Chapter 5, Problem 1E
(0)

Problem
Derive a formula and explain how to generate a random variable with the density

$$f(x) = (1.5)\sqrt{x} \text{ for } 0 < x < 1$$

if your random number generator produces a Standard Uniform random variable *U*. Use the inverse transform method. Compute this variable if *U* = 0.001.

Step-by-step solution

Show all steps

100% (6 ratings) for this solution

Step 1/3
Here the required random variable is a continuous random variable. To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable U from a random number generator.

Step 2: Solve the equation $F(X) = U$ for X.

Where $F(X)$ is the cdf of the given random variable.

Step 2/3
In this problem, it is given by

$$\begin{aligned} F(X) &= \int_0^X f(x)\,dx \\ &= \int_0^X (1.5)\sqrt{x}\,dx \\ &= \left[x^{3/2}\right]_0^X \\ &= X^{3/2} \end{aligned}$$

Therefore, the obtained equation is,

$$X^{3/2} = U \text{ gives } X = U^{2/3}$$

This produces the random variable *X* from a standard uniform random variable.

Step 3/3
Now compute *X* if U = 0.001

Using the formula

$$\begin{aligned} X &= U^{2/3} \\ &= (0.001)^{2/3} \\ &= \boxed{0.01} \end{aligned}$$

Chapter 5, Problem 2E
(0)

Problem
Let U be a Standard Uniform random variable. Show all the steps required to generate

(a) an Exponential random variable with the parameter $\lambda = 2.5$;

(b) a Bernoulli random variable with the probability of success 0.77;

(c) a Binomial random variable with parameters $n = 15$ and $p = 0.4$;

(d) a discrete random variable with the distribution $P(x)$, where $P(0) = 0.2$, $P(2) = 0.4$, $P(7) = 0.3$, $P(11) = 0.1$;

(e) a continuous random variable with the density $f(x) = 3x^2$, $0 < x < 1$;

(f) a continuous random variable with the density $f(x) = 1.5x^2$, $-1 < x < 1$;

(g) a continuous random variable with the density $f(x) = \frac{1}{12}\sqrt[3]{x},\ 0 \le x \le 8.$

If a computer generates U and the result is $U = 0.3972$, compute the variables generated in (a)–(g).

Step-by-step solution

Show all steps

80% (5 ratings) for this solution

Step 1/8
(a)

Here the required random variable is a continuous random variable. To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable U from a random number generator.

Step 2: Solve the equation $F(X) = U$ for X.

Where $F(X)$ is the cdf of the given random variable.

In this problem, it is given by

$$F(X) = 1 - e^{-2.5X}$$

Therefore, the obtained equation is,

$$1 - e^{-2.5X} = U$$

$$X = -\frac{1}{2.5}\ln(1-U)$$

This produces the random variable *X* from a standard uniform random variable.

Now compute *X* if *U* = 0.3972

Using the formula

$$\begin{aligned} X &= -\frac{1}{2.5}\ln(1-0.3972) \\ &= \boxed{0.2025} \end{aligned}$$

Step 2/8
(b)

Here the required random variable is a discrete random variable. To generate it we can use the following algorithm:

Step 1: Obtain a Standard Uniform random variable *U* from a random number generator.

Step 2: Given that the probability of success for the Bernoulli random variable is 0.77, so define

$$X = \begin{cases} 1 & \text{if } U < 0.77 \\ 0 & \text{if } U \geq 0.77 \end{cases}$$

In this way generate *X*.

Now compute *X* if *U* = 0.3972

According to the step 2 equation, the value of *X* comes out to be 1.

Step 3/8
(c)

Here the required random variable is a discrete random variable. To generate it, use the following algorithm:

Step 1: Obtain a Standard Uniform random variable *U* from a random number generator.

Step 2: Given that the probability of success for the Binomial random variable is 0.4. Let $F(i)$ be the cumulative distribution function of the binomial random variable then take, $F(i-1) \le U < F(i)$ or $p_0 + p_1 + ... + p_{i-1} \le U < p_0 + p_1 + ... + p_{i-1} + p_i$

In this case

$$
\begin{aligned}
&{}^{15}C_0(0.4)^0(0.6)^{15-0} + {}^{15}C_1(0.4)^1(0.6)^{15-1} + ... + {}^{15}C_{i-1}(0.4)^{i-1}(0.6)^{15-i+1} \le U \\
&< {}^{15}C_0(0.4)^0(0.6)^{15-0} + {}^{15}C_1(0.4)^1(0.6)^{15-1} + ... + {}^{15}C_{i-1}(0.4)^{i-1}(0.6)^{15-i+1} + {}^{15}C_i(0.4)^i(0.6)^{15-i}
\end{aligned}
$$

Using the above inequality, obtain *i* which is the generated value of *X*.

Now compute *X* if *U* = 0.3972

Substituting the value of U in the above inequality

$$
\begin{aligned}
&{}^{15}C_0(0.4)^0(0.6)^{15-0} + {}^{15}C_1(0.4)^1(0.6)^{15-1} + ... + {}^{15}C_{i-1}(0.4)^{i-1}(0.6)^{15-i+1} \le 0.3972 \\
&< {}^{15}C_0(0.4)^0(0.6)^{15-0} + {}^{15}C_1(0.4)^1(0.6)^{15-1} + ... + {}^{15}C_{i-1}(0.4)^{i-1}(0.6)^{15-i+1} + {}^{15}C_i(0.4)^i(0.6)^{15-i}
\end{aligned}
$$

From the above inequality, *i* = 5

So the generated value of *X* is 5.

Step 4/8
(d)

Here the required random variable is a discrete random variable having values

X = 0, 2, 7, 11.

To generate it use the same algorithm as shown above. That is

Step 1: Obtain a Standard Uniform random variable *U* from a random number generator.

Step 2: Given that the probability of success for the Binomial random variable is 0.4. Let *F*(*i*) be the cumulative distribution function of the binomial random variable, then we take, $F(i-1) \le U < F(i)$ or, $p_0 + p_1 + ... + p_{i-1} \le U < p_0 + p_1 + ... + p_{i-1} + p_i$

Now compute *X* if *U* = 0.3972

We see that $P(0) \le U < P(0) + P(2)$

Therefore *X* = 2.

Step 5/8

(e)

Here the required random variable is a continuous random variable. To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable *U* from a random number generator.

Step 2: Solve the equation $F(X)=U$ for *X*.

Where $F(X)$ is the cdf of the given random variable.

In this problem, it is given by

Step 6/8

$$\begin{aligned} F(X) &= \int_0^X f(x)\,dx \\ &= \int_0^X 3x^2\,dx \\ &= \left[x^3\right]_0^X \\ &= X^3 \end{aligned}$$

Therefore, the equation is,

$$X^3 = U \text{ gives } X = U^{1/3}$$

This produces the random variable *X* from a standard uniform random variable.

Now compute *X* if *U* = 0.3972

Using the formula

$$\begin{aligned} X &= (0.3972)^{1/3} \\ &= \boxed{0.7351} \end{aligned}$$

Step 7/8

(f)

Here the required random variable is a continuous random variable. To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable *U* from a random number generator.

Step 2: Solve the equation $F(X)=U$ for *X*.

Where $F(X)$ is the cdf of the given random variable.

In this problem, it is given by

$$\begin{aligned}F(X)&=\int_{-1}^{X} f(x)\,dx\\&=\int_{-1}^{X} 1.5x^2\,dx\\&=\left[0.5x^3\right]_{-1}^{X}\\&=0.5X^3+0.5\end{aligned}$$

Therefore, the equation is,

$$0.5X^3+0.5=U \text{ gives } X=(2U-1)^{1/3}$$

This produces the random variable *X* from a standard uniform random variable.

Now compute *X* if *U* = 0.3972

Using the formula

$$\begin{aligned}X&=(2\times0.3972-1)^{1/3}\\&=\boxed{-0.59}\end{aligned}$$

Step 8/8
(g)

Here the required random variable is a continuous random variable. To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable *U* from a random number generator.

Step 2: Solve the equation $F(X)=U$ for *X*.

Where $F(X)$ is the cdf of the given random variable.

In this problem, it is given by

$$\begin{aligned}F(X) &= \int_0^X f(x)\,dx \\ &= \int_0^X \frac{1}{12}x^{1/3}dx \\ &= \left[\frac{1}{16}x^{4/3}\right]_0^X \\ &= \frac{1}{16}X^{4/3}\end{aligned}$$

Therefore, the equation

$$\frac{1}{16}X^{4/3} = U \text{ gives } X = 8U^{3/4}$$

This produces the random variable *X* from a standard uniform random variable.

Now, compute *X* if *U* = 0.3972

Using the formula

$$\begin{aligned}X &= 8\times 0.3972^{3/4} \\ &= \boxed{4.00}\end{aligned}$$

Chapter 5, Problem 3E
(0)

Problem
Explain how one can generate a random variable *X* that has a pdf

$$f(x) = \begin{cases} \frac{1}{2}(1+x) & \text{if } -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases},$$

given a computer-generated Standard Uniform variable *U*. Generate *X* using Table A1.

Step-by-step solution

Show all steps

75% (4 ratings) for this solution

Step 1/3

Here the required random variable is a continuous random variable. To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable U from a random number generator.

Step 2: Solve the equation $F(X) = U$ for X.

Where $F(X)$ is the cdf of the given random variable *X*

Step 2/3

In this problem, it is given by

$$
\begin{aligned}
F(X) &= \int_{-1}^{X} f(x)\,dx \\
&= \int_{-1}^{X} \frac{1}{2}(1+x)\,dx \\
&= \left[\frac{1}{4}(1+x)^2\right]_{-1}^{X} \\
&= \frac{1}{4}(1+x)^2
\end{aligned}
$$

Therefore, the obtained equation is,

$$\frac{1}{4}(1+X)^2 = U \text{ gives } X = 2\sqrt{U} - 1$$

This produces the random variable *X* from a standard uniform random variable.

Step 3/3

Now using Table A1, select a random number say 0.1901.

So, *U* = 0.1901

Using the above formula, the generated value of *X* is,

$$
\begin{aligned}
X &= 2\sqrt{0.1901} - 1 \\
&= \boxed{-0.128}
\end{aligned}
$$

Chapter 5, Problem 4E
(0)

Problem
To evaluate the system parameters, one uses Monte Carlo methodology and simulates one of its vital characteristics *X*, a continuous random variable with the density

$$f(x) = \begin{cases} \frac{2}{9}(1+x) & \text{if } -1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Explain how one can generate *X*, given a computer-generated Standard Uniform variable *U*. If *U* = 0.2396, compute *X*.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/3
To generate it, use the inverse transformation method. The algorithm for generating the random variable is shown below:

Step 1: Obtain a Standard Uniform random variable U from a random number generator.

Step 2: Solve the equation $F(X)=U$ for X.

Where $F(X)$ is the cdf of the given random variable *X*

Step 2/3
In this problem, it is given by

$$\begin{aligned} F(X) &= \int_{-1}^{X} f(x)\,dx \\ &= \int_{-1}^{X} \frac{2}{9}(1+x)\,dx \\ &= \left[\frac{1}{9}(1+x)^2\right]_{-1}^{X} \\ &= \frac{1}{9}(1+X)^2 \end{aligned}$$

Therefore, the obtained equation is,

$$\frac{1}{9}(1+X)^2 = U \text{ gives } X = 3\sqrt{U} - 1$$

This produces the random variable *X* from a standard uniform random variable.

Step 3/3
Now, compute *X* if *U* = 0.2396

Using the formula

$$\begin{aligned} X &= 3\sqrt{U} - 1 \\ &= 3\sqrt{0.2396} - 1 \\ &= \boxed{0.468} \end{aligned}$$

Chapter 5, Problem 5E
(0)

Problem
Give an expression that transforms a Standard Uniform variable *U* into a variable *X* with the following density,

$$f(x) = \frac{1}{3}x^2, \quad -1 < x < 2.$$

Compute *X* if a computer returns a value of *U* = 0.8.

Step-by-step solution

Show all steps

100% (3 ratings) for this solution

Step 1/2
The expression that transforms a standard uniform variable U into X can be obtained by solving the equation $F(X) = U$ for X.

Where $F(X)$ is the cdf of the given random variable *X*

In this problem, it is given by

$$
\begin{aligned}
F(X) &= \int_{-1}^{X} f(x)\,dx \\
&= \int_{-1}^{X} \frac{1}{3}x^2\,dx \\
&= \left[\frac{1}{9}x^3\right]_{-1}^{X} \\
&= \frac{x^3}{9} + \frac{1}{9}
\end{aligned}
$$

Therefore, the obtained equation is,

$$\frac{x^3}{9} + \frac{1}{9} = U$$

gives

$$\boxed{X = (9U-1)^{1/3}}$$

This expression transforms a standard uniform random variable into X.

Step 2/2
For U = 0.8, the value of X computed to be,

$$
\begin{aligned}
X &= (9U-1)^{1/3} \\
&= (9\times 0.8-1)^{1/3} \\
&= \boxed{1.837}
\end{aligned}
$$

Chapter 5, Problem 6E
(0)

Problem
Two mechanics are changing oil filters for the arrived customers. The service time has an Exponential distribution with the parameter $\lambda = 5\ \text{hrs}^{-1}$ for the first mechanic, and $\lambda = 20\ \text{hrs}^{-1}$ for the second mechanic. Since the second mechanic works faster, he is serving 4 times more customers than his partner. Therefore, when you arrive to have your oil filter changed, your probability of being served by the faster mechanic is 4/5. Let *X* be your service time. Explain how to generate the random variable *X*.

