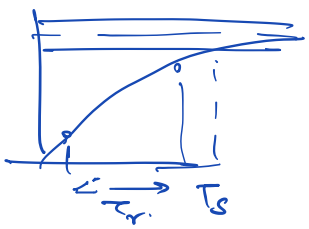
Last Time:

Dommanit pole approx.

- 1st order ←
- 2nd order



2nd order system

$$\frac{d^2c(t)}{dt^2} + a_1\frac{dc(t)}{dt} + a_0c(t) = b_0r(t)$$

↑ No Zeros.

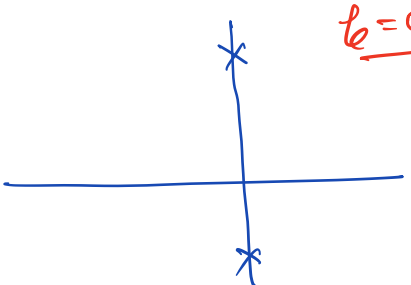
$$G(s) = \frac{b_0}{s^2+a_1s+a_0}$$

$$G(s) = K\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

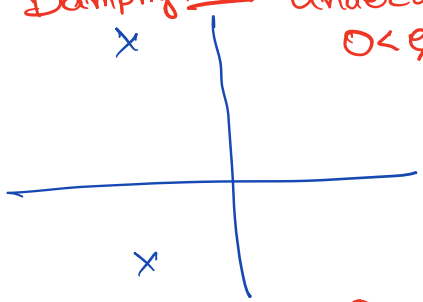
Natural freq

Damping ratio

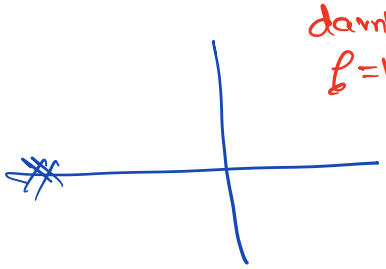
Undamped $\zeta = 0$



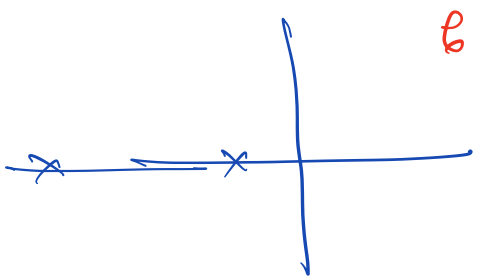
Underdamped. $0 < \zeta < 1$



Critically damped $\zeta = 1$



Overdamped. $\zeta > 1$



$s^2 + 2\zeta\omega_n s + \omega_n^2$

$\Delta = b^2 - 4ac$

$= 4\zeta^2\omega_n^2 - 4\omega_n^2$

$= 4\omega_n^2(\zeta^2 - 1)$.

$\zeta > 1$

$\zeta^2 - 1 > 0 \Rightarrow$ Real roots

$\zeta < 1$

$\zeta^2 - 1 < 0 \Rightarrow$ Complex conjugate roots

$\zeta = 1$

$\zeta^2 - 1 = 0 \Rightarrow$ Repeated roots

Stability $\Rightarrow \zeta \geq 0$

Example

a) $\dfrac{12}{s^2 + 8s + 12}$

ω_n, ζ.

Underdamped / - - -

$\omega_n = \sqrt{12}$

$\zeta = \dfrac{4}{\sqrt{12}} = \sqrt{\dfrac{16}{12}} > 1$

$\zeta \geq 1 \Rightarrow$ overdamped.

b) $\dfrac{16}{s^2 + 8s + 16}$

$\omega_n = 4$

$\zeta = 1$

$\Rightarrow$ Critically Damped.

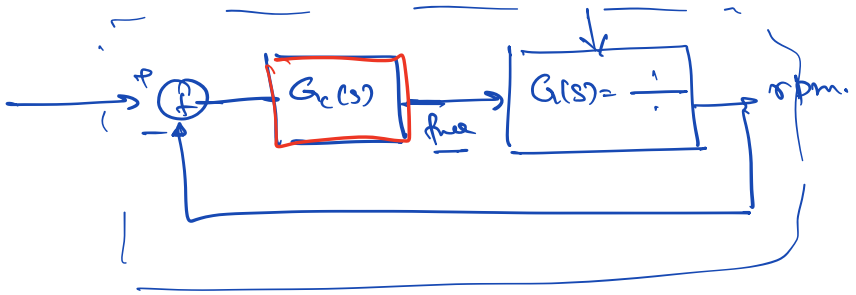
c) $\dfrac{20}{s^2+8s+20}$

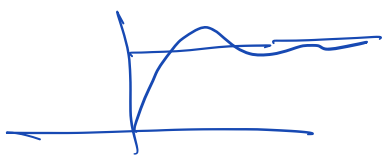
$\omega_n = \sqrt{20}$

$\zeta = \dfrac{4}{\sqrt{20}} < 1$

Underdamped.

