Laboratory Work 4

PRE-WORK

You will use an analog filter that passes sinusoids of frequencies of about 0 Hz (DC) and 3 kHz and blocks frequencies of about 1 kHz and infinity (high frequency). The 1 kHz and 3 kHz frequencies are in the band of audio sounds we perceive, and the effect of the filter can be heard from speakers. The input to the filter in this work is a triangular waveform which is equal to a summation of sinusoids of frequency 1 kHz, 3 kHz, 5 kHz, etc. The filter alters the shape of the triangular waveform by blocking the 1 kHz component.

In this laboratory work, you will design a frequency-selective circuit based on the circuit shown in Fig. 1. This circuit will strongly reject one frequency (1 kHz) and strongly pass another frequency (3 kHz), with intermediate response for other frequencies. This circuit is thus a combination of a band-pass and a band-reject filter.

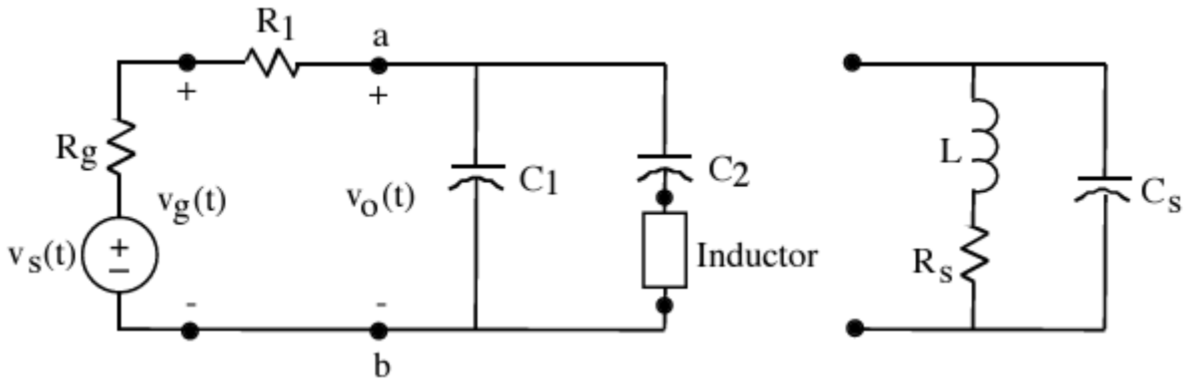

Figure 1: Circuit for combined band-pass and band-reject filter (on left)

The frequency response function, $H(j\omega)$, of the filter is given by the ratio of the phasor for the output, $V_o(j\omega)$, to the phasor for the input, $V_i(j\omega)$. This ratio is found by transforming the circuit to the frequency domain and finding the value of the voltage divider formed by the impedance to the right of a, b divided by the sum of R_1 plus the impedance to the right of a, b. To achieve the behavior of a band-reject filter centered at 1 kHz, the impedance of L and C_2 is designed to be zero at 1 kHz. This turns out to be equivalent to setting the resonant frequency of L and C_2 to 1 kHz. At resonance, the impedances of L and C_2 are equal but opposite—one is positive imaginary and the other is negative imaginary. To achieve the behavior of a band-pass filter that passes 3 kHz, the parallel combination of C_1 and L plus C_2 must look like an open circuit at 3 kHz. How this can be achieved may be understood by first considering a simpler situation. If C_1 were in parallel with just L, then the parallel impedance would look like an open circuit (i.e., infinity ohms) at the resonant frequency of C_1 and L. At the resonant frequency the sum of the impedances is zero, and the sum appears in the denominator of the parallel impedance, which equals the product of impedances over the sum of impedances. The divide by zero means the parallel impedance appears to be infinite or an open circuit. It is though the parallel C_1 and L would seem to disappear! The inclusion of C_2 alters the situation slightly by lowering the apparent impedance of L plus C_2. (The impedance of C_2 is negative imaginary, whereas the impedance of L is positive imaginary.) Nevertheless, the combination of L and C_2 looks like the impedance of an inductor (positive imaginary) above the resonant frequency of

L plus C_2, which is 1 kHz by design. Thus, we can find a value for C_1 that will yield an impedance that is the negative of the impedance of L and C_2 at 3 kHz.

