Last Time

PZ Plots — BIBO Stability
└→ Form of the step resp.

- Dominant Pole Approx

Example

a) $G_1(s) = \dfrac{1}{s+2}$

b) $G_2(s) = \dfrac{1}{s+2} \cdot \dfrac{20}{s+20}$

Last class, we saw ~~that~~ they have similar step response

$$C_1(s) = R(s)G_1(s) = \frac{1}{s(s+2)}$$

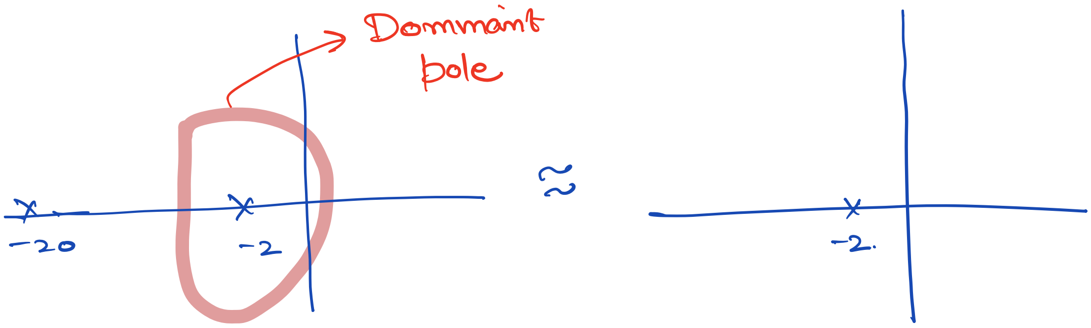
$$= \frac{0.5}{s} - \frac{0.5}{s+2}$$

$$C_2(s) = R(s)G_2(s) = \frac{20}{s(s+2)(s+20)}$$

$$= \frac{0.5}{s} - \frac{0.55}{s+2} + \boxed{\frac{0.055}{s+20}}$$

$$c_1(t) = 0.5 - 0.5e^{-2t}$$

$$c_2(t) = 0.5 - 0.55e^{-2t} + \boxed{0.055\,e^{-20t}}$$



