Problem

Out of six computer chips, two are defective. If two chips are randomly chosen for testing (without replacement), compute the probability that both of them are defective. List all the outcomes in the sample space.

**Step-by-step solution**

**Show all steps**

100% (16 ratings) for this solution

**Step 1/3**

Out of six computer chips, two are defective. The objective is to compute the probability if two chips are randomly chosen for testing (without replacement), both of them are defective and have to list all the outcomes of the sample space.

**Step 2/3**

Let *A* be the event that the first chip is defective and let *B* be the event that the second chip is defective without replacement of the first chip. Clearly the events *A* and *B* are mutually exclusive. Probability of event *A*, *P (A)* = 2/6 as there are two defective chips in the six chips. Probability of even *B*, *P (B)* = 1/5 as the remaining number of defective chip is one and remaining total number of chips is 5.The required probability is $P(A \cap B)$. As the events *A* and *B* are mutually exclusive,

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= \frac{2}{6} \times \frac{1}{5} \\ &= \frac{2}{30} \\ &= \frac{1}{15} \end{aligned}$$

If two chips are randomly chosen for testing without replacement from six computer chips having two defectives, the probability that the two chips are defective is $\boxed{\frac{1}{15}}$.

**Step 3/3**

Let *S* denotes the sample space of choosing two chips randomly from the six computer chips in which two chips are defective. Let *D* denotes the defective chip and *G* denotes the non defective chip. Then the sample space S = {DD, GD, DG, GG}.

Problem
Suppose that after 10 years of service, 40% of computers have problems with motherboards (MB), 30% have problems with hard drives (HD), and 15% have problems with both MB and HD. What is the probability that a 10-year old computer still has fully functioning MB and HD?

**Step-by-step solution**

**Show all steps**

100% (8 ratings) for this solution

**Step 1/5**
Let *A* denotes the event that computers have problems with motherboards after 10 years of service. Let *B* denotes the event that computers have problems with hard drives after 10 years of service. From the given details, $P(A)$ = 0.4, $P(B)$ = 0.3 and $P(A \cap B)$ = 0.15 as 15% of the computers have problems with both mother boards and hard disks after 10 years of service. We require that probability that a 10-year old computer still has fully functioning Mother Board and Hard disk.

**Step 2/5**
We require the probability $P(A \cup B)^c$ where $A \cup B$ denotes the event that either a computer have problem with mother board or with hard disk after 10 years of service. So the required probability is the complement of this event.

**Step 3/5**
From the law of addition of probability, we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Substitute the values in this formula.

$$\begin{aligned} P(A \cup B) &= 0.4 + 0.3 - 0.15 \\ &= 0.55 \end{aligned}$$

The formula to find the compliment is

$$\begin{aligned} P(A \cup B)^c &= 1 - P(A \cup B) \\ &= 1 - 0.55 \\ &= 0.45 \end{aligned}$$

**Step 4/5**
The probability that a 10-year old computer still has fully functioning Mother Board and Hard disk is 0.45.

**Step 5/5**
The following diagram illustrates the probabilities.

Problem

A new computer virus can enter the system through e-mail or through the internet. There is a 30% chance of receiving this virus through e-mail. There is a 40% chance of receiving it through the internet. Also, the virus enters the system simultaneously through email and the internet with probability 0.15. What is the probability that the virus does not enter the system at all?

**Step-by-step solution**

**Show all steps**

100% (9 ratings) for this solution

**Step 1/5**

Let *A* denotes the event that virus enter the system through e-mail. Let *B* denotes the event that virus enter the system through internet. From the given details, $P(A)$= 0.3, $P(B)$= 0.4 and $P(A\cap B)$= 0.15 as there is a 15% chance that the virus enter the system simultaneously through email and internet. We require that probability the virus does not enter the system at all.

**Step 2/5**

We require the probability $P(A\cup B)^c$ where $A\cup B$ denotes the event that virus enter the system through either email or internet. So the required probability is the complement of this event.

**Step 3/5**

From the law of addition of probability of two events, we have $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. Substitute the values in this formula.

$$\begin{aligned} P(A\cup B) &= 0.3+0.4-0.15 \\ &= 0.55 \end{aligned}$$

The formula to find the compliment is

$$\begin{aligned} P(A\cup B)^c &= 1-P(A\cup B) \\ &= 1-0.55 \\ &= 0.45 \end{aligned}$$

**Step 4/5**

The probability that the virus does not enter the system at all is 0.45.

Problem

A computer program is tested by 3 *independent* tests. When there is an error, these tests will discover it with probabilities 0.2, 0.3, and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found by at least one test?

**Step-by-step solution**

**Show all steps**

100% (10 ratings) for this solution

**Step 1/3**

Given that a computer program is tested by 3 independent tests and if there is an error, these tests will discover it with probabilities 0.2, 0.3 and 0.5 respectively. We have to find the probability that if the program contains error, it will be found by at least one test.

**Step 2/3**

Let *A* denotes the event that error is discovered by test one, *B* denotes the event that error is discovered by test two and *C* denotes the event that error is discovered by test three. Then from the given details, $P(A)=0.2$ , $P(B)=0.3$ and $P(C)=0.5$ . We require the probability that $P(A\cup B\cup C)$ as it is the probability that the error is found by at least one test.

**Step 3/3**

From the addition rule for probability for three events,

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

As the three events are independent to each other,

$$\begin{aligned}P(A\cap B)&=P(A)\times P(B)\\&=0.2\times 0.3\\&=0.06\end{aligned}$$

$$\begin{aligned}P(A\cap C)&=P(A)\times P(C)\\&=0.2\times 0.5\\&=0.10\end{aligned}$$

$$\begin{aligned}P(B\cap C)&=P(B)\times P(C)\\&=0.3\times 0.5\\&=0.15\end{aligned}$$

$$\begin{aligned}P(A\cap B\cap C)&=P(A)\times P(B)\times P(C)\\&=0.2\times 0.3\times 0.5\\&=0.03\end{aligned}$$

Substitute the above values in the addition rule.

$$\begin{aligned} P(A \cup B \cup C) &= 0.2 + 0.3 + 0.5 - 0.06 - 0.1 - 0.15 + 0.03 \\ &= 0.72 \end{aligned}$$

The probability that if the program contains error, it will be found by at least one test is 0.72.

**Chapter 2, Problem 6E**
(0)

Problem
Under good weather conditions, 80% of flights arrive on time. During bad weather, only 30% of flights arrive on time. Tomorrow, the chance of good weather is 60%. What is the probability that your flight will arrive on time?

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/3**
Given that 80% of flights arrive on time under good weather conditions and only 30% of flights arrive on time under bad weather conditions. Tomorrow, the chance of good weather is 60%. We have to find the probability that the flight will arrive on time.

**Step 2/3**
Let A denotes that event that the weather conditions will be good and B denotes the event that flights arrive on time. The given probabilities are $P(B \mid A) = 0.8$, $P(B \mid A^c) = 0.3$ and $P(A) = 0.6$. We require the probability $P(B)$.

**Step 3/3**
By law of total probability

$$P(B) = P(A \cap B) + P(A^c \cap B).$$

By conditional probabilities,

$P(A \cap B) = P(B \mid A)P(A)$ and $P(A^c \cap B) = P(B \mid A^c)P(A^c)$

So

$$\begin{aligned} P(B) &= P(B \mid A)P(A) + P(B \mid A^c)P(A^c) \\ &= 0.8(0.6) + 0.3(1 - 0.6) \\ &= 0.48 + 0.12 \\ &= 0.60 \end{aligned}$$

The probability that the flight will arrive on time is 0.60.

**Chapter 2, Problem 7E**
(0)

Problem
A system may become infected by some spyware through the internet or e-mail. Seventy percent of the time the spyware arrives via the internet, thirty percent of the time via e-mail. If it enters via the internet, the system detects it immediately with probability 0.6. If via e-mail, it is detected with probability 0.8. What percentage of times is this spyware detected?

**Step-by-step solution**

**Show all steps**

100% (8 ratings) for this solution

**Step 1/3**
Given that 70% of time the spyware arrives via the internet and 30% via email. The system detects immediately with probability 0.6 if it arrives through internet and with probability 0.8 if it arrives through email. We have to find the percentage of times is the spyware is detected by the system.

**Step 2/3**
Let A denotes that event that the spyware arrives through internet and let B denotes the event that the system detects it immediately. The given probabilities are $P(B \mid A) = 0.6$, $P(B \mid A^c) = 0.8$, $P(A) = 0.7$ and $P(A^c) = 0.3$ where $A^c$ denotes the event that the spyware arrives through email. We require the probability $P(B)$.

**Step 3/3**
By law of total probability

$P(B) = P(A \cap B) + P(A^c \cap B)$.

By conditional probabilities,

$P(A \cap B) = P(B \mid A)P(A)$ and $P(A^c \cap B) = P(B \mid A^c)P(A^c)$

So

$$\begin{aligned} P(B) &= P(B \mid A)P(A) + P(B \mid A^c)P(A^c) \\ &= 0.6(0.7) + 0.8(0.3) \\ &= 0.42 + 0.24 \\ &= 0.66 \end{aligned}$$

The percentage of times is the spyware is detected by the system is 66%.

**Chapter 2, Problem 8E**
(0)

Problem
A shuttle's launch depends on three key devices that may fail independently of each other with probabilities 0.01, 0.02, and 0.02, respectively. If any of the key devices fails, the launch will be postponed. Compute the probability for the shuttle to be launched on time, according to its schedule.

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/3**
Given that a shuttle's launch depends on three key devices, having independent probabilities of failing are 0.01, 0.02 and 0.02 respectively. If any one of the device fails, then the launch is postponed. We have to compute the probability for the shuttle to be launched on time.

**Step 2/3**
Let *A*, *B* and *C* denote the events that devices 1, 2 and 3 fail respectively. Then $P(A) = 0.01$, $P(B) = 0.02$ and $P(C) = 0.02$. The shuttle will be launched if none of the three devices fail. So we require the probability that none of the three devices will fail. It is the probability $P(A \cup B \cup C)^c$ which is equivalent to 1 - $P(A \cup B \cup C)$ by complement rule. By addition probability rule for three events,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$P(A \cap B) = P(A).P(B)$, $P(A \cap C) = P(A).P(C)$, $P(B \cap C) = P(B).P(C)$ and $P(A \cap B \cap C) = P(A).P(B).P(C)$ as the events are independent to each other. Substitute the values in the formula.

$P(A \cup B \cup C)$ = 0.01 + 0.02 + 0.02 - (0.01) (0.02) - (0.01) (0.02) - (0.02) (0.02) + (0.01) (0.02) (0.02)

= 0.05 - 0.0002 - 0.0002 - 0.0004 + 0.000004

= 0.049204

**Step 3/3**
$P(A \cup B \cup C)^c$ = 1 - 0.049204 = 0.9508.

The probability that the shuttle will be launched on time is 0.9508.

**Chapter 2, Problem 9E**
(0)

Problem
Successful implementation of a new system is based on three independent modules. Module 1 works properly with probability 0.96. For modules 2 and 3, these probabilities equal 0.95 and 0.90. Compute the probability that at least one of these three modules fails to work properly.

**Step-by-step solution**

**Show all steps**

100% (3 ratings) for this solution

**Step 1/3**
Given that the probability that module 1 works properly is 0.96, module 2 works properly is 0.95 and module 3 works properly is 0.90. The modules are independent and they are important for successful implementation of a new system. We have to compute the probability that at least one of these three modules fails to work properly.

**Step 2/3**
Let *A*, *B* and *C* denote the events that module 1, module 2 and module 3 works properly respectively. Then $P(A)=0.96$, $P(B)=0.95$ and $P(C)=0.9$. The required probability is $P(A^c \cup B^c \cup C^c)$ as we require the probability that at least one module fails to work properly. By addition probability rule for three events,

$$P(A^c \cup B^c \cup C^c) = P(A^c)+P(B^c)+P(C^c)-P(A^c \cap B^c)-P(A^c \cap C^c)-P(B^c \cap C^c) \\ +P(A^c \cap B^c \cap C^c)$$

$P(A^c \cap B^c) = P(A^c).P(B^c)$, $P(A^c \cap C^c) = P(A^c).P(C^c)$, $P(B^c \cap C^c) = P(B^c).P(C^c)$ and $P(A^c \cap B^c \cap C^c) = P(A^c).P(B^c).P(C^c)$ as the events are independent to each other. By using the complement formula,

$P(A^c)$= 1 - 0.96 = 0.04

$P(B^c)$= 1 - 0.95 = 0.05

$P(C^c)$= 1 - 0.90 = 0.10

Substitute the values in the formula.

$P(A^c \cup B^c \cup C^c)$= 0.04 + 0.05 + 0.1 - (0.04) (0.05) - (0.04) (0.1) - (0.05) (0.1) + (0.04) (0.05) (0.1)

= 0.19 - 0.002 - 0.004 - 0.005 + 0.0002

$= 0.1792$

**Step 3/3**
The probability that at least one of these three modules fails to work properly is 0.1792.

**Chapter 2, Problem 10E**
(0)

Problem
Three computer viruses arrived as an e-mail attachment. Virus A damages the system with probability 0.4. Independently of it, virus B damages the system with probability 0.5. Independently of A and B, virus C damages the system with probability 0.2. What is the probability that the system gets damaged?

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/3**
Given that the probability that Virus A damages the system is 0.4, Virus B damages the system is 0.5 and Virus C damages the system is 0.2 and these are independent to each other. We have to find the probability that the system get damaged.

**Step 2/3**
Let *A*, *B* and *C* denote the events that Virus A, Virus B and Virus C damage the system respectively. Then $P(A)=0.4$, $P(B)=0.5$ and $P(C)=0.2$. The system will get damaged if at least one of the three Virus damages the system. It is the probability $P(A\cup B\cup C)$. By addition probability rule for three events,

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

$P(A\cap B)=P(A).P(B)$, $P(A\cap C)=P(A).P(C)$, $P(B\cap C)=P(B).P(C)$ and $P(A\cap B\cap C)=P(A).P(B).P(C)$ as the events are independent to each other. Substitute the values in the formula.

$P(A\cup B\cup C)$ = 0.4 + 0.5 + 0.2 - (0.4) (0.5) - (0.4) (0.2) - (0.5) (0.2) + (0.4) (0.5) (0.2)

= 1.1 - 0.2 - 0.08 - 0.1 + 0.04

= 0.76

**Step 3/3**
The probability that the system gets damaged is 0.76.

**Chapter 2, Problem 11E**
(0)

Problem
A computer program is tested by 5 independent tests. If there is an error, these tests will discover it with probabilities 0.1, 0.2, 0.3, 0.4, and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found

(a) by at least one test?

(b) by at least two tests?

(c) by all five tests?

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/6**
Given that a computer program is tested by five independent tests and if there is an error, these tests will discover it with probabilities 0.1, 0.2, 0.3, 0.4 and 0.5 respectively. Let *A*, *B*, *C*, *D* and *E* denote the events that each of the five independent tests discover the error respectively. So $P(A)$ = 0.1, $P(B)$ = 0.2, $P(C)$ = 0.3, $P(D)$ = 0.4 and $P(E)$ = 0.5.

**Step 2/6**
(a)

Probability that the error will be found by at least one test is given by the probability $P(A \cup B \cup C \cup D \cup E)$. It is equivalent to the complement of the event that all the five tests do not discover the error. So

$P(A \cup B \cup C \cup D \cup E) = 1 - P(A^c \cap B^c \cap C^c \cap D^c \cap E^c)$. Using the rule of complement,

$P(A^c)$ = 1 - 0.1 = 0.9

$P(B^c)$ = 1 - 0.2 = 0.8

$P(C^c)$ = 1 - 0.3 = 0.7

$P(D^c)$ = 1 - 0.4 = 0.6

$P(E^c)$ = 1 - 0.5 = 0.5

As the events are all independent,

$$P(A^c \cap B^c \cap C^c \cap D^c \cap E^c) = P(A^c).P(B^c).P(C^c).P(D^c).P(E^c)$$

Substitute the values in the formula.

$$\begin{aligned} P(A \cup B \cup C \cup D \cup E) &= 1-(0.9)(0.8)(0.7)(0.6)(0.5) \\ &= 1-0.1512 \\ &= 0.8488 \end{aligned}$$

Probability that the error will be found by at least one test is 0.8488.

**Step 3/6**
(b)

Probability that the error will be found by at least two tests can be found indirectly by subtracting the probability that the error will be found by exactly one test from the probability that the error will be found by at least one test.

**Step 4/6**

Probability that the error will be found by exactly one test = $P(A \cap B^c \cap C^c \cap D^c \cap E^c)$ + $P(A^c \cap B \cap C^c \cap D^c \cap E^c)$ + $P(A^c \cap B^c \cap C \cap D^c \cap E^c)$ + $P(A^c \cap B^c \cap C^c \cap D \cap E^c)$ + $P(A^c \cap B^c \cap C^c \cap D^c \cap E)$.

As all the events are independent,

$$\begin{aligned} P(A \cap B^c \cap C^c \cap D^c \cap E^c) &= P(A).P(B^c).P(C^c).P(D^c).P(E^c) \\ &= (0.1)(0.8)(0.7)(0.6)(0.5) \\ &= 0.0168 \end{aligned}$$

$$\begin{aligned} P(A^c \cap B \cap C^c \cap D^c \cap E^c) &= P(A^c).P(B).P(C^c).P(D^c).P(E^c) \\ &= (0.9)(0.2)(0.7)(0.6)(0.5) \\ &= 0.0378 \end{aligned}$$

$$\begin{aligned} P(A^c \cap B^c \cap C \cap D^c \cap E^c) &= P(A^c).P(B^c).P(C).P(D^c).P(E^c) \\ &= (0.9)(0.8)(0.3)(0.6)(0.5) \\ &= 0.0648 \end{aligned}$$

$$\begin{aligned} P(A^c \cap B^c \cap C^c \cap D \cap E^c) &= P(A^c).P(B^c).P(C^c).P(D).P(E^c) \\ &= (0.9)(0.8)(0.7)(0.4)(0.5) \\ &= 0.1008 \end{aligned}$$

$$\begin{aligned} P(A^c \cap B^c \cap C^c \cap D^c \cap E) &= P(A^c).P(B^c).P(C^c).P(D^c).P(E) \\ &= (0.9)(0.8)(0.7)(0.6)(0.5) \\ &= 0.1512 \end{aligned}$$

Probability that the error will be found by exactly one test = 0.0168 + 0.0378 + 0.0648 + 0.1008 + 0.1512 = 0.3714

**Step 5/6**
Probability that the error will be found by at least one test is 0.8488 as from (a).
Probability that the error will be found by at least two tests = 0.8488 - 0.3714 = 0.4774.
Probability that the error will be found by at least two tests is 0.4774.

**Step 6/6**
(c)

Probability that the error will be found by all five tests is given by the probability $P(A \cap B \cap C \cap D \cap E)$ which is given by the product $P(A).P(B).P(C).P(D).P(E)$ as all the events are independent. Substitute the values in the above product.

$$\begin{aligned} P(A \cap B \cap C \cap D \cap E) &= P(A).P(B).P(C).P(D).P(E) \\ &= (0.1)(0.2)(0.3)(0.4)(0.5) \\ &= 0.0012 \end{aligned}$$

Probability that the error will be found by all five tests is 0.0012.

**Chapter 2, Problem 12E**
(0)

Problem
A building is examined by policemen with four dogs that are trained to detect the scent of explosives. If there are explosives in a certain building, and each dog detects them with probability 0.6, independently of other dogs, what is the probability that the explosives will be detected by at least one dog?

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/2**
Given that four dogs are trained by policemen to detect the scent of explosives and the probability that each dog will detect the scent of explosives is 0.6 when the explosives are present in a certain building. We have to find the probability that the explosives will be detected by at least one dog.

**Step 2/2**
Let *A*, *B*, *C* and *D* are the events that the four dogs detect the explosives present in certain building respectively. Then we have $P(A)=P(B)=P(C)=P(D)=0.6$. Probability that the explosives will be detected by at least one dog is given by the probability $P(A\cup B\cup C\cup D)$. It is equivalent to the complement of the event that all the four dogs do not detect the explosives present in the building.

So, $P(A\cup B\cup C\cup D)=1-P(A^c\cap B^c\cap C^c\cap D^c)$.

Using the rule of complement, $P(A^c)=P(B^c)=P(C^c)=P(D^c)=0.4$

As the events are all independent, $P(A^c\cap B^c\cap C^c\cap D^c)=P(A^c).P(B^c).P(C^c).P(D^c)$

Substitute the values in the formula.

$$\begin{aligned}P(A\cup B\cup C\cup D)&=1-(0.4)(0.4)(0.4)(0.4)\\&=1-0.0256\\&=0.9744\end{aligned}$$

Probability that the explosives will be detected by at least one dog is 0.9744.

**Chapter 2, Problem 13E**
(0)

Problem
An important module is tested by three independent teams of inspectors. Each team detects a problem in a defective module with probability 0.8. What is the probability that at least one team of inspectors detects a problem in a defective module?

**Step-by-step solution**

**Show all steps**

100% (13 ratings) for this solution

**Step 1/2**
Given that an important module is tested by three independent teams of inspectors and each team detects a problem in a defective module with probability 0.8. We have to find the probability that at least one team of inspectors detects a problem in a defective module.

**Step 2/2**
Let *A*, *B* and *C* are the events that the three teams of inspectors detect a problem in a defective module respectively. Then $P(A)=P(B)=P(C)=0.8$. Probability that the problem in a defective module will be detected by at least one team of inspectors is given by the probability $P(A\cup B\cup C)$. It is equivalent to the complement of the event that all the three teams do not detect the problem in a defective module.

So $P(A\cup B\cup C)=1-P(A^c\cap B^c\cap C^c)$.

Using the rule of complement, $P(A^c)=P(B^c)=P(C^c)=0.2$

As the events are all independent, $P(A^c\cap B^c\cap C^c)=P(A^c).P(B^c).P(C^c)$

Substitute the values in the formula.

$$\begin{aligned} P(A\cup B\cup C) &= 1-(0.2)(0.2)(0.2) \\ &= 1-0.008 \\ &= 0.992 \end{aligned}$$

Probability that the problem in a defective module will be detected by at least one team of inspectors is 0.992.

**Chapter 2, Problem 14E**
(0)

Problem
A spyware is trying to break into a system by guessing its password. It does not give up until it tries 1 million different passwords. What is the probability that it will guess the password and break in if by rules, the password must consist of

(a) 6 different lower-case letters?

(b) 6 different letters, some may be upper-case, and it is casesensitive?

(c) any 6 letters, upper- or lower-case, and it is case-sensitive?

(d) any 6 characters including letters and digits?

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/5**
Given that a spyware is trying to break into a system by guessing its password. It does not give up until it tries 1 million different passwords.

**Step 2/5**
(a)

The probability that the spyware will guess the password and break in if by rules the password must consists of 6 different lower-case letters is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the total number of outcomes. Here $N_F$ = 1,000,000 as the spyware tries 1 million different passwords. The total number of outcomes is given by $26P_6$ as permutation without replacement is to be used and the password must be six different lower-case letters.

Probability that the spyware will guess the password and break in is

$$\begin{aligned}\frac{N_F}{N_T} &= \frac{1,000,000}{26P_6}\\ &= \frac{1,000,000}{165,765,600}\\ &= 0.006032615\end{aligned}$$

The probability that the spyware will guess the password and break in if by rules the password must consists of 6 different lower-case letters is 0.006032615.

**Step 3/5**
(b)

The probability that the spyware will guess the password and break in if by rules the password must consists of 6 different letters, some may be upper case and it is case-sensitive is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the total number of outcomes. Here $N_F$ = 1,000,000 as the spyware tries 1 million different passwords. The total number of outcomes is given by $52P_6$ as permutation without replacement is to be used and the password must be six different letters with some upper case and the password is case sensitive.

Probability that the spyware will guess the password and break in is

$$\frac{N_F}{N_T} = \frac{1{,}000{,}000}{52P_6}$$
$$= 0.000068222$$

The probability that the spyware will guess the password and break in if by rules the password must consists of 6 different letters, some may be upper case and it is case-sensitive is 0.000068222.

**Step 4/5**
(c)

The probability that the spyware will guess the password and break in if by rules the password must consists of any six letters, upper or lower case and it is case-sensitive is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the total number of outcomes. Here $N_F$ = 1,000,000 as the spyware tries 1 million different passwords. The total number of outcomes is given by $52^6$ as permutation with replacement is to be used as the password must be any six letters of upper case or lower case and the password is case sensitive.

Probability that the spyware will guess the password and break in is

$$\frac{N_F}{N_T} = \frac{1{,}000{,}000}{52^6}$$
$$= 0.00005058$$

The probability that the spyware will guess the password and break in if by rules the password must consists of any six letters, upper or lower case and it is case-sensitive is 0.00005058.

**Step 5/5**

(d)

The probability that the spyware will guess the password and break in if by rules the password must consists of any six characters including letters and digits is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the total number of outcomes. Here $N_F$ = 1,000,000 as the spyware tries 1 million different passwords. The total number of outcomes is given by $62^6$ as permutation with replacement is to be used as the password must be any six characters including letters and digits.

Probability that the spyware will guess the password and break in is

$$\begin{aligned}\frac{N_F}{N_T} &= \frac{1,000,000}{62^6}\\ &= 0.000017606\end{aligned}$$

The probability that the spyware will guess the password and break in if by rules the password must consists of any six characters including letters and digits is 0.000017606.

**Chapter 2, Problem 15E**
(3)

Problem
A computer program consists of two blocks written independently by two different programmers. The first block has an error with probability 0.2. The second block has an error with probability 0.3. If the program returns an error, what is the probability that there is an error in both blocks?

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/4**
Given that a computer program consists of two blocks written independently by two different programmers. The first block has an error with probability 0.2 and the second block has an error with probability 0.3. We have to find the probability that if the program returns an error, that there is an error in both blocks.

**Step 2/4**
Let *A* is the event that there is an error in the first block and let *B* is the event that there is an error in the second block and *C* is the event that the computer program returns error. Given that $P(A)$= 0.2 and $P(B)$= 0.3 and the events *A* and *B* are independent.
The probability that the computer program will return error is $P(C)$.

$$\begin{aligned} P(C) &= P(A \cap B^c) + P(A^c \cap B) + P(A \cap B) \\ &= P(A).P(B^c) + P(A^c).P(B) + P(A).P(B) \\ &= 0.2(0.7) + 0.8(0.3) + (0.3)(0.2) \\ &= 0.44 \end{aligned}$$

**Step 3/4**
The required probability is that there is an error in both blocks given that the program returns an error. Using Baye’s rule, the probability is given as $\frac{P(A \cap B)}{P(C)} = \frac{0.06}{0.44} = 0.1364$

**Step 4/4**
If the program returns an error, the probability that there is an error in both blocks is 0.1364.

**Chapter 2, Problem 16E**
(3)

Problem
A computer maker receives parts from three suppliers, S1, S2, and S3. Fifty percent come from S1, twenty percent from S2, and thirty percent from S3. Among all the parts supplied by S1, 5% are defective. For S2 and S3, the portion of defective parts is 3% and 6%, respectively.

(a) What portion of all the parts is defective?

(b) A customer complains that a certain part in her recently purchased computer is defective. What is the probability that it was supplied by S1?

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/4**
Given that a computer maker receives parts from three suppliers, *S1*, *S2* and *S3*. The percentages of supply from these suppliers are 0.5, 0.2 and 0.3 respectively. *S1* supplies 5% defective, *S2* supplies 3% defective and *S3* supplies 6% defective parts.

**Step 2/4**
Let *D* represents the event that a randomly selected part is defective. The given probabilities are $P(S1)$= 0.5, $P(S2)$= 0.2, $P(S3)$= 0.3, $P(D|S1)$= 0.05, $P(D|S2)$= 0.03 and $P(D|S3)$= 0.06.

**Step 3/4**
(a)

The portion of all parts is defective is given by

$$P(S1)P(D|S1)+P(S2)P(D|S2)+P(S3)P(D|S3)$$

= (0.5) (0.05) + (0.2) (0.03) + (0.3) (0.06)

= 0.025 + 0.006 + 0.018

= 0.049

The portion of all parts is defective is 4.9%.

**Step 4/4**
(b)

The probability that the defective part is supplied by S1 given that a customer complains that a certain part in her recently purchased computer is defective is given by the probability $P(S1 \mid D)$.

Using Baye’s rule, $P(S1 \mid D) = \frac{P(D \mid S1)P(S1)}{P(D)}$

Substitute the values in the above formula.

$$\begin{aligned}\frac{P(D \mid S1)P(S1)}{P(D)} &= \frac{0.05(0.5)}{0.049} \\ &= \frac{0.025}{0.049} \\ &= 0.5102\end{aligned}$$

A customer complains that a certain part in her recently purchased computer is defective. The probability that it was supplied by *S1* is 0.5102.

**Chapter 2, Problem 17E**
(0)

Problem
A computer assembling company receives 24% of parts from supplier X, 36% of parts from supplier Y, and the remaining 40% of parts from supplier Z. Five percent of parts supplied by X, ten percent of parts supplied by Y, and six percent of parts supplied by Z are defective. If an assembled computer has a defective part in it, what is the probability that this part was received from supplier Z?

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/5**
Given that a computer assembling company receives parts from three suppliers *X*, *Y* and *Z*. The percentages of supply from these suppliers are 24%, 36% and 40% respectively. *X* supplies 5% defective, *Y* supplies 10% defective and *Z* supplies 6% defective parts.

**Step 2/5**
Let *D* represents the event that a randomly selected part is defective. The given probabilities are $P(X)$= 0.24, $P(Y)$= 0.36, $P(Z)$= 0.4, $P(D|X)$= 0.05, $P(D|Y)$= 0.1 and $P(D|Z)$= 0.06.

**Step 3/5**
The probability that the defective part is supplied by *Z* given that an assembled computer has a defective part in it is given by the probability $P(Z|D)$.

Using Baye's rule, $P(Z|D) = \dfrac{P(D|Z)P(Z)}{P(D)}$

**Step 4/5**
The portion of all parts is defective $P(D)$ is given by

$P(X)P(D|X) + P(Y)P(D|Y) + P(Z)P(D|Z)$

= (0.24) (0.05) + (0.36) (0.1) + (0.4) (0.06)

= 0.012 + 0.036 + 0.024

= 0.072

The portion of all parts is defective $P(D)$is 7.2%.

**Step 5/5**
Substitute the values in the formula.

$$\begin{aligned}\frac{P(D\mid Z)P(Z)}{P(D)} &= \frac{0.06(0.4)}{0.072}\\ &= \frac{0.024}{0.072}\\ &= 0.333\end{aligned}$$

If an assembled computer has a defective part in it, the probability that this part was received from supplier *Z* is 0.333.

**Chapter 2, Problem 18E**
(3)

Problem
A problem on a multiple-choice quiz is answered correctly with probability 0.9 if a student is prepared. An unprepared student guesses between 4 possible answers, so the probability of choosing the right answer is 1/4. Seventy-five percent of students prepare for the quiz. If Mr. X gives a correct answer to this problem, what is the chance that he did not prepare for the quiz?

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/4**
Given that a problem on a multiple-choice quiz is answered correctly with probability 0.9 if a student is prepared and answered correctly with probability 0.25 if not prepared. The probability of prepared students is 0.75.

**Step 2/4**
Let *C* denotes the event that a student answers a question correctly. Let *Y* denotes the event that a student is prepared for the quiz and let *N* denotes the event that a student is not prepared for the quiz. Then the given probabilities are $P(Y)$= 0.75, $P(N)$= 0.25, $P(C \mid Y)$= 0.9 and $P(C \mid N)$= 0.25.

**Step 3/4**
We have to find that probability that if Mr. *X* gives a correct answer to a problem, the chance that he did not prepare for the quiz. This probability is given by $P(N \mid C)$ and using Baye's rule, $P(N \mid C) = \dfrac{P(C \mid N)P(N)}{P(C)}$ where

$$\begin{aligned} P(C) &= P(N)P(C \mid N) + P(Y)P(C \mid Y) \\ &= 0.25(0.25) + (0.75)(0.9) \\ &= 0.0625 + 0.675 \\ &= 0.7375 \end{aligned}$$

**Step 4/4**
Substitute the values in the Baye's formula $P(N \mid C) = \dfrac{P(C \mid N)P(N)}{P(C)}$.

$$\frac{P(C \mid N)P(N)}{P(C)} = \frac{0.25(0.25)}{0.7375}$$
$$= \frac{0.0625}{0.7375}$$
$$= 0.08475$$

If Mr. *X* gives a correct answer to a problem, the chance that he did not prepare for the quiz is 0.08475

**Chapter 2, Problem 19E**
(0)

Problem
At a plant, 20% of all the produced parts are subject to a special electronic inspection. It is known that any produced part which was inspected electronically has no defects with probability 0.95. For a part that was not inspected electronically this probability is only 0.7. A customer receives a part and find defects in it. What is the probability that this part went through an electronic inspection?

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/3**
Given that at a plant, 20% of all the produced parts are subject to a special electronic inspection.

Let *D* denotes the event that a part is defective.

Let *Y* denotes the event that a part is inspected electronically and

let $Y'$ denotes the event that a part is not inspected electronically.

The probability that any produced part which was inspected electronically has no defect is 0.95

This means,

$$\begin{aligned} P(D \mid Y) &= 1 - 0.95 \\ &= 0.05 \end{aligned}$$

The probability that any produced part which was no inspected electronically has no defect is 0.7. This means,

$$\begin{aligned} P(D \mid Y') &= 1 - 0.7 \\ &= 0.3 \end{aligned}$$

The 20% of all the produced parts are subject to a special electronic inspection. This in mathematical notation $P(Y) = 0.2$

The remaining 80% are not electronically inspected. This means $P(Y') = 0.80$

**Step 2/3**
Now find the probability of getting a defective part from the production.

$$\begin{aligned}P(D) &= P(Y')P(D \mid Y') + P(Y)P(D \mid Y) \\ &= 0.80(0.3) + (0.2)(0.05) \\ &= 0.24 + 0.01 \\ &= 0.25\end{aligned}$$

**Step 3/3**

It is required to find the probability that the selected part went through an electronic inspection found defective.

Substitute the values in the Bayes' formula:

$$\begin{aligned}P(Y \mid D) &= \frac{P(Y)P(D \mid Y)}{P(D)} \\ &= \frac{0.20(0.05)}{0.25} \\ &= \frac{0.01}{0.25} \\ &= 0.04\end{aligned}$$

Hence, the probability that the selected part is defective that went through an electronic inspection is 4%.

**Chapter 2, Problem 20E**
(1)

Problem
All athletes at the Olympic games are tested for performanceenhancing steroid drug use. The imperfect test gives positive results (indicating drug use) for 90% of all steroid-users but also (and incorrectly) for 2% of those who do not use steroids. Suppose that 5% of all registered athletes use steroids. If an athlete is tested negative, what is the probability that he/she uses steroids?

**Step-by-step solution**

**Show all steps**

84% (6 ratings) for this solution

**Step 1/4**
Given that the imperfect steroid drug use test gives positive results for 90% of all steroid users and 2% of all non-steroid users. Also 5% of all registered athletes use steroids. If an athlete is tested negative, we have to find the probability that he/she uses steroids.

**Step 2/4**
Let *Y* denotes the event that an athlete uses steroids and let *N* denotes the event that an athlete does not use steroids. Let *A* denotes the event that the steroid test gives positive results and let *B* denotes the event that the steroid test gives negative results.
Then the given probabilities are $P(Y)$ = 0.05, $P(N)$ = 0.95, $P(A \mid Y)$ = 0.9, $P(A \mid N) = 0.02$, $P(B \mid Y)$ = 0.1 and $P(B \mid N) = 0.98$

**Step 3/4**
The required probability is given by $P(Y \mid B)$ and using Baye's rule,

$$P(Y \mid B) = \frac{P(B \mid Y)P(Y)}{P(B)}$$ where

$$\begin{aligned} P(B) &= P(N)P(B \mid N) + P(Y)P(B \mid Y) \\ &= 0.95(0.98) + 0.05(0.1) \\ &= 0.931 + 0.005 \\ &= 0.936 \end{aligned}$$

**Step 4/4**
Substitute the values in the Baye's formula $P(Y \mid B) = \frac{P(B \mid Y)P(Y)}{P(B)}$.

$$\frac{P(B|Y)P(Y)}{P(B)} = \frac{0.1(0.05)}{0.936}$$
$$= \frac{0.005}{0.936}$$
$$= 0.00534$$

If an athlete is tested negative, the probability that he/she uses steroids is 0.00534.

