Q1

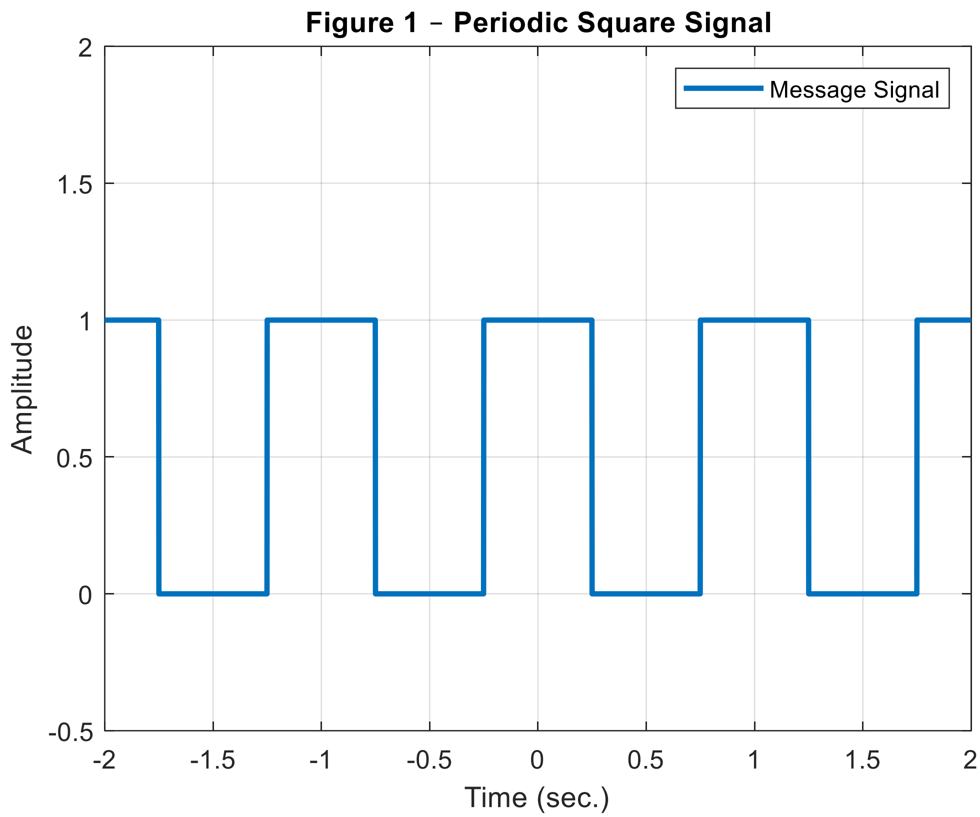

Figure 1 - Periodic Square Signal

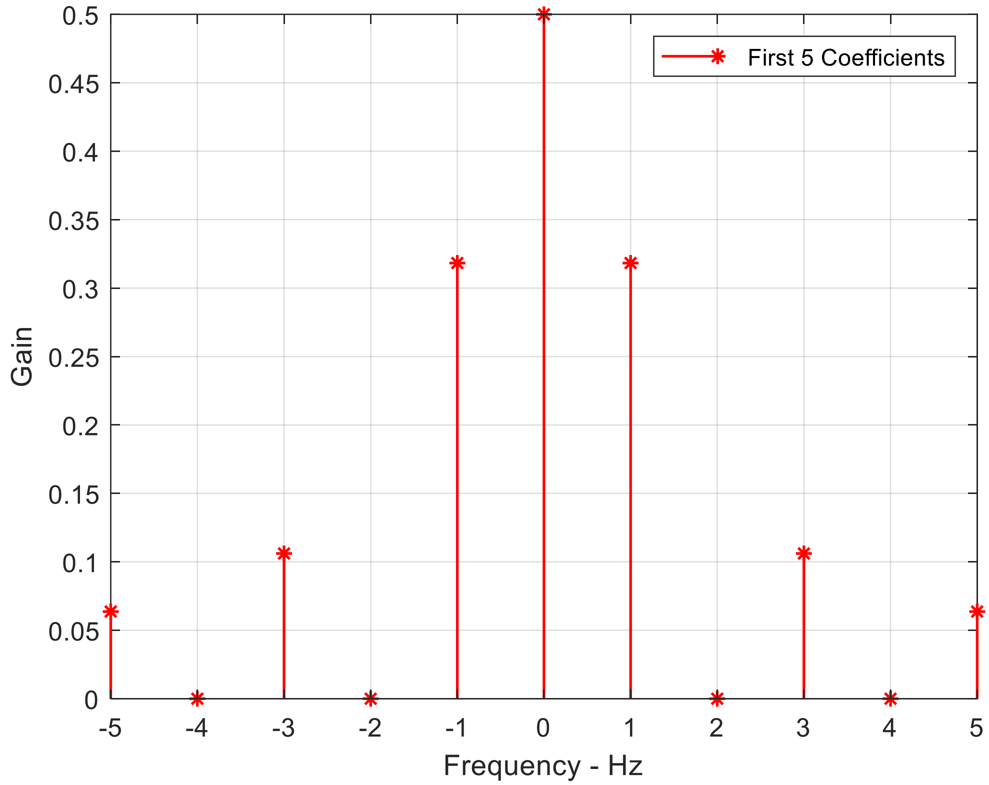

Figure 2 - First 5 Coefficients

A) Plot the first five of x(t) as a function of the frequency

When the first five harmonics (the fundamental frequency and the following four harmonics) are examined, it is seen that odd harmonics dominate in the Fourier series of the square wave. In the amplitude spectrum, it is observed that the amplitudes are higher at certain harmonics (especially odd harmonics) and close to zero (or very small) at even harmonics. This is a characteristic feature of square waves, because the amplitude typically decreases in the form "4/(nπ)" (only for odd n values).

