remain to be solved? Can we prevent this new more sophisticated species of CA-based models from evolving into computationally inscrutable and theoretically vacuous simulation behemoths?

In this paper I will discuss some of these questions and propose both a theoretical and a technical answer to some major dilemmas. I will examine what is required, conceptually and practically, in order to go beyond games in the area of urban and regional modeling with CA. It is shown that generalized CA models are more robust conceptually than one might think, thanks to their strong theoretical foundation in the notion of *proximal space;* and it is argued that they can become more consistently generalizeable and more easily implementable through integration with *geo-algebra,* a new formal tool expressing some critical properties of proximal space.

Interactivity and realism in CA-based models

In this section the two major requirements for CA-based urban models that can serve as instruments for serious qualitative forecasts are examined. *Interactivity* is essential for the exploration of options. Good forecasts are about probing the boundaries of possibilities and assessing probabilities within the feasible regions; they are also about developing a keen intuition as to what might be the case—something much harder to express in numbers. In exploring possibilities, urban and regional CA-based models must allow all sorts of "what if ..." questions to be asked and answered, preferably at a pace resembling that of a conversational exchange. This immediacy is necessary when understanding is to be built up one little idea at a time. This is especially true in the case of CA where, as with other models of complexity, small changes in the rules or initial conditions can sometimes make a dramatic difference in the outcomes.

An integral aspect of the required interactive immediacy is of course the capacity to visualize outcomes. This is especially true when dealing with qualitative forecasts, as numbers are not the best medium for conveying qualitative information. A picture may be worth a thousand words, but in this case a pattern is worth a million numbers. Who would even consider CA for formal or applied research were it not for their spectacular graphic behavior. But visualization does not stop with the display of the CA pattern itself. Aggregate outcomes, statistical trends, comparative measures, and the space of possibilities itself should be interactively reproducible in graphic or even animated format. To take full advantage of CA models as qualitative forecasting tools, planners and others need to rely as much on their right-brain powers of pattern recognition and relationship perception as on left-brain analyses of the inevitably inaccurate quantitative outputs.

This kind of advanced interactivity and visualization is well within the reach of urban or regional modelers today, thanks to GIS technology and related research on human–computer interfaces for geographic information processing (Mark and Frank, 1992). Indeed CA-based models, because of their natural affinity with the data structures of raster GIS, have been quick in capitalizing on these new possibilities (Batty and Xie, 1994a; 1994b; 1997; White and Engelen, 1992; Xie, 1994). But although the necessary technical tools appear to be already there, the question of the most appropriate interactivity and visualization formats for forecasting and general planning purposes remains open. In search of partial answers to this question we have been working recently on the idea of SUSS or 'spatial understanding support system' (Couclelis and Monmonier, 1995). A discussion of these issues here would take us too far off course. Suffice it to say that, if the structure of the underlying model is too ad hoc and messy, the chances of smooth interactivity and enlightening visualization should be slim indeed.

Even more imperative than the interactivity requirement is the need for CA-based models that are more *realistic.* No models based on toy values and the homogeneity, uniformity, universality, etc, assumptions of classic CA can have a claim to the status of exploratory tools for real-world applications. There are thus at least two dimensions to model realism that need to be addressed here: realism with respect to *data,* and realism with respect to model *structure.*

Here again, the several recent efforts to integrate CA-based models with GIS have gone a long way towards meeting the data realism requirement. CA-based models have the remarkable property of producing dynamic large-scale patterns while maintaining the high resolution of location-specific information (White and Engelen, 1992). Along with their more traditional cousins, the partial difference equations, CA are capable of exploiting fine-grained spatial information very efficiently. 'Draping' a CA model over a realistic terrain representation and using actual vegetation or land-use data is no longer a novelty (Batty and Xie, 1994a; Clarke et al, 1994; White and Engelen, 1992). The fact that this can now be done relatively easily in the context of a GIS is a major factor in the current revival of interest in CA as models (rather than metaphors) of actual geographic processes.