**Chapter 2, Problem 21E**
(0)

Problem
In the system in Figure 2.7, each component fails with probability 0.3 independently of other components. Compute the system's reliability.

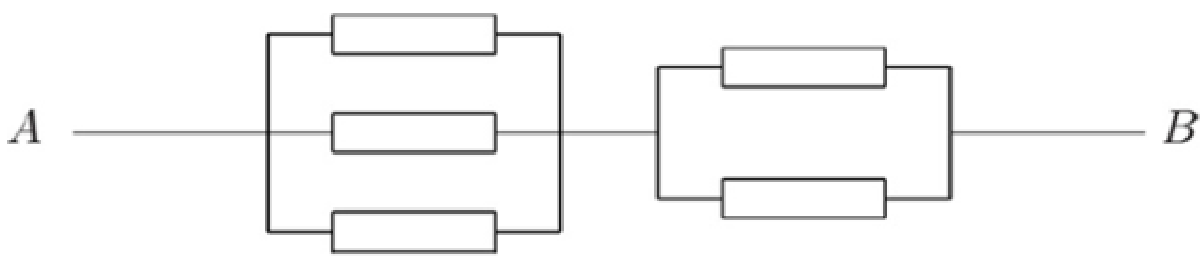


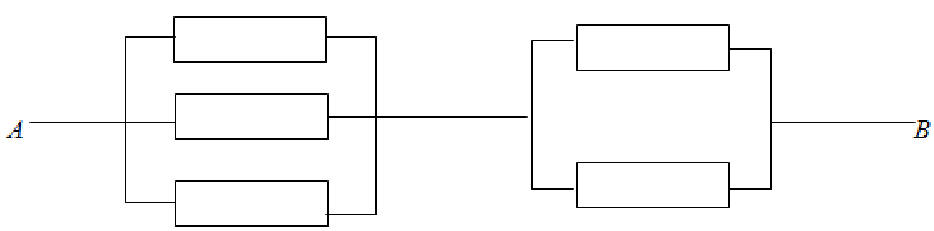

**FIGURE 2.7**: *Calculate reliability of this system (Exercise 2.21)*.

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/3**
Consider the diagram of the system in which every component fails with probability $0.30$ independent of other components.




**Step 2/3**
The objective is to calculate the reliability of the system. This means that the probability that the system will not fail is required.

Define the event *A* as the first circuit to be success and event *B* as the second circuit to be success. There are three components in the first circuit and two components in the second circuit. It is given that each component fails independently. This means that the probability gives the fail percentage and also the probabilities of no failure of the components are also independent. So, the probability of *A* is calculated as,

$$P(A) = P(\text{comp } 1 \cup \text{comp } 2 \cup \text{comp } 3)$$

Using the complement rule, the probability of *A* can be written as,

$$\begin{aligned} P(A) &= 1 - P(\text{comp } 1 \cup \text{comp } 2 \cup \text{comp } 3)^c \\ &= 1 - P(\text{comp } 1^c \cap \text{comp } 2^c \cap \text{comp } 3^c) \end{aligned}$$

Since, the probabilities of failures of system is independent, the probability of *A* is written as,

$$P(A) = 1 - \left[ P(\text{comp } 1^c) \cdot P(\text{comp } 2^c) \cdot P(\text{comp } 3^c) \right]$$

Substitute the probability of failure which is 0.30. So, the probability that system *A* will not fail is calculated as,

$$\begin{aligned} P(A) &= 1 - \left[ (0.30) \cdot (0.30) \cdot (0.30) \right] \\ &= 1 - 0.027 \\ &= 0.973 \end{aligned}$$

**Step 3/3**

Similarly, the probability that the system *B* with two components will not fail is calculated as,

$$\begin{aligned} P(B) &= 1 - \left[ (0.30) \cdot (0.30) \right] \\ &= 1 - 0.09 \\ &= 0.91 \end{aligned}$$

The two systems *A* and *B* are connected in sequel and hence the entire system will not fail if both the systems will not fail. So, the probability that both the systems will not fail is calculated as,

$$\begin{aligned} P(A) \cdot P(B) &= (0.973)(0.91) \\ &= 0.88543 \end{aligned}$$

Therefore, the reliability of the system is $\boxed{88.54\%}$.

**Chapter 2, Problem 22E**
(0)

Problem
Three highways connect city A with city B. Two highways connect city B with city C. During a rush hour, each highway is blocked by a traffic accident with probability 0.2, independently of other highways.

(a) Compute the probability that there is at least one open route from A to C.

(b) How will a new highway, also blocked with probability 0.2 independently of other highways, change the probability in (a) if it is built

*(α)* between A and B?

*(β)* between B and C?

*(γ)* between A and C?

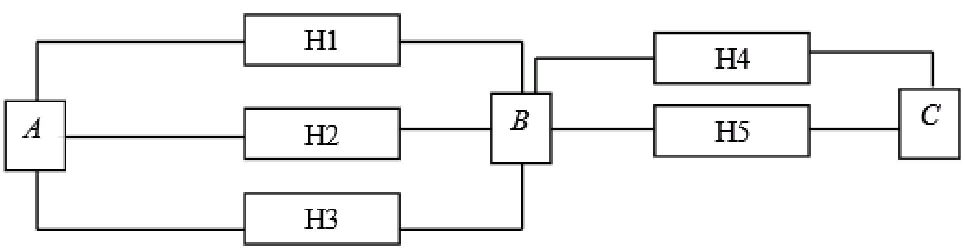
**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/9**
Given that three highways connect city *A* with city *B* and two high ways connect city *B* with city *C*. During a rush hour, each highway is blocked by a traffic accident with probability 0.2, independently of other highways.

**Step 2/9**
The scenario is present in the diagram as follows.




**Step 3/9**
(a)

The probability that there is at least one open route from *A* to *C* can be computed using the concept of reliability. Cities *A* and *B* are connected parallel by three highways and cities *B* and *C* are connected by two highways. Cities *A* and *C* are connected sequel by city *B*. Let *H1*, *H2*, *H3*, *H4* and *H5* denotes the event that the five highways are not blocked by a traffic accident respectively. So the probabilities of these five events are all 0.8 each. Also these events are all independent.

**Step 4/9**

First we can calculate the probability that there is at least one open route between cities *A* and *B*. It is the probability given by $P(H1 \cup H2 \cup H3)$ as the three highways are parallel.

$$\begin{aligned} &P(H1 \cup H2 \cup H3) \\ &= 1 - P(H1 \cup H2 \cup H3)^c \text{(using complement rule)} \\ &= 1 - P(H1^c \cap H2^c \cap H3^c) \text{(using complement rule for combined events)} \\ &= 1 - \left\{P(H1^c).P(H2^c).P(H1^c)\right\} \text{(as the events are independent)} \end{aligned}$$

$$\begin{aligned} &= 1 - (0.2^3) \\ &= 0.992 \end{aligned}$$

So the probability that there is at least one open route between cities *A* and *B* is 0.992.

**Step 5/9**

Similarly, the probability that there is at least one open route between cities *B* and *C* is given by

$$\begin{aligned} &P(H4 \cup H5) \\ &= 1 - P(H4 \cup H5)^c \text{(using complement rule)} \\ &= 1 - P(H4^c \cap H5^c) \text{(using complement rule for combined events)} \\ &= 1 - \left\{P(H4^c).P(H5^c)\right\} \text{(as the events are independent)} \end{aligned}$$

$$\begin{aligned} &= 1 - (0.2^2) \\ &= 0.96 \end{aligned}$$

**Step 6/9**

The probability that there is at least one open route between cities *A* and *C* is given by $P(H1 \cup H2 \cup H3) \cap P(H4 \cup H5)$ as the cities are connected in sequel. Substitute the probabilities in above formula.

$$\begin{aligned} P(H1 \cup H2 \cup H3) \cap P(H4 \cup H5) &= 0.992(0.96) \\ &= 0.95232 \end{aligned}$$

The probability that there is at least one open route from *A* to *C* is 0.95232.

**Step 7/9**

(b)

Let *N* denotes the new highway and $P(N)$ denotes that independent probability that the new highway is not blocked by traffic accident. Then $P(N)$ = 1 - 0.2 = 0.8.

**Step 8/9**

$(\alpha)$

The change in probability (a) if *N* is between *A* and *B* is given by
$P(H1 \cup H2 \cup H3 \cup N) \cap P(H4 \cup H5)$

= (1 - $0.2^4$) (0.96)

= 0.958464

So the probability in (a) increases from 0.95232 to 0.958464, if the new highway is between *A* and *B*.

**Step 9/9**
$(\beta)$

The change in probability (a) if *N* is between *B* and *C* is given by

$P(H1 \cup H2 \cup H3) \cap P(H4 \cup H5 \cup N)$

= (0.992) (1 - $0.2^3$)

= 0.984064

So the probability in (a) increases from 0.95232 to 0.984064, if the new highway is between *B* and *C*.

$(\gamma)$

The change in probability (a) if *N* is between *A* and *C* is given by

$$
\begin{aligned}
&\left[P(H1 \cup H2 \cup H3) \cap P(H4 \cup H5)\right] \cup P(N) \\
&= \left[P(H1 \cup H2 \cup H3) \cap P(H4 \cup H5)\right] + P(N) \\
&- \left(\left[P(H1 \cup H2 \cup H3) \cap P(H4 \cup H5)\right] \cap P(N)\right) \text{(Using addition rule)} \\
&= 0.95232 + 0.8 - (0.95232)(0.8) \\
&= 0.95232 + 0.8 - 0.761856 \\
&= 0.990464
\end{aligned}
$$

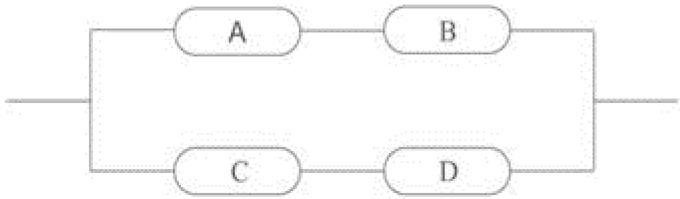
So the probability in (a) increases from 0.95232 to 0.990464, if the new highway is between *A* and *C*.

**Chapter 2, Problem 23E**
(2)

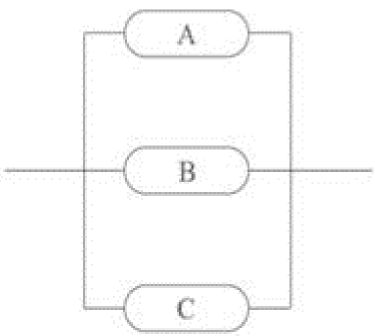
Problem

Calculate the reliability of each system shown in Figure 2.8, if components A, B, C, D, and E function properly with probabilities 0.9, 0.8, 0.7, 0.6, and 0.5, respectively.

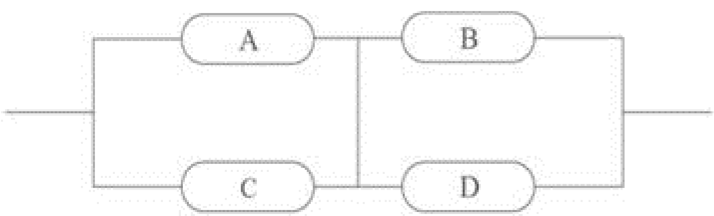

(a)



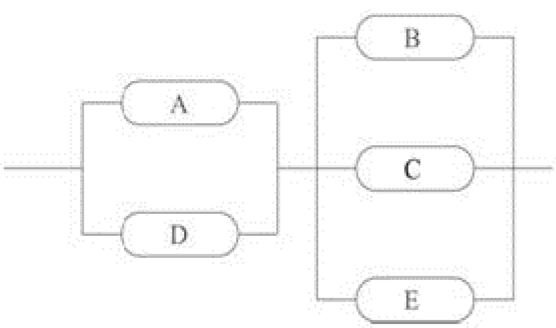

(b)




(c)




(d)




(e)

**FIGURE 2.8**: *Calculate reliability of each system (Exercise 2.23)*.

**Step-by-step solution**

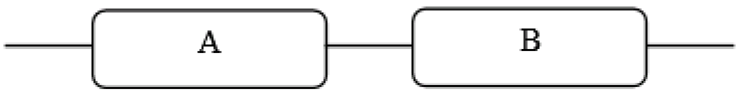
**Show all steps**

88% (8 ratings) for this solution

**Step 1/8**
Given that the probabilities that components *A*, *B*, *C*, *D* and *E* function properly are 0.9, 0.8, 0.7, 0.6 and 0.5 respectively. Let the name of the components itself denote the event that the corresponding component works properly. So $P(A)$ = 0.9, $P(B)$ = 0.8, $P(C)$ = 0.7, $P(D)$ = 0.6 and $P(E)$ = 0.5. We have to find the reliability of the given structure of combination of above said components.

**Step 2/8**
(a)

The diagrammatic representation of the system is as follows.



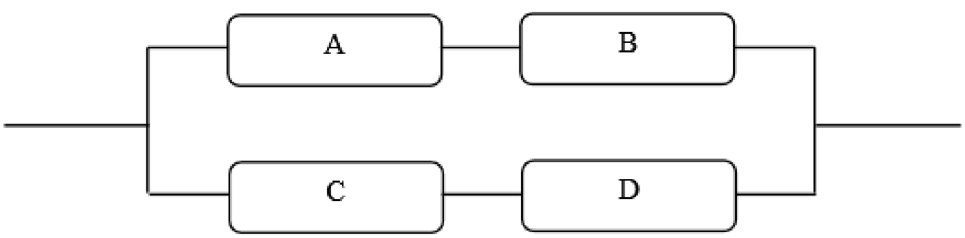

The above system consists of components *A* and *B* connected in sequel. So the reliability of the system is the probability that both *A* and *B* function properly. This probability is given by $P(A \cap B) = P(A).P(B)$ as the events are independent. Substitute the probabilities in the above formula.

$$\begin{aligned} P(A \cap B) &= (0.9)(0.8) \\ &= 0.72 \end{aligned}$$

Reliability of this system is 0.72.

**Step 3/8**
(b)

The diagrammatic representation of the system is as follows.




In the above system, the components *A* and *B* and components *C* and *D* are connected in sequel. The entire system is connected in parallel with *A*, *B* and *C*, *D*. The reliability of this system is given by the probability $P(A \cap B) \cup P(C \cap D)$.

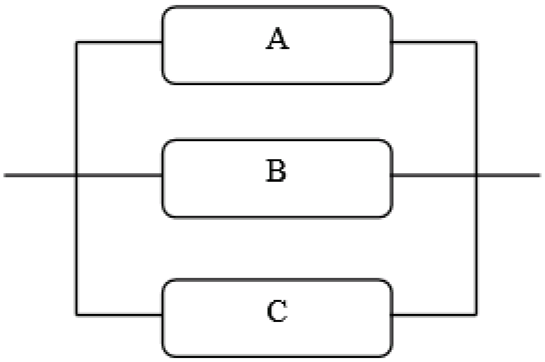
$$\begin{aligned} &P(A \cap B) \cup P(C \cap D) \\ &= P(A \cap B) + P(C \cap D) - P(A \cap B) \cap P(C \cap D) \\ &= P(A).P(B) + P(C).P(D) - P(A).P(B).P(C).P(D) \\ &= 0.9(0.8) + 0.7(0.6) - (0.9)(0.8)(0.7)(0.6) \end{aligned}$$

$= 0.72 + 0.42 - 0.3024$
$= 0.8376$

Reliability of this system is 0.8376.

**Step 4/8**
(c)

The diagrammatic representation of the system is as follows.




In the above system, the three components *A*, *B* and *C* are connected in parallel. So the reliability of the system is the probability $P(A \cup B \cup C)$. This probability is equivalent to the probability 1 - $P(A \cup B \cup C)^c$.

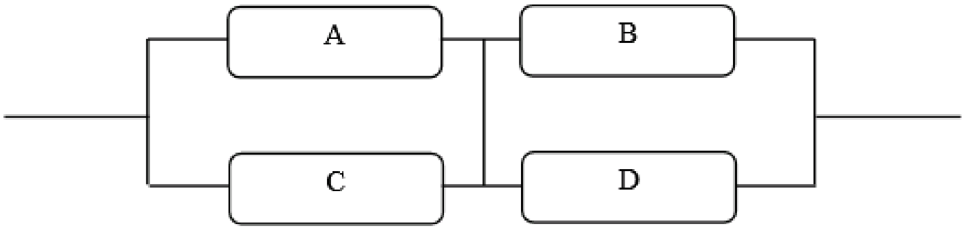
$1 - P(A \cup B \cup C)^c$
$= 1 - P(A^c \cap B^c \cap C^c)$
$= 1 - \left[ P(A^c).P(B^c).P(C^c) \right]$
$= 1 - \left[ (1 - 0.9)(1 - 0.8)(1 - 0.7) \right]$

$= 1 - \left[ (0.1)(0.2)(0.3) \right]$
$= 1 - 0.006$
$= 0.994$

Reliability of this system is 0.994.

**Step 5/8**
(d)

The diagrammatic representation of the system is as follows.




This system is same as the system in part (b) but having an extra link in the center. It enables the system additional pair of components *AD* and *CB* than in the system in part (b). The reliability of this system is given by the probability

$P(A\cap B)\cup P(C\cap D)\cup P(A\cap D)\cup P(C\cap B)$.

$$P(A\cap B)\cup P(C\cap D)$$
$$=0.8376$$

$$P(A\cap D)\cup P(C\cap B)$$
$$=P(A\cap D)+P(C\cap B)-P(A\cap D)\cap P(C\cap B)$$
$$=P(A).P(D)+P(C).P(B)-P(A).P(D).P(C).P(B)$$
$$=0.9(0.6)+0.7(0.8)-(0.9)(0.6)(0.7)(0.8)$$

$$=0.54+0.56-0.3024$$
$$=0.7976$$

**Step 6/8**
Now, the reliability of the entire system is given as follows.

$$P(A\cap B)\cup P(C\cap D)\cup P(A\cap D)\cup P(C\cap B)$$
$$=P(A\cap B)\cup P(C\cap D)+P(A\cap D)\cup P(C\cap B)$$
$$-\left(P(A\cap B)\cup P(C\cap D)\cap P(A\cap D)\cup P(C\cap B)\right)$$

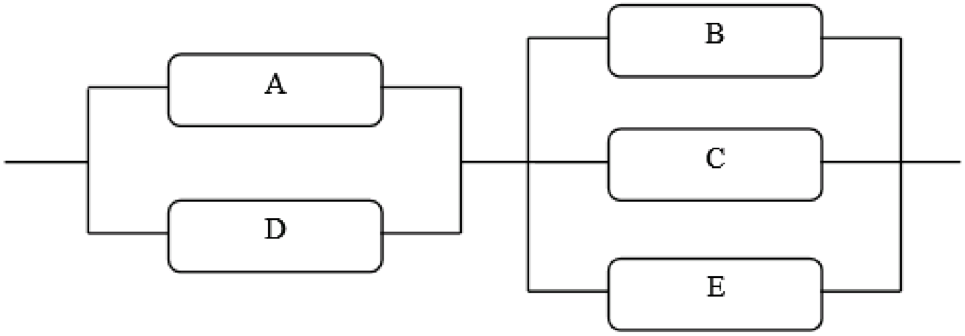
= 0.8376 + 0.7976 - (0.8376) (0.7976)

= 0.9671

Reliability of the system is 0.9671.

**Step 7/8**
(e)

The diagrammatic representation of the system is as follows.




This system contains two sequel systems *AD* and *BCE*. The reliability of this system is given by the probability $P(A \cup D) \cap P(B \cup C \cup E)$.

$$\begin{aligned} P(A \cup D) &= P(A) + P(D) - P(A).P(D) \\ &= 0.9 + 0.6 - (0.9)(0.6) \\ &= 1.5 - 0.54 \\ &= 0.96 \end{aligned}$$

$$\begin{aligned} P(B \cup C \cup E) &= 1 - P(B \cup C \cup E)^c \\ &= 1 - P(B^c \cap C^c \cap E^c) \\ &= 1 - \left[ P(B^c).P(C^c).P(E^c) \right] \\ &= 1 - \left[ (1-0.8)(1-0.7)(1-0.5) \right] \end{aligned}$$

$$\begin{aligned} &= 1 - \left[ (0.2)(0.3)(0.5) \right] \\ &= 1 - 0.03 \\ &= 0.97 \end{aligned}$$

**Step 8/8**

Now, the reliability of the entire system is given as follows.

$$\begin{aligned} P(A \cup D) \cap P(B \cup C \cup E) &= P(A \cup D).P(B \cup C \cup E) \\ &= 0.96(0.97) \\ &= 0.9312 \end{aligned}$$

Reliability of this system is 0.9312.

**Chapter 2, Problem 24E**

(2)

Problem
Among 10 laptop computers, five are good and five have defects. Unaware of this, a customer buys 6 laptops.

(a) What is the probability of exactly 2 defective laptops among them?

(b) Given that *at least* 2 purchased laptops are defective, what is the probability that *exactly* 2 are defective?

**Step-by-step solution**

**Show all steps**

100% (8 ratings) for this solution

**Step 1/8**
Given that among 10 laptop computers, five are good and five have defects and unaware of this, a customer buys 6 laptops.

**Step 2/8**
(a)

The probability that exactly 2 defective laptops among them is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the number of total outcomes. Here $N_T$ is the number of ways of choosing 6 laptops from 10 laptops without replacement. It is given by the combination $C(n,k)=\frac{n!}{k!(n-k)!}$. Substitute $n$ = 10 and $k$ = 6 in the combination formula.

$$\begin{aligned} C(10,6) &= \frac{10!}{6!(10-6)!} \\ &= \frac{3,628,800}{720(24)} \\ &= 210 \end{aligned}$$

**Step 3/8**
Here $N_F$ is the number of favourable outcomes which is given as $C(5,2)\times C(5,4)$.

$$\begin{aligned} C(5,2)\times C(5,4) &= \frac{5!}{2!3!}\times\frac{5!}{4!1!} \\ &= \frac{120}{2(6)}\times\frac{120}{24} \\ &= 50 \end{aligned}$$

**Step 4/8**

The probability that exactly 2 defective laptops among them is given by $\frac{50}{210} = 0.2381$

**Step 5/8**
(b)

The probability that exactly 2 are defective given that at least 2 purchased laptops are defective is given by the conditional probability

$P(\text{Exactly two are defective }|\text{At least two are defective})$.

We already computed the probability that exactly two laptops are defective in part (a) as 0.2381. We need the probability that at least two are defective.

**Step 6/8**
The probability that at least two are defective is the complement of probability that at most one laptop is defective, that is 1 - $P(\text{Atmost one is defective})$.

$$
\begin{aligned}
&= 1 - \left(P(\text{exactly one defective})\right) \\
&= 1 - \left(\frac{C(5,1)}{C(10,6)}\right) \\
&= 1 - \frac{5}{210} \\
&= 0.9762
\end{aligned}
$$

**Step 7/8**
Now the conditional probability is computed as follows.

$$
\begin{aligned}
&P(\text{Exactly two are defective }|\text{At least two are defective}) \\
&= \frac{P(\text{Exactly two are defective}) \cap P(\text{At least two are defective})}{P(\text{At least two are defective})} \\
&= \frac{P(\text{Exactly two are defective})}{P(\text{At least two are defective})} \\
&= \frac{0.2381}{0.9762} \\
&= 0.2439
\end{aligned}
$$

**Step 8/8**
The probability that exactly 2 are defective given that at least 2 purchased laptops are defective is 0.2439.

**Chapter 2, Problem 25E**
(0)

Problem
Two out of six computers in a lab have problems with hard drives. If three computers are selected at random for inspection, what is the probability that none of them has hard drive problems?

**Step-by-step solution**

**Show all steps**

**Step 1/4**
Given that two out of six computers in a lab have problems with hard drives. Three computers are selected at random for inspection. We have to find the probability that none of them has hard drive problems.

**Step 2/4**

The probability that none of them has hard drive problems is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the number of total outcomes. Here $N_T$ is the number of ways of choosing three computers from six computers without replacement. It is given by the combination $C(n,k)=\frac{n!}{k!(n-k)!}$. Substitute $n$ = 6 and $k$ = 3 in the combination formula.

$$\begin{aligned} C(6,3) &= \frac{6!}{3!(6-3)!} \\ &= \frac{720}{36} \\ &= 20 \end{aligned}$$

**Step 3/4**
Here $N_F$ is the number of favourable outcomes which is given as $C(4,3)$.

$$\begin{aligned} C(4,3) &= \frac{4!}{3!1!} \\ &= \frac{24}{6} \\ &= 4 \end{aligned}$$

**Step 4/4**

The probability that none of them has hard drive problems is given by $\frac{N_F}{N_T}=\frac{4}{20}=0.2$.

**Chapter 2, Problem 26E**
(1)

Problem
For the University Cheese Club, Danielle and Anthony buy four brands of cheese, three brands of crackers, and two types of grapes. Each club member has to taste two different cheeses, two types of crackers, and one bunch of grapes. If 40 club members show up, can they all have different meals?

**Step-by-step solution**

**Show all steps**

**Step 1/15**
Given Information:

• The number of available brands of cheese, crackers, and grapes is 4, 3, and 2, respectively.

• The number of brands of cheese, crackers, and grapes to be tasted is 2, 2, and 1, respectively.

• 40 members come for the meal.

**Step 2/15**
The number of ways to choose $k$ objects out of $n$ available is calculated using the combinations, for which the formula is stated below:

**Step 3/15**
$$\binom{n}{k}=\frac{n!}{k!(n-k)!}$$

**Step 4/15**
The ways to choose 2 cheese brands out of 4 available are:

**Step 5/15**
$$\begin{aligned}\binom{4}{2}&=\frac{4!}{2!(4-2)!}\\&=\frac{4\times3\times2!}{2!\times2!}\\&=2\times3\\&=6\end{aligned}$$

**Step 6/15**
Thus, there are 6 different ways to choose cheese brands.

**Step 7/15**
The ways to choose 2 cracker brands out of the 3 available are:

**Step 8/15**

$$\begin{aligned}\binom{3}{2} &= \frac{3!}{2!(3-2)!} \\ &= \frac{3\times 2!}{2!\times 1!} \\ &= 3\end{aligned}$$

**Step 9/15**
Thus, there are 3 different ways to choose cracker brands.

**Step 10/15**
The ways to choose one bunch of grapes from 2 available are:

**Step 11/15**

$$\begin{aligned}\binom{2}{1} &= \frac{2!}{1!(2-1)!} \\ &= \frac{2!}{1!\times 1!} \\ &= 2\end{aligned}$$

**Step 12/15**
Thus, there are only 2 different ways to choose the bunch of grapes.

**Step 13/15**
The different categories are chosen independently, so the total possible ways are calculated by multiplying them as:

$$\begin{aligned}n &= 6\times 3\times 2 \\ &= 36\end{aligned}$$

**Step 14/15**
It is found that there are 36 different possibilities for meals.

Thus, 40 club members cannot have all different meals because there are only 36 different choices possible.

**Step 15/15**
No, the 40 club members cannot have different meals altogether.

**Chapter 2, Problem 27E**
(0)

Problem
This is known as *the Birthday Problem*.

(a) Consider a class with 30 students. Compute the probability that at least two of them have their birthdays on the same day. (For simplicity, ignore the leap year.)

(b) How many students should be in class in order to have this probability above 0.5?

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/4**
(a)

We have to compute the probability that at least two of 30 students in a class have their birthdays on the same day. The probability can be computed using the complement rules. The probability that at least two of 30 students in a class have their birthdays on the same day will be the complement of the probability that none of the students have their birthdays on same day. This is the probability given by $1-P(\text{Distinct birthdays})$.

$$
\begin{aligned}
P(\text{Distinct birthdays}) &= \frac{365}{365}\times\frac{364}{365}\times\frac{363}{365}\times\ldots\times\frac{336}{365} \\
&= \frac{P(365,30)}{365^{30}} \\
&= 0.293683757
\end{aligned}
$$

The probability that at least two of 30 students in a class have their birthdays on the same day = 1 - 0.294 = 0.706.

**Step 2/4**
(b)

We have to find the number of students in class in order to have this probability above 0.5. So we should have $P(\text{Distinct birthdays}) < 0.5$.

$$\frac{P(365,n)}{365^{n}} < 0.5$$

We have $\frac{P(365,30)}{365^{30}} = 0.294$

For $n$ = 20, $\frac{P(365,20)}{365^{20}} = 0.589$

So $n$ is between 20 and 30. We can use linear interpolation as follows.

$$\frac{n-20}{30-20} = \frac{0.5-0.589}{0.294-0.589}$$

$$n = \frac{-0.089}{-0.295}(10)+20$$
$$= 23.017$$

**Step 3/4**

So $n$ = 23 will give $\frac{P(365,n)}{365^n} < 0.5$.

$$\frac{P(365,23)}{365^{23}} = 0.4927 < 0.5$$

**Step 4/4**

So at least 23 students should be in the class in order to keep the probability of at least two students share their birthday above 0.5.

**Chapter 2, Problem 28E**

(0)

Problem

Among eighteen computers in some store, six have defects. Five randomly selected computers are bought for the university lab. Compute the probability that all five computers have no defects.

**Step-by-step solution**

**Show all steps**

100% (3 ratings) for this solution

**Step 1/4**

Given that among eighteen computers in some store, six have defects. Five randomly selected computers are bought for the university lab.

**Step 2/4**

The probability that all five computers have no defects is given by $\frac{N_F}{N_T}$ where $N_F$ is the number of favourable outcomes and $N_T$ is the number of total outcomes. Here $N_T$ is the number of ways of choosing five computers from eighteen computers without replacement. It is given by the combination $C(n,k)=\dfrac{n!}{k!(n-k)!}$. Substitute *n* = 18 and *k* = 5 in the combination formula.

$$\begin{aligned} C(18,5) &= \frac{18!}{5!(18-5)!} \\ &= 8{,}568 \end{aligned}$$

**Step 3/4**

Here $N_F$ is the number of favourable outcomes which is given as $C(12,5)$.

$$\begin{aligned} C(12,5) &= \frac{12!}{5!7!} \\ &= 792 \end{aligned}$$

**Step 4/4**

The probability that none of them has hard drive problems is given by

$$\frac{N_F}{N_T}=\frac{792}{8{,}568}=0.0924.$$

**Chapter 2, Problem 29E**

(1)

Problem

A quiz consists of 6 multiple-choice questions. Each question has 4 possible answers. A student is unprepared, and he has no choice but guessing answers completely at random. He passes the quiz if he gets at least 3 questions correctly. What is the probability that he will pass?

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/3**

Given that a quiz consists of 6 multiple-choice questions and each question has 4 possible answers. A student is unprepared and guesses answers completely at random. He passes the quiz if he gets at least 3 questions correctly.

**Step 2/3**
The probability that the student passes is the probability that he guess at least three questions correctly. As there are four possible answers for a question, the probability of guessing the correct answer is 0.25.

$$P(\text{guessing at least three questions correct})$$
$$=P(\text{guessing three questions correct}) + P(\text{guessing four questions correct})$$
$$+ P(\text{guessing five questions correct}) + P(\text{guessing six questions correct})$$

$$P(\text{guessing three questions correct}) = \mathrm{C}(6,3) \times 0.25^3 \times 0.75^3 = 0.13184$$

$$P(\text{guessing four questions correct}) = \mathrm{C}(6,4) \times 0.25^4 \times 0.75^2 = 0.03296$$

$$P(\text{guessing five questions correct}) = \mathrm{C}(6,5) \times 0.25^5 \times 0.75^1 = 0.004394$$

$$P(\text{guessing six questions correct}) = \mathrm{C}(6,6) \times 0.25^6 \times 0.75^0 = 0.000244$$

**Step 3/3**
Add the above probabilities to get the required probability.

0.13184 + 0.03296 + 0.004394 + 0.000244 = 0.1694.

So the probability that the student passes the quiz by guessing is 0.1694.

**Chapter 2, Problem 30E**
(0)

Problem
An internet search engine looks for a keyword in 9 databases, searching them in a random order. Only 5 of these databases contain the given keyword. Find the probability that it will be found in at least 2 of the first 4 searched databases.

**Step-by-step solution**

**Show all steps**

**Step 1/3**
Given that an internet search engine looks for a keyword in 9 databases, searching them in a random order. Only 5 of these databases contain the given keyword. We have to find the probability that it will be found in at least 2 of the first 4 searched databases.

**Step 2/3**

Let A be the event that the database contain the given keyword. Then $P(A) = \frac{5}{9}.$
Probability that the keyword will be found in at least 2 of the first 4 searched databases is given by

= *P* (found in exactly two of the first 4 searched databases) + *P* (found in exactly three of the first 4 searched databases) + *P* (found in exactly four of the first 4 searched databases)

= $C$ (4, 2) $(5/9)^2$ $(4/9)^2$ + $C$ (4, 3) $(5/9)^3$ $(4/9)^1$ + $C$ (4, 4) $(5/9)^4$ $(4/9)^0$

= 6 (0.060966316) + 4 (0.076207895) + 1 (0.095259868)

= 0.766

**Step 3/3**
The probability that the internet search engine will find the given keyword in at least 2 of the first 4 searched databases is 0.766.

**Chapter 2, Problem 31E**
(0)

Problem
Consider the situation described in Example 2.24 on p. 22, but this time let us define the sample space clearly. Suppose that one child is older, and the other is younger, their gender is independent of their age, and the child you met is one or the other with probabilities 1/2 and 1/2.

(a) List all the outcomes in this sample space. Each outcome should tell the children's gender, which child is older, and which child you have met.

(b) Show that *unconditional* probabilities of outcomes *BB, BG*, and *GB* are equal.

(c) Show that *conditional* probabilities of *BB, BG*, and *GB*, after you met Lev, are not equal.

(d) Show that Lev has a brother with *conditional* probability 1/2.

**Step-by-step solution**

**Show all steps**

**Step 1/11**
Given that a family has two children and we met one to them. In the two children, one is older and the other is younger. Their gender is independent of their age, and the child we meet is one or the other with probabilities 1/2 and 1/2.

**Step 2/11**
(a)

Let *YG* denotes the younger girl, *YB* denotes the younger boy, *EG* denotes the elder girl and *EB* denotes the elder boy. Let the first two letters of an event denotes the child we met. Then the sample space is

{*YGEG*, *YGEB*, *EGYG*, *EGYB*, *YBEG*, *YBEB*, *EBYG*, *EBYB*}

**Step 3/11**
(b)

Unconditional probability of *BB*, which is the probability of two boys, is 2 / 8 as there are two events of getting two boys namely *YBEB* and *EBYB* and the total number of events in the sample space is 8.

**Step 4/11**
Unconditional probability of *BG*, which is the probability of elder boy and younger girl, is also 2 / 8 as there are two events namely *EBYG* and *YGEB* out of 8 events.

**Step 5/11**
Unconditional probability of *GB* which is the probability of elder girl younger boy is also 2 / 8 as there are two events namely *EGYB* and *YBEG* out of 8 events. So the unconditional probabilities of outcomes *BB, BG* and *GB* are equal.

**Step 6/11**
Hence it is shown that unconditional probabilities of *BB*, *BG* and *GB* are equal.

**Step 7/11**
(c)

Conditional probability of *BB* after we met Leo is given by $P(BB \mid \text{first met child is boy})$ which is given by formula

$$\frac{P(BB \cap \text{first met Leo})}{P(\text{first met Leo})}$$

*P* (first met Leo) = 4 / 8 as there are four events namely *YBEG*, *YBEB*, *EBYG*, and *EBYB* out of 8 events.

$$\frac{P(BB \cap \text{first met Leo})}{P(\text{first met Leo})}$$
$$= \frac{P(BB)}{P(\text{first met Leo})}$$
$$= \frac{2/8}{4/8}\left[\text{As } P(BB) = 2/8 \text{ from previous part}\right]$$
$$= 0.5$$

**Step 8/11**

Conditional probability of *BG* after we met Leo is given by $P(BG \mid \text{first met Leo})$ which is given by formula

$$\frac{P(BG \cap \text{first met Leo})}{P(\text{first met Leo})}$$

$$= \frac{P(BG \cap \text{first met Leo})}{4/8}$$
$$= \frac{1/8}{4/8}\left[\text{As } (BG \cap \text{first met Leo}) = EBYG\right]$$
$$= 0.25$$

**Step 9/11**

Conditional probability of *GB* after we met Leo is given by $P(GB \mid \text{first met Leo})$ which is given by formula

$$\frac{P(GB \cap \text{first met Leo})}{P(\text{first met Leo})}$$

$$= \frac{P(GB \cap \text{first met Leo})}{4/8}$$
$$= \frac{1/8}{4/8}\left[\text{As } (GB \cap \text{first met Leo}) = YBEG\right]$$
$$= 0.25$$

**Step 10/11**

Here the three probabilities are not equal. Hence it is shown that conditional probabilities of *BB*, *BG* and *GB* are not equal.

