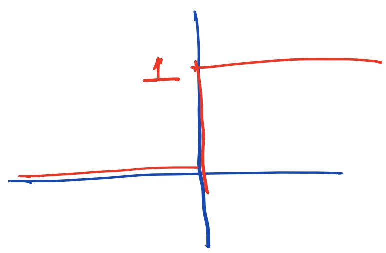
Announcements

- Labs → Installing Matlab.
- Communication
- Assignments.

Notational remark

Step fn $u(t)$



$u(t)$ for control input

$\mathbb{1}(t)$ for step fn.

Today's Lecture

- Laplace Transforms.
- Transfer fn of LTI systems
- Pole-zero plots of TFs.

Laplace Transform

1) Bilateral LT : $\int_{-\infty}^{\infty}$ ——

2) Unilateral LT $\int_{0^-}^{\infty}$ ——

→ No need to keep track of ROC.

$$f(t) \xleftrightarrow{\mathcal{L}} F(s)$$

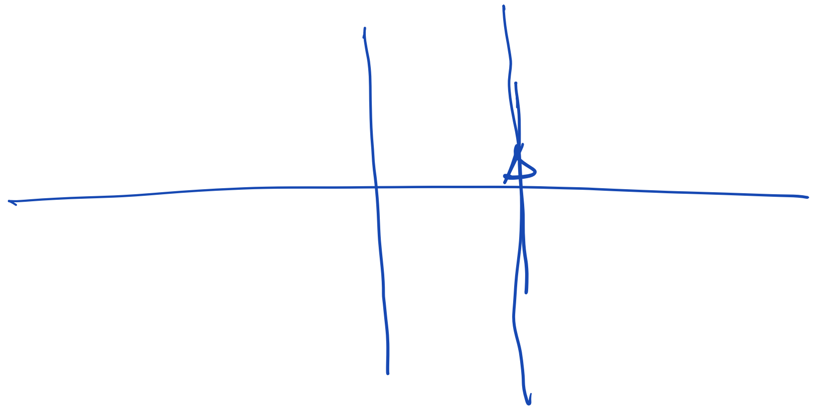
$$F(s) = \mathcal{L}(f(t))$$

$$f(t) = \mathcal{L}^{-1}(F(s)).$$

$$F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt.$$

$$f(t) = \frac{1}{2\pi j} \oint_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st}\, ds.$$

Sec 2.1 – 2.2 of textbook



TRANSFER FN (TF)

$$r(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow c(t)$$

Impulse response: $g(t)$

TF. $G(s) = \mathcal{L}(g(t))$

Constant Coefficient Linear diff. eqn. (LDE)

$$2\frac{d^2c(t)}{dt^2} + 4\frac{dc(t)}{dt} + 1c(t)$$

$$= \boxed{3}\frac{dr(t)}{dt} + \boxed{5}r(t)$$

Coeff of each term does not depend on time.

$$\frac{dc(t)}{dt} = t\,r(t)$$

Non-linear $\left(\frac{dc(t)}{dt}\right)^2$ $\quad \frac{d^2c(t)}{dt^2} \cdot \frac{dc(t)}{dt}$

$$G(s) = \frac{3s + 5}{2s^2 + 4s + 1}$$

$$2\frac{d^2c(t)}{dt^2} + 4\frac{dc(t)}{dt} + 1c(t)$$

$$= \boxed{3}\frac{dr(t)}{dt} + \boxed{5}r(t)$$

Take LT of both sides (assuming 0 initial cond).

$$c(t) \xleftrightarrow{\mathcal{L}} C(s)$$

$$\frac{d\,c(t)}{dt} \xleftrightarrow{\mathcal{L}} s\,C(s) - \cancel{c(0^-)}^{\,0}$$

$$\frac{d^2c(t)}{dt} \xleftrightarrow{\mathcal{L}} s^2 C(s)$$

$$\Rightarrow \quad 2s^2C(s) + 4sC(s) + C(s)$$

$$= 3sR(s) + 5R(s).$$

$$\Rightarrow \quad C(s)\left[2s^2+4s+1\right] = [3s+5]\,R(s).$$

$$\Rightarrow \quad C(s) = \underbrace{\left(\frac{3s+5}{2s^2+4s+1}\right)}_{G(s)} R(s)$$

$$G(s) = \frac{C(s)}{R(s)}$$

Ex $$\frac{d\,c(t)}{dt} + 2c(t) = r(t)$$

$$G(s) = \frac{1}{s+2}$$

Ex $$G(s) = \frac{s+1}{s^2+2s+3}.$$

Write the DE ?

$$1\,\frac{d^2c(t)}{dt^2} + 2\,\frac{dc(t)}{dt} + 3\,c(t)$$

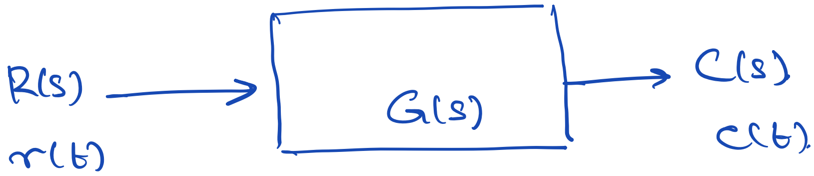
$$= 0\,\frac{d^2r(t)}{dt^2} + 1\,\frac{dr(t)}{dt} + 1\,r(t)$$

$$G(s) = \frac{N(s)}{D(s)} \begin{array}{l} \rightarrow \text{poly of } s \quad \deg(N) = m \\ \rightarrow \text{poly of } s \quad \deg(D) = n. \end{array}$$

