Last class

- LDE $\longleftrightarrow$ TF
- Step response of a TF $\quad [r(t) = \mathbb{1}(t)]$
- Pole-Zero Plots.

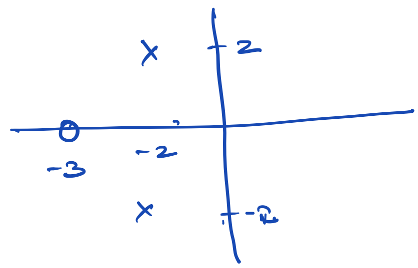
Today

Pole-Zero Plots (PZ Plots)

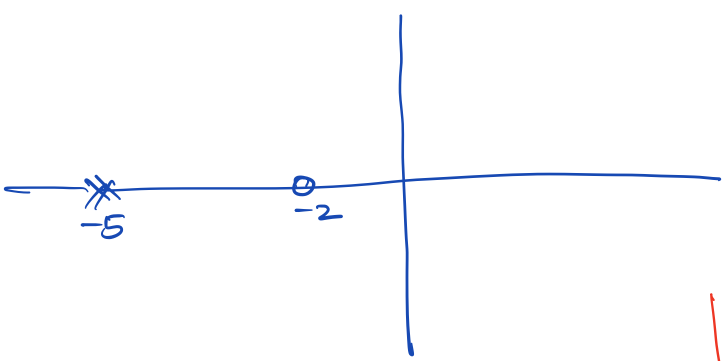
1. BIBO stability
2. Step response

Gain

$$G(s) = 10 \frac{s+3}{((s+2)^2 + 2^2)}$$



Ex $\quad G(s) = \dfrac{s+2}{s^2 + 10s + 25} = \dfrac{s+2}{(s+5)^2}$

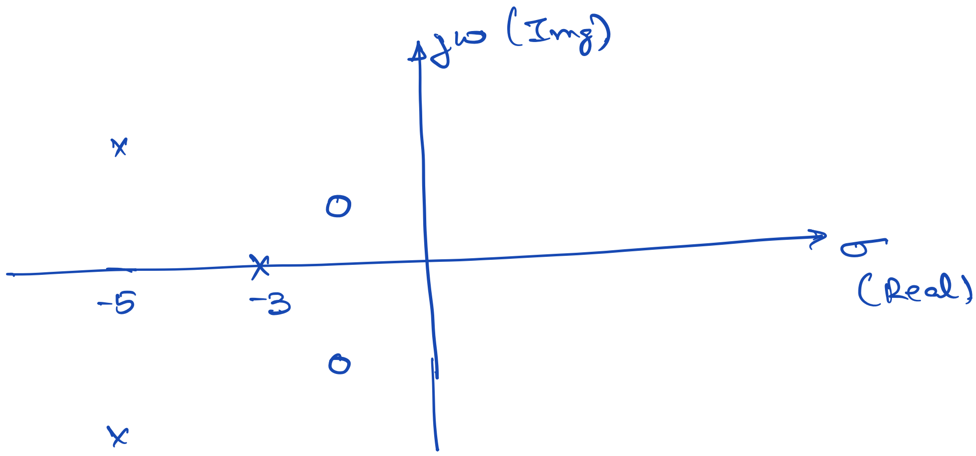


$$G(s) = \frac{(s+2)^2 + 1^2}{(s+3)\left[(s+5)^2 + 2^2\right]}$$

$$= \frac{(s+2+j)(s+2-j)}{(s+3)(s+5+2j)(s+5-2j)}$$

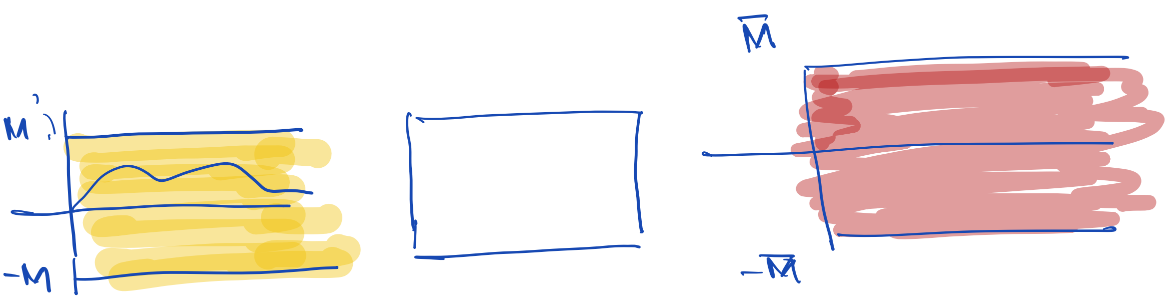
$(s+a)^2 + b^2$

$= (s+a+jb)(s+a-jb)$

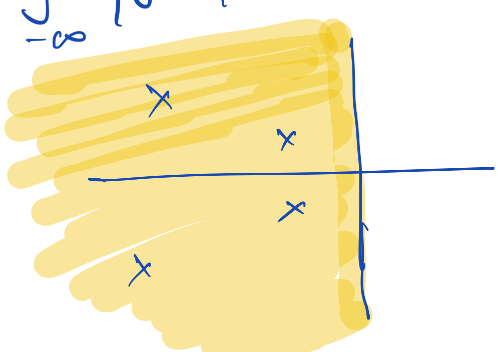


Complex plane

① BIBO stability



$$\int_{-\infty}^{\infty} |g(\tau)| \, d\tau < \infty$$



All poles lie in the open left hand plane (OLHP)

