You're your own best teacher: A Self-Supervised Learning Approach For Expressive Representations

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TODO: relate our work to TimeVQVAE, Neural Representation, Barlow, MaskGIT,

TODO: Include something on time series generation / representation learning

0.1 MaskGIT

The Masked Generative Image Transformer (MaskGIT)[3] is a generative transformer model for image synthesis developed by Google Research. The novelty of the model lies in the token generation. Unlike popular autoregressive generative transformers, who treat images as a sequence of tokens, MaskGIT introduces an image synthesis paradigm using a bi-directional transformer decoder. This means that during training MaskGIT learns to predict tokens in all directions, an intuitively more natural way to consider images. At inference time MaskGIT starts out with a blank canvas and predicts the entire image, and iteratively keeps and conditions on the most confident pixels.

MaskGIT assumes a tokenization procedure for stage 1. In the original paper [3] VQGAN [2] was used and the actual contribution of the work revolved around improving stage 2, hence we present that part only.

0.1.1 Masked Visual Token Modeling (Prior learning)

For some image X in the dataset $\mathcal{D}$, let $Y = \{y_i\}_{i=1}^N$ denote the latent tokens obtained by passing X through the VQ-Encoder and denote the corresponding binary mask by $M = \{m_i\}_{i=1}^N$. During training a subset of Y is replaced by a special masking token we denote by $\texttt{M}$ according to the binary mask M. This is done by

$$Y_{\text{Mask}} = Y \odot (1_N - M) + M \cdot \texttt{M}, \tag{1}$$

where $\odot$ is the Hadamard product, i.e point wise multiplication, and 1_N is a vector with the same shape as M and Y.

The sampling procedure, or choice number of tokens to mask, is parameterized by a mask scheduling function γ. The sampling can be summarized as follows

- Sample $r \sim U(0, 1]$.
- Sample $\lceil \gamma(r) \cdot N \rceil$ indices I uniformly from $\{0, \ldots, N-1\}$ without replacement.
- Create M by setting $m_i = 1$ if $i \in I$, and $m_i = 0$ otherwise.

The training objective is to minimize the negative log likelihood of the masked tokes, conditional on the unmasked

$$\mathcal{L}_{\text{Mask}} = -\mathbb{E}_{Y \in \mathcal{D}} \left[\sum_{i \in I} p(y_i | Y_{\text{Mask}}) \right] \tag{2}$$

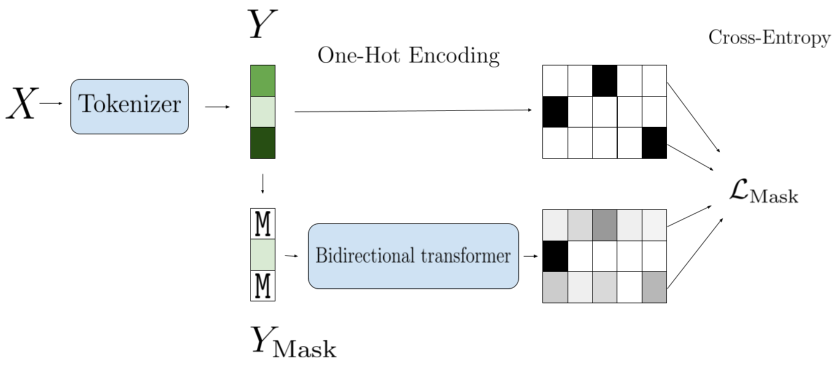

Figure 1: MaskGIT forward computation.

A bidirectional transformer is used to predict the probabilities $p(y_i|Y_{\text{Mask}})$ of each masked token, and $\mathcal{L}_{\text{Mask}}$ is computed as the cross entropy between the ground truth one-hot token and the predicted token.

0.1.2 Iterative decoding (Image generation)

The bi-directional transformer could in principle predict all `M` tokens and generate a sample in a single pass by simply sampling from the predicted probabilites $p(y_i|Y_{\text{Mask}})$ from a forward pass of an all masked sequence. However, there are challenges with this approach. In their original article [3] proposes a novel non-autoregressive decoding method to sythesize samples in a constant number of steps.