**Step 11/11**

(d)

Conditional probability that Leo has a brother is given by $P(\text{Leo has brother} \mid \text{First met Leo})$.

$$P(\text{Leo has brother} \mid \text{First met Leo}) = \frac{\mathrm{P}(\text{Leo has brother } \cap \text{ First met Leo})}{P(\text{First met Leo})}$$
$$= \frac{2/8}{4/8}\ \left(\text{As (Leo has brother } \cap \text{ First met Leo)} = EBYB \text{ and } YBEB\right)$$
$$= 1/2$$

It is shown that the conditional probability that Leo has a brother is 1/2.

**Chapter 2, Problem 32E**
(0)

Problem

Show that events $A,\ \ B,\ \ C,\ \ \ldots$ are disjoint if and only if $A, \overline{B}, C, \ldots$ are exhaustive.

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/4**

To show that the events *A*, *B*, *C*… are disjoint if and only if $\overline{A},\ \overline{B},\ \overline{C}\ldots$ are exhaustive.

**Step 2/4**

First assume that *A*, *B*, *C*… are disjoint. By the definition for disjoint events, $A\cap B\cap C\cap\ldots=\{\varnothing\}$. This implies that $\left(\overline{A\cap B\cap C\cap\ldots}\right)=\Omega$. By the rules of complement for intersection of events,

$$\left(\overline{A\cap B\cap C\cap\ldots}\right)=\left(\overline{A}\cup\overline{B}\cup\overline{C}\cup\ldots\right)=\Omega$$

It is the definition for the exhaustive events. So the events $\overline{A},\ \overline{B},\ \overline{C}\ldots$ are exhaustive if *A*, *B*, *C*… are disjoint.

**Step 3/4**

Now assume that the events $\overline{A},\ \overline{B},\ \overline{C}\ldots$ are exhaustive. Then by the definition for exhaustive events, $\left(\overline{A}\cup\overline{B}\cup\overline{C}\cup\ldots\right)=\Omega$. By the rule of union of complement events, $\left(\overline{A}\cup\overline{B}\cup\overline{C}\cup\ldots\right)=\left(\overline{A\cap B\cap C\cap\ldots}\right)=\Omega$ which implies that $A\cap B\cap C\cap\ldots=\{\varnothing\}$. It is the definition for disjoint events. So the events *A*, *B*, *C*… are disjoint if $\overline{A},\ \overline{B},\ \overline{C}\ldots$ are exhaustive.

**Step 4/4**

So it is shown that the events *A*, *B*, *C*… are disjoint if and only if $\overline{A},\ \overline{B},\ \overline{C}\ldots$ are exhaustive.

**Chapter 2, Problem 33E**
(0)

Problem
Events *A* and *B* are independent. Show, intuitively and mathematically, that:

(a) Their complements are also independent.

(b) If they are disjoint, then ***P***{*A*} = 0 or ***P***{*B*} = 0.

(c) If they are exhaustive, then ***P***{*A*} = 1 or ***P***{*B*} = 1.

**Step-by-step solution**

**Show all steps**

**Step 1/4**
Given that the events *A* and *B* are independent.

**Step 2/4**
(a)

We have to show that the complements of *A* and *B* are also independent. Events *A* and *B* are independent if the probability of occurrence of *A* is not affected by event *B* and vice versa. If *A* and *B* are independent then $P(A \cap B) = P(A).P(B)$. We have to prove that $P(\overline{A} \cap \overline{B}) = P(\overline{A}).P(\overline{B})$.

$$\begin{aligned}
& P(\overline{A} \cap \overline{B}) \\
& = P(\overline{A \cup B}) \\
& = 1 - P(A \cup B) \\
& = 1 - P(A) - P(B) + P(A \cap B) \\
& = 1 - P(A) - P(B) + P(A).P(B) \\
& = P(\overline{A}) - P(B)\left[1 - P(A)\right] \\
& = P(\overline{A}) - P(B)\left[P(\overline{A})\right] \\
& = P(\overline{A})\left[1 - P(B)\right] \\
& = P(\overline{A}).P(\overline{B})
\end{aligned}$$

So we have shown that $P(\overline{A} \cap \overline{B}) = P(\overline{A}).P(\overline{B})$. So the complements of *A* and *B* are independent if the events *A* and *B* are independent.

**Step 3/4**
(b)

We have to show that if the events *A* and *B* are disjoint, then $P(A)=0 \text{ or } P(B)=0$. As the events *A* and *B* are independent, $P(A\cap B)=P(A).P(B)$. Also as the events *A* and *B* are disjoint, $P(A\cap B)$ = 0 as $(A\cap B)=\{\varnothing\}$ for disjoint events. So combining these two rules will give as $P(A\cap B)=P(A).P(B)$ = 0. So $P(A)=0 \text{ or } P(B)=0$ if the events *A* and *B* are independent and disjoint.

**Step 4/4**
(c)

We have to show that if the events *A* and *B* are exhaustive, then $P(A)=1 \text{ or } P(B)=1$. As the events *A* and *B* are exhaustive, $P(A\cup B)=1 \text{ as } A\cup B=\Omega$. Also as the events *A* and *B* are independent, $P(A\cap B)=P(A).P(B)$ and $P(A\cup B)=P(A)+P(B)-P(A).(B)$. So combining these two rules will give as $P(A\cup B)=P(A)+P(B)-P(A).P(B)=1$.

$$P(A)+P(B)-P(A).P(B)=1$$
$$P(A)\left[1-P(B)\right]=\left[1-P(B)\right]$$
$$P(A)=1$$

There is another chance is that

$$P(A)+P(B)-P(A).P(B)=1$$
$$P(B)\left[1-P(A)\right]=\left[1-P(A)\right]$$
$$P(B)=1$$

So $P(A)=1 \text{ or } P(B)=1$ if the events *A* and *B* are independent and exhaustive.

**Chapter 2, Problem 34E**
(0)

Problem
Derive a computational formula for the probability of a union of *N* arbitrary events. Assume that probabilities of all individual events and their intersections are given.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/2**
We have to derive a computational formula for the probability of a union of *N* arbitrary events assuming that probabilities of all individual events and their intersections are given.

**Step 2/2**
Let $A_1$, $A_2$, $A_3 \ldots A_N$ denotes *N* arbitrary events. We require the formula to find the probability $P(A_1 \cup A_2 \cup A_3 \cup ... \cup A_N)$. For two events, we have the probability $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$ and for three events, we have the probability

$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) = {} & P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) \\ & + P(A_1 \cap A_2 \cap A_3) \end{aligned}$$

Generalizing this will give,

$$\begin{aligned} & P(A_1 \cup A_2 \cup A_3 \cup ... \cup A_N) \\ & = P(A_1) + P(A_2) + P(A_3) + ... + P(A_N) \\ & -P(A_1 \cap A_2) - P(A_1 \cap A_3) - ... - P(A_1 \cap A_N) \\ & -P(A_2 \cap A_3) - P(A_2 \cap A_4) - ... - P(A_2 \cap A_N) \\ & -P(A_3 \cap A_4) - P(A_3 \cap A_5) - ... - P(A_3 \cap A_N) \\ & ... \\ & ... \\ & ... \\ & -P(A_{N-3} \cap A_{N-2}) - P(A_{N-3} \cap A_{N-1}) - ... - P(A_{N-3} \cap A_N) \\ & -P(A_{N-2} \cap A_{N-1}) - P(A_{N-2} \cap A_N) \\ & -P(A_{N-1} \cap A_N) \\ & +P(A_1 \cap A_2 \cap A_3 \cap ... \cap A_N) \end{aligned}$$

This is the formula to compute the probability of a union of *N* arbitrary events.

**Chapter 2, Problem 35E**
(2)

Problem
Prove that

$$\overline{E_1 \cap \ldots \cap E_n} = \overline{E_1} \cup \ldots \cup \overline{E_n}$$

for arbitrary events $E_1, \ldots, E_n.$

**Step-by-step solution**

**Show all steps**

**Step 1/2**

We have to prove that $\overline{E_1 \cap \ldots \cap E_n} = \overline{E}_1 \cup \ldots \cup \overline{E}_n$ for arbitrary events $E_1 \ldots E_n$.

$$\begin{aligned}\overline{E_1 \cap \ldots \cap E_n} &= 1 - (E_1 \cap \ldots \cap E_n) \\ &= 1 - \{\text{all events occur}\} \\ &= \text{complement of At least one event occur} \\ &= \overline{E}_1 \cup \ldots \cup \overline{E}_n\end{aligned}$$

**Step 2/2**

So we have proved that $\overline{E_1 \cap \ldots \cap E_n} = \overline{E}_1 \cup \ldots \cup \overline{E}_n$ for arbitrary events $E_1 \ldots E_n$.

**Chapter 2, Problem 35E**
(2)

Problem
Prove that

$$\overline{E_1 \cap \ldots \cap E_n} = \overline{E}_1 \cup \ldots \cup \overline{E}_n$$

for arbitrary events $E_1, \ldots, E_n.$

**Step-by-step solution**

**Show all steps**

**Step 1/2**

We have to prove that $\overline{E_1 \cap \ldots \cap E_n} = \overline{E}_1 \cup \ldots \cup \overline{E}_n$ for arbitrary events $E_1 \ldots E_n$.

$$\begin{aligned}\overline{E_1 \cap \ldots \cap E_n} &= 1 - (E_1 \cap \ldots \cap E_n) \\ &= 1 - \{\text{all events occur}\} \\ &= \text{complement of At least one event occur} \\ &= \overline{E}_1 \cup \ldots \cup \overline{E}_n\end{aligned}$$

**Step 2/2**

So we have proved that $\overline{E_1 \cap ... \cap E_n} = \overline{E}_1 \cup ... \cup \overline{E}_n$ for arbitrary events $E_1 ... E_n$.

**Chapter 2, Problem 37E**
(0)

Problem

Prove “subadditivity”: $\boldsymbol{P}\{E_1 \cup E_2 \cup \ldots\} \leq \sum \boldsymbol{P}\{E_i\}$ for *any* events $E_1, E_2, \ldots \in \mathfrak{M}$.

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/3**

We have to prove the sub additive: $P(E_1 \cup E_2 \cup ...) \leq \sum P(E_i)$ for any events $E_1, E_2 ... \in \Re$.

**Step 2/3**

If $E_1, E_2 ... \in \Re$ then $(E_1 \cup E_2 \cup ...) \in \Re$, by the definition for sigma algebra. From the Sigma - additive rule for mutually exclusive events,

$$P(E_1 \cup E_2 \cup ...) = \sum P(E_i)$$

For any events of $E_1, E_2 ... \in \Re$, the left hand side of the above equation will become as follows.

$$\begin{aligned}
&P(E_1 \cup E_2 \cup ...) \\
&= P(E_1) + P(E_2) + ... \\
&-P(E_1 \cap E_2) - P(E_1 \cap E_3) - ... \\
&-P(E_2 \cap E_3) - P(E_2 \cap E_4) - ... \\
&-P(E_3 \cap E_4) - P(E_3 \cap E_5) - ... \\
&... \\
&... \\
&... \\
&+P(E_1 \cap E_2 \cap ...)
\end{aligned}$$

$$P(E_1 \cup E_2 \cup ...) + P(E_1 \cap E_2) + P(E_1 \cap E_3) + ... - P(E_1 \cap E_2 \cap ...)$$
$$= \sum P(E_i)$$

$$P(E_1 \cup E_2 \cup ...) < \sum P(E_i)$$

So $P(E_1 \cup E_2 \cup ...) \leq \sum P(E_i)$ for any events as $P(E_1 \cup E_2 \cup ...) = \sum P(E_i)$ for mutually exclusive events.

**Step 3/3**

We have proved the sub additive $P(E_1 \cup E_2 \cup ...) \leq \sum P(E_i)$ for any events $E_1, E_2 ... \in \Re$.

Problem

A computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4. Independently of it, the second file will be corrupted with probability 0.3.

(a) Compute the probability mass function (pmf) of *X*, the number of corrupted files.

(b) Draw a graph of its cumulative distribution function (cdf).

**Step-by-step solution**

**Show all steps**

95% (20 ratings) for this solution

**Step 1/6**

Given that a computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4 and independently of it; the second file will be corrupted with probability 0.3.

**Step 2/6**

(a)

We have to compute the probability mass function (pmf) of $X$, the number of corrupted files. The possible outcomes of the value of the random variable $X$ is 0, 1 and 2 where $(X = 0)$ denotes no files corrupted, $(X = 1)$ denotes one file corrupted and $(X = 2)$ denotes two files corrupted.

**Step 3/6**

The probability mass function is the collection of probabilities of all possible outcomes of the random variable.

$P(X = 0)$

$=$ Probability that first file not corrupted $\times$ Probability that second file not corrupted

$= (1 - 0.4) \times (1 - 0.3) = 0.6 \times 0.7 = 0.42$

$P(X = 1)$

$=$ (Probability that first file corrupted $\times$ Probability that second file not corrupted) $+$ (Probability that first file not corrupted $\times$ Probability that second file corrupted)

$= 0.4(1 - 0.3) + 0.3(1 - 0.4) = (0.4 \times 0.7) + (0.3 \times 0.6)$

$= 0.28 + 0.18$

$= 0.46$

$P(X = 2)$

$=$ Probability that first file corrupted $\times$ Probability that second file corrupted

$= 0.4 \times 0.3 = 0.12$

## Step 4/6

The probability mass function of $X$, the number of corrupted files is as follows.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(x)$ | 0.42 | 0.46 | 0.12 |

## Step 5/6

(b)

The cumulative distribution function (cdf) of $F(x)$ is given as follows. The cumulative probabilities are calculated by adding the corresponding probability with the previous cumulative probabilities.

$F(x)=0$ is 0.42.

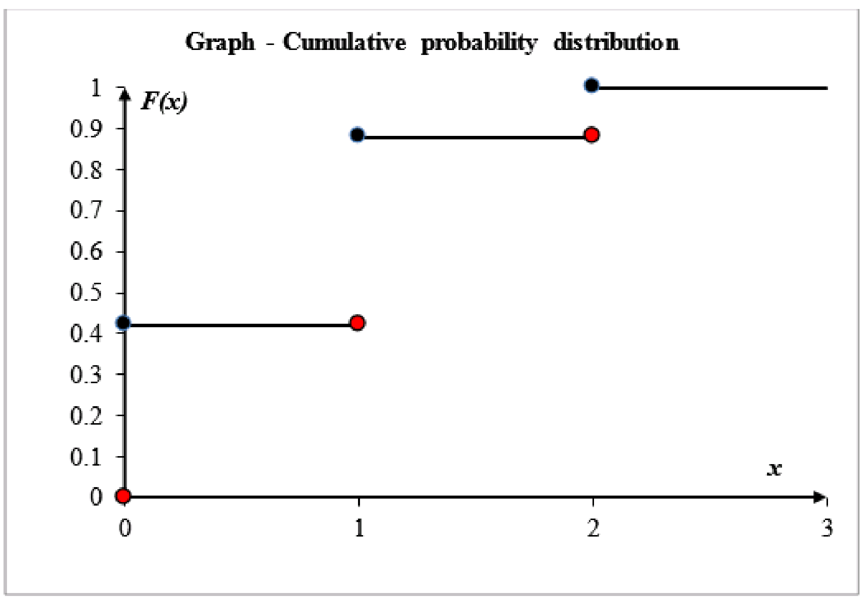
$F(x)=1$ is $0.42+0.46=0.88$

$F(x)=2$ is $0.88+0.12=1.00$.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $F(x)$ | 0.42 | 0.88 | 1.00 |

## Step 6/6

The graph of the cumulative probability of $X$ is as follows.




In the above graph, the red circles denote excluded points.

## Chapter 3, Problem 2E

(0)

Problem

Every day, the number of network blackouts has a distribution (probability mass function)

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(x)$ | 0.7 | 0.2 | 0.1 |

A small internet trading company estimates that each network blackout results in a $500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

**Step-by-step solution**

**Show all steps**

100% (10 ratings) for this solution

**Step 1/4**

Given that the number of network blackouts has a distribution (probability mass function) given in the table and a small internet trading company estimates that each network blackout results in $500 loss.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(x)$ | 0.7 | 0.2 | 0.1 |

We have to compute the expectation and variance of this company's daily loss due to blackouts.

**Step 2/4**

Expectation of *X* is given as $E(X) = \sum_x xP(x)$.

$$\sum_x xP(x) = (0\times 0.7) + (1\times 0.2) + (2\times 0.1)$$
$$= 0 + 0.2 + 0.2$$
$$= 0.4$$

Expectation of the company's daily loss due to network blackouts = *E* (500*X*)

= 500 *E*(*X*) as *E*(*aX*) = *aE*(X)

500 *E*(*X*) = $500 × 0.4 = $200.

**Step 3/4**

Variance of *X* is given as $Var(X) = E(X^2) - [E(X)]^2$.

$$E(X^2) = \sum_x x^2P(x) = (0^2\times 0.7) + (1^2\times 0.2) + (2^2\times 0.1)$$
$$= 0 + 0.2 + 0.4$$
$$= 0.6$$

$$\begin{aligned} Var(X) &= 0.6-[0.4]^2 \\ &= 0.6-0.16 \\ &= 0.44 \end{aligned}$$

Variance of the company's daily loss due to network blackouts = *Var* (500*X*)

*Var* (500*X*) = $500^2$ *Var*(*X*) as *Var* (a*X*) = $a^2$ *Var* (*X*).

$500^2$ *Var*(*X*) = 250,000(0.44)

= $110, 000

**Step 4/4**
Expectation of the company's daily loss due to network blackouts is $200 and the variance of the company's daily loss due to network blackouts is $110,000.

**Chapter 3, Problem 3E**
(0)

Problem
There is one error in one of five blocks of a program. To find the error, we test three randomly selected blocks. Let *X* be the number of errors in these three blocks. Compute **E**(*X*) and Var(*X*).

**Step-by-step solution**

**Show all steps**

100% (13 ratings) for this solution

**Step 1/3**
Given that there is one error in one of five blocks of a program. To find the error, three randomly selected blocks are tested and *X* denotes the number of errors in these three blocks. We have to compute *E*(*X*) and *Var*(*X*).

**Step 2/3**
The possible values of the random variable *X* are 0 and 1.

$P(X=0)$

= Probability that all the three blocks do not contain error

= $\frac{C(4,3)}{C(5,3)}$ as there are *C* (4, 3) ways to choose three blocks without error from four error -free blocks and *C* (5, 3) ways to choose three blocks from five blocks.

= 0.4.

$P(X=1)$

= Probability that one of the three blocks contain error

= $\frac{C(4,2)}{C(5,3)}$ as there are *C* (4, 2) ways to choose two blocks without error from four error -free blocks and *C* (5, 3) ways to choose three blocks from five blocks.

= 0.6.

The probability mass function of *X* is as follows.

| | | |
|---|---|---|
| *x* | 0 | 1 |
| *P*(*x*) | 0.4 | 0.6 |

Expectation of *X* is given as $E(X)=\sum_x xP(x)$.

$$\sum_x xP(x) = (0\times 0.4)+(1\times 0.6)$$
$$= 0+0.6$$
$$= 0.6$$

*E*(X) = 0.6

**Step 3/3**

Variance of *X* is given as $Var(X)=E(X^2)-\left[E(X)\right]^2$.

$$E(X^2)=\sum_x x^2P(x)=(0^2\times 0.4)+(1^2\times 0.6)$$
$$=0.6$$

$$Var(X)=0.6-\left[0.6\right]^2$$
$$=0.6-0.36$$
$$=0.24$$

*Var*(*X*) = 0.24

**Chapter 3, Problem 4E**

(0)

Problem

Tossing a fair die is an experiment that can result in any integer number from 1 to 6 with equal probabilities. Let *X* be the number of dots on the top face of a die. Compute **E**(*X*) and Var(*X*).

**Step-by-step solution**

**Show all steps**

100% (9 ratings) for this solution

**Step 1/4**
Given that a fair die is tossed and *X* denotes the number of dots on top face of a die. We have to compute *E*(*X*) and *Var*(*X*).

**Step 2/4**
The possible values of the random variable *X* are 1, 2, 3, 4, 5 and 6. The probability of getting any number from 1 to 6 is 1/6 as getting the numbers is equal likely.

$$P(X=1)=P(X=2)=P(X=3)=P(X=4)=P(X=5)=P(X=6)=1/6$$

The probability mass function of *X* is as follows.

| *x* | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| *P*(*x*) | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |

**Step 3/4**
Expectation of *X* is given as $E(X)=\sum_x xP(x)$.

$$\begin{aligned}\sum_x xP(x) &= \left(1\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right)+\left(3\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)+\left(5\times\frac{1}{6}\right)+\left(6\times\frac{1}{6}\right)\\ &= \left(\frac{1}{6}\right)+\left(\frac{2}{6}\right)+\left(\frac{3}{6}\right)+\left(\frac{4}{6}\right)+\left(\frac{5}{6}\right)+\left(\frac{6}{6}\right)\\ &= \frac{21}{6}\\ &= 3.5\end{aligned}$$

*E*(X) = 3.5

**Step 4/4**
Variance of *X* is given as $Var(X)=E(X^2)-\left[E(X)\right]^2$.

$$\begin{aligned}E(X^2)=\sum_x x^2P(x) &= \left(1^2\times\frac{1}{6}\right)+\left(2^2\times\frac{1}{6}\right)+\left(3^2\times\frac{1}{6}\right)+\left(4^2\times\frac{1}{6}\right)+\left(5^2\times\frac{1}{6}\right)+\left(6^2\times\frac{1}{6}\right)\\ &= \left(\frac{1}{6}\right)+\left(\frac{4}{6}\right)+\left(\frac{9}{6}\right)+\left(\frac{16}{6}\right)+\left(\frac{25}{6}\right)+\left(\frac{36}{6}\right)\\ &= \frac{91}{6}\end{aligned}$$

$$Var(X) = \frac{91}{6} - \left(\frac{21}{6}\right)^2$$
$$= \frac{35}{1}$$
$$= 2.916$$

*Var*(*X*) = 2.916

**Chapter 3, Problem 5E**
(0)

Problem
A software package consists of 12 programs, five of which must be upgraded. If 4 programs are randomly chosen for testing,

(a) What is the probability that at least two of them must be upgraded?

(b) What is the expected number of programs, out of the chosen four, that must be upgraded?

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/4**
Given that a software package consists of 12 programs, five of which must be upgraded. Four programs are randomly chosen for testing.

**Step 2/4**
(a)

Let *X* be the random variable denotes the number of programs to be upgraded in the randomly chosen four tests. The possible values of the *X* are 0, 1, 2, 3 and 4.

$P(X = 0)$

= Probability that all the four programs not require upgrade.

= $\frac{C(5,0)C(7,4)}{C(12,4)}$ as there are *C* (7, 4) ways to choose four programs out of seven programs not requiring upgrade and *C* (12, 4) ways to choose five programs from twelve programs.

= 0.0707

$P(X=1)$

= Probability that one of the four programs requires upgrade.

= $\frac{C(5,1)C(7,3)}{C(12,4)}$ as there are *C* (5, 1) ways to choose one program out of five programs to be upgraded and *C* (7, 3) ways to choose three programs out of seven programs not requiring upgrade and *C* (12, 5) ways to choose five programs from twelve programs.

= 0.3535

$P(X=2)$

= Probability that two of the four programs requires upgrade.

= $\frac{C(5,2)C(7,2)}{C(12,4)}$

= 0.4242

$P(X=3)$

= Probability that three of the four programs requires upgrade.

= $\frac{C(5,3)C(7,1)}{C(12,4)}$

= 0.1414

$P(X=4)$

= Probability that all the four programs require upgrade.

= $\frac{C(5,4)C(7,0)}{C(12,4)}$

= 0.0101

The probability mass function of *X* is as follows.

| | | | | | |
|---|---|---|---|---|---|
| *x* | 0 | 1 | 2 | 3 | 4 |
| *P*(*x*) | 0.0707 | 0.3535 | 0.4242 | 0.1414 | 0.0101 |

**Step 3/4**

Probability that at least two of them must be upgraded = $P(X \geq 2)$.

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - 0.0707 - 0.3535 \\ &= 0.5758 \end{aligned}$$

**Step 4/4**
(b)

Expected number of programs, out of the chosen four, that must be upgraded is given as $E(X) = \sum_x xP(x)$.

$$\begin{aligned} \sum_x xP(x) &= (0 \times 0.0707) + (1 \times 0.3535) + (2 \times 0.4242) + (3 \times 0.1414) + (4 \times 0.0101) \\ &= 0 + 0.3535 + 0.8484 + 0.4242 + 0.0404 \\ &= 1.6665 \end{aligned}$$

The expected number of Expected number of programs, out of the chosen four, that must be upgraded is 1.6665.

**Chapter 3, Problem 6E**
(0)

Problem
A computer program contains one error. In order to find the error, we split the program into 6 blocks and test two of them, selected at random. Let *X* be the number of errors in these blocks. Compute **E**(*X*).

**Step-by-step solution**

**Show all steps**

100% (9 ratings) for this solution

**Step 1/2**
Given that a computer program contains one error. To find the error, the program is split into 6 blocks and two blocks are selected at random and tested. Here *X* denotes the number of errors in these two blocks. We have to compute *E*(*X*). As there are is only one error in the program, any one block of the six blocks contain error.

**Step 2/2**
The possible values of the random variable *X* are 0 and 1.

$P(X = 0)$

= Probability that all the two blocks do not contain error

$= \frac{C(5,2)}{C(6,2)}$ as there are *C* (5, 2) ways to choose two blocks without error from five error - free blocks and *C* (6, 2) ways to choose two blocks from six blocks.

= 0.667.

$P(X=1)$

= Probability that one of the two blocks contain error

$= \frac{C(5,1)}{C(6,2)}$ as there are *C* (5, 1) ways to choose one blocks without error from five error - free blocks and *C* (6, 2) ways to choose two blocks from six blocks.

= 0.333.

The probability mass function of *X* is as follows.

| *x* | 0 | 1 |
|---|---|---|
| *P*(*x*) | 0.667 | 0.333 |

Expectation of *X* is given as $E(X)=\sum_x xP(x)$.

$$\begin{aligned}\sum_x xP(x) &= (0\times 0.667)+(1\times 0.333)\\ &= 0+0.333\\ &= 0.333\end{aligned}$$

*E*(X) = 0.333

**Chapter 3, Problem 7E**
(0)

Problem
The number of home runs scored by a certain team in one baseball game is a random variable with the distribution

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(x)$ | 0.4 | 0.4 | 0.2 |

The team plays 2 games. The number of home runs scored in one game is independent of the number of home runs in the other game. Let *Y* be the *total* number of home runs. Find **E**(*Y*) and Var(*Y*).

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/4**

Given that the number of home runs scored by a certain team in one baseball game is a random variable with the distribution

| | | | |
|---|---|---|---|
| *x* | 0 | 1 | 2 |
| *P*(*x*) | 0.4 | 0.4 | 0.2 |

The team plays two games. The number of home runs scored in one game is independent of the number of home runs in the other game. Let *Y* be the total number of home runs. We have to find *E*(*Y*) and *Var*(*Y*).

**Step 2/4**

Let *X1* denotes the home runs scored by the baseball team in the first game let *X2* denotes the home runs scored by the team in the second game. Clearly *X1* and *X2* are discrete random variables which take values 0, 1 and 2. As *Y* is the total number of home runs, *Y* = *X1* + *X2*. The variable *Y* is also a random variable as the sum of two random variables is another random variable.

$$
\begin{aligned}
E(Y) &= E(X1 + X2) \\
&= E(X1) + E(X2)
\end{aligned}
$$

$$E(X1) = \sum_x xP(x)$$ .

$$
\begin{aligned}
\sum_x xP(x) &= (0 \times 0.4) + (1 \times 0.4) + (2 \times 0.2) \\
&= 0 + 0.4 + 0.4 \\
&= 0.8
\end{aligned}
$$

So $E(X1) = E(X2) = 0.8$.

*E*(*Y*) = 0.8 + 0.8 = 1.6

**Step 3/4**

Now we have to find the variance.

$$
\begin{aligned}
&VarY = Var(X1 + X2) \\
&= Var(X1) + Var(X2) \text{ (as both are independent)}
\end{aligned}
$$

Variance of *X1* is given as $Var(X1) = E(X1^2) - [E(X1)]^2$ .

$$E(X1^2) = \sum_x x^2 P(x) = (0^2 \times 0.4) + (1^2 \times 0.4) + (2^2 \times 0.2)$$
$$= 0 + 0.4 + 0.8$$
$$= 1.2$$

$$Var(X1) = 1.2 - [0.8]^2$$
$$= 1.2 - 0.64$$
$$= 0.56$$

So $Var(X1) = Var(X2) = 0.56$

*Var*(*Y*) = 0.56 + 0.56 = 1.12.

**Step 4/4**
The expectation *E*(*Y*) is 1.6 and the variance *Var*(*Y*) is 1.12.

**Chapter 3, Problem 8E**
(0)

Problem
A computer user tries to recall her password. She knows it can be one of 4 possible passwords. She tries her passwords until she finds the right one. Let *X* be the number of wrong passwords she uses before she finds the right one. Find **E**(*X*) and Var(*X*).

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/3**
Find $E(X)$ and *Var*(*X*).

The possible values of *X* are 0, 1, 2 and 3 as these are the possible number of wrong passwords the user tries unit she finds the right one.

The probability that the user find the correct password is $\frac{1}{4}$ as the correct password is in among four passwords.

Obtain $P(X=0)$.

$$\begin{aligned}P(X=0)&=\text{User finds the correct password at first attempt}\\&=\frac{1}{4}\\&=0.25\end{aligned}$$

Obtain $P(X=1)$.

$$\begin{aligned}P(X=1)&=\text{User finds the correct password in the second attempt}\\&=\frac{3}{4}\times\frac{1}{3}\\&=\frac{3}{12}\\&=\frac{1}{4}(\text{or }0.25)\end{aligned}$$

Obtain $P(X=2)$.

$$\begin{aligned}P(X=2)&=\text{User finds the correct password in the third attempt}\\&=\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}\\&=\frac{6}{24}\\&=\frac{1}{4}(\text{or }0.25)\end{aligned}$$

Obtain $P(X=3)$.

$$\begin{aligned}P(X=3)&=\text{User finds the correct password in the fourth attempt}\\&=\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}\times\frac{1}{1}\\&=\frac{6}{24}\\&=\frac{1}{4}(\text{or }0.25)\end{aligned}$$

From the results, the probability distribution function of *X* is tabulated below:

| | | | | |
|---|---|---|---|---|
| *x* | 0 | 1 | 2 | 3 |
| *P*(*x*) | 0.25 | 0.25 | 0.25 | 0.25 |

**Step 2/3**

The mean is obtained below:

The formula for $E(X)$ is,

$$E(X) = \sum_x xP(x)$$

Therefore,

$$\begin{aligned}\sum_x xP(x) &= (0\times 0.25)+(1\times 0.25)+(2\times 0.25)+(3\times 0.25)\\ &= 0+0.25+0.5+0.75\\ &= 1.5\end{aligned}$$

Thus, the expected mean is, $E(X) = \boxed{1.5}$

**Step 3/3**

The variance of *X* is obtained below:

The formula for variance of *X* is,

$$Var(X) = E(X^2) - [E(X)]^2$$

Therefore,

$$\begin{aligned}Var(X) &= \left((0^2\times 0.25)+(1^2\times 0.25)+(2^2\times 0.25)+(3^2\times 0.25)\right)-(1.5)^2\\ &= (0+0.25+1.0+2.25)-2.25\\ &= 3.5-2.25\\ &= 1.25\end{aligned}$$

Thus, the variance of *X* is, $Var(X) = \boxed{1.25}$.

**Chapter 3, Problem 9E**

(0)

Problem

It takes an average of 40 seconds to download a certain file, with a standard deviation of 5 seconds. The actual distribution of the download time is unknown. Using Chebyshev’s inequality, what can be said about the probability of spending more than 1 minute for this download?

**Step-by-step solution**

**Show all steps**

75% (4 ratings) for this solution

**Step 1/4**
Given that it takes an average of 40 seconds to download a certain file, with a standard deviation of 5 seconds and the actual distribution of download time is unknown. Using Chebyshev's inequality we have to say about the probability of spending more than 1 minute for this download.

**Step 2/4**
The Russian Mathematician Chebyshev's showed that any random variable *X* with expectation $\mu = E(X)$ and standard deviation $\sigma^2 = Var(X)$ belongs to the interval $\mu \pm \varepsilon = [\mu - \varepsilon, \mu + \varepsilon]$ with probability of at least $1 - (\sigma/\varepsilon)^2$, that is $P\{|X - \mu| > \varepsilon\} \leq \left(\frac{\sigma}{\varepsilon}\right)^2$ for any distribution with expectation $\mu$ and variance $\sigma^2$ and for any positive $\varepsilon$.

**Step 3/4**
Also Chebyshev's inequality shows that only a large variance may allow a variable *X* to differ significantly from its expectation $\mu$. In this case, the risk of seeing an extremely low or extremely high value of *X* increases.

**Step 4/4**
Here the probability of spending more than 1 minute for this download is given by $P(X > 60)$. By Chebyshev's inequality, setting $\varepsilon = 20$ will give

$$P(|X - \mu| > 20) \leq \left(\frac{\sigma}{20}\right)^2$$
$$= P(|X - 40| > 20) \leq \left(\frac{5}{20}\right)^2$$
$$= P(X > 60) \leq \left(\frac{1}{4}\right)^2$$
$$= P(X > 60) \leq \left(\frac{1}{16}\right)$$

So the probability of spending more than 1 minute for the download of the file will be lesser than (1/16).

**Chapter 3, Problem 10E**
(0)

Problem
Every day, the number of traffic accidents has the probability mass function

| $x$ | 0 | 1 | 2 | more than 2 |
|---|---|---|---|---|
| $P(x)$ | 0.6 | 0.2 | 0.2 | 0 |

independently of other days. What is the probability that there are more accidents on Friday than on Thursday?

**Step-by-step solution**

**Show all steps**

100% (9 ratings) for this solution

**Step 1/6**
Every day, the number of traffic accidents has the probability mass function independently of other days.

| $x$ | 0 | 1 | 2 | more than 2 |
|---|---|---|---|---|
| $P(x)$ | 0.6 | 0.2 | 0.2 | 0 |

We have to find the probability that there are more accidents on Friday than on Thursday.

**Step 2/6**
Let $X1$ denotes the number of accidents in Thursday and $X2$ denotes the number of accidents in Friday. The possible events for more accidents on Friday than Thursday are as follows.

$X1$ = 0 and $X2$ = 1 or 2 or more than 2

$X1$ = 1 and $X2$ = 2 or more than 2

$X1$ = 2 and $X2$ = more than 2.

**Step 3/6**
Probability for the event $X1$ = 0 and $X2$ = 1 or 2 or more than 2 is calculated as follows.

$$\begin{aligned}
&P(X1=0 \text{ and } X2=1 \text{ or } 2 \text{ or more than } 2)\\
&=P(X1=0)\cap\left[P(X2=1)\cup P(X2=2)\cup P(X2=\text{more than } 2)\right]\\
&=P(X1=0).\left[P(X2=1)+P(X2=2)+P(X2=\text{more than } 2)\right]\\
&=0.6(0.2+0.2+0)\\
&=0.24
\end{aligned}$$

**Step 4/6**
Probability for the event $X1$ = 1 and $X2$ = 2 or more than 2 is calculated as follows.

$$
\begin{aligned}
&P(X1=1 \text{ and } X2=2 \text{ or more than } 2)\\
&=P(X1=1)\cap\left[P(X2=2)\cup P(X2=\text{more than } 2)\right]\\
&=P(X1=1).\left[P(X2=2)+P(X2=\text{more than } 2)\right]\\
&=0.2(0.2+0)\\
&=0.04
\end{aligned}
$$

**Step 5/6**
Probability for the event *X1* = 1 and *X2* = 2 or more than 2 is calculated as follows.

$$
\begin{aligned}
&P(X1=2 \text{ and } X2=\text{more than } 2)\\
&=P(X1=2)\cap\left[P(X2=\text{more than } 2)\right]\\
&=P(X1=2).\left[P(X2=\text{more than } 2)\right]\\
&=0.2(0)\\
&=0
\end{aligned}
$$

**Step 6/6**
The probability that there are more accidents on Friday than on Thursday is 0.24 + 0.04 + 0 = 0.28.

**Chapter 3, Problem 11E**
(0)

Problem
Two dice are tossed. Let *X* be *the smaller* number of points. Let *Y* be *the larger* number of points. If both dice show the same number, say, *z* points, then *X* = *Y* = *z*.

