Q1

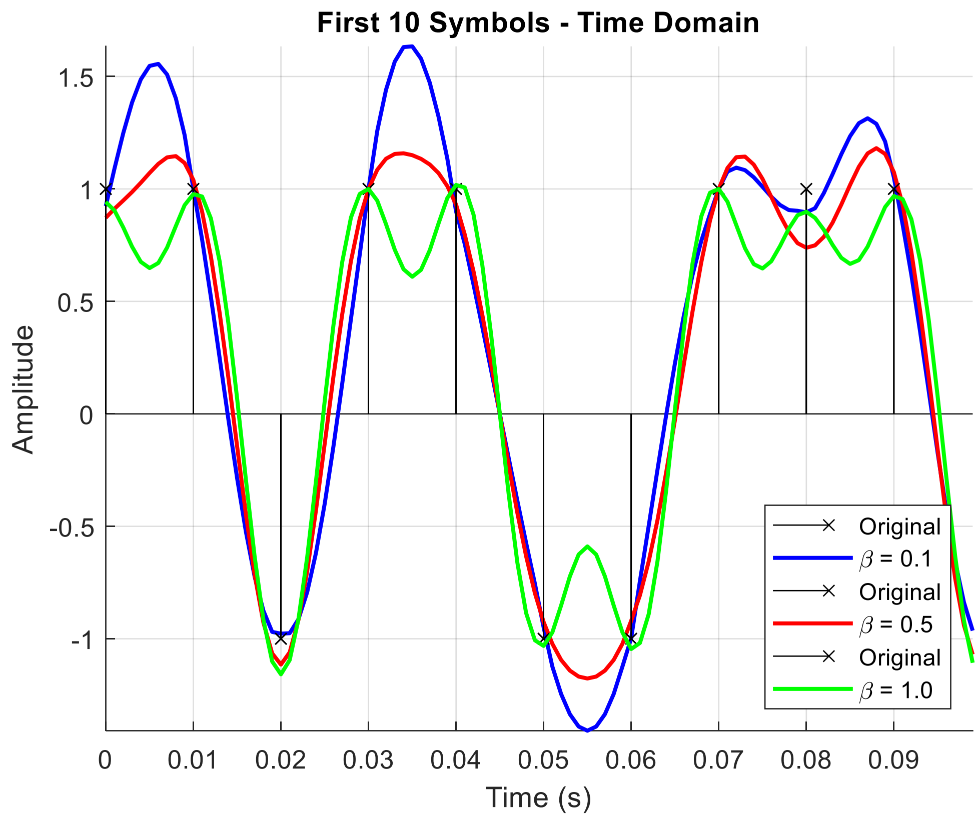

Figure 1 – First 10 Symbols

At β=0.1 each pulse is very narrow, so the reconstructed curve only “lights up” right at the sample instants (the black “×” marks). You’ll see perfect agreement at t ≈ 0.01s, 0.03s, 0.05s, etc., but in between the trace drops almost to zero far from the smooth waveform you’d expect. In other words, apart from those isolated points, the overall shape is badly under-represented.

At β=0.5 the pulses widen enough to start filling the gaps. The curve now approximates the peaks and troughs more faithfully, and the segment between, say, 0.02s and 0.04s looks much closer to the original. However you’ll still notice slight “dips” or “bumps” where two pulses meet so it’s better than β=0.1 but not yet a perfect interpolation.

At β=1.0 pulses overlap heavily, producing a much smoother envelope. The reconstructed waveform now follows the general sine-like shape, and intermediate amplitudes are far more accurate. On the downside, neighboring peaks bleed into each other so around t ≈ 0.08s you may see the crest a bit flattened or shifted compared to the ideal. In short, best overall fit so far, but with some amplitude distortion at the joins.