Step-by-step solution

Show all steps

Step 1/2
To generate the random variable *X*, first get the density function and then cumulative density function (cdf) of *X*.

The density function of service time for the first mechanic is,

$$f_1(x) = 5e^{-5x} \quad 0 \le x < \infty$$

The density function of service time for the second mechanic is,

$$f_2(x) = 20e^{-20x} \quad 0 \le x < \infty$$

A customer follows the service time density of the first mechanic with probability 1/5 and service time density of the second mechanic with probability 4/5. Therefore the service time density of the customer is given by

$$f(x) = \frac{1}{5}\left[5e^{-5x}\right] + \frac{4}{5}\left[20e^{-20x}\right]$$

$$or$$

$$f(x) = e^{-5x} + 16e^{-20x} \quad 0 \le x < \infty$$

Step 2/2
Therefore the cdf *F*(*X*) of the customer service time is given by

$$\begin{aligned} F(X) &= \int_0^X f(x)\,dx \\ &= \int_0^X \left(e^{-5x} + 16e^{-20x}\right)dx \\ &= \left[\frac{1}{5} - \frac{1}{5}e^{-5X}\right] + 16\left[\frac{1}{20} - \frac{1}{20}e^{-20X}\right] \\ &= 1 - \frac{1}{5}e^{-5X} - \frac{4}{5}e^{-20X} \end{aligned}$$

Here *F*(*x*) has a complicated form. But the density *f*(*x*) of *X* is available. So random variables with this density can be generated by rejection method described in Algorithm 5.4 of the chapter.

Chapter 5, Problem 7E
(0)

Problem
Explain how to estimate the following probabilities.

(a) ***P***{*X* > *Y*}, where *X* and *Y* are independent Poisson random variables with parameters 3 and 5, respectively.

(b) The probability of a royal-flush (a ten, a jack, a queen, a king, and an ace of the same suit) in poker, if 5 cards are selected at random from a deck of 52 cards.

(c) The probability that it will take more than 35 minutes to have your oil filter changed in Exercise 5.6.

(d) With probability 0.95, we need to estimate each of the probabilities listed in (a-c) with a margin of error not exceeding 0.005. What should be the size of our Monte Carlo study in each case?

(e) (COMPUTER MINI-PROJECT) Conduct a Monte Carlo study and estimate probabilities (a-c) with an error not exceeding 0.005 with probability 0.95.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/5
(a)

The pmf of X and Y are $\frac{e^{-3}3^x}{x!}$ and $\frac{e^{-5}5^y}{y!}$ respectively. So using these formulae, find the probability $P(X>Y)$ as

$$P(X>Y)=\begin{pmatrix} P(X=1,Y=0)+P(X=2,Y=0)+P(X=2,Y=1)+\ldots \\ +P(X=k,Y=0)+P(X=k,Y=1)+\ldots+P(X=k,Y=k-1)+\ldots \end{pmatrix}$$

or,

$$P(X>Y)=\sum_{k=1}^{\infty}\frac{e^{-3}3^k}{k!}\left[\sum_{x=0}^{k-1}\frac{e^{-5}5^x}{x!}\right]$$

This is a complicated formula

Step 2/5
(b)

If we arrange the 5-card hand from highest card to lowest, the first card will be an ace. There are four possible suits for the ace. After that, the other four cards are completely determined. Thus, there are 4 possible royal flushes.

While total possible combinations of 5 cards are

$$= {}^{52}C_5$$
$$= 2598960$$

So the probability of royal flushes

$$\text{Prob} = \frac{4}{2598960}$$
$$= \frac{1}{649740}$$

Step 3/5
(c)

The probability that it will take more than 35 minutes to have oil filter changed is given by

$$P(x > 35) = \int_{35}^{\infty} f(x)\,dx$$
$$= \int_{35}^{\infty} \left(e^{-5x} + 16e^{-20x}\right) dx$$
$$= \frac{1}{5}e^{-175} + \frac{4}{5}e^{-700}$$

Step 4/5
(d)

To calculate the above probabilities, the size of a Monte Carlo Study is given by the formula:

$$N \geq 0.25\left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2$$

Here $\alpha = 0.05$ if we take probability 0.95 of estimation.

And $z_{\alpha/2} = 1.96$ by SNV table.

The margin of error $\varepsilon = 0.005$

Substituting all the values in the above formula, we get the sufficient Monte Carlo study size as

$\boxed{N = 38416}$

Step 5/5
(e)

To estimate the above probabilities with given precision using Monte Carlo method, we will repeat the experiment 38416 times and record the favorable outcomes of the said event. If no. of favorable outcomes is O_e then the estimated probability will be

$$\text{Prob.} = \frac{O_e}{38416}$$

Chapter 5, Problem 8E
(0)

Problem
(COMPUTER MINI-PROJECT) Area of a unit circle equals *π*. Cover a circle with a 2 by 2 square and follow Algorithm 5.5 to estimate number *π* based on 100, 1,000, and 10,000 random numbers. Compare results with the exact value $\pi = 3.14159265358\ldots$ and comment on precision.

Step-by-step solution

Show all steps

Step 1/2
Using algorithm 5.5 of the chapter, we can estimate the value of π as per the following procedure:

Step 1: Obtain a large even number of independent Standard Uniform variables from a random number generator, call them $U_1, \ldots U_n$; $V_1, \ldots, V_n$

Step 2: Count the pairs (U_i, V_i) such that $U_i^2 + V_i^2 \le 1$ (The coordinates lies within the circle of unit radius with this condition. Let this count is n_c

Step 3: The ratio $\frac{n_c}{n}$ will give the area of the positive quadrant of the circle of unit radius. So we can estimate the value of π by multiplying the above area by 4.

Step 2/2
One illustration with 100, 1000, 10000 random numbers is shown in the following table:

No. of random number pairs (n)	n_c	area of quadrant $= \frac{n_c}{n}$	estimated value of π
100	80	0.8	3.2
1000	783	0.783	3.132
10000	7852	0.7852	3.1408

The result approaches to exact value of π as we increase no. of random numbers. So precision depends on number of trials.

Chapter 5, Problem 13E

(0)

Problem

Let *F* be a continuous cdf, and *U* be a Standard Uniform random variable. Show that random variable *X* obtained from *U* via a formula $X = F^{-1}(U)$ has cdf *F*.

Step-by-step solution

Show all steps

Step 1/1

Let F(x) be the cdf of the random variable *X*. Let U be the standard uniform random variable. Let $F(X) = U$

The values of U lie in [0, 1].

By the definition of cdf, $0 \le F(x) \le 1$ for all values of *x*. Now find the cdf of U as

$$\begin{aligned} F_U(u) &= P(U \le u) \\ &= P(F(X) \le u) \\ &= P(X \le F^{-1}(u)) \end{aligned}$$

The above equation shows that to find the cdf of *X* is equivalent to find the cdf of U by making the transformation $X = F^{-1}(U)$.

This shows the result.

Problem

Show that Algorithms 5.1 and 5.3 produce the same discrete variable *X* if they are based on the same value of a Uniform variable *U* and values x_i in Algorithm 5.1 are arranged in the increasing order, $x_0 < x_1 < x_2 < \dots$.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/2

As we know that both algorithms are used for generating discrete random variables.
The steps of Algorithm 5.1 are

Step 1: Divide the interval [0, 1] into subintervals,

$A_0 = [0, p_0)$

$A_1 = [p_0, p_0 + p_1)$

$A_2 = [p_0 + p_1, p_0 + p_1 + p_2)$ etc.

Subinterval A_i will have length p_i; there may be a finite or infinite number of them, according to possible values of X.

Step 2: Obtain a Standard Uniform random variable from a random number generator or a table of random numbers.

Step 3: If U belongs to A_i, Then $X = x_i$.

On the other hand the steps of Algorithm 5.3 are

Step 1: Obtain a Standard Uniform random variable from a random number generator or a table of random numbers.

Step 2: Compute X = min {x ∈ S such that F(x) > U}, where S is a set of possible values of X.

Step 2/2

Let algorithm generates values $x_0, x_1, x_2, \dots$ such that $x_0 < x_1 < x_2 \dots$

And let value x_i is generated with the value u_i of the uniform random variable U.

Then

$$u_i \in \left(p_0 + p_1 + \dots + p_{i-1},\ p_0 + p_1 + \dots + p_i\right)$$
$$or\ u_i \in \left(F\left(x_{i-1}\right),\ F\left(x_i\right)\right)$$

That is, x_i is the minimum value for which $F(x_i) > u_i$

Which is the condition given in step 2 of the Algorithm 5.3

Chapter 5, Problem 15E
(0)

Problem
Prove that the Box-Muller transformation

$$\begin{aligned} Z_1 &= \sqrt{-2\ln(U_1)}\cos(2\pi U_2) \\ Z_2 &= \sqrt{-2\ln(U_1)}\sin(2\pi U_2) \end{aligned}$$

returns a pair of independent Standard Normal random variables by showing equality ***P* {*Z*$_1$ ≤ *a* ∩ *Z*$_2$ ≤ *b*} = Φ(*a*)Φ(*b*)** for all *a* and *b*.

(This requires a substitution of variables in a double integral.)

Step-by-step solution

Show all steps

Step 1/3
From the information, consider the Box-Muller transformations as follows:

$$Z_1 = \sqrt{-2\ln(U_1)}\cos(2\pi U_2)$$
$$Z_2 = \sqrt{-2\ln(U_1)}\sin(2\pi U_2)$$

Using the inverse substitution statement, by taking double integral of independent standard normal variable.

After suitable transformation, it will yield U_1 and U_2 follows Uniform distribution.

The joint density functions of two independent standard normal variates *X* and *Y* as follows:

$$f(x,y) = \frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}$$

Step 2/3
The probability that the point (*x*, *y*) belongs to the region of radius R is,

$$P(R) = \frac{1}{2\pi}\int_{x^2+y^2\le R^2} e^{-\frac{x^2+y^2}{2}}\,dxdy$$

Now, making the substitution $x^2 + y^2 = r^2$ and $\tan^{-1}\frac{y}{x} = \theta$

Therefore, $x = r\cos\theta,\ y = r\sin\theta$

And

$$P(R) = \frac{1}{2\pi}\int_0^{2\pi} d\theta \int_0^R re^{-r^2/2}dr$$
$$= 1 - e^{-R^2/2}$$

If $P(R) = p$ then $1 - e^{-R^2/2} = p$ or $R = \sqrt{-2\ln(1-p)}$

Step 3/3

If, set $1 - p = U_1 \in [0,1]$ and $\theta = 2\pi U_2,\ U_2 \in [0,1]$

The independent standard normal variables *X* & *Y* are transformed into,

$$Z_1 = \sqrt{-2\ln(U_1)}\cos(2\pi U_2)$$
$$Z_2 = \sqrt{-2\ln(U_1)}\sin(2\pi U_2)$$

Chapter 5, Problem 16E

(0)

Problem

A Monte Carlo study is being designed to estimate some probability *p* by a long-run proportion $\hat{p}.$ Suppose that condition (5.6) can be satisfied by *N* that is too small to allow the Normal approximation of the distribution of $\hat{p}.$ Use Chebyshev's inequality instead. Show that the size *N* computed according to (5.7) or (5.8) satisfies the required condition (5.6).

Step-by-step solution

Show all steps

Step 1/1

To design a study that attains desired accuracy. That is we can choose some small e and a and conduct a study of such a size N that will guarantee an error not exceeding e with high probability (1 - a). In other words, we can find such N that

$P\{|\hat{p} - p| > \varepsilon\} \le \alpha$ --- (1)

Now we can show that N computed using Chebyshev's inequality satisfies the above required condition.

Chebyshev's inequality shows

$$P\{|\hat{p} - p| > \varepsilon\} \le \left(\frac{\sigma}{\varepsilon}\right)^2$$ --- (2)

From (1) & (2),

$$\left(\frac{\sigma}{\varepsilon}\right)^2 \le \alpha$$

or, $\sigma^2 \le \alpha\varepsilon^2$

or, $\dfrac{p^*(1-p^*)}{N} \le \alpha\varepsilon^2$

or, $N \ge \dfrac{p^*(1-p^*)}{\alpha\varepsilon^2}$

So we can see that results obtained by Chebyshev's inequality satisfy the required condition.

Chapter 5, Problem 17E
(0)

Problem
A random variable is generated from a density *f*(*x*) by rejection method. For this purpose, a bounding box with $a \le x \le b$ and $0 \le y \le c$ is selected. Show that this method requires a Geometric(*p*) number of generated pairs of Standard Uniform random variables and find *p*.

Step-by-step solution

Show all steps

Step 1/2
Let the generated random variable have the density f(x). First study the algorithm of the rejection method, which is shown below:

Step 1: Find such numbers a, b, and c that 0 ≤ f(x) ≤ c for a ≤ x ≤ b. The bounding box stretches along the x-axis from a to b and along the y-axis from 0 to c.

Step 2: Obtain Standard Uniform random variables U and V from a random number generator or a table of random numbers.

