HBM538E Mathematical Methods in Data Analysis and Machine Learning

Assignment 4

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Due: 17.01.2024, 23:00

Problem 1: The function

Consider the N-D convex function

$$f(x_1, \ldots, x_N) = 2.4 \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{x_i x_j}{i+j-1} - \sum_{i=1}^{N} i\, x_i - 1.47 \tag{1}$$

and plot the graph of the function for $N = 2$.

Problem 2: Optimization problem

Express the function (1) in the form $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T S\mathbf{x} - \mathbf{b}^T\mathbf{x} + c$ for $N = 8$ where $S = S^T$.

Problem 3: Properties of S

For $N = 8$, show that S is positive definite by checking its eigenvalues.

Problem 4: Implementation of the Gradient Descent

Find the global minimum of the function in (1) for $N = 8$ using the Gradient Descent algorithm. Take the initial point as $x^{(0)} = [1.5\ 1.5\ \cdots\ 1.5]^T$ and apply the following learning rate selection approaches:

1. In-exact line search with $s_0 = 0.1$, 0.01 and 0.001, respectively.
2. Backtracking with $s_0 = 0.5$.

3. Momentum with the optimal parameter.
4. Nesterov method with the optimal parameters.
5. ADAM method with the default parameters.

Problem 5: The Stochastic Gradient Descent

Find the global minimum of the function in (1) for $N = 8$ using the Stocastic Gradient Descent algorithm with mini-batch size is 2.

Problem 6: Compare and justify your results

Plot a 2-D graph including the values of the function in (1) vs the iteration number for each approach and method in Problem 4 and Problem 5. Discuss your findings.