Part 1 Pre-Work

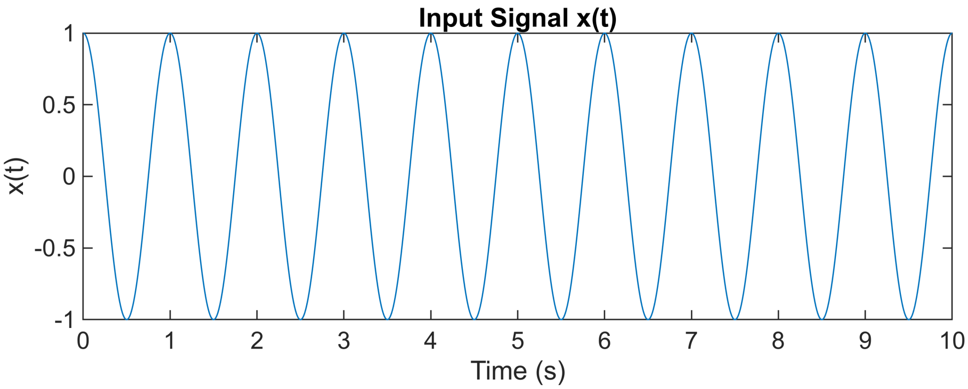
```
R = 1;
C = 0.1;
f0 = 1;

s = tf('s');
H = 1 / (s * R * C + 1);

disp('Transfer Function H(s):');
```

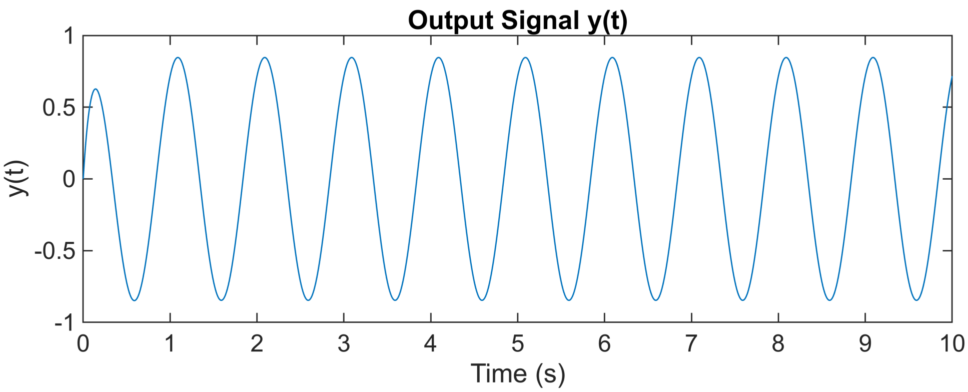
```
Transfer Function H(s):
```

```
H
```

```
H =
 
      1
  ---------
  0.1 s + 1
 
Continuous-time transfer function.
Model Properties
```

```

t = 0:0.01:10;
x_t = cos(2 * pi * f0 * t);

y_t = lsim(H, x_t, t);

figure;
subplot(2, 1, 1);
plot(t, x_t);
title('Input Signal x(t)');
xlabel('Time (s)');
ylabel('x(t)');

subplot(2, 1, 2);
plot(t, y_t);
title('Output Signal y(t)');
xlabel('Time (s)');
ylabel('y(t)');
```





Part 2 Example

```
syms t;
tau = 1;
T0 = 5;
k_vec = [-3:3];
% !!!IMPORTANT!!!: the signal definition must cover [0 to T0]
% the signal is defined over [-T0, 2T0], which covers [0, T0]
xt = heaviside(t+tau/2)-heaviside(t-tau/2) + heaviside(t-(T0-tau/2))-heaviside(t-
(T0+tau/2));
[Xw, w] = FourierSeries(xt, T0, k_vec);
```

X1 =

$$\frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}$$

X1 =

$$\left(\frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}\quad \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}\right)$$