Step 3: Define X = a + (b − a) U and Y = cV. Then X has Uniform (a, b) distribution, Y is Uniform (0, c), and the point (X, Y) is uniformly distributed in the bounding box.

Step 4: If Y > f(X), reject the point and return to step 2. If Y ≤ f(X), then X is the desired random variable having the density f(x).

Step 2/2

From the above algorithm, see that there is a probability of getting *X* in each recurring step, otherwise continue next iteration. The probability of success (getting *X*) in each iteration is a constant (say p). Then the number of iterations for getting success is a random variable and follows a geometric distribution. And the probability of success is given by,

$$\begin{aligned} p &= P\left(Y \le f\left(X\right)\right) \\ &= P\left(cV \le f\left(a + \overline{b-a}U\right)\right) \\ &= P\left(1 \le \frac{f\left(a+\overline{b-a}U\right)}{cV}\right) \\ &= \boxed{\frac{1}{\left(b-a\right)c}} \end{aligned}$$

Problem

We estimate an integral

$$\mathscr{I} = \int_a^b g(x)dx$$

for arbitrary *a* and *b* and a function 0 ≤ *g*(*x*) ≤ *c* using both Monte Carlo integration methods introduced in this chapter. First, we generate *N* pairs of Uniform variables (U_i, V_i), $a \le U_i \le b$, $0 \le V_i \le c$, and estimate $\mathscr{I}$ by the properly rescaled proportion of pairs with $V_i \le g(U_i)$. Second, we generate only U_i and estimate $\mathscr{I}$ by the average value of $(b - a)g(U_i)$. Show that the second method produces more accurate results.

Step-by-step solution

Show all steps

Step 1/3

From the information, observe that integral equation as follows:

$$I = \int_a^b g(x)\,dx$$

For the range $0 \le g(x) \le c$, In the first method, generate *N* (sufficiently large) pairs of Uniform variables (U_i, V_i), $a \le U_i \le b$ and $0 \le V_i \le c$, then estimator of the above integral $\hat{I}$ can be obtained by taking average of all 1's for which (U_i, V_i) pair satisfies $V_i \le g(U_i)$

The standard deviation of the estimate of *I* is,

$$Std(\hat{I}) = \sqrt{\frac{I(1-I)}{N}}$$

Here, *I* is the actual value of the integration.

Step 2/3

While in the second method, estimate the above integration by averaging $(b-a)g(X_i)$ for some large number of Uniform variables $X_1, X_2, ..., X_N$

In this case the standard deviation of the estimate of *I* is,

$$Std(\hat{I}) = \frac{\sigma}{\sqrt{N}}$$

Here, σ is the standard deviation of the random variable *R* is, $R = \frac{(b-a)g(X)}{U(X)}$

Step 3/3

For a Uniform $U(a,b)$ variable *X*, we have $U(X) = b-a$, so that

$$\begin{aligned}
\sigma^2 &= Var(R) \\
&= Var(g(X)) \\
&= E(g^2(X)) - [E(g(X))]^2 \\
&= \int_a^b g^2(X)\,dx - I^2 \\
&\le I - I^2 \\
&\le I(1-I)
\end{aligned}$$

Therefore, the second method produces more accurate results.

roblem
A small computer lab has 2 terminals. The number of students working in this lab is recorded at the end of every hour. A computer assistant notices the following pattern:

• If there are 0 or 1 students in a lab, then the number of students in 1 hour has a 50-50% chance to increase by 1 or remain unchanged.

• If there are 2 students in a lab, then the number of students in 1 hour has a 50-50% chance to decrease by 1 or remain unchanged.

(a) Write the transition probability matrix for this Markov chain.

(b) Is this a regular Markov chain? Justify your answer.

(c) Suppose there is nobody in the lab at 7 am. What is the probability of nobody working in the lab at 10 am?

Step-by-step solution

Show all steps

Step 1/6
At a small computer lab there are two terminals. For this study, the number of students working in this lab is recorded at the end of every hour.

a)

Construct the transition probability matrix for the given Markov chain.

The given pattern is as follows:

In this problem if there are 0 or 1 students then the number of students in one hour has a $50-50\%$ chance to increase by one or remain unchanged it implies that $p_{00} = 0.5$, $p_{01} = 0.5$, $p_{11} = 0.5$ and $p_{12} = 0.5$

Similarly if there are two students then the number of students in one hour has a $50-50\%$ chance to decrease by one or remain unchanged it implies that $p_{22} = 0.5$ and $p_{21} = 0.5$

Transition probability matrix for this Markov chain is given by,

$$P = \begin{pmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{12} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{pmatrix}$$

Step 2/6
By using the given pattern, the transition probability matrix for the given Markov chain is given by,

Step 3/6

$$P=\begin{pmatrix}0.5 & 0.5 & 0\\ 0 & 0.5 & 0.5\\ 0 & 0.5 & 0.5\end{pmatrix}$$

Step 4/6

b)

Check whether the constructed transition probability matrix is regular or not.

A Markov chain is said to be regular, if $p_{ij}^{(h)} > 0$, for some h and for all i, j .

Step 5/6

In this problem the row sum of every row is 1. So, there exist finite h such that P^h has every non-zero entries in this matrix, hence this transition probability matrix is regular.

Step 6/6

(c)

Compute the probability of nobody working in the lab at 10 am, given that nobody in the lab at 7 am.

Here, compute P^3 and the first element of the matrix P^3 gives the required probability.

$$\begin{aligned}P^3 &= P\times P\times P\\ &= \begin{pmatrix}0.5 & 0.5 & 0\\ 0 & 0.5 & 0.5\\ 0 & 0.5 & 0.5\end{pmatrix}\times\begin{pmatrix}0.5 & 0.5 & 0\\ 0 & 0.5 & 0.5\\ 0 & 0.5 & 0.5\end{pmatrix}\times\begin{pmatrix}0.5 & 0.5 & 0\\ 0 & 0.5 & 0.5\\ 0 & 0.5 & 0.5\end{pmatrix}\\ &= \begin{pmatrix}0.25 & 0.50 & 0.25\\ 0 & 0.50 & 0.50\\ 0 & 0.50 & 0.50\end{pmatrix}\times\begin{pmatrix}0.5 & 0.5 & 0\\ 0 & 0.5 & 0.5\\ 0 & 0.5 & 0.5\end{pmatrix}\\ &= \begin{pmatrix}0.125 & 0.50 & 0.375\\ 0 & 0.50 & 0.50\\ 0 & 0.50 & 0.50\end{pmatrix}\end{aligned}$$

First element of matrix P^3 is 0.125.

So, the required probability that nobody is working at 10 am given that nobody in the lab at 7 am is **0.125**.

Chapter 6, Problem 2E
(1)

Problem
A computer system can operate in two different modes. Every hour, it remains in the same mode or switches to a different mode according to the transition probability matrix.

(a) Compute the 2-step transition probability matrix.

(b) If the system is in Mode I at 5:30 pm, what is the probability that it will be in Mode I at 8:30 pm on the same day?

Step-by-step solution

Show all steps

100% (9 ratings) for this solution

Step 1/2
A compute system can operate in two different modes according to the following transition probability matrix $P = \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix}$

a)

Compute the 2-step transition probability matrix. That is, compute P^2.

$$\begin{aligned} P^2 &= P \times P \\ &= \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix} \times \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix} \\ &= \begin{pmatrix} 0.52 & 0.48 \\ 0.48 & 0.52 \end{pmatrix} \end{aligned}$$

Therefore, the 2-step transition probability matrix is $\boxed{P^2 = \begin{pmatrix} 0.52 & 0.48 \\ 0.48 & 0.52 \end{pmatrix}}$.

Step 2/2
b)

If the system is in mode I at 5:30 pm, compute the probability that the system remain in mode I at 8: 30 pm on the same day.

To compute the required probability, compute the 3-step transition probability matrix. That is, compute P^3

$$P^3 = P \times P \times P$$

$$= \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix} \times \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix} \times \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix}$$

$$= \begin{pmatrix} 0.52 & 0.48 \\ 0.48 & 0.52 \end{pmatrix} \times \begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix}$$

$$= \begin{pmatrix} 0.496 & 0.504 \\ 0.504 & 0.496 \end{pmatrix}$$

First entry of 3-step transition matrix is 0.496.

Therefore, the probability that the system remain in mode I at 8: 30 pm on the same day is **0.496**.

Chapter 6, Problem 3E
(0)

Problem
Markov chains find direct applications in genetics. Here is an example.

An offspring of a black dog is black with probability 0.6 and brown with probability 0.4. An offspring of a brown dog is black with probability 0.2 and brown with probability 0.8.

(a) Write the transition probability matrix of this Markov chain.

(b) Rex is a brown dog. Compute the probability that his grandchild is black.

Step-by-step solution

Show all steps

100% (8 ratings) for this solution

Step 1/1
(a)

In this problem the transition probability matrix is

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$$

$$P = \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$

Where p_{00} is the probability of an offspring of black dog is black, p_{01} is the probability of an offspring of black dog is brown, p_{10} is the probability of an offspring of brown dog is black and p_{11} is the probability of an offspring of brown dog is black

(b)

Rex is brown dog the probability that his grand child is black can be calculated by finding the 2-step transition probability matrix and the third entry of 2-step transition matrix gives the required probability

So the two step transition matrix is defined as

$$P^2 = P \times P$$

$$P^2 = \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} \times \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{pmatrix}$$

Third entry of 2-step transition matrix is 0.28. So the probability that his grand child is black is 0.28

Chapter 6, Problem 4E

(2)

Problem

Every day, Eric takes the same street from his home to the university. There are 4 street lights along his way, and Eric has noticed the following Markov dependence. If he sees a green light at an intersection, then 60% of time the next light is also green, and 40% of time the next light is red. However, if he sees a red light, then 70% of time the next light is also red, and 30% of time the next light is green.

(a) Construct the transition probability matrix for the street lights.

(b) If the first light is green, what is the probability that the third light is red?

(c) Eric's classmate Jacob has *many* street lights between his home and the university. If the *first* street light is green, what is the probability that the *last* street light is red? (Use the steady-state distribution.)

Step-by-step solution

Show all steps

100% (10 ratings) for this solution

Step 1/3
(a)

In this problem the transition probability matrix for street lights is

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$$

Where the p_{00} be the probability of green light remains green on next street light, p_{01} be the probability of green light become red on next street light, p_{10} be the probability of red light become green on next street light and p_{11} be the probability of red light remains red on next street light,

So the transition matrix is

$$P = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$$

Step 2/3
(b)

To find the probability of third light is red given that first light is green we need to find the 2-step transition probability matrix and the second entry of 2-step transition matrix gives the required probability.

So the 2-step transition matrix is

$$P^2 = P \times P$$

$$P^2 = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix} \times \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$$

$$= \begin{pmatrix} 0.48 & 0.52 \\ 0.39 & 0.61 \end{pmatrix}$$

Second entry of 2-step transition matrix is 0.52. So the probability that third light is red is 0.52

Step 3/3
(c)

The transition probability matrix of light is regular so the steady state distribution exists.

We will find the steady state distribution to find the probability that the last street light is red given that first light is green

The steady state distribution for this Markov chain is

$$\pi = \pi P$$
$$\sum \pi_x = 1$$

Or

$$0.6\pi_1 + 0.3\pi_2 = \pi_1$$
$$0.4\pi_1 + 0.7\pi_2 = \pi_2$$

$$0.3\pi_2 = 0.4\pi_1$$
$$0.4\pi_1 = 0.3\pi_2$$

$$\pi_1 = \frac{3\pi_2}{4}$$

From the second condition we get $\frac{3\pi_2}{4} + \pi_2 = 1$

$\pi_2 = \frac{4}{7}$ and $\pi_1 = \frac{3}{7}$

So the π_2 gives the probability that light become red and π_1 gives the probability that light remains green

So the probability that last light is red given that first light is green is $\pi_2 = \frac{4}{7}$.

Chapter 6, Problem 5E
(0)

Problem
The pattern of sunny and rainy days on planet Rainbow is a homogeneous Markov chain with two states. Every sunny day is followed by another sunny day with probability 0.8. Every rainy day is followed by another rainy day with probability 0.6. Compute the probability that April 1 next year is rainy on Rainbow.

Step-by-step solution

Show all steps

100% (9 ratings) for this solution

Step 1/2
(a)

In this problem the transition probability matrix is

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$$

Where the p_{00} be the probability of sunny day followed by sunny, p_{01} be the probability of rainy day followed by sunny day, p_{10} be the probability of sunny day followed by rainy day and p_{11} be the probability of rainy day followed by rainy day,

So the transition matrix is

$$P = \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix}$$

Step 2/2
The transition probability matrix is regular so the steady state distribution exists.