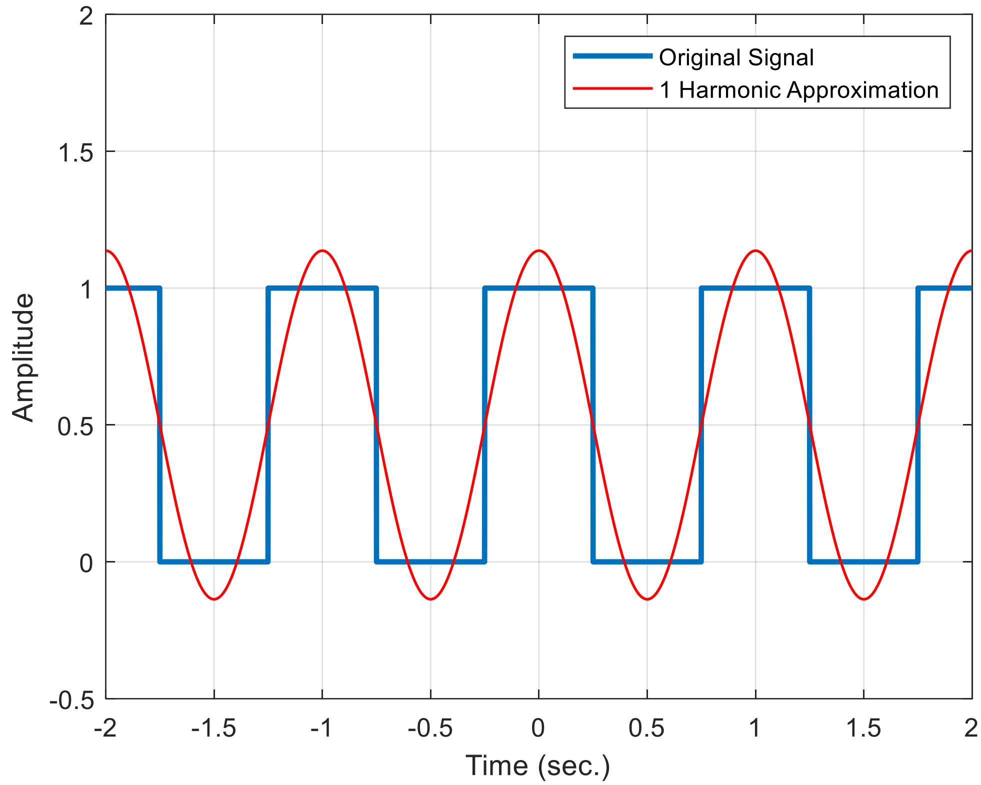

Figure 3 - First 1 Harmonic Approximation

B) Plot the first harmonic approximation of x(t) and the original signal on top of each other as a function of time and comment on your results

When only the first harmonic (the fundamental frequency component) is used, the obtained approximation resembles a sine wave rather than a square wave. Since the sharp transitions in the wave are almost not captured at all, there are significant differences compared to the original square signal. Therefore, it is not possible to accurately represent the square wave using only the fundamental frequency.

