1.) Newsvendor Problem

2.) Blackwell's principle of irrelevant

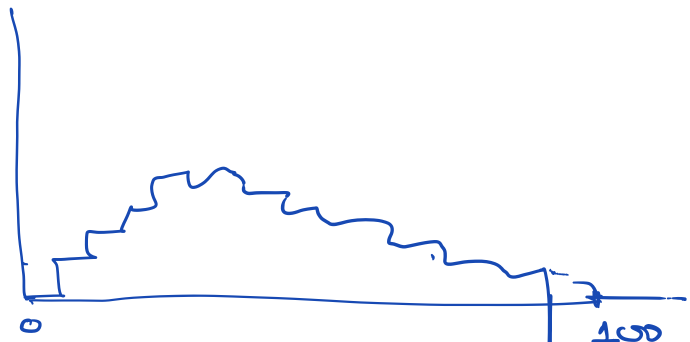
$\$p < \$q$

Action: a Demand: W

PMF W

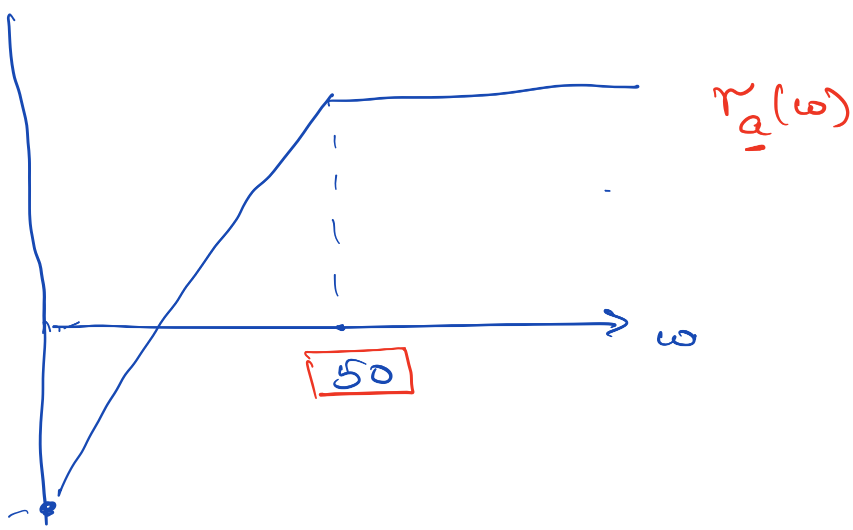
↑

Known



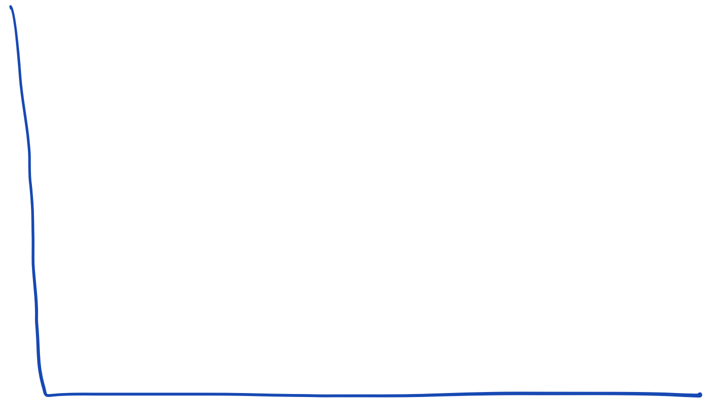
$$r(a, w) = \begin{cases} qw - pa & w \leq a \\ qa - pa & w > a \end{cases}$$

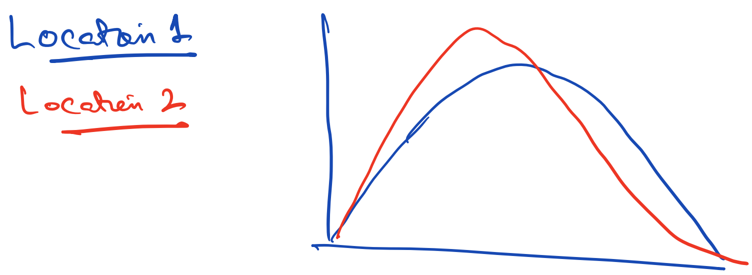
$r(a,w)$



$$J(a) = \mathbb{E}[r(a, W)]$$

$$J(a) = \sum_{\omega \in W} P_W(\omega)\, r(a, \omega)$$



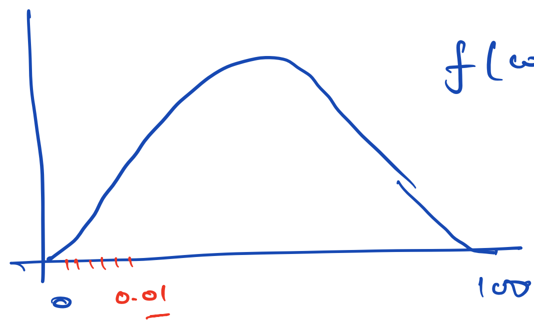


$$J^* = \max_{a \in A} J(a)$$ Opt. perf.

