# Application of Mathematical Models on Smart Grid

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## I. Abstract

The mathematical model is the explanation of the determined problem through mathematics. It can be used for engineering and other sciences. The whole process of creating a mathematical model is called mathematical modeling **[1].**

While creating the mathematical model of the events taking place in nature, a mathematical model can also be created for human situations. Also in this study, the goal was to create a mathematical model for the Smart Grid. We know that every work to be done on this subject is very important these days when the use of Smart Grid has become increased.

It is also known that the creation of the mathematical model is very successful and important in terms of supporting other scientific stages and understanding them. It is very important what the parameters are and for what purposes they will be used together with the model created. We have completed our mathematical model with the parameters that can be changed in our project and the constraints that we have created as a result of careful research.

Our mathematical model has been completed by considering all the needs and demands, especially for HEMS. As can be seen in the explanations we have given in detail in the following chapters, we have created a mathematical model with applicable, changeable parameters as a result of our studies.

## II. Abbrevıatıons

WoutLSNOPV: Without Load Shifting No Photovoltaic

WithLSNOPV: With Load Shifting No Photovoltaic

WoutLSPV: Without Load Shifting and Photovoltaic

WithLSPV: With Load Shifting and Photovoltaic

PAR: Peak to Average Ratio

PV: Photovoltaic

HEMS: Home Energy Management System

## III. Introduction

Smart grids enable two-way communication between the consumer and the supplier manufacturer with the help of smart meters and other smart tools. With these smart tools, it is ensured that the communication between the producer and the consumer is fast and secure. Considering the advantages of the smart grid, it has been observed that the demand for it has increased. **[2]**

Recently, with the development of technological studies, changes have occurred in the supply-demand balance. Here, the demand for fast and reliable has also manifested itself in grid applications. It is known that the importance of the smart grid, where classical grid applications are insufficient, is increasing day by day, especially in developed countries. According to reference with the number **[3]**, the importance given to Smart grid applications by developed countries is given.

In the paper **[4]**, the importance of the locations of the grids is mentioned. Accordingly, it was stated that centralized grids cause more environmental pollution than dispersed grids. It is also known that centralized grids cause more losses than distributed grids. However, the Smart grid has become even more important to minimize the losses in electrical energy coming to the consumer. In addition to these, it is obvious that innovative studies are needed because classical grids cause unplanned consumption.

Considering both cases, the idea of smart grids has emerged to distribute electricity and use it optimally. With these ideas, the importance of the smart grid has increased in recent days, as the losses in transmission decrease and it enables consumers to save electricity effectively. With smart grids, the dependency on non-renewable energy sources is reduced, thus minimizing the high carbon emission problem. It also enables the home to have its own Home Energy Management System (HEMS) by connecting to smart homes with micro-grids. The production of solar energy in the places formed by these connections and the management of its relationship with the battery and the grid is done by HEMS. On the other hand, the HEMS operating system requires forecasts, self-decision mechanisms, and smart management. Model network mechanism and planning are needed for the formation of this forecasting model **[5].**

The authors have ensured that the developed model is applicable to the constraints which are determined previously. In this study, the possible demand situation of the customers has been determined and constraints have been determined in a way to offer optimum solutions for these demands. Our approach was to investigate the articles which include mathematical model derivations used in smart grid applications from reliable databases such as IEEE Xplore, Web of Science, Scopus et cetera by making those examinations, we improved our understanding of the smart grid day by day.

At the very beginning, we searched the world wide web to acquire information about smart grid applications, then, by the instructions of our instructor, we focused on the mathematical models developed for the smart grid.

In one of the guideline papers, for the first time, we observed the terms ‘peak clipping’, ‘valley filling’, and ‘load shifting’ which are the most common approaches used in smart grid systems. **[6]**. In the same paper, we observed a brief mathematical model and how it was represented. Additionally, we learned some of the methods being applied in the smart grid to organize demand response.

Then, we have been tasked to examine one specific mathematical model by explaining its objective function, its constraints, and its additional algorithm. To complete this task, we searched for various papers which will be cited in the literature survey section, and finally, we presented what we have found. This process also helped us to improve our understanding of the mathematical model developing for the HEMS field of the smart grid. After all these efforts, we managed to come up with mathematical models with their proper objective functions and constraints.

For our mathematical model, we first made a general literature search and then compared similar topics. Then, as a result of the research, we carefully determined the constraints and determined the stages to be used while creating the mathematical model in further study. After this process, a mathematical model was created with the determined constraints. In addition, we have reached multiple sources for the data we would include in our study, and we have added the most appropriate and reliable ones to our article.

The benefit of modeling the methods is that they provide us with a way of representing real-life applications, and we can adjust the parameters of those models to adapt them for the benefit of the customer.

With these studies, it is aimed to create the most suitable solution-oriented mathematical model for the consumer and the supplier manufacturer, and this aim was achieved at the end of the project.

# IV. Literature Survey

Before starting our work on designing the models we read and analyzed various research papers from scientific journals such as IEEE, Science Direct, and various university libraries.

| References | Field of Study | Objective Functions | Additional Algorithms | Applied Case | Results | Notes |
|---|---|---|---|---|---|---|
| Home Energy Management Strategy for DR Accomplishment Considering PV Uncertainties and Battery Energy Storage System" by Waseem, M.; Waqas, A.B.; Ali, Y.; Khan, D.; Faheem, | HEMS | Minimize Electricity Cost | CS Optimization | 5 Different simutlation. Variables: PV, ES, H2G | %71.4 Cost Reduction | |
| Optimal Time of Use Electricity Pricing Model and Its Application to Electrical Distribution System HEJUN YANG , (Member, IEEE), LEI WANG, AND YINGHAO MA, (Member, IEEE) | | Minimizing the peak-valley difference, the voltage fluctuation, and the power loss. | particle swarm optimization (PSO) optimal period partitioning algorithm | IEEE 14-bus system | %22.2 smoother load curve | |
| Integrated Optimization of Smart Home Appliances with Cost-effective Energy Management System Tesfahun Molla, Baseem Khan, Member, IEEE, Bezabih Moges, Hassan Haes Alhelou, Reza Zamani, and Pierluigi Siano, Senior Member, IEEE | HEMS | Minimizing energy cost, peak-to-average ratio (PAR) and peak load demand | grey wolf optimizer (GWO) particle swarm optimization (PSO) | Eight different cases are considered (Letting user decide on times, ignoring user etc.) | Ignoring user dissatisfaction and letting algorithm assign the appliances to the best times, PAR is almost 1 with GWO and 2.8 with PSO, which also minimizes the energy cost a lot | A rooftop photovoltaic (PV) system is integrated with the system to show the cost effectiveness of the appliances. |
| DETERMINATION OF OPTIMAL DIRECT LOAD CONTROL STRATEGY USING LINEAR PROGRAMMING | Load Management | Minimize peak load ($P_p$) | Linear Programming | Different real power system cases where water heaters and air conditioners are controlled at the same time. | In the first case, controlled 100000 air conditioners and the same number of water heaters where reduction is 3.3%. In the second case, controlled 200000 air conditioners and water heaters, this reduction is 5,5 %. In the third case, controlled 300000 air conditioners and water heaters, the reduction amounts to 7,2%. | As a result of mathematical model that is applied to the power grid, the power demand load curve is more linear, in other words stable. Therefore, the peak is reduced and shifted to other hours of day. |
| A NEW HEURISTICALLY OPTIMIZED HOME ENERGY MANAGEMENT CONTROLLER FOR SMART GRİD | HEM System | Minimize Electricity Cost | BFOA, GA, BPSO, WDO,GBPSO | | BFOA =6.99%, GA = 4.2%, BPSO=36.87%, WDO =32.29%, GBPSO=37.76% Cost Reduction | |
| Optimization in Energy Management Systems,mathworks | HEMS and generalized EMS | Minimize Electricity Cost | Linear programming,parse optimization | Matlab code application in simulink, where the power from the grid, the energy from the battery, the energy from the solar panel is taken as variable and the cost is optimized. | 25% cost reduction | |
| A Mathematical Programming Formulation for Optimal Load Shifting of Electricity Demand for the Smart Grid----R. Lily Hu , Member, IEEE, Ryan Skorupski, Robert Entriken, Senior Member, IEEE, and Yinyu Ye---- IEEE Xplore | HEMS | Minimizing the Electricty Cost by Shifting the End-Use Loads from On-Peak Times to Off-Peak Times | Linear Programming | The real electricity data of Boston, MA has been used. | At the end of one week, a total of 4.63% money saving observed | In this paper, there are 4 more mathematical models which can be investigated later. |