X1 =

$$\left(\frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}\quad \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}\quad \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}\right)$$

X1 =

$$\left(\frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}\ \ \frac{1}{5}\right)$$

X1 =

$$\left(\frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}\ \ \frac{1}{5}\ \ \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}\right)$$

X1 =

$$\left(\frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}\ \ \frac{1}{5}\ \ \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}\ \ \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}\right)$$

X1 =

$$\left(\sigma_1\ \ \sigma_2\ \ \sigma_3\ \ \frac{1}{5}\ \ \sigma_3\ \ \sigma_2\ \ \sigma_1\right)$$

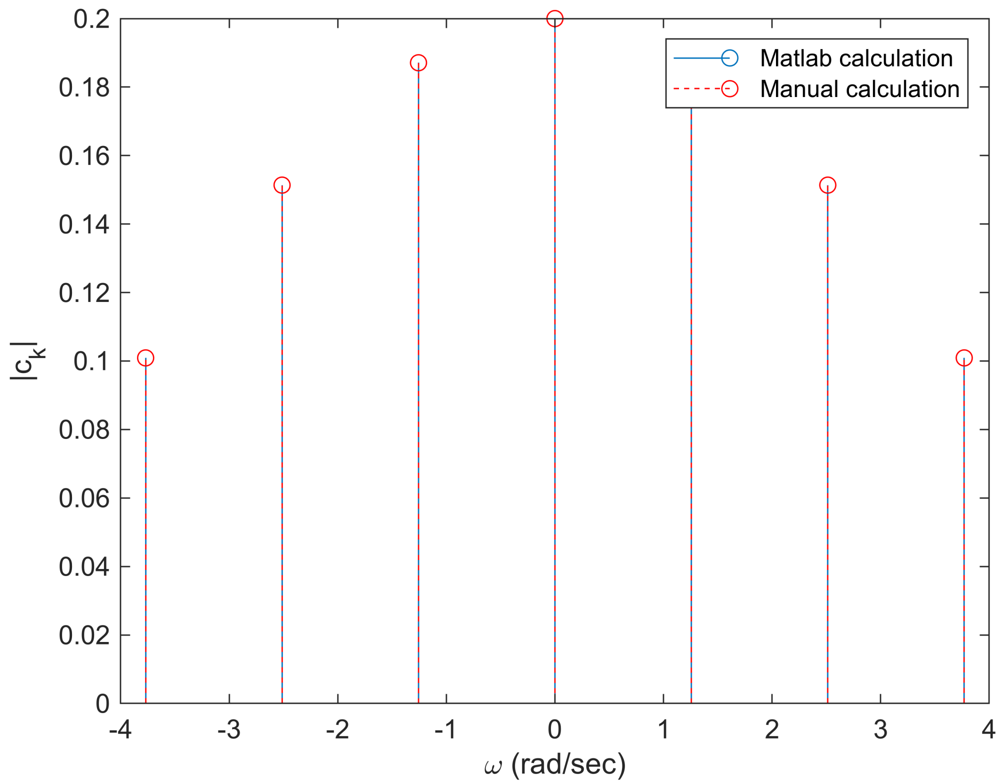
where

$$\sigma_1 = \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{12\,\pi}$$

$$\sigma_2 = \frac{\sqrt{2}\ \sqrt{\sqrt{5}+5}}{8\,\pi}$$

$$\sigma_3 = \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4\,\pi}$$

```
% plot the results from Matlab calculation
figure;
stem(w,abs(Xw), 'o-');
hold on;
% the results based on manual calculation (slides # 16)
Xw_manual = tau/T0*sinc(k_vec*tau/T0); stem(w, abs(Xw_manual), 'r--');
legend('Matlab calculation', 'Manual calculation'); xlabel('\omega (rad/sec)');
ylabel('|c_k|');
```



