Q1

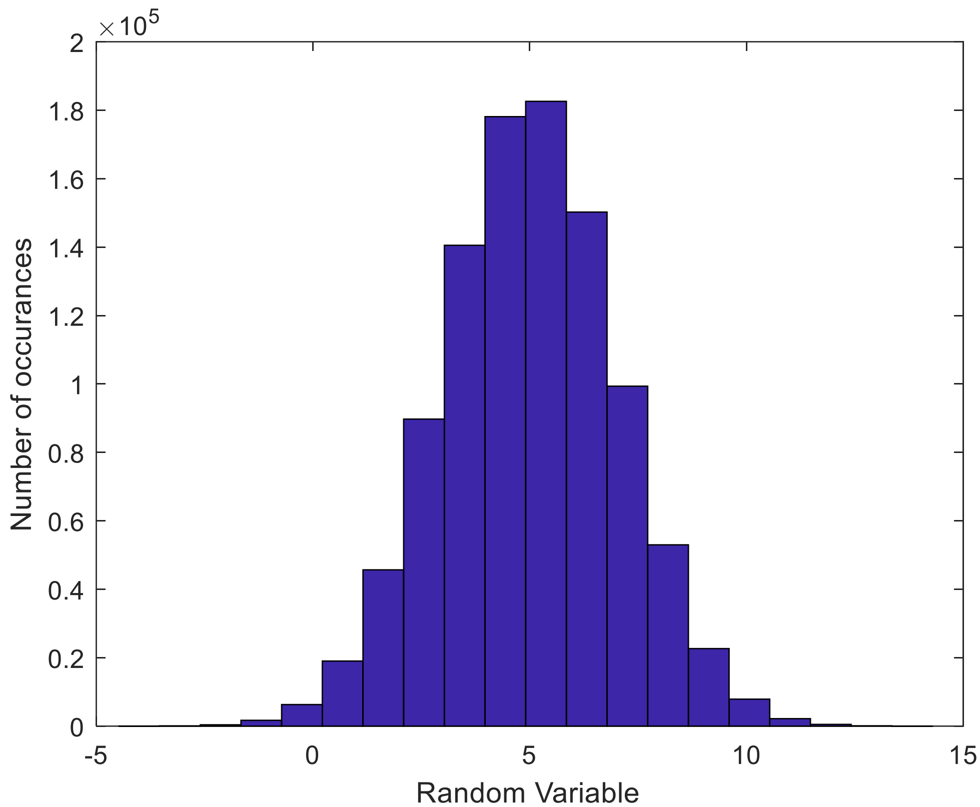

Figure 1 – Gaussian Distribution with mean 5 and variance 4

	Matlab Simulation	Q-Table
P(X > 7.34)	0.1210	0.1210
P(X < 3.81)	0.2759	0.2776
P(1.87 < X < 3.49)	0.1663	0.1672

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P(X > 7.34) = 1.2100e-01
P(X < 3.81) = 2.7592e-01
P(1.87 < X < 3.49) = 1.6633e-01
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$$A = 5, \sigma^2 = 4$$

$$P(X > T) = Q\left(\frac{T - A}{\sqrt{\sigma^2}}\right)$$

$$P(X > 7.34) = Q\left(\frac{7.34 - 5}{2}\right) = Q(1.17) = 0.1210$$

$$P(X < 3.81) = 1 - P(X > 3.81) = 1 - Q\left(\frac{3.81 - 5}{2}\right) = 1 - Q(-0.595)$$
$$= 1 - \left(1 - Q(0.595)\right) = Q(0.595) = 0.2776$$

$$P(1.87 < X < 3.49) = P(X > 1.87) - P(X > 3.49) = Q\left(\frac{1.87 - 5}{2}\right) - Q\left(\frac{3.49 - 5}{2}\right)$$
$$= Q(-1.565) - Q(-0.755) = \left(1 - Q(1.565)\right) - \left(1 - Q(0.755)\right)$$
$$= Q(0.755) - Q(1.565) = 0.2266 - 0.0594 = 0.1672$$

There may be some discrepancies between simulated and theoretical probabilities. These differences primarily arise because theoretical probabilities are typically calculated under ideal assumptions and controlled conditions. For instance, Q-Table values are often generated based on uniform or Gaussian (normal) distribution assumptions. However, real-world data may not perfectly follow these distributions. Factors such as sample size in the simulation, randomness in number generation, or external environmental influences can cause deviations between simulated outcomes and theoretical expectations.

