

Homework #3

CS 169/268 Optimization

Fall 2023

Due: Thursday October 30 11:59pm on Canvas

Reading:

Undergrads:

Kochenderfer and Walker, Chapter 6, and Chapter 7 Sections 7.1-7.5;

Also Belegundu and Chandrupatla, Chapter 3: Sections 3.7-3.9. Chapter 7: 7.3 and 7.6

Grads:

Kochenderfer and Walker, Chapters 6 and 7;

Also Belegundu and Chandrupatla (3rd ed.), Chapter 3: Sections 3.7-3.9; Chapter 7.

Bertsekas 2.2, 2.7

Also please have a brief look at *every page* of (but you *will not be tested on!*):

Shewchuck on Preconditioned Conjugate Gradient (just Section 12, pp. 39-41);

Stüben, Algebraic Multigrid review paper: just Sections 1 and 2.

Review Questions (study for midterm - don't turn in):

K&W derive eqs. 5.16; also 6.7 (also covered in B&C Chapter 3 assigned reading);

Also K&W 6.13, incl exercise 6.10.

Recall the Homework Ground Rules in HW0, point #1. In accordance with those rules, ...

Problem 1: (undergrads and grads) *Test* a Quasi-Newton method on your problem of HW2, in the manner of HW2, Problem 1c. *Explore* numerically, and *report*, as usual.

Problem 2: (grads only - extra credit up to 10% for undergrads)

Write down mathematically (in your report) and *implement* an unconstrained optimization problem with regular 2D graph structure (any tiling of 2D is OK):

$$f(\mathbf{x}) = \sum_{(i,j) \in G} g(x_i, x_j) + \sum_i h(x_i)$$
$$\frac{\partial f(\mathbf{x})}{\partial x_k} = \sum_{j|(k,j) \in G} \frac{\partial g(x_k, x_j)}{\partial x_k} + \sum_{i|(i,k) \in G} \frac{\partial g(x_i, x_k)}{\partial x_k} + \frac{\partial h(x_k)}{\partial x_k}$$

where G is the graph (without self-edges). (Examples: Electrical resistance network with current source and sink; nonlinear spring network with frustration as in B&C section 3.1; Finite element method as in B&C + outside references; electrostatics in 2D with fully connected graph.) Note that often, especially in physics examples, $g(x, y) = g(y, x)$, but this is not necessary.

Numerically study the scaling of Quasi-Newton and/or CG and/or PCG algorithm performance as the size of the graph increases. *Report*, as in previous homework problems.