Along with the integration with real data came the technical possibility to relax any or all of the assumptions of standard CA that do not fit our experience of cities and regions. Space no longer needs to be homogeneous either in its properties or in its structure; neighborhoods need not be uniform across the space, and transition functions need not be universal (that is, equally applicable at every point); the system need not be closed to outside influences, and so on (figure 1). Conceptually such extensions are straightforward and mathematically they are also quite easy to express in the language of formal model theory (Couclelis, 1985). Distance-decay effects can be built into CA neighborhoods, the transition rules can be probabilistic rather than deterministic, and variable time steps can be used to fit some external schedule (for example, the seasonal variations in the rate of vegetation growth). There is practically no defining characteristic of standard CA that researchers, in their efforts to make CA-based models more realistic, have not been able to discard.

These developments have been mostly in the right direction but problems still remain, both theoretical and practical. Endless ad hoc tinkering with the original CA framework could yield model structures almost as complicated and inscrutable as the reality they purport to represent and that are as difficult to understand and interpret in a meaningful fashion. We would be back to the megasimulations of the 1960s and 1970s, and all the subsequent problems and criticisms that all but killed that particular line of research (Lee, 1973). The lesson learned is that, once complexity degenerates into complication, the game is lost.

Another problem with the CA generalization trend is theoretical. Much of the beauty and suggestive power of standard CA models lies in the contrast between the extreme simplicity of their structure and the unexpected complexity of their behavior; and most analytic results on CA reported in the literature pertain only to the simplest standard formats (Wolfram, 1986). Although it is unlikely that such formal results will ever find direct application in urban and regional modeling, they attest to the existence of a consistent mathematical and computational framework with well-defined properties. It is questionable how much of that formal integrity still remains once one starts dismantling every single CA premise. In other words, what, if anything, is left to

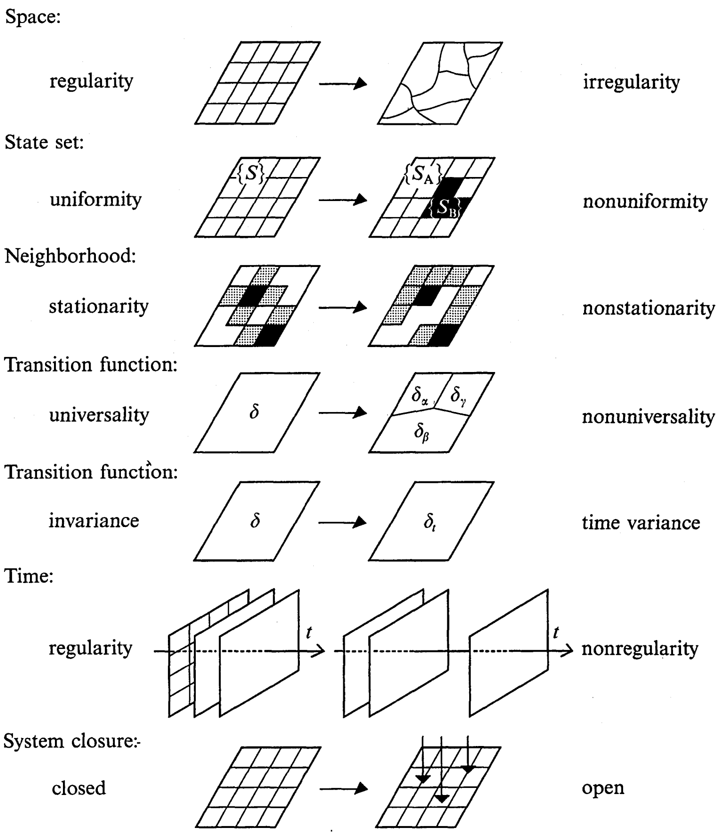


Figure 1. Common generalizations of cellular automata (CA).

hold these generalized CA models together once every distinguishing characteristic of the original framework has been stripped away.