Rational poly of s

Proper rational poly $\quad m < n$

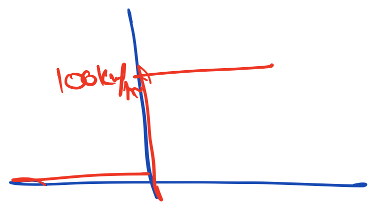
$$\boxed{m = n}$$



Focus on: $r(t) = \mathbb{1}(t)$

$$\updownarrow$$

$$\frac{1}{s}$$



$$\frac{d\,c(t)}{dt} + 2c(t) = r(t)$$

$$G(s) = \frac{1}{s+2}$$

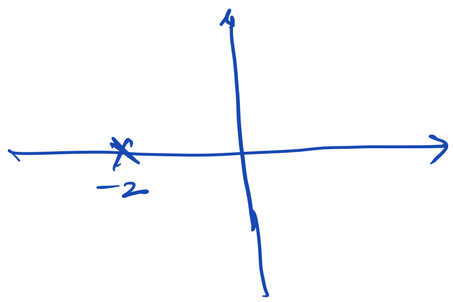
$$C(s) = G(s)\cdot R(s) = \frac{1}{s(s+2)}$$

$$= \left[\frac{1/2}{s} - \frac{1/2}{s+2}\right]$$

$$c(t) = \left[\frac{1}{2} - \frac{1}{2}e^{-2t}\right]\mathbb{1}(t)$$

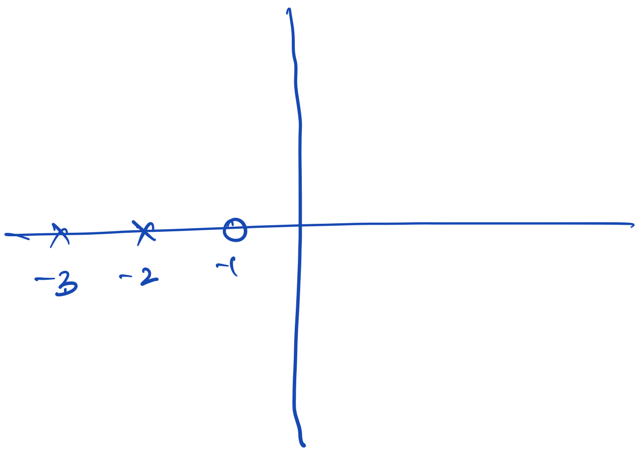
Pole-Zero Plot

$$G(s) = \frac{1}{s+2}$$



S-plane.

$$G(s) = \frac{(s+1)}{(s+2)(s+3)}$$



Repeated poles

— zeros

Start of → All coeffs are real.

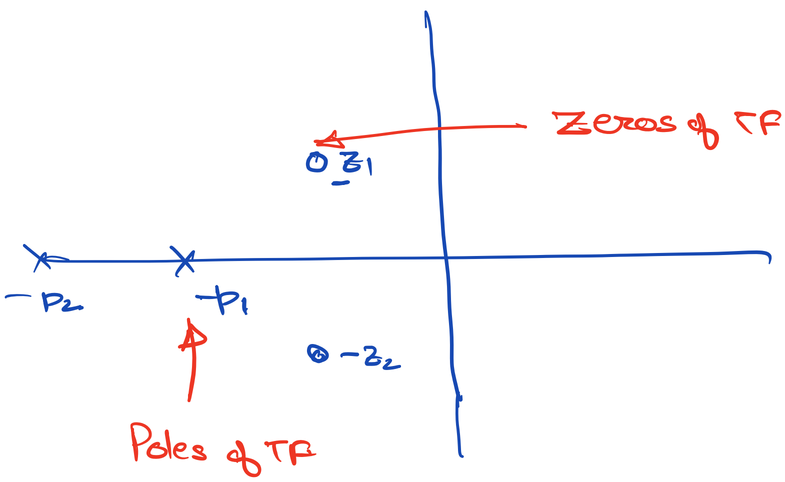
$$a_n \frac{d^n c(t)}{dt^n} + \dots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + \dots + b_0 r(t)$$

$$G(s) = \frac{b_m s^m + \dots + b_0}{a_n s^n + \dots + a_0}$$

All roots (of num/den) are either real or occur in complex conjugate pairs ⇐

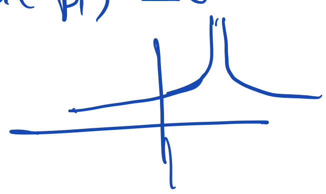
$$= \boxed{K} \frac{(s+z_1)(s+z_2)\dots(s+z_m)}{(s+p_1)\dots(s+p_n)}$$

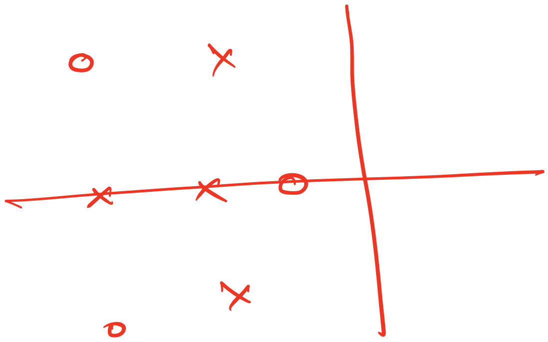
Gain of TF



$G(-z_1) = 0$

$G(-p_1) = \infty$





$x^2 + 4x + 4.$

$(x+2)^2$ Roots are real.

$x^2 + 4 = 0$

$(x + j2)(x - j2)$

$\mathrm{Im}\left((x+2)(x+j2)\right) \neq 0$

$\left((x+a) + jb\right)\left((x+a) - jb\right)$

$\left[(x+a)^2 + b^2\right]$

$x^2 = 2 + 2j$

$\pm\sqrt{2+2j}$