Q2 AND Q3

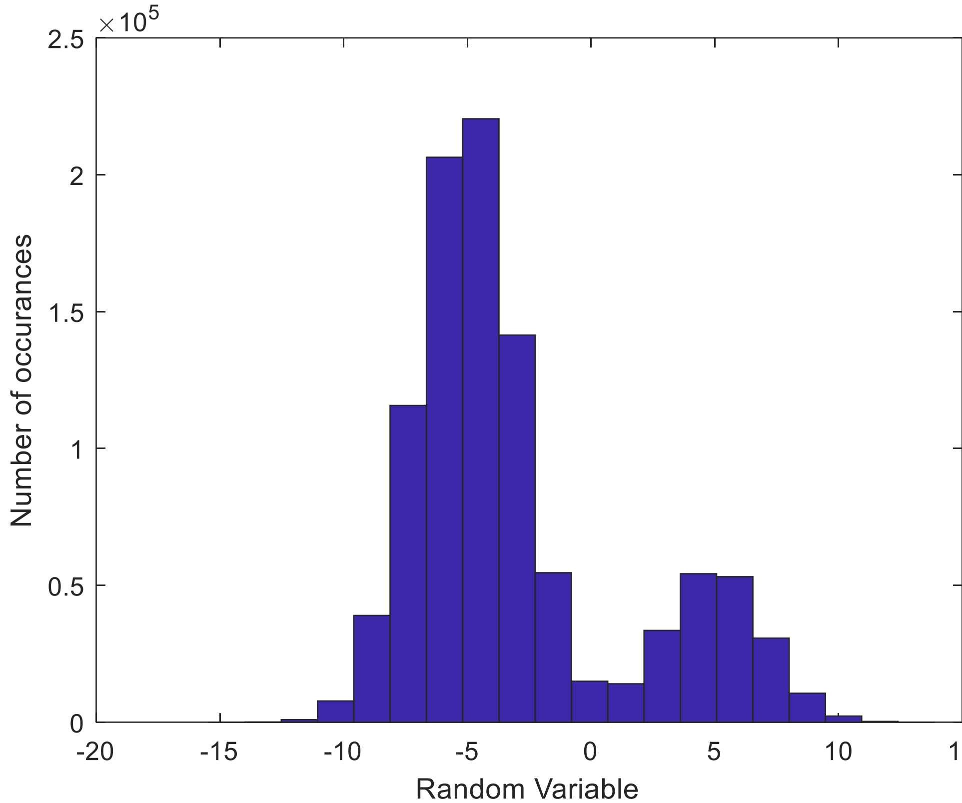

Figure 2 - Gaussian Distribution with mean 5 and variance 4 (q = 0.2 for bit-1)

