# Experiment 2

# Analysis of Discrete-Time Systems

1. Purpose
   The main purpose of this experiment is to study the interrelation between the transfer function, difference equation and the impulse response of a discrete-time system. One other purpose of this experiment is to study the maximum/minimum phase property.

2. Laboratory Work
   Consider the transfer functions of two systems as beow:

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}, \qquad k = 1,2$$

where

$$\boldsymbol{b_1} = [b_{10}, b_{11}, b_{12}] = [1, -0.4944, 0.64]$$
$$\boldsymbol{a_1} = [1, a_{11}, a_{12}] = [1, -1.3315, 0.49]$$
$$\boldsymbol{b_2} = [b_{20}, b_{21}, b_{22}] = [1, 0.4944, 0.64]$$
$$\boldsymbol{a_2} = [1, a_{21}, a_{22}] = [1, 1.3315, 0.49]$$

Assume that $x[n]$ is the input and $y[n]$ is the output of these systems. Also, assume that $x[n]$ is given for $n = 0, 1, \dots, N-1$ and $x[n]$ and $y[n]$ are zero for $n < 0$

A. Write the parametric difference equation showing the input-output relationship
B. Plot the pole-zero diagrams for both systems in the z-plane
C. Using the difference equation obtained write a MATLAB function in the following format
$$\boldsymbol{y} = input(\boldsymbol{b}, \boldsymbol{a}, \boldsymbol{x})$$
where $\boldsymbol{x}$ and $\boldsymbol{y}$ are input output vectors, respectively, and a and b are defined as
$$\boldsymbol{a} = [1, a_1, a_2]; \quad \boldsymbol{b} = [b_0, b_1, b_2]$$
Although $y[n]$ may be nonzero for $n > N-1$ (Why?), your function should compute only the first $N$ samples of $\boldsymbol{y}$
D. Generate the signal $x[n] = \boldsymbol{randn}(1, 256)$. This is a Gaussian distributed random signal. This signal is chosen becuse it has components in all frequencies.
E. Uzing $x[n]$ as the input of both systems, obtain the output of each system and plot input and output spectra. Comment on the plots
F. Combine the two systems in parallel and repeat part (e)
G. Combine the two systems in a cascade manner and repeat part (e)
H. For the transfer function H_k(z), which is given above, let the numerator be equal to one. Determine the transfer function when you place the poles of the system at:

   a. $z_{1,2} = 0.8e^{\pm j0.1\pi}$
   b. $z_{1,2} = 0.8e^{\pm j0.5\pi}$
   c. $z_{1,2} = 0.8e^{\pm j0.9\pi}$
   d. $z_{1,2} = 0.1e^{\pm j0.5\pi}$
   e. $z_{1,2} = 0.95e^{\pm j0.5\pi}$

For each of the cases, compute and plot the magnitude responses of the system by making use of the MATLAB function **freqz**. Comment the frequency response at 100 points.

I. For each of the transfer functions below; determine the pole-zero locations (you my use the MATLB function **roots**), compute and plot the impulse and unit step response of the system, and state whether the system is stable/or max(min) phase. Compute the system responses fort he first 30 samples.

a. $H(z) = \frac{1+0.7264z^{-1}+0.64z^{-2}}{1-0.6356z^{-1}+0.49z^{-2}}$

b. $H(z) = \frac{1+1.1350z^{-1}+1.5625z^{-2}}{1-0.6356z^{-1}+0.49z^{-2}}$

c. $H(z) = \frac{1+0.7264z^{-1}+0.64z^{-2}}{1-1.3620z^{-1}+2.25z^{-2}}$

J. In the above question, compare the magnitude (frequency) response of the systems (a) and (b) and comment on it. Support your answer with theoretical reasoning.