*Table 1: Literature Survey Table*

# V.Parameters

$P_a(h)$=*How much power an appliance consumes each time interval,*

$p_{ij}^{k}$=*Energy assigned to energy phase j of appliance i during the whole period of time slot k*

$E_{ij}$=*Energy requirement for energy phase j for appliance i*

$B_a$=*Working hours array of an appliance a day.*

$B_a(h)$=*Working or not each time interval*

$B_a1$=*Number of ones there are in Ba(h)*

$E_a(h)$=*How much energy an appliance consumes each time interval*

$E_{SI}(h)$=*Total Energy Consumption of Shiftable and Interruptible Appliances Each Time Interval*

$E_{SNI}(h)$=*Total Energy Consumption of Shiftable and Non-Interruptible Appliances Each Time Interval*

$E_R(h)$=*Total Energy Consumption of Regular Appliances Each Time Interval*

$E_{final}(h)$=*Total Energy Consumption of All Appliances of Home Each Time Interval*

$S_n(h)$=*Solar Power Generation Capacity*

$E_{solar}(h)$=*Energy Generated by Photovoltaic Cell each time interval*

$E_{battery}(h)$=*Stored energy in battery each time interval*

$E_{grid}(h)$=*Total Energy Consumed from Grid*

$E_{gridmax}(h)$=*Maximum Energy Capacity of Timeslot 'h'*

$E_{load}(h)$=*Total Energy Consumed by User in each time interval*

$E_{battery}^{cap}$=*Battery Capacity*

$E_{battery}^{cap}$=*Total Energy in Battery*

$C_a(h)$ = *How much money an appliance makes each time interval*

*CSI = Cost of Shiftable and Interruptible appliances*

*CSNI = Cost of Shiftable and Non-Interruptible appliances*

*CR = Cost of Regular Appliances*

*M = Array of Average Sunbathing Time in Hours*

$S_p$= *Array of different cells with different power capacities(kW)*

*n= Number of solar panels*

$T_{app}$ = *How many hours an appliance Works*

$P_{battery}$ = *Power supplied by the battery*

$P_{solar}$ = *Power produced by solar panels*

$P_{invAC}$ = *Power on the inverter's AC bus*

$\mu_{dc2ac}$=*Performance of inverter*

$P_{grid}$ = *Power supplied from the grid*

$Pl_{oads}$ = *Power consumed by appliances*

$SOC_{gel}$ = *State of Charge for Gel Battery*

$SOC_{li}$= *State of Charge for Li-Ion Battery*

$SOC_{Pb}$ = *State of Charge for Lead Acid Battery*

$f_{diss}$=*Total Dissatisfaction Level Caused by All Appliances*

$f_{diss}^{SI}$ =*Dissatisfaction Level Caused by Shiftable and Interruptable Devices*

$f_{diss}^{SNI}$=*Dissatisfaction Level Caused by Shiftable and Non-Interruptable Devices*

$S_a(h)$=*Score Evaluating the Dissatisfaction Level Caused*

*by Operating Shiftable Appliance "a" at Timeslot "h"* (*0-5*)

$Pr_a$=*User Preference Array for* $a^{th}$ *Appliance*

*Final=Scalar Multiplication Array of User Preference Arrayand "Working Hours of an Appliance"*

$\beta_{ch}$ = *Battery's charge efficiency*

$\beta_{dch}$ = *Battery's discharge efficiency*

$P_{battery}^{ch}(h)$= *Battery charging power*

$P_{battery}^{dch}(h)$= *Battery discharging power*

$P_{ch,min}$= *Minimum battery charging power*

$P_{ch,max}$= *Maximum battery charging power*

$P_{dch,min}$ = *Minimum battery discharging power*

$P_{dch,max}$ = *Maximum battery discharging power*

## VI. Case Studies

In this section, we divided it into 2 different cases using a mathematical model and a PV system for the design of smart energy home systems. The tools that are used in these models are different from the non-smart energy home systems, with the mathematical model we created, we tried to shift the adjustable household appliances from the time when the electricity prices were high to the times when they were lower (CASE-1). In addition, we aimed to benefit from solar energy, to reduce energy costs, and with the energy storage system, we aimed to store the energy obtained from the sun for use to drive household appliances later (CASE-2).

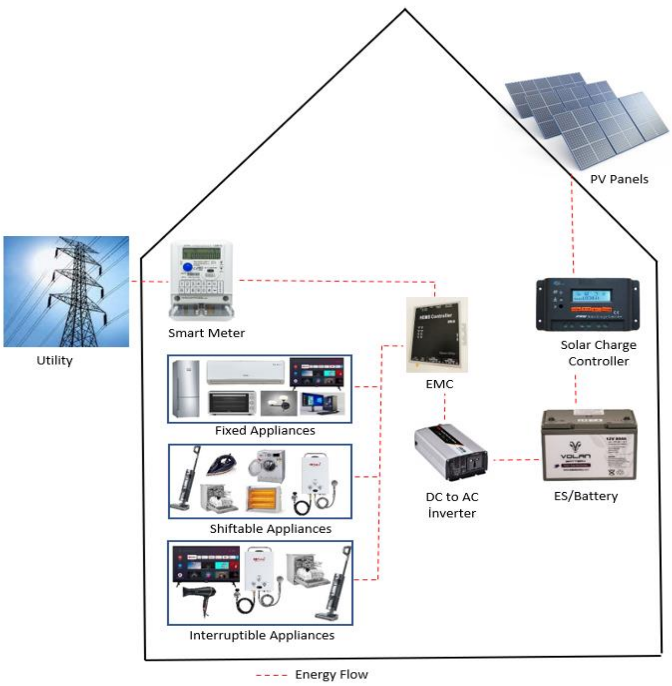


*Figure 1: HEMS with PV Cell and Storage*

## a. Case-1: Only Utility

In the first case, using the mathematical model we proposed in the first case, we aimed to shift the electricity consumed by users from times when electricity demand is high, that is, where electricity prices are high, to times where electricity prices are low. By using the proposed mathematical model, it is aimed to operate the adjustable household appliances in accordance with the 3-time electricity tariff in Turkey. Thus, it has been studied on the minimization of the cost that will be reflected on the electricity bill.