In the remainder of the paper these two issues in CA generalization will be addressed: first, the question of the theoretical foundation of the new generation of more realistic CA-based urban and regional models; second, the more technical problem of preventing urban system complexity from turning into unwieldy model complication. In the next section an answer will be proposed to the first point through a discussion of the notion of proximal space, and in the following section a new theoretically grounded methodology, called geo-algebra, that should help keep these extended CA models theoretically transparent, internally consistent, and easily implementable in GIS, will be outlined.

Proximal space: a foundation for generalized cellular automata

Underlying every spatial representation is an implicit model of space. We do not normally need to worry about this until we reach some conceptual dilemma of the kind presented by the generalization of CA as discussed above. I will argue that the strongest theoretical justification for using CA models in urban and regional modeling, in the original or an expanded form, is their embodiment of an extremely important model of space which I will call *proximal* (Couclelis, 1991). Proximal space is the smile of the Cheshire cat, the intangible feature which, as in Alice's Wonderland, still lingers after all characteristic aspects of the CA framework have disappeared.

In this section I will discuss proximal space in connection with two other, more familiar, notions: absolute space and relative space.

Discussions of absolute space versus relative space have been numerous in geography, going back at least to the 1960s (see Harvey, 1969). *Absolute,* or Cartesian, or Newtonian, space is a neutral container of things and events. In Newton's view, as interpreted by Nagel (1961, page 207),

> "Absolute space is thus nonsensible and is not a material object or relation between such objects. It is an amorphous receptacle within which all physical processes occur and to which physical motions must be referred".

The most characteristic expression of absolute space is the Cartesian coordinate space, an a priori frame of reference for locating points, trajectories, and objects. *Relative,* or Leibnitzian, space, on the other hand, is space as constituted through the spatial relations arising among things and events and has no meaning apart from these relations. Thus, although the primitive notion of absolute space is the (referenced) point or location, the primitive of relative space is the (spatial) relation (figure 2).

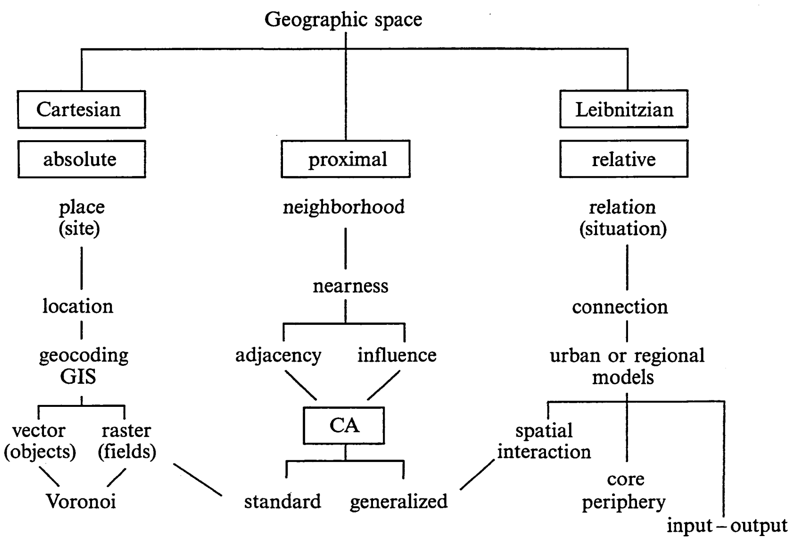


Figure 2. A map of proximal space. (Note: CA, cellular automata; GIS, geographic information systems.)

The distinction between absolute and relative space is important for urban and regional modeling because it corresponds to the basic geographic distinction between the properties of *site* (a place in and of itself) and *situation* (the position of a place relative to other relevant places). The major conceptual contribution of the quantitative revolution in geography has been to move the discipline away from an almost exclusive preoccupation with site (the 'what's where on the globe' of old-fashioned regional geography) to the study of situation as reflected in the relative distances and resulting interactions and flows among places. Virtually all common classes of mathematical urban or regional models (the gravity and spatial interaction family, the location – allocation models, the regional econometric models, the core – periphery models) are models of relative space. This trend away from site and towards situation went so far as to prompt accusations that quantitative geography was studying 'the geography of nowhere' (Johnston, 1985). In reaction to this, the movement for a 'new regional geography' (that is, place-oriented, though not oblivious to spatial relations) is currently gathering strength in the discipline.

