Markowitz efficient frontier states investors should consider multiple securities in a portfolio rather than individually. A portfolio that contains combination of securities with low correlation can benefit from a diversification effect. Meaning investors can optimize their return without assuming additional risk. Markowitz

```
In [5]: import numpy as np
   import pandas as pd
   from pandas_datareader import data as web
   import matplotlib.pyplot as plt
   %matplotlib inline
```

WE will download the data on PG stock and ^GSPC

```
In [2]: tickers = ["PG", "^GSPC"]
    data = pd.DataFrame()
    for t in tickers:
        data[t] = web.DataReader(t, data_source = "yahoo", start = "2010-1-
1")["Adj Close"]
```

```
In [4]: #normalize the data
  (data/data.iloc[0]*100).plot(figsize = (16,8))
  plt.show()
```



Calculate the daily change, returns of both securties

```
In [51]: simple_returns = (data/data.shift(1)) - 1
```

```
In [52]: #we will check if the data matches and have equal values - > 2494 PG and
          2494 ^GSPC
          simple_returns.info()
          <class 'pandas.core.frame.DataFrame'>
         DatetimeIndex: 2495 entries, 2010-01-04 to 2019-11-29
         Data columns (total 2 columns):
         PG
                   2494 non-null float64
          ^GSPC
                   2494 non-null float64
         dtypes: float64(2)
         memory usage: 58.5 KB
In [53]: #check the tail end of the data to check for most current date
          simple_returns.tail()
Out[53]:
                         PG
                              ^GSPC
               Date
          2019-11-22 -0.000415
                             0.002175
          2019-11-25 0.001829
                             0.007507
          2019-11-26 0.014522
                             0.002196
          2019-11-27 -0.004090
                             0.004174
          2019-11-29 0.002464 -0.004011
         simple_returns.cov() * 250
In [11]:
Out[11]:
                          ^GSPC
                     PG
             PG 0.021868 0.011354
          ^GSPC 0.011354 0.021763
In [54]: #the correlation between PF and ^GSPC is positive but low so the portfol
          io should benefit from
          #markowitz diversification effect
          simple returns.corr()
Out[54]:
                     PG
                          ^GSPC
             PG 1.000000 0.520464
          ^GSPC 0.520464 1.000000
In [57]: # portfolio optimization -> We will need the count of securities in the
           portfolio
          port asset = len(tickers)
          print(f"The number of securties in the portfolio is {port_asset}")
         The number of securties in the portfolio is 2
```

WE will need the expected returns and the volatility to simulate a mean variance combination with 1000 simulations. WE are considering 1000 combinations of the same 2 assets of their weight values not 1000 different investments.

```
In [ ]:
         #Bellow we will run a simulation of 1000 differenct portfolio that conta
In [58]:
          in PG and ^GSPC to test Markowitz theory.
          #This Will provide us with both 1000 different expected returns and 1000
          volatility values
         portfolio returns = []
         portfolio_volatilities = []
          for x in range(1000):
              weights = np.random.random(port_asset)
              weights /= np.sum(weights)
              portfolio returns.append(np.sum(weights * simple returns.mean()) * 2
          50)
              portfolio volatilities.append(np.sqrt(np.dot(weights.T, np.dot(simpl
          e_returns.cov() * 250, weights))))
              # we will need to convert the volatilities and and the expected retu
          rns into a numpy array
         port_Returns = np.array(portfolio_returns)
         port Vol = np.array(portfolio volatilities)
 In [ ]:
 In [ ]:
In [60]:
         #lets create a data fram containing the data
          portfolios = pd.DataFrame({"Returns": port_Returns, "Volatility": portfo
          lio volatilities})
In [61]:
         portfolios.head()
Out[61]:
                    Volatility
             Returns
          0 0.112761 0.132366
          1 0.112363 0.129450
          2 0.112482 0.128782
          3 0.112670 0.130430
          4 0.112502 0.128800
```

```
In [62]:
           portfolios.tail()
Out[62]:
                 Returns
                         Volatility
                         0.128836
                0.112449
            995
           996
                0.112493 0.128787
                0.112543 0.128953
           997
                0.112838 0.134600
                0.112159 0.133605
In [63]:
           portfolios.info()
           <class 'pandas.core.frame.DataFrame'>
           RangeIndex: 1000 entries, 0 to 999
           Data columns (total 2 columns):
                           1000 non-null float64
           Returns
                           1000 non-null float64
           Volatility
           dtypes: float64(2)
           memory usage: 15.7 KB
          portfolios.plot(x = "Volatility", y = "Returns", kind = "scatter", figsize
In [66]:
           = (16,8)
           plt.title("Markowitz portfolio Theory\n The Efficient Frontier")
           plt.show()
                                               Markowitz portfolio Theory
                                                 The Efficient Frontier
            0.118
            0.116
            0.114
            0.112
            0.110
            0.108
```

The above graph shows a set of 1000 portfolios of different weights containing PG & ^GSPC, and displays the typical shape of Markowitz efficient portfolio. There are a set of efficient portfolios that can provide a higher rate of return for the same or lower risk. The starting point is the minimum variance portfolio.

0.135

0.140

Volatility

0.130

0.150

0.145