(a) Find the joint probability mass function of (*X*, *Y*).

(b) Are *X* and *Y* independent? Explain.

(c) Find the probability mass function of *X*.

(d) If *X* = 2, what is the probability that *Y* = 5?

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/5**
Given that two dice are tossed and let *X* be the smaller number of points, *Y* be the larger number of points. If both dice show the same number, say, *z* points, then *X* = *Y* = *z*.

**Step 2/5**
(a)

The following is the sample space of tossing two dice.

$$\Omega = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

The joint probability function of *X* and *Y* is as below.

| **(x,y)** | ***y*** | | | | | |
|---|---|---|---|---|---|---|
| ***x*** | **1** | **2** | **3** | **4** | **5** | **6** |
| **1** | **1/36** | **2/36** | **2/36** | **2/36** | **2/36** | **2/36** |
| **2** | **0** | **1/36** | **2/36** | **2/36** | **2/36** | **2/36** |
| **3** | **0** | **0** | **1/36** | **2/36** | **2/36** | **2/36** |
| **4** | **0** | **0** | **0** | **1/36** | **2/36** | **2/36** |
| **5** | **0** | **0** | **0** | **0** | **1/36** | **2/36** |
| **6** | **0** | **0** | **0** | **0** | **0** | **1/36** |

**Step 3/5**
(b)

Two events *X* and *Y* are independent in a joint distribution then $P_{x,y}(X = x, Y = y) = P_x(X = x)P_y(Y = y)$. We can check this for any *x* and *y*. Let us take the probability $P_{x,y}(X = 1, Y = 1)$. Its value is 1/36 as from the above table. $P_x(X = 1)$ = 1/36 and $P_x(Y = 1)$= 1/36. So $P_x(X = x)P_y(Y = y) = (1/36) \times (1/36) = (1/1296)$. It is clear that $P_{x,y}(X = x, Y = y) \neq P_x(X = x)P_y(Y = y)$. So the events *X* and *Y* are not independent.

**Step 4/5**
(c)

Probability mass function of *X* is given by $P_x(X = x) = \sum_y P_{x,y}(X = x, Y = y)$.

$$P_x(X = 1) = (1/36) + (2/36) + (2/36) + (2/36) + (2/36) + (2/36) = (11/36)$$

$$P_x(X = 2) = (1/36) + (2/36) + (2/36) + (2/36) + (2/36) = (9/36)$$

$$P_x(X = 3) = (1/36) + (2/36) + (2/36) + (2/36) = (7/36)$$

$P_x(X=4)=(1/36)+(2/36)+(2/36)=(5/36)$

$P_x(X=5)=(1/36)+(2/36)=(3/36)$

$P_x(X=6)=(1/36)$

The probability mass function in table is as follows.

| *x* | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| *P(x)* | 11/36 | 9/36 | 7/36 | 5/36 | 3/36 | 1/36 |

**Step 5/5**

(d)

If *X* = 2, the probability that *Y* = 5 is given by the conditional probability $P_{x,y}(Y=5\,|\,X=2)$.

$$
\begin{aligned}
&P_{x,y}(Y=5\,|\,X=2)\\
&=\frac{P_{x,y}(Y=5,X=2)}{P(X=2)}\\
&=\frac{2/36}{9/36}\text{(from joint distribution table and marginal distribution of } x)\\
&=2/9
\end{aligned}
$$

If *X* = 2, the probability that *Y* = 5 is 2/9.

**Chapter 3, Problem 12E**

(0)

Problem

Two random variables, *X* and *Y*, have the joint distribution *P*(*x*,*y*),

| $P(x,y)$ | | $x=0$ | $x=1$ |
|---|---|---|---|
| $y$ | 0 | 0.5 | 0.2 |
| | 1 | 0.2 | 0.1 |

(a) Are *X* and *Y* independent? Explain.

(b) Are (*X* + *Y*) and (*X* − *Y*) independent? Explain.

**Step-by-step solution**

**Show all steps**

**Step 1/16**

Consider two random variables *X* and *Y* with the joint distribution $P(x,y)$.

| $P(x,y)$ | | $x$ | |
|---|---|---|---|
| | | 0 | 1 |
| $y$ | 0 | 0.5 | 0.2 |
| | 1 | 0.2 | 0.1 |

**Step 2/16**
(a)

The objective is to determine whether *X* and *Y* are independent.

**Step 3/16**
The joint probability mass function of *X* and *Y* is defined as,

**Step 4/16**

$$\begin{aligned} P(x,y) &= P\{(X,Y)=(x,y)\} \\ &= P\{X=x \cap Y=y\} \end{aligned}$$

**Step 5/16**
In a joint distribution, two events *X* and *Y* are independent if

**Step 6/16**

$$P_{x,y}(X=x,Y=y)=P_x(X=x)P_y(Y=y)$$

**Step 7/16**

Assume that $X=1,Y=1$. The table shows that the value of $P_{1,1}(X=1,Y=1)=0.1$. The individual probabilities are calculated as,

**Step 8/16**

$$\begin{aligned} P_x(X=1) &= P(X=1,Y=0)+P(X=1,Y=1) \\ &= 0.2+0.1 \\ &= 0.3 \end{aligned}$$

**Step 9/16**

$$\begin{aligned} P_y(Y=1) &= P(X=0,Y=1)+P(X=1,Y=0) \\ &= 0.2+0.1 \\ &= 0.3 \end{aligned}$$

**Step 10/16**
So, the product of these probabilities is obtained as,

**Step 11/16**

$$P_1(X=1)P_1(Y=1)=(0.3)(0.3)$$
$$=0.09$$
$$\neq P_{1,1}(X=1,Y=1)$$

**Step 12/16**

This shows that $P_{x,y}(X=x,Y=y)\neq P_x(X=x)P_y(Y=y)$. Therefore, the random variables *X* and *Y* are not independent.

**Step 13/16**

(b)

The objective is to determine whether $(X+Y)$ and $(X-Y)$ are independent.

**Step 14/16**

Assume $U=X+Y$ and $V=X-Y$. This means that the possible values of *U* are 0, 1, and 2. The possible values of *V* are $-1,0,\text{ and }1$. To verify whether *U* and *V* are independent, it has to be proved that,

**Step 15/16**

$$P_{u,v}(U=u,V=v)=P_u(U=u)P_v(V=v)$$

Assume that $U=0,V=0$. So, the joint probability is calculated as,

**Step 16/16**

$$P_{u,v}(U=0,V=0)=\frac{P(U=0\text{ and }V=0)}{P(V=0)}$$
$$=\frac{P(X=0\cap Y=0)}{P(X=0\cap Y=0)+P(X=1\cap Y=1)}$$
$$=\frac{0.5}{0.5+0.1}$$
$$\approx 0.83$$

Now, calculate $P_u(U=0)$ and $P_v(V=0)$.

$$P_u(U=0)=P(X=0,Y=0)$$
$$=0.5$$

$$P_v(V=0)=P(X=0,Y=0)+P(X=1,Y=1)$$
$$=0.5+0.1$$
$$=0.6$$

So,

$$\begin{aligned} P_u(U=0)\cdot P_v(V=0) &= (0.5)(0.6) \\ &= 0.30 \\ &\neq P_{u,v}(U=0, V=0) \end{aligned}$$

This shows that $P_{u,v}(U=u, V=v) \neq P_u(U=u)P_v(V=v)$. Therefore, $(X+Y)$ and $(X-Y)$ are not independent.

**Chapter 3, Problem 13E**
(0)

Problem
Two random variables *X* and *Y* have the joint distribution, *P*(0,0) = 0.2, *P*(0,2) = 0.3, *P*(1,1) = 0.1, *P*(2,0) = 0.3, *P*(2,2) = 0.1, and *P*(*x*,*y*) = 0 for all other pairs (*x*,*y*).

(a) Find the probability mass function of *Z* = *X* + *Y*.

(b) Find the probability mass function of *U* = *X* − *Y*.

(c) Find the probability mass function of *V* = *XY*.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/7**
Given the two random variables *X* and *Y* have the joint distribution, *P* (0, 0) = 0.2, *P* (0, 2) = 0.3, *P* (1, 1) = 0.1, *P* (2, 0) = 0.3, *P* (2, 2) = 0.1 and *P* (x, y) = 0 for all other pairs. These joint probabilities can be displayed in the table as follows.

| *P*(*x*, *y*) | *x* y\X | | | |
|---|---|---|---|---|
| | 0 | ~~1~~ 0 | ~~2~~ 1 | 2 |
| *y* | 0 0 | 0.2 | 0 | 0.3 |
| | 1 1 | 0 | 0.1 | 0 |
| | 2 2 | 0.3 | 0 | 0.1 |

**Step 2/7**
(a)

The probability mass function of *Z* = *X* + *Y* is found as follows. For the given values of *X* and *Y*, the possible values of *Z* are 0, 1, 2, 3 and 4. The collection of probabilities *Z* will give the probability mass function of *Z*.

$P(Z=0)$
$= P(X=0, Y=0)$
$= 0.2$

$P(Z=1)$
$= P(X=0, Y=1) + P(X=1, Y=0)$
$= 0+0$
$= 0$

$P(Z=2)$
$= P(X=0, Y=2) + P(X=2, Y=0) + P(X=1, Y=1)$
$= 0.3+0.3+0.1$
$= 0.7$

$P(Z=3)$
$= P(X=1, Y=2) + P(X=2, Y=1)$
$= 0+0$
$= 0$

$P(Z=4)$
$= P(X=2, Y=2)$
$= 0.1$

**Step 3/7**
The probability mass function of *Z* is as follows.

| | | | | | |
|---|---|---|---|---|---|
| *z* | 0 | 1 | 2 | 3 | 4 |
| *P*(*Z* = *z*) | 0.2 | 0 | 0.7 | 0 | 0.1 |

The distribution is valid as $\sum P(Z=z)$ = 0.2 + 0.7 + 0.1 = 1

**Step 4/7**
(b)

The probability mass function of *U* = *X* - *Y* is found as follows. For the given values of *X* and *Y*, the possible values of *U* are -2, -1, 0, 1, and 2. The collection of probabilities *U* will give the probability mass function of *U*.

$P(U=-2)$
$= P(X=0, Y=2)$
$= 0.3$

$P(U=-1)$
$=P(X=0,Y=1)+P(X=1,Y=2)$
$=0+0$
$=0$

$P(U=0)$
$=P(X=0,Y=0)+P(X=1,Y=1)+P(X=2,Y=2)$
$=0.2+0.1+0.1$
$=0.4$

$P(U=1)$
$=P(X=1,Y=0)+P(X=2,Y=1)$
$=0+0$
$=0$

$P(U=2)$
$=P(X=2,Y=0)$
$=0.30$

**Step 5/7**
The probability mass function of *U* is as follows.

| *u* | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *P*(*U* = *u*) | 0.3 | 0 | 0.4 | 0 | 0.3 |

The distribution is valid as $\sum P(U=u)$ = 0.30 + 0.40 + 0.30 = 1

**Step 6/7**
(c)

The probability mass function of *V* = *XY* is found as follows. For the given values of *X* and *Y*, the possible values of *V* are 0, 1, 2 and 4. The collection of probabilities *V* will give the probability mass function of *V*.

$P(V=0)$
$=P(X=0,Y=0)+P(X=0,Y=1)+P(X=0,Y=2)$
$+P(X=1,Y=0)+P(X=2,Y=0)$
$=0.20+0+0.30+0+0.3$
$=0.80$

$P(V=1)$
$=P(X=1,Y=1)$
$=0.1$

$$
\begin{aligned}
&P(V=2)\\
&=P(X=1,Y=2)+P(X=2,Y=1)\\
&=0+0\\
&=0
\end{aligned}
$$

$$
\begin{aligned}
&P(V=4)\\
&=P(X=2,Y=2)\\
&=0.1
\end{aligned}
$$

**Step 7/7**

The probability mass function of *V* is as follows.

| *v* | 0 | 1 | 2 | 4 |
|---|---|---|---|---|
| *P*(*V* = *v*) | 0.8 | 0.1 | 0 | 0.1 |

The distribution is valid as $\sum P(V=v)$ = 0.80 + 0.10 + 0.10 = 1

**Chapter 3, Problem 14E**

(0)

Problem

An internet service provider charges its customers for the time of the internet use rounding it up to the nearest hour. The joint distribution of the used time (*X*, hours) and the charge per hour (*Y*, cents) is given in the table below.

| $P(x,y)$ | | $x$ | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| $y$ | 1 | 0 | 0.06 | 0.06 | 0.10 |
| | 2 | 0.10 | 0.10 | 0.04 | 0.04 |
| | 3 | 0.40 | 0.10 | 0 | 0 |

Each customer is charged *Z* = *X* · *Y* cents, which is the number of hours multiplied by the price of each hour. Find the distribution of *Z*.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/4**

Given that an internet service provider charges its customer for the time of the internet use rounding it up to the nearest hour. The joint distribution of the time (*X* , hours) and the charge per hour (*Y*, cents) is given in the table below.

| *P*(*x*, *y*) | *x* | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | |
| *y* | 1 | 0 | 0.06 | 0.06 | 0.10 |
| | 2 | 0.10 | 0.10 | 0.04 | 0.04 |
| | 3 | 0.40 | 0.10 | 0 | 0 |

**Step 2/4**

Each customer is charged *Z* = *X*. *Y* cents, which is the number of hours multiplied by the price of each hour. We have to find the distribution of *Z*.

**Step 3/4**

For the given values of *X* and *Y*, the possible values of *Z* are 1, 2, 3, 4, 6, 8, 9 and 12. The collection of probabilities *Z* will give the distribution of *Z*.

$$P(Z=1)$$
$$=P(X=1, Y=1)$$
$$=0$$

$$P(Z=2)$$
$$=P(X=1, Y=2)+P(X=2, Y=1)$$
$$=0.10+0.06$$
$$=0.16$$

$$P(Z=3)$$
$$=P(X=1, Y=3)+P(X=3, Y=1)$$
$$=0.40+0.06$$
$$=0.46$$

$$P(Z=4)$$
$$=P(X=2, Y=2)+P(X=4, Y=1)$$
$$=0.10+0.10$$
$$=0.20$$

$$P(Z=6)$$
$$=P(X=2, Y=3)+P(X=3, Y=2)$$
$$=0.10+0.04$$
$$=0.14$$

$P(Z=8)$
$=P(X=4,Y=2)$
$=0.04$

$P(Z=9)$
$=P(X=3,Y=3)$
$=0$

$P(Z=12)$
$=P(X=4,Y=3)$
$=0$

**Step 4/4**
The probability distribution of *Z* is as follows.

| *z* | 1 | 2 | 3 | 4 | 6 | 8 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|
| *P*(*Z* = *z*) | 0 | 0.16 | 0.46 | 0.20 | 0.14 | 0.04 | 0 | 0 |

The distribution is valid as $\sum P(Z=z)$ = 0.16 + 0.46 + 0.20 + 0.14 + 0.04 = 1

**Chapter 3, Problem 15E**
(0)

Problem
Let *X* and *Y* be the number of hardware failures in two computer labs in a given month. The joint distribution of *X* and *Y* is given in the table below.

(a) Compute the probability of at least one hardware failure.

(b) From the given distribution, are *X* and *Y* independent? Why or why not?

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/3**
Given that *X* and *Y* the number of hardware failures in two-computer labs in a given month. The joint distribution *P*(*x*, *y*) is as follows.

| *P*(*x*, *y*) | *x* | | | |
|---|---|---|---|---|
| | 0 | 1 | 2 | |
| *y* | 0 | 0.52 | 0.20 | 0.04 |
| | 1 | 0.14 | 0.02 | 0.01 |
| | 2 | 0.06 | 0.01 | 0 |

**Step 2/3**
(a)

Probability of at least one hardware failure is given as 1 - Probability of no failures. Probability of no failures is $P(X=0,Y=0)=0.52$. So the probability of at least one hardware failure = 1 - 0.52 = 0.48.

**Step 3/3**
(b)

Two events *X* and *Y* are independent in a joint distribution then $P_{x,y}(X=x,Y=y)=P_x(X=x)P_y(Y=y)$. We can check this for any *x* and *y* in the joint distribution. Let us take the probability $P_{x,y}(X=1,Y=1)$. Its value is 0.02 as from the above table. $P_x(X=1)$ = 0.2 + 0.02 + 0.01 = 0.23 and $P_x(Y=1)$= 0.14 + 0.02 + 0.01 = 0.17. So $P_x(X=x)P_y(Y=y)=(0.23)\times(0.17)=0.0391$. It is clear that $P_{x,y}(X=x,Y=y)\neq P_x(X=x)P_y(Y=y)$. So the events *X* and *Y* are not independent.

**Chapter 3, Problem 16E**
(0)

Problem
The number of hardware failures, *X*, and the number of software failures, *Y*, on any day in a small computer lab have the joint distribution *P*(*x*,*y*), where *P*(0,0) = 0.6, *P*(0,1) = 0.1, *P*(1,0) = 0.1, *P*(1,1) = 0.2. Based on this information,

(a) Are *X* and *Y* (hardware and software failures) independent?

(b) Compute **E**(*X* + *Y*), i.e., the expected total number of failures during 1 day.

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/5**

Given that the numbers of hardware failures *X*, and the numbers of software failures *Y*, on any day in a small computer lab have the joint distribution *P*(*x*, *y*) is as follows.

| *P*(*x*, *y*) | *x* | | |
|---|---|---|---|
| | 0 | 1 | |
| *y* | 0 | 0.60 | 0.10 |
| | 1 | 0.10 | 0.20 |

**Step 2/5**

(a)

Two events *X* and *Y* are independent in a joint distribution then $P_{x,y}(X = x, Y = y) = P_x(X = x)P_y(Y = y)$. We can check this for any *x* and *y* in the joint distribution. Let us take the probability $P_{x,y}(X = 1, Y = 1)$. Its value is 0.20 as from the above table. $P_x(X = 1)$ = 0.1 + 0.2 = 0.3 and $P_x(Y = 1)$= 0.10 + 0.20 = 0.30. So $P_x(X = x)P_y(Y = y) = (0.3 \times 0.3) = 0.09$.

It is clear that $P_{x,y}(X = x, Y = y) \neq P_x(X = x)P_y(Y = y)$. So the events *X* and *Y* are not independent. That is hardware and software failures are dependent.

**Step 3/5**

(b)

To find *E* (*X* + *Y*), the distribution of *X* + *Y* is required first. Let *Z* = *X* + *Y*. Then the possible values of *Z* are 0, 1 and 2. The distribution of *Z* is the collection of probabilities of values of *Z*.

$$\begin{aligned} &P(Z = 0) \\ &= P(X = 0, Y = 0) \\ &= 0.60 \end{aligned}$$

$$\begin{aligned} &P(Z = 1) \\ &= P(X = 0, Y = 1) + P(X = 1, Y = 0) \\ &= 0.10 + 0.10 \\ &= 0.20 \end{aligned}$$

$$P(Z=2)$$
$$=P(X=1, Y=1)$$
$$=0.20$$

The probability distribution of $Z$ is

| $z$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(Z = z)$ | 0.60 | 0.20 | 0.20 |

The distribution is valid as $\sum P(Z=z)$ = 0.60 + 0.20 + 0.20 = 1

**Step 4/5**

The value is $E$ ($X$ + $Y$) is given as $E$ ($Z$) = $\sum z.P(Z=z)$.

$$\begin{aligned}\sum z.P(Z=z) &= (0\times 0.60)+(1\times 0.20)+(2\times 0.20)\\ &= 0+0.20+0.40\\ &= 0.60\end{aligned}$$

**Step 5/5**

The expected total number of failures during 1 day is 0.60.

**Chapter 3, Problem 17E**

(0)

Problem

Shares of company A are sold at $10 per share. Shares of company B are sold at $50 per share. According to a market analyst, 1 share of each company can either gain $1, with probability 0.5, or lose $1, with probability 0.5, independently of the other company. Which of the following portfolios has the lowest risk:

(a) 100 shares of A

(b) 50 shares of A + 10 shares of B

(c) 40 shares of A + 12 shares of B

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/8**
Given that shares of company A are sold at $10 per share and shares of company B are sold at $50 per shares. According to a market analyst, 1 share of each company can either gain $1, with probability 0.5 or lose $1 with probability 0.5 independently of the other company.

**Step 2/8**
Let *A* denotes the random variable of gain from shares of company A and let *B* denotes the random variable of gain from shares of company B. A negative value will indicate loss. The probability distribution of gain from shares of company A is given in the following table.

| | | |
|---|---|---|
| $a$ | -1 | 1 |
| $P(A = a)$ | 0.5 | 0.5 |

Similarly the probability distribution of gain from shares of company B is given as

| | | |
|---|---|---|
| $b$ | -1 | 1 |
| $P(B = b)$ | 0.5 | 0.5 |

**Step 3/8**
The expected return from shares of A is given as $\sum_a a.P(A=a)$ = (-1) (0.5) + (1) (0.5) = 0. Similarly the expected return from Portfolio of shares of B is also 0.

**Step 4/8**
The variance of return from shares of A is given as $(-1)^2$ (0.5) + $(1)^2$ (0.5) = 1.0 and the variance of return from shares of B is also 1.0.

**Step 5/8**
(a)

As the expected return is zero for one share, it will be zero for any number of shares. The variance of returns from portfolio of 100 shares of A is given as $(100)^2$ *VAR* (*A*) = $10,000 (1) = $10,000.

**Step 6/8**
(b)

As the expected return is zero for one share of both A and B, it will be zero for any number of combination of shares for both A and B. The variance of returns from portfolio of 50 shares of A and 10 shares of B is given as $(50)^2$ *VAR* (*A*) + $(10)^2$ *VAR* (*B*) = \$2,500 (1) + \$100 (1) = \$2,600.

**Step 7/8**
(c)

As the expected return is zero for one share of both A and B, it will be zero for any number of combination of shares for both A and B. The variance of returns from portfolio of 40 shares of A and 12 shares of B is given as $(40)^2$ *VAR* (*A*) $(12)^2$ + *VAR* (*B*) = \$1,600 (1) + \$144 (1) = \$1,744.

**Step 8/8**
We can see that the expected returns are all same for the three portfolios but the variance is the lowest for the third portfolio. So the portfolio with lowest risk is the third portfolio of buying 40 shares of A and 12 shares of B.

**Chapter 3, Problem 18E**
(0)

Problem
Shares of company A cost \$10 per share and give a profit of X%. Independently of A, shares of company B cost \$50 per share and give a profit of Y%. Deciding how to invest \$1,000, Mr. X chooses between 3 portfolios:

(a) 100 shares of A,

(b) 50 shares of A and 10 shares of B,

(c) 20 shares of B.

The distribution of *X* is given by probabilities:

$$P\{X=-3\}=0.3, P\{X=0\}=0.2, P\{X=3\}=0.5..$$

The distribution of *Y* is given by probabilities:

$$P\{Y=-3\}=0.4, P\{Y=3\}=0.6.$$

Compute expectations and variances of the total dollar profit generated by portfolios (a), (b), and (c). What is the least risky portfolio? What is the most risky portfolio?

**Step-by-step solution**

**Show all steps**

**Step 1/6**
From the given information,

Shares of company A are sold at $10 per share and give a profit of *X*%. Independently of A, shares of company B are sold at $50 per shares and give a profit of *Y*%. Mr. *X* has to choose a portfolio to invest $1,000 in the given three portfolios.

The probability mass function of *X* is given as,

| $x$ | -3 | 0 | 3 |
|---|---|---|---|
| $P\ (X = x)$ | 0.3 | 0.2 | 0.5 |

The probability mass function of *Y* is given as,

| $y$ | -3 | 0 | 3 |
|---|---|---|---|
| $P(Y = y)$ | 0.4 | 0 | 0.6 |

**Step 2/6**
Compute expected value and variance of the total dollar profit generate by each portfolio.

Instead of looking at % return dollar, it can more accessibly, look at dollar return for each of the companies. Let *X* be the random variable for each share in company *X* and let *Y* be the dollar return for each share of company *Y*.

Then it,

| | | | |
|---|---|---|---|
| *x* | -$0.3 | $0 | $0.3 |
| *P* (*X* = *x*) | 0.3 | 0.2 | 0.5 |
| *y* | -$1.5 | $0 | $1.5 |
| *P* (*Y* = *y*) | 0.4 | 0 | 0.6 |

Then, write the return in dollars for each of the three portfolios $P_1, P_2$ and $P_3$ as linear combinations of the random variable *X* and *Y*:

$$P_1 = 100X$$
$$P_2 = 50X + 10Y$$
$$P_3 = 20Y$$

**Step 3/6**
To find the expected values and variances on the dollar return for each of the portfolios, it needs to calculate the expected values and variances for random variable *X* and *Y*.

So, the expected values and variances for random variable *X* is,

$$\begin{aligned}E[Y] &= -\$0.3\times 0.3+\$0\times 0.2+\$0.3\times 0.5\\ &= 0.06\end{aligned}$$

$$\begin{aligned}V[X] &= E\left[X^2\right]-\left(E[X]\right)^2\\ &= \left(-\$0.3\right)^2\times 0.3+\$0^2\times 0.2+\left(\$0.3\right)^2\times 0.5-0.06^2\\ &= 0.0684\end{aligned}$$

The expected values and variances for random variable *Y* is,

$$\begin{aligned}E[Y] &= -\$1.5\times 0.4+\$1.5\times 0.6\\ &= 0.3\end{aligned}$$

$$\begin{aligned}V[Y] &= E\left[Y^2\right]-\left(E[Y]\right)^2\\ &= \left(-\$1.5\right)^2\times 0.4+\left(\$1.5\right)^2\times 0.6-0.3^2\\ &= 2.16\end{aligned}$$

**Step 4/6**
(a)

Using the available $1,000, Mr. X can buy 100 shares of A. The expected returns from the portfolio of 100 shares of A is given as,

$100\ E(X) = 100\ (0.06) = \$6.$

The variance of return from the portfolio of 100 shares of A is given as,

$100^2 VAR\ (X) = 10{,}000\ (0.0684) = \$684.$

**Step 5/6**
(b)

Using the available $1,000, Mr. X can buy 50 shares of A and 10 shares of B. The expected returns from the portfolio of 50 shares of A and 10 shares of B is given as,

$50E(X) + 10\ E(Y) = 50\ (0.06) + 10\ (0.3) = \$6.$

The variance of return from the portfolio of 50 shares of A and 10 shares of B is given as,

$50^2 VAR\ (X) + 10^2 VAR\ (Y) = 2{,}500\ (0.0684) + 100(2.16) = \$387.$

**Step 6/6**
(c)

Using the available $1,000, Mr. X can buy 20 shares of B. The expected returns from the portfolio of 20 shares of B is given as,

$20\ E(Y) = 20\ (0.3) = \$6.$

The variance of return from the portfolio of 20 shares of B is given as,

$20^2 VAR(Y) = 400\,(2.16) = \$864$.

The expected returns are all same for the three portfolios but the variance is lowest for the second portfolio and highest for the third portfolio. So, the portfolio with lowest risk is the second portfolio of buying 50 shares of A and 10 shares of B.

The portfolio with highest risk is the third portfolio of buying 20 shares of B.

**Chapter 3, Problem 19E**
(0)

Problem
A and B are two competing companies. An investor decides whether to buy

(a) 100 shares of A, or

(b) 100 shares of B, or

(c) 50 shares of A and 50 shares of B.

A profit made on 1 share of A is a random variable $X$ with the distribution $P(X = 2) = P(X = -2) = 0.5$.

A profit made on 1 share of B is a random variable $Y$ with the distribution $P(Y = 4) = 0.2$, $P(Y = -1) = 0.8$.

If $X$ and $Y$ are independent, compute the expected value and variance of the total profit for strategies (a), (b), and (c).

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/6**
Given that A and B are competing companies and a profit made on 1 share of A is a random variable $X$ with the distribution

| | | |
|---|---|---|
| $x$ | 2 | -2 |
| $P(X = x)$ | 0.5 | 0.5 |

A profit made on 1 share of B is a random variable $Y$ with the distribution

| | | |
|---|---|---|
| $y$ | -1 | 4 |
| $P(Y = y)$ | 0.8 | 0.2 |

**Step 2/6**

The expected value of $X$ is given as $\sum_{x} x.P(X = x)$ = (2) (0.5) + (-2) (0.5) = 0. The variance of $X$ is given as $[(-2)^2 (0.5) + (2)^2 (0.5)] - 0^2 = 4$.

**Step 3/6**

The expected value of $Y$ is given as $\sum_{y} y.P(Y = y)$ = (-1) (0.8) + (4) (0.2) = 0. The variance of $Y$ is given as $[(-1)^2 (0.8) + (4)^2 (0.2)] - 0^2 = 4$.

**Step 4/6**

(a)

The expected value of total profit from the portfolio of 100 shares of A is given as 100 $E(X)$ = 100 (0) = \$0. The variance of total profit from the portfolio of 100 shares of A is given as $100^2 VAR(X)$ = 10,000 (4) = \$40,000.

**Step 5/6**

(b)

The expected value of total profit from the portfolio of 100 shares of B is given as 100 $E(Y)$ = 100 (0) = \$0. The variance of total profit from the portfolio of 100 shares of B is given as $100^2 VAR(Y)$ = 10,000 (4) = \$40,000.

**Step 6/6**

(c)

The expected value of total profit from the portfolio of 50 shares of A and 50 shares of B is given as 50 $E(X)$ + 50 $E(Y)$ = 0 + 0 = \$0. The variance of total profit from the portfolio of 50 shares of A and 50 shares of B is given as $50^2 VAR(X) + 50^2 VAR(X)$ = 2,500 (4) + 2,500 (4) = \$10,000 + \$10,000 = \$20,000.

**Chapter 3, Problem 20E**

(0)

Problem

A quality control engineer tests the quality of produced computers. Suppose that 5% of computers have defects, and defects occur independently of each other.

(a) Find the probability of exactly 3 defective computers in a shipment of twenty.

(b) Find the probability that the engineer has to test at least 5 computers in order to find 2 defective ones.

**Step-by-step solution**

**Show all steps**

**Step 1/6**
Given that a quality control engineer tests the quality of produced computers. Given that 5% of the computers are supposed to be defective and the defects occur independently to each other.

**Step 2/6**
Let *X* denotes the number of defectives in the given samples. Then *X* is a binomial random variable with parameters *n* and *p* = 0.05. The probability mass function of *X* is given as $P(X=x)=\binom{n}{x}p^x q^{n-x}$ where *q* = 1 - *p* and *x* = 0, 1 … n.

**Step 3/6**
(a)

The probability of exactly 3 defective computers in shipment of twenty is given by the probability $P(X=3)$. Here *n* = 20, *x* = 3 and *p* = 0.05. Substitute these values in the probability mass function.

$$
\begin{aligned}
&P(X=3) \\
&=\binom{20}{3}(0.05)^3(1-0.05)^{20-3} \\
&=1{,}140(0.000125)(0.418120335) \\
&=0.0596
\end{aligned}
$$

The probability of exactly 3 defective computers in shipment of twenty is 0.0596.

**Step 4/6**
(b)

The probability that the engineer has to test at least 5 computers in order to find 2 defective ones is given by the negative binomial distribution as here we require 2 defectives from at least five computers. The probability mass function of negative binomial distribution is given as $P(X=x)=\binom{x-1}{k-1}p^k q^{x-k}$ where *q* = 1 - *p* and *x* = *k*, *k*+1 … *n*. Here *X* denotes the number of trials required to achieve *k* number of successes.

**Step 5/6**

Here $x = 5$, $k = 2$ and $p = 0.05$. We require the probability $P(X \geq 5)$. Substitute the values in the probability mass function of negative binomial distribution

$$P(X = x) = \binom{x-1}{k-1} p^k q^{x-k}$$
.

$$\begin{aligned}
&P(X \geq 5) \\
&= 1 - P(X \leq 4) \\
&= 1 - P(X = 2) - P(X = 3) - P(X = 4) \\
&= 1 - \binom{2-1}{2-1}(0.05)^2(0.95)^0 - \binom{3-1}{2-1}(0.05)^2(0.95)^1 - \binom{4-1}{2-1}(0.05)^2(0.95)^2 \\
&= 1 - 0.0025 - 0.007125 - 0.009025 \\
&= 0.98135
\end{aligned}$$

**Step 6/6**

The probability that the engineer has to test at least 5 computers in order to find 2 defective ones is 0.981.

**Chapter 3, Problem 21E**

(1)

Problem

A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the probability that it entered at least 10 computers.

**Step-by-step solution**

**Show all steps**

100% (10 ratings) for this solution

**Step 1/3**

Given that a lab network consisting of 20 computers was attacked by a computer virus. The probability that this virus entering a computer is 0.4 that is independently of other computers. Determine the probability that the virus entered at least 10 computers.

**Step 2/3**

Let $X$ denotes the number of computers attacked by the virus in the given 20 computers. Then $X$ is a binomial random variable with parameters $n = 20$ and $p = 0.4$.

The probability mass function of $X$ is given as $P(X = x) = \binom{20}{x}(0.4)^x(0.6)^{20-x}$ for $x$ = 0, 1 …, 20.

The probability that the virus entered at least 10 computers is given as $P(X \geq 10)$ which is equivalent to $1 - P(X \leq 9)$. The value of the cumulative distribution function $P(X \leq 9)$ can be found from Table A2 which is 0.7553. So $1 - P(X \leq 9)$ = 1 - 0.7553 = 0.2447.

**Step 3/3**
Other method:

$$\begin{aligned} P(X \geq 10) &= 1 - P(X < 10) \\ &= 1 - P(X \leq 9) \\ &= 1 - \sum_{x=0}^{9} P(X = x) \end{aligned}$$

$$\begin{aligned} &= 1 - \sum_{x=0}^{9} \binom{20}{x} (0.4)^x (0.6)^{20-x} \\ &= 1 - 0.755337 \\ &= 0.244663 \\ &\approx 0.2447 \end{aligned}$$

The probability that the virus attacks at least 10 computers is **0.2447**.

**Chapter 3, Problem 22E**
(0)

Problem
Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains more than 3 defective ones?

**Step-by-step solution**

**Show all steps**

100% (3 ratings) for this solution

**Step 1/4**
Given that five percent of computer parts produced by a certain supplier are defective. We have to find the probability that a sample of 16 parts contains more than 3 defective ones.

**Step 2/4**
Let *X* denotes the number of computer parts that are defective in the sample of 16. Then *X* is a binomial random variable with parameters *n* =16 and *p* = 0.05. The probability mass function of *X* is given as $P(X=x)=\binom{16}{x}(0.05)^x(0.95)^{16-x}$ for *x* = 0, 1 … 16.

**Step 3/4**
The probability that more than 3 defective ones are present in the sample of 16 is given by the probability $P(X>3)$ which is equivalent to $1-P(X\le 3)$.

$$\begin{aligned}P(X\le 3)&=P(X=0)+P(X=1)+P(X=2)+P(X=3)\\&=\binom{16}{0}(0.05)^0(0.95)^{16}+\binom{16}{1}(0.05)^1(0.95)^{15}+\binom{16}{2}(0.05)^2(0.95)^{14}+\binom{16}{3}(0.05)^3(0.95)^{13}\\&=0.440126668+0.370632984+0.146302493+0.035933945\\&=0.99299609\end{aligned}$$

$$\begin{aligned}&1-P(X\le 3)\\&=1-0.99299609\\&=0.007\end{aligned}$$

**Step 4/4**
The probability that more than 3 defective ones are present in the sample of 16 is 0.007.

**Chapter 3, Problem 23E**
(1)

Problem
Every day, a lecture may be canceled due to inclement weather with probability 0.05. Class cancelations on different days are independent.