The place of GIS in this controversy is peculiar because, although it is a technology borne out of the (mostly relative-space-oriented) quantitative approach to geography, it is a straightforward embodiment of the absolute view of space (Couclelis, 1991). Whether expressed in raster format as a field of measurements or in vector format as a collection of geometric objects, the essence of the space of GIS is the georeferenced location. There have been discussions of this issue in the GIS literature suggesting that the raster–vector dichotomy in GIS corresponds to the absolute space–relative space distinction (Peuquet, 1994). This we believe is incorrect. The ease with which vector and raster GIS can be interconverted or even merged together into something like a Voronoi diagram-based GIS, which combines advantages from both, testifies to the full compatibility of the underlying space models (Gold, 1992). This discrepancy in underlying models of space between GIS, on the one hand, and traditional urban and regional models, on the other, is in our view the main reason why their integration has proved so difficult. This is also where the notion of proximal space, and its embodiment in CA-based urban and regional models, can help fill a critical conceptual and technical gap.

Proximal space is the bridge between absolute and relative space. The key notion in absolute space is the georeferenced item, and in relative space it is the spatial relation, but the key notion in proximal space is the *neighborhood*. The neighborhood, along with the associated notion of nearness, is the most fundamental concept in topology but the intuitive interpretation of the term (which is well compatible with its formal meaning) will suffice for now. A neighborhood surrounds a localized item or place but it also embodies the notion of proximity to that place, which is a relation. There are two meanings of proximity that are relevant here: basic spatial proximity, or the property of being 'next to'; and functional proximity, or influence. This second meaning is less general but it is also obvious: given that, as we know from classical physics, there can be no effect over distance, something at a given location can be influenced only by what is, at least functionally, 'next to' it.

The notion of neighborhood is central to the CA paradigm and appears to be the only defining characteristic of the CA framework that has survived all attempts at generalization. Keep the standard CA specification intact but take the neighborhood condition out and nothing remotely recognizable as a cellular automaton remains. Standard CA neighborhoods are based on the spatial adjacency meaning of the term but, from an early on in CA history, extensions to second-order and nth-order neighbors (that is, to cells not physically adjacent to the reference cell) have paved the way towards extension to the influence-based sense. In general, influence-based neighborhoods need not be spatially contiguous and there is no more of a theoretical problem with this generalization of the CA framework than with any other kind of generalization already undertaken (Couclelis, 1985; 1989).

The traditional compact neighborhoods of standard CA build a bridge with absolute-space structures such as the Voronoi diagrams supported by GIS (for an excellent discussion of the notion of neighbor in this context, see Gold, 1992) whereas neighborhoods based on spatial influence are the natural link with relative space and its relation-based urban and regional models. Thus, the neighborhood of each location of interest can be the configuration of all other locations it is influenced by. This is a straightforward extension of the original notion of CA neighborhood as the spatially localized set of influencer cells but it reaches right into the area of geographic potential and spatial interaction models. The question now is whether this conceptually simple extension can be realized in a way that is both formally consistent and practically useful for urban and regional modeling in the age of GIS.

Geo-algebra: an algebra for proximal space

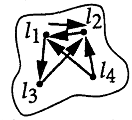
In this section the basic ideas behind geo-algebra, a formalism for describing proximal space developed by Takeyama (Takeyama and Couclelis, 1997), are presented. Though the term may not be familiar, the notion of proximal space is of course nothing new in mathematics. Differential calculus is probably the most thorough treatment of its properties, fractal geometry is a surprising new perspective on it, and the formalism of CA (just like the closely related partial difference equations) is a particular expression of discretized proximal space. This is to be contrasted with analytic geometry, which describes absolute space, and general geometry, combinatorial topology, and matrix algebra, which deal directly with relative-space relations. The fact that all mathematics is ultimately equivalent reinforces our intuition that there is but one space in our experience, whereas the astounding variety of existing mathematical structures underlines the benefits of exploring space from different formal viewpoints. Significant for our discussion of proximal space is the fact that the associated notions of neighborhood and continuity are considered the foundation of topology (and by extension arguably of all of mathematics) and also that the calculi best capable of expressing dynamics are most likely to be found in that realm.