The decoding process goes from $t = 0$ to T. To generate a sample at inference time one starts out with a all masked sequence which we denote by $Y^{(0)}_{\text{Mask}}$. At iteration t the model predicts the probabilities for all the mask tokens, $p(y_i|Y^{(t)}_{\text{Mask}})$, in parallel. At each masked index i a token $y_i^{(t)}$ is sampled according to the predicted distribution, and the corresponding probability $c_i^{(t)}$ is used as a measure of the confidence in the sample. For the unmasked tokens a confidence of 1 is assigned to the true position. The number of $y_i^{(0)}$ with highest confidence kept for the next iteration is determined by the mask scheduling function. We mask $n = \lceil \gamma(t/T) \cdot N \rceil$ of the lower confidence tokens by calculating $M^{(t+1)}$ by

$$m_i^{(t+1)} = \begin{cases} 1, \text{ if } c_i < \text{Sort}([c_1^{(t)}, \ldots, c_N^{(t)}])[n] \\ 0, \text{ otherwise} \end{cases} \tag{3}$$

The algorithm synthesizes a full image in T steps.

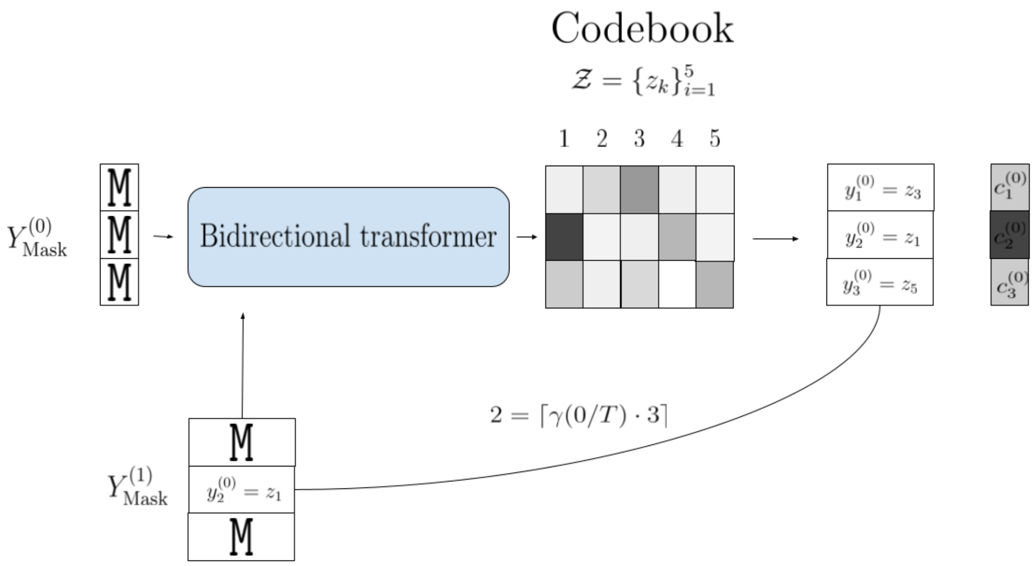

Figure 2: Illustration of first pass of the iterative decoding algorithm.

Implementation details

0.2 TimeVQVAE

TimeVQVAE is a time series generation model based on VQVAE and MaskGIT. It is the first to our and the authors knowledge that utilizes vector quantization (VQ) to address the TSG problem. It leverages a two stage approach similar to VQVAE and uses a bidirectional transformer akin to MaskGIT for prior learning. Additionally, they propose VQ modeling in time-frequency domain, separating data into high and low frequency components to better retain temporal consistencies and generate higher quality samples.

TimeVQVAE provides class-guided conditional sampling.

0.2.1 Tokenization

0.2.2 Prior learning

TimeVQVAE uses a modified version of MaskGIT in order to learn the prior. As the original MaskGIT has no means of jointly sample from the two modalities introduced by the high-low frequency split.

For some X in the dataset $\mathcal{D}$, let $Y = \{y_i\}_{i=1}^N$ denote the latent tokens obtained by passing X through the VQ-Encoder and denote the corresponding binary mask