(a) There are 15 classes left this semester. Compute the probability that at least 4 of them get canceled.

(b) Compute the probability that the tenth class this semester is the third class that gets canceled.

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/5**
Given that every day, a lecture may be cancelled due to inclement weather with probability 0.05 and class cancellations on different days are independent.

**Step 2/5**
Let *X* denotes the number of cancelled classes for given *n*. Then *X* is a binomial random variable with parameters *n* and *p* = 0.05. The probability mass function of *X* is given as

$$P(X=x)=\binom{n}{x}(0.05)^x(0.95)^{n-x}$$ for *x* = 0, 1 … *n*.

**Step 3/5**
(a)

The probability that at least 4 of 15 classes get cancelled is given by the probability $P(X \geq 4)$ which is equivalent to $1-P(X \leq 3)$.

$$P(X \leq 3)=P(X=0)+P(X=1)+P(X=2)+P(X=3)$$
$$=\binom{15}{0}(0.05)^0(0.95)^{15}+\binom{15}{1}(0.05)^1(0.95)^{14}+\binom{15}{2}(0.05)^2(0.95)^{13}+\binom{15}{3}(0.05)^3(0.95)^{12}$$
$$=0.46329123+0.365756234+0.134752296+0.030732979$$
$$=0.99453274$$

$$1-P(X \leq 3)$$
$$=1-0.99453274$$
$$=0.0055$$

The probability that at least 4 of 15 classes get cancelled due to inclement weather is 0.0055.

**Step 4/5**
(b)

Probability that the tenth class this semester is the third class gets cancelled is given by the negative binomial distribution. The probability mass function of negative binomial distribution is given as $P(X=x)=\binom{x-1}{k-1}p^k q^{x-k}$ where *q* = 1 - *p* and *x* = *k*, *k*+1 … *n*. Here *X* denotes the number of trials required to achieve *k* number of successes. Here *X* = 10 and *k* = 3. Substitute these values in the probability mass function. We require the probability *P*(*X* = 10) for *k* = 3.

$$P(X=10)$$
$$=\binom{10-1}{3-1}(0.05)^3(0.95)^7$$
$$=0.00314$$

**Step 5/5**
Probability that the tenth class this semester is the third class gets cancelled is 0.00314.

**Chapter 3, Problem 24E**
(0)

Problem

An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword.

(a) Compute the probability that at least 5 of the first 10 sites contain the given keyword.

(b) Compute the probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword.

**Step-by-step solution**

**Show all steps**

91% (11 ratings) for this solution

**Step 1/4**

Given that an internet search engine looks for a certain keyword in a sequence of independent websites. It is believed that 20% of the sites contain this keyword.

**Step 2/4**

Let *X* denotes the number of contain the keyword in the first 10 websites. Then *X* is a binomial random variable with parameters *n* = 10 and *p* = 0.2. The probability mass function of *X* is given as $P(X = x) = \binom{10}{x}(0.2)^x(0.8)^{10-x}$ for *x* = 0, 1 … 10.

**Step 3/4**

(a)

The probability that at least 5 of 10 sites contain the keyword is given by the probability $P(X \geq 5)$ which is equivalent to $1 - P(X \leq 4)$. The value of the cumulative distribution function $P(X \leq 4)$ can be found from Table A2 which is 0.9672. So $1 - P(X \leq 4)$ = 1 - 0.9672 = 0.0328.

**Step 4/4**

(b)

Probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword is given by $P(X \geq 5)$ where *X* is a geometric random variable with parameter *p* = 0.2. The probability mass function of *X* is given by $P(X = x) = p(1-p)^{x-1}$ for *x* = 1, 2….

$$
\begin{aligned}
&P(X \geq 5) \\
&= 1 - P(X \leq 4) \\
&= 1 - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4) \\
&= 1 - (0.8^0)(0.2) - (0.8^1)(0.2) - (0.8^2)(0.2) - (0.8^3)(0.2) \\
&= 1 - 0.2 - 0.16 - 0.128 - 0.1024
\end{aligned}
$$

= 0.4096

Probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword is 0.4096.

**Chapter 3, Problem 25E**
(0)

Problem
About ten percent of users do not close Windows properly. Suppose that Windows is installed in a public library that is used by random people in a random order.

(a) On the average, how many users of this computer *do not* close Windows properly before someone *does* close it properly?

(b) What is the probability that exactly 8 of the next 10 users will close Windows properly?

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/10**
Consider that Windows is installed in a public library which is used randomly in a random order. It is given that about ten percent of users do not close Windows properly.

**Step 2/10**
(a)

The number of users of this computer who do not close Windows properly before someone does close it properly is required.

**Step 3/10**
In this experiment, each trial has exactly two outcomes, “closing Windows properly” and “closing Windows not properly”. The experiment is based on the number of failures before the first success. Hence it follows a Geometric distribution.

**Step 4/10**
Define *X* to be the number of users who do not close Windows properly before some person does close it properly. The probability that the user do not close the Windows properly is given as $0.10$. So, the probability that the user closes Windows properly is calculated as,

**Step 5/10**

$$p = 1 - 0.10$$
$$= 0.90$$

The expected value of the random variable *X* is defined as $\frac{1}{p}$. So, the expected value is calculated as,

**Step 6/10**

$$E(X) = \frac{1}{0.90}$$
$$\approx 1.1111$$

**Step 7/10**

But the required expected number of users excludes the last person who closes the window properly. So, the required expectation is the expectation of $X - 1$. So, it is calculated as,

**Step 8/10**

$$E(X-1) = E(X) - 1$$
$$= 1.1111 - 1$$
$$= 0.1111$$

**Step 9/10**

Therefore, the average number of users of this computer who do not close Windows properly before someone who does closes it properly $\boxed{0.11}$.

**Step 10/10**

(b)

The probability that exactly 8 of the next 10 users will close Windows properly is required.

Define *X* as the number of users who close Windows properly. Hence this follows a binomial distribution with the success defined as closing Windows properly.

The probability function of binomial distribution is defined as,

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

The required probability is $P(X = 8)$. So, substitute $x = 8, p = 0.90, n = 10$ in the above formula.

$$
\begin{aligned}
P(X=8) &= \binom{10}{8}(0.9)^8(1-0.9)^{10-8} \\
&= (45)(0.43046721)(0.01) \\
&\approx 0.194
\end{aligned}
$$

Therefore, the probability that exactly 8 out of the next 10 users will close Windows properly is $\boxed{0.194}$.

**Chapter 3, Problem 26E**
(0)

Problem
After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files.

(a) Compute the probability that at least 5 of the first 20 files are damaged.

(b) Compute the probability that the manager has to check at least 6 files in order to find 3 undamaged files.

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/6**
Given that after a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files.

**Step 2/6**
Let *X* denotes the number of files of this computers affected by virus in the first 20 files. So *X* is a binomial random variable with $n$ = 20 and $p$ = 0.2.

**Step 3/6**
(a)

Probability that at least 5 of the first 20 files are damaged is given by $P(X \geq 5)$. The probability mass function of binomial distribution is given as $P(X=x)=\binom{20}{x}p^x q^{20-x}$

where $q = 1 - p$ and $x = 0, 1 \ldots n$. The cumulative probability can be found using Table A2.

$$P(X \geq 5) = 1 - P(X \leq 4)$$
$$= 1 - 0.6296$$
$$= 0.3704$$

Probability that at least 5 of the first 20 files are damaged is 0.3704.

**Step 4/6**
(b)

The probability that the manager has to check at least 6 files in order to find 3 undamaged files is given by the negative binomial distribution. The probability mass function of negative binomial distribution is given as $P(X = x) = \binom{x-1}{k-1} p^k q^{x-k}$ where $q = 1 - p$ and $x = k, k+1 \ldots n$. Here $X$ denotes the number of trials required to achieve $k$ number of successes.

**Step 5/6**
Here $x = 6$, $k = 3$ and $p = 0.8$. We require the probability $P(X \geq 6)$. Substitute the values in the probability mass function of negative binomial distribution

$$P(X = x) = \binom{x-1}{3-1}(0.8)^3(0.2)^{x-3}$$
.

$$P(X \geq 6)$$
$$= 1 - P(X \leq 5)$$
$$= 1 - P(X = 3) - P(X = 4) - P(X = 5)$$
$$= 1 - \binom{3-1}{3-1}(0.8)^3(0.2)^0 - \binom{4-1}{3-1}(0.8)^3(0.2)^1 - \binom{5-1}{3-1}(0.8)^3(0.2)^2$$
$$= 1 - 0.512 - 0.3072 - 0.12288$$
$$= 0.05792$$

**Step 6/6**
The probability that the manager has to check at least 6 files in order to find 3 undamaged files is s 0.05792.

**Chapter 3, Problem 27E**
(0)

Problem

Natasha is learning a poem for the New Year show. She recites it until she can finally read the whole verse without a single mistake. However, each time there is only a 20% probability of that, and it seems independent of previous recitations.

(a) Compute the probability that it will take Natasha more than 10 recitations.

(b) What is the expected number of times Natasha will recite the poem?

(c) Masha suggests that Natasha should keep working on this until she reads the whole verse without a single mistake three times. Repeat (a) and (b) assuming that Natasha agrees with this recommendation.

**Step-by-step solution**

**Show all steps**

**Step 1/21**

Given Information:

• Success is reading the whole verse with no mistakes.

• The probability of one success is $p = 0.2$.

**Step 2/21**

(a)

Find the probability that more than 10 trials are needed for the first success.

**Step 3/21**

Let the variable that records the number of trials until she reads the whole verse correctly be denoted by $X$.

**Step 4/21**

From the nature of the variable, the distribution of $X$ can be written as:

**Step 5/21**

$X \sim Geometric\left(p = 0.2\right)$

**Step 6/21**

The probability mass function of this variable is:

**Step 7/21**

$$\begin{aligned} P\left(X = x\right) &= \left(1 - p\right)^{x-1} p \\ &= \left(1 - 0.2\right)^{x-1} 0.2 \\ &= 0.8^{x-1}\left(0.2\right) \end{aligned}$$

**Step 8/21**

The probability that $X$ is more than 10 can be computed as:

**Step 9/21**

$$\begin{aligned} P(X>10) &= 1-P(X\le 10) \\ &= 1-P(X=1)-P(X=2)-\ldots-P(X=10) \\ &= 1-\left(0.8^{1-1}\right)(0.2)-\left(0.8^{2-1}\right)(0.2)-\ldots-\left(0.8^{10-1}\right)(0.2) \\ &\approx 0.1074 \end{aligned}$$

**Step 10/21**

Thus, there is approximately 0.1074 probability that she will need to recite it more than 10 times.

**Step 11/21**

(b)

The expected value of the geometric variable is:

**Step 12/21**

$$E(X)=\frac{1}{p}$$

**Step 13/21**

The expected value of $X$ is computed as:

$$\begin{aligned} E(X) &= \frac{1}{p} \\ &= \frac{1}{0.2} \\ &= 5 \end{aligned}$$

Thus, five repetitions are expected for the first success.

**Step 14/21**

(c)

Let the variable that records the number of trials until she reads the whole verse $k=3$ times correctly be denoted by $Y$.

From the nature of the variable, the distribution of $Y$ can be written as:

$$Y \sim Negative\ Binomial(k=3, p=0.2)$$

The probability mass function of this variable is:

$$\begin{aligned}P(Y=y)&=\binom{y-1}{k-1}(1-p)^{y-k}\,p^{k}\\&=\binom{y-1}{3-1}(1-0.2)^{y-3}(0.2)^{3}\\&=\binom{y-1}{2}(0.8)^{y-3}(0.2)^{3}\end{aligned}$$

**Step 15/21**
The probability that $Y$ is more than 10 can be computed as:

**Step 16/21**

$$\begin{aligned}P(Y>10)&=1-P(Y\le 10)\\&=1-P(Y=3)-P(Y=4)-...-P(Y=10)\\&=1-\binom{3-1}{2}(0.8)^{3-3}(0.2)^{3}-\binom{4-1}{2}(0.8)^{4-3}(0.2)^{3}-...-\binom{10-1}{2}(0.8)^{10-3}(0.2)^{3}\\&=0.6778\end{aligned}$$

**Step 17/21**
Thus, there is a 0.6778 probability that she will need to recite it more than 10 times for getting it correct three times.

**Step 18/21**
The expected value of the negative binomial variable is:

**Step 19/21**

$$E(Y)=\frac{k}{p}$$

**Step 20/21**
The expected value of $Y$ is computed as:

$$\begin{aligned}E(Y)&=\frac{3}{p}\\&=\frac{3}{0.2}\\&=15\end{aligned}$$

Thus, fifteen repetitions are expected for three successes.

**Step 21/21**
Thus, the probability for the first success taking more than 10 repetitions is 0.1074 and that with the recommendation is 0.6778. Similarly, the expected number of repetitions for one success is 5 and that with the recommendation is 15.

**Chapter 3, Problem 28E**
(0)

Problem
Messages arrive at an electronic message center at random times, with an average of 9 messages per hour.

(a) What is the probability of receiving *at least* five messages during the next hour?

(b) What is the probability of receiving *exactly* five messages during the next hour?

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/4**
Given that messages arrive at an electronic message centre at random times, with an average of 9 messages per hour.

**Step 2/4**
Let *X* denotes the number of messages arrive at the electronic message centre in an hour. So *X* is a Poisson random variable with parameter $\lambda = 9$. The probability mass function for *X* is given as $P(X = x) = e^{-\lambda}\dfrac{\lambda^x}{x!}$ for *x* = 0, 1, 2…

**Step 3/4**
(a)

Probability of receiving at least five messages during the next hour is given as $P(X \geq 5)$. It is equivalent to $1 - P(X \leq 4)$. The cumulative probability $P(X \leq 4)$ can be found from Table A3 which is 0.055. So $1 - P(X \leq 4)$ = 1 - 0.055 = 0.945.

Probability of receiving at least five messages during the next hour is 0.945.

**Step 4/4**
(b)

Probability of receiving exactly five messages during the next hour is given as $P(X = 5)$. Now we can find this probability using the probability mass function. Substitute *X* = 5 and $\lambda = 9$ in the probability mass function.

$$P(X = 5) = e^{-9}\frac{9^5}{5!} = 0.06073$$

Probability of receiving exactly five messages during the next hour is 0.06073.

**Chapter 3, Problem 29E**
(0)

Problem
The number of received electronic messages has Poisson distribution with some parameter $\lambda$. Using Chebyshev inequality, show that the probability of receiving more than $4\lambda$ messages does not exceed $1/(9\lambda)$.

**Step-by-step solution**

**Show all steps**

100% (3 ratings) for this solution

**Step 1/3**
Given that the number of received messages has Poisson distribution with some parameter $\lambda$. We have to show that the probability of receiving more than $4\lambda$ messages does not exceed 1/$9\lambda$ using Chebyshev's inequality.

**Step 2/3**
Let *X* denotes the number of messages arrive at the electronic message centre in an hour. So *X* is a Poisson random variable with parameter $\lambda$. The probability mass function for *X* is given as $P(X = x) = e^{-\lambda}\dfrac{\lambda^x}{x!}$ for *x* = 0, 1, 2…

**Step 3/3**
The Chebyshev's inequality states that $P\{|X-\mu|>\varepsilon\} \le \left(\dfrac{\sigma}{\varepsilon}\right)^2$ for any distribution with expectation $\mu$ and variance $\sigma^2$ and for any positive $\varepsilon$. Here $\mu = \sigma^2 = \lambda$. The probability of receiving more than $4\lambda$ is given as $P(X > 4\lambda)$. This probability can be written as $P(X > 4\lambda) = P(X - \lambda > 3\lambda)$. By Chebyshev's inequality it satisfies the inequality $P(|X-\lambda|>3\lambda) \le \left(\dfrac{\lambda}{9\lambda^2}\right) = \left(\dfrac{1}{9\lambda}\right)$ where $\varepsilon = 3\lambda$. Now it is shown that the probability of receiving more than $4\lambda$ messages does not exceed 1/$9\lambda$.

**Chapter 3, Problem 30E**
(0)

Problem
An insurance company divides its customers into 2 groups. Twenty percent of customers are in the high-risk group, and eighty percent are in the low-risk group. The high-risk customers make an average of 1 accident per year while the low-risk customers make an average of 0.1 accidents per year. Eric had no accidents last year. What is the probability that he is a high-risk driver?

**Step-by-step solution**

**Show all steps**

**Step 1/5**
Given that an insurance company divides its customers into 2 groups. Twenty percent of customers are in high-risk group and eighty percent are in the low-risk group. The high-risk customers make an average of 1 accident per year while the low-risk customers make an average of 0.1 accidents per year. Eric had no accidents last year. We have to find the probability that he is a high risk driver.

**Step 2/5**
Let *X* denotes the number of accidents made by the high-risk customer per year. So *X* is a Poisson random variable with parameter $\lambda = 1$. The probability mass function for *X* is given as $P(X = x) = \frac{e^{-1}}{x!}$ for *x* = 0, 1, 2… Let *Y* denotes the number of accidents made by the low-risk customer per year. So *Y* is a Poisson random variable with parameter $\lambda = 0.1$. The probability mass function for *Y* is given as $P(Y = y) = \frac{e^{-0.1}(0.1)^y}{y!}$ for *y* = 0, 1, 2… Let *N* denotes the combined event of no accident in a year, *H* denotes the event of selecting a customer from high risk group and *L* denotes the event of selecting a customer from low risk group.

**Step 3/5**
Probability that a high-risk driver makes no accident last year

$$
\begin{aligned}
&P(N \mid H) \\
&= 0.2P(X = 0) \\
&= 0.2e^{-1} \\
&= 0.073575888
\end{aligned}
$$

Probability that a low-risk driver makes no accident last year

$$P(N \mid L)$$
$$= 0.8P(Y = 0)$$
$$= 0.8e^{-0.1}$$
$$= 0.723869934$$

**Step 4/5**

We require the probability $P(H \mid N)$. By Baye's theorem,

$$P(H \mid N) = \frac{P(N \mid H)}{P(N \mid H) + P(N \mid L)}$$

Substitute the probabilities.

$$P(H \mid N)$$
$$= \frac{0.073575888}{0.073575888 + 0.723869934}$$
$$= 0.0923$$

**Step 5/5**

The probability that Eric had no accidents last year, he is a high-risk driver is 0.0923.

**Chapter 3, Problem 31E**

(0)

Problem

Eric from Exercise 3.30 continues driving. After three years, he still has no traffic accidents. Now, what is the conditional probability that he is a high-risk driver?

**Step-by-step solution**

**Show all steps**

**Step 1/4**

Given that Eric from Exercise 3.29 continues driving and after three years, he still has no traffic accidents. We have to find the conditional probability that he is a high-risk driver.

**Step 2/4**

We require the probability $P(H \mid N)$. Here *N* denotes the event of no accidents for three years. Let *X* denotes the number of accidents made by the high-risk customer in last three years. So *X* is a Poisson random variable with parameter $\lambda = 3$. The probability mass function for $X$ is given as $P(X=x)=\frac{e^{-3}(3)^x}{x!}$ for $x$ = 0, 1, 2… Let $Y$ denotes the number of accidents made by the low-risk customer in last three years. So $Y$ is a Poisson random variable with parameter $\lambda = 0.3$. The probability mass function for $Y$ is given as $P(Y=y)=\frac{e^{-0.3}(0.3)^y}{y!}$ for $y$ = 0, 1, 2…

Probability that a high-risk driver makes no accidents in last three years

$$
\begin{aligned}
&P(N \mid H) \\
&= 0.2P(X=0) \\
&= 0.2e^{-3} \\
&= 0.009957414
\end{aligned}
$$

Probability that a low-risk driver makes no accident last year

$$
\begin{aligned}
&P(N \mid L) \\
&= 0.8P(Y=0) \\
&= 0.8e^{-0.3} \\
&= 0.592654576
\end{aligned}
$$

**Step 3/4**

We require the probability $P(H \mid N)$. By Baye’s theorem,

$$P(H \mid N)=\frac{P(N \mid H)}{P(N \mid H)+P(L \mid H)}$$

Substitute the probabilities.

$$
\begin{aligned}
&P(H \mid N) \\
&= \frac{0.009957414}{0.009957414+0.592654576} \\
&= 0.0165
\end{aligned}
$$

**Step 4/4**

The probability that Eric had no accidents in last three years, he is a high-risk driver is 0.0165.

**Chapter 3, Problem 32E**
(1)

Problem
Before the computer is assembled, its vital component (motherboard) goes through a special inspection. Only 80% of components pass this inspection.

(a) What is the probability that at least 18 of the next 20 components pass inspection?

(b) On the average, how many components should be inspected until a component that passes inspection is found?

**Step-by-step solution**

**Show all steps**

100% (8 ratings) for this solution

**Step 1/4**
Given that before the computer is assembled, its motherboard goes through a special inspection. Only 80% of components pass this inspection.

**Step 2/4**
Let *X* denotes the number of components pass inspection in the next 20 components. Then *X* is a binomial random variable with parameters *n* = 20 and *p* = 0.8. The probability mass function of *X* is given as $P(X=x)=\binom{20}{x}(0.8)^x(0.2)^{20-x}$ for *x* = 0, 1 … 20.

**Step 3/4**
(a)

The probability that at least 18 of the next 20 components pass inspection is given by the probability $P(X \geq 18)$.

$$
\begin{aligned}
&P(X \geq 18) \\
&= P(X=18)+P(X=19)+P(X=20) \\
&= \binom{20}{18}(0.8)^{18}(0.2)^{20-18}+\binom{20}{19}(0.8)^{19}(0.2)^{20-19}+\binom{20}{20}(0.8)^{20}(0.2)^{20-20} \\
&= 0.136909428+0.057646075+0.011529215 \\
&= 0.2061
\end{aligned}
$$

The probability that at least 18 of the next 20 components pass inspection is 0.2061.

**Step 4/4**
(b)

On the average, the number of components should be inspected until a component that passes inspection is found can be computed using the Geometric distribution. Let *X* be the number of components should be inspected until a component that passes inspection is found. Then *X* follows Geometric distribution with *p* = 0.8. The probability mass function is given as $P(X=x)=(1-p)^{x}p$ for *x* = 1, 2 … The expectation of *X*, is given as 1/*p*. So the average number of components should be inspected until a component that passes inspection is 1/0.8 = 1.25.

**Chapter 3, Problem 33E**

(0)

Problem

On the average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.

(a) Compute the probability that less than 10 computers crashed.

(b) Compute the probability that exactly 10 computers crashed. You may want to use a suitable approximation.

**Step-by-step solution**

**Show all steps**

80% (5 ratings) for this solution

**Step 1/4**

Given that on average 1 in 800 computers crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.

**Step 2/4**

Let *X* denotes the number of computers crash in the 4,000 working computers during severe thunderstorm. Then *X* is a binomial random variable with parameters *n* = 4,000 and *p* = 1/800. Since *n* is large and *p* is very small, Poisson approximation can be used. If *X* is a binomial random variable with parameters *n* and *p* then *X* approximately follows Poisson distribution with parameter *np*. Here *X* follows Poisson distribution with parameter $\lambda=(4,000\times 1/800)=5$. Then $P(X=x)=e^{-5}\frac{5^{x}}{x!}$ for *x* = 1, 2, 3…

**Step 3/4**

(a)

The probability that less than 10 computers crashed is given as $P(X < 10)$.

$P(X < 10) = P(X \le 9)$.

From table A3, the value of $P(X \le 9)$ is 0.968.

The probability that less than 10 computers crashed is 0.968.

**Step 4/4**
(b)

The probability that exactly 10 computers crashed is given as $P(X = 10)$.

$$P(X = 10) = e^{-5}\frac{5^{10}}{10!} = 0.01813$$.

The probability that exactly 10 computers crashed is 0.01813.

**Chapter 3, Problem 34E**
(0)

Problem
The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

(a) What is the probability of at least 3 computer shutdowns during the next year?

(b) During the next year, what is the probability of at least 3 months (out of 12) with exactly 1 computer shutdown in each?

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/6**
Given that, the number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

**Step 2/6**
Let *X* denotes the computer shutdowns during any month. Here *X* follows Poisson distribution with parameter $\lambda = 0.25$. Then $P(X = x) = e^{-0.25}\frac{0.25^x}{x!}$ for *x* = 1, 2, 3… Let *Y* denotes the computer shutdowns during any year. The computer shut downs during a year follows Poisson distribution with parameter $\lambda = 0.25(12) = 3$.

Then $P(Y=y)=e^{-3}\frac{3^y}{y!}$ or $y$ = 1, 2, 3…

**Step 3/6**
(a)

The probability that, at least 3 computer shutdowns during the next year is given by the probability $P(Y\geq 3)$.

$$P(Y\geq 3)=1-P(Y\leq 2)$$

From table A3, the value of $P(X\leq 2)$ is 0.423.

$$1-P(Y\leq 2)=1-0.423=0.577$$

The probability that at least 3 computer shutdowns during the next year is 0.577.

**Step 4/6**
(b)

The probability of at least 3 months with exactly one computer shutdown in each is given computed as follows.

Probability that there is exactly one computer shutdown in one month is given by

$P(X=1)$ which is given as $P(X=x)=e^{-0.25}\frac{0.25^1}{1!}=0.1947$.

**Step 5/6**
Let $Z$ denotes the number of months out of 12 having exactly one computer shutdown. Then $Z$ is a binomial random variable with parameters $n$ = 12 and $p$ = 0.1947. The probability mass function of $Z$ is given as $P(Z=z)=\binom{12}{z}(0.1947)^z(1-0.1947)^{12-z}$ for $z$ = 0, 1, 2…12.

**Step 6/6**
The probability of at least 3 months out of 12 months with exactly one computer shutdown is each is given by the probability $P(Z\geq 3)$.

$$\begin{aligned}
&P(Z\geq 3)\\
&=1-P(Z\leq 2)\\
&=1-P(Z=0)-P(Z=1)-P(Z=2)\\
&=1-\binom{12}{0}(0.1947)^0(0.8053)^{12}-\binom{12}{1}(0.1947)^1(0.8053)^{11}-\binom{12}{2}(0.1947)^2(0.8053)^{10}\\
&=1-0.074386202-0.215815129-0.286980791
\end{aligned}$$

$$=0.423$$

The probability of at least 3 months out of 12 months with exactly one computer shutdown is each is 0.423.

**Chapter 3, Problem 35E**

(0)

Problem

A dangerous computer virus attacks a folder consisting of 250 files. Files are affected by the virus independently of one another. Each file is affected with the probability 0.032. What is the probability that more than 7 files are affected by this virus?

**Step-by-step solution**

**Show all steps**

75% (4 ratings) for this solution

**Step 1/3**

Given that, a dangerous computer virus attacks a folder consisting of 250 files. Files are affected by the virus independently of one another. Each file is affected with the probability 0.032. We have to find the probability that more than 7 files are affected by the virus.

**Step 2/3**

Let *X* denotes the number of files affected by the virus in the 250 files. Then *X* is a binomial random variable with parameters *n* = 250 and *p* = 0.032. Since *n* is large and *p* is very small, Poisson approximation can be used. If *X* is a binomial random variable with parameters *n* and *p* then *X* approximately follows Poisson distribution with parameter *np*. Here *X* follows Poisson distribution with parameter $\lambda = (250 \times 0.032) = 8$.

Then $P(X = x) = e^{-8}\dfrac{8^x}{x!}$ for *x* = 1, 2, 3…

**Step 3/3**

(a)

The probability that more than 7 files are attacked by the virus is given by $P(X > 7)$.

$P(X > 7) = 1 - P(X \leq 7)$.

From table A3, the value of $P(X \leq 7)$ is 0.4530.

$$\begin{aligned} P(X > 7) &= 1 - P(X \leq 7) \\ &= 1 - 0.4530 \\ &= 0.5470 \end{aligned}$$

The probability that more than 7 files are attacked by the virus is 0.5470.

**Chapter 3, Problem 36E**
(0)

Problem
In some city, the probability of a thunderstorm on any day is 0.6. During a thunderstorm, the number of traffic accidents has Poisson distribution with parameter 10. Otherwise, the number of traffic accidents has Poisson distribution with parameter 4. If there were 7 accidents yesterday, what is the probability that there was a thunderstorm?

**Step-by-step solution**

**Show all steps**

**Step 1/5**
Given that in some city, the probability of a thunderstorm on any day is 0.6. During the thunderstorm, the number of traffic accidents has Poisson distribution with parameter 10. Otherwise the number of traffic accidents has Poisson distribution with parameter 4. We have to find if there were 7 accidents yesterday, the probability that there was a thunderstorm.

**Step 2/5**
Let *X* denotes the number of accidents during thunderstorm. So *X* is a Poisson random variable with parameter $\lambda = 10$. The probability mass function for *X* is given as $P(X = x) = \frac{e^{-10}(10)^x}{x!}$ for *x* = 0, 1, 2… Let *Y* denotes the number of accidents during non-thunder storm day. So *Y* is a Poisson random variable with parameter $\lambda = 4$. The probability mass function for *Y* is given as $P(Y = y) = \frac{e^{-4}(4)^y}{y!}$ for *y* = 0, 1, 2… Let *N* denotes the event of 7 accidents in a day. Let *S* denotes the event of thunder storm.

**Step 3/5**
Probability that there are 7 accidents during a thunderstorm day is given as follows.

$$\begin{aligned} &P(N \mid S) \\ &= 0.6P(X = 7) \\ &= 0.6\frac{e^{-10}(10)^7}{7!} \\ &= 0.054047535 \end{aligned}$$

Probability that there are 7 accidents during a non thunderstorm day is given as follows.

$$P(N \mid S^c)$$
$$= 0.4P(X = 7)$$
$$= 0.4\frac{e^{-4}(4)^7}{7!}$$
$$= 0.023816145$$

**Step 4/5**

We require the probability $P(S \mid N)$. By Baye's theorem,

$$P(S \mid N) = \frac{P(N \mid S)}{P(N \mid S) + P(N \mid S^c)}$$

Substitute the probabilities.

$$P(S \mid N)$$
$$= \frac{0.054047535}{0.054047535 + 0.023816145}$$
$$= 0.694$$

**Step 5/5**

If there were 7 accidents yesterday, the probability that there was a thunderstorm is 0.694.

**Chapter 3, Problem 37E**

(0)

Problem

An interactive system consists of ten terminals that are connected to the central computer. At any time, each terminal is ready to transmit a message with probability 0.7, independently of other terminals. Find the probability that exactly 6 terminals are ready to transmit at 8 o'clock.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/4**

Given that an interactive system consists of ten terminals that are connected to the central computer. At any time, each terminal is ready to transmit message with probability 0.7, independently of other terminals. We have to find the probability that exactly 6 terminals are ready to transmit at 8'o clock.

**Step 2/4**
Let *X* denotes the number of terminals ready to transmit message in the 10 terminals. Then *X* is a binomial random variable with parameters *n* = 10 and *p* = 0.7. The probability mass function of *X* is given as $P(X = x) = \binom{10}{x}(0.7)^x(0.3)^{10-x}$ for *x* = 0, 1 … 20.

**Step 3/4**
The probability that exactly 6 terminals are ready to transmit at 8'o clock is given as $P(X = 6)$ which is given by $P(X = 6) = \binom{10}{6}(0.7)^6(0.3)^4$ = 0.2

**Step 4/4**
The probability that exactly 6 terminals are ready to transmit at 8'o clock is 0.2.

**Chapter 3, Problem 38E**
(1)

Problem
Network breakdowns are unexpected rare events that occur every 3 weeks, on the average. Compute the probability of more than 4 breakdowns during a 21-week period.

**Step-by-step solution**

**Show all steps**

100% (3 ratings) for this solution

**Step 1/4**
Given that network breakdowns are unexpected rare events that occur every 3 weeks, on the average. We have to compute the probability of more than 4 break downs during a 21-week period.

**Step 2/4**
Let *X* denotes the number of network breakdowns during 3 weeks period. So *X* is a Poisson random variable with parameter $\lambda = 1$. The probability mass function for *X* is given as $P(X = x) = \frac{e^{-1}(1)^x}{x!}$ for *x* = 0, 1, 2… Let *Y* denotes the number of network breakdowns during 21 week period. So *Y* is a Poisson random variable with parameter $\lambda = 7$. The probability mass function for *Y* is given as $P(Y = y) = \frac{e^{-7}(7)^y}{y!}$ for *y* = 0, 1, 2…

**Step 3/4**

Probability of more than 4 breakdowns during a 21-week period is given by $P(Y > 4)$.
$P(Y > 4) = 1 - P(Y \leq 4)$. The value of $P(Y \leq 4)$ is 0.173 as from the table A3. So
$1 - P(Y \leq 4) = 1 - 0.173 = 0.827$.

**Step 4/4**

Probability of more than 4 breakdowns during a 21-week period is 0.827.

**Chapter 3, Problem 39E**

(0)

Problem

Simplifying expressions, derive from the definitions of variance and covariance that

(a) $\mathrm{Var}(X) = \mathbf{E}(X^2) - \mathbf{E}^2(X)$;

(b) $\mathrm{Cov}(X, Y) = \mathbf{E}(XY) - \mathbf{E}(X)\mathbf{E}(Y)$.

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/2**

(a)

To derive from definition that $Var(X) = E(X^2) - E^2(X)$. The definition of variance of variance of *X* is $Var(X) = E\left(X - E(X)\right)^2$.

$$
\begin{aligned}
&E\left(X - E(X)\right)^2 \\
&= E\left(X^2 + E^2(X) - 2.X.E(X)\right) \\
&= E(X^2) + E^2(X) - 2E(X).E(X) \\
&= E(X^2) + E^2(X) - 2E^2(X) \\
&= E(X^2) - E^2(X)
\end{aligned}
$$

Hence it is derived that $Var(X) = E(X^2) - E^2(X)$ from its definition
$Var(X) = E\left(X - E(X)\right)^2$.

**Step 2/2**

(b)

To derive from definition that $Cov(X,Y)=E(XY)-E(X)E(Y)$. The definition of covariance of (*X*, *Y*) is $Cov(X,Y)=E\left(\left(X-E(X)\right)\left(Y-E(Y)\right)\right)$.

$$
\begin{aligned}
&E\left(\left(X-E(X)\right)\left(Y-E(Y)\right)\right)\\
&=E\left(XY-XE(Y)-YE(X)+E(X)E(Y)\right)\\
&=E(XY)-E(X)E(Y)-E(Y)E(X)+E(X)E(Y)\\
&=E(XY)-E(X)E(Y)
\end{aligned}
$$

Hence it is derived that $Cov(X,Y)=E(XY)-E(X)E(Y)$ from its definition $Cov(X,Y)=E\left(\left(X-E(X)\right)\left(Y-E(Y)\right)\right)$.

**Chapter 3, Problem 40E**
(0)

Problem
Show that

$$
\begin{aligned}
&\mathrm{Cov}(aX+bY+c,dZ+eW+F)\\
&\quad=ad\ \mathrm{Cov}(\mathrm{X},\mathrm{Z})+ae\ \mathrm{Cov}(\mathrm{X},\mathrm{W})+bd\ \mathrm{Cov}(\mathrm{Y},\mathrm{Z})+be\ \mathrm{Cov}(\mathrm{Y},\mathrm{W})
\end{aligned}
$$

for any random variables *X*, *Y*, *Z*, *W*, and any non-random numbers *a*, *b*, *c*, *d*, *e*, *f*.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/2**
To show that

$$
\begin{aligned}
Cov(aX+bY+c,dZ+eW+f)&=adCov(X,Z)+aeCov(X,W)+bdCov(Y,Z)\\
&\quad+beCov(Y,W)
\end{aligned}
$$

for any random variables *X*, *Y*, *Z* and *W* and any non-random number *a*, *b*, *c*, *d*, *e* and *f*.

The definition of covariance of (*X*, *Y*) is $Cov(X,Y)=E(XY)-E(X)E(Y)$.