Part 3 Lab Assignments

A: Symbolic Fourier Series Calculation

```
syms t;
T0 = 2;
k_vec = -3:3;
t_signal = -5:0.01:5;

xt = piecewise(mod(t, T0) < 1, 1, ...
               mod(t, T0) < 2, -1);

[Xw, w] = FourierSeries(xt, T0, k_vec);
```

```
X1 =
```

$$\frac{\displaystyle\int_0^2 \begin{cases} e^{3\pi t i} & \text{if } t \bmod 2 < 1 \\ -e^{3\pi t i} & \text{if } t \bmod 2 \in [1,2) \\ 0 & \text{otherwise} \end{cases} \, dt}{2}$$

```
X1 =
```

$$\left(\frac{\int_0^2 \begin{cases} e^{3\pi t i} & \text{if } t \bmod 2 < 1 \\ -e^{3\pi t i} & \text{if } t \bmod 2 \in [1,2) \\ 0 & \text{otherwise} \end{cases} dt}{2} \quad \frac{\int_0^2 \begin{cases} e^{2\pi t i} & \text{if } t \bmod 2 < 1 \\ -e^{2\pi t i} & \text{if } t \bmod 2 \in [1,2) \\ 0 & \text{otherwise} \end{cases} dt}{2}\right)$$

```
X1 =
```

$$\left(\frac{\int_0^2 \begin{cases} e^{3\pi t i} & \sigma_2 \\ -e^{3\pi t i} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} dt}{2} \quad \frac{\int_0^2 \begin{cases} e^{2\pi t i} & \sigma_2 \\ -e^{2\pi t i} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} dt}{2} \quad \frac{\int_0^2 \begin{cases} e^{\pi t i} & \sigma_2 \\ -e^{\pi t i} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} dt}{2}\right)$$

where

$$\sigma_1 = \text{ if } t \bmod 2 \in [1,2)$$

$$\sigma_2 = \text{ if } t \bmod 2 < 1$$

```
X1 =
```

$$\left(\frac{\int_0^2 \begin{cases} \sigma_3 & \sigma_2 \\ -\sigma_3 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} dt}{2} \quad \frac{\int_0^2 \begin{cases} \sigma_4 & \sigma_2 \\ -\sigma_4 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} dt}{2} \quad \frac{\int_0^2 \begin{cases} e^{\pi t i} & \sigma_2 \\ -e^{\pi t i} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} dt}{2} \quad \frac{\int_0^2 \begin{cases} 1 & \sigma_2 \\ -1 & \sigma_1 \\ 0 & \text{otherwis} \end{cases}}{2}\right.$$

where

$$\sigma_1 = \text{ if } t \bmod 2 \in [1,2)$$

$$\sigma_2 = \text{ if } t \bmod 2 < 1$$

$$\sigma_3 = e^{3\pi t i}$$

$$\sigma_4 = e^{2\pi t i}$$

```
X1 =
```

$$\left(\frac{\int_0^2 \begin{cases} \sigma_3 & \sigma_2 \\ -\sigma_3 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} \sigma_4 & \sigma_2 \\ -\sigma_4 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} e^{\pi t i} & \sigma_2 \\ -e^{\pi t i} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} 1 & \sigma_2 \\ -1 & \sigma_1 \\ 0 & \text{otherwis} \end{cases}}{2}\right.$$

where

$$\sigma_1 = \text{ if } t \bmod 2 \in [1, 2)$$

$$\sigma_2 = \text{ if } t \bmod 2 < 1$$

$$\sigma_3 = e^{3 \pi t i}$$

$$\sigma_4 = e^{2 \pi t i}$$

$$\sigma_5 = e^{-\pi t i}$$

```
X1 =
```

$$\left(\frac{\int_0^2 \begin{cases} \sigma_4 & \sigma_2 \\ -\sigma_4 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} \sigma_5 & \sigma_2 \\ -\sigma_5 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} e^{\pi t i} & \sigma_2 \\ -e^{\pi t i} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} 1 & \sigma_2 \\ -1 & \sigma_1 \\ 0 & \text{otherwis} \end{cases}}{2}\right.$$

