

Homework 3 (Due October 24, 2016, 9:00 AM)

BLM 1541: Statistics and Probability — Fall 2016

Print family (or last) name: _____

Print given (or first) name: _____

Print given student number: _____

I see that this homework has 5 questions in total.

I agree that I have to submit my homework solution before the deadline (October 24, 2016, 9 AM) otherwise my homework solution will not be graded.

I accept that ***I will add the signed version of this instruction page as a first page into my homework solution***; otherwise my homework solution will not be graded.

I know that ***I have to give my solutions written on white A4-sized pages that are stapled on the left-up corner***; otherwise my homework solution will not be graded.

I will take care of the readability of my solutions, from which I may lose 10 points.

For any proofs, I am sure to provide a step-by-step argument, with justifications for every step.

I understand that, during solving this homework, it is prohibited to exchange information about solutions with any other person in any way, including by talking or ex-changing solutions / papers.

I know that the course book is “Probability and Statistics for Computer Scientists, by Michael Baron (2nd Edition) ”.

I have read, understand and accept all of the instructions above. On my honor, I pledge that I have not violated the provisions of the Academic Integrity Code of Yıldız Technical University.

Signature and Date

Problem 1 [20 Points]: By the end of the course BLM1541: Statistics and Probability (Fall'16), a group of students, consisting 4 male and 2 female students, is selected according to student's performance. As a researcher, I would like to pick three students from this group in order to study and analyze the performance of unmanned air vehicles (UAVs) in extreme situations (*Hint: Three students are picked from the group without any replacement*).

- a) Two male students are randomly selected without replacement from the group. Find the probability that at least two of the selected students are male students.

Let X be the random variable that represents the number of picked male students.

- b) Find and plot the probability mass function (PMF) of the random variable X .
 c) Find and the cumulative distribution function, F , of the random variable X .
 d) Compute the expected value and variance of X .
 e) Compute the expected value of $\ln(X)$, which is the entropy obtained for diversity benefits in the project.

Problem 2 [20 Points]: In Davutpaşa housing compound of Yıldız Technical University, the heating systems are provided from three companies.

- Company A has installed its system into 60% of houses and 8% of its products are defective
- Company B has installed its system into 30% of houses and 6% of its products are defective
- Company C has installed its system into 10% of houses and 10% of its products are defective

With these information,

- a) What's the probability that the heating system in a house is defective?
 b) Given that it is defective, how likely was it to have been produced by Company C?

Problem 3 [20 Points]: Toss a coin 20 times, and let X be the number of heads that turn up. What is the probability that X is divisible by 2, given that X is divisible by 3?

Problem 4 [20 Points]: In Beşiktaş, when I asked the traffic police what the average number of car accidents is in Beşiktaş per day, he answered me 5 car accidents per day. What is the probability that in any given day the number of car accidents in Beşiktaş will exceed 4? Be sure to explain your reasoning.

Problem 5 [20 Points]: Decide about whether each of the following statements is TRUE or FALSE, and shortly explain your explanation of why:

- a) _____: If events X and Y are independent, then knowing $P(X)$ and $P(Y)$ allows one to compute $P(X \cup Y)$.
 b) _____: Mutually exclusive events are not always (in fact almost never) independent.
 c) _____: A student comes up a discrete function in his/her research like that

$$f(x_i) = \begin{cases} \alpha (1 + x_i - x_i^2), & x_i = -3, -2, -1, 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

with $\alpha \in \mathbb{R}$. Could this function be a valid PMF, with an appropriate choice of α .

- d) _____: If discrete random variables X and Y are independent, then $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$.
 e) _____: Let X be a random variable with sample space Ω . If $P(A \cup B) = \Omega$, can A and B be independent?
 f) _____: If X is a discrete random variable and c is a constant, the following all properties of the variance are valid:
- $\text{var}(c) = 0$
 - $\text{var}(X + c) = \text{var}(X)$
 - $\text{var}(cX) = c^2 \text{var}(X)$