**Step 2/2**

$$
\begin{aligned}
&Cov(aX+bY+c,dZ+eW+f)\\
&=E\left[(aX+bY+c)(dZ+eW+f)\right]-E(aX+bY+c).E(dZ+eW+f)\\
&=E(adXZ+aeXW+afX+bdYZ+beYW+bfY+cdZ+ceW+cf)-\\
&\left(E(aX)+E(bY)+E(c)\right).\left(E(dZ)+E(eW)+E(f)\right)\\
&=E(adXZ)+E(aeXW)+E(afX)+E(bdYZ)+E(beYW)+E(bfY)+E(cdZ)+E(ceW)\\
&+E(cf)\\
&-\begin{bmatrix}E(aX).E(dZ)+E(aX).E(eW)+E(aX).E(f)+E(bY).E(dZ)+E(bY).E(eW)\\+E(bY).E(f)+E(c).E(dZ)+E(c).E(eW)+E(c)E(f)\end{bmatrix}\\
&=adE(XZ)+aeE(XW)+afE(X)+bdE(YZ)+beE(YW)+bfE(Y)+cdE(Z)\\
&+ceE(W)+cf\\
&-\begin{bmatrix}adE(X).E(Z)+aeE(X).E(W)+afE(X)+bdE(Y).E(Z)+beE(Y).E(W)\\+bfE(Y)+cdE(Z)+ceE(W)+cf\end{bmatrix}\\
&=ad\left[E(XZ)-E(X).E(Z)\right]+ae\left[E(XW)-E(X).E(W)\right]\\
&+bd\left[E(YZ)-E(Y).E(Z)\right]+be\left[E(YW)-E(Y).E(W)\right]\\
&+afE(X)-afE(X)+bfE(Y)-bfE(Y)+cdE(Z)-cdE(Z)\\
&+ceE(W)-cdE(W)+cf-cf\\
&=adCov(X,Z)+aeCov(X,W)+bdCov(Y,Z)+ceE(W)+beCov(Y,W)
\end{aligned}
$$

So we have showed that

$$
\begin{aligned}
Cov(aX+bY+c,dZ+eW+f)=&\,adCov(X,Z)+aeCov(X,W)+bdCov(Y,Z)\\
&+beCov(Y,W)
\end{aligned}
$$

for any random variables *X*, *Y*, *Z* and *W* and any non-random number *a*, *b*, *c*, *d*, *e* and *f*.

**Chapter 4, Problem 1E**
(0)

Problem
The lifetime, in years, of some electronic component is a continuous random variable with the density

$$f(x) = \begin{cases} \frac{k}{x^4} & \text{for} \quad x \geq 1 \\ 0 & \text{for} \quad x < 1. \end{cases}$$

Find *k*, the cumulative distribution function, and the probability for the lifetime to exceed 2 years.

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/4**
The given density function is as shown in below,

$$f(x) = \begin{cases} \dfrac{k}{x^4} \text{ for } x \geq 1 \\ 0 \text{ for } x < 1 \end{cases}$$

**Step 2/4**
Find the value of $k$.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{1}^{\infty} \frac{k}{x^4}\,dx = 1$$

$$k\int_{1}^{\infty} \frac{1}{x^4}\,dx = 1$$

$$\frac{k}{3} = 1$$

$$k = 3$$

Hence, the required $k$ value is, $\boxed{3}$.

**Step 3/4**
Find the cumulative distribution function.

$$
\begin{aligned}
F(X) &= P(X < x) \\
&= \int_1^x \left(\frac{3}{t^4}\right) dx \\
&= 3\int_1^x \left(t^{-4}\right) dx \\
&= 3\left(\frac{t^{-4+1}}{-4+1}\right)_1^x
\end{aligned}
$$

$$
\begin{aligned}
&= 3\left(\frac{t^{-3}}{-3}\right)_1^x \\
&= \left(\frac{-1}{t^3}\right)_1^x \\
&= \left(\frac{-1}{x^3} - \frac{-1}{1^3}\right) \\
&= \left(\frac{-1}{x^3} + 1\right) \\
&= \left(1 - \frac{1}{x^3}\right)
\end{aligned}
$$

Hence, the required cumulative distribution function is, $\left(1 - \frac{1}{x^3}\right)$.

**Step 4/4**

Find the probability for the life time to exceed two years.

$$
\begin{aligned}
P(X > 2) &= 1 - P(X < 2) \\
&= 1 - \left(1 - \frac{1}{2^3}\right) \\
&= 1 - \frac{7}{8} \\
&= 0.125
\end{aligned}
$$

Hence, the required probability is, $\boxed{0.125}$.

**Chapter 4, Problem 2E**
(0)

Problem
The time, in minutes, it takes to reboot a certain system is a continuous variable with the density

$$f(x) = \begin{cases} C(10-x)^2, & \text{if } 0 < x < 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) Compute *C*.

(b) Compute the probability that it takes between 1 and 2 minutes to reboot.

**Step-by-step solution**

**Show all steps**

100% (11 ratings) for this solution

**Step 1/3**
The given density is shown below,

$$f(x) = \begin{cases} C(10-x)^2 & \text{if } 0 < x < 10 \\ 0 & \text{otherwise} \end{cases}$$

**Step 2/3**
a)

Find the value of $C.$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{10} c(10-x)^2\,dx = 1$$

$$c\int_{0}^{10} (10-x)^2\,dx = 1$$

$$c\left(\frac{x^3}{3} - 10x^2 + 100x\right) = 1$$

$$\frac{1000c}{3} = 1$$

$$c = \frac{3}{1000}$$

$$c = 0.003$$

Hence, the required $c$ value is, $\boxed{0.003}$.

**Step 3/3**
b)

Find the probability it takes 1 to 2 minutes to reboot the system.

$$\begin{aligned} P(1 < X < 2) &= \int_1^2 0.003(10-x)^2\,dx \\ &= 0.003\left(100x + \frac{x^3}{3} - 10x^2\right)_1^2 \\ &= 0.003\left(100(2) + \frac{2^3}{3} - 10(2)^2 - 100(1) - \frac{1^3}{3} + 10(1)^2\right) \\ &= 0.217 \end{aligned}$$

Hence, the required probability is, $\boxed{0.217}$.

**Chapter 4, Problem 3E**
(0)

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/3**
The given density function is as shown below,

$$f(x) = k\left(1-x^3\right) \text{ for } 0 < x < 1$$
.

**Step 2/3**
Find the value of $k$.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 k(1-x^3)dx = 1$$

$$k\int_0^1 (1-x^3)dx = 1$$

$$k\left(x-\frac{x^4}{4}\right)_0^1 = 1$$

$$k\left(1-\frac{1^4}{4}-0\right) = 1$$

$$k\left(\frac{3}{4}\right) = 1$$

$$k = \frac{4}{3} = 1.33$$

Hence, the required value of *k* is, $\boxed{1.33}$.

**Step 3/3**

Find the probability that it takes less than 0.5 hour to install the module.

$$\begin{aligned} P\left(X<\frac{1}{2}\right) &= \int_0^{0.5} 1.33\left(1-x^3\right)dx \\ &= 1.33\left(x-\frac{x^4}{4}\right)_0^{0.5} \\ &= 1.33\left(0.5-\frac{(0.5)^4}{4}\right) \\ &= 1.33(0.484375) \\ &= 0.644219 \\ &= 0.644 \qquad (\text{Round to 3 decimal place}) \end{aligned}$$

Hence, the required probability is, $\boxed{0.644}$.

**Chapter 4, Problem 4E**

(0)

Problem

Lifetime of a certain hardware is a continuous random variable with density

$$f(x) = \begin{cases} K - x/50 & \text{for } 0 < x < 10 \text{ years} \\ 0 & \text{for all other} x \end{cases}$$

(a) Find *K*.

(b) What is the probability of a failure within the first 5 years?

(c) What is the expectation of the lifetime?

**Step-by-step solution**

**Show all steps**

100% (10 ratings) for this solution

**Step 1/4**

The density function of lifetime of the certain hardware is,

$$f(x) = \begin{cases} K - \dfrac{x}{50} & \text{if } 0 < x < 10\, years \\ 0 & \text{for all other } x \end{cases}$$

**Step 2/4**

a)

Find the value of $k$.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{10} \left(k - \frac{x}{50}\right) dx = 1$$

$$\left(kx - \frac{x^2}{100}\right)_0^{10} = 1$$

$$\left(k(10) - \frac{10^2}{100} - 0\right) = 1$$

$$10k - 1 = 1$$

$$10k = 2$$

$$k = 0.2$$

Hence, the required $k$ value is, $\boxed{0.2}$.

**Step 3/4**

b)

Find the probability of failure within the first five years.

$$\begin{aligned} P(0 < X < 5) &= \int_0^5 \left(0.2 - \frac{x}{50}\right) dx \\ &= \left(0.2x - \frac{x^2}{100}\right)_0^5 \\ &= \left(0.2(5) - \frac{5^2}{100} - 0\right) \\ &= 0.75 \end{aligned}$$

Hence, the required probability is, $\boxed{0.75}$.

**Step 4/4**
c)

Find the expectation of the lifetime.

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^{10} \left(0.2x - \frac{x^2}{50}\right) dx \\ &= \left(\frac{x^2}{10} - \frac{x^3}{150}\right)_0^{10} \\ &= \left(\frac{10^2}{10} - \frac{10^3}{150} - 0\right) \\ &= 3.33 \end{aligned}$$

Hence, the required expectation of the lifetime is, $\boxed{3.33 \text{ years}}$.

**Chapter 4, Problem 5E**
(0)

Problem
Two continuous random variables *X* and *Y* have the joint density

$$f(x, y) = C(x^2 + y), \quad -1 \leq x \leq 1, \quad 0 \leq y \leq 1.$$

(a) Compute the constant *C*.

(b) Find the marginal densities of *X* and *Y*. Are these two variables independent?

(c) Compute probabilities $P\{Y < 0.6\}$ and $P\{Y < 0.6 \mid X < 0.5\}$.

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/10**

The joint density function of the *X* and *Y* random variables is as shown in below,

$$f(x,y) = C(x^2 + y) \quad -1 \le x \le 1,\ 0 \le y \le 1.$$

**Step 2/10**

a)

Find the value of constant $c$.

$$\int_y \int_x f(x,y)dxdy = 1$$

$$\int_0^1 \int_{-1}^1 C(x^2 + y)dxdy = 1$$

$$C\int_0^1 \left(\frac{x^3}{3} + xy\right)_{-1}^1 dy = 1$$

$$C\int_0^1 \left(\frac{1}{3} + y + \frac{1}{3} + y\right)dy = 1$$

$$C\int_0^1 \left(\frac{2}{3} + 2y\right)dy = 1$$

$$C\left(\frac{2y}{3} + y^2\right)_0^1 = 1$$

$$C\left(\frac{2}{3} + 1 - 0\right) = 1$$

$$C\left(\frac{5}{3}\right) = 1$$

$$C = \frac{3}{5}$$

Hence, the required value of *C* is, $\boxed{\frac{3}{5} = 0.6}$.

**Step 3/10**

b)

Find the marginal density of *X*.

$$f(x) = \int_y f(x,y)dy$$

$$= \int_0^1 0.6\left(x^2 + y\right)dy$$

$$= 0.6\left(x^2 y + \frac{y^2}{2}\right)_0^1$$

$$= 0.6\left(x^2 + \frac{1}{2} - 0\right)$$

$$= 0.6x^2 + 0.3$$

Hence, the required marginal density of *X* is, $\boxed{f_X(x) = \int 0.6x^3 + 0.3 \quad \text{for} \quad -1 \le x \le 1}$.

**Step 4/10**
Find the marginal density of *Y.*

$$f(y) = \int_x f(x,y)dx$$

$$= \int_{-1}^1 0.6(x^2 + y)dx$$

$$= 0.6\left(\frac{x^3}{3} + xy\right)_{-1}^1$$

$$= 0.6\left(\frac{1}{3} + y + \frac{1}{3} + y\right)$$

$$= 0.6\left(\frac{2}{3} + 2y\right)$$

$$= 0.4 + 1.2y$$

Hence, the required marginal density of *Y* is $\boxed{f_Y(y) = (0.4 + 1.2y) \quad \text{for} \quad 0 \le y \le 1}$.

**Step 5/10**
If two continuous variables *X* and *Y* are independent, then follow the below rule,

$$f(x,y) = f(x).f(y)$$

$$\begin{aligned} f(x).f(y) &= \left(0.6x^2+0.3\right)\left(0.4+1.2y\right) \\ &= \left(0.24x^2+0.12+0.72x^2y+0.36y\right) \\ &\neq 0.6\left(x^2+y\right) \\ &\neq f(x,y) \end{aligned}$$

Hence, the random variables X and *Y* are not independent.

**Step 6/10**
c)

The individual probabilities can be computed using the marginal density functions. The probability $P\left(Y<0.6\right)$ is computed from the marginal density function of *Y* as follows.

$$\begin{aligned} P\left(Y<0.6\right) &= \int_0^{0.6}(0.4+1.2y)dy \\ &= \left(0.4y+0.6y^2\right)_0^{0.6} \\ &= \left(0.4(0.6)+0.6(0.6)^2-0\right) \\ &= 0.456 \end{aligned}$$

$$P\left(Y<0.6\right)=0.456$$

**Step 7/10**

Next we have to compute the conditional probability $P\left(Y<0.6 \mid X<0.5\right)$. Using the formula for conditional probabilities,

$$P\left(Y<0.6 \mid X<0.5\right)=\frac{P(Y<0.6\cap X<0.5)}{P(X<0.5)}$$

**Step 8/10**
Find the probability of numerator first.

$$\begin{aligned} P\left(Y<0.6\cap X<0.5\right) &= \int_0^{0.6}\int_{-1}^{0.5}0.6(x^2+y)dxdy \\ &= \int_0^{0.6}0.6\left(\frac{x^3}{3}+xy\right)_{-1}^{0.5}dy \\ &= \int_0^{0.6}0.6\left(\frac{0.5^3}{3}+0.5y+\frac{1}{3}+y\right)dy \end{aligned}$$

$$= \int_0^{0.6} 0.6\left(\frac{1.125}{3} + 1.5y\right)dy$$
$$= 0.6\left(\frac{1.125}{3}y + 0.75y^2\right)_0^{0.6}$$
$$= 0.6\left(\frac{1.125}{3}(0.6) + 0.75(0.6)^2 - 0\right)$$
$$= 0.297$$

**Step 9/10**
Find the probability of denominator.

$$P(X < 0.5) = \int_{-1}^{0.5} \left(0.6x^2 + 0.3\right)dx$$
$$= \left(0.2x^3 + 0.3x\right)_{-1}^{0.5}$$
$$= \left(0.2(0.5)^3 + 0.3(0.5) + 0.2 + 0.3\right)$$
$$= 0.675$$

**Step 10/10**
Now substitute these probabilities in the conditional probability.

$$P(Y < 0.6 \mid X < 0.5) = \frac{P(Y < 0.6 \cap X < 0.5)}{P(X < 0.5)}$$
$$= \frac{0.297}{0.675}$$
$$= 0.44$$

Hence, the required probability is, $\boxed{0.44}$.

**Chapter 4, Problem 6E**
(1)

Problem
A program is divided into 3 blocks that are being compiled on 3 parallel computers. Each block takes an Exponential amount of time, 5 minutes on the average, independently of other blocks. The program is completed when *all* the blocks are compiled. Compute the expected time it takes the program to be compiled.

**Step-by-step solution**

**Show all steps**

**Step 1/2**

Let $X_1$, $X_2$ and $X_3$ denotes the time to compile a block by the three computers each respectively. Then $X_i \sim Exp(1/5)$ for *i* = 1, 2 and 3 as average = $1/\lambda$ = 5.

The expected time it takes the program to be compiled. It is the expected time of combined distribution of time taken to compile all the three blocks. We know that the linear combination of *n* independent exponential variables follows Gamma distribution with parameters $\alpha$ = *n* and $\lambda$.

**Step 2/2**

Let $Y = X_1 + X_2 + X_3$ Then *Y* follows Gamma distribution with parameters $\alpha = 3 \text{ and } \lambda = \frac{1}{5}.$

Now, find the expected value of *Y*.

$$
\begin{aligned}
E(Y) &= \frac{\alpha}{\lambda} \\
&= \frac{3}{1/5} \\
&= \frac{15}{1} \\
&= 15
\end{aligned}
$$

Hence, the expected time it takes the program to be compiled is, $\boxed{15 \text{ minutes}}$.

**Chapter 4, Problem 7E**

(0)

Problem

The time it takes a printer to print a job is an Exponential random variable with the expectation of 12 seconds. You send a job to the printer at 10:00 am, and it appears to be third in line. What is the probability that your job will be ready before 10:01?

**Step-by-step solution**

**Show all steps**

**Step 1/3**

Let *T* denotes the time taken by the printer to print a job. Then $T_i \sim Exp(1/12)$ for *i* = 1, 2…*n* as average = $1/\lambda$ = 12 seconds$^{-1}$. We need probability that our job which is third in line will be printed within 60 seconds that is $P\left(\{T = T_1 + T_2 + T_3\} < 60\right)$ . We know that the linear combination of *n* independent exponential variables follows Gamma distribution with parameters $\alpha$ = *n* and $\lambda$.

**Step 2/3**

Let *T* follows Gamma distribution with parameters $\alpha$ = 3 and $\lambda = 1/12$ sec$^{-1}$ as it is the probability of printing three jobs.

The probability $P(T < 60)$ can be computed using the Gamma-Poisson formula $P(T \le t) = P(X \ge \alpha)$ where *X* is a Poisson random variable with parameter $\lambda t$.

**Step 3/3**

Find the probability that your job will be ready before 10:01.

$$\begin{aligned} P(T < 60) &= P(X \ge 3) \\ &= 1 - P(X \le 2) \quad (\text{From table } A-3) \\ &= 1 - 0.125 \\ &= 0.875 \end{aligned}$$

Hence, the required probability is, $\boxed{0.875}$.

**Chapter 4, Problem 8E**

(0)

Problem

For some electronic component, the time until failure has Gamma distribution with parameters *α* = 2 and *λ* = 2 (years$^{-1}$). Compute the probability that the component fails within the first 6 months.

**Step-by-step solution**

**Show all steps**

100% (4 ratings) for this solution

**Step 1/2**

Let *T* denotes the time until failure of the electronic component. Then $T \sim Gamma(2,2)$. We need probability that the component fails within first six months is given by $P(T < 0.5)$. The probability $P(T < 0.5)$ can be computed using the Gamma-Poisson formula $P(T \le t) = P(X \ge \alpha)$ where *X* is a Poisson random variable with parameter $\lambda t$.

**Step 2/2**

Using the Gamma-Poisson formula,

Here, *X* is the Poisson random variable with parameter $\lambda t = 2(0.5) = 1$.

$$\begin{aligned} P(T < 0.5) = P(X \geq 2) \\ &= 1 - P(X < 2) \\ &= 1 - \left(P(X=0) + P(X=1)\right) \\ &= 1 - \left(e^{-1}\frac{(1)^0}{0!} + e^{-1}\frac{(1)^1}{1!}\right) \\ &= 1 - \left(0.367879441 + 0.367879441\right) \\ &= 1 - 0.735758882 \\ &= 0.264 \end{aligned}$$

.

Hence, the required probability that the component fails within first 6 months is, $\boxed{0.264}$.

**Chapter 4, Problem 9E**

(0)

Problem

On the average, a computer experiences breakdowns every 5 months. The time until the first breakdown and the times between any two consecutive breakdowns are independent Exponential random variables. After the third breakdown, a computer requires a special maintenance.

(a) Compute the probability that a special maintenance is required within the next 9 months.

(b) Given that a special maintenance was not required during the first 12 months, what is the probability that it will not be required within the next 4 months?

**Step-by-step solution**

**Show all steps**

**Step 1/4**

a)

Find the probability that the computer requires special maintenance within the next 9 months.

Let *X* denotes the time until first break down and *Y* denotes the time between any consecutive break downs. Then $X \sim Exp(1/5)$ and $Y \sim Exp(1/5)$. Let *T* denotes the time that the computer experience third breakdown. So $T = X + Y + Y$ and as *X* and *Y* are independent exponential variables and *T* has a Gamma distribution with parameters $\alpha = 3$ and $\lambda = 1/5$.

The probability $P(T < 9)$ can be computed using the Gamma-Poisson formula $P(T \le t) = P(X \ge \alpha)$ where *X* is a Poisson random variable with parameter $\lambda t$.

**Step 2/4**
Using the Gamma-Poisson formula,

Let *X* is the Poisson random variable with parameter $\lambda t = 0.2(9) = 1.8$.

$$\begin{aligned} P(T < 9) &= P(X \ge 3) \\ &= 1 - P(X \le 2) \\ &= 1 - 0.731 \qquad (\text{From Table } A3) \\ &= 0.269 \end{aligned}$$

Hence, the required probability is, $\boxed{0.269}$.

**Step 3/4**
b)

Find the probability that it will not be required within next 4 months.

It is the conditional probability given by $P(T < 16 \mid T < 12)$.

$$\begin{aligned} P(T < 16 \mid T < 12) &= \frac{P(T < 16 \cap T < 12)}{P(T < 12)} \\ &= \frac{P(T < 16)P(T < 12)}{P(T < 12)} \\ &= P(T < 16) \end{aligned}$$

This is a consequence of the memory less property of the exponential distribution.

**Step 4/4**
Using the Gamma-Poisson formula,

$$\begin{aligned} P(T < 16) &= P(X \ge 3) \\ &= 1 - P(X \le 2) \end{aligned}$$

Let *X* is the Poisson random variable with parameter $\lambda t = 0.2(16) = 3.2$.

$$\begin{aligned} P(T<16) &= 1-P(X\le 2) \\ &= 1-\left(e^{-3.2}\frac{3.2^0}{0!}-e^{-3.2}\frac{3.2^1}{1!}-e^{-3.2}\frac{3.2^2}{2!}\right) \\ &= 1-(0.040762203-0.130439052-0.208702484) \\ &= 0.62 \end{aligned}$$

Hence, the required probability is, $\boxed{0.62}$.

**Chapter 4, Problem 10E**

(0)

Problem

Two computer specialists are completing work orders. The first specialist receives 60% of all orders. Each order takes her Exponential amount of time with parameter $\lambda_1$ = 3 hrs$^{-1}$. The second specialist receives the remaining 40% of orders. Each order takes him Exponential amount of time with parameter $\lambda_2$ = 2 hrs$^{-1}$.

A certain order was submitted 30 minutes ago, and it is still not ready. What is the probability that the first specialist is working on it?

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/4**

Consider two computer specialists who are completing work orders. The first specialist received $60\%$ of total orders for which she takes exponential amount of time with parameter $\lambda_1 = 3\text{ hrs}^{-1}$. The second specialist received the remaining $40\%$ of total orders for which she takes exponential amount of time with parameter $\lambda_2 = 2\text{ hrs}^{-1}$.

**Step 2/4**

An order was submitted 30 minutes ago, but still it is not ready. The probability that the first specialist is working on the order is required.

Define *A* as the event that the first specialist receives the order and *B* as the event that the second specialist receives the order. So, $P(A)=0.60, P(B)=0.40$.

Define *X* to be the time taken to complete an order. The exponential time with parameter taken by the first specialist is $\lambda = 3$.

This means that $X|A$ follows an exponential distribution with parameter $\lambda = 3$. The cumulative distributive function of $X|A$ is given by,

$$F(X|A) = 1 - e^{-3x}$$

**Step 3/4**

The exponential time with parameter taken by the second specialist is $\lambda = 2$.

This means that $X|B$ follows an exponential distribution with parameter $\lambda = 2$. The cumulative distributive function of $X|B$ is given by,

$$F(X|B) = 1 - e^{-2x}$$

**Step 4/4**

It is given that the order which was submitted 30 minutes ago is still not ready. So, the time is $30 \text{ minutes} = \frac{1}{2}$.

Use Baye's rule to calculate the probability that the first specialist is working on the order. So, the required probability is calculated as,

$$\begin{aligned}
P\left(A \middle| X \geq \frac{1}{2}\right) &= \frac{P\left(X \geq \frac{1}{2}|A\right)P(A)}{P\left(X \geq \frac{1}{2}|A\right)P(A) + P\left(X \geq \frac{1}{2}|B\right)P(B)} \\
&= \frac{\left[1 - P\left(X < \frac{1}{2}|A\right)\right]P(A)}{\left[1 - P\left(X < \frac{1}{2}|A\right)\right]P(A) + \left[1 - P\left(X < \frac{1}{2}|B\right)\right]P(B)} \\
&= \frac{\left[1 - F\left(\frac{1}{2}|A\right)\right]P(A)}{\left[1 - F\left(\frac{1}{2}|A\right)\right]P(A) + \left[1 - F\left(\frac{1}{2}|B\right)\right]P(B)}
\end{aligned}$$

Use the cumulative distribution function and the probability is calculated as,

$$
\begin{aligned}
P\left(A \middle| X \geq \frac{1}{2}\right) &= \frac{\left[1-\left(1-e^{-3/2}\right)\right](0.60)}{\left[1-\left(1-e^{-3/2}\right)\right](0.60)+\left[1-\left(1-e^{-2/2}\right)\right](0.40)} \\
&= \frac{\left(e^{-1.5}\right)(0.60)}{\left(e^{-1.5}\right)(0.60)+\left(e^{-1}\right)(0.40)} \\
&= \frac{0.133878}{0.133878+0.147152} \\
&\approx 0.4764
\end{aligned}
$$

Therefore, probability that the first specialist is working on the order is $\boxed{0.4764}$.

**Chapter 4, Problem 11E**
(0)

Problem
Consider a satellite whose work is based on a certain block A. This block has an independent backup B. The satellite performs its task until both A and B fail. The lifetimes of A and B are exponentially distributed with the mean lifetime of 10 years.

(a) What is the probability that the satellite will work for more than 10 years?

(b) Compute the expected lifetime of the satellite.

**Step-by-step solution**

**Show all steps**

100% (6 ratings) for this solution

**Step 1/5**
A satellite whose work is based on a certain block A. This block has an independent backup B. The satellite performs its task until both A and B fail. The lifetimes of A and B are exponentially distributed with the mean lifetime of 10 years.

Let $X_1$ denotes the lifetime of A and $X_2$ denotes the lifetime of B. Then $X_1 \sim Exp(0.1)$ and $X_2 \sim Exp(0.1)$.

**Step 2/5**
(a)

Compute the probability that the satellite will work for more than 10 years. This is given by the probability $P\left(X_1 > 10 \cup X_2 > 10\right)$.

$$\begin{aligned}P\left(X_1>10\cup X_2>10\right) &= P(X_1>10)+P(X_2>10)-P(X_1>10\cap X_2>10)\\ &= P(X_1>10)+P(X_2>10)-P(X_1>10)P(X_2>10)\\ &= \left\{\begin{matrix}1-P(X_1\le 10)+1-P(X_2\le 10)\\ -\left(1-P(X_1\le 10)\right)\left(1-P(X_2\le 10)\right)\end{matrix}\right\}\\ &= 2-2P(X_1\le 10)-\left(1-P(X_1\le 10)\right)^2\end{aligned}$$

The last step is the consequence that both $X_1$ and $X_2$ are identical and independent exponential random variables.

**Step 3/5**

$$\begin{aligned}P\left(X_1\le 10\right) &= \int_0^{10}(1/10)e^{-x_1/10}dx_1\\ &= (1/10)\left(\frac{e^{-x_1/10}}{-1/10}\right)_0^{10}\\ &= -\left(e^{-x_1/10}\right)_0^{10}\\ &= -\left(e^{-1}-e^{0}\right)\end{aligned}$$

$$= -(0.367879441-1)$$
$$= 0.632$$

Substituting in the previous result,

$$\begin{aligned}P\left(X_1>10\cup X_2>10\right) &= 2-2P(X_1\le 10)-\left(1-P(X_1\le 10)\right)^2\\ &= 2-2(0.632)-(1-0.632)^2\\ &= 2-1.264-0.135424\\ &= 0.6\end{aligned}$$

So the probability that the satellite will work for more than 10 years is $\boxed{0.6}$.

**Step 4/5**

(b)

Compute the expected lifetime of the satellite. Let *T* denotes the lifetime of the satellite. The lifetime of the satellite is the maximum of the lifetimes of A and B that is $T = \max(X_1, X_2)$. Then *T* has the cumulative distribution function is given as follows.

$$\begin{aligned}P\left(\max\{X_1,X_2\}\le t\right) &= F(X_1)F(X_2)\\ &= \left(1-e^{-0.1t}\right)^2\\ &= 1+e^{-0.2t}-2e^{-0.1t}\end{aligned}$$

Differentiating it with respect to *t* will give its probability density function.

$$f(t) = 0.2e^{-0.1t} - 0.2e^{-0.2t}$$

**Step 5/5**

The expected lifetime of the satellite is given by $\int_0^\infty tf(t)dt$

$$\begin{aligned} E(T) &= \int_0^\infty tf(t)dt \\ &= \int_0^\infty t\left(0.2e^{-0.1t} - 0.2e^{-0.2t}\right)dt \\ &= 0.2\int_0^\infty \left(te^{-0.1t} - te^{-0.2t}\right)dt \end{aligned}$$

$$\begin{aligned} &= 0.2\left(\frac{\Gamma(2)}{0.1^2} - \frac{\Gamma(2)}{0.2^2}\right) \\ &= 0.2(100 - 25) \\ &= 15 \end{aligned}$$

The expected lifetime of the satellite is $\boxed{15 \text{ years}}$.

**Chapter 4, Problem 12E**
(0)

Problem
A computer processes tasks in the order they are received. Each task takes an Exponential amount of time with the average of 2 minutes. Compute the probability that a package of 5 tasks is processed in less than 8 minutes.

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/2**
Given that a computer processes tasks in the order they are received. Each task takes an Exponential amount of time with the average of 2 minutes. We have to compute the probability that a package of 5 tasks is processed in less than 8 minutes.

Let *T* denotes the time taken by the computer to process a task. Then $T_i \sim Exp\left(\frac{1}{2}\right)$ for *i* = 1, 2…*n* as average = $1/\lambda$ = 2 minutes$^{-1}$. We need probability that a package of five tasks is processed in less than 8 minutes that is $P\left(\{T = T_1 + T_2 + T_3 + T_4 + T_5\} < 8\right)$. We know that the linear combination of *n* independent exponential variables follows Gamma distribution with parameters $\alpha$ = *n* and $\lambda$.

**Step 2/2**

Here *T* follows Gamma distribution with parameters $\alpha$ = 5 and $\lambda = 1/2$ min$^{-1}$ as it is the probability of processing five jobs. The probability $P(T<8)$ can be computed using the Gamma-Poisson formula $P\left(T \le t\right) = P(X \ge \alpha)$ where *X* is a Poisson random variable with parameter $\lambda t$.

Using the Gamma-Poisson formula, $P(T<8) = P(X \ge 5) = 1 - P(X \le 4)$ where *X* is the Poisson random variable with parameter $\lambda t = \frac{8}{2} = 4$. From table A3, the value of $P(X \le 4)$ is 0.629.

Then,

$$\begin{aligned} P\left(T<8\right) &= 1 - P(X \le 4) \\ &= 1 - 0.629 \\ &= 0.371 \end{aligned}$$

The probability that a package of 5 tasks is processed in less than 8 minutes is $\boxed{0.371}$.

**Chapter 4, Problem 13E**

(0)

Problem

On the average, it takes 25 seconds to download a file from the internet. If it takes an Exponential amount of time to download one file, then what is the probability that it will take more than 70 seconds to download 3 independent files?

**Step-by-step solution**

**Show all steps**

100% (2 ratings) for this solution

**Step 1/2**

On average, takes 25 seconds to download a file from the internet. If it takes an Exponential amount of time to download one file, then we have to compute the probability that it will take more than 70 seconds to download 3 independent files.

Let *T* denotes the time taken to download a file from the internet. Then $T_i \sim Exp(1/25)$ for *i* = 1, 2…*n* as average = $1/\lambda$ = 25 $sec^{-1}$. We need probability that three independent files take more than 70 seconds to download that is $P\left(\{T = T_1 + T_2 + T_3\} > 70\right)$. We know that the linear combination of *n* independent exponential variables follows Gamma distribution with parameters $\alpha$ = *n* and $\lambda$.

**Step 2/2**

Here *T* follows Gamma distribution with parameters $\alpha$ = 3 and $\lambda = 1/25$ $sec^{-1}$ as it is the probability of downloading three files. The probability $P(T > 70)$ can be computed using the Gamma-Poisson formula $P\left(T > t\right) = P(X < \alpha)$ where *X* is a Poisson random variable with parameter $\lambda t$.

Using the Gamma-Poisson formula, $P(T > 70) = P(X < 3) = P(X \le 2)$ where *X* is the Poisson random variable with parameter $\lambda t = \frac{70}{25} = 2.8$. From table A3, the value of $P(X \le 2)$ is 0.469.

Then,

$$\begin{aligned} P\left(T > 70\right) &= P(X < 3) \\ &= P(X \le 2) \\ &= 0.469 \end{aligned}$$

Hence, the probability that it will take more than 70 seconds to download 3 independent files $\boxed{0.469}$

**Chapter 4, Problem 14E**

(1)

Problem

The time *X* it takes to reboot a certain system has Gamma distribution with ***E***(*X*) = 20 min and Std(*X*) = 10 min.

(a) Compute parameters of this distribution.

(b) What is the probability that it takes less than 15 minutes to reboot this system?

**Step-by-step solution**

**Show all steps**

100% (3 ratings) for this solution

**Step 1/2**

Time *X* it takes to reboot a certain system has Gamma distribution with E(*X*) = 20 min and Std(*X*) = 10 min.

(a)

The mean and variance of Gamma distribution is given as $\frac{\alpha}{\lambda}$ and $\frac{\alpha}{\lambda^2}$ respectively. Substitute the respective values in the above formulae.

Then, $\frac{\alpha}{\lambda} = 20$ and $\frac{\alpha}{\lambda^2} = 100$

Consider,

$$\frac{\alpha}{\lambda^2} = 100$$
$$\frac{\alpha}{\lambda}\left(\frac{1}{\lambda}\right) = 100$$
$$20\left(\frac{1}{\lambda}\right) = 100$$

$$\frac{1}{\lambda} = 5$$
$$\lambda = 0.2$$

And,

$$\frac{\alpha}{0.2} = 20$$
$$\alpha = 4$$

So the parameters of the Gamma distribution are $\alpha = 4$ and $\lambda = 0.2$.

**Step 2/2**

(b)

Let *T* denotes the time of the system to reboot, then *T* ~ Gamma (4, 0.2). The probability that it takes less than 15 minutes to reboot the system is given by the formula $P(T < 15)$. The probability $P(T < 15)$ can be computed using the Gamma-Poisson formula $P(T \le t) = P(X \ge \alpha)$ where *X* is a Poisson random variable with parameter $\lambda t$.

Using the Gamma-Poisson formula, $P(T < 15) = P(X \ge 4) = 1 - P(X \le 3)$ where *X* is the Poisson random variable with parameter $\lambda t = \frac{15}{5} = 3$. From table A3, the value of $P(X \le 3)$ is 0.647.

Then,

$$\begin{aligned} P(T < 15) &= P(X \ge 4) \\ &= 1 - P(X \le 3) \\ &= 1 - 0.647 \\ &= 0.353 \end{aligned}$$
.

The probability that it takes less than 15 minutes to reboot the system is $\boxed{0.353}$.

**Chapter 4, Problem 15E**
(0)

Problem
A certain system is based on two independent modules, A and B. A failure of any module causes a failure of the whole system. The lifetime of each module has a Gamma distribution, with parameters *α* and *λ* given in the table,

| Component | $\alpha$ | $\lambda$(years$^{-1}$) |
|---|---|---|
| *A* | 3 | 1 |
| *B* | 2 | 2 |

(a) What is the probability that the system works at least 2 years without a failure?

(b) Given that the system failed during the first 2 years, what is the probability that it failed due to the failure of component B (but not component A)?

**Step-by-step solution**

**Show all steps**

75% (4 ratings) for this solution

**Step 1/4**

Consider a certain system which is based on two independent modules, *A* and *B*. A failure of one module results in the failure of whole system. The lifetime of each module has a gamma distribution with parameters $\alpha$ and $\lambda$ is given as,

| Component | $\alpha$ | $\lambda\left(\text{years}^{-1}\right)$ |
|---|---|---|
| $A$ | 3 | 1 |
| $B$ | 2 | 2 |

Define $T_1$ as the lifetime of component A and $T_2$ as the lifetime of component B. Then $T_1$ defines a gamma distribution with parameters 3 and 1. And, $T_2$ defines a gamma distribution with parameters 2 and 2.

**Step 2/4**

The probability that the system works for at least 2 years without a failure is required.

The probability that the system works for at least 2 years without a failure is $P\{(T_1 > 2)\cap(T_2 > 2)\}$.