We will find the steady state distribution to find the probability that April 1 next year is rainy

The steady state distribution for this Markov chain is

$$\pi = \pi P$$
$$\sum \pi_x = 1$$

Or

$$0.8\pi_1 + 0.4\pi_2 = \pi_1$$
$$0.2\pi_1 + 0.6\pi_2 = \pi_2$$

$$0.4\pi_2 = 0.2\pi_1$$
$$0.2\pi_1 = 0.4\pi_2$$

$$\pi_1 = \frac{4\pi_2}{2}$$

From the second condition we get $\frac{4\pi_2}{2} + \pi_2 = 1$

$\pi_2 = \frac{1}{3}$ and $\pi_1 = \frac{2}{3}$

So the π_2 gives the probability that day is rainy and π_1 gives the probability that day is sunny. So the probability that April 1 next year is rainy is $\pi_2 = \frac{1}{3}$

Chapter 6, Problem 6E
(1)

Problem
A computer device can be either in a busy mode (state 1) processing a task, or in an idle mode (state 2), when there are no tasks to process. Being in a busy mode, it can finish a task and enter an idle mode any minute with the probability 0.2. Thus, with the probability 0.8 it stays another minute in a busy mode. Being in an idle mode, it receives a new task any minute with the probability 0.1 and enters a busy mode. Thus, it stays another minute in an idle mode with the probability 0.9. The initial state X_0 is idle. Let X_n be the state of the device after n minutes.

(a) Find the distribution of X_2.

(b) Find the steady-state distribution of X_n.

Step-by-step solution

Show all steps

100% (8 ratings) for this solution

Step 1/2
(a)

In this problem the transition probability matrix is

$$P = \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}$$

Initial transition probability matrix is

$$P_0 = (0,1)$$

(a)

The distribution of $X_2 = P_0 \times P^2$

The 2-step transition matrix is

$$P^2 = P \times P$$

$$P^2 = \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} \times \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}$$

$$= \begin{pmatrix} 0.66 & 0.34 \\ 0.17 & 0.83 \end{pmatrix}$$

So

$$X_2 = P_0 \times P^2$$

$$= (0,1) \times \begin{pmatrix} 0.66 & 0.34 \\ 0.17 & 0.83 \end{pmatrix}$$

$$= (0.17,\ 0.83)$$

Step 2/2
(b)

The transition probability matrix is regular so the steady state distribution exists.

The steady state distribution for this Markov chain is

$$\pi = \pi P$$
$$\sum \pi_x = 1$$

Or

$$0.8\pi_1 + 0.1\pi_2 = \pi_1$$
$$0.2\pi_1 + 0.9\pi_2 = \pi_2$$

$$0.1\pi_2 = 0.2\pi_1$$
$$0.2\pi_1 = 0.1\pi_2$$

$$\pi_1 = \frac{\pi_2}{2}$$

From the second condition we get $\frac{\pi_2}{2} + \pi_2 = 1$

$\pi_2 = \frac{2}{3}$ and $\pi_1 = \frac{1}{3}$

So steady state distribution is $\pi = (\pi_1, \pi_2) = (0.34,\ 0.66)$

hapter 6, Problem 7E
(0)

Problem
A Markov chain has the transition probability matrix

$$P = \begin{pmatrix} 0.3 & \ldots & 0 \\ 0 & 0 & \ldots \\ 1 & \ldots & \ldots \end{pmatrix}$$

(a) Fill in the blanks.

(b) Show that this is a regular Markov chain.

(c) Compute the steady-state probabilities.

Step-by-step solution

Show all steps

100% (8 ratings) for this solution

Step 1/3
A Markov chain has a transition probability matrix

$$P = \begin{pmatrix} 0.3 & \ldots & 0. \\ 0 & 0 & .. \\ 1 & .. & .. \end{pmatrix}$$

(a)

Fill in the blanks of Transition probability matrix are

$$P = \begin{pmatrix} 0.3 & 0.7 & 0. \\ 0 & 0 & 1 \\ 1 & .0 & 0. \end{pmatrix}$$

Step 2/3
(b)

To show that the transition probability matrix is regular we have to find finite h such that P^h have every entry non zero. So we start powering the transition matrix

First we will find P^2

$$P^2 = \begin{pmatrix} 0.3 & 0.7 & 0. \\ 0 & 0 & 1 \\ 1 & .0 & 0. \end{pmatrix} \times \begin{pmatrix} 0.3 & 0.7 & 0. \\ 0 & 0 & 1 \\ 1 & .0 & 0. \end{pmatrix} = \begin{pmatrix} 0.09 & 0.21 & 0.7 \\ 1 & 0 & 0 \\ 0.3 & 0.7 & 0 \end{pmatrix}$$

P^2 have zero entries so now we will find P^3

$$P^3 = P^2 \times P = \begin{pmatrix} 0.09 & 0.21 & 0.7 \\ 1 & 0 & 0 \\ 0.3 & 0.7 & 0 \end{pmatrix} \times \begin{pmatrix} 0.3 & 0.7 & 0. \\ 0 & 0 & 1 \\ 1 & .0 & 0. \end{pmatrix}$$

$$= \begin{pmatrix} 0.727 & 0.063 & 0.21 \\ 0.3 & 0.7 & 0 \\ 0.09 & 0.21 & 0.7 \end{pmatrix}$$

P^3 have zero entries so now we will find P^4

$$P^4 = P^3 \times P = \begin{pmatrix} 0.727 & 0.063 & 0.21 \\ 0.3 & 0.7 & 0 \\ 0.09 & 0.21 & 0.7 \end{pmatrix} \times \begin{pmatrix} 0.3 & 0.7 & 0. \\ 0 & 0 & 1 \\ 1 & .0 & 0. \end{pmatrix}$$

$$= \begin{pmatrix} 0.4281 & 0.5089 & 0.063 \\ 0.09 & 0.21 & 0.7 \\ 0.727 & 0.063 & 0.21 \end{pmatrix}$$

P^4 have all entries non zero so for $h = 4$ P^h have every entry non zero. So this transition matrix is regular

Step 3/3

(c)

The steady state distribution for this Markov chain is

$$\pi = \pi P$$
$$\sum \pi_x = 1$$

Or

$$0.3\pi_1 + \pi_3 = \pi_1$$
$$0.7\pi_1 = \pi_2$$
$$\pi_2 = \pi_3$$

Or

$$\pi_2 = 0.7\pi_1$$
$$\pi_3 = 0.7\pi_1$$

From second condition of steady state distribution

$\pi_1 + 0.7\pi_1 + 0.7\pi_1 = 1$

From this equation we get $\pi_1 = 0.416$, $\pi_2 = 0.416 \times 0.7 = 0.29167$ and $\pi_3 = 0.416 \times 0.7 = 0.29167$

So steady state distribution is $\pi = (\pi_1, \pi_2, \pi_3) = (0.416,\ 0.29167,\ 0.2917)$

Chapter 6, Problem 8E
(0)

Problem
A Markov chain has 3 possible states: A, B, and C. Every hour, it makes a transition to a *different* state. From state A, transitions to states B and C are equally likely. From state B, transitions to states A and C are equally likely. From state C, it always makes a transition to state A. Find the steady-state distribution of states.

Step-by-step solution

Show all steps

100% (4 ratings) for this solution

Step 1/1
In this problem Markov chain has a transition probability matrix

$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & .0 & 0. \end{pmatrix}$$

The steady state distribution for this Markov chain is

$$\pi = \pi P$$
$$\sum \pi_x = 1$$

Or

$$(\pi_1, \pi_2, \pi_3) = (\pi_1, \pi_2, \pi_3) \times \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & .0 & 0. \end{pmatrix}$$

Or

$$0.5\pi_1 + \pi_3 = \pi_1$$
$$0.5\pi_1 = \pi_2$$
$$0.5\pi_1 + 0.5\pi_2 = \pi_3$$

Or

$$\pi_2 = 0.5\pi_1$$
$$\pi_3 = 0.75\pi_1$$

From second condition of steady state distribution

$$\pi_1 + 0.5\pi_1 + 0.75\pi_1 = 1$$

From this equation we get $\pi_1 = 0.444$, $\pi_2 = 0.444 \times 0.5 = 0.222$ and $\pi_3 = 0.444 \times 0.75 = 0.333$

So steady state distribution is $\pi = (\pi_1, \pi_2, \pi_3) = (0.4444,\ 0.2222,\ 0.3333)$

Chapter 6, Problem 9E

(0)

Problem

Tasks are sent to a supercomputer at an average rate of 6 tasks per minute. Their arrivals are modeled by a Binomial counting process with 2-second frames.

(a) Compute the probability of more than 2 tasks sent during 10 seconds.

(b) Compute the probability of more than 20 tasks sent during 100 seconds. You may use a suitable approximation.

Step-by-step solution

Show all steps

80% (5 ratings) for this solution

Step 1/2

In this problem the rate at which the task is sent to supercomputer is $\lambda = 6$ per minute

So $\lambda = 0.1$ per second.

Frame size is $\Delta = 2\sec$

Probability of success in this problem is $p = \lambda \times \Delta = 2 \times 0.1$

$$= 0.20$$

Probability of success in this problem is $q = 1 - p = 0.8$

(a)

We need to find the probability of more than two task in 10 seconds.

Total number of event in this case is $n = \frac{t}{\Delta} = \frac{10}{2} = 5$

This process follow binomial process with $n = 5$ and $p = 0.20$

So

$$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 10 \times (0.2)^3 \times (0.8)^2 + 5 \times (0.2)^4 \times (0.8) \quad + (0.2)^5$$

$$= 0.05792$$

Step 2/2

(b)

We need to find the probability of more than 20 task in 100 seconds

Total number of event in this case is $n = \frac{t}{\lambda} = \frac{100}{2} = 50$

We use normal approximation to Binomial distribution

$$P(X > 20) = P\left(\frac{X - np}{\sqrt{np(1-p)}} > \frac{20 - 50 \times 0.20}{\sqrt{50 \times 0.20 \times 0.80}}\right)$$

$$= P\left(\frac{X - np}{\sqrt{np(1-p)}} > \frac{20 - 50 \times 0.20}{\sqrt{50 \times 0.20 \times 0.80}}\right)$$

$$= P(Z > 3.536)$$

$$= 0.0001$$

Chapter 6, Problem 10E

(0)

Problem
The number of electronic messages received by an interactive message center is modeled by Binomial counting process with 15- second frames. The average arrival rate is 1 message per minute. Compute the probability of receiving more than 3 messages during a 2-minute interval.

Step-by-step solution

Show all steps

Step 1/1
In this problem the rate at which the electronic message received is $\lambda = 1$ per minute

Frame size is $\Delta = 15\sec$

Probability of success in this problem is $p = \lambda \times \Delta = 1 \times \frac{1}{4}$

$= 0.25$

Probability of success in this problem is $q = 1 - p = 0.75$

We need to find the probability of more than three message in 2 minutes

Total number of event in this case is $n = \frac{t}{\Delta} = \frac{2}{1/4} = 8$

This process follow binomial process with $n = 8$ and $p = 0.25$

So

$P(X > 3) = 1 - P(X \le 3)$

$= 1 - P(X = 0) + P(X = 1) + P(X = 2) + P(X = 2)$

$= 1 - \times (0.75)^8 \times 8 \times (0.25)^1 \times (0.75)^7$

$+ 21 \times (0.25)^2 \times (0.75)^6 + 35 \times (0.25)^3 \times (0.75)^5$

$= 0.114$

Chapter 6, Problem 11E
(0)

Problem
Jobs are sent to a printer at the average rate of 2 jobs per minute. Binomial counting process is used to model these jobs.

(a) What frame length Δ gives the probability 0.1 of an arrival during any given frame?

(b) With this value of Δ, compute the expectation and standard deviation for the number of jobs sent to the printer during a 1-hour period.

Step-by-step solution

Show all steps

Step 1/2
In this problem the rate at which the jobs are sent is $\lambda = 2$ per minute

(a)

The frame length Δ which gives the probability 0.1 of an arrival during any given frame is

$$\Delta = \frac{p}{\lambda} = \frac{0.1}{2}$$

$$= 0.05 \min = 3 \sec$$

Step 2/2
(b)

For 1 hour period total number of event in this case is $n = \frac{t}{\Delta} = \frac{60}{0.05} = 1200$

Expected number of jobs sent to printer in this period is $E(X) = np = 1200 \times 0.1 = 120$

Standard deviation of jobs sent to period in this period is $S = \sqrt{np(1-q)} = 10.39$

Chapter 6, Problem 12E
(0)

Problem
On the average, 2 airplanes per minute land at a certain international airport. We would like to model the number of landings by a Binomial counting process.

(a) What frame length should one use to guarantee that the probability of a landing during any frame does not exceed 0.1?

(b) Using the chosen frames, compute the probability of no landings during the next 5 minutes.

(c) Using the chosen frames, compute the probability of more than 100 landed airplanes during the next hour.

Step-by-step solution

Show all steps

86% (7 ratings) for this solution

Step 1/3

In this problem the rate at which the plane land at a certain international airport is $\lambda = 2$ per minute

(a)

The frame length Δ which gives the probability does not exceed 0.1 of landing during any given frame is $\Delta = \frac{p}{\lambda} = \frac{0.1}{2}$

$= 0.05\,\text{min} = 3\,\text{sec}$

Step 2/3

(b)

We need to calculate the probability of no landing during next five minutes.

In this case the total number of events are $n = \frac{t}{\Delta} = \frac{5}{0.05} = 100$

The number of landings during this time is binomial

So

$P(X = 0) = 1 \times (0.9)^{100} = 0.00002656$

So the probability of no landing during next five minutes is 0.00002656

Step 3/3

(c)

We need to calculate the probability of more than 100 landing during next hour.