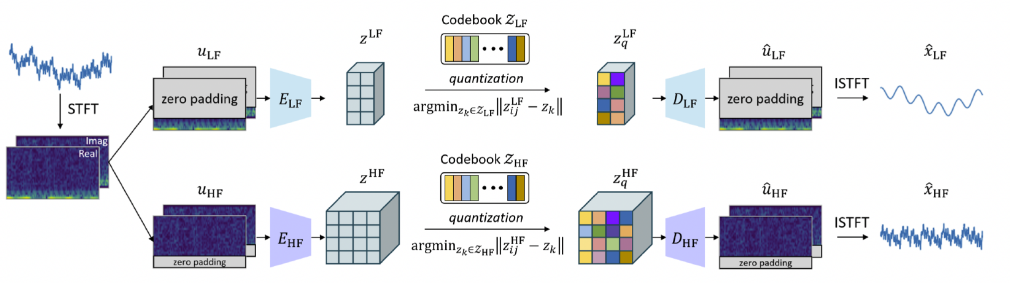

Figure 3: Stage 1: Tokenization. Figure taken with permission from [1]

by $M = \{m_i\}_{i=1}^N$.

A discrete latent representation $z_q \in \mathbb{R}^{d \times h \times w}$ can equivalently be represented as a sequence of codebook indices by

$$s_{ij} = k, \text{ whenever } (z_q)_{ij} = z_k \tag{4}$$

Class Conditional Sampling

0.3 SSL

Our model leverages SSL algorithms in order to learn more expressive latent representations. Here we present the relevant algorithms for our work.

0.3.1 Barlow Twins

Barlow Twins is a non-contrastive SSL method based on applying the *redundancy-reduction principle* (or efficient coding hypothesis) [5] from the neuroscientist H. Barlow to a pair of identical networks.

In essence the model wants to encourage representations of similar samples to be similar, while simultaneously reducing the amount of redundancy between the components of the vectors. This is enforced by producing two augmented views of each sample and projecting the their representations onto a vast feature space, in such a way that their cross-correlation is close to the identity.

The Barlow Twins algorithm starts out by creating two different augmented views for each datapoint in a batch D. The augmentations are selected by sampling from a collection of augmentations $\mathcal{T}$. We denote the batches of augmented views $T(D) = X$ and $T'(D) = X'$, for augmentations $T, T' \sim \mathcal{T}$. The batches are then passed through an encoder (give representations Y and Y') and a *projector* to produce batches of embeddings Z and Z'. The embeddings are assumed to be

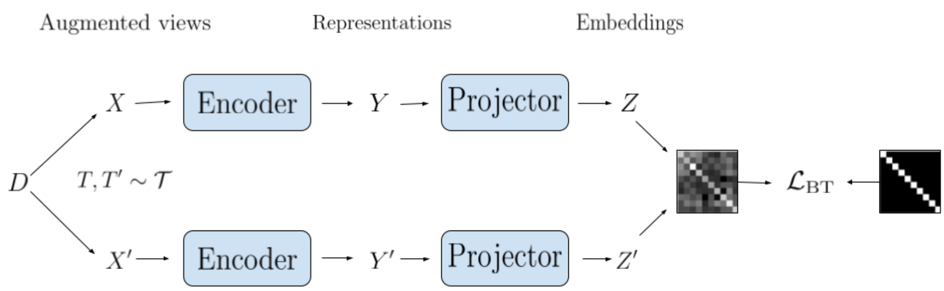

Figure 4: Overview of the Barlow Twins architecture. Figure inspired by [6]

mean centered across the batch dimension.

The loss function is calculated using the cross correlation matrix $\mathcal{C}$ between Z and Z', and measuring its deviance from the identity. In particular the Barlow Twins loss is defined as

$$\mathcal{L}_{\mathrm{BT}} = \overbrace{\sum_i (1 - \mathcal{C}_{ii})^2}^{\text{Invariance}} + \lambda \overbrace{\sum_i \sum_{j \neq i} \mathcal{C}_{ij}^2}^{\text{Redundancy reduction}} , \tag{5}$$

where

$$\mathcal{C}_{ij} = \frac{\sum_b z_{b,i} z'_{b,j}}{\sqrt{\sum_b (z_{b,i})^2} \sqrt{\sum_b (z'_{b,j})^2}} . \tag{6}$$

The *invariance term* assists in making the embedding invariant to the distortions introduced by the augmentations, hence pushes the representations closer together. The *redundancy reduction term* decorrelates the different vector components, which reduces the information redundancy.

0.3.2 VIbCReg

VIbCReg [7] is a non-contrastive SSL model with siamese architecture based on VICReg [8]. It can be seen as VICReg with better covariance regularization and IterNorm [9]. Overall the architecture is similar to Barlow Twins, but a key difference is that variance/covariance regularization is done in each branch individually.