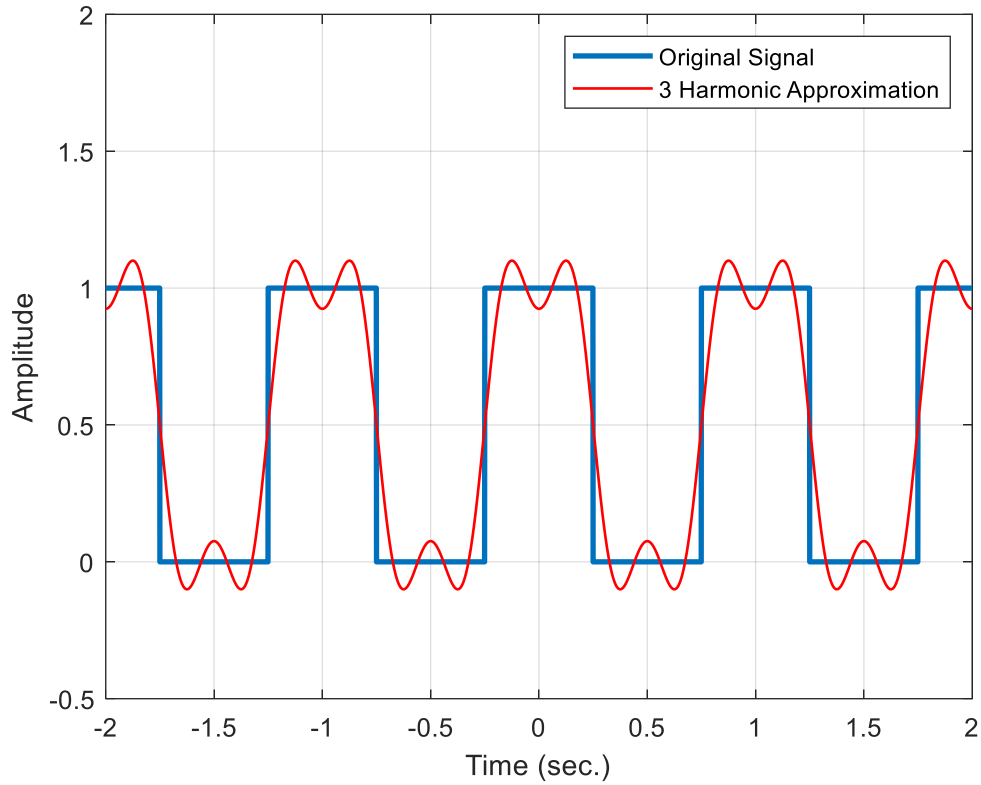

Figure 4 - First 3 Harmonic Approximation

C) Plot the first three harmonics approximation of x(t) and the original signal on top of each other as a function of time and comment on your results

When the first three harmonics are added (the fundamental, 3rd, and 5th harmonics, i.e., the odd harmonics), the shape of the signal in the time domain becomes somewhat closer to the square wave. In particular, there is a more pronounced “flattening” in the peak and trough regions of the wave, and a sharper curve in the transition regions. Nevertheless, the sharp transitions are still not fully captured, and it is observed that there are oscillations similar to the “Gibbs phenomenon” at the corners of the wave, and a fully square shape is not formed.