Geo-algebra is designed to express proximal space and its properties at a level that, on the one hand, builds on the spatial data manipulation capabilities of GIS and, on the other, supports the mathematical modeling of spatial relations characteristic of the relative-space perspective. It thus leads to a logically consistent integration of urban and regional modeling with GIS databases and operations, something that has proved unexpectedly difficult to achieve by other means. Geo-algebra is described in detail in Takeyama and Couclelis (1997). Here I shall only give a brief outline of its main concepts.

Geo-algebra is an extension and generalization of map algebra, as developed by Tomlin (1990), and also incorporates concepts from image algebra (Ritter et al, 1990). Map algebra, built on the data structures of raster GIS, makes use of (raster) GIS map layers as operands on which to perform a variety of map analysis operations. The approach is also known as cartographic modeling because of its emphasis on processing information presented in cartographic form. It cannot deal easily with situational information pertaining to phenomena not naturally describable in absolute space, such as interaction flows and true dynamics (as opposed to kinematics).

Geo-algebra streamlines and generalizes map algebra. The key idea is the expansion of the set of operands of the algebra to include, in addition to maps, two other classes of constructs called *relational maps* and *metarelational maps*. Relational maps embody the idea of the influence-based neighborhood specification discussed above. Each location *(l)* in the space has a relational map associated with it which represents the configuration of all other locations that relate to or influence the reference location. A metarelational map is then the set of all relational maps corresponding to each of the relevant locations. Thus figures 3(d) and 3(e) show the relational map(s) and metarelational map corresponding to the interactions shown on the map [figure 3(a)] and the connectivity matrix [figure 3(b)].

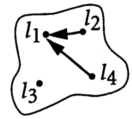
These three classes of operands of geo-algebra (maps, relational maps, and metarelational maps) are manipulated through a range of well-defined operations, distinguished into *local* and *nonlocal*. Local operations modify values at a location independently of the values at any other location, whereas nonlocal operations involve functions of the values and relative positions of other locations influencing the reference location. In geo-algebra, any nonlocal operation is computed in the following three steps. First, for each location of a map, all locations influencing that location are defined. Second, the values at these influencing locations are computed.



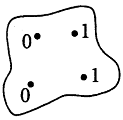
(a) Map showing interactions

	l_1	l_2	l_3	l_4
l_1	0	1	1	0
l_2	1	0	0	0
l_3	0	1	0	0
l_4	1	1	0	0

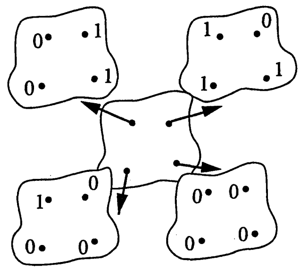
(b) Connectivity matrix for map (a)