Bounded input

$|r(t)| \le M \, , \forall t.$

$$c(t) = r(t) * g(t)$$

$$= \int_{-\infty}^{\infty} r(t-\tau)\, g(\tau)\, d\tau$$

$$|c(t)| = \left| \int_{-\infty}^{\infty} \cdots \right| \le \int_{-\infty}^{\infty} | \cdots | \, dt$$

$$\le \int_{-\infty}^{\infty} \underbrace{|r(t-\tau)|}_{\le M} |g(\tau)| \, d\tau \le \int_{-\infty}^{\infty} M |g(\tau)| \, d\tau$$

$$\le M \boxed{\int_{-\infty}^{\infty} |g(\tau)| \, d\tau} \rightarrow K. < \infty$$

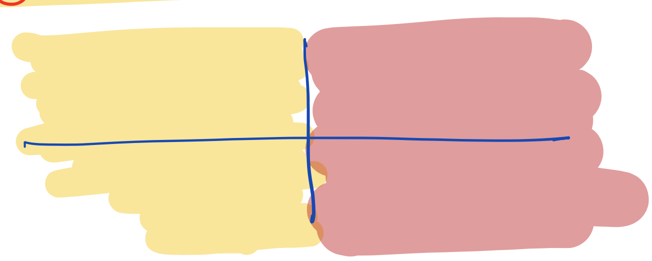
Causal systems $\quad g(t) = 0 \, , \quad t < 0$

BIBO stability $\equiv \int_{0^-}^{\infty} |g(\tau)| \, d\tau < \infty$

$$\sigma \ge 0 \Rightarrow e^{-\sigma t} \le 1 , \quad t \ge 0$$

$$|g(\tau)| e^{-\sigma \tau} \le |g(\tau)|$$

$$\int_{0^-}^{\infty} |g(\tau)| e^{-\sigma \tau} d\tau \le \int_{0^-}^{\infty} |g(\tau)| \, d\tau < \infty$$



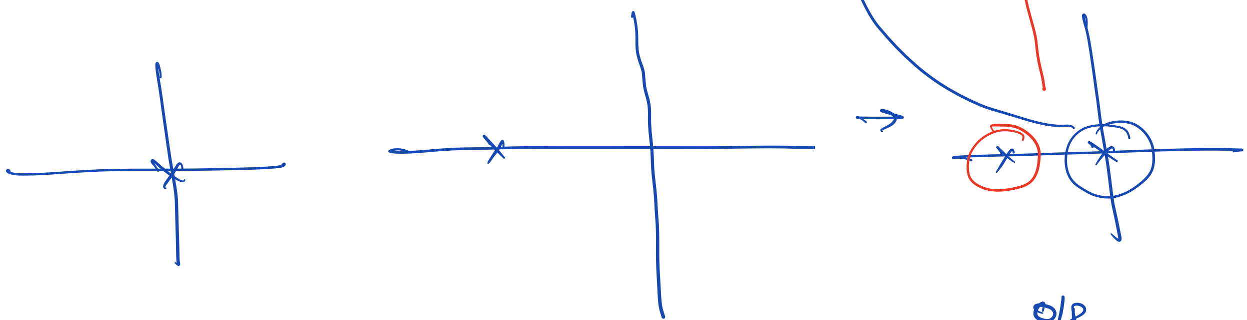
$\sigma = 0$

If there is a single pole (no repeated pole) $\Rightarrow$ marginally stable.

Pz Plot and step response

$$\frac{1}{s} \rightarrow \boxed{\frac{1}{s+2}} \rightarrow \left[\frac{1}{2} - \frac{1}{2}e^{-2t}\right]\mathbb{1}(t)$$

$C(s) = R(s) \cdot G(s)$



I/P   TF   O/P.

$$\frac{1}{2} - \frac{1}{2}e^{-2t}$$

Forced response.   Natural resp.

Form of o/p

(s+1)

$$\frac{1}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$$

$$\left[K_1 + K_2 e^{-2t}\right]\mathbb{1}(t)$$

Ex $G(s) = \frac{s+3}{(s+2)(s+4)}$

Form

Step response ?

$$\left[k_1 + k_2 e^{-2t} + k_3 e^{-4t} \right] \mathbb{1}(t)$$

(b) $G(s) = \frac{10}{} \frac{s+1}{(s+2)(s+4)}.$

$$= \left[k_1 + k_2 e^{-2t} + k_3 e^{-4t} \right] \mathbb{1}(t)$$

(c) $G(s) = \frac{s+1}{(s+2)^2 + 1^2}$

$$k_1 + k_2 e^{-2t} \left[\cos(1t + \phi) \right]$$

(d) $G(s) = \frac{s+1}{\left[(s+2)^2 + 1^2\right](s+3)}$

$$\left[k_1 + k_2 e^{-2t} \left[\cos(t + \phi)\right] + k_3 e^{-3t} \right] \mathbb{1}(t)$$

$$\frac{s+1}{s\left((s+2)^2+1^2\right)} = \frac{A}{s} + \frac{B(s+1)}{(s+2)^2+1} + \frac{C}{(s+2)^2+1}$$

$$\downarrow$$

$$A + Be^{-2t}\underline{\cos t} + Ce^{-2t}\underline{\sin t}$$

$\cos a \cos b + \sin a \sin b$

$\cos(t+\phi)$

$$G_1(s) = \frac{1}{s+2} \cdot \frac{20}{s+20}$$

$$G_2(s) = \frac{1}{s+2}$$