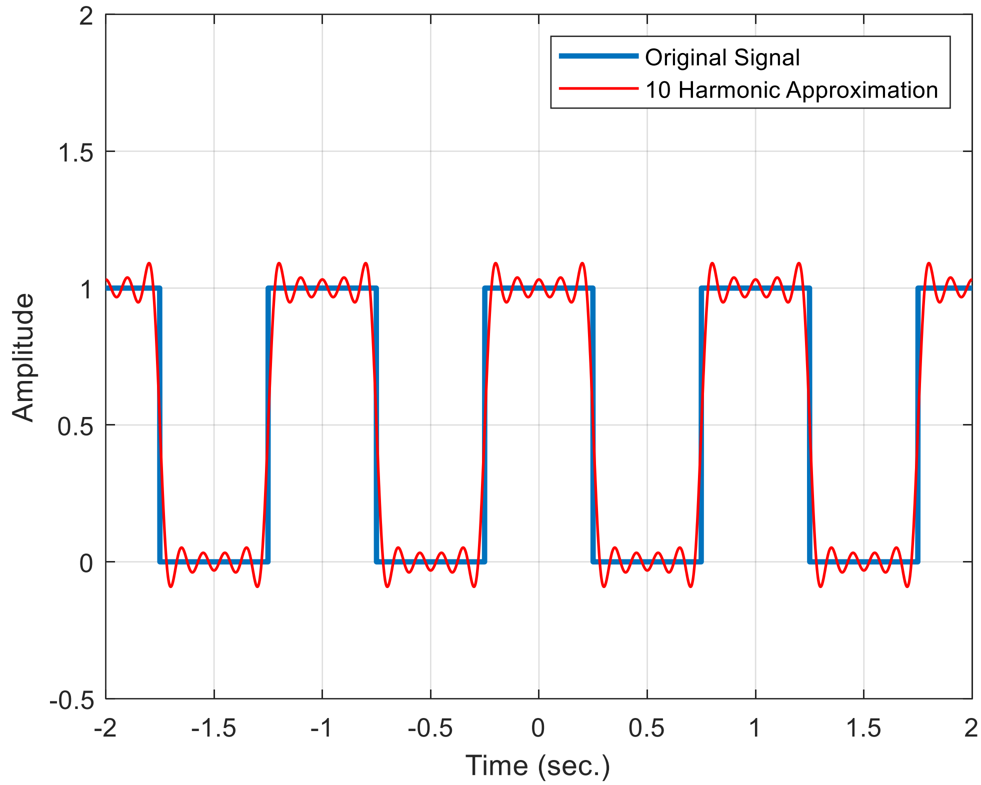

Figure 5 - First 10 Harmonic Approximation

D) Plot the first ten harmonics approximation of x(t) and the original signal on top of each other as a function of time and comment on your results

When ten harmonics are used (especially recalling that the odd harmonics are dominant), a shape quite close to the square wave is obtained. Although the signal still does not form perfectly vertical corners during the rising and falling edges, it becomes very similar to the original. Small oscillations the “Gibbs phenomenon” can still be observed at the corners, but the increasing number of harmonics makes the signal approach a clearer square form.