where

$$\sigma_1 = \text{ if } t \bmod 2 \in [1, 2)$$

$$\sigma_2 = \text{ if } t \bmod 2 < 1$$

$$\sigma_3 = e^{-2 \pi t i}$$

$$\sigma_4 = e^{3 \pi t i}$$

$$\sigma_5 = e^{2 \pi t i}$$

$$\sigma_6 = e^{-\pi t i}$$

```
X1 =
```

$$\left(\frac{\int_0^2 \begin{cases} \sigma_5 & \sigma_2 \\ -\sigma_5 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} \sigma_6 & \sigma_2 \\ -\sigma_6 & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} \mathrm{e}^{\pi\, t\, \mathrm{i}} & \sigma_2 \\ -\mathrm{e}^{\pi\, t\, \mathrm{i}} & \sigma_1 \\ 0 & \text{otherwise} \end{cases} \mathrm{d}t}{2} \quad \frac{\int_0^2 \begin{cases} 1 & \sigma_2 \\ -1 & \sigma_1 \\ 0 & \text{otherwis} \end{cases}}{2}\right.$$

where

$$\sigma_1 = \text{ if } \; t \bmod 2 \in [1, 2)$$

$$\sigma_2 = \text{ if } \; t \bmod 2 < 1$$

$$\sigma_3 = \mathrm{e}^{-3\,\pi\, t\, \mathrm{i}}$$

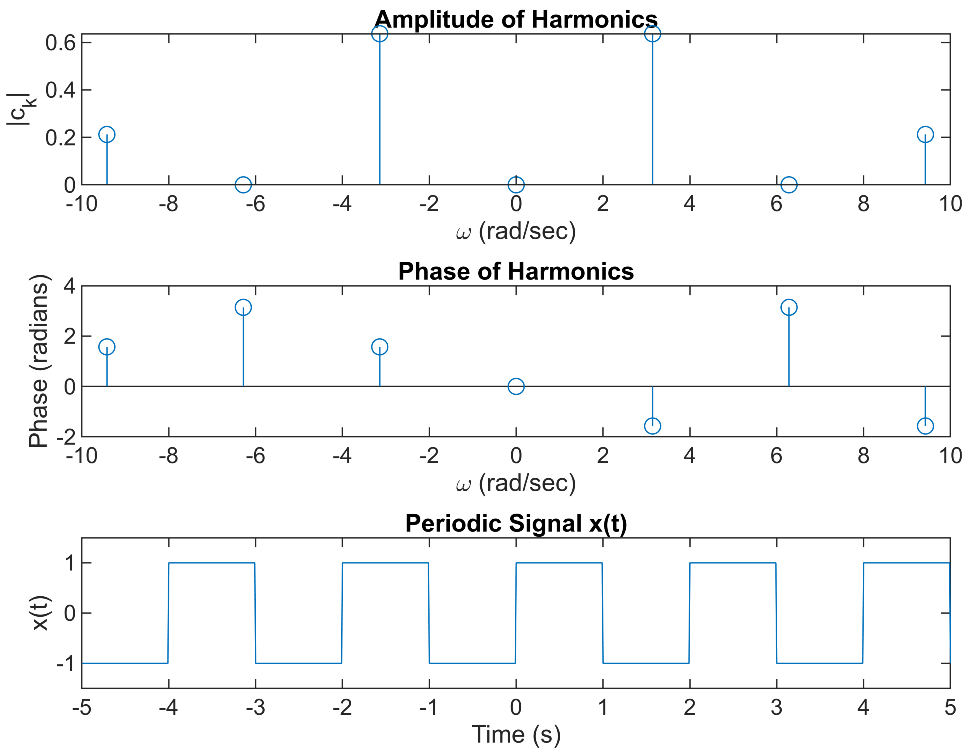
$$\sigma_4 = \mathrm{e}^{-2\,\pi\, t\, \mathrm{i}}$$