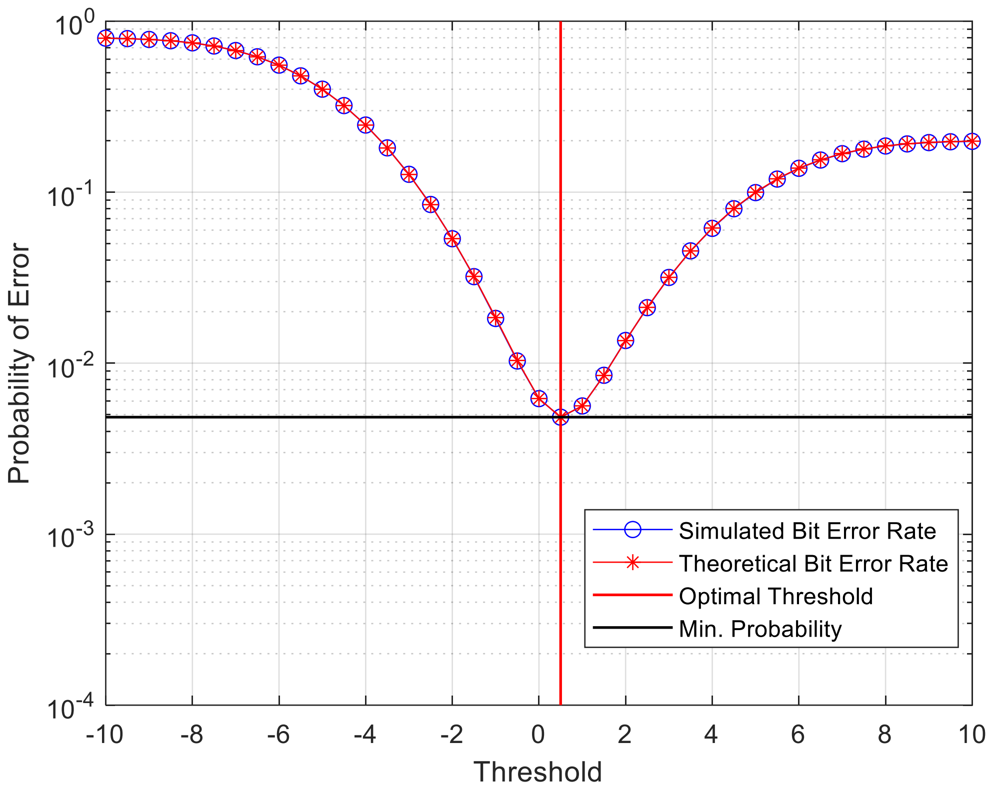

Figure 3 – Threshold Versus Probability Of Error For q = 0.2

$$P_e = Q\left(\frac{T + A}{\sqrt{\sigma^2}}\right)(1 - q) + \left(1 - Q\left(\frac{T - A}{\sqrt{\sigma^2}}\right)\right)q$$

$$T^* = \frac{\sigma^2}{2A} ln\left(\frac{1 - q}{q}\right)$$

$$T^* = \frac{4}{2 * 5} ln\left(\frac{1 - 0.2}{0.2}\right) = 0.554518$$

$$T^*_{sim} = 0.5$$

As seen, the values of T^* = 0.5545 and $T_{sim}{}^*$ = 0.5 are not very close. This difference can be explained by considering the nature of theoretical vs. simulation-based thresholds. The theoretical threshold T^* is derived analytically under ideal assumptions such as perfect knowledge of prior probabilities and noise statistics using the likelihood ratio test. In contrast, $T_{sim}{}^*$ is often estimated from numerical simulations, which may involve approximations, finite sample sizes, and practical limitations. Additionally, the value 0.5 used in the simulation might be based on a simplified assumption (like equal priors or symmetry) rather than the actual prior probability of 0.2, which affects the optimal decision threshold. Therefore, the discrepancy arises mainly due to differences in assumptions and estimation methods.

In communication systems, selecting an appropriate threshold value is crucial for ensuring optimal system performance. The primary goal is to minimize potential losses by accurately detecting the transmitted signals and reducing the bit error rate (BER). If the threshold is set too low or too high, the system may misinterpret the received signals, resulting in incorrect data decisions and information loss. This becomes especially important in noisy environments, where determining the optimal threshold based on the statistical characteristics of the signal helps improve detection accuracy. As a result, not only is the reliability of data transmission enhanced, but system resources such as power and bandwidth are used more efficiently. Therefore, the threshold value is a critical parameter that directly impacts both the error rate and overall system efficiency.