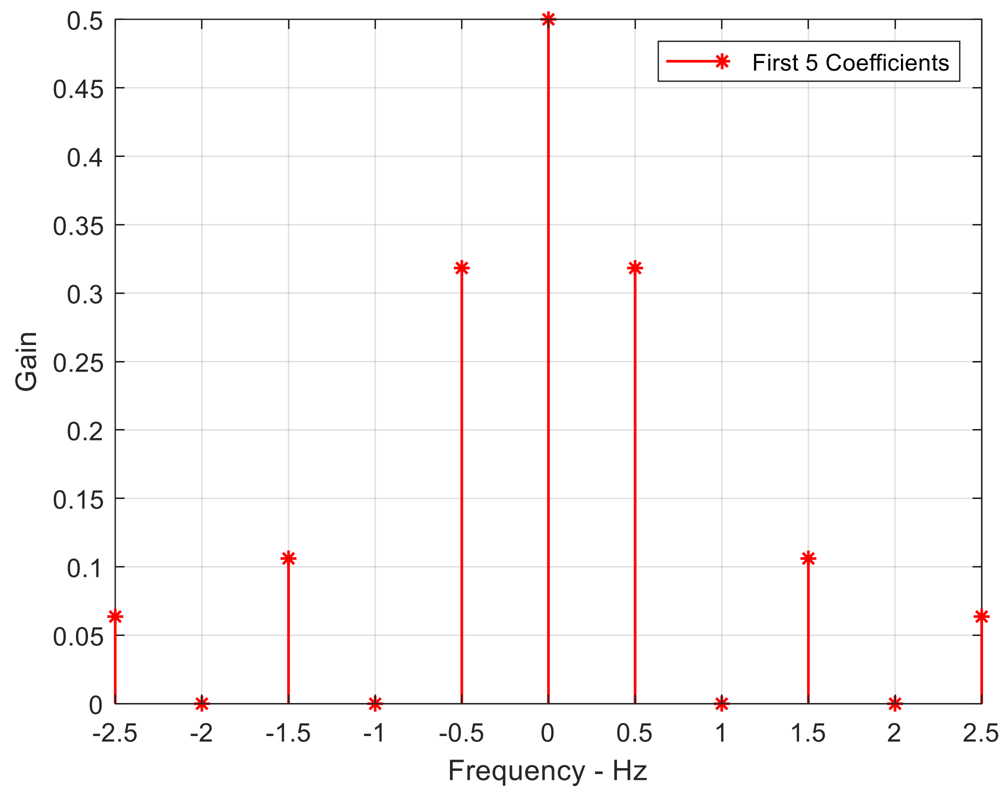

Figure 6 - First 5 Coefficients (0.5 Hz)

E) In part (a), what happens when you change the fundamental frequency to 0.5 Hz? Comment on your results

Reducing the fundamental frequency from 1 Hz to 0.5 Hz doubles the period of the signal. Therefore, in the frequency spectrum, instead of being more closely spaced, the harmonic components appear more spread out (closer together in frequency terms), because the harmonics are now located at 0.5 Hz, 1 Hz, 1.5 Hz, 2 Hz, etc. Since the amplitude values belong to the same square wave form, they show a similar distribution (such as the dominance of odd harmonics), but their positions on the frequency axis are halved. As a result, the wave oscillates more “slowly,” and accordingly, the period of the square wave in the time domain becomes longer.