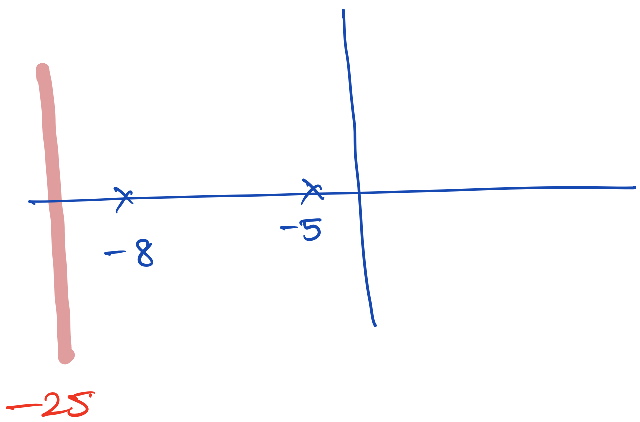
In the notes, it says 10

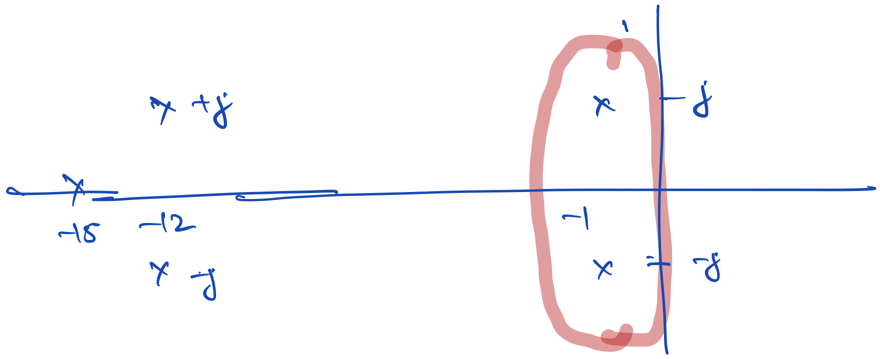
$p_1 = -\sigma_1 \pm j\omega_1$

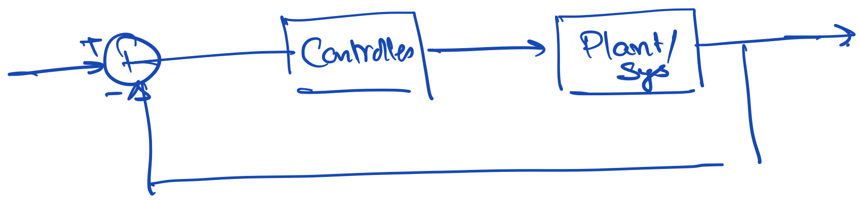
$p_2 = -\sigma_2 \pm j\omega_2$

p_1 dominates p_2 if $\sigma_2 \geq 5\sigma_1$

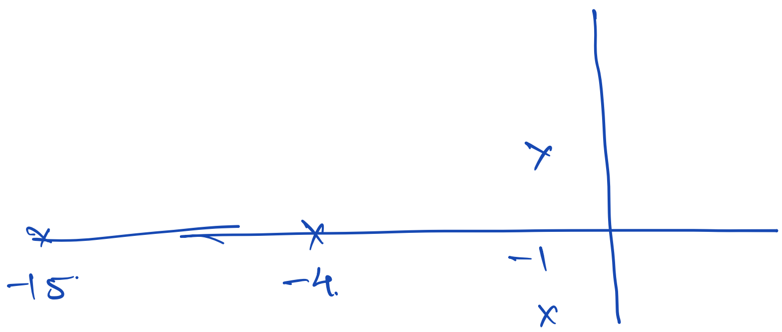
Is dominant pole approx. valid?

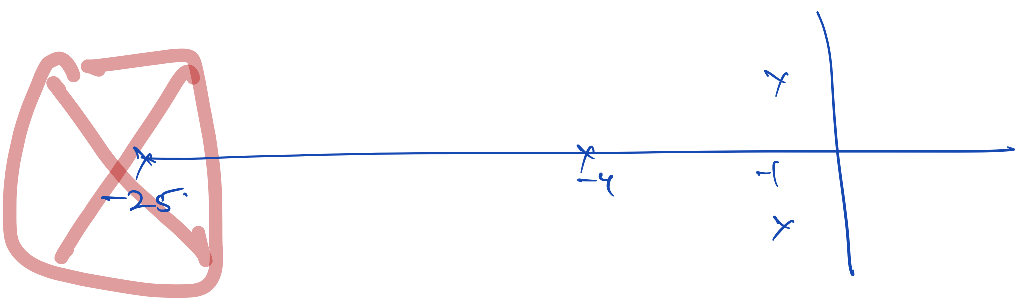


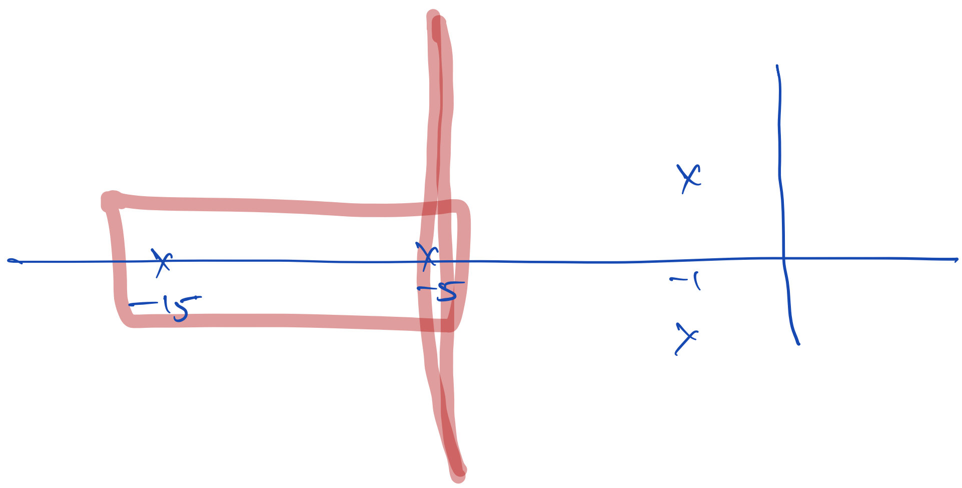




Spec. $\Longrightarrow$ Location of Dominant poles $\Longrightarrow$ Place dominant poles at desired loc' & others "far away"