Q2

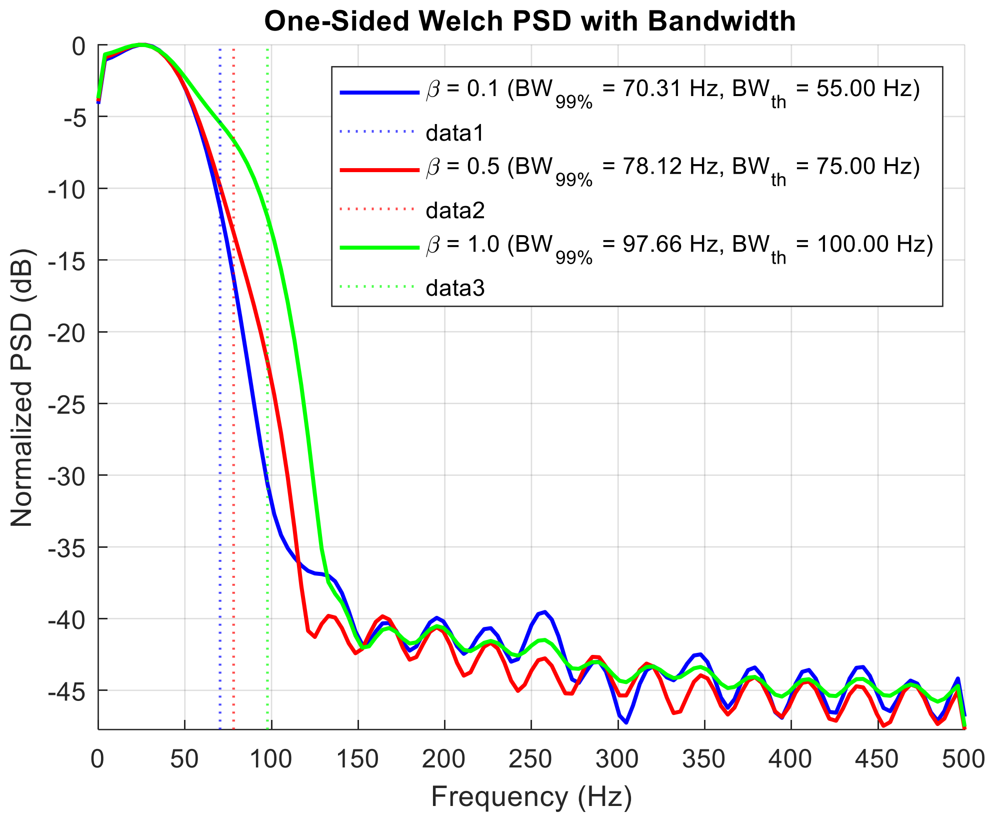

Figure 2 – One-Sided Welch PSD with Bandwidth

β	99% Empirical BW (Hz)	Theoretical BW (Hz)
0.1	70.31	55
0.5	78.12	75
1.0	97.66	100

The theoretical bandwidth using:

$$BW_{th} = \frac{1+\beta}{2T}$$

Both empirical and theoretical bandwidth increase linearly with β, confirming our earlier observation that wider pulses (larger β) occupy more spectrum.

For $\beta = 0.1$, the empirical BW (70.3 Hz) exceeds the theoretical 55 Hz by ∼28%. This is because a low roll off pulse has relatively high sidelobes, so you need a wider band to capture 99% of its energy.

At $\beta = 0.5$, empirical (78.1 Hz) and theoretical (75 Hz) agree within ∼4%. By this point the sidelobes are lower, so the simple formula becomes much more accurate.

For $\beta = 1.0$, the empirical BW (97.7 Hz) is just 2.3% below the theoretical 100 Hz. Here the main lobe dominates and the roll-off transition perfectly matches the $(1+\beta)/2$ prediction.

Theoretical bandwidth predictions provide an excellent guide errors drop from ∼28% at very small β to only a few percent for moderate and high β validating both our pulse-width/overlap discussion and the roll off formula.