Since the modules are independent,

$$P\{(T_1 > 2)\cap(T_2 > 2)\} = P(T_1 > 2)P(T_2 > 2)$$

Use the Gamma-Poisson formula $P(T > t) = P(X < \alpha)$. Here *X* is a Poisson random variable with parameter $\lambda t$. So,

$$P(T_1 > 2) = P(X \le 2)$$

Here, *X* is the Poisson random variable with parameter $\lambda t = 2(1) = 2$. From the table of Poisson distribution, the value of $P(X \le 2)$ is obtained as 0.677.

Similarly, $P(T_2 > 2) = P(X \le 1)$. Here, *X* is the Poisson random variable with parameter $\lambda t = 2(2) = 4$. From the table of Poisson distribution, the value of $P(X \le 2)$ is obtained as 0.092.

Hence, the probability that the system works for at least 2 years without a failure is calculated as,

$$\begin{aligned} P\{(T_1 > 2)\cap(T_2 > 2)\} &= P(T_1 > 2)P(T_2 > 2) \\ &= (0.677)(0.092) \\ &= 0.062 \end{aligned}$$

Therefore, probability that that the system works for at least 2 years without a failure is $\boxed{0.062}$.

**Step 3/4**

(b)

The probability that the system failed due to failure of component *B* (but not component *A* ) given that the system failed during the first 2 years is required. So, the required probability is,

$$\begin{aligned}P\{(T_2<2)\cap(T_1>2)\}\big|P\{(T_1<2)\cup(T_2<2)\} &= \frac{\left[P\{(T_2<2)\cap(T_1>2)\}\right]\cap\left[P\{(T_1<2)\cup(T_2<2)\}\right]}{\left[P\{(T_1<2)\cup(T_2<2)\}\right]}\\ &= \frac{P(\{T_2<2\}\cap\{T_1>2\})}{P(\{T_1<2\}\cup\{T_2<2\})}\end{aligned}$$

The probabilities are calculated as follows:

$$\begin{aligned}P(\{T_1<2\}\cup\{T_2<2\}) &= P(T_1<2)+P(T_2<2)-P(T_1<2)P(T_2<2)\\ &= \begin{bmatrix}\{1-P(T_1\geq 2)\}+\{1-P(T_2\geq 2)\}\\ -\{1-P(T_1\geq 2)\}\{1-P(T_2\geq 2)\}\end{bmatrix}\\ &= (1-0.677)+(1-0.092)-(1-0.677)(1-0.092)\\ &= 0.9377\end{aligned}$$

**Step 4/4**

The probability $P(\{T_2<2\}\cap\{T_1>2\})$ is calculated as,

$$\begin{aligned}P(\{T_2<2\}\cap\{T_1>2\}) &= P(T_2<2)P(T_1>2)\\ &= \{1-P(T_2<2)\}P(T_1>2)\\ &= (1-0.092)(0.677)\\ &= 0.6147\end{aligned}$$

Substitute these probabilities in the formula for conditional probability.

$$\begin{aligned}P\{(T_2<2)\cap(T_1>2)\}\big|P\{(T_1<2)\cup(T_2<2)\} &= \frac{P(\{T_2<2\}\cap\{T_1>2\})}{P(\{T_1<2\}\cup\{T_2<2\})}\\ &= \frac{0.6147}{0.9377}\\ &= 0.656\end{aligned}$$

Therefore, probability that that the system failed due to failure of component *B* (but not component *A* ) given that the system failed during the first 2 years is $\boxed{0.656}$.

**Chapter 4, Problem 16E**
(1)

**Step-by-step solution**

**Show all steps**

**Step 1/8**
Given that *Z* is a standard Normal random variable and we have to compute the given probabilities. The probabilities can be obtained from the standard normal distribution table given in table A4.

(a)

Compute the probability $P(Z < 1.25)$

$$\begin{aligned} P(Z < 1.25) &= \Phi(1.25) \\ &= NORMSDIST(1.25) \qquad \{\text{Excel function}\} \\ &= 0.8944 \end{aligned}$$

Therefore, $P(Z < 1.25) = \boxed{0.8944}$

**Step 2/8**
(b)

Compute the probability $P(Z \le 1.25)$

$$\begin{aligned} P(Z \le 1.25) &= P(Z < 1.25) \\ &= \Phi(1.25) \\ &= NORMSDIST(1.25) \\ &= 0.8944 \end{aligned}$$

Therefore, $P(Z \le 1.25) = \boxed{0.8944}$

**Step 3/8**
(c)

Compute the probability, $P(Z > 1.25)$

$$\begin{aligned} P(Z > 1.25) &= 1 - \Phi(1.25) \\ &= 1 - NORMSDIST(1.25) \\ &= 1 - 0.8944 \\ &= 0.1056 \end{aligned}$$

**Step 4/8**

Therefore, $P(Z > 1.25) = \boxed{0.1056}$

**Step 5/8**

(d)

Compute the probability, $P(|Z| \le 1.25)$

$$\begin{aligned} P(|Z| \le 1.25) &= P(-1.25 < Z < 1.25) \\ &= 2 \times \Phi(1.25) - 1 \\ &= 2 \times NORMSDIST(1.25) - 1 \end{aligned}$$

$$= 2 \times 0.8944 - 1$$
$$= 0.7888$$

Therefore, $P(|Z| \le 1.25) = \boxed{0.7888}$

**Step 6/8**

(e)

Compute the probability, $P(Z < 6)$

$$\begin{aligned} P(Z < 6) &= \Phi(6) \\ &= NORMSDIST(6) \\ &\approx 1 \end{aligned}$$

Therefore, $P(Z < 6) \approx \boxed{1}$

**Step 7/8**

(f)

Compute the probability $P(Z > 6)$

$$
\begin{aligned}
P(Z > 6) &= 1 - \Phi(6) \\
&= 1 - NORMSDIST(6) \\
&\approx 1 - 1 \\
&\approx 0
\end{aligned}
$$

Therefore, $P(Z > 6) \approx \boxed{0}$

**Step 8/8**

(g)

Determine the value of $z_1$ such that $P(Z \leq z_1) = 0.8$

Consider,

$$
\begin{aligned}
P(Z \leq z_1) &= 0.8 \\
\Phi(z_1) &= 0.8
\end{aligned}
$$

$$
\begin{aligned}
z_1 &= \Phi^{-1}(0.8) \\
z_1 &= NORMSINV(0.8) \qquad \{\text{Excel function}\} \\
z_1 &= 0.8416
\end{aligned}
$$

With probability 0.8, variable *Z* does not exceed the value $\boxed{0.84}$.

**Chapter 4, Problem 17E**

(0)

**Step-by-step solution**

**Show all steps**

**Step 1/14**

a)

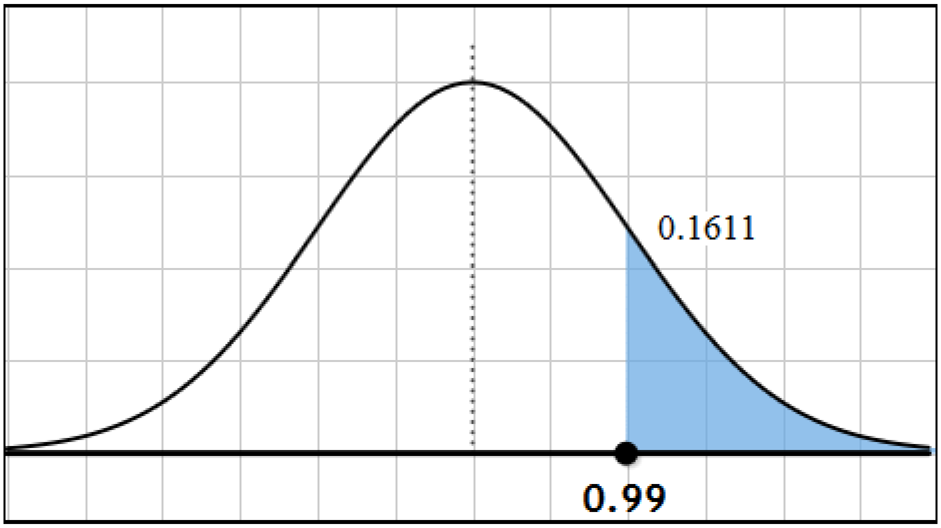
Calculate the probability at $P(Z \geq 0.99)$.

$$
\begin{aligned}
P(Z \geq 0.99) &= 1 - P(Z < 0.99) \\
&= 1 - (\text{=NORMSDIST(0.99)}) \qquad (\text{Use MS Excel function}) \\
&= 1 - 0.838913 \\
&= 0.161087 \\
&= 0.1611 \qquad (\text{Round to 4 decimal place})
\end{aligned}
$$

Hence, the required probability is $\boxed{0.1611}$.

**Step 2/14**

The graphical representation is as shown in below:




**Step 3/14**

b)

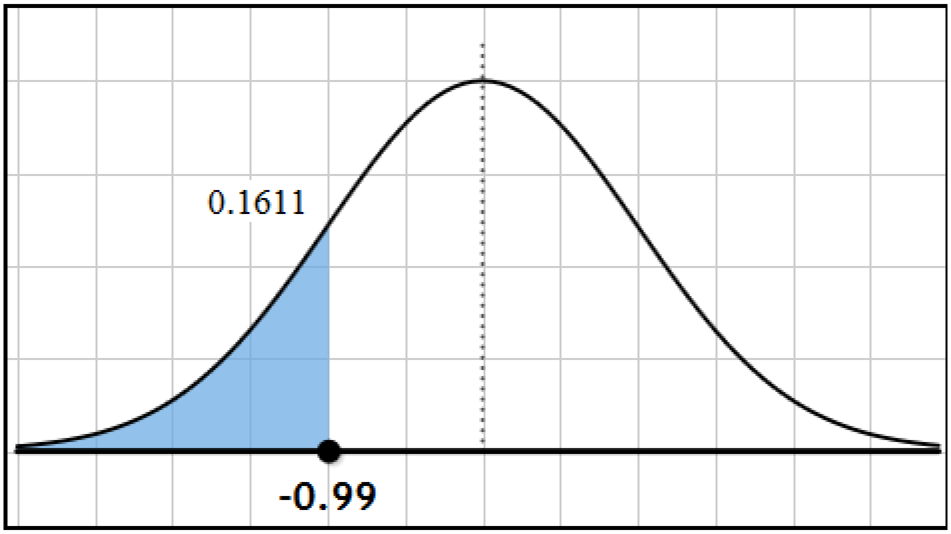
Calculate the probability at $P(Z \le -0.99)$.

$$\begin{aligned} P(Z \le -0.99) &= (\text{=NORMSDIST(-0.99)}) \qquad (\text{Use MS Excel function}) \\ &= 0.161087 \\ &= 0.1611 \qquad (\text{Round to 4 decimal place}) \end{aligned}$$

Hence, the required probability is $\boxed{0.1611}$.

**Step 4/14**

The graphical representation is as shown in below:



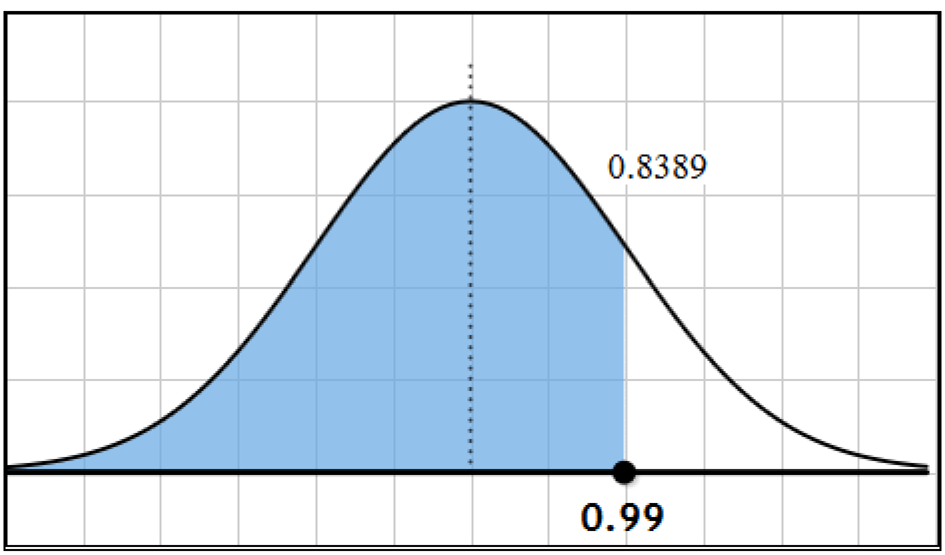

**Step 5/14**
c)

Calculate the probability at $P(Z < 0.99)$.

$$\begin{aligned} P(Z < 0.99) &= (=\text{NORMSDIST}(0.99)) \qquad (\text{Use MS Excel function}) \\ &= 0.838913 \\ &= 0.8389 \qquad (\text{Round to 4 decimal place}) \end{aligned}$$

Hence, the required probability is $\boxed{0.8389}$.

**Step 6/14**
The graphical representation is as shown in below:



**Step 7/14**

d)

Calculate the probability $P(|Z|>0.99)$

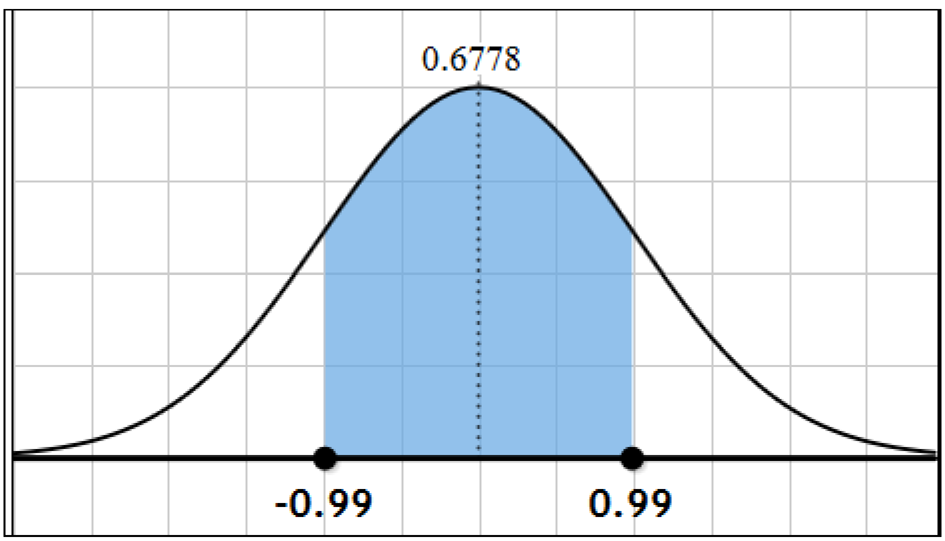
$$
\begin{aligned}
P(|Z|>0.99) &= P(-0.99<Z<0.99) \\
&= P(Z<0.99)-P(Z<-0.99) \\
&= \begin{bmatrix} (=\text{NORMSDIST}(0.99))- \\ (=\text{NORMSDIST}(\text{-}0.99)) \end{bmatrix} \qquad (\text{Use MS Excel function})
\end{aligned}
$$

$$
\begin{aligned}
&= 0.838913-0.161087 \\
&= 0.677826 \\
&= 0.6778 \qquad (\text{Round to 4 decimal place})
\end{aligned}
$$

Hence, the required probability is $\boxed{0.6778}$.

**Step 8/14**

The graphical representation is as shown in below:




**Step 9/14**

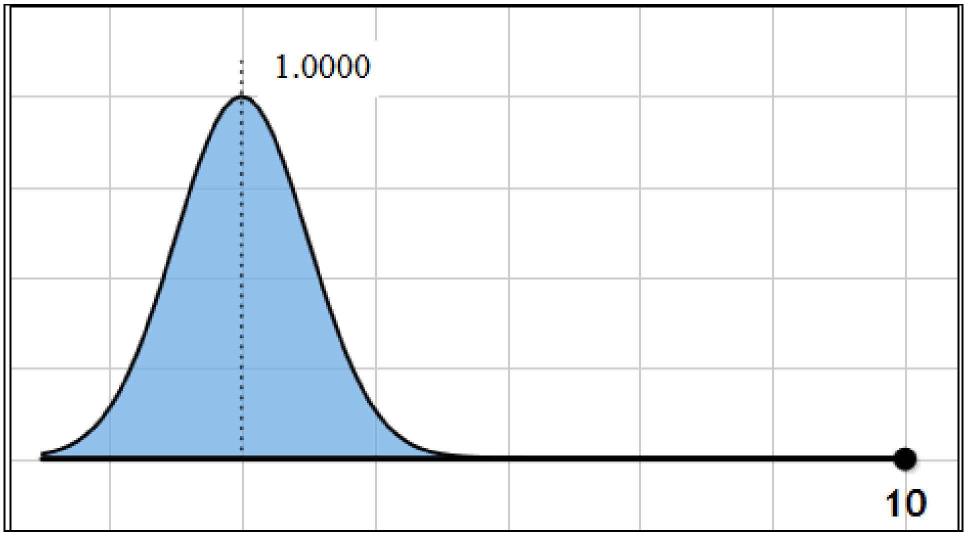
e)

Calculate the probability that $P(Z<10)$.

$$
\begin{aligned}
P(Z<10) &= (=\text{NORMSDIST}(10)) \qquad (\text{Use MS Excel function}) \\
&= 0.9999999999 \\
&= 1.0000 \qquad (\text{Round to 4 decimal place})
\end{aligned}
$$

Hence, the required probability is $\boxed{1.0000}$.

**Step 10/14**
The graphical representation is as shown in below:



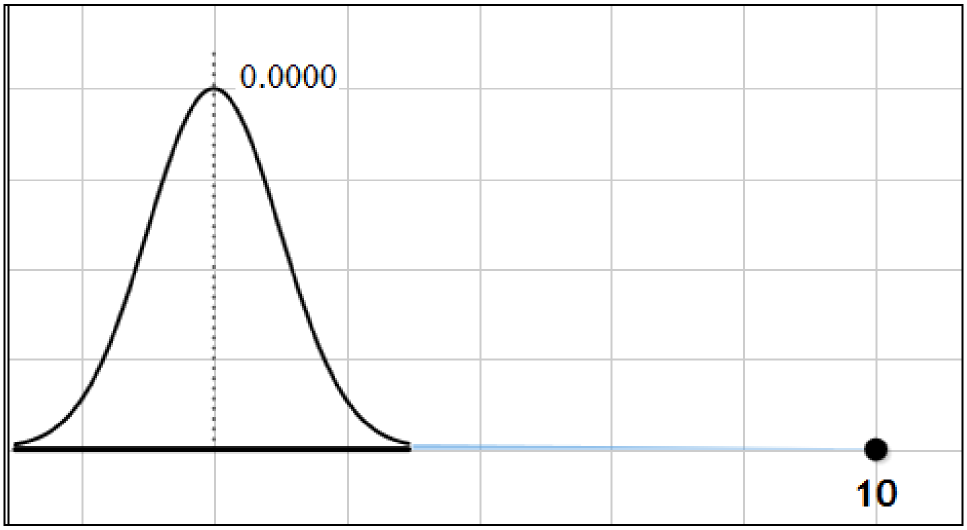

**Step 11/14**
f)

Calculate the probability that $P(Z>10)$.

$$\begin{aligned} P(Z>10) &= 1-P(Z\le 10) \\ &= 1-(=\text{NORMSDIST}(10)) \qquad (\text{Use MS Excel function}) \\ &= 1-1 \\ &= 0.0000000000 \\ &= 0.0000 \qquad (\text{Round to 4 decimal place}) \end{aligned}$$

Hence, the required probability is $\boxed{0.0000}$.

**Step 12/14**
The graphical representation is as shown in below:




**Step 13/14**

g)

Determine the value $z_1$ such that $P(Z < z_1) = 0.90$

Consider,

**Step 14/14**

$$P(Z < z_1) = 0.90$$

$$\frac{z_1 - 0}{1} = (=\text{NORMSINV}(0.9)) \qquad (\text{Use MS Excel function})$$

$$\frac{z_1 - 0}{1} = 1.281552$$

$$z_1 - 0 = 1.281552$$

$$z_1 = 1.28 \qquad (\text{Round to 2 decimal place})$$

Hence, the required answer is $\boxed{1.28}$.

**Step-by-step solution**

**Show all steps**

**Step 1/9**
For a random variable *X* with E(*X*) = -3 and Var(*X*) = 4, compute the following probabilities. Standardize *X* by transforming it to standard normal variable using the transforming $Z = \frac{X-\mu}{\sigma}$ to find probabilities involving *X*.

**Step 2/9**
(a)

Probability $P(X \le 2.39)$ can be computed by standardizing it.

$$P(X \le 2.39) = P\left(\frac{X-\mu}{\sigma} \le \frac{2.39+3}{2}\right)$$
$$= P(Z < 2.70)$$

$$= \Phi(2.70)$$
$$= NORMSDIST(2.70) \qquad \text{[Using Excel function]}$$
$$= 0.9965$$

Therefore, $P(X \le 2.39) = \boxed{0.9965}$

**Step 3/9**
(b)

Compute the probability, $P(Z \ge -2.39)$

$$P(Z \ge -2.39) = 1 - P(Z < -2.39)$$
$$= 1 - \Phi(-2.39)$$
$$= 1 - NORMSDIST(-2.39)$$
$$= 1 - 0.0084$$
$$= 0.9916$$

Therefore, $P(Z \ge -2.39) = \boxed{0.9916}$

**Step 4/9**
(c)

Compute the probability, $P(|X| \ge 2.39)$

$$\begin{aligned} P(|X| \geq 2.39) &= 1 - P(|X| < 2.39) \\ &= 1 - P(-2.39 < X < 2.39) \\ &= 1 - P\left(\frac{-2.39+3}{2} < Z < \frac{2.39+3}{2}\right) \\ &= 1 - P(0.31 < Z < 2.70) \end{aligned}$$

$$\begin{aligned} &= 1 - (\Phi(2.70) - \Phi(0.31)) \\ &= 1 - (0.9965 - 0.6217) \\ &= 0.3748 \\ &= 0.6252 \end{aligned}$$

**Step 5/9**

Therefore, $P(|X| \geq 2.39) = \boxed{0.6252}$

**Step 6/9**
(d)

Compute the probability, $P(|X+3| \geq 2.39)$

$$\begin{aligned} P(|X+3| \geq 2.39) &= 1 - P(|X+3| < 2.39) \\ &= 1 - P(-2.39 < X+3 < 2.39) \\ &= 1 - P\left(\frac{-2.39}{\sqrt{4}} < \frac{X+3}{\sqrt{4}} < \frac{2.39}{\sqrt{4}}\right) \\ &= 1 - P(-1.195 < Z < 1.195) \end{aligned}$$

$$\begin{aligned} &= 1 - \{2P(Z < 1.195) - 1\} \\ &= 1 - \{2 \times NORMSDIST(1.195) - 1\} \\ &= 2 - 2 \times 0.884 \\ &= 0.232 \end{aligned}$$

Therefore, $P(|X+3| \geq 2.39) = \boxed{0.232}$

**Step 7/9**
(e)

Compute the probability $P(X < 5)$

$$
\begin{aligned}
P(X<5) &= P\left(Z<\frac{5+3}{2}\right) \\
&= P(Z<4) \\
&= NORMSDIST(4) \\
&\approx 1
\end{aligned}
$$

Therefore, $P(X<5) \approx \boxed{1}$

**Step 8/9**
(f)

Compute the probability $P(|X|<5)$

$$
\begin{aligned}
P(|X|<5) &= P(-5<X<5) \\
&= P\left(\frac{-5+3}{2}<Z<\frac{5+3}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= P(-1<Z<4) \\
&= \Phi(4)-\Phi(-1) \\
&= 1-0.1587 \\
&= 0.8413
\end{aligned}
$$

Therefore, $P(|X|<5) = \boxed{0.8413}$

**Step 9/9**
(g)

With probability 0.33, let variable *X* exceeds the value *x*.

That is, find $x$ such that $P(X>x)=0.33$

Consider,

$$
\begin{aligned}
&P(X>x)=0.33 \\
&1-P(X\le x)=0.33 \\
&1-P\left(Z<\frac{x+3}{2}\right)=0.33 \\
&P\left(Z<\frac{x+3}{2}\right)=0.67
\end{aligned}
$$

From table A4, $\Phi(0.44)=0.67$.

Then,

$$\frac{x+3}{2} = 0.44$$
$$x = 2(0.44) - 3$$
$$x = -2.12$$

With probability 0.33, let variable *X* exceeds the value $\boxed{-2.12}$ .

**Chapter 4, Problem 19E**
(0)

Problem
According to one of the Western Electric rules for quality control, a produced item is considered conforming if its measurement falls within three standard deviations from the target value. Suppose that the process is in control so that the expected value of each measurement equals the target value. What percent of items will be considered conforming, if the distribution of measurements is

(a) Normal ($\mu, \sigma$)?

(b) Uniform($a, b$)?

**Step-by-step solution**

**Show all steps**

**Step 1/2**
(a)

Let *X* is the random variable denoting the measurements of the items. The measurements are of normal distribution with parameters $\mu$ and $\sigma^2$. As the expected value of each measurement equals the target value, $E(X) = \mu$. The percent of items fall within three standard deviations from the target value is given as $P(\mu - 3\sigma < X < \mu + 3\sigma)$.

$$\begin{aligned} P(\mu - 3\sigma < X < \mu + 3\sigma) &= P\left(\frac{\mu - 3\sigma - \mu}{\sigma} < Z < \frac{\mu + 3\sigma - \mu}{\sigma}\right) \\ &= P(-3 < Z < 3) \\ &= 2\Phi(3) - 1 \end{aligned}$$

$$= 2 \times NORMSDIST(3) - 1$$
$$= 2 \times 0.9987 - 1$$
$$= \boxed{0.9973}$$

If the measurements follow normal distribution then 99.73% of the item will be conforming.

**Step 2/2**
(b)

Consider $X$ is the random variable that follows Uniform distribution with parameters $a$ and $b$.

The mean and standard deviation of the Uniform distribution are as follows:

$$E(X) = \frac{b+a}{2}$$
$$V(X) = \frac{b-a}{\sqrt{12}}$$

Compute the percent of items fall within three standard deviations from the target value.

$$\begin{aligned}
P(\mu - 3\sigma < X < \mu + 3\sigma) &= P\left[E(X) - 3SD(X) < X < E(X) + 3SD(X)\right] \\
&= P\begin{bmatrix} E(X) - 3SD(X) - E(X) < X - E(X) < \\ E(X) + 3SD(X) - E(X) \end{bmatrix} \\
&= P\left[-3SD(X) < X - E(X) < 3SD(X)\right] \\
&= P\left(\frac{-3SD(X)}{SD(X)} < \frac{X - E(X)}{SD(X)} < \frac{3SD(X)}{SD(X)}\right) \\
&= P(-3 < Z < 3) \\
&= 2P(Z < 3) - 1 \\
&= 2(0.9987) - 1 \\
&= 1.9973 - 1 \\
&= 0.9973
\end{aligned}$$

If the measurements follow uniform distribution then 99.73% of the item will be conforming.

**Chapter 4, Problem 20E**
(0)

Problem
Refer to Exercise 4.19. What percent of items falls *beyond* 1.5 standard deviations from the mean, if the distribution of measurements is

(a) Normal ($\mu, \sigma$)?

(b) Uniform($a, b$)?

**Step-by-step solution**

**Show all steps**

100% (5 ratings) for this solution

**Step 1/3**
Found the percentage of items fall beyond 1.5 standard deviations from mean if the measurements follow normal distribution and uniform distribution.

**Step 2/3**
(a)

Let $X$ is the random variable denoting the measurements of the items. The measurements are of normal distribution with parameters $\mu$ and $\sigma^2$. The percent of items fall beyond 1.5 standard deviations from mean is given as 1- $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$.

$$\begin{aligned}
P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) &= P\left(\frac{\mu - 1.5\sigma - \mu}{\sigma} < Z < \frac{\mu + 1.5\sigma - \mu}{\sigma}\right) \\
&= P(-1.5 < Z < 1.5) \\
&= \Phi(1.5) - \Phi(-1.5) \\
&= 0.9332 - 0.0668 \quad [\text{From z-table}] \\
&= 0.8664
\end{aligned}$$

The percent of items fall beyond 1.5 standard deviations from mean is

$$1 - 0.8664 = \boxed{0.1336}$$

**Step 3/3**
(b)

The measurements are of Uniform distribution. The percent of items fall beyond 1.5 standard deviations from mean is given as

$$1 - P\left(\frac{a+b}{2} - 1.5\frac{(b-a)}{\sqrt{12}} < X < \frac{a+b}{2} + 1.5\frac{(b-a)}{\sqrt{12}}\right)$$

Consider,

$$P\left(\frac{a+b}{2}-1.5\frac{(b-a)}{\sqrt{12}}<X<\frac{a+b}{2}+1.5\frac{(b-a)}{\sqrt{12}}\right)$$

$$=\frac{\frac{a+b}{2}+1.5\frac{(b-a)}{\sqrt{12}}-\frac{a+b}{2}+1.5\frac{(b-a)}{\sqrt{12}}}{(b-a)}$$

$$=3\frac{(b-a)}{(b-a)\sqrt{12}}$$

$$=0.866$$

The percent of items fall beyond 1.5 standard deviations from mean is

$$1-0.866=\boxed{0.134}$$

**Chapter 4, Problem 21E**
(0)

Problem
The average height of professional basketball players is around 6 feet 7 inches, and the standard deviation is 3.89 inches. Assuming Normal distribution of heights within this group,

(a) What percent of professional basketball players are taller than 7 feet?

(b) If your favorite player is within the tallest 20% of all players, what can his height be?

**Step-by-step solution**

**Show all steps**

**Step 1/3**
Let the random variable *X* denotes the heights of professional basketball players.

Given that the random variable *X* follows normal distribution with mean $\mu=79$ inches and standard deviation $\sigma$ = 3.9 inches.

**Step 2/3**
(a)

Percent of professional basketball players taller than 7 feet (84 inches) is calculated as follows:

$$\begin{aligned}P(X>84)&=P\left(\frac{X-\mu}{\sigma}>\frac{84-79}{3.89}\right)\\&=P(z>1.29)\\&=1-P(z\le 1.29)\\&=1-0.9015\\&=0.0985\end{aligned}$$

Therefore, the percent of professional basketball players taller than 7 feet is 9.85%.

**Step 3/3**
(b)

The height of the favourite player whose is within the tallest 20% of all players is calculated as follows:

$$\begin{aligned}P(X\le x)&=0.2\\P\left(\frac{X-\mu}{\sigma}\le\frac{x-79}{3.89}\right)&=0.2\\P\left(Z\le\frac{x-79}{3.89}\right)&=0.2\\\frac{x-79}{3.89}&=-0.84 \qquad \text{(From normal tables)}\\x-79&=-3.27\\x&=\boxed{75.73}\end{aligned}$$

If our favourite player is within the tallest 20% of all players, then the height of the player should be at least 6 feet and 3.7 inches.

**Chapter 4, Problem 22E**
(0)

Problem
Refer to the country in Example 4.11 on p. 91, where household incomes follow Normal distribution with $\mu$ = 900 coins and $\sigma$ = 200 coins.

(a) A recent economic reform made households with the income below 640 coins qualify for a free bottle of milk at every breakfast. What portion of the population qualifies for a free bottle of milk?

(b) Moreover, households with an income within the lowest 5% of the population are entitled to a free sandwich. What income qualifies a household to receive free sandwiches?

**Step-by-step solution**

**Show all steps**

84% (6 ratings) for this solution

**Step 1/3**

Consider that the average household income follows Normal distribution with $\mu = 900 \text{ coins}$ and $\sigma = 200 \text{ coins}$.

**Step 2/3**

(a)

Households with income below 640 coins are eligible to get free bottle of milk at every breakfast. The portion of the population which is qualified for a free bottle of milk is required. So, the required probability is $P(X < 640)$.

Use the formula for probability of Normal distribution $P(X < x) = P\left(Z < \frac{X-\mu}{\sigma}\right)$. So, the required probability is calculated as,

$$\begin{aligned} P(X < 640) &= P\left(Z < \frac{640-900}{200}\right) \\ &= P(Z < -1.3) \end{aligned}$$

Use the Normal distribution table and calculate the probability. The probability that corresponds to $z = -1.3$ is obtained as 0.0968.

Therefore, portion of the population which is qualified for a free bottle of milk is $\boxed{9.68\%}$.

**Step 3/3**

(b)

Households with income within the lowest 5% of the population are eligible to get a free sandwich. The income that qualifies to receive free sandwiches is required.

Define *X* as the income which qualifies to receive free sandwiches. Use the formula for probability of Normal distribution $P(X < x) = P\left(Z < \frac{X-\mu}{\sigma}\right)$.

$$P\left(Z < \frac{X-900}{200}\right) = 0.05$$

Use the normal distribution tables and locate the probability $0.05$ which gives the value as $-1.64$. So,

$$\begin{aligned}
\frac{X-900}{200} &= -1.64 \\
X-900 &= -328 \\
X &= -328+900 \\
X &= 572
\end{aligned}$$

Therefore, the income that qualifies to receive free sandwiches is less than $\boxed{572\text{ coins}}$

**Chapter 4, Problem 23E**
(0)

Problem
The lifetime of a certain electronic component is a random variable with the expectation of 5000 hours and a standard deviation of 100 hours. What is the probability that the average lifetime of 400 components is less than 5012 hours?

**Step-by-step solution**

**Show all steps**

100% (8 ratings) for this solution

**Step 1/2**
The life time of a certain electronic component is a random variable with mean 5000 hours and standard deviation is 100 hours.

Let $X$ represents the average life time of a certain electronic component and follows a normal distribution with parameters $(\mu = 5000, \sigma = 100)$.

**Step 2/2**
Calculate the probability that the average life time of 400 components is less than 5012 hours.

$$\begin{aligned}
P(X<5012) &= P\left(Z<\frac{X-\mu}{\sigma/\sqrt{n}}\right) \\
&= P\left(Z<\frac{5012-5000}{100/\sqrt{400}}\right) \\
&= P(Z<2.4)
\end{aligned}$$

$$= (=\text{NORMSDIST}(2.4)) \qquad (\text{Use MS Excel function})$$
$$= 0.991802$$
$$= 0.9918 \qquad (\text{Round to 4 decimal place})$$

Hence, the required probability is $\boxed{0.9918}$.

**Chapter 4, Problem 24E**
(0)

Problem
Installation of some software package requires downloading 82 files. On the average, it takes 15 sec to download one file, with a variance of 16 sec$^2$. What is the probability that the software is installed in less than 20 minutes?

**Step-by-step solution**

**Show all steps**

100% (17 ratings) for this solution

**Step 1/3**
Consider a software package which requires download of 82 files. It takes 15 seconds on average to download one file with a variance of $16 \text{ sec}^2$.

**Step 2/3**
The probability that the software is installed in less than 20 minutes is required. Define *X* as the random variable that denotes the time required to download one file and define *S* as the total time required to download all the 82 files. So, the random variable *S* is the sum of all the 82 random variables of *X*.

By central limit theorem, for large samples, the distribution is normal with the mean $n\mu$ and variance $n\sigma^2$.

**Step 3/3**
So, the probability that the software is installed in less than 20 minutes (or 1200 seconds) is calculated as,

$$\begin{aligned} P(S<1200) &= P\left(Z<\frac{S-n\mu}{\sqrt{n\sigma^2}}\right) \\ &= P\left(Z<\frac{1200-(82)(15)}{\sqrt{(82)(16)}}\right) \\ &= P(Z<-0.8283) \end{aligned}$$

Use the Normal distribution table and calculate the probability. The probability that corresponds to $z=-0.83$ is obtained as 0.2033.

Therefore, the probability that the software is installed in less than 20 minutes is $\boxed{0.2033}$

**Chapter 4, Problem 25E**
(0)

Problem
Among all the computer chips produced by a certain factory, 6 percent are defective. A sample of 400 chips is selected for inspection.

(a) What is the probability that this sample contains between 20 and 25 defective chips (including 20 and 25)?

(b) Suppose that each of 40 inspectors collects a sample of 400 chips. What is the probability that at least 8 inspectors will find between 20 and 25 defective chips in their samples?

**Step-by-step solution**

**Show all steps**

100% (9 ratings) for this solution

**Step 1/3**
Let *X* denotes the number of defectives in the sample of 400 chips.

Given that the random variable *X* follows binomial distribution with parameters *p* = 0.06 and *n* = 400.