Q2

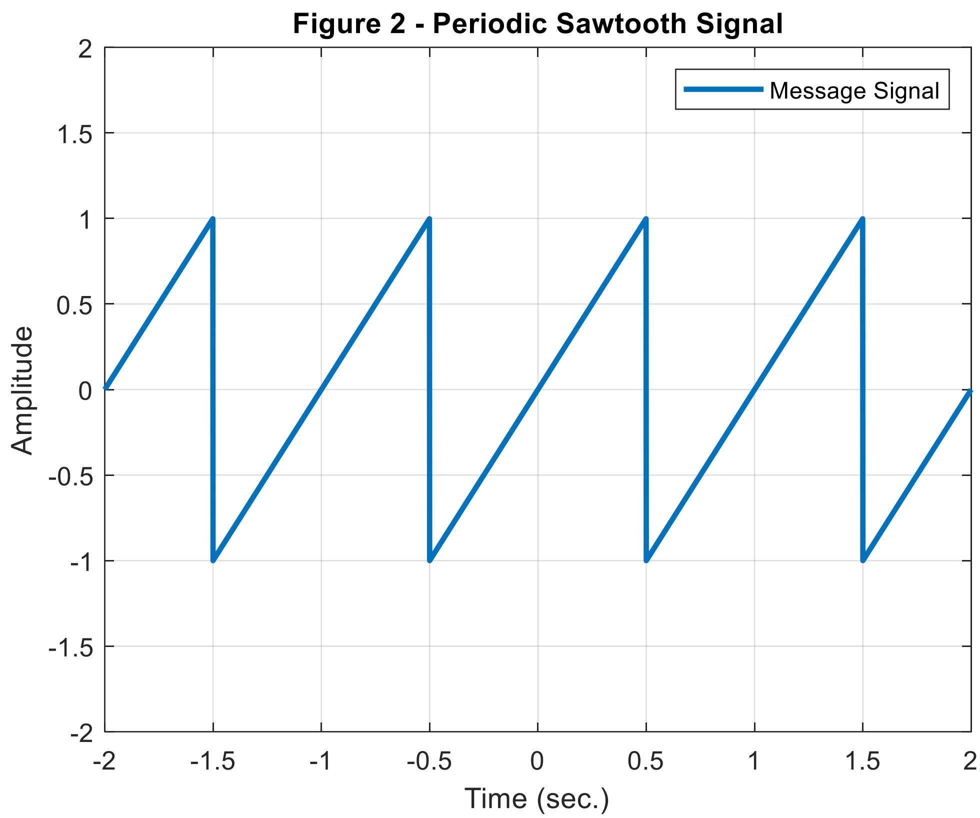

Figure 7 - Periodic Sawtooth Signal

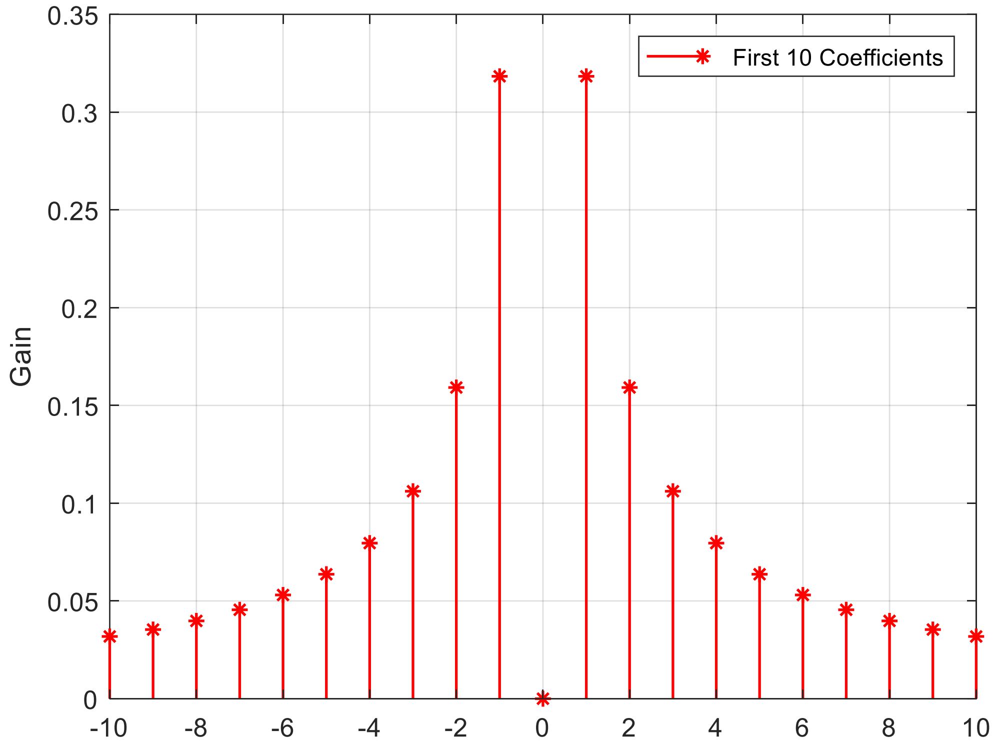

Figure 8 - First 10 Coefficients

A) Plot the first ten of y(t) as a function of the frequency

In this section, we examine the first ten harmonic components (including the fundamental frequency) of the sawtooth wave on the frequency axis. In the Fourier series of the sawtooth wave, each harmonic (both odd and even) has a certain amplitude, and this amplitude typically tends to decrease on the order of “1/n”. Therefore, when looking at the frequency spectrum, it is observed that the amplitude values decrease as the harmonic order increases. Also, non-zero coefficients can be seen for both even and odd harmonics (unlike the square wave, which only has odd harmonics).