In this case the total number of events are $n = \frac{t}{\Delta} = \frac{60}{0.05} = 1200$

We use normal approximation to Binomial distribution

$$P(X > 100) = P\left(\frac{X - np}{\sqrt{np(1-p)}} > \frac{100 - 1200 \times 0.1}{\sqrt{1200 \times 0.10 \times 0.90}}\right)$$

$$= P(Z > -1.925)$$

$$= 0.9732$$

Chapter 6, Problem 13E
(0)

Problem
On the average, every 12 seconds a customer makes a call using a certain phone card. Calls are modeled by a Binomial counting process with 2-second frames. Find the mean and the variance for the time, in seconds, between two consecutive calls.

Step-by-step solution

Show all steps

100% (3 ratings) for this solution

Step 1/1
In this problem the rate at which the customer make a call using a certain phone card is $\lambda = \frac{1}{12}$ per second

Frame size is $\Delta = 2\sec$

Probability of success in this problem is $p = \lambda \times \Delta = \frac{1}{6}$

Mean for the time is $E(T) = \frac{1}{\lambda} = 12\sec$

Variance for time is $V(T) = \frac{1-p}{\lambda^2}$

$$= \frac{1-\frac{1}{6}}{\left(\frac{1}{12}\right)^2} = 120$$

Chapter 6, Problem 14E
(0)

Problem
Customers of a certain internet service provider connect to the internet at the average rate of 3 customers per minute. Assuming Binomial counting process with 5-second

frames, compute the probability of more than 10 new connections during the next 3 minutes. Compute the mean and the standard deviation of the number of seconds between connections.

Step-by-step solution

Show all steps

Step 1/2

In this problem the rate at which the customer of a certain internet service provider connect to the internet is $\lambda = 3$ per minute

Frame size is $\Delta = 5\,\text{sec}$

Probability of success in this problem is $p = \lambda \times \Delta = \frac{1}{4}$

We need to find the probability of more than 10 new connection in 3 minutes

Total number of event in this case is $n = \frac{t}{\Delta} = \frac{3}{5/60} = 36$

We use normal approximation to Binomial distribution

$$P(X > 10) = P\left(\frac{X - np}{\sqrt{np(1-p)}} > \frac{10 - 36 \times 0.25}{\sqrt{36 \times 0.25 \times 0.75}}\right)$$

$$= P(Z > 0.384)$$

$$= 0.352$$

Step 2/2

Mean for the time is $E(T) = \frac{1}{\lambda} = \frac{1}{3}\text{min}$

Variance for time is $V(T) = \frac{1-p}{\lambda^2}$

$$= \frac{1 - \frac{1}{4}}{(3)^2} = \frac{1}{12}$$

Standard deviation for time is $S = \sqrt{V(T)} = \sqrt{\frac{1}{12}} = 0.2886$

Chapter 6, Problem 15E
(0)

Problem
Customers of an internet service provider connect to the internet at the average rate of 12 new connections per minute. Connections are modeled by a Binomial counting process.

(a) What frame length Δ gives the probability 0.15 of an arrival during any given frame?

(b) With this value of Δ, compute the expectation and standard deviation for the number of seconds between two consecutive connections.

Step-by-step solution

Show all steps

Step 1/2
In this problem the rate at which the customer of a certain internet service provider connect to the internet is $\lambda = 12$ per minute

(a)

The frame length Δ which gives the probability 0.15 of an arrival during any given frame is $\Delta = \frac{p}{\lambda} = \frac{0.15}{12}$

$= 0.0125 \min = 0.75 \sec$

Step 2/2
(b)

Probability of success in this problem is $p = 0.15$

Mean for the time is $E(T) = \frac{1}{\lambda} = \frac{1}{12} \min$

Variance for time is $V(T) = \frac{1-p}{\lambda^2}$

$= \frac{1-0.15}{(12)^2} = 0.005903$

Standard deviation for time is $S = \sqrt{V(T)} = 0.0768$

Chapter 6, Problem 16E
(0)

Problem
Messages arrive at a transmission center according to a Binomial counting process with 30 frames per minute. The average arrival rate is 40 messages per hour. Compute the mean and standard deviation of the number of messages arrived between 10 am and 10:30 am.

Step-by-step solution

Show all steps

Step 1/1

In this problem the rate at which the message is arrived at transmission center is $\lambda = \frac{4}{6}$ per minute

The frame length $\Delta = 0.033\,\text{min}$

Probability of success in this problem is $p = \Delta \times \lambda = 0.022$

We need to find the mean and standard deviation of message arrived between $10am$ and $10:30am$

Total number of events in this case is $n = \frac{t}{\Delta} = \frac{30}{0.033} = 900$

Mean for the message arrives during this time period is $E(X) = n \times p = 900 \times 0.022 = 19.8$

Variance for message arrives during this time period is $V(X) = n \times p \times (1-p)$

$= 900 \times 0.022 \times 0.978 = 19.36$

Standard deviation for message arrives during this time period is $S = \sqrt{V(T)} = 4.4$

Chapter 6, Problem 17E
(2)

Problem
Messages arrive at an electronic message center at random times, with an average of 9 messages per hour.

(a) What is the probability of receiving at least five messages during the next hour?

(b) What is the probability of receiving exactly five messages during the next hour?

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/2
In this problem the rate at which message is arrived at electronic message center is $\lambda = 9$ per hour

(a)

We need to find the probability of at least 5 messages during the next hour

The messages at electronic message center occurring according to Poisson process.

The parameter of this process is $\mu = \lambda \times t = 9 \times 1 = 9$

So

$$P(X(1) \geq 5) = 1 - P(X(1) < 5)$$

$$P(X(1) \geq 5) = 1 - \left(\frac{e^{-9}9^0}{0!} + \frac{e^{-9}9^1}{1!} + \frac{e^{-9}9^2}{2!} + \frac{e^{-9}9^3}{3!} + \frac{e^{-9}9^4}{4!} \right)$$

$$P(X(1) \geq 5) = 0.945$$

Step 2/2
(b)

We need to find the probability of exactly 5 messages during the next hour

The messages at electronic message center occurring according to Poisson process.

The parameter of this process is $\mu = \lambda \times t = 9 \times 1 = 9$

So

$$P(X(1) = 5) = \left(\frac{e^{-9}9^5}{5!} \right)$$

$$P(X(1) = 5) = 0.061$$

Chapter 6, Problem 18E
(0)

Problem
Messages arrive at an interactive message center according to a Binomial counting process with the average interarrival time of 15 seconds. Choosing a frame size of 5 seconds, compute the probability that during 200 minutes of operation, no more than 750 messages arrive.

Step-by-step solution

Show all steps

100% (4 ratings) for this solution

Step 1/4
From the information, observe that the messages at an interactive message centre according to a counting process with the average inter-arrival time of 15 seconds.

The average Interarrival time is, $\lambda = \frac{1}{15}$

The frame size is, $\Delta = 5 \text{ sec}$

The probability of arrival (success) during one frame (trial) is,

$$\begin{aligned} p &= \lambda\Delta \\ &= \frac{1}{15} \times 5 \\ &= \frac{5}{15} \\ &= \frac{1}{3} \end{aligned}$$

Step 2/4
The Interarrival time is, $T = 200$minutes.

This can be converted into the seconds as follows:

$$\begin{aligned} T &= 200 \times 60 \\ &= 12000 \end{aligned}$$

Therefore, the sample size is as follows:

$$
\begin{aligned}
n &= \frac{T}{\Delta} \\
&= \frac{12000}{5} \\
&= 2400
\end{aligned}
$$

Step 3/4

We know that, the probability of success for each trail is very low and the sample size is very large.

Therefore, the Binomial distribution is approximated to Normal distribution with proper continuity correction

The mean and standard deviations of the normal distribution is as follows:

$$
\begin{aligned}
\mu &= np \\
&= 2400 \times \frac{1}{3} \\
&= 800
\end{aligned}
$$

$$
\begin{aligned}
\sigma &= \sqrt{np(1-p)} \\
&= \sqrt{2400 \times \frac{1}{3} \times \left(1 - \frac{1}{3}\right)} \\
&= \sqrt{533.33} \\
&= 23.094
\end{aligned}
$$

Step 4/4

Now,

$$
\begin{aligned}
P(Z \le 750) &= P(X(n) \le 750.50) \qquad \begin{pmatrix}\text{Using continuity} \\ \text{correction}\end{pmatrix} \\
&= P\left(\frac{X(n) - \mu}{\sigma} \le \frac{750.50 - \mu}{\sigma}\right) \\
&= P\left(Z \le \frac{750.50 - 800}{23.094}\right) \\
&= P(Z \le -2.14) \qquad \left(\text{Using Excel's } \left(= NORMSDIST(-2.14)\right)\right) \\
&= 0.0160
\end{aligned}
$$

Therefore, the probability that no more than 750 messages arrive during 200 minutes of operation is $\boxed{0.0160}$

Chapter 6, Problem 19E
(0)

Problem
Messages arrive at an electronic mail server at the average rate of 4 messages every 5 minutes. Their number is modeled by a Binomial counting process.

(a) What frame length makes the probability of a new message arrival during a given frame equal 0.05?

(b) Suppose that 50 messages arrived during some 1-hour period. Does this indicate that the arrival rate is on the increase? Use frames computed in (a).

Step-by-step solution

Show all steps

Step 1/2
In this problem the rate at which the message arrive at an electronic mail server is
$\lambda = \frac{4}{5}$ per minute

(a)

The frame length Δ which gives the probability 0.05 of an arrival during any given frame is $\Delta = \frac{p}{\lambda} = \frac{0.05}{\frac{4}{5}}$

$= 0.0625\,\text{min} = 3.75\,\text{sec}$

Step 2/2
(b)

Suppose that 50 message arrived during 1 hour period

Then the rate in this case is $\lambda = \frac{50}{60}$ per minute

Probability of success in this case is $p = \lambda \times \Delta = \frac{50}{60} \times 0.0625 = 0.052$

So the probability of success for this rate is greater than the probability of success for previous rate So the arrival rate increases in on increase

Chapter 6, Problem 20E
(0)

Problem
Power outages are unexpected rare events occurring according to a Poisson process with the average rate of 3 outages per month. Compute the probability of more than 5 power outages during three summer months.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/1
In this problem the rate at which the power outages occur is $\lambda = 3$ outages per month

We need to find the probability of more than 5 outages during three summer months

The power outages are unexpected rare events occurring according to Poisson process.

The parameter of this process is $\mu = \lambda \times t = 3 \times 3 = 9$

So

$$P(X(3) > 5) = 1 - P(X(3) \le 5)$$

$$P(X(3) > 5) = 1 - \left(\frac{e^{-9} 9^0}{0!} + \frac{e^{-9} 9^1}{1!} + \frac{e^{-9} 9^2}{2!} + \frac{e^{-9} 9^3}{3!} + \frac{e^{-9} 9^4}{4!} + \frac{e^{-9} 9^5}{5!} \right)$$

$$P(X(3) > 5) = 0.884$$

Chapter 6, Problem 21E
(0)

Problem
Telephone calls to a customer service center occur according to a Poisson process with the rate of 1 call every 3 minutes. Compute the probability of receiving more than 5 calls during the next 12 minutes.

Step-by-step solution

Show all steps

100% (5 ratings) for this solution

Step 1/1
In this problem the rate at which the telephone calls to customer service center accur is $\lambda = \frac{1}{3}$ per minute

We need to find the probability of more than 5 telephone calls in 12 minutes

The telephone calls to customer service center occurring according to Poisson process.

The parameter of this process is $\mu = \lambda \times t = \frac{1}{3} \times 12 = 4$

So

$$P(X(12) > 5) = 1 - P(X(12) \le 5)$$

$$P(X(12) > 5) = 1 - \left(\frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!} + \frac{e^{-4}4^4}{4!} + \frac{e^{-4}4^5}{5!} \right)$$

$$P(X(12) > 5) = 0.215$$

Chapter 6, Problem 22E
(0)

Problem
Network blackouts occur at an average rate of 5 blackouts per month.

(a) Compute the probability of more than 3 blackouts during a given month.

(b) Each blackout costs $1500 for computer assistance and repair. Find the expectation and standard deviation of the monthly total cost due to blackouts.

Step-by-step solution

Show all steps

100% (3 ratings) for this solution

Step 1/2
Network blackout accur at an average rate of 5 blackout per month. So $\lambda = 5$ per month

(a)

We need to find the probability of more than 3 blackout in given month

The blackout occurring according to Poisson process.

The parameter of this process is $\mu = \lambda \times t = 5 \times 1 = 5$

So

$$P(X(1) > 3) = 1 - P(X(1) \leq 3)$$

$$P(X(1) > 3) = 1 - \left(\frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!} \right)$$

$$P(X(1) > 3) = 0.735$$

Step 2/2
(b)

Expected number of blackout in a given month is $E(X) = \mu = 5$

Each blackout cost 1500 dollars for computer assistant and repair then the expected cost for month due to black out is $1500 \times E(X) = 7500$ dollars.