$$a^* = \arg\max_{a \in A} J(a)$$ Opt. action

$W \subseteq \mathbb{R}$

$a \in W = [0, 100]$



$f(\omega)$ pdf

$$\boxed{J(a) = \int_0^{100} f(\omega)\, r(a,\omega)\, d\omega}$$

$$a^* = \underset{a \in A}{\arg\max}\ J(a)$$

$\hookrightarrow$ Plot it appears J(a) is concave.

$$\frac{d}{da} J(a) = 0 \qquad \frac{d^2}{da^2} J(a) < 0$$

$$r(a,\omega) = \begin{cases} q\omega - pa & \omega \le a \\ qa - pa & \omega > a \end{cases}$$

$$J(a) = \int_0^a f(\omega)\,[q\omega - pa]\, d\omega + \int_a^\infty f(\omega)\,[qa - pa]\, d\omega.$$

$$\frac{d}{da} J(a) = \cancel{[qa - pa] f(a)} + \int_0^a [-p] f(\omega)\, d\omega$$
$$- \cancel{[qa - pa] f(a)} + \int_a^\infty f(\omega)\,(q - p)\, d\omega.$$

Leibniz integral rule.

$$\frac{d}{dx}\left(\int_{p(x)}^{q(x)} f(x,t)\,dt\right) = f(x, q(x))\frac{d}{dx}q(x) - f(x, p(x))\frac{d}{dx}p(x) + \int_{p(x)}^{q(x)} \frac{\partial}{\partial x} f(x,t)\,dt.$$

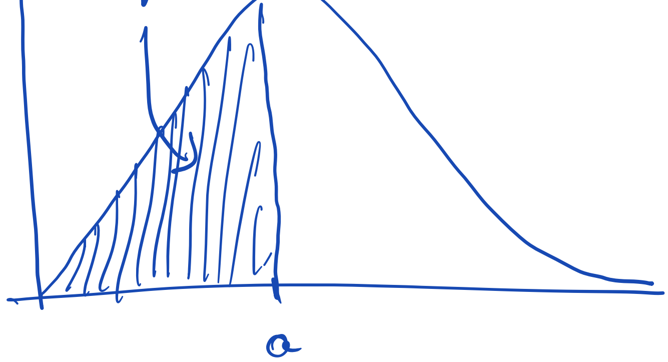
$$= -p\,F(a) + (q-p)\left[1 - F(a)\right]$$