As before a batch D is augmented to create two views and passed through an encoder and projector. The embedding Z and Z' are *whitened* using IterNorm [9].

define or elaborate on this

The loss consists of a similarity loss between the branches, and feature decoration (FD) loss together with a feature component expressiveness (FcE) term at each branch. Input data is processed in batches. Let $Z \in \mathbb{R}^{B \times F}$ where B and F

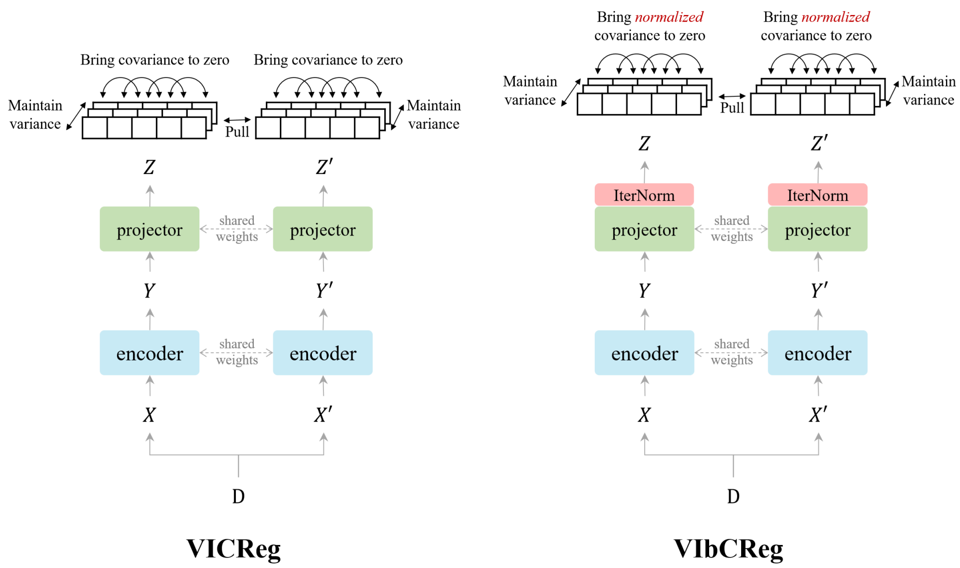

Figure 5: Overview of VIbCReg, and comparison with VICReg. Taken with permission from [**lee2024computer**]

denotes the batch and feature sizes respectively. We denote a row in Z by Z_b and column by Z_f, and similarly for Z'.

The similarity loss is defined as the MSE of the two embeddings

$$s(Z, Z') = \frac{1}{B} \sum_{b=1}^{B} ||Z_b - Z'_b||_2^2, \tag{7}$$

which encourages them to be similar.

The FcE/Variance term acts on each branch separately and encourages the variation across a batch to stay large. It is defined as

$$v(Z) = \frac{1}{F} \sum_{f=1}^{F} \max(0, \gamma - \sqrt{\mathrm{Var}(Z_f) + \epsilon}), \tag{8}$$

where Var() is a variance estimator, γ is a target value for the standard deviation, which both in VIbCReg and VICReg is set to 1. ϵ is a small scalar preventing numerical instabilities.

Finally the FD/covariance term pushes the normalized covariance
First we mean shift and normalize along the batch dimension

$$\widehat{Z}_b = \frac{Z_b - \bar{Z}}{||Z_b - \bar{Z}||_2} \text{ where } \bar{Z} = \frac{1}{B} \sum_{b=1}^{B} Z_b, \tag{9}$$

$$\widehat{Z} = [\widehat{Z}_1, \ldots, \widehat{Z}_B]^T, \tag{10}$$

compute the normalized covariance matrix

$$C(Z) = \frac{1}{B-1}\widehat{Z}^T\widehat{Z}, \tag{11}$$

and take the mean square across all off-diagonal elements to obtain the FD loss

$$c(Z) = \frac{1}{F^2}\sum_{i \neq j} C(Z)_{ij}^2. \tag{12}$$

The total loss is then given by

$$\mathcal{L}_{\text{VIbCReg}} = \lambda s(Z, Z') + \mu[v(Z) + v(Z')] + \nu[c(Z) + c(Z')] \tag{13}$$

where λ, μ and ν are hyperparameters determining the importance of each term. The normalization of the covariance matrix keeps the range of the FD loss small, independent of data, and eases hyperparameter tuning across datasets.