Q3

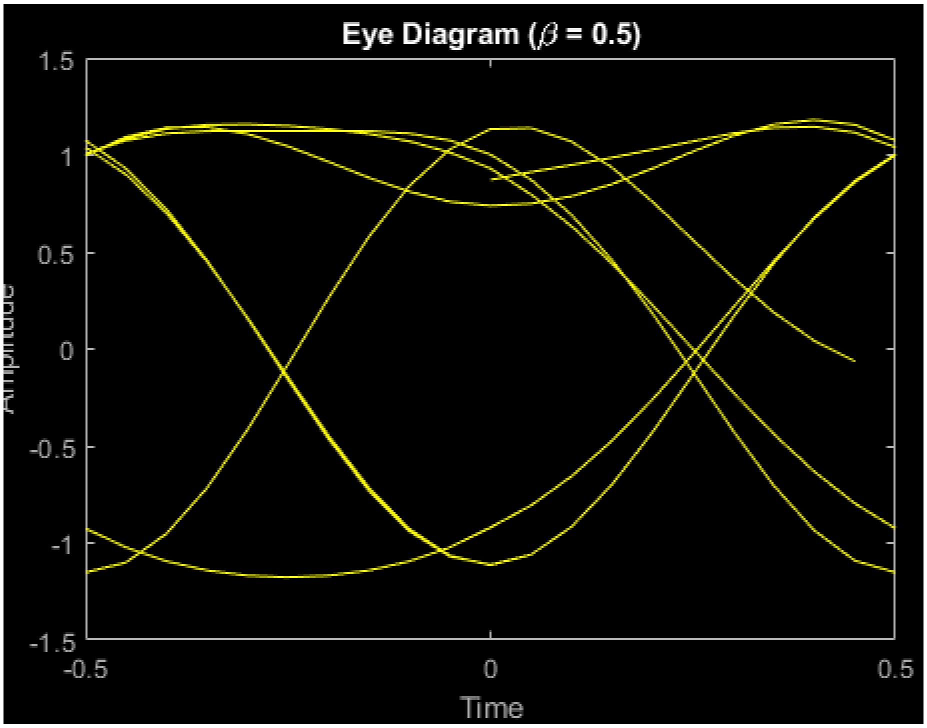

Figure 3 – Eye Diagram (β = 0.5)

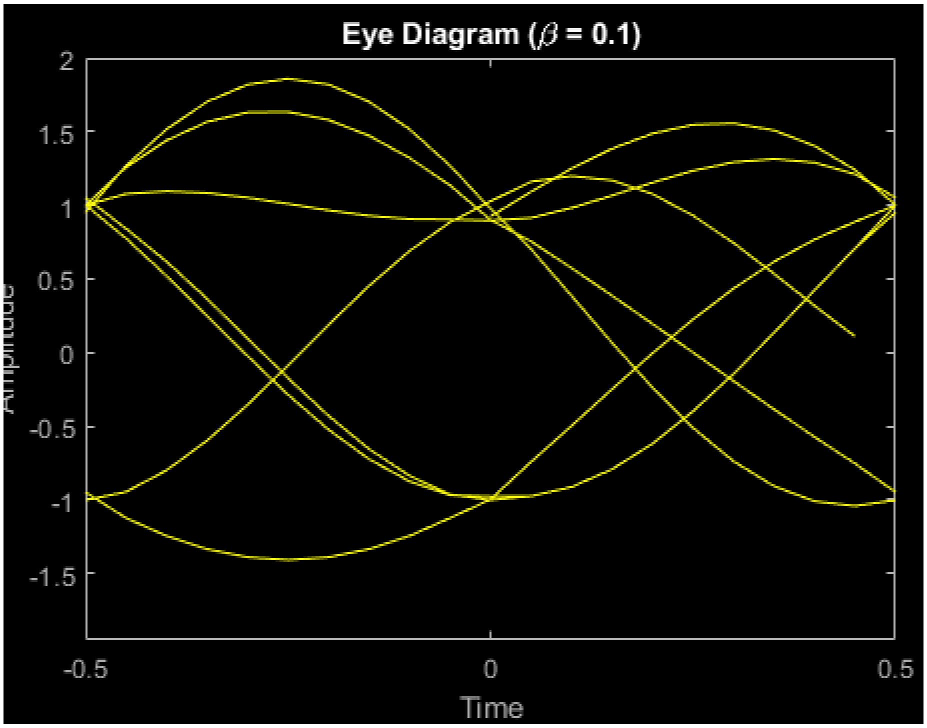

Figure 4 - Eye Diagram (β = 0.1)