$$\sigma_5 = \mathrm{e}^{3\,\pi\, t\, \mathrm{i}}$$

$$\sigma_6 = \mathrm{e}^{2\,\pi\, t\, \mathrm{i}}$$

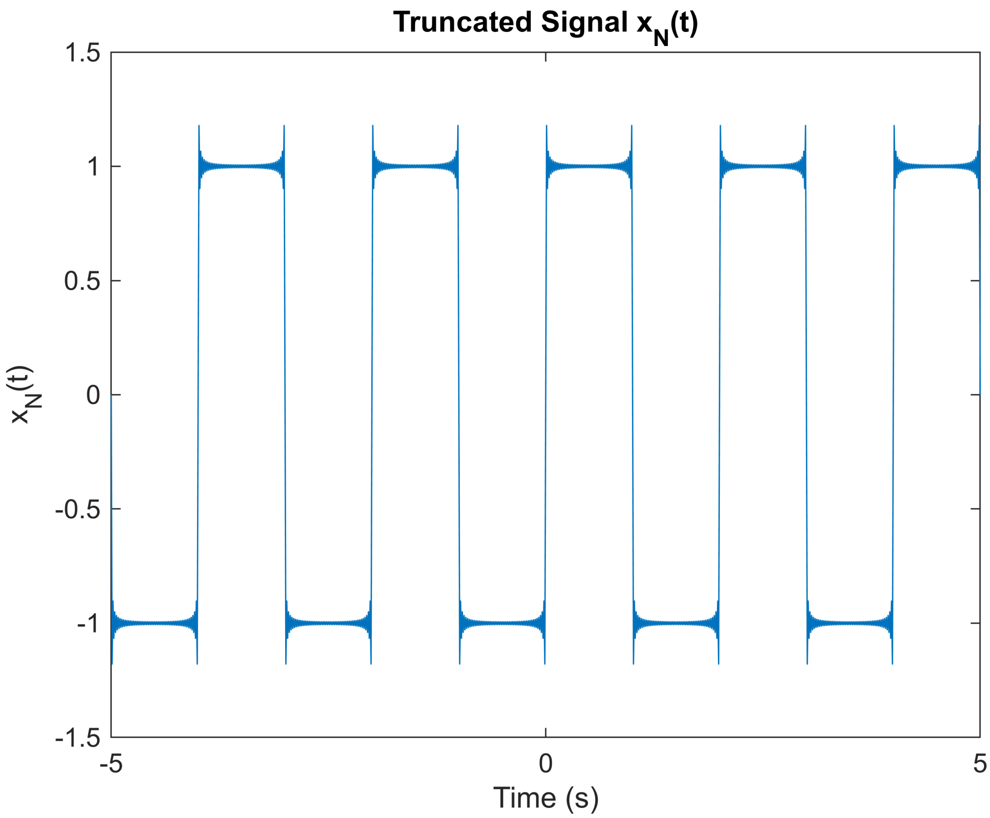
$$\sigma_7 = \mathrm{e}^{-\pi\, t\, \mathrm{i}}$$

```
x_signal = double(subs(xt, t, t_signal));

figure;
subplot(3, 1, 1);
stem(w, abs(Xw), 'o-');
title('Amplitude of Harmonics');
xlabel('\omega (rad/sec)');
ylabel('|c_k|');

subplot(3, 1, 2);
stem(w, angle(Xw), 'o-');
title('Phase of Harmonics');
xlabel('\omega (rad/sec)');
ylabel('Phase (radians)');

subplot(3, 1, 3);
plot(t_signal, x_signal);
title('Periodic Signal x(t)');
xlabel('Time (s)');
ylabel('x(t)');
xlim([-5 5])
ylim([-1.5 1.5])
```