Step response of 1st & 2nd order systems

1st ~~cor~~ order [Proper TF]

$$\frac{d\,c(t)}{dt} + a\,c(t) = \underline{b\,r(t)}$$

$$G(s) = \frac{b}{s+a} = \frac{b}{a}\,\frac{a}{s+a}$$

$$= K\,\frac{a}{s+a}$$

[Canonical form of 1st order sys]

$$G(s) = \frac{a}{s+a}$$

[Normalized TF)

Step response : → $1/s$

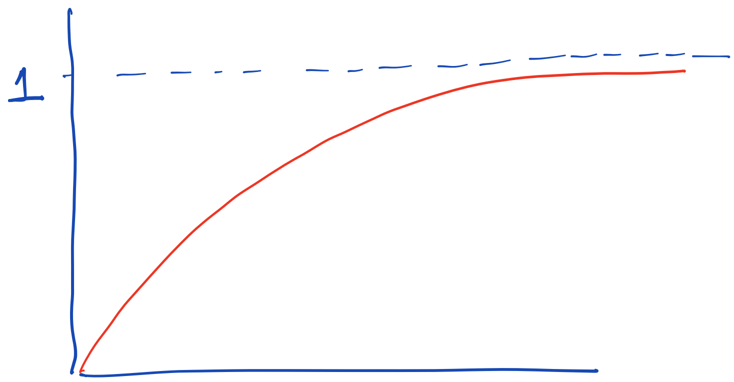
$$C(s) = R(s)\,G(s) = \frac{a}{s(s+a)}$$

$$= \frac{1}{s} - \frac{1}{s+a}$$

$$c(t) = 1 - e^{-at}$$

General,

$$c(t) = K\left[1 - e^{-at}\right]$$



Final value Thm

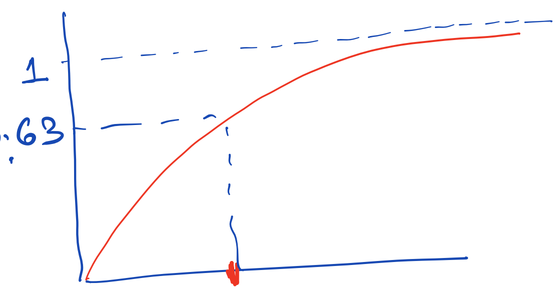
$$c(\infty) = \lim_{s \to 0} s\, C(s)$$

$$= \lim_{s \to 0} \cancel{s} \cdot \overset{R(s)}{\frac{1}{\cancel{s}}}\, G(s)$$

$$= G(0)$$

Time Const

$$\tau = \frac{1}{a}$$

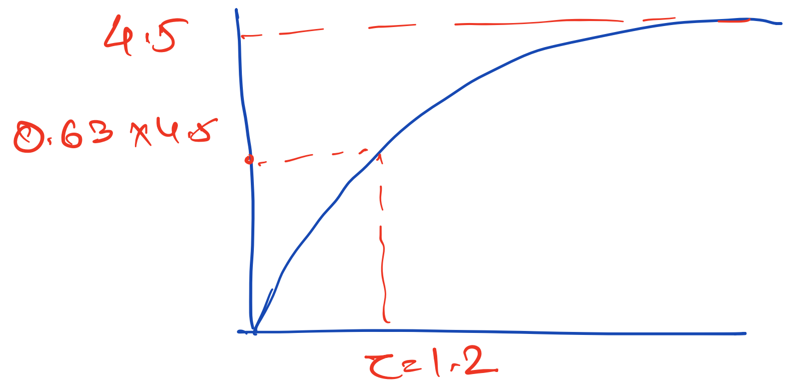


$c(\tau) = 0.63$

$1 - e^{-1}$

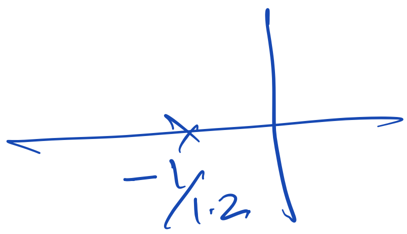
$\tau = 1/a$

SYSTEM ID.