<u>*Appliance Types*</u>

*Shiftable and Interruptible **SI***

These appliances can be shifted to any time slot and can be interrupted when required. These appliances include. **[7].**

- Vacuum Cleaner
- Water Heater
- Water Pump
- Dish Washer
- Hair Dryer

*Shiftable and Uninterruptible **SNI***

These appliances can be shifted to any time slot but once start their operation, they must complete their operation without interruption. These appliances include. **[7].**

- Washing Machine
- Cloth Dryer

*Regular Appliances **R***

These appliances are also called thermostatically controlled appliances because their operation depends on temperature. These appliances include. **[7].**

- Air Conditioner
- Refrigerator
- Oven (stove)

*Mathematical Models: Load Shifting, Minimization of Dissatisfaction*

**<u>Objective Functions</u>**

$$Min\ Cost = Min\ [CSI\ +\ CSNI\ +\ CR] = min\ \sum_{h=0}^{23}\left(E_{final}(h) * c(h)\right)$$

$$min\ f_{diss} = min\ (\ f_{diss}^{SI} + f_{diss}^{SNI}\ )$$

## Constraints

*Electrical demand-supply balance:*

Considering load shifting,

$$E_{final}(h) \le E_{grid}(h) \quad (\text{load always met by grid}) \qquad (1)$$

$$E_{final}(h) = E_{SI}(h) + E_{SNI}(h) + E_{R}(h) \qquad (2)$$

$$CSI = \sum_{h=0}^{23} E_{SI}(h) \times c(h) \qquad (3)$$

$$CSNI = \sum_{h=0}^{23} E_{SNI}(h) \times c(h) \qquad (4)$$

$$CR = \sum_{h=0}^{23} E_{R}(h) \times c(h) \qquad (5)$$

$$E_{SI}(h) = \sum_{a=1}^{a=amax} \left(P_a(h) * B_a(h)\right) \qquad (6)$$

$$E_{SNI}(h) = \sum_{a=1}^{a=amax} \left(P_a(h) * B_a(h)\right) \qquad (7)$$

$$E_{R}(h) = \sum_{a=1}^{a=amax} \left(P_a(h) * B_a(h)\right) \qquad (8)$$

$$E_{load}(h) = E_{final}(h) \quad (\text{ illegal electric usage}) \qquad (9)$$

*Grid Constraints*

$$0 \le E_{grid}(h) < E_{gridmax}(h) \quad (\text{supplied energy amount}) \qquad (10)$$

$$0 \le \sum_{a}^{\text{all appliances}} E_a(h) * Ba(h) \le E_{gridmax}(h) \quad (\text{supplied energy amount}) \qquad (11)$$

$$ceil\left(T_{app}\right) = Ba1 \qquad (12)$$

Phase wise energy requirement of appliances,

*Appliance operation cycle energy:* $\sum_{k=1}^{m} p_{ij}^{k} = E_{ij}, \quad \forall i, j \qquad (13)$

*Power Safety*

*Total required energy*: $\sum_{h=0}^{23} E_{final}(h) \le E_{grid,max}(h) \qquad (14)$

**User Dissatisfaction:**

Here the aim is to minimize the dissatisfaction level of the users caused by shifting the devices.

$$f_{diss} = f_{diss}^{SI} + f_{diss}^{SNI} \qquad (15)$$

Where $f_{diss}^{SI}$ represents the dissatisfaction level caused by shiftable-interruptible devices and $f_{diss}^{SNI}$ represents the dissatisfaction level caused by shiftable-uninterruptible devices.

$$f_{diss}^{SI} = \sum_{a=1}^{total\ \#\ of\ SI\ devices} \sum_{h=1}^{h=24} S_a(h)*Ba(h) \qquad (16)$$

$$f_{diss}^{SNI} = \sum_{a=1}^{total\ \#\ of\ SNI\ devices} \sum_{h=1}^{h=24} S_a(h)*Ba(h) \qquad (17)$$

*Appliances work or not*

$B_a(h)= [b_a^1, b_a^2 \ldots\ldots B_a^{24}]$ -> Every Shiftable appliance has this binary array.

$B_a(h)$ = if the appliance works 1, if not 0 between (0 AM – 1 AM)

*Dissatisfaction Score*

$S_a(h)$ = This is a score evaluating the dissatisfaction level caused by operating shiftable appliance “a” at timeslot “h”, which is determined by customers according to their living arrangement.

$S_a(h)$ ranges from 0 to 5 (integer values). A larger value of $S_a(h)$ means a higher dissatisfaction level **[8]**.

As a summary, the objective function of this mathematical model (15) is to minimize the summation of the two equations that lies above (16) and (17).

$$Pr_a * Ba = Final \qquad (user\ preference\ arrayed) \qquad (18)$$

### b. Case-2: Utility + PV Panels + Storage (Battery)

**Solar Package System**

In today’s world, the prosumer term is a popular word that means the consumer who also is a producer. In other words, each consumer that consumes electricity can produce its own need by integrating photovoltaic (PV) cells et cetera. **[9].**

Also, the electricity produced by PV panels can be stored in batteries to use at efficient time intervals to decrease the electricity bill of the day. In the scope of the second case, the primary aim is to minimize the daily cost of electricity by using PV panels and their solar batteries with corresponding capacity. We called the system consisting of *PV panels and solar battery* as “solar package system”.

To actualize the second case, there will be some models and concepts.

*Mathematical Models: Load shifting, Piece-wise Energy Distribution, Minimization of Dissatisfaction*

Like the first case, load shifting method will be used to control using of efficient (cost effective) time interval via SI and SNI. Piece-wise energy distribution is used to create an original formula to minimize the electricity to be drawn from the grid and the daily cost of dependent energy. Minimization of dissatisfaction used to evaluate users' preferences.