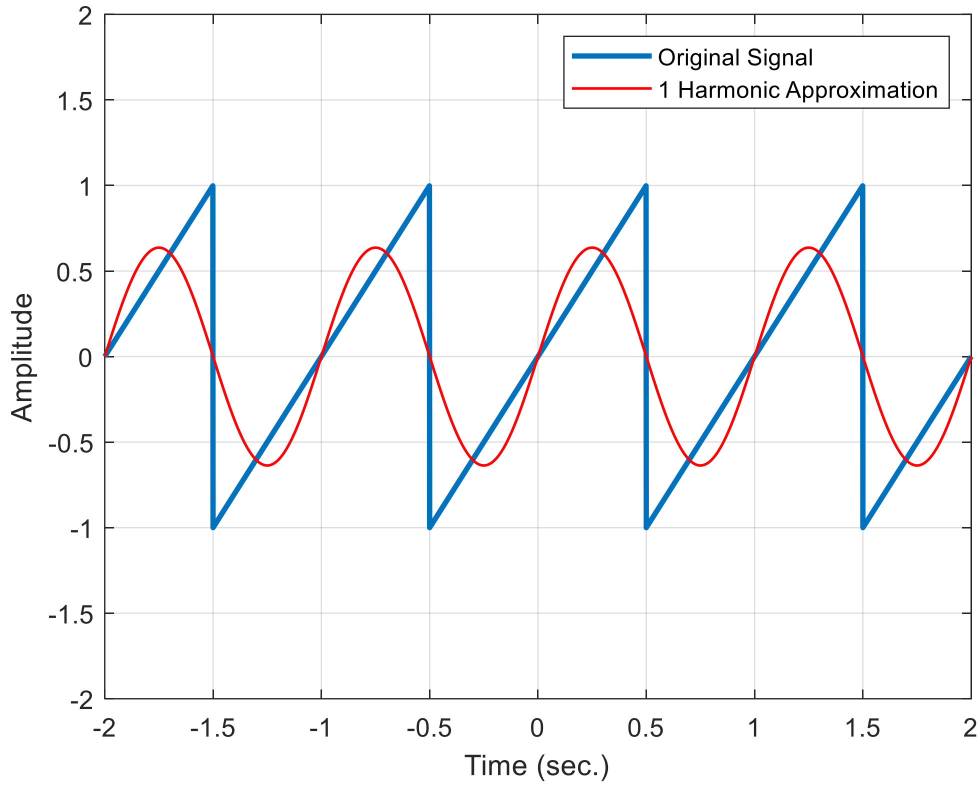

Figure 9 - First 1 Harmonic Approximation

B) Plot the first harmonic approximation of y(t) and the original signal on top of each other as a function of time and comment on your results

When only the first harmonic (the fundamental frequency) is used, the obtained approximation is a single sine wave. The sawtooth wave is a wave that increases linearly and makes a sudden jump (discontinuity) at the end of the period. This linear increase and sudden transition behavior cannot be captured with a single sine component. Therefore, when plotted on top of each other in the time domain, a noticeable difference is observed compared to the original signal (especially the slope of the rising edge and the jump points are inconsistent).