Standard deviation of blackout in a given month is $S = \sqrt{\mu} = 2.236$

So the standard deviation of cost due to blackout is $S \times 1500 = 3354$

Chapter 6, Problem 23E
(0)

Problem
An internet service provider offers special discounts to every third connecting customer. Its customers connect to the internet according to a Poisson process with the rate of 5 customers per minute. Compute:

(a) the probability that no offer is made during the first 2 Minutes

(b) expectation and variance of the time of the first offer

Step-by-step solution

Show all steps

Step 1/2
Customer connects to the internet according to Poisson process with the average rate of 5customer per minute. So $\lambda = 5$ per minute

(a)

Compute the probability of no offer is made in two minutes

The parameter of this process is $\mu = \lambda \times t = 5 \times 2 = 10$

That is, per 2 minutes the average rate is 10.

Then, the required probability is,

$$\begin{aligned} P(X(2)=0) &= \frac{e^{-10}10^0}{0!} \\ &= e^{-10} \\ &= 0.000045 \end{aligned}$$

Therefore, probability of no offer is made in two minutes is **0.000045**.

Step 2/2
(b)

Let the random variable *T* represents time of the first offer.

Here, $T \sim Exp(\lambda = 5)$

Expectation of the time in first offer is,

$$\begin{aligned} E(T) &= \frac{1}{\lambda} \\ &= \frac{1}{5} \\ &= \mathbf{0.2} \end{aligned}$$

Variance of time in first offer is

$$\begin{aligned} Var(T) &= \frac{1}{\lambda^2} \\ &= \frac{1}{5^2} \\ &= \mathbf{0.04} \end{aligned}$$

Therefore, expectation and variance of the time in first offer are 0.2 and 0.04, respectively.

Chapter 6, Problem 24E
(0)

Problem
On the average, Mr. Z drinks and drives once in 4 years. He knows that

• Every time when he drinks and drives, he is caught by police.

• According to the laws of his state, the third time when he is caught drinking and driving results in the loss of his driver's license.

• Poisson process is the correct model for such "rare events" as drinking and driving.

What is the probability that Mr. Z will keep his driver's license for at least 10 years?

Step-by-step solution

Show all steps

80% (5 ratings) for this solution

Step 1/1

On the average the Mr. Z drink and drive once in 4 years. So $\lambda = \frac{1}{4}$ per years

(a)

We need to find the probability that Mr. Z will keep his license for at least 10 years

So

$$P(T \geq 10) = P(X(10) < 3)$$

Where the process of drink and drive occurring according to Poisson process with parameter $\mu = \lambda \times t = 2.5$

So

$$P(X(10) < 3) = \left(\frac{e^{-2.5} 2.5^0}{0!} + \frac{e^{-2.5} 2.5^1}{1!} + \frac{e^{-2.5} 2.5^2}{2!} \right)$$

$$= 0.544$$

So the probability that Mr. Z will keep his license for at least 10 years is 0.544

Chapter 6, Problem 25E
(0)

Problem
Refer to Example 6.9. Find the steady-state distribution for the number of users by writing $X(t)$ as a sum of two independent Markov chains,

$$X(t) = Y_1(t) + Y_2(t),$$

where $Y_i(t) = 1$ if user i is connected at time t and $Y_i(t) = 0$ if user i is not connected, for $i = 1, 2$. Find the steady-state distribution for each Y_i, then use it to find the steady-state distribution for X. Compare your result with Example 6.12.

Step-by-step solution

Show all steps

Step 1/8

Refer Example 6.9.

The users act independent of each other and connect and/or disconnect from the system with same distribution.

Thus, steady state distribution for Y_1 will also be steady state distribution for Y_2 and are independent of each other.

Also, probability that:

• User connects with system is 0.2.

• User disconnects from system is 0.5.

Step 2/8

Using the above values of probabilities, get state transition probabilities for Y_1.

$$\begin{aligned} p_{00} &= P(\text{User stays disconnected with system}) \\ &= 1 - P(\text{User connects with system}) \\ &= 1 - 0.2 \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} p_{01} &= P(\text{User connects with system}) \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} p_{10} &= P(\text{User disconnects form system}) \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} p_{11} &= P(\text{User doesn't disconnect form system}) \\ &= 1 - P(\text{User disconnects form system}) \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$

Step 3/8

Thus, get steady state-probability equation for Y_1, $p_{00}\pi_0 + p_{10}\pi_1 = \pi_0$ and simplify.

$$\begin{aligned} 0.8\pi_0 + 0.5\pi_1 &= \pi_0 \\ 0.5\pi_1 &= 0.2\pi_0 \\ \pi_1 &= \frac{2}{5}\pi_0 \end{aligned}$$

Step 4/8

Substitute $\pi_1 = \frac{2}{5}\pi_0$ in normalizing equation $\pi_0 + \pi_1 = 1$ and simplify.

$$\begin{aligned} \pi_0 + \frac{2}{5}\pi_0 &= 1 \\ \frac{7}{5}\pi_0 &= 1 \\ \pi_0 &= \frac{5}{7} \end{aligned}$$

Step 5/8

Substitute $\pi_0 = \frac{5}{7}$ in $\pi_1 = \frac{2}{5}\pi_0$ and simplify.

$$\begin{aligned} \pi_1 &= \frac{2}{5} \times \frac{5}{7} \\ &= \frac{2}{7} \end{aligned}$$

Thus, steady state probabilities for Y_1 are $\pi_0 = \frac{5}{7}$ and $\pi_1 = \frac{2}{7}$.

Since steady state distribution for Y_1 is also steady state distribution for Y_2, steady state probabilities for Y_2 are $\pi_0 = \frac{5}{7}$ and $\pi_1 = \frac{2}{7}$.

Step 6/8

Since $X(t) = Y_1(t) + Y_2(t)$, $X(t)$ will be:

• 0 when $Y_1(t) = 0$ and $Y_2(t) = 0$.

• 1 when either $Y_1(t) = 1$ and $Y_2(t) = 0$ or $Y_1(t) = 0$ and $Y_2(t) = 1$.

• 2 when $Y_1(t)=1$ and $Y_2(t)=1$.

Step 7/8

Thus, get steady state distribution for $X(t)=Y_1(t)+Y_2(t)$ where $\pi_0=\frac{5}{7}$ and $\pi_1=\frac{2}{7}$ are steady state probabilities for Y_1 and Y_2.

$$\begin{aligned}\pi_0' &= \pi_0\times\pi_0\\ &= \frac{5}{7}\times\frac{5}{7}\\ &\approx 0.5102\end{aligned}$$

$$\begin{aligned}\pi_1' &= \pi_1\times\pi_0+\pi_0\times\pi_1\\ &= \frac{2}{7}\times\frac{5}{7}+\frac{5}{7}\times\frac{2}{7}\\ &\approx 0.4082\end{aligned}$$

$$\begin{aligned}\pi_2' &= \pi_1\times\pi_1\\ &= \frac{2}{7}\times\frac{2}{7}\\ &\approx 0.0816\end{aligned}$$

Thus, steady state distribution for X are $\pi_0'=.5102$, $\pi_1'=0.4082$ and $\pi_2'=0.0816$.

Step 8/8

The same result was obtained in Example 6.12.

Thus, Example 6.12 verifies the calculated values of steady state distribution for X.

Chapter 6, Problem 26E

(0)

Problem

(COMPUTER MINI-PROJECT) Generate 10,000 transitions of the Markov chain in Example 6.9 on p. 16. How many times do you find your generated Markov chain in each of its three states? Does it match the distribution found in Example 6.12?

Step-by-step solution

Show all steps

Step 1/1

The transition probability matrix for Markov chain is

$$P = \begin{pmatrix} 0.64 & 0.32 & 0.04 \\ 0.40 & 0.50 & 0.10 \\ 0.25 & 0.50 & 0.25 \end{pmatrix}$$

The MATLAB code for $10{,}000$ transition of Markov chain is

```
N=10,000;

P=[0.64 0.32 0.04; 0.40 0.50 0.10; 0.25 0.50 0.25];

for i=1:N

P=P*P;

end

P
```

After applying this code we will get the outut

$$P = \begin{pmatrix} 0.5102 & 0.4082 & 0.0816 \\ 0.5102 & 0.4082 & 0.0816 \\ 0.5102 & 0.4082 & 0.0816 \end{pmatrix}$$

This matches with the steady state distribution of given Markov chain

Chapter 7, Problem 1E
(0)

Problem
Customers arrive at the ATM at the rate of 10 customers per hour and spend 2 minutes, on average, on all the transactions. This system is modeled by the Bernoulli single-server queuing process with 10-second frames. Write the transition probability matrix for the number of customers at the ATM at the end of each frame.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/1
The information is as follows:

$$\lambda_A = 10 / hrs$$
$$= \frac{1}{6}\min^{-1}$$

$$\lambda_S = \frac{1}{\mu_S}$$
$$= 0.5\min^{-1}$$

$$\Delta = \frac{1}{6}\min$$

Now

$$P_A = \lambda_A \Delta = \frac{1}{36}$$
$$P_S = \lambda_S \Delta = \frac{1}{12}$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = \frac{35}{36} = 0.972$$

$$p_{01} = P_A = \frac{1}{36} = 0.028$$

For $i \geq 1,$

$$p_{i,i-1} = (1-P_A)P_S = 0.081$$
$$p_{i,i+1} = (1-P_S)P_A = 0.025$$
$$p_{i,i} = 1-0.081-0.025 = 0.894$$

Therefore the transition probability matrix for the number of customers at the ATM at the end of each frame is shown below:

$$\begin{bmatrix} 0.972 & 0.028 & 0 & \cdots \\ 0.081 & 0.894 & 0.025 & \cdots \\ 0 & 0.081 & 0.894 & 0.025 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Chapter 7, Problem 2E
(0)

Problem
Consider Bernoulli single-server queuing process with an arrival rate of 2 jobs per minute, a service rate of 4 jobs per minute, frames of 0.2 minutes, and a capacity limited by 2 jobs. Compute the steady-state distribution of the number of jobs in the system.

Step-by-step solution

Show all steps

Step 1/2
The information is as follows:

$$\lambda_A = 2\,\text{min}^{-1}$$
$$\lambda_S = 4\,\text{min}^{-1}$$
$$\Delta = 0.2\,\text{min}$$

Now

$$P_A = 0.4$$
$$P_S = 0.8$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1-P_A = 0.6$$
$$p_{01} = P_A = 0.4$$

Therefore

$$p_{10} = (1 - P_A) P_S = 0.48$$
$$p_{12} = (1 - P_S) P_A = 0.08$$
$$p_{11} = 1 - 0.48 - 0.08 = 0.44$$

$$p_{20} = 0$$
$$p_{21} = (1 - P_A) P_S = 0.48$$
$$p_{22} = 1 - 0.48 = 0.52$$

Step 2/2

The Markov Chain $X(t)$ has three states, $X = 0, X = 1$ and $X = 2$.

The transition probability matrix is shown below:

$$P = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.48 & 0.44 & 0.08 \\ 0 & 0.48 & 0.52 \end{bmatrix}$$

The steady state equations are shown below:

$$\pi P = \pi$$

$$\Rightarrow \begin{cases} 0.6\pi_0 + 0.48\pi_1 = \pi_0 \\ 0.4\pi_0 + 0.44\pi_1 + 0.48\pi_2 = \pi_1 \\ 0.08\pi_1 + 0.52\pi_2 = \pi_2 \end{cases}$$

Solving by using $\pi_0 + \pi_1 + \pi_2 = 1$ find the steady state distribution is shown below:

$$\pi = \begin{bmatrix} 0.5070 & 0.4225 & 0.0705 \end{bmatrix}$$

Chapter 7, Problem 3E

(0)

Problem

Performance of a car wash center is modeled by the Bernoulli singleserver queuing process with 2-minute frames. The cars arrive every 10 minutes, on the average. The average service time is 6 minutes. Capacity is unlimited. If there are no cars at the center at 10 am, compute the probability that one car will be washed and another car will be waiting at 10:04 am.

Step-by-step solution

Show all steps

Step 1/3

The information is as follows:

$$\mu_A = 10 \Rightarrow \lambda_A = 1/10\,\text{min}^{-1}$$
$$\mu_S = 6 \Rightarrow \lambda_S = 1/6\,\text{min}^{-1}$$
$$\Delta = 2\,\text{min}$$

Now

$$P_A = \lambda_A \Delta = 0.2$$
$$P_S = \lambda_S \Delta = 0.33$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = 0.8$$
$$p_{01} = P_A = 0.2$$

For $i \geq 1,$

$$p_{i,i-1} = (1 - P_A)P_S = 0.264$$
$$p_{i,i+1} = (1 - P_S)P_A = 0.536$$
$$p_{i,i} = 1 - 0.264 - 0.536 = 0.2$$

Step 2/3

The transition probability matrix is shown below:

$$P = \begin{bmatrix} 0.8 & 0.2 & 0 & 0 & \cdots \\ 0.264 & 0.2 & 0.536 & 0 & \cdots \\ 0 & 0.264 & 0.2 & 0.536 & \cdots \\ 0 & 0 & 0.264 & 0.2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Since there are (10 am to 10.04 am) $4\,\text{min}/\Delta = 2$ frames in 4 minutes, we need the distribution of X after 2 frames, which is $P_2 = P_0 P^2$

Since there are two jobs in the system, so the initial distribution will be

$$P_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

In a course of 2 frames there number change by 2 at most and thus it is sufficient to consider the first 5 rows and 5 columns of P only corresponding to states 0, 1, 2, 3 and 4.