DESIGN OF FILTER CIRCUIT

The design objective is a circuit that will reject a 1 kHz sinusoid and pass a 3 kHz sinusoid. Based on the above discussion, derive an equation that the value of C_2 must satisfy in terms of L and ω so that the circuit will reject frequency ω. Then derive an equation that the value of C_1 must satisfy in terms of L, C_2, and ω so that the circuit will pass frequency 3ω. Assuming L = 100 mH, calculate the values of C_1 and C_2. Procure L, C_1, and C_2 from the stockroom or another source. If necessary, use several capacitors in parallel to achieve approximately the desired value of C_1 and C_2.

CHARACTERIZATION OF COMPONENTS

A. Resistor

Procure a 10 kΩ resistor for R_1.

B. Inductor

The inductor you are using has a series resistance, Rs, and a parasitic capacitance, Cs. Fig. 2 shows a model of the inductor that includes these non-ideal characteristics. In order to make accurate predictions of the filter's response to different frequencies, we must include Rs and Cs.

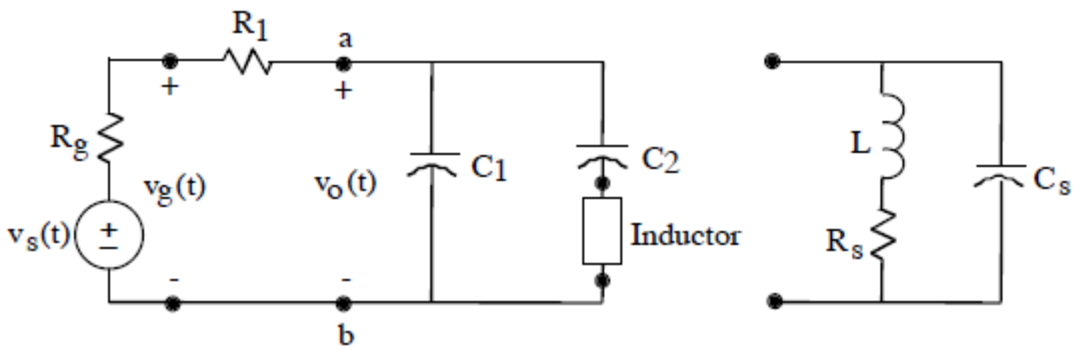

Figure 2: Model of inductor (shown at far right)

C. Capacitors

Determine capacitance of your capacitors according to required specifications of the filter.

PLOTS OF FILTER RESPONSE

A. Motivation

With a sufficiently accurate model, we can predict the output of the filter circuit. In this part of the lab, you will write a Matlab® program to calculate the filter output.

B. Transfer Function

As the first step in calculating the output of the filter, write a function that calculates the transfer function, H(jω) = Vo/Vi, of the circuit in Fig. 1 as a function of ω.

C. Frequency Response
   As the second step in calculating the output of the filter, write a script file that calculates the filter transfer function for frequencies from ω = 0 rad/s to ω = 2π·6000 rad/s. Save the values in an array for plotting.

D. Plot of Frequency Response
   Create superimposed plots of the magnitude of the transfer function versus frequency for the filter in Fig. 1.