B: Gibbs Phenomenon

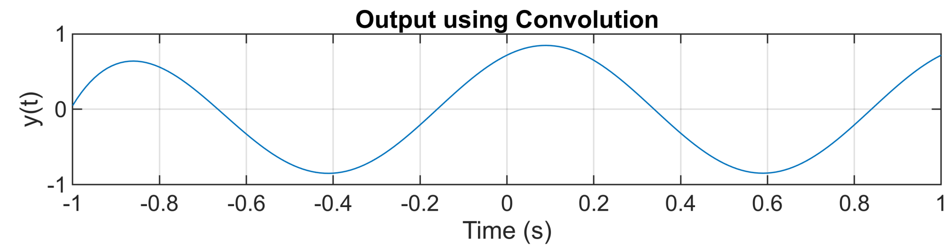
```
n_vec = -99:99;
Omega0 = 2 * pi / T0;
Cn = zeros(size(n_vec));

Cn(1:2:end) = 2 ./ (1j * n_vec(1:2:end) * pi);
Cn(2:2:end) = 0;

t = -5:0.01:5;
xt_truncated = zeros(size(t));

for m = 1:length(n_vec)
   xt_truncated = xt_truncated + Cn(m) * exp(1j * n_vec(m) * Omega0 * t);
end

figure;
plot(t, real(xt_truncated));
title('Truncated Signal x_N(t)');
xlabel('Time (s)');
ylabel('x_N(t)');
```

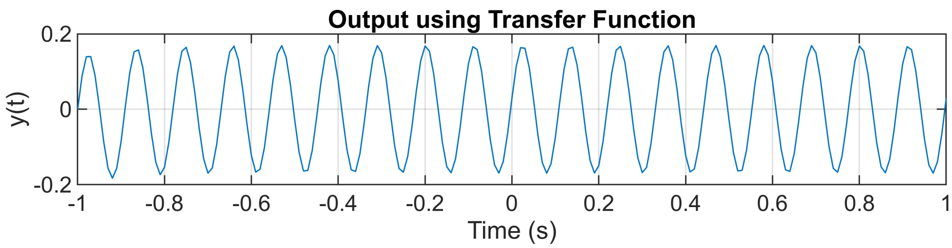


C: Linear Time Invariant System with Periodic Inputs

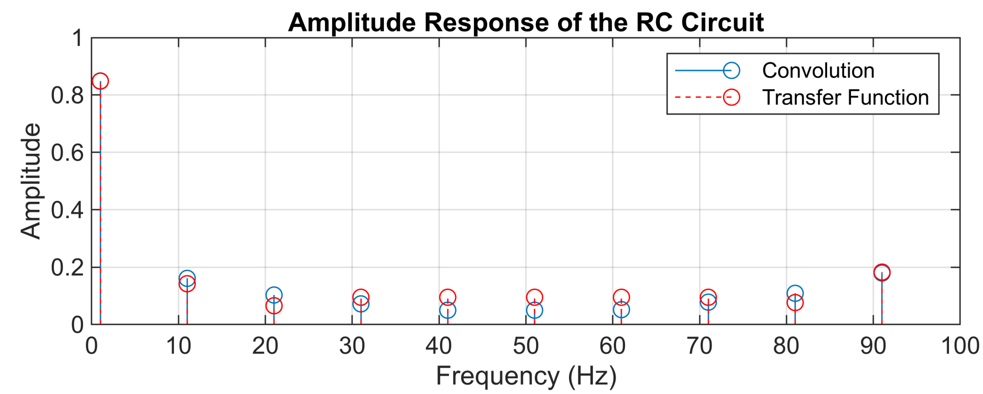
```
R = 1;
C = 0.1;
f0 = 1;
dt = 0.01;
t = -1:dt:1;

x_t = cos(2 * pi * f0 * t);

s = tf('s');
H = 1 / (s * R * C + 1);

h_t = (1/(R*C)) * exp(-t/(R*C)) .* heaviside(t);
y_convolution = conv(x_t, h_t, 'same') * dt;

frequencies = 1:10:100;
amplitude_response_conv = zeros(size(frequencies));

for i = 1:length(frequencies)
    f = frequencies(i);
    x_t = cos(2 * pi * f * t);
    y_conv = conv(x_t, h_t, 'same') * dt;
    amplitude_response_conv(i) = max(abs(y_conv));
end
```