d) $\dfrac{20}{s^2+20}$

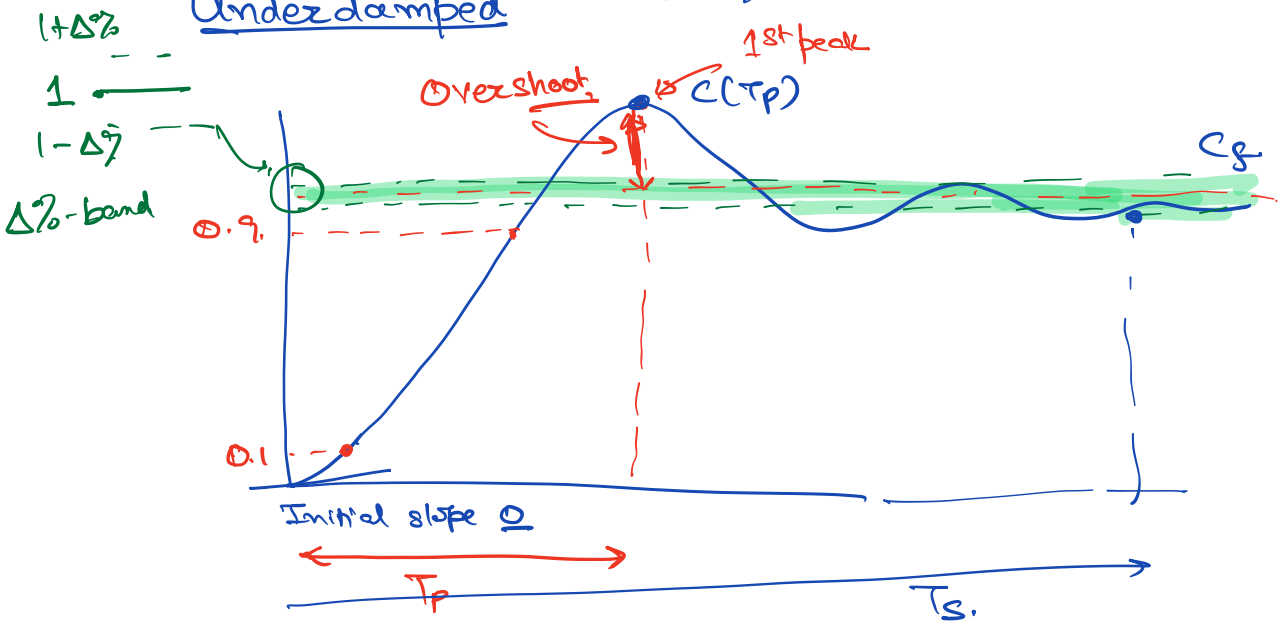




%OS ≤ —

T_s ≤ —

Underdamped $0 < \zeta < 1$



Peak Time: Time of 1st peak

% OS =

T_r Rise Time: 0.1 of final value to 0.9 of final value

Δ%- settling time.

$$\% OS = \frac{C(T_p) - C_f}{C_f} \times 100.$$

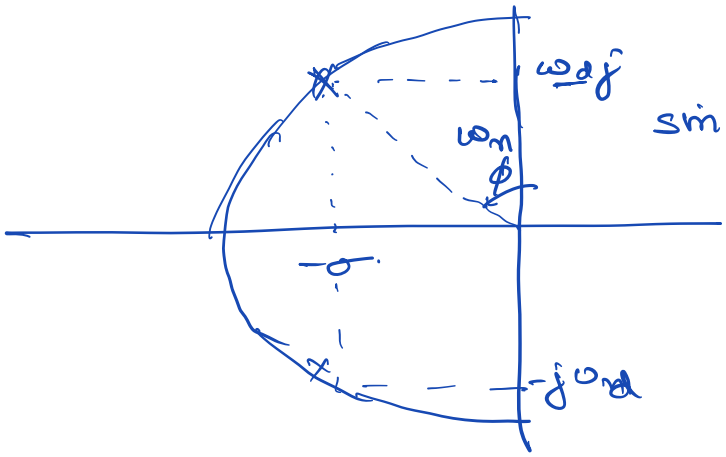
$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_d^2 + \sigma^2}{s^2 + 2\sigma s + \omega_d^2 + \sigma^2}$$

$$\sigma = \zeta\omega_n$$

$$\omega_n^2 = \sigma^2 + \omega_d^2$$

↳ Damped freq.

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$



$\sin\phi = \zeta$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$$

$$= e^{-\pi\sigma/\omega_d} \times 100$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

$$T_s(1\%) = \frac{4.6}{\sigma}$$

$$T_s(2\%) = \frac{4}{\sigma}$$

$$T_s(5\%) = \frac{3}{\sigma}$$

$\omega_n T_r$ = function (ζ)

$\omega_n T_r$ \ ζ	0.1	0.2	. . .	0.9
$\omega_n T_r$	1.104	1.203		

$\zeta = 0.15$

$T_r = \dfrac{\omega_n T_r}{\omega_n}$ ← Read from table