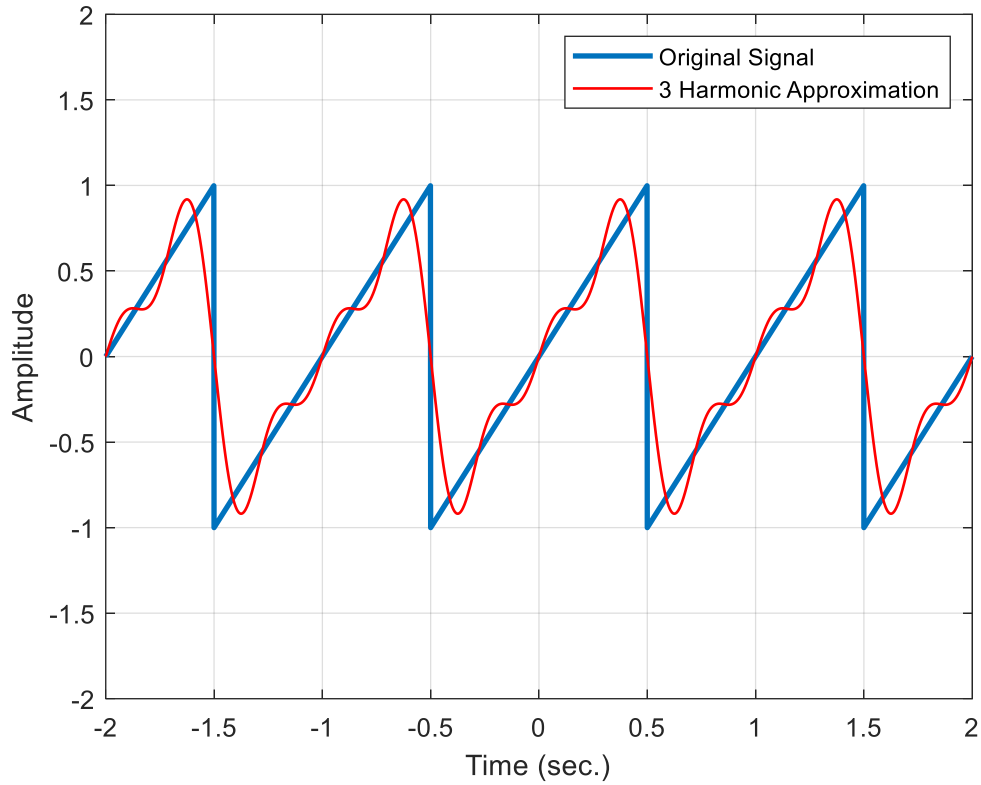

Figure 10 - First 3 Harmonic Approximation

C) Plot the first three harmonics approximation of y(t) and the original signal on top of each other as a function of time and comment on your results

When the first three harmonics (the fundamental, 2nd, and 3rd harmonics) are used, the shape of the sawtooth wave is represented somewhat better. In particular, the linear part of the rising slope is captured more accurately, and compared to the initial case, the signal begins to resemble the sawtooth wave more. However, it still does not achieve full accuracy at the point where the wave "breaks" periodically (where it jumps from the highest value to the lowest), and a visible difference remains.

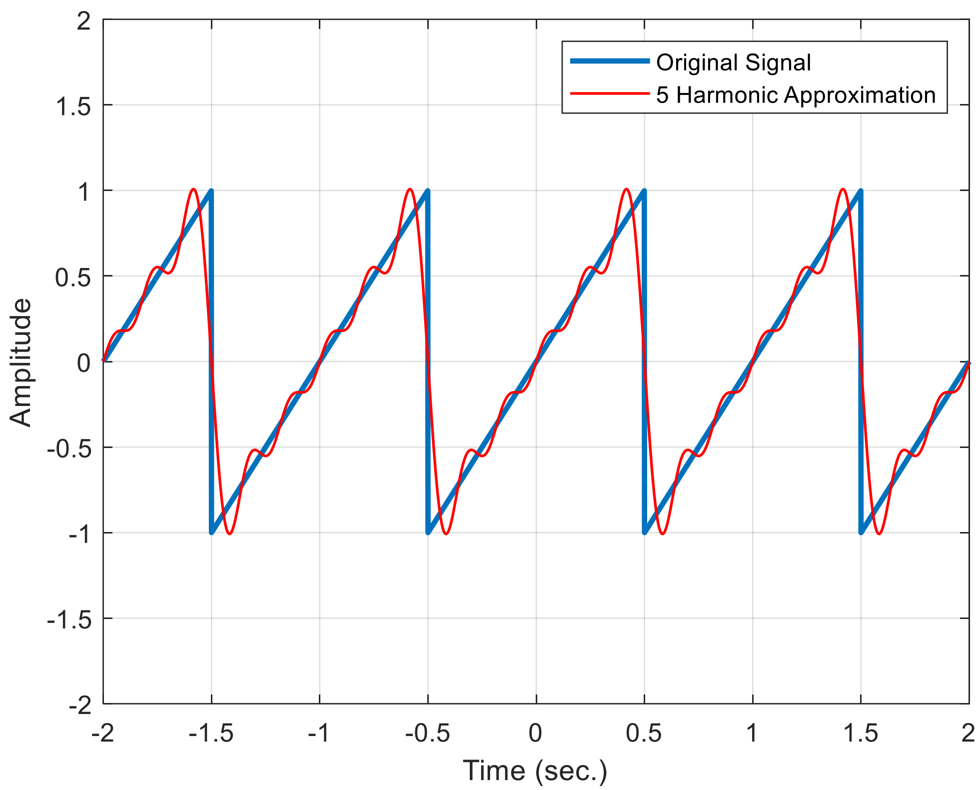

Figure 11 - First 5 Harmonic Approximation

D) Plot the first five harmonics approximation of y(t) and the original signal on top of each other as a function of time and comment on your results

The approximation using five harmonics yields a form much closer to the sawtooth wave. The linear rising slope of the wave becomes smoother; however, at the sharp transition point at the end of the period, there is still not a completely "vertical" jump, but rather a slight softening (or small oscillations). Nevertheless, it is clearly observed that the signal is closer to the original sawtooth wave compared to cases with fewer harmonics. As the number of harmonics increases, both the rising slope is represented more accurately and the deviation at the jump point decreases.