**Objective Functions**

$$Min\ Cost = Min\ [CSI\ +\ CSNI\ +\ CR] = min\ \sum_{h=0}^{23}\left(E_{grid}(h) * c(h)\right)$$

$$min\ f_{diss} = min\ (\ f_{diss}^{SI} + f_{diss}^{SNI}\ )$$

**Constraints**

*Electrical demand-supply balance:*

Considering load shifting,

$$E_{final}(h) \leq E_{grid}(h)\ \ (load\ always\ met\ by\ grid) \qquad (1)$$

$$E_{final}(h) = \quad E_{SI}(h) + E_{SNI}(h) + E_{R}(h) \qquad (2)$$

$$CSI=\sum_{h=0}^{23} E_{SI}(h)\times c(h) \qquad (3)$$

$$CSNI=\sum_{h=0}^{23} E_{SNI}(h)\times c(h) \qquad (4)$$

$$CR=\sum_{h=0}^{23} E_{R}(h)\times c(h) \qquad (5)$$

$$E_{SI}(h)= \sum_{a=1}^{a=amax} \left(P_a(h)*B_a(h)\right) \qquad (6)$$

$$E_{SNI}(h)= \sum_{a=1}^{a=amax} \left(P_a(h)*B_a(h)\right) \qquad (7)$$

$$E_{R}(h)= \sum_{a=1}^{a=amax} \left(P_a(h)*B_a(h)\right) \qquad (8)$$

$$E_{load}(h)=E_{final}(h)\ \ (\ illegal\ electric\ usage) \qquad (9)$$

*Grid Constraints*

$$0\leq E_{grid}(h)<E_{gridmax}(h) \quad (supplied\ energy\ amount) \qquad (10)$$

$$0 \leq \sum_{a}^{all\ appliances} E_a(h)*Ba(h) \leq E_{gridmax}(h) \quad (supplied\ energy\ amount) \qquad (11)$$

$$ceil\left(T_{app}\right)=Ba1 \qquad (12)$$

Phase wise energy requirement of appliances,

*Appliance operation cycle energy:* $\sum_{k=1}^{m} p_{ij}^{k} = E_{ij}\ , \quad \forall\, i, j$ (13)

*Power Safety*

*Total required energy*: $\sum_{h=0}^{23} E_{final}(h) \leq E_{grid,max}(h)$ (14)

*PV Solar Panels*

First of all, it is known that a PV panel needs solar energy to convert this into electricity energy, that is, the sunbathing time of a solar cell is important. Knowing that an array can be defined as consisting of 12 elements which represents daily solar power capability of twelve months of a year. The solar power capability index is calculated by taking the daily average of sunbathing time in units of hours.

Then, let us say that elements of array are denoted by M = $m_1$ (January), $m_2$ (February), …, $m_{12}$ (December) and the values of array for İstanbul case are:

$$\mathbf{M_{1x12}} = [2.4, 3.2, 4.4, 6.1, 8.3, 10.2, 10.9, 10.1, 8.1, 5.5, 3.6, 2.5]$$

Hours (Average) (İstanbul) **[10]**

For instance, the $m_1$ value is the daily average of sunbath hours of the month January. Therefore, $m_1$ = 2.4 hours means that the daily average of the sunbath of İstanbul in January.

After that, solar cell manufacturers produce different cells with different power capacities. Therefore, the power generation capacity of different PV cells needs to be expressed using the following array:

$\mathbf{S_{p4x1}}$ = [0.3, 0.5, 1, 2]’ kWatts (Different Panels) **[11]**

The energy produced by the solar cell can be modeled using the energy-power equation as follows assuming same types of panels are used in terms of efficiency: **[12]**

Energy (kWh) = Number of Panels (n units) x Solar Panel Power (kWatts) x Time (Hours)

$$\mathbf{(E_{solar})_{4x12}} = n \times (S_p)_{4x1} \times (M)_{1x12}$$

For instance, let the number of panels be 4. Then, energy matrix produced by these panels can be calculated as:

$$(E_{solar})_{4x12} = 4 \times [0.3, 0.5, 1, 2]' \times [2.4, 3.2, 4.4, 6.1, 8.3, 10.2, 10.9, 10.1, 8.1, 5.5, 3.6, 2.5]$$

$(E_{solar})_{4x12}$ =

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $M_6$ | $M_7$ | $M_8$ | $M_9$ | $M_{10}$ | $M_{11}$ | $M_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.88 | 3.84 | 5.28 | 7.32 | 9.96 | 12.24 | 13.08 | 12.12 | 9.72 | 6.6 | 4.32 | 3 |
| 2 | 4.8 | 6.4 | 8.8 | 12.2 | 16.6 | 20.4 | 21.8 | 20.2 | 16.2 | 11 | 7.2 | 5 |
| 3 | 9.6 | 12.8 | 17.6 | 24.4 | 33.2 | 40.8 | 43.6 | 40.4 | 32.4 | 22 | 14.4 | 10 |
| 4 | 19.2 | 25.6 | 35.2 | 48.8 | 66.4 | 81.6 | 87.2 | 80.8 | 64.8 | 44 | 28.8 | 20 |

In the matrix given above each row represents the energy production using the related solar panel(s) in the $S_p$ vector. For example, the first row shows us the monthly energy production of 4 units of 0.3-kwatt solar panels and so on.

Then, we can summarize the PV model as:

$$\textbf{PV Model: } (E_{solar})_{ixj} = n \times (S_p)_{ix1} \times (M)_{1xj} \qquad (15)$$

Authorized power limit of solar panels in Turkey is limited to 10 kW power per apartment. Thereby, this constraint can be shown as: **[13]**

*Solar panels authorized power limit* $n \, .Sp_{ix1} \leq 10kW \qquad (16)$

As mentioned above, the solar system is a package of PV panels, solar battery and maybe an AC inverter. As a result of this, the solar energy provided by PV panels cannot exceed its corresponding solar battery capacity.

$0 \leq E_{solar}(h) \leq E_{battery}(h)$ *(battery capacity-solar energy matching)* $\qquad (17)$

**Solar Battery Storage**

Every solar battery has a charge capacity that restrain the maximum storage from PV cell energy.

*Battery Capacity:* $SOC(h) = E^{h}_{battery} / E^{cap}_{battery} \qquad (18)$

In case of overcharging and undercharging, the lifespan of the batteries decreases. Therefore, in this research, authors avoided overcharging and undercharging by keeping the energy stored in the battery within a certain range. The interval values we determined in equations in 19;

*Battery level:* $\%40 \leq SOC_{Pb}(h) \leq \%75$ (*Max and min rate of state of charge in Lead Acid Battery)*

$\%40 \leq SOC_{gel}(h) \leq \%75$ (*Max and min rate of state of charge in Gel Battery*)

$\%20 \leq SOC_{li}(h) \leq \%80$ (*Max and min rate of state of charge in Li-Ion Battery*)

*(19)*

These below values (Equations 20, and 21) were adjusted according to the appliances that work constantly at home. The lowest power value that should be in the batteries has been calculated by considering the refrigerator and deep freezer that are used constantly. The highest value was made by considering the power consumed by most household appliances in case of operation.**[14]**

*Battery max. charging power limit:*

$$P_{ch,min} = 0.04kW < P^{ch}_{battery}(h)/ß_{ch} < 0.35kW = P_{ch,max} \qquad (20)$$

*Battery max. discharging power limit:*

$$P_{dch,min} = 0.04kW < P^{dch}_{battery}(h).ß_{dch} < 0.35kW = P_{dch,max} \qquad (21)$$

If we assume that there is AC inverter in the "solar package system" (solar panels, solar battery, AC inverter), then the calculation needs to be considered.

*Battery with AC inverter*

$P_{invAC} = P_{invDC}\, x\, \mu_{ac2dc}$ (*Power lost in inverter- when energy moving battery to loads*) (22)

$P_{battery} + P_{solar} + P_{invDC} = 0$ *(in case of inverter power loss)* *(23)*

**Piecewise Energy Distribution via Load Shifting**

As mentioned before, prosumers produce electricity while they are consuming it. Many of them prefer to store the excess energy in solar battery storage systems. Photovoltaic solar panel systems consist of photovoltaic panels, solar charge controllers and batteries. These solar batteries are an essential component in renewable energy systems. Solar batteries require energy storage produced by a wind turbine, solar panel, or a hydroelectric system to be connected to the electricity grid. Solar batteries are very useful if you want to use an independent power grid or in the event of a power outage.