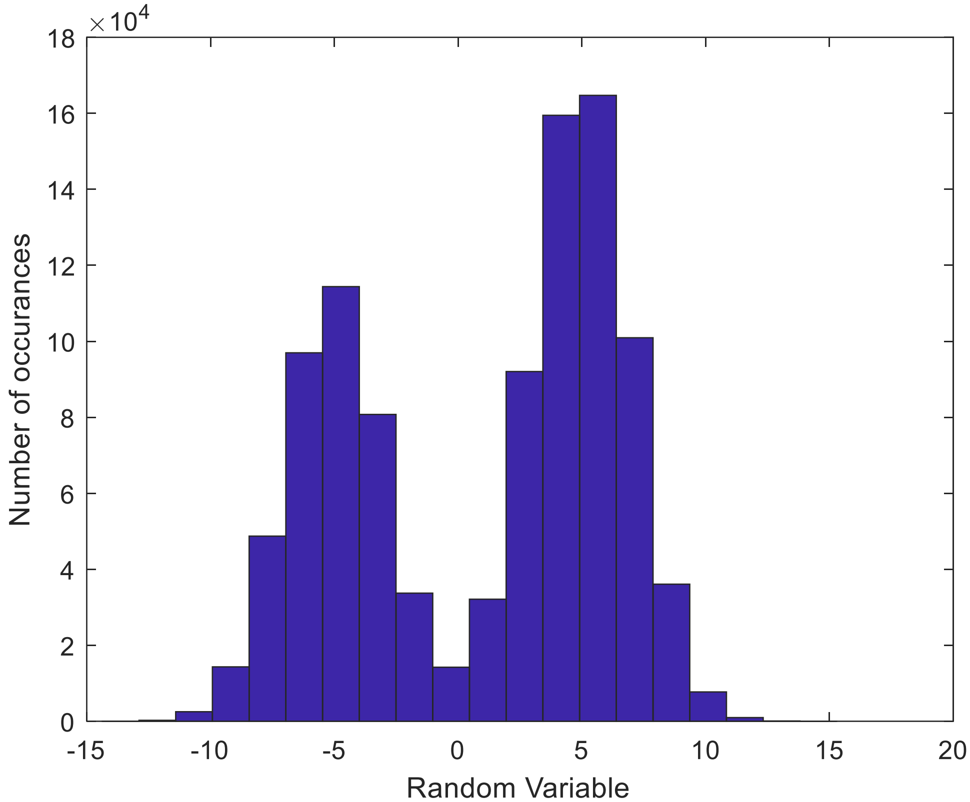

Figure 5 - Gaussian Distribution with mean 5 and variance 4 (q = 0.6 for bit-1)

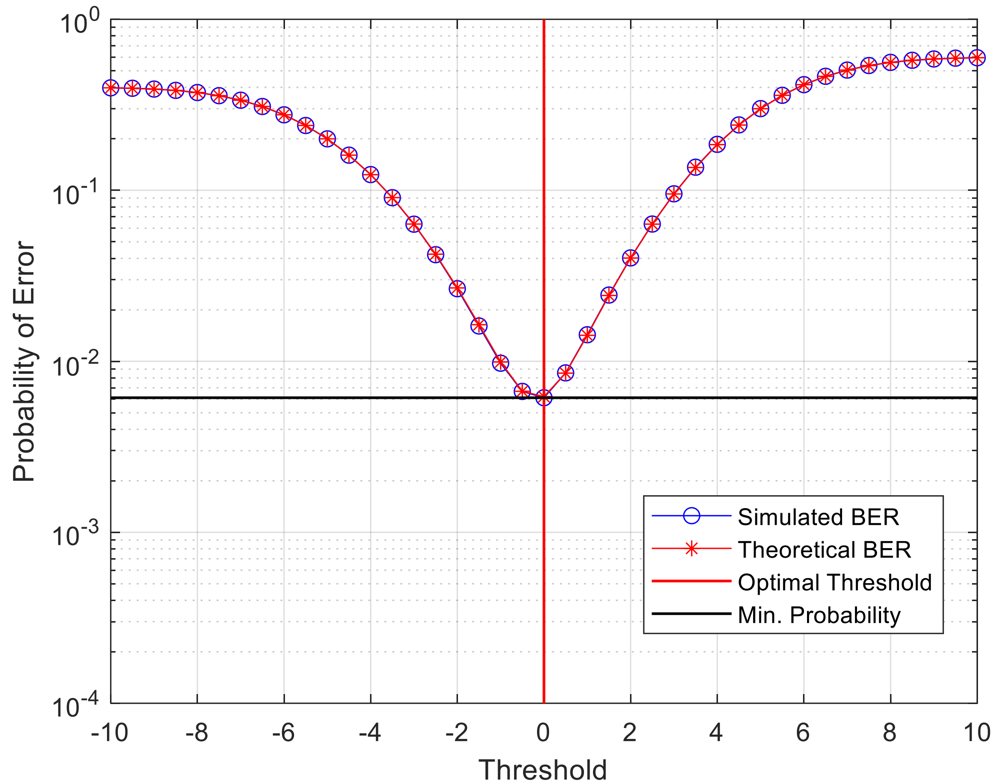

Figure 6 - Threshold Versus Probability Of Error For q = 0.6