(c) Influences on location l_1

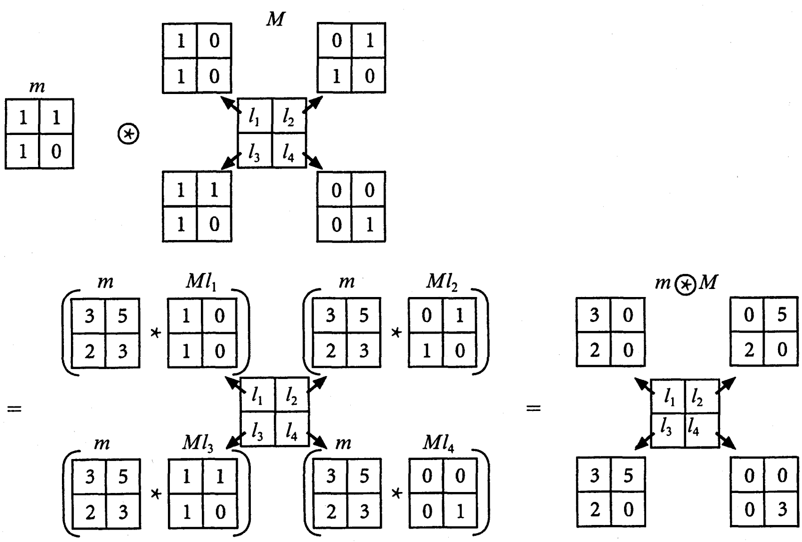


(d) Relational map for location l_1

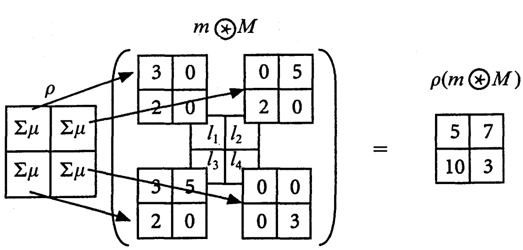


(e) Metarelational map for map (a)

Figure 3. Map, relational map, and metarelational map.



(a) Multiplication between a map and a relational map



$$\Sigma\mu \; \begin{array}{|c|c|}\hline 3 & 0 \\ \hline 2 & 0 \\ \hline \end{array} = 3+0+2+0 = 5$$

(b) Application of a global influence function

Figure 4. Nonlocal operations in geo-algebra.

Third, a new value for each location is computed as a function of the values at the influencing locations. In other words, the computation of a nonlocal operation is decomposed into finding (a) which are the interacting locations (represented by a metarelational map), (b) how much there is to interact (computed as the multiplication between a map and a metarelational map), and (c) what is the result of the interaction (computed through the application of a 'global influence function'). Figure 4 shows graphically the procedure for deriving a new map out of an initial map and a given set of interactions among locations, as modified by a simple global interaction function whereby each relational map is collapsed into the sum of its values.

Thus the key notion in the formulation of nonlocal operations is that of the relational map. The application of the neighborhood transition function in CA is a specific form of nonlocal operation in geo-algebra, involving a particularly simple kind of relational map—the neighborhood template. In Takeyama and Couclelis (1997) we provide a proof that every cellular automaton model can be reformulated in the language of geo-algebra. We believe this to be true of any reasonable generalization of CA models, including the generalization to noncontiguous (influence-based) neighborhoods. Specifically, we discuss extensions to nonuniform time intervals and open system dynamics (evolution with external input). Other work along these lines demonstrates the applicability of the formalism to the reformulation of conventional gravity-based urban and regional models. It thus appears that an algebra of proximal space is now available. Although this work is still in progress, we believe that any geographic model can in principle be formulated in similar terms. To what extent it would also be computationally efficient to do so is not yet clear at this point.

Conclusions

Our work with geo-algebra has shown that the theoretical bridge between absolute and relative space provided by proximal space can be supported technically by a consistent formal language with several properties of interest to urban and regional modelers. The possibility for seamless integration of traditional interaction-based models, generalized CA models, and GIS should contribute greatly to progress towards the interactivity and the realism requirements as discussed earlier. As a lingua franca across diverse modeling approaches, the language of geo-algebra can facilitate model comparison, extension, combination and modification across a wide range of relevant model structures, while keeping these experiments implemented with available GIS databases and operations.

Particularly attractive at the moment are the theoretical connections forged by geo-algebra. On the one hand we have absolute space and the concrete georeferenced location or object with attributes, which knows nothing of its surroundings; on the other we have relative space and the rarefied complex spatial relation which is so innocent of the specifics of place as to be dubbed 'the geography of nowhere'. In between lie proximal space and the generalized CA models, which can now partake systematically of the earthy data-richness of GIS while probing hypotheses about the large-scale effects of microscale interactions. The fact that these models can now also be extended into the realm of relative space, where the most robust theories of urban and regional development are still to be found, is a particularly exciting prospect. The burden is on us to show that this theoretical prospect is also practically realizable in the form of an efficient transparent GIS implementation. This would be one of several ways of making sure that CA models in the hands of urban and regional modelers are much more than a game.