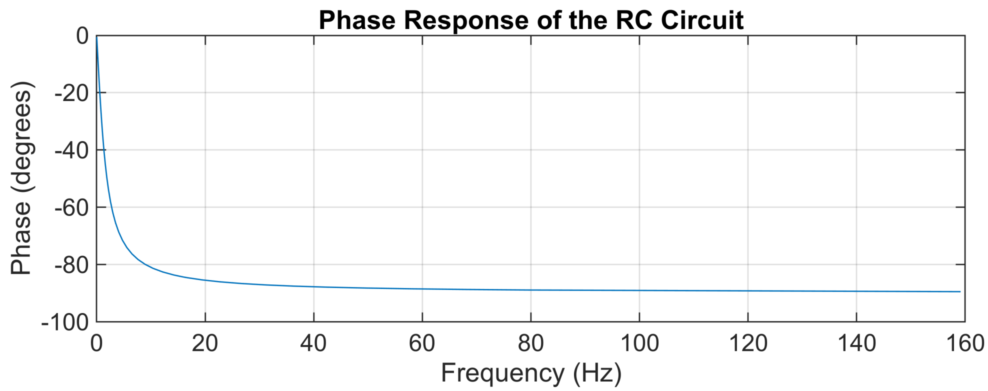
```
y_transfer = lsim(H, x_t, t);

amplitude_response_transfer = zeros(size(frequencies));

for i = 1:length(frequencies)
    f = frequencies(i);
    x_t = cos(2 * pi * f * t);
    y_transfer_temp = lsim(H, x_t, t);
    amplitude_response_transfer(i) = max(abs(y_transfer_temp));
end

[mag, phase, w] = bode(H);
mag = squeeze(mag);
phase = squeeze(phase);

figure;
subplot(3, 1, 1);
plot(t, y_convolution);
title('Output using Convolution');
xlabel('Time (s)');
ylabel('y(t)');
grid on;

subplot(3, 1, 2);
plot(t, y_transfer);
title('Output using Transfer Function');
xlabel('Time (s)');
ylabel('y(t)');
grid on;
```






```
figure;
subplot(2, 1, 1);
stem(frequencies, amplitude_response_conv, 'o-', 'DisplayName', 'Convolution');
hold on;
stem(frequencies, amplitude_response_transfer, 'r--', 'DisplayName', 'Transfer
Function');
title('Amplitude Response of the RC Circuit');
xlabel('Frequency (Hz)');
ylabel('Amplitude');
legend;
grid on;

subplot(2, 1, 2);
plot(w/(2*pi), phase);
title('Phase Response of the RC Circuit');
xlabel('Frequency (Hz)');
ylabel('Phase (degrees)');
grid on;
```





Functions

```
function [X, w] = FourierSeries(x, T0, k_vec)
    % function [X, w] = FourierSeries(x, T0, k_vec) %
    % symbolically calculate the Fourier Series, and return the % numerical results
    %
    % x: the time domain signal within one period;
    %     it must have definition over [0, T0]
    %
    %     it must be a symbolic function of t
    %
%
% T0: the period of the signal
% k_vec: the range of Harmonics to be calculated
    syms t
    for mm = 1:length(k_vec)
        k = k_vec(mm);
        % Fourier series
        X1(mm) = int(x*exp(-j*2*pi*k*t/T0), t, 0, T0)/T0
        % change the symbolic value to numerical value
        X(mm) = subs(X1(mm));
        % angular frequency
        w(mm) = k*2*pi/T0;
    end
end
```