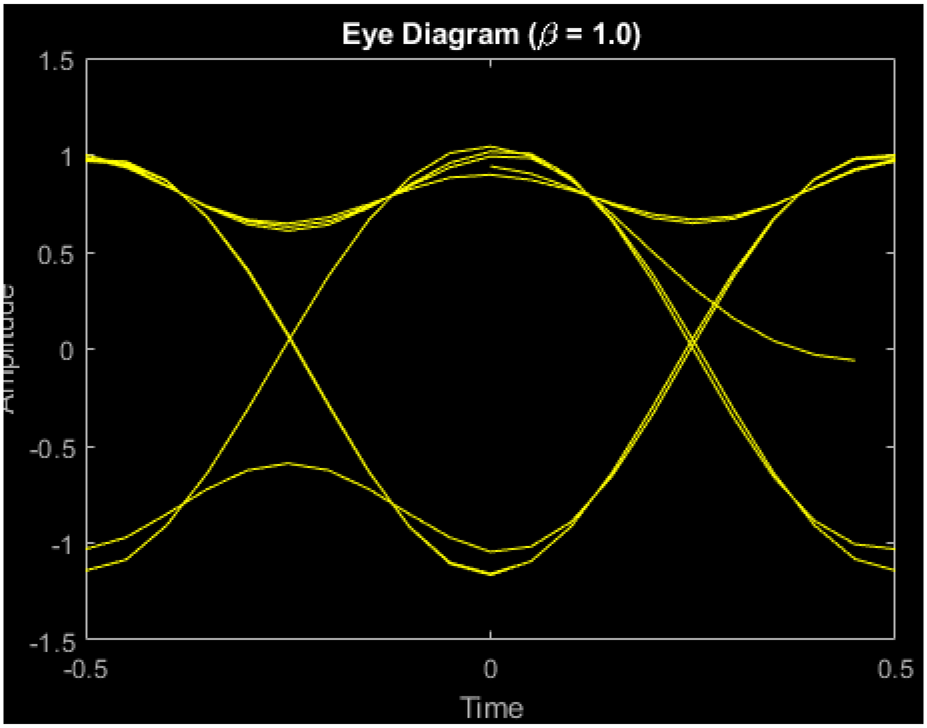

Figure 5 - Eye Diagram (β = 1.0)

The β = 1.0 eye diagram shows by far the largest vertical and horizontal opening. You can see that at the sampling instant (t = 0) the traces cluster tightly around +1 and -1 with minimal spread, and the zero crossing region remains wide. In contrast, for β = 0.1 the curves fan out dramatically both the amplitude "eye height" and timing "eye width" are much smaller signaling severe intersymbol interference (ISI). The β = 0.5 case sits in between.

A wide vertical opening (amplitude margin) guarantees that channel noise or amplitude jitter must be very large before symbols are mis detected. A wide horizontal opening (time margin) means the receiver has more leeway in sampling time before crossing into an adjacent bit interval. An open eye directly correlates with lower bit error rates.

Low β (0.1)

- Bandwidth is minimal, but the pulse's long, oscillatory side-lobes stretch far in time (high ISI).
- Eye is almost closed: adjacent symbols smear into one another, harming both timing and amplitude discrimination.
- Detectability is poor: small noise or timing errors lead to bit-errors.

Moderate β (0.5)

- Pulses widen (main lobe grows) and side lobes decay faster (ISI reduced).
- Eye opens up noticeably, improving margins, but still some ripple at the edges of the eye.

High β (1.0)

- Excess bandwidth is highest, giving the shortest impulse response in time with minimal side lobes (lowest ISI).
- Eye is maximally open: best amplitude and timing margins (most robust symbol detection).

In short, increasing β trades extra spectrum for a shorter, more time localized pulse, dramatically reducing ISI and yielding a more open eye hence better detectability at the receiver.