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References

Batty M, Xie Y, 1994a, "Urban analysis in a GIS environment: population density modeling using ARC/INFO", in *GIS and Spatial Analysis* Eds S Fotheringham, P Rogerson (Taylor and Francis, London) pp 189–219

Batty M, Xie Y, 1994b, "Modeling inside GIS. Part I: model structures, exploratory spatial data analysis, and aggregation" *International Journal of Geographical Information Systems* **8** 291–307

Batty M, Xie Y, 1997, "Modeling inside GIS. Part II: selecting and calibrating urban models using ARC/INFO" *International Journal of Geographical Information Systems* forthcoming

Clarke K C, Brass J A, Riggan P J, 1994, "A cellular automaton model of wildfire propagation and extinction" *Photogrammetric Engineering and Remote Sensing* **60** 1355–1367

Couclelis H, 1985, "Cellular worlds: a framework for modeling micro–macro dynamics" *Environment and Planning A* **16** 141–154

Couclelis H, 1989, "Macrostructure and microbehavior in a metropolitan area" *Environment and Planning B: Planning and Design* **16** 141–154

Couclelis H, 1991, "Requirements for planning relevant GIS: a spatial perspective" *Papers in Regional Science* **70**(1) 9–19

Couclelis H, Monmonier M, 1995, "Using SUSS to resolve NIMBY: How spatial understanding support systems can help with the 'Not in my back yard' syndrome" *Geographical Systems* **2** 83–101

Eigen M, Winkler R, 1981 *Laws of the Game: How the Principles of Nature Govern Chance* (Alfred A Knopf, New York)

Gold C M, 1992, "The meaning of 'neighbor', in Theories and Methods of Spatiotemporal Reasoning in Geographic Space", in *Lecture Notes in Computer Science* **639** Eds A U Frank, I Campari, U Formentini (Springer, Berlin) pp 220–235

Harvey D, 1969 *Explanation in Geography* (Edward Arnold, Sevenoaks, Kent)

Johnston R J, 1985, "To the ends of the Earth", in *The Future of Geography* Ed. R J Johnston (Methuen, London) pp 26–338

Lee D B, 1973, "Requiem for large scale models" *Journal of the American Institute of Planners* **31** 158–166

Mark D, Frank A (Eds), 1992, "User interfaces for geographic information systems: report on the initiative 13 specialist meeting", NCGIA technical report 92-3, University of California, Santa Barbara, CA

Nagel E, 1961 *The Structure of Science* (Routledge and Kegan Paul, London)

Peuquet D J, 1994, "It's about time: a conceptual framework for the representation of temporal dynamics in geographic information systems" *Annals of the Association of American Geographers* **84** 441–461

Ritter G X, Wilson J, Davidson J, 1990, "Image algebra: an overview" *Computer Vision, Graphics, and Image Processing* **49** 297–331

Takeyama M, 1996 *Geo-Algebra: A Mathematical Approach to Integrating Spatial Modeling and GIS* PhD thesis, Department of Geography, University of California, Santa Barbara, CA

Takeyama M, Couclelis H, 1997, "Map dynamics: integrating cellular automata and GIS through geo-algebra" *International Journal of Geographical Information Systems* forthcoming

Tobler W, 1979, "Cellular geography", in *Philosophy in Geography* Eds S Gale, G Olsson (D Reidel, Dordrecht) pp 379–386

Tomlin D C, 1990 *Geographic Information Systems and Cartographic Modeling* (Prentice-Hall, Englewood Cliffs, NJ)

White R, Engelen G, 1992, "Cellular automata and fractal urban form: a cellular modeling approach to the evolution of urban land use patterns", technical report 92064, Research Institute for Knowledge Systems, Maastricht, The Netherlands

Wittgenstein L, 1953 *Philosophical Investigations* (Macmillan, New York)

Wolfram S (Ed.), 1986 *Theory and Applications of Cellular Automata* (World Scientific, Singapore)

Xie Y, 1994 *Analytical Models and Algorithms for Cellular Urban Dynamics* PhD thesis, Department of Geography, University of Buffalo, New York