Figure 1 shows the general structure of HEMS with photovoltaic cells and battery. In this system, PV cells charge the batteries which store electrical energy to use later. We will be able to supply electricity from the battery to the grid when prices are high, and the house does not need electricity. Usage of pre-generated electricity at the interval when the electricity cost is higher than the other intervals reduces the bills more efficiently. Therefore, in our model PV panels charge the batteries and then the stored energy ($E_{battery}$) is used in high-cost intervals.

The objective function here is to minimize energy consumption from the grid ($E_{grid}$) and electricity dependency as mentioned before.

$$E_{grid} = \{E_{load} - E_{battery}, E_{battery} > 0\ kWh\ and\ E_{grid} > 0\ kWh\ and\ h_{22} \geq h \geq h_{17}$$

$$E_{load}, E_{grid} > 0kWh\ and \left(E_{battery} = 0kWh\ or\ h_{17(nextday)} > h > h_{22}\right)$$

$$0, E_{battery} > 0\ kWh\ and\ E_{grid} = 0\ kWh \quad \}$$

**User Dissatisfaction:**

Here the aim is to minimize the dissatisfaction level of the users caused by shifting the devices.

$f_{diss} = f_{diss}^{SI} + f_{diss}^{SNI}$ *(24)*

Where $f_{diss}^{SI}$ represents the dissatisfaction level caused by shiftable-interruptible devices and $f_{diss}^{SNI}$ represents the dissatisfaction level caused by shiftable-uninterruptible devices.

$$f_{diss}^{SI} = \sum_{a=1}^{total\ \#\ of\ SI\ devices} \sum_{h=1}^{h=24} S_a(h) * Ba(h) \qquad (25)$$

$$f_{diss}^{SNI} = \sum_{a=1}^{total\ \#\ of\ SNI\ devices} \sum_{h=1}^{h=24} S_a(h) * Ba(h) \qquad (26)$$

*Appliances work or not*

$B_a(h)$= [$b_a^1$, $b_a^2$ ……. $B_a^{24}$] -> Every Shiftable appliance has this binary array.

$B_a(h)$ = if the appliance works 1, if not 0 between (0 AM – 1 AM)

*Dissatisfaction Score*

$S_a(h)$ = This is a score evaluating the dissatisfaction level caused by operating shiftable appliance "a" at timeslot "h", which is determined by customers according to their living arrangement.

$S_a(h)$ ranges from 0 to 5 (integer values). A larger value of $S_a(h)$ means a higher dissatisfaction level **[8]**.

As a summary, the objective function of this mathematical model (24) is to minimize the summation of the two equations that lies above (25) and (26).

$$Pr_a * Ba = Final \qquad (user\ preference\ arrayed) \qquad (27)$$

## VII. Implementation of Optimization Model

**Load Shifting Algorithm Explanation**

1) Initialize constraints, tariff, appliance list

2) Calculate the hourly load array for only fixed devices. This array will be used to check if we exceed the hourly allowed max load

3) Iterate over shiftable and dissatisfaction allowed devices. First, take how much hour a device works a day then assign this device an empty array with 24 elements (each element corresponding an hour). We are going to fill this array with ones.

4) Iterate over the sorted tariff array. It gives us which hours cost less and we start to put ones into a temporary array starting from the first element of the sorted array. If an appliance works 3 hours a day, we put 3 ones adjacent to each other.

5) Then we first multiply this temp array with the power of the device and add this array to our power array to see if any hour exceeds the max hourly load. If not, we change the device's hourly array, but if it exceeds, we try to find another hour when the tariff costs less and do the same procedure.