Since, *n* is large, use normal approximation to find the probability using Central limit theorem which states that if *X* follows binomial distribution with parameters *n* and *p.*

Then the random variable *X* approximately follows normal distribution with parameters $\mu = np$ and $\sigma = \sqrt{npq}$.

So here *X* approximately follows normal distribution with parameters $\mu = 24$ and $\sigma = 4.75$.

**Step 2/3**
(a)

The probability that this sample contains between 20 and 25 defective chips (including 20 and 25) is given by $P(-19.5 < X < 25.5)$ using the continuity correction

$$
\begin{aligned}
P(19.5 < X < 25.5) &= P\left(\frac{19.5-24}{4.75} < Z < \frac{25.5-24}{4.75}\right) \\
&= P(-0.95 < Z < 0.32) \\
&= \Phi(0.32) - \Phi(-0.95) \\
&= 0.6255 - 0.1711 \\
&= 0.45
\end{aligned}
$$

Therefore, the probability that this sample contains between 20 and 25 defective chips (including 20 and 25) is 0.45.

**Step 3/3**
(b)

Suppose that each of 40 inspectors collects a sample of 400 chips.

Let *Y* denotes the number of inspectors who find between 20 and 25 defective chips in their samples.

Then the random variable *Y* follows binomial distribution with parameters *n* = 40 and *p* = 0.45.

The probability that at least 8 inspectors will find between 20 and 25 defective chips in their samples is given by the probability $P(Y \geq 8)$. Using the normal approximation, *Y* follows normal distribution with parameters $\mu = 18$ and $\sigma = 3.15$.

$$
\begin{aligned}
P(Y \geq 8) &= P(Y \geq 7.5) \\
&= P\left(\frac{Y-\mu}{\sigma} \geq 7.5\right) \\
&= P\left(Z \geq \frac{7.5-18}{3.15}\right) \\
&= P(Z \geq -3.33)
\end{aligned}
$$

$$= 1 - P(Z < -3.33)$$
$$= 1 - 0.0004$$
$$= 0.9996$$

The probability that at least 8 inspectors will find between 20 and 25 defective chips in their samples is 0.9996.

**Chapter 4, Problem 26E**
(0)

Problem
An average scanned image occupies 0.6 megabytes of memory with a standard deviation of 0.4 megabytes. If you plan to publish 80 images on your web site, what is the probability that their total size is between 47 megabytes and 50 megabytes?

**Step-by-step solution**

**Show all steps**

100% (10 ratings) for this solution

**Step 1/3**
Given that an average scanned image occupies 0.6 megabytes of memory with a standard deviation of 0.4 megabytes.

If we plan to publish 80 images on our web site, we have to compute the probability that their total size is between 47 megabytes and 50 megabytes.

**Step 2/3**
Let the random variable *X* denotes the memory size required for a scanned image.

Let *S* denotes the total memory size required to publish 80 images on our website.

Then the random variable *S* can be expressed as follows:

$$S = X_1 + X_2 + ... + X_{80}$$

By Central limit theorem, for large *n*

$$S \sim N(n\mu, n\sigma^2)$$

**Step 3/3**
The required probability is $P(47 < S < 50)$ given that $\mu$ = 0.6 and $\sigma$ = 0.4 is calculated as follows:

$$\begin{aligned}P(47 < S < 50) &= P(S < 50) - P(S < 47)\\&= P\left(\frac{s-\mu}{\sigma} < \frac{50-(80)0.6}{(0.4)\sqrt{80}}\right) - P\left(\frac{s-\mu}{\sigma} < \frac{47-(80)0.6}{(0.4)\sqrt{80}}\right)\\&= P(Z < 0.56) - P(Z < -0.28)\\&= 0.7123 - 0.3898\\&= 0.3225\end{aligned}$$

The probability that their total size is between 47 megabytes and 50 megabytes is 0.3225.

**Chapter 4, Problem 27E**
(0)

Problem
A certain computer virus can damage any file with probability 35%, independently of other files. Suppose this virus enters a folder containing 2400 files. Compute the probability that between 800 and 850 files get damaged (including 800 and 850).

**Step-by-step solution**

**Show all steps**

100% (8 ratings) for this solution

**Step 1/3**
Given that a certain computer virus can damage any file with probability 35% independently of other files.

Suppose this virus enters a folder containing 2,400 files and so we have to find the probability that between 800 and 850 files get damaged (including 800 and 850).

**Step 2/3**
Let *X* denotes the number of files damaged by the virus in the 2,400 files.

Then *X* is a binomial random variable with parameters *p* = 0.35 and *n* = 2,400.

Since, *n* is large, use normal approximation to find the probability using Central limit theorem.

which states that if *X* follows binomial distribution with parameters *n* and *p* then *X* approximately follows normal distribution with parameters $\mu = np$ and $\sigma = \sqrt{npq}$.

So here *X* approximately follows normal distribution with parameters $\mu = 840$ and $\sigma = 23.4$.

**Step 3/3**
The probability that between 800 and 850 files get damaged (including 800 and 850) is calculated as follows:

$$\begin{aligned} P(799.5 < X < 850.5) &= P\left(\frac{799.5-840}{23.4} < Z < \frac{850.5-840}{23.4}\right) \\ &= P(-1.73 < Z < 0.45) \\ &= \Phi(0.45) - \Phi(-1.73) \\ &= 0.6736 - 0.0418 \\ &= \boxed{0.6318} \end{aligned}$$

Therefore, the probability that between 800 and 850 files get damaged (including 800 and 850) is 0.6318.

**Chapter 4, Problem 28E**
(0)

Problem
For the inaugural meeting of the University Cheese Club, Danielle and Anthony plan to have at least 1/4 pounds of cheese for each attending club member to taste. They anticipate at least 100 club members at the meeting, although the exact number is unknown. Members are recommended to bring some cheese to the meeting. It is estimated that each attending club member, independently of others, will bring 1 pound of cheese with probability 0.1, 1/2 pounds with probability 0.2, 1/4 pounds with probability 0.4, and will bring no cheese with probability 0.3.

(a) Can the club leaders conclude that with a probability of 0.95 or higher, the amount of cheese brought to the meeting is at least 1/4 pounds per each attending club member?

(b) How many club members must attend the meeting to make sure that there is at least 1/4 pounds of cheese per each attending club member with a probability of 0.95 or higher?

**Step-by-step solution**

**Show all steps**

**Step 1/10**
Let $X_i$ be amount of cheese brought by member $i$.

Calculate expectation of $X_i$.

$$\begin{aligned}\mu &= E(X_i)\\ &= 1\times P(X_i=1)+\frac{1}{2}\times P\left(X_i=\frac{1}{2}\right)+\frac{1}{4}\times P\left(X_i=\frac{1}{4}\right)+0\times P(X_i=0)\\ &= 1\times 0.1+\frac{1}{2}\times 0.2+\frac{1}{4}\times 0.4+0\times 0.3\\ &= 0.3 \text{ pounds}\end{aligned}$$

**Step 2/10**

Calculate variance of $X_i$.

$$\begin{aligned}\sigma^2 &= E(X_i^2)-(E(X_i))^2\\ &= 1^2\times P(X_i=1)+\left(\frac{1}{2}\right)^2\times P\left(X_i=\frac{1}{2}\right)+\left(\frac{1}{4}\right)^2\times P\left(X_i=\frac{1}{4}\right)+0^2\times P(X_i=0)-0.3^2\\ &= 1^2\times 0.1+\frac{1}{4}\times 0.2+\frac{1}{16}\times 0.4+0\times 0.3-0.09\\ &= 0.085 \text{ sq pounds}\end{aligned}$$

**Step 3/10**

Let $S_n=\sum_{i=1}^{n} X_i$ be total cheese brought by $n$ club members.

So, average amount of cheese brought by each club member is $\frac{S_n}{n}$.

**Step 4/10**

Calculate probability that average amount of cheese brought in the meeting is at least $\frac{1}{4}$ pounds per club member.

$$P\begin{pmatrix}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{pmatrix} = P\left(\frac{S_n}{n} > \frac{1}{4}\right)$$
$$= P\left(S_n > 0.25n\right)$$
$$= P\left(\frac{S_n - n\mu}{\sqrt{n\sigma^2}} > \frac{0.25n - n\mu}{\sqrt{n\sigma^2}}\right)$$
$$= P\left(\frac{S_n - n\mu}{\sqrt{n\sigma^2}} > \frac{0.25n - 0.3n}{\sqrt{0.085n}}\right)$$
$$= P\left(\frac{S_n - n\mu}{\sqrt{n\sigma^2}} > -0.1715\sqrt{n}\right)$$

**Step 5/10**

Suppose $n$ is a large number, using central limit theorem, $\frac{S_n - n\mu}{\sqrt{n\sigma^2}}$ can be approximated as standard normal random variable.

Thus, simplify $P\begin{pmatrix}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{pmatrix} = P\left(\frac{S_n - n\mu}{\sqrt{n\sigma^2}} > -0.1715\sqrt{n}\right)$ where $\Phi(\ )$ is cumulative distribution function.

$$P\begin{pmatrix}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{pmatrix} = P\left(\frac{S_n - n\mu}{\sqrt{n\sigma^2}} > -0.1715\sqrt{n}\right)$$
$$= P\left(\frac{S_n - n\mu}{\sqrt{n\sigma^2}} \le 0.1715\sqrt{n}\right)$$
$$= \Phi\left(0.1715\sqrt{n}\right)$$

**Step 6/10**

(a)

Substitute $n = 100$ in $P\begin{pmatrix}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{pmatrix} = \Phi\left(0.1715\sqrt{n}\right)$.

$$P\left(\begin{array}{l}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{array}\right)=\Phi\left(0.1715\sqrt{100}\right)$$
$$=\Phi(1.715)$$

**Step 7/10**

See Table A4 in Appendix to get probability values for corresponding $z$-values.

For $z$-value of $1.715$, probability is $0.9568$.

Thus, simplify $P\left(\begin{array}{l}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{array}\right)=\Phi(1.715)$.

$$P\left(\begin{array}{l}\text{amount of cheese is atleast}\\ \frac{1}{4}\text{ pounds per club member}\end{array}\right)=\Phi(1.715)$$
$$=0.9568$$

**Step 8/10**

Since the probability is more than $0.95$, club leaders can conclude that with a probability of $0.95$ or higher, the amount of cheese brought to the meeting is at least $\frac{1}{4}\text{ pounds per club member}$.

**Step 9/10**

(b)

See Table A4 in Appendix to get $z$-values for probability values.

For probability of $0.95$, $z$-value is $1.645$.

Thus, $\Phi(1.645)=0.95$.

**Step 10/10**

Comparing $\Phi(1.645)=0.95$ with $\Phi\left(0.1715\sqrt{n}\right)$,

$0.1715\sqrt{n}=1.645$.

Solve $0.1715\sqrt{n}=1.645$.

$$0.1715\sqrt{n} = 1.645$$
$$\sqrt{n} = \frac{1.645}{0.1715}$$
$$n = \left(\frac{1.645}{0.1715}\right)^2$$
$$n \approx 92$$

Thus, at least 92 members must attend so that probability the amount of cheese brought to the meeting is at least $\frac{1}{4}$ pounds per club member is $0.95$ or higher.

**Chapter 4, Problem 29E**

(0)

Problem

Natasha has recently learned to write. Now, she can write each letter in a readable way with probability 0.8. Compute the probability that

(a) You can read your name, when Natasha writes it.

(b) You can read at least 90% of Natasha’s 20-letter phrase.

(c) You can read at least 90% of Natasha’s 100-letter verse.

(d) You can read at least 90% of Natasha’s 500-letter story.

**Step-by-step solution**

**Show all steps**

**Step 1/6**

The given question deals with the study of the probability of a number of events pertaining to an experiment under consideration.

The experiment involves the writing of a person, named Natasha. The probability that a letter written by her will be legible or will be readable is $0.8$. Hence, we can consider the event, that a letter will be legible, then we can say,

$$P(\textit{letter is legible}) = 0.8$$

**Step 2/6**

(a).

The first event under consideration is whether a name written by Natasha will be readable or legible or not. Let us consider the name Rwen. Thus, the name Rwen will be legible or readable if all the letters are legible. Thus, the probability that the name Rwen written by Natasha will be legible is,

$$\begin{aligned} P(\textit{Rwen will be legible}) &= P(\textit{all letters are legible}) \\ &= (0.8)^4 \\ &= 0.4096 \end{aligned}$$

Hence, the probability that the Rwen written down by Natasha will be readable is, $0.4096$.

**Step 3/6**
(b).

The next event is in a $20$-letter phrase written by Natasha, at least $90\%$ of the letters will be readable. The probability that a letter written by Natasha is readable is, $0.8$. Thus, it can be considered that the random variable $X$ to be number of letters that are legible follows a Binomial probability with success probability of $0.8$. Now, consider a random variable $X$ to be number of letters that are legible, then according to the condition given, more than or equal $18$ letters has to be legible. Since,

$$\begin{aligned} &90\% \textit{ of } 20 \\ &= \frac{90}{100} * 20 \\ &= 18 \end{aligned}$$

Thus, the probability is obtained by,

$$\begin{aligned} P(X \geq 18) &= P(X = 18) + P(X = 19) + P(X = 20) \\ &= \binom{20}{18}(0.8)^{18}(0.2)^2 + \binom{20}{19}(0.8)^{19}(0.2)^1 + \binom{20}{20}(0.8)^{20}(0.2)^0 \\ &= 0.2061 \end{aligned}$$

Hence, the required probability that in a $20$-letter phrase written by Natasha, at least $90\%$ of letters will be legible is, $0.2061$.

**Step 4/6**
(c).

The next event is in a $100$-letter phrase written by Natasha, at least $90\%$ of the letters will be readable. The probability that a letter written by Natasha is readable is, $0.8$. Thus, consider that the random variable $X$ to be number of letters that are legible follows a Binomial probability with success probability of $0.8$. Now, consider a random variable $X$ to be number of letters that are legible, then according to the condition given, more than or equal $90$ letters has to be legible. Since,

$$\begin{aligned} &90\% \; of \; 100 \\ &= \frac{90}{100} * 100 \\ &= 90 \end{aligned}$$

Thus, the probability is obtained by,

$$\begin{aligned} P(X \geq 90) &= P(X=90) + P(X=91) + P(X=92) + \ldots + P(X=100) \\ &= \binom{100}{90}(0.8)^{90}(0.2)^{10} + \binom{100}{91}(0.8)^{91}(0.2)^{9} + \binom{100}{92}(0.8)^{92}(0.2)^{8} + \ldots \\ &\quad + \binom{100}{100}(0.8)^{100}(0.2) \\ &= 0.0057 \end{aligned}$$

Hence, the required probability that in a $100$-letter phrase written by Natasha, at least $90\%$ of letters will be legible is, $0.0057$.

**Step 5/6**
(d).

The next event is in a $500$-letter phrase written by Natasha, at least $90\%$ of the letters will be readable. The probability that a letter written by Natasha is readable is, $0.8$. Thus, consider that the random variable $X$ to be number of letters that are legible follows a Binomial probability with success probability of $0.8$. Now, consider a random variable $X$ to be number of letters that are legible, then according to the condition given, more than or equal $450$ letters has to be legible. Since,

**Step 6/6**

$$\begin{aligned} &90\% \; of \; 500 \\ &= \frac{90}{100} * 500 \\ &= 450 \end{aligned}$$

Now, the mean and the variance of the random variable, $X$ is given by,

$$\begin{aligned} E(X) &= 500 \times 0.8 \\ &= 400 \\ Var(X) &= 500 \times 0.8 \times 0.2 \\ &= 80 \end{aligned}$$

Thus, the probability using the central limit theorem is obtained by,

$$\begin{aligned}P(X \geq 450) &= 1 - P(X < 450) \\ &= 1 - P\left(\frac{X-400}{\sqrt{80}} < \frac{450-400}{\sqrt{80}}\right) \\ &= 1 - \Phi(5.590) \\ &\sim 0\end{aligned}$$

Hence, the required probability that in a $500$-letter phrase written by Natasha, at least $90\%$ of letters will be legible is, $0$.

**Chapter 4, Problem 30E**
(0)

Problem
Seventy independent messages are sent from an electronic transmission center. Messages are processed sequentially, one after another. Transmission time of each message is Exponential with parameter $\lambda = 5$ min$^{-1}$. Find the probability that all 70 messages are transmitted in less than 12 minutes. Use the Central Limit Theorem.

**Step-by-step solution**

**Show all steps**

100% (13 ratings) for this solution

**Step 1/3**
From the given information 70 independent messages are sent from an electronic transmission centre.

Messages are processed sequentially, one after another.

Transmission time of each message is Exponential with parameter $\lambda$ = 5 min$^{-1}$.

The objective is to find the probability that all 70 messages are transmitted in less than 12 minutes using Central limit theorem.

**Step 2/3**
Let *X* denotes the transmission time of each message.

Let *S* denotes the total time required to send seventy independent messages.

Then the random variable *S* can be expressed as follows:

$$S = X_1 + X_2 + \ldots + X_{70}$$

Here, *S* is the sum of independent and identical exponential random variables.

By Central limit theorem, for large *n*,

$$S \sim N\left(n\mu, n\sigma^2\right)$$

Given that $\mu = 1/5 = 0.2$ and $\sigma = \sqrt{\left(1/5^2\right)} = \left(1/5\right)$ = 0.2.

**Step 3/3**
Then the probability that all 70 messages are transmitted in less than 12 is calculated as follows:

$$\begin{aligned}
P(S<12) &= P\left(\frac{S-\mu}{\sigma} < \frac{12-70(0.2)}{(0.2)\sqrt{70}}\right) \\
&= P\left(Z < \frac{12-(70)0.2}{(0.2)\sqrt{70}}\right) \\
&= P(Z<-1.2) \\
&= 1-P(Z<1.2) \\
&= 1-0.8849 \\
&= \boxed{0.1151}
\end{aligned}$$

Therefore, probability that all 70 messages are transmitted in less than 12 minutes is 0.1151.

**Chapter 4, Problem 31E**
(1)

Problem
A computer lab has two printers. Printer I handles 40% of all the jobs. Its printing time is Exponential with the mean of 2 minutes. Printer II handles the remaining 60% of jobs. Its printing time is Uniform between 0 minutes and 5 minutes. A job was printed in less than 1 minute. What is the probability that it was printed by Printer I?

**Step-by-step solution**

**Show all steps**

100% (7 ratings) for this solution

**Step 1/5**
Let *A* denotes the event that a job is printed by Printer I.

Let *B* denotes the event that a job is printer by Printer II.

Let $P(A)=0.4$ represent the probability of the event *A.*

Let $P(B)=0.6$ represent the probability of the event *B*.

Let $T_1$ denotes the time taken by Printer I to print a job and $T_2$ denotes the time taken by Printer II to print a job.

Then $T_1 \sim Exp(1/2)$ and $T_2 \sim U(0,5)$.

Let *W* denotes the event that a job is printed in less than one minute.

**Step 2/5**

The required probability is given by $P(A|W)$ which is given by the Baye's rule as follows:

$$P(A|W)=\frac{P(W|A).P(A)}{P(W|A).P(A)+P(W|B).P(B)}$$

**Step 3/5**

The probability $P(W|A)=P(T_1<1)$ is calculated as follows:

$$\begin{aligned}P(T_1<1)&=1-e^{-1/2(1)}\\&=1-0.60530659\\&=0.39346934\end{aligned}$$

**Step 4/5**

The conditional probability $P(W|B)=P(T_2<1)$ is calculated as follows.

$$\begin{aligned}P(T_2<1)&=\int_0^1\frac{1}{5-0}dt_2\\&=\frac{1}{5}(t_2)_0^1\\&=0.2\end{aligned}$$

**Step 5/5**

Substitute the probabilities in the conditional probability as follows:

$$\begin{aligned}P(A|W)&=\frac{0.39346934(0.4)}{0.39346934(0.4)+0.2(0.6)}\\&=\frac{0.157387736}{0.277387736}\\&=0.567\end{aligned}$$

A job was printed in less than one minute then the probability it was printed by Printer I is 0.567.

**Chapter 4, Problem 32E**

(0)

Problem

An internet service provider has two connection lines for its customers. Eighty percent of customers are connected through Line I, and twenty percent are connected through Line II. Line I has a Gamma connection time with parameters $\alpha = 3$ and $\lambda = 2\ \text{min}^{-1}$. Line II has a Uniform(*a, b*) connection time with parameters $a = 20$ sec and $b = 50$ sec. Compute the probability that it takes a randomly selected customer more than 30 seconds to connect to the internet.

**Step-by-step solution**

**Show all steps**

**Step 1/5**

The objective is to compute the probability that it takes a randomly selected customer more than 30 seconds to connect to the internet.

From the given information,

Eighty percent of customers are connected through Line I.

Twenty percent are connected through Line II.

Given that Line I has a Gamma connection time with parameters $\alpha = 3$ and $\lambda = 2$ $\text{min}^{-1}$. Line II has a Uniform (20sec, 50sec) connection time.

**Step 2/5**

Let the event *A* denotes that a customer connects to internet through Line I.

Let the event *B* denotes the event that a customer connects to internet through Line II.

Then $P(A) = 0.8$ and $P(B) = 0.2$.

Let $T_1$ denotes the time taken by Line I to connect to internet and $T_2$ denotes the time taken by line II to connect to internet. Then $T_1 \sim Gamma(3,2)$ and $T_2 \sim U(20,50)$.

Let *W* denotes the event that its takes 30 seconds to connect to the internet.

The required probability $P(W)$ is calculated by using the following formula:

$$P(W) = P(A).P(W \mid A) + P(B).P(W \mid B) \quad \rightarrow (1)$$

**Step 3/5**

Using the Gamma-Poisson formula $P(T_1 > 0.5) = P(X \le 3)$, where *X* follows Poisson distribution with parameter $\lambda t = 2(0.5) = 1$.

$$\begin{aligned} P(W \mid A) &= P(T_1 > 0.5) \\ &= P(X \le 3) \quad \text{here,X follows } P(\lambda) \\ &= P(X=0)+P(X=1)+P(X=2)+P(X=3) \\ &= 0.3679+0.3678+0.1839+0.0613 \\ &= 0.9810 \end{aligned}$$

**Step 4/5**

The conditional probability $P(W \mid B)$ is calculated as follows:

$$\begin{aligned} P(W \mid B) &= P(T_2 > 30) \\ &= \int_{30}^{50} \frac{1}{50-20} dt_2 \\ &= \frac{1}{30}(t_2)_{30}^{50} \\ &= 0.667 \end{aligned}$$

**Step 5/5**

Substitute the probabilities in (1) to obtain the required probability.

$$\begin{aligned} P(W) &= P(A).P(W \mid A)+P(B).P(W \mid B) \\ &= 0.8(0.981)+0.2(0.667) \\ &= 0.7848+0.1334 \\ &= \boxed{0.9182} \end{aligned}$$

Therefore, the probability that it takes a randomly selected customer more than 30 seconds to connect to the internet is 0.9182.

**Chapter 4, Problem 33E**

(0)

Problem

Upgrading a certain software package requires installation of 68 new files. Files are installed consecutively. The installation time is random, but on the average, it takes 15 sec to install one file, with a variance of 11 $sec^2$.

(a) What is the probability that the whole package is upgraded in less than 12 minutes?

(b) A new version of the package is released. It requires only *N* new files to be installed, and it is promised that 95% of the time upgrading takes less than 10 minutes. Given this information, compute *N*.

**Step-by-step solution**

**Show all steps**

**Step 1/2**

The average installation time of 68 new files is 15 seconds and variance is 11 seconds.

Let $X$ follows a normal distribution with parameters is, $\left(\mu = 15 \text{ and } \sigma = \sqrt{11} = 3.32\right)$.

a)

Find the probability that the whole package is upgraded in less than 12 minutes.

Here, $n = 68, \mu = 15\,\text{sec}, \sigma^2 = 11\,\text{sec}^2 \text{ and } x = 12\,\text{min} \Rightarrow 720\,\text{sec}$

$$\begin{aligned} P(X < 12) &= P\left(Z < \frac{720 - 15 \times 68}{\sqrt{11 \times 68}}\right) \\ &= P\left(Z < \frac{720 - 1020}{\sqrt{748}}\right) \\ &= P(Z < -10.97) \\ &= 2.66362 \times 10^{-28} \\ &= 0.0000 \end{aligned}$$

Hence, the required probability is, $\boxed{0.0000}$.

**Step 2/2**

b)

Find the 95% of the time upgrading takes less than 10 minutes.

Here, $n = 68, \mu = 15\,\text{sec}, \sigma^2 = 11\,\text{sec}^2 \text{ and } x = 10\,\text{min} \Rightarrow 600\,\text{sec}$

For $N = 38.$

$$\begin{aligned}
P(X \le 600) &= P\left(Z < \frac{600-15N}{\sqrt{11N}}\right) \\
&= P\left(Z < \frac{600-15N}{\sqrt{11N}}\right) \\
&= P\left(Z < \frac{600-15\times 38}{\sqrt{11\times 38}}\right) \\
&= P(Z < 1.467) \\
&= 0.929
\end{aligned}$$

For $N = 40.$

$$\begin{aligned}
P(X \le 600) &= P\left(Z < \frac{600-15N}{\sqrt{11N}}\right) \\
&= P\left(Z < \frac{600-15N}{\sqrt{11N}}\right) \\
&= P\left(Z < \frac{600-15\times 39}{\sqrt{11\times 39}}\right) \\
&= P(Z < 0.724) \\
&= 0.765
\end{aligned}$$

Hence, the value of *N* is $\boxed{38}$ so that 95% of the time upgrading takes less than 10 minutes.

**Chapter 4, Problem 34E**
(0)

Problem
Two independent customers are scheduled to arrive in the afternoon. Their arrival times are uniformly distributed between 2 pm and 8 pm. Compute

(a) the expected time of the first (earlier) arrival;

(b) the expected time of the last (later) arrival.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/2**
(a)

The expected time of the first (earlier) arrival is required. The earlier arrival is given by $\min(T_1, T_2)$. So we require the density of $\min(T_1, T_2)$ to find its expectation. By definition,

$$P\left(\min\{T_1, T_2\} \le t\right) = \left(1 - [1 - F(T_1)][1 - F(T_1)]\right)$$
$$= 1 - \left[1 - \frac{t}{360}\right]^2$$

So the probability density of $\min(T_1, T_2)$ can be found by differentiating $1 - \left[1 - \frac{t}{360}\right]^2$ which is equal to $-2\left[1 - \frac{t}{360}\right]\left(\frac{-1}{360}\right) = \frac{1}{180}\left[1 - \frac{t}{360}\right]$. So the expectation is,

$$\int_0^{360}\left(\frac{1}{180}\left[t - \frac{t^2}{360}\right]\right)dt = \frac{1}{180}\left(\frac{t^2}{2} - \frac{t^3}{1,080}\right)_0^{360}$$
$$= \frac{1}{180}\left(\frac{360^2}{2} - \frac{360^3}{1,080} - 0\right)$$
$$= 120$$

Hence, the expected time of the first (earlier) arrival is 4 pm.

**Step 2/2**

(b)

The expected time of the last (later) arrival is required. The later arrival is given by $\max(T_1, T_2)$. We require the density of $\max(T_1, T_2)$ to find its expectation. By definition,

$$P\left(\max\{T_1, T_2\} \le t\right) = F(T_1)F(T_2)$$
$$= \left(\frac{t}{360}\right)^2$$

So the probability density of $\max(T_1, T_2)$ can be found by differentiating $\left(\frac{t}{360}\right)^2$ which is equal to $\left(\frac{t}{64,800}\right)$. So the expectation is,

$$\int_0^{360}\left(\frac{t^2}{64,800}\right)dt = \left(\frac{t^3}{194,400}\right)_0^{360}$$
$$= \frac{360^3}{194,400} - 0$$
$$= 240$$

Hence, the expected time of the last (later) arrival is at 6 pm.

**Chapter 4, Problem 35E**
(0)

**Step-by-step solution**

**Show all steps**

**Step 1/6**
Let *X* and *Y* be independent Standard Uniform random variables.

Probability density function of *X* and *Y* is given below:

$$f(x)=\begin{cases}1 & ,\ 0\le x\le 1\\ 0 & ,\ \text{otherwise}\end{cases} \qquad ......(1)$$

$$f(y)=\begin{cases}1 & ,\ 0\le y\le 1\\ 0 & ,\ \text{otherwise}\end{cases} \qquad ......(2)$$

As, *X* and *Y* be independent Standard Uniform random variables.

$$\begin{aligned}f_{X,Y}(x,y)&=f_X(x)f_Y(y) \qquad ,\ 0\le y\le 1, 0\le x\le 1\\ &=1\times 1\\ &=1\end{aligned}$$

**Step 2/6**
(a)

The objective is to find the probability of an event $\{0.5<(X+Y)<1.5\}$.

Now, to find the probability density function of $Z=X+Y$.

Let, $Z=X+Y$ and $V=X$

To find the probability density function of *Z*, first find the joint probability density function of *Z* and *V.*

Write *X* and *Y* in the form of *Z* and *V*.

$X=V$ and $Y=Z-V$

Now, to find the Jacobian of *X* and *Y* with respect of *Z* and *V*

$$|J| = \frac{\delta(X,Y)}{\delta(Z,V)}$$
$$= \begin{pmatrix} \frac{\delta X}{\delta Z} & \frac{\delta X}{\delta V} \\ \frac{\delta Y}{\delta Z} & \frac{\delta Y}{\delta V} \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$
$$= |(0-1)|$$
$$=1$$

Now, to find the limits of Z and V with the help of graph (Shaded region).

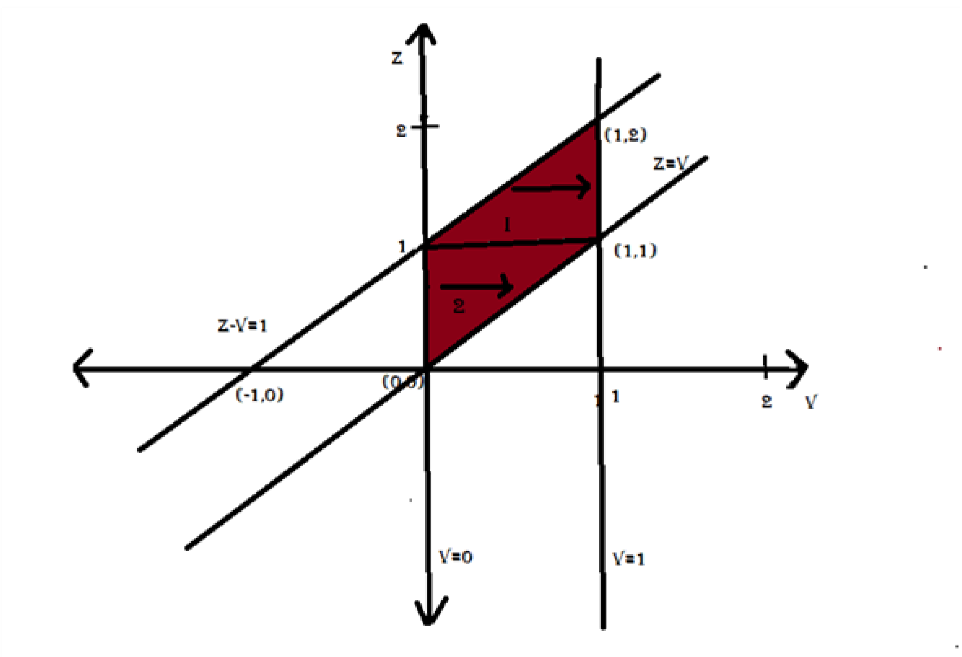
$0 < X = V < 1$ from (1)

$0 < Y = Z - V < 1$ from (2)

That is, $0 < V < 1$ and $0 < Z - V < 1$

Draw lines, $V = 1$, $V = 0$, $Z - V = 0$ and $Z - V = 1$, find the feasible region.

**Step 3/6**
The graph is given below, and feasible region is shaded with brown colour,




The feasible region is divided in to two region (1 and 2).

Now, to find the limits of *Z* and *V* in region 1:

For $0 < Z < 1$, limits of V can be found along the arrow in the region 1, that is,

$0 < V < Z$

**Step 4/6**

Therefore, in region 1, $0 < Z < 1$ and $0 < V < Z$

Now, to find the limits of *Z* and *V* in region 2:

For $1 < Z < 2$, limits of *V* can be found along the arrow in the region 2, that is,

$Z - 1 < V < 1$

Therefore, in region 1, $1 < Z < 2$ and $Z - 1 < V < 1$

$$\begin{aligned} f_{Z,V}(z,v) &= f_{X,Y}(x,y) \times |J| \\ &= 1 \times 1 \\ &= 1 \end{aligned}$$

With above provided limits of *Z* and *V*

**Step 5/6**

Now, to find the marginal probability density function of *Z*.

For, $0 < Z < 1$

$$\begin{aligned} f_Z(z) &= \int_0^z f_{Z,V}(z,v)\,dv \\ &= \int_0^z (1)\,dv \\ &= z \qquad \qquad \text{......}(3) \end{aligned}$$

For, $1 < Z < 2$

$$\begin{aligned} f_Z(z) &= \int_{z-1}^1 f_{Z,V}(z,v)\,dv \\ &= \int_{z-1}^1 (1)\,dv \\ &= 2 - z \qquad \qquad \text{......}(4) \end{aligned}$$

$$
\begin{aligned}
P(0.5 < X+Y < 1.5) &= P(0.5 < Z < 1.5) \\
&= P(0.5 < Z < 1) + P(1 < Z < 1.5) \\
&= \int_{0.5}^{1} f_Z(z)\,dz + \int_{1}^{1.5} f_Z(z)\,dz \\
&= \int_{0.5}^{1} (z)\,dz + \int_{1}^{1.5} (2-z)\,dz \\
&= \left.\frac{z^2}{2}\right|_{0.5}^{1} + \left. 2z - \frac{z^2}{2}\right|_{1}^{1.5} \\
&= \left(\frac{1}{2} - \frac{0.5^2}{2}\right) + \left[\left(2\times 1.5 - \frac{1.5^2}{2}\right) - \left(2\times 1 - \frac{1}{2}\right)\right] \\
&= 0.375 + 0.375 \\
&= 0.75
\end{aligned}
$$

Therefore, the probability of an event $\{0.5 < (X+Y) < 1.5\}$ is 0.75.

**Step 6/6**
(b)

The objective is to find the probability of an event $\{0.3 < X < 0.7 \mid Y > 0.5\}$.

It is given that *X* and *Y* are independent, therefore, the conditional probability

$P(0.3 < X < 0.7 \mid Y > 0.5)$ is obtained as follow:

$$
\begin{aligned}
P(0.3 < X < 0.7 \mid Y > 0.5) &= P(0.3 < X < 0.7) \\
&= \int_{0.3}^{0.7} (1)\,dx \\
&= x\Big|_{0.3}^{0.7} \\
&= 0.4
\end{aligned}
$$

Therefore, the probability of an event $\{0.3 < X < 0.7 \mid Y > 0.5\}$ is 0.4.

**Chapter 4, Problem 36E**
(0)

Problem

Prove the memoryless property of Geometric distribution. That is, if *X* has Geometric distribution with parameter *p*, show that

$$\boldsymbol{P}\{X > x + y \mid X > y\} = \boldsymbol{P}\{X > x\}$$

for any integer *x, y* ≥ 0.

**Step-by-step solution**

**Show all steps**

100% (1 rating) for this solution

**Step 1/2**

We have to prove the memory less property of Geometric distribution that is if *X* has Geometric distribution with parameter *p*, we have to show that

$P(X > x + y \mid X > y) = P(X > x)$ for any integer *x*, $y \geq 0$.

**Step 2/2**

From the definition,

$$\begin{aligned} P(X > x + y \mid X > y) &= \frac{P(X > x + y \cap X > y)}{P(X > y)} \\ &= \frac{P(X > x + y)}{P(X > y)} \\ &= \frac{(1-p)^{x+y}}{(1-p)^y} \\ &= \frac{(1-p)^x (1-p)^y}{(1-p)^y} \end{aligned}$$

$$= (1-p)^x$$

$$= P(X > x)$$

Hence, it is proved that the memory less property of Geometric distribution that is if *X* has Geometric distribution with parameter *p*, we have to show that

$P(X > x + y \mid X > y) = P(X > x)$ for any integer *x*, $y \geq 0$.