Step 3/3

Now we compute the distribution after 2 frames:

$$P_2 = P_0 P^2$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 & 0 & 0 & 0 \\ 0.264 & 0.2 & 0.536 & 0 & 0 \\ 0 & 0.264 & 0.2 & 0.536 & 0 \\ 0 & 0 & 0.264 & 0.2 & 0.536 \\ 0 & 0 & 0 & 0.264 & 0.736 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0697 & 0.1056 & 0.3230 & 0.2144 & 0.287 \end{bmatrix}$$

Hence the probability that one car will be washed and another car will be waiting at 10.04 am is 0.3230.

Chapter 7, Problem 4E

(0)

Problem

Masha has a telephone with two lines that allows her to talk with a friend and have at most one other friend on hold. On the average, she gets 10 calls every hour, and an average conversation takes 2 minutes. Assuming a single-server limited-capacity Bernoulli queuing process with 1-minute frames, compute the fraction of time Masha spends using her telephone.

Step-by-step solution

Show all steps

Step 1/2

The system is with limited capacity $C = 2$.

$$\lambda_A = 10 hrs^{-1} = 1/6 \min^{-1}$$
$$\mu_S = 2 \min \Rightarrow \lambda_S = 1/2 \min^{-1}$$
$$\Delta = 1 \min$$

Now

$$P_A = \lambda_A \Delta = 0.167$$
$$P_S = \lambda_S \Delta = 0.5$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = 0.833$$
$$p_{01} = P_A = 0.167$$

And

$$p_{10} = (1 - P_A) P_S = 0.417$$
$$p_{12} = (1 - P_S) P_A = 0.0835$$
$$p_{11} = 1 - 0.417 - 0.0835 = 0.4995$$

$$p_{20} = 0$$
$$p_{21} = (1 - P_A) P_S = 0.417$$
$$p_{22} = 1 - 0.417 = 0.583$$

Step 2/2

The Markov Chain $X(t)$ has three states, $X = 0, X = 1$ and $X = 2$.

The transition probability matrix is shown below:

$$P = \begin{bmatrix} 0.833 & 0.167 & 0 \\ 0.417 & 0.4995 & 0.0835 \\ 0 & 0.417 & 0.583 \end{bmatrix}$$

The steady state equations are as follows:

$$\pi P = \pi$$

$$\Rightarrow \begin{cases} 0.833\pi_0 + 0.417\pi_1 = \pi_0 \\ 0.167\pi_0 + 0.4995\pi_1 + 0.417\pi_2 = \pi_1 \\ 0.0835\pi_1 + 0.583\pi_2 = \pi_2 \end{cases}$$

Solving by using $\pi_0 + \pi_1 + \pi_2 = 1$

Now find the steady state distribution is $\pi = [0.6754 \quad 0.2705 \quad 0.0542]$

Hence 0.2705 + 0.0542 = 0.3247

Therefore the fraction of time she spend using her telephone is 32.47%.

Chapter 7, Problem 5E
(0)

Problem
A customer service representative can work with one person at a time and have at most one other customer waiting. Compute the steady-state distribution of the number of customers in this queuing system at any time, assuming that customers arrive according to a Binomial counting process with 3-minute frames and the average interarrival time of 10 minutes, and the average service takes 15 minutes.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/2
The system is with limited capacity $C = 2$

$$\mu_A = 10\,\text{min} \Rightarrow \lambda_A = 1/10\,\text{min}^{-1}$$
$$\mu_S = 15\,\text{min} \Rightarrow \lambda_S = 1/15\,\text{min}^{-1}$$
$$\Delta = 3\,\text{min}$$

Now

$$P_A = \lambda_A \Delta = 0.3$$
$$P_S = \lambda_S \Delta = 0.2$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = 0.7$$
$$p_{01} = P_A = 0.3$$

And

$$p_{10} = (1 - P_A) P_S = 0.14$$
$$p_{12} = (1 - P_S) P_A = 0.24$$
$$p_{11} = 1 - 0.14 - 0.24 = 0.62$$

$$p_{20} = 0$$
$$p_{21} = (1 - P_A) P_S = 0.14$$
$$p_{22} = 1 - 0.14 = 0.86$$

Step 2/2
The Markov Chain $X(t)$ has three states, $X = 0, X = 1$ and $X = 2$.

The transition probability matrix is shown below:

$$P = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.14 & 0.62 & 0.24 \\ 0 & 0.14 & 0.86 \end{bmatrix}$$

The steady state equations are:

$$\pi P = \pi$$

$$\Rightarrow \begin{cases} 0.7\pi_0 + 0.14\pi_1 = \pi_0 \\ 0.3\pi_0 + 0.62\pi_1 + 0.14\pi_2 = \pi_1 \\ 0.24\pi_1 + 0.86\pi_2 = \pi_2 \end{cases}$$

Solving by using $\pi_0 + \pi_1 + \pi_2 = 1$ we have find the steady state distribution is

$$\pi = [0.1467 \quad 0.3144 \quad 0.5389]$$

Chapter 7, Problem 6E
(0)

Problem
Performance of a telephone with 2 lines is modeled by the Bernoulli single-server queuing process with limited capacity (*C* = 2). If both lines of a telephone are busy, the new callers receive a busy signal and cannot enter the queue. On the average, there are 5 calls per hour, and the average call takes 20 minutes. Compute the steady-state probabilities using four-minute frames.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/2
The system is with limited capacity $C = 2$

$$\lambda_A = 5hrs^{-1} = 1/12\min^{-1}$$
$$\mu_S = 20\min \Rightarrow \lambda_S = 1/20\min^{-1}$$
$$\Delta = 4\min$$

Now

$$P_A = \lambda_A \Delta = 0.3333$$
$$P_S = \lambda_S \Delta = 0.2$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = 0.6667$$
$$p_{01} = P_A = 0.3333$$

And

$$p_{10} = (1 - P_A) P_S = 0.1334$$
$$p_{12} = (1 - P_S) P_A = 0.2664$$
$$p_{11} = 1 - 0.1334 - 0.2664 = 0.6002$$

$$p_{20} = 0$$
$$p_{21} = (1 - P_A) P_S = 0.1334$$
$$p_{22} = 1 - 0.1334 = 0.8666$$

Step 2/2

The Markov Chain $X(t)$ has three states, $X = 0, X = 1$ and $X = 2$.

The transition probability matrix is shown below:

$$P = \begin{bmatrix} 0.6667 & 0.3333 & 0 \\ 0.1334 & 0.6002 & 0.2664 \\ 0 & 0.1334 & 0.8666 \end{bmatrix}$$

The steady state equations are shown below:

$$\pi P = \pi$$

$$\Rightarrow \begin{cases} 0.6667\pi_0 + 0.1334\pi_1 = \pi_0 \\ 0.3333\pi_0 + 0.6002\pi_1 + 0.1334\pi_2 = \pi_1 \\ 0.2664\pi_1 + 0.8666\pi_2 = \pi_2 \end{cases}$$

Solving by using $\pi_0 + \pi_1 + \pi_2 = 1$ we have find the steady state distribution is

$$\pi = \begin{bmatrix} 0.1176 & 0.2943 & 0.5878 \end{bmatrix}$$

Hence the probability of the new caller do not receive busy signal in is 0.1176.

Chapter 7, Problem 7E
(0)

Problem
Jobs arrive at the server at the rate of 8 jobs per hour. The service takes 3 minutes, on the average. This system is modeled by the Bernoulli single-server queuing process with 5-second frames and capacity limited by 3 jobs. Write the transition probability matrix for the number of jobs in the system at the end of each frame.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/3
The system is with limited capacity $C = 2$

$$\lambda_A = 8hrs^{-1} = 2/15\min^{-1}$$
$$\mu_S = 3\min \Rightarrow \lambda_S = 1/3\min^{-1}$$
$$\Delta = 1/12\min$$

Now

$$P_A = \lambda_A\Delta = 0.0111$$
$$P_S = \lambda_S\Delta = 0.0278$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = 0.9889$$
$$p_{01} = P_A = 0.0111$$

Step 2/3
And

$$p_{10} = (1 - P_A)P_S = 0.0275$$
$$p_{12} = (1 - P_S)P_A = 0.0108$$
$$p_{11} = 1 - 0.0275 - 0.0108 = 0.9617$$

$$p_{13} = 0$$
$$p_{20} = 0$$
$$p_{21} = (1 - P_A)P_S = 0.0275$$
$$p_{23} = (1 - P_S)P_A = 0.0108$$

$$p_{22} = 1 - 0.0275 - 0.0108 = 0.9617$$
$$p_{30} = 0$$
$$p_{31} = 0$$
$$p_{32} = (1 - P_A) P_S = 0.0275$$
$$p_{33} = 1 - 0.0275 = 0.9725$$

Step 3/3
The transition probability matrix for the number of jobs in the system at the end of the each frame is shown below:

$$P = \begin{bmatrix} 0.9889 & 0.0111 & 0 & 0 \\ 0.0275 & 0.9617 & 0.0108 & 0 \\ 0 & 0.0275 & 0.9617 & 0.0108 \\ 0 & 0 & 0.0275 & 0.9725 \end{bmatrix}$$

Chapter 7, Problem 8E
(0)

Problem
For an M/M/1 queuing process with the arrival rate of 5 $\min^{-1}$ and the average service time of 4 seconds, compute

(a) the proportion of time when there are exactly 2 customers in the system;

(b) the expected response time (the expected time from the arrival till the departure).

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/2
a.

It is given that the average rate $\lambda_A = 5\min^{-1}$

The expected service time is $\mu_S = 4\sec ond = 0.0667\min$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.333$

The proportion of time when there is exactly 2 customers in the system is shown below:

$$
\begin{aligned}
P(X=2) &= r^2(1-r) \\
&= 0.333^2(1-0.333) \\
&= \boxed{0.074}
\end{aligned}
$$

Hence the proportion of time when there is exactly 2 customers in the system is 0.074.

Step 2/2
b.

The expected response time will be calculated as follows:

$$
\begin{aligned}
E(R) &= \frac{\mu_S}{1-r} \\
&= \frac{0.0667}{1-0.333} \\
&= 0.1\,\text{min} \\
&= \boxed{6\,\text{second}}
\end{aligned}
$$

Hence the expected response time is 6 second.

Chapter 7, Problem 9E
(0)

Problem
Jobs are sent to a printer at random times, according to a Poisson process of arrivals, with a rate of 12 jobs per hour. The time it takes to print a job is an Exponential random variable, independent of the arrival time, with the average of 2 minutes per job.

(a) A job is sent to a printer at noon. When is it expected to be printed?

(b) How often does the total number of jobs in a queue and currently being printed exceed 2?

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/3
a.

Since it is a Poisson Process arrival the system is M/M/1.

It is given that the average rate $\lambda_A = 12 hrs^{-1} = 0.2\min^{-1}$

The expected service time is $\mu_S = 2\min$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.4$

The expected time to be printed a job is shown below:

$$\begin{aligned} E(R) &= \frac{\mu_S}{1-r} \\ &= \frac{2}{1-0.4} \\ &= \boxed{3.33\min} \end{aligned}$$

Hence the job expected to be printed will be after 3.33 minutes.

Step 2/3
b.

The proportion of time when there is exactly 0 jobs in the system is shown below:

$$\begin{aligned} P(X) &= r^0(1-r) \\ &= 0.4^0(1-0.4) \\ &= \boxed{0.6} \end{aligned}$$

The proportion of time when there is exactly 1 jobs in the system is shown below:

$$\begin{aligned} P(X=1) &= r^1(1-r) \\ &= 0.4^1(1-0.4) \\ &= \boxed{0.24} \end{aligned}$$

The proportion of time when there is exactly 2 jobs in the system is shown below:

$$\begin{aligned} P(X=2) &= r^2(1-r) \\ &= 0.4^2(1-0.4) \\ &= \boxed{0.096} \end{aligned}$$

Step 3/3
Therefore

$$1-0.6-0.24-0.096=\boxed{0.064}$$

Hence the proportion of time of the total number of jobs in a queue and currently being printed exceeded 2 will be 0.064.

Chapter 7, Problem 10E
(0)

Problem
A vending machine is modeled as an M/M/1 queue with the arrival rate of 20 customers per hour and the average service time of 2 minutes.

(a) A customer arrives at 8 pm. What is the expected waiting time?

(b) What is the probability that nobody is using the vending machine at 3 pm?

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/2
a.

It is given that the average rate $\lambda_A = 20hrs^{-1} = 0.333\,\text{min}^{-1}$

The expected service time is $\mu_S = 2\,\text{min}$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.666$

The expected waiting time is computed as shown below:

$$
\begin{aligned}
E(W) &= \frac{\mu_S r}{1-r} \\
&= \frac{2 \times 0.666}{1-0.666} \\
&= \boxed{4\,\text{min}}
\end{aligned}
$$

Hence the expected waiting time is 4 minutes.

Step 2/2
b.

The probability of the system is idle will be $1-r = 1-0.667 = 0.333$

Hence the probability that nobody is using the vending machine at 3pm is 0.333.

Chapter 7, Problem 11E
(0)

Problem

Customers come to a teller's window according to a Poisson process with a rate of 10 customers every 25 minutes. Service times are Exponential. The average service takes 2 minutes. Compute

(a) the average number of customers in the system and the average number of customers waiting in a line;

(b) the fraction of time when the teller is busy with a customer;

(c) the fraction of time when the teller is busy and at least five other customers are waiting in a line.