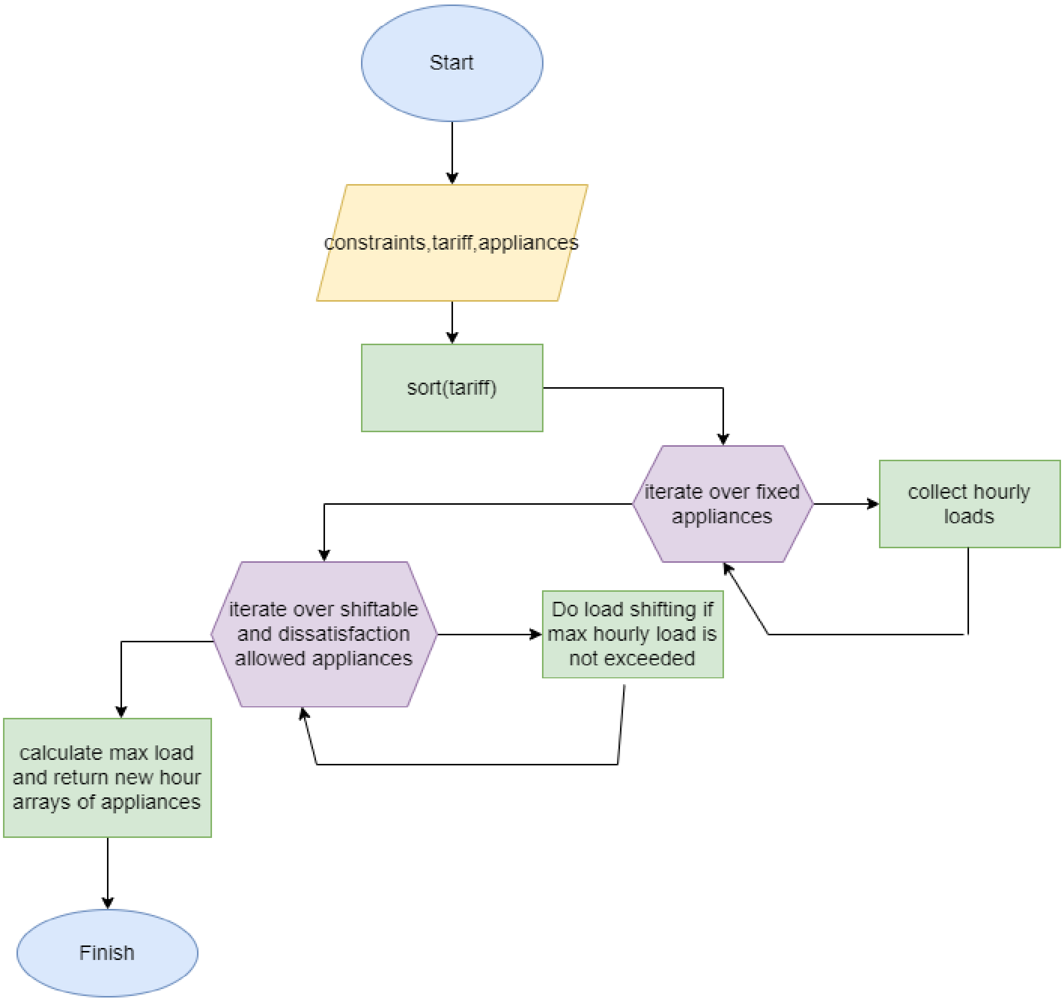


*Figure 2: Flowchart of Load Shifting Algorithm*

| APPLIANCE NAME | OLD WORK HOURS | NEW WORK HOURS |
|---|---|---|
| **Vacuum Cleaner** | 1 p.m. | 1 a.m. |
| **Water Heater** | 3 p.m. – 6 p.m. | 3 a.m. – 6 a.m. |
| **Water Pump** | 1 p.m. – 3 p.m. | 1 a.m. – 3 a.m. |
| **Dish Washer** | 7 p.m. – 9 p.m. | 9 p.m. – 11 p.m. |
| **Hair Dryer** | 9 p.m. | 11 p.m. |
| **Washing Machine** | 4 p.m. – 6 p.m. | 4 a.m. – 6 a.m. |
| **Clothes Dryer** | 7 p.m. – 9 p.m. | 7 a.m. – 9 a.m. |
| **Desktop Computer** | 7 p.m. – 10 p.m. | 6 a.m. – 9 a.m. |
| **Iron** | 8 p.m. – 9 p.m. | 2 p.m. – 3 p.m. |
| **Electrical Shaver** | 8 a.m. | 1 p.m. |
| **Phone Charger** | 6 p.m. – 7 p.m. | 3 p.m. – 4 p.m. |

*Table 2: Work hour of shiftable appliances*

After applying our load shifting algorithm, Table 2 shows the differences between when an appliance works before load shift and after load shift.

## VIII. Result and Analysis

As stated in the model part, we have two cases. In case1, we only considered the load shifting, and no PV is included. In case-2 PV and home to grid situations are included.

**Electricity Tariff**

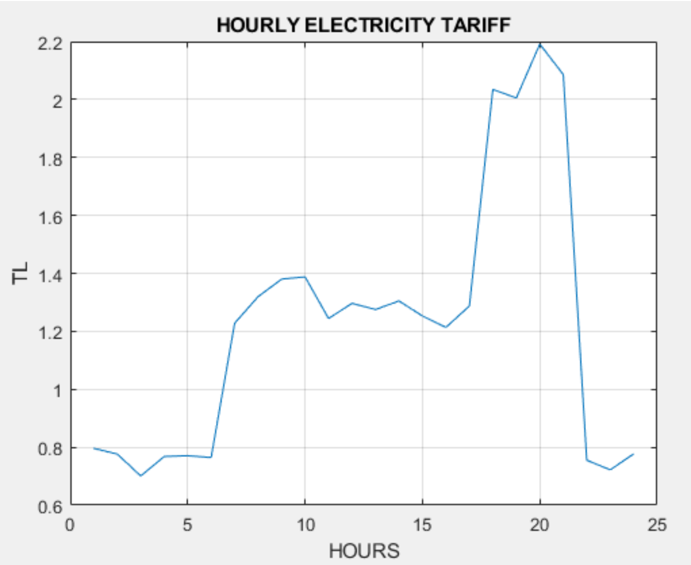


*Figure 3: Electricity Tariff*

A typical electricity tariff is given in Figure 3. It costs much more when there is more load demand between 5 pm – 10 pm. This data is taken from a website which gives information about the electricity tariff in Turkey **[15].**

**Case-1**

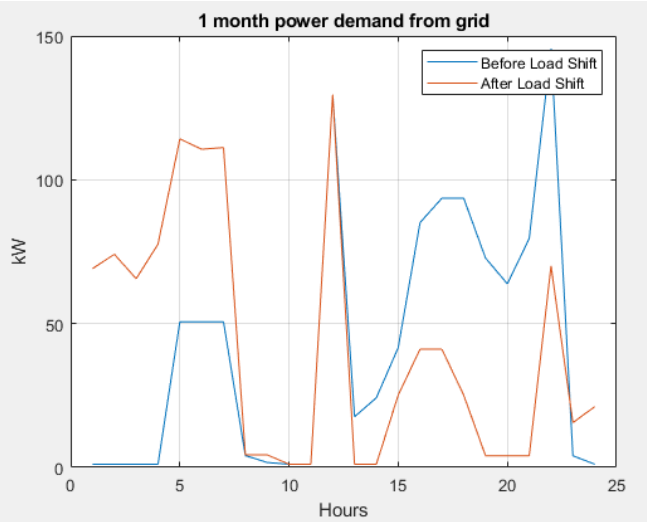


*Figure 4: A Typical Load Demand*

A typical load demand of a home is shown with the blue curve in Figure 4. As seen in the Figure 4, our load demand reaches the highest point between 5 pm – 10 pm. This load demand negatively affects our electricity bill. Our aim will be to shift the loads by considering dissatisfaction level of residents so that their electricity bill will decrease.

As shown with the red curve in Figure 4, after applying our load shifting algorithm, load demand curve changes. Load demands move to other hours when the electricity tariff costs less which decreases our total bill. Total cost decreased as you can see in the Figure 6.

## Case-2

After obtaining expected results in case1, we now added PV to our model. It should have a positive impact on our total bill. Furthermore, load demand from grid should decrease when it supplies energy to our system.

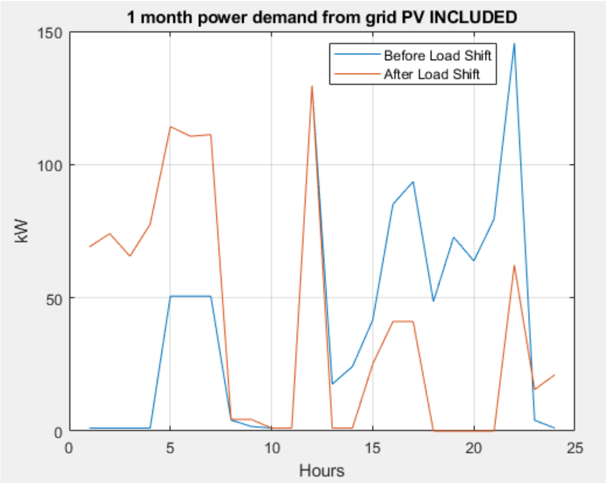


*Figure 5: Load demand of a home before load shift and PV included*

Our load demand before load shift is shown with blue curve in Figure 5. If we compare the blue curves in Figure 4 and Figure 5 load curves are almost similar but when we look closely at 5 p.m. the decrease in kW is seen thanks to PV. Our total cost decreases compared to the case where we have no PV and load shifting.

Red line in Figure 5 shows the load demand where we have load shifting and PV effect. As known, PVs work and produce electricity effectively between 1 pm to 6 pm. However, we also use storage devices to store the energy produced by PV and use it for the times when the electricity tariff costs more which is around 8 pm. If we compare the red lines in Figure 5 with Figure 4, there is a kW difference around 8 pm when the storage devices supply energy and our dependency on grid decreases.