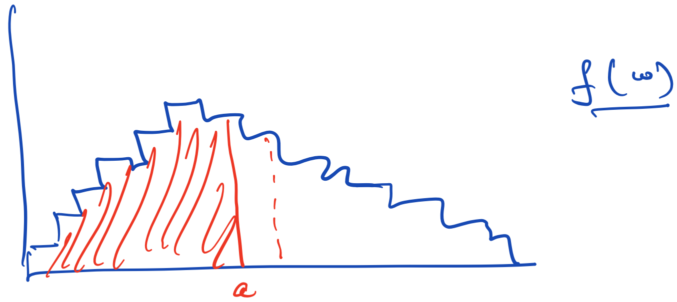
$$= \underline{0}$$

$$\boxed{F(a) = \frac{q-p}{q}} \equiv a = F^{-1}\left(\frac{q-p}{q}\right)$$

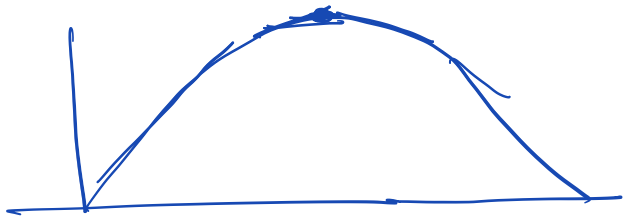
Critical fractile

$$\frac{q-p}{q} \equiv 1 - p/q$$





$$J(i+1) - J(i) \gtrless 0$$

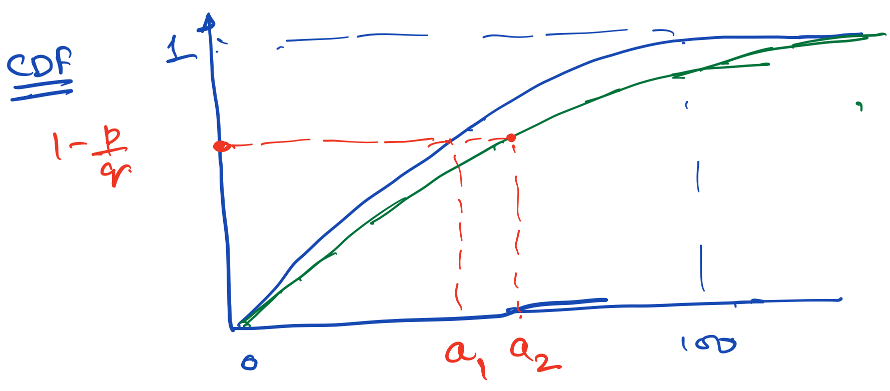


$$\max_i \{ i : J(i+1) - J(i) \geq 0 \}$$

$$\max \{ i : F(i) \leq \frac{q-p}{q} \} \rightarrow \text{opt action}$$

Comparing a for multiple location

Stochastic Monotonicity.



$$\boxed{F_2(\omega) \leq F_1(\omega)}$$

a_2 vs a_1

$a_2 \geq a_1$

Making decision w/ info

$c_i \geq 0$

$\sum_{i=0}^{\infty} c_i \to$ limit
$\hookrightarrow \infty$
$\hookrightarrow +\infty - \infty$
Not define

$$c : \underline{S} \times A \times W \to \underline{\mathbb{R}_{\geq 0}}$$

Info avail to DM "state"

Action

Uncertainty $\to$ known PMF

$(\Omega, \mathcal{F}, P)$ Underlying prob. space

(S, W) are rv of $(\Omega, \mathcal{F}, P)$

$P_{S,W}$ $P_{W|S}$

Policy $\pi : S \to A$

$$J(\pi) = \mathbb{E}\left[c(S, \pi(S), W) \right]$$

$$= \sum_{(s,\omega) \in S \times W} P(s,\omega)\, c(s, \pi(s), \omega)$$

Finitely many possibilities of π. $|A|^{|S|}$

$$\pi^* = \arg\min_{\pi \in \Pi} J(\pi) \quad \leftarrow \quad (P1)$$

Opt. policy

Functional optimization prob.

$\forall s \in S$ (P2)

$$\pi^{0}(s) = \arg\min_{a \in A} \underbrace{\mathbb{E}\left[c(s, a, W) \mid S = s \right]}_{Q(s,a)}$$

$\forall s, a$ $\quad Q(s,a) = \mathbb{E}\left[c(s,a,W) \mid S = s \right]$ Comp $|S||A|$

$$= \sum_{w \in W} P_{W|S}(w|s)\, c(s,a,w)$$

$$\pi^o(s) = \arg\min_{a \in A} Q(s,a)$$

Claim 1 $\quad \pi^o$ is optimal for P1

Proof $\quad$ Let π be any policy,

$$J(\pi) = \mathbb{E}\left[c(S, \pi(S), W) \right]$$

$$= \mathbb{E}\left[\underbrace{\mathbb{E}\left[c(S, \pi(S), W) \mid S \right]} \right]$$

$$\geq \mathbb{E}\left[\mathbb{E}\left[c(S, \pi^o(S), W) \mid S \right] \right]$$

$$= J(\pi^o)$$

$\Rightarrow \pi^o$ is opt.

Claim 2 $\quad P_S(s) > 0, \ \forall s.$

Then all opt. soln of (P1) are also soln of (P2).

Pf $\quad$ Proof by contradiction.

Let μ^* is a opt policy that does not satisfy (P2) (SS)

(P2)

$$\pi^0(s) = \arg\min_{a \in A} \underbrace{\mathbb{E}[\, c(s, a, W) \mid S = s}_{Q(s,a)}$$

$$Q(s, \pi^0(s)) \leq Q(s, \mu^*(s))$$

$$\exists\, s^0 \ \text{s.t} \quad Q(s^0, \pi^0(s^0)) < Q(s^0, \mu^*(s^0))$$

$$\mathbb{E}[\, c(s, \pi^0(s), W) \mid S=s] \leq \mathbb{E}[\, c(s, \mu^*(s), W) \mid S=s]$$

strict for s^0

$$\sum_{s \in S} P_S(s)\, \mathbb{E}[\ \cdots\ ] < \sum_{s \in S} P_S(s)\, \mathbb{E}[\ \cdots\ ]$$

$P_S(s^0) > 0$

$$\mathbb{E}[\, c(s, \pi^0(s), W)] < \mathbb{E}[c(s, \mu^*(s), W)]$$