LAB WORK

EFFECT OF FILTER ON TRIANGLE WAVE

To illustrate the effect of the filter on a non-sinusoidal waveform, set the input signal to be a 1 kHz triangular waveform. According to Fourier theory, the triangular waveform at 1 kHz is equal to a summation of sinusoids of frequencies 1 kHz, 3 kHz, 5 kHz, etc. The formula for a triangular waveform is as follows:

$$v(t) \;=\; \frac{8}{\pi^2}\left[\sin(2\pi\;1000\;t) - \frac{1}{3^2}\sin(2\pi\;3000\;t) + \frac{1}{5^2}\sin(2\pi\;5000\;t) - \cdots\right] \qquad (1)$$

According to the principle of superposition, the output of the filter will be the summation of the response of the filter to each of the sinusoids in (1). The output amplitude will be the input amplitude times the frequency response for that frequency. Since our filter is designed to eliminate the 1 kHz signal and pass the 3 kHz signal, we would expect that the output signal would look more like the 3 kHz signal. (Note that the 5 kHz signal is small going into the filter and, as the filter frequency response curve shows, the response of the filter makes the small signal even smaller. If possible capture the filter input and output waveforms using an oscilloscope and plot them in Matlab®. Otherwise, sketch the input and output waveform of the filter circuit as shown on the oscilloscope. Comment on whether it appears that the 1 kHz signal is eliminated.

EFFECT OF FILTER ON AUDIO SIGNALS (OPTIONAL)

If you are interested in hearing how the filter circuit affects an audio waveform, you may build the circuit in Fig. 3. (Substitute the filter circuit in Fig. 1 for the RLC bandpass circuit shown in the middle section of Fig. 3.) You may use the output from your mp3 player as the audio input with the output out going to your earphones. Because we are used to hearing distorted sounds, the music may not sound very different to you when it goes through the filter. To exaggerate the effect, try using a 1 kHz triangular waveform as the input. (Use a small-amplitude signal and slowly ramp it up until the output is audible in your earphones.) After listening to the filtered sound, bypass R_1 to hear the unfiltered sound. The sound should be much lower, since it will contain a stronger 1 kHz component.

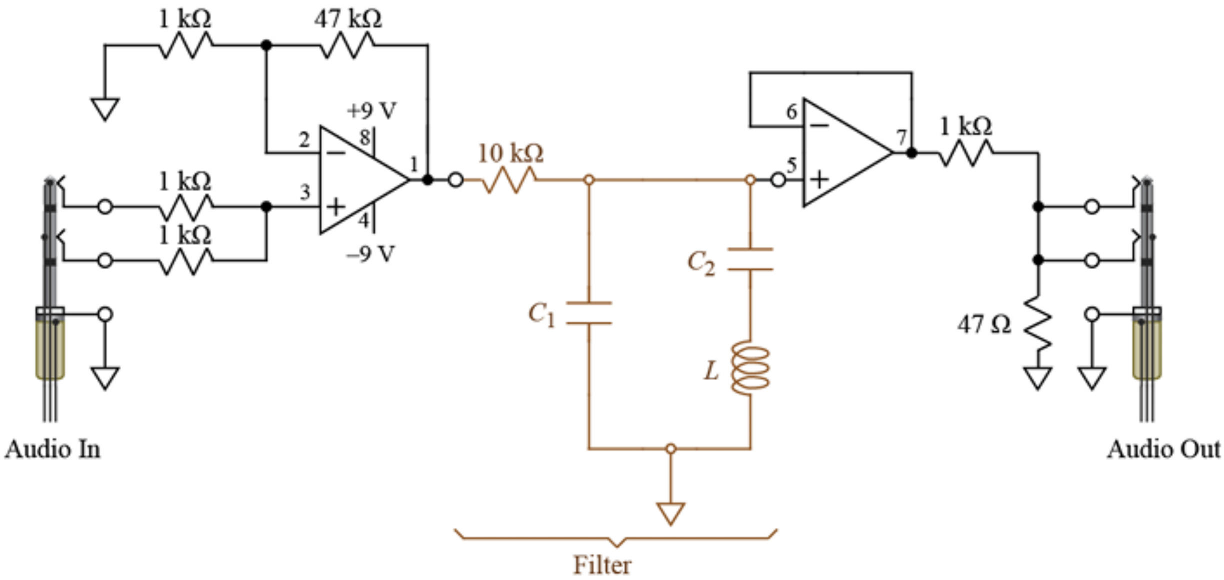

Figure 3: Circuit for audio signals