## Total Cost

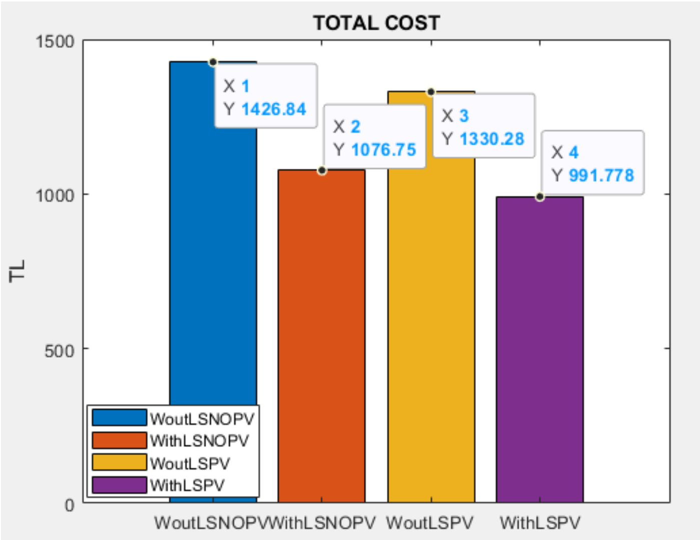


*Figure 6: Total Cost*

As shown in Figure 6 Total Cost, the most efficient way to decrease the electricity cost is addition PV and doing Load Shifting. Because in that case, we decrease the energy provided by grid. That means we buy less energy with Money. This affects our electricity bill directly.

**Before Load Shift NO PV**

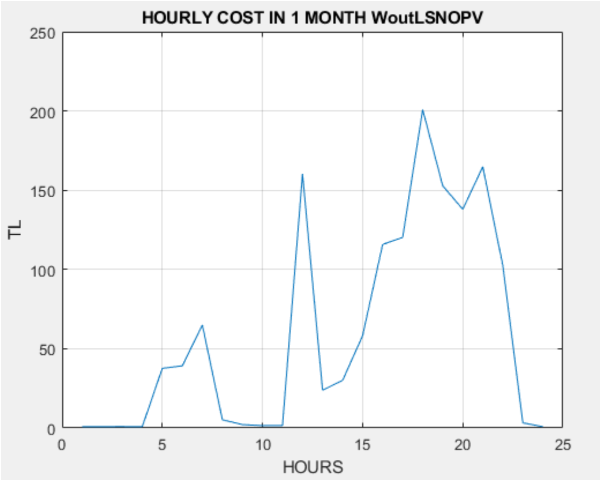


*Figure 7: Hourly cost in 1 Month Without Load Shifting No PV*

As shown Figure 7, the most effective hours in our electricity bill is 5 p.m.-9-pm . Because, power demand is the biggest point at that time intervals. People come to their home from the business.

**After load shift no PV**

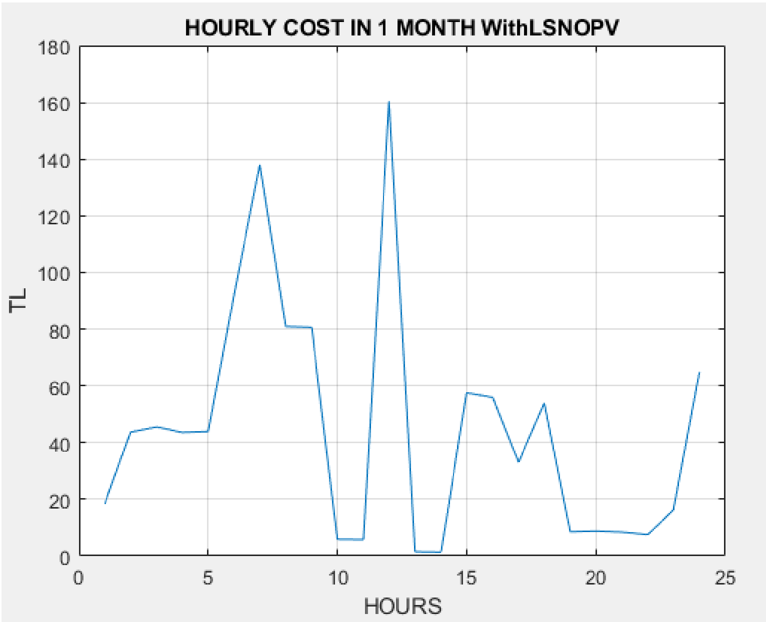


*Figure 8: Hourly cost in 1 Month with Load Shifting No PV*

As shown in Figure 8, we distributed the power demand in Figure 4. As you can see, we multiply each time interval with the cost of each time interval. As you can see the graph the most part of the electricity bill belongs to the noon time.

**PV included before load shift**

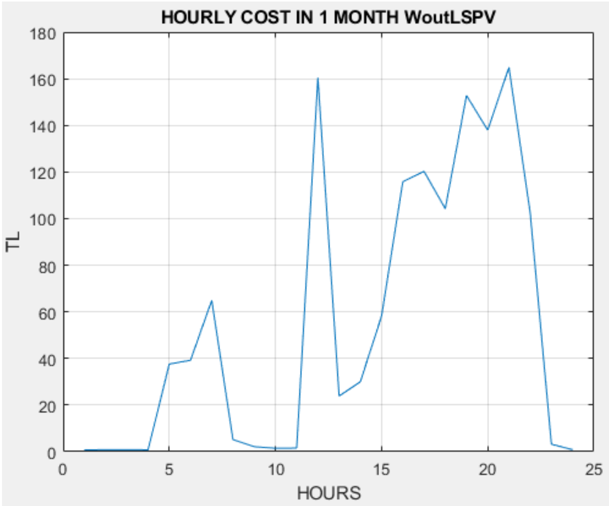


*Figure 9: Hourly cost in 1 Month without Load Shifting No PV*

As shown in Figure 9, the most expensive hours have great impact to our bill. Because we did not implement the Load Shifting Algorithm yet. However, PV affects the cost compared with the 7.

**After load shift PV included**

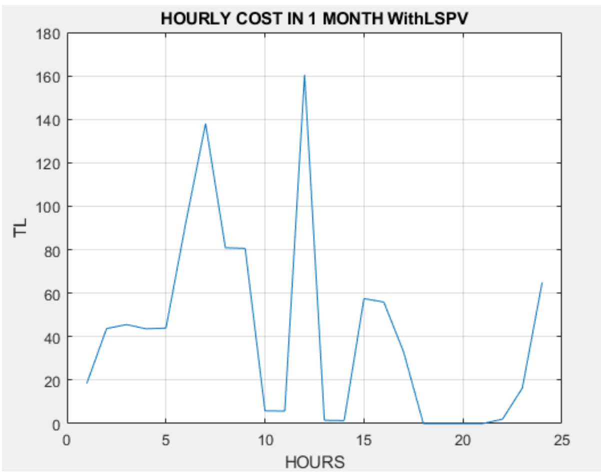


*Figure 10: Hourly cost in 1 Month with Load Shifting and PV*

The most efficient curve is shown in Figure 10 for the electricity bill. Because we implement the Load Shifting Algorithm in that case. As you can see the evening hours, cost of that time intervals is zero. Because we stored the electricity produced by the PV in Storage devices, and at these times our storage acts like a generator and supply power to our system. Because of that reason we do not have any demand from grid.

