Instructions

- Answer all questions clearly and completely.
- Use appropriate figures, labels, and legends where necessary.
- Report all calculated probabilities up to 4 decimal places.
- Discuss any observed discrepancies between theoretical and simulated results.

Question 1: Probability Estimation via Histogram and Thresholding

You are given a Gaussian random variable:

$$x \sim \mathcal{N}(A, \sigma^2)$$

where $A = 5$ and $\sigma^2 = 4$.

1. Simulate $N = 10^6$ samples of x and plot the histogram using 20 bins.
2. Compute the simulated probability P_{sim} and theoretical probability P_{the} that $x > 7.34$.
3. Compute the simulated and theoretical probability that $x < 3.81$.
4. Compute the simulated and theoretical probability that $1.87 < x < 3.49$.
5. Discuss any discrepancies between simulated and theoretical probabilities. What factors may cause the differences?

Question 2: Binary Detection and Error Probability Analysis

Consider binary communication where the transmitted symbols are $+A$ and $-A$ based on bit values 1 and 0, respectively.

1. Simulate $N = 10^6$ transmitted bits with prior probability $q = 0.2$ for bit-0 and $1 - q$ for bit-1.
2. Add Gaussian noise of variance $\sigma^2 = 4$ to the transmitted symbols.
3. For thresholds T varying from -10 to 10 (with a step size of 0.5), detect the bits and calculate the **simulated probability of error** for each threshold.
4. Derive and plot the **theoretical probability of error** for each threshold.
5. Plot both curves (simulated and theoretical) on a semi-logarithmic (semilogy) scale.
6. Explain the behavior of probability of error as a function of threshold.

Question 3: Optimal Threshold Estimation

1. Derive the optimal threshold T^* that minimizes the probability of error theoretically, given by:

$$T^* = \frac{\sigma^2}{2A} \log\left(\frac{1-q}{q}\right)$$

2. From your simulation results, find the threshold T^*_{sim} that minimizes the simulated probability of error.
3. Compare T^* and T^*_{sim}. Are they close? If not, explain why.
4. Mark both T^* and T^*_{sim} on your semilogy plot from Question 2.
5. Discuss the practical importance of selecting an optimal threshold in communication systems.