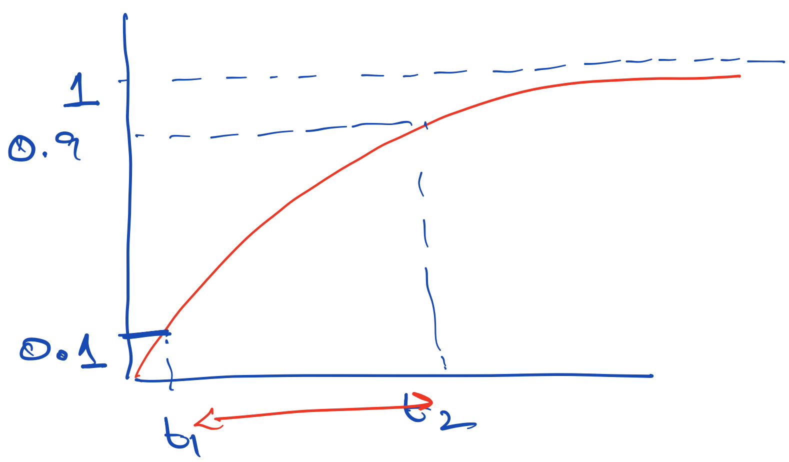


$$K \frac{a}{s+a}$$

$$\tau = 1/a \Rightarrow a = 1/\tau = \frac{1}{1.2}$$



Rise Time



$$T_r = t_2 - t_1$$

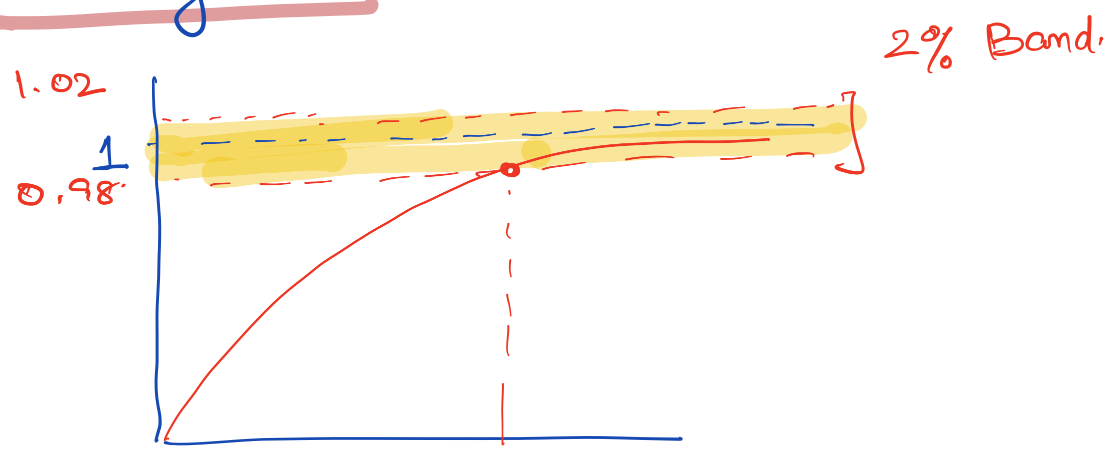
$$c(t) = 1 - e^{-at}$$

$$0.1 = 1 - e^{-at_1} \Rightarrow t_1 = 0.11/a$$

$$0.9 = 1 - e^{-at_2} \Rightarrow t_2 = \frac{2.31}{a}$$

$$\boxed{T_r = t_2 - t_1 = \frac{2.2}{a}}$$

Settling Time



$$0.98 = 1 - e^{-aT_s} \Rightarrow \boxed{T_s = \frac{4}{a}}$$

Example $\quad G(s) = \dfrac{100}{s+50} = K\dfrac{a}{s+a}$

$\tau = \dfrac{1}{a} = \dfrac{1}{50} = 0.02 \Rightarrow K=2, \; a=50$

$T_r = \dfrac{2.2}{a} = 2.2\tau = 0.044 \text{ s.}$

$T_s = \dfrac{4}{a} = 4\tau = 0.08 \text{ s}$