**PAR**

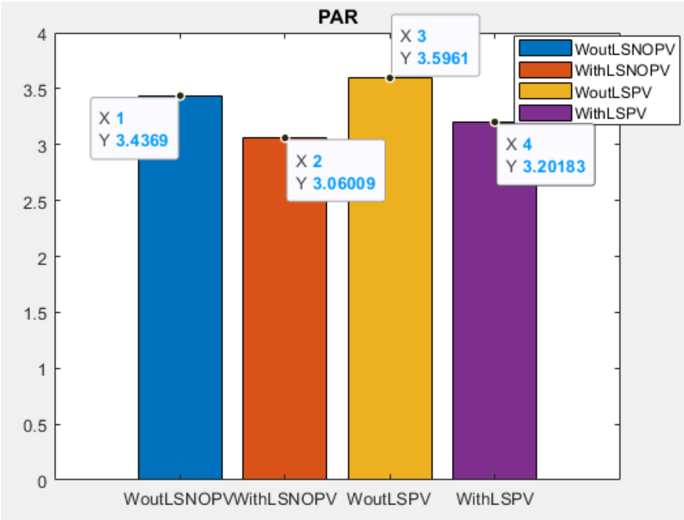


*Figure 11: PAR for Each Case*

PAR ratio is seen in Figure 11 We see a slight change between PAR ratios. Since our first aim was to cut off the cost, we did not entirely consider decreasing the par ratio.

## IX. Conclusion

At the very beginning of our research, we had no idea about the smart grid systems. But in time, as exploring the world of these systems and surveying the literature, we have managed to achieve a level such that we set up our own mathematical models that may have been applied in smart grid systems and we managed the optimize those models in MATLAB with their corresponding constraints.

We have separated our research into two cases, in other words scenarios, where in the first one we only considered the load shifting whereas in the other one we considered both load shifting and PV panels.

In the first case, the power demand from the grid is being compared for before load shifting and after load shifting situations. Before applying the load shifting algorithm, the power demand of a resident was at the maximum around evening time that is the time range where the electricity price becomes maximum. However, after applying our load shifting algorithm, we managed to shift the loads from high-priced intervals to low-priced intervals considering the user satisfaction as a constraint. Thus, as a result of that we managed to reduce the monthly electricity bill of a residence around 25%.

Then in our second case, we included the PV panels and the energy storage devices. At first, we tried PV panels without applying the load shifting and then we included the load shifting and we observed the results. As a result of these operations, we show that the case where PV and storage included decreased the monthly electricity cost around 30% when it's compared with no load shifting and no PV cases. As a downside, we didn't observe a change in the PAR level because decreasing the PAR level was not our main goal.

So as a result of our studies, we managed to decrease the electricity cost without dissatisfying the users by adding the corresponding constraints to our implementations. And by doing that we concluded our research.

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## XII. Appendix

| # | Name | Fixed | Shiftable | Interruptable | Power(kw) | Duration | Usage Frequency |
|---|---|---|---|---|---|---|---|
| 1 | TV | X | | X | A+:0.098 | 5 hours | Every day |
| 2 | Vacuum Cleaner | | X | X | A class: 0.7A+ Class:0.6A++ Class: 0.55 | 0.8 Hours | 3 days a week |
| 3 | Water Heater | | X | X | A class: 2.2-10A+ Class:1.5-7.6 | 4 Hours | Every day |
| 4 | Water Pump | | X | X | A class: 0.8A+ Class: 0.55-2.2 | 2.5 Hours | Every day |
| 5 | Dish Washer | | X | X | A class: 1.77A+ Class:0.79A++ Class: 0.72 | 2.16 Hours | 4 days a week |
| 6 | Hair Dryer | | X | X | 2.2 | 0.2 Hours | Every Day |
| 7 | Washing Machine | | X | | A+ class: 1.8A++: 0.53 | 2.9 Hours | 4 days a week |
| 8 | Clothes Dryer | | X | | A : 4 - 6 KWA+ : 3.5 KWA++ : 3 KW | 2.7 Hours | 4 days a week |
| 9 | Air Conditioner | | X | | Fan only mode : 0.75 KWWarmer days : 3.5 KWAvg : 3 – 5 KW | 3 Hours | Every Day |
| 10 | Refrigerator | X | | | A Class: 0.062A+ Class:0.046A++ Class: 0.021 | 14 Hours | Every Day |
| 11 | Oven | X | | | A : 1.5 KW – 2.5 KWA+ : 1.5 KW – 2.5 KWA++ : 0.8 – 0.9 KW | 7 Hours | Every Day |
| 12 | Desktop Computer | X | | | Computer Screen is ON :0.06-0.25 Computer Screen is SLEEP 0.001-0.006 | 4 Hours | Every Day |
| 13 | Laptop | X | | | 0.09 | 4 Hours | Every Day |
| 14 | Iron | | X | | 2-2.5 | 2 Hours | 2 Days a week |
| 15 | LED | X | | | 12 | 5 Hours | Every Day |
| 16 | Incandescent | X | | | 0.18 | 5 Hours | Every Day |
| 17 | Toast Machine | X | | | 2 | 0.25 Hours | 2 Days a week |
| 18 | Electric Heater | | X | X | 2 | 3 | Every Day |
| 19 | Freezer | X | | | A++ Class: 0.01575, A+ class:0.025 | 20 Hours | Every Day |
| 20 | Microwave | X | | | 0.8 | 0.25 hours | Every Day |
| 21 | Coffee Maker | X | | | 0.75 | 0.5 Hours | Every Day |
| 22 | Electric Kettle | X | | | 2.2 | 0.5 Hours | Every Day |
| 23 | Safety Camera | X | | | 5 | 24 Hours | Every Day |
| 24 | Sewing Machine | | X | X | 0.15 | 1 Hour | 1 day a week |
| 25 | Smoke Detector | X | | | 1 | 24 Hours | Every Day |
| 26 | Electric Shaver | | X | | 0.02 | 0.08 Hours | Every Day |
| 27 | Phone Charger | | X | | fast charge:0.02 charge: 0.01 | 2 Hours | 4 days a week |
| 28 | Printer | X | | | 150 | 0.25 Hours | 2 days a week |
| 29 | Gaming Console | X | | | min: 0.05 max:0.22 | 3 hours | 5 days a week |

*Figure 12: Appliance List*

| Home-type solar panel power (kW) | |
|---|---|
| Gesper Polycrystal | 0.3 |
| | 0.5 |
| | 1 |
| | 2 |

*Table 3: PV panel power (with brand)*

| Home-type solar battery (Wh)(Vah) | | | |
|---|---|---|---|
| 800 cycle | | 1200 cycle | |
| Sonnenschein S12/27 G5 | 12V/27Ah | Sonnenschein S12/32 G6 | 12V/32Ah |
| Sonnenschein S12/41 A | 12V/41Ah | Sonnenschein S12/60 A | 12V/60Ah |
| Sonnenschein S12/90 A | 12V/90Ah | Sonnenschein S12/100 A | 12V/100Ah |
| Sonnenschein S12/230 A | 12V/230Ah | Sonnenschein S12/185 A | 12V/185Ah |

*Table 4: Corresponding Solar Battery (with brand)*