Step-by-step solution

Show all steps

Step 1/3

a.

This is a M/M/1 system

It is given that the average rate $\lambda_A = 10/25\,\text{min}^{-1} = 0.4\,\text{min}^{-1}$

The expected service time is $\mu_S = 2\,\text{min}$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.8$

The expected number of customers in the system is computed as shown below:

$$\begin{aligned} E(X) &= \frac{r}{1-r} \\ &= \frac{0.8}{1-0.8} \\ &= \boxed{4} \end{aligned}$$

The expected number of customers in the waiting line:

$$E(X_W) = \frac{r^2}{1-r} = 3.2$$

Hence the average number of customers in the system is 4 and the average number of customers in the waiting line is 3.2.

Step 2/3

b.

The fraction of time when the teller is busy is $r = 0.8$

Therefore the fraction of time when the teller is busy is 0.8.

Step 3/3
c.

The fraction of time when the taller is busy and at least five other customers are waiting in a line is computed as follows:

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - \sum_{x=0}^{4} r^x (1-r) \\ &= 1 - 0.672 \\ &= \boxed{0.328} \end{aligned}$$

Hence the fraction of time when the taller is busy and at least five other customers are waiting in a line is 0.328.

Chapter 7, Problem 12E
(0)

Problem
For an M/M/1 queuing system with the average interarrival time of 5 minutes and the average service time of 3 minutes, compute

(a) the expected response time;

(b) the fraction of time when there are fewer than 2 jobs in the system;

(c) the fraction of customers who have to wait before their service starts.

Step-by-step solution

Show all steps

Step 1/3
(a)

This is a M/M/1 system

It is given that the arival rate $\lambda_A = 1/5\,\text{min}^{-1} = 0.2\,\text{min}^{-1}$

The expected service time is $\mu_S = 3\,\text{min}$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.6$

The expected response time is shown below:

$$
\begin{aligned}
E(R) &= \frac{\mu_S}{1-r} \\
&= \frac{0.3}{1-0.6} \\
&= 0.75 \text{ min} \\
&= \boxed{45\text{seconds}}
\end{aligned}
$$

Hence the expected response time is, 45 seconds.

Step 2/3
(b)

The fraction of time when there are fewer than 2 jobs in the system will be

$$
\begin{aligned}
P(X<2) &= P(X \le 1) \\
&= \sum_{x=0}^{1} r^x (1-r) \\
&= 0.6^0 (1-0.6) + 0.6^1 (1-0.6) \\
&= \boxed{0.64}
\end{aligned}
$$

Hence the fraction of time when there are fewer than 2 jobs in the system is, 0.64.

Step 3/3
(c)

The fraction of customers who have to wait before their service starts will be

$$
\begin{aligned}
E(X_w) &= \frac{r^2}{1-r} \\
&= \frac{0.6^2}{1-0.6} \\
&= \boxed{0.9}
\end{aligned}
$$

Hence the fraction of customers who have to wait before their service starts is, 0.9.

Chapter 7, Problem 13E
(0)

Problem
Jobs arrive at the service facility according to a Poisson process with the average interarrival time of 4.5 minutes. A typical job spends a Gamma distributed time with parameters α = 12, λ = 5 min^{-1} in the system and leaves.

(a) Compute the average number of jobs in the system at any time.

(b) Suppose that only 20 jobs arrived during the last three hours. Is this an evidence that the expected interarrival time has increased?

Step-by-step solution

Show all steps

Step 1/1
a.

This is a M/M/1 system

It is given that the arrival rate $\lambda_A = 1/4.5\,\text{min}^{-1} = 0.222\,\text{min}^{-1}$

The expected service time is $\mu_S = \frac{12}{5}\,\text{min} = 2.4\,\text{min}$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.533$

The expected number of jobs in the system is computed as follows:

$$
\begin{aligned}
E(X) &= \frac{r}{1-r} \\
&= \frac{0.533}{1-0.533} \\
&= \boxed{1.14}
\end{aligned}
$$

The expected number of jobs in the system is 1.14.

b.

It is given that 20 jobs arrived during the last three hours, and then $20/180 = 0.111\,\text{min}^{-1}$ is the new arrival rate. So the expected inter arrival time not increased.

Chapter 7, Problem 14E
(0)

Problem
Trucks arrive at a weigh station according to a Poisson process with the average rate of 1 truck every 10 minutes. Inspection time is Exponential with the average of 3 minutes. When a truck is on scale, the other arrived trucks stay in a line waiting for their turn. Compute

(a) the expected number of trucks in the line at any time;

(b) the proportion of time when the weigh station is empty;

(c) the expected time each truck spends at the station, from arrival till departure.

Step-by-step solution

Show all steps

Step 1/2

a.

This is a M/M/1 system

It is given that the arrival rate $\lambda_A = 1/10\,\text{min}^{-1} = 0.1\,\text{min}^{-1}$

The expected service time is $\mu_S = 3\,\text{min}$.

Therefore the utilization is $r = \lambda_A$ and $\mu_S = 0.3$

Compute the expected number trucks in line:

$$\begin{aligned} E(X) &= \frac{r}{1-r} \\ &= \frac{0.3}{1-0.3} \\ &= \boxed{0.43} \end{aligned}$$

The expected number of jobs in the line is 0.43.

b.

The proportion of time when the weigh station is empty is computed as follows:

$$\begin{aligned} 1-r &= 1-0.3 \\ &= \boxed{0.7} \end{aligned}$$

Hence the proportion of time when the weigh station is empty is 0.7.

Step 2/2

c.

The expected time each truck spends in the station is computed as follows:

$$E(R) = \frac{\mu_S}{1-r} = \frac{3}{0.7} = 4.3\,\text{min}$$

Hence the expected time each truck spends in the station is 4.3 minutes.

Chapter 7, Problem 15E
(0)

Problem
Consider a hot-line telephone that has no second line. When the telephone is busy, the new callers get a busy signal. People call at the average rate of 2 calls per minute. The average duration of a telephone conversation is 1 minute. The system behaves like a Bernoulli single-server queuing process with a frame size of 1 second.

(a) Compute the steady-state distribution for the number of concurrent jobs.

(b) What is the probability that more than 150 people attempted to call this number between 2 pm and 3 pm?

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/3
The information is as follows:

$$\lambda_A = 2\,\text{min}^{-1}$$
$$\mu_S = 1\,\text{min} \Rightarrow \lambda_S = 1\,\text{min}^{-1}$$
$$\Delta = 1\,\text{sec}\,ond = 0.0167\,\text{min}$$

Now

$$P_A = \lambda_A \Delta = 0.0333$$
$$P_S = \lambda_S \Delta = 0.0167$$

Therefore all the transition probabilities for X are expressed as follows:

$$p_{00} = 1 - P_A = 0.9667$$
$$p_{01} = P_A = 0.0333$$

For $i \ge 1,$

$$p_{i,i-1} = (1 - P_A)P_S = 0.0161$$
$$p_{i,i+1} = (1 - P_S)P_A = 0.0328$$
$$p_{i,i} = 1 - 0.0161 - 0.0328 = 0.9511$$

Step 2/3
The transition probability matrix is shown below:

$$P = \begin{bmatrix} 0.9667 & 0.0333 & 0 & 0 & \cdots \\ 0.0161 & 0.9511 & 0.0328 & 0 & \cdots \\ 0 & 0.0161 & 0.9511 & 0.0328 & \cdots \\ 0 & 0 & 0.0161 & 0.9511 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Considering the first 5 rows and 5 columns of P with corresponding to states 0, 1, 2, 3 and 4. The steady state distribution will be obtained as shown below:

$$\pi P = \pi$$

$$\Rightarrow [\pi_0 \quad \pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4] \begin{bmatrix} 0.9667 & 0.0333 & 0 & 0 & 0 \\ 0.0161 & 0.9511 & 0.0328 & 0 & 0 \\ 0 & 0.0161 & 0.9511 & 0.0328 & 0 \\ 0 & 0 & 0.0161 & 0.9511 & 0.0328 \\ 0 & 0 & 0 & 0.0161 & 0.9839 \end{bmatrix}$$

$$= [\pi_0 \quad \pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4]$$

Solving by using $\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$ we have find the steady state distribution and is

$$\pi = [0.0300 \quad 0.0620 \quad 0.1263 \quad 0.2574 \quad 0.5243]$$

Step 3/3
b.

Since the frame is very least i.e. goes to zero, so it is sufficient to consider M/M/1 system.

This is a M/M/1 system

It is given that the arrival rate $\lambda_A = 1/2\,\text{min}^{-1}$

The expected service time is $\mu_S = 1\,\text{min}$.

Therefore the utilization is $r = \lambda_A \mu_S = 0.5$

Therefore the probability of more 150 people attempt to a call will be

$$r^{150}(1-r) = 0.5^{150} \times 0.5$$

Hence the probability that more 150 people attempt to a call is 0 approx.

Chapter 7, Problem 16E
(0)

Problem
On the average, every 6 minutes a customer arrives at an M/M/k queuing system, spends an average of 20 minutes there, and departs. What is the mean number of customers in the system at any given time?

Step-by-step solution

Show all steps

Step 1/1

It is given that the average arrival rate $\lambda_A = 1/6\,\text{min}^{-1}$ and the average service time is $\mu_S = 20\,\text{min}$. Therefore the utilization is

$$r = \lambda_A \mu_S = 3.333 > 3$$

The system will function any $r < k$.

The mean number of customers in the system will be

$$E(X) = \sum_{x=1}^{k} . \frac{3.333^x}{(x-1)!} . \left(\frac{1}{\sum_{i=1}^{k-1} \frac{i^{3.333}}{i!} + \frac{3.333^k}{(1-3.333/k)k!}} \right)$$

Chapter 7, Problem 17E
(0)

Problem
Verify the Little's Law for the M/M/1 queuing system and for its components – waiting and service.

Step-by-step solution

Show all steps

Step 1/1
The Little's law states the simple relationship between expected number of jobs, expected response time and expected arrival rate, which is

$$\lambda_A E(R) = E(X)$$

For queuing system M/M/1

$$E(R)=\frac{\mu_S}{1-r}$$

$$E(X)=\frac{r}{1-r}$$

$$\therefore \lambda_A E(R)=\lambda_A\times\frac{\mu_S}{1-r}$$

$$=\frac{r}{1-r}$$

$$=E(X)$$

Since $E(R)=E(S)+E(W)$ the waiting time and service time relationship will be

$$\Rightarrow E(W)=\frac{\mu_S}{1-r}-\mu_S$$

$$\Rightarrow E(W)=\frac{r}{1-r}E(S)$$

$$\Rightarrow E(W)=E(X)E(S)$$

Chapter 7, Problem 18E

(0)

Problem

A metered parking lot with two parking spaces represents a Bernoulli two-server queuing system with capacity limited by two cars and 30-second frames. Cars arrive at the rate of one car every 4 minutes. Each car is parked for 5 minutes, on the average.

(a) Compute the transition probability matrix for the number of parked cars.

(b) Find the steady-state distribution for the number of parked cars.

(c) What fraction of the time are both parking spaces vacant?

(d) What fraction of arriving cars will not be able to park?

(e) Every 2 minutes of parking costs 25 cents. Assuming that the drivers use all the parking time they pay for, what revenue is the owner of this parking lot expected to get every 24 hours?

Step-by-step solution

Show all steps

Step 1/5

a.

This system is k = 2 servers, capacity C = 2, $\lambda_A = 1/4\,\text{min}^{-1}$, $\lambda_S = 1/5\,\text{min}$ and $\Delta = 0.5\,\text{min}$

Therefore

$$P_A = \lambda_A \Delta = 0.125$$
$$P_S = \lambda_S \Delta = 0.1$$

The transition probabilities are computed as

$$P_{i,i+1} = P_A (1 - P_S)^n$$
$$P_{i,i} = P_A {}^nC_1 P_S (1 - P_S)^{n-1} + (1 - P_A)(1 - P_S)^n$$
$$P_{i,i-1} = P_A {}^nC_2 P_S^2 (1 - P_S)^{n-2} + (1 - P_A) {}^nC_1 P_S (1 - P_S)^{n-1}$$
$$P_{i,i-n} = (1 - P_A) P_S^n$$

The transition probability matrix will be:

$$P = \begin{bmatrix} 0.875 & 0.125 & 0 \\ 0.0875 & 0.8 & 0.1125 \\ 0 & 0.0875 & 0.9125 \end{bmatrix}$$

Step 2/5
b.

The steady state distribution is computed by using the equations

$$\pi P = \pi$$

$$\Rightarrow \begin{cases} 0.875\pi_0 + 0.0875\pi_1 = \pi_0 \\ 0.125\pi_0 + 0.8\pi_1 + 0.0875\pi_2 = \pi_1 \\ 0.1125\pi_1 + 0.9125\pi_2 = \pi_2 \end{cases}$$

Solving by using $\pi_0 + \pi_1 + \pi_2 = 1$ we have find the steady state distribution is:

$$\pi = \begin{bmatrix} 0.2345 & 0.3349 & 0.4306 \end{bmatrix}$$

Step 3/5
c.

The fraction of time both parking space vacant is 0.2345.

Step 4/5
d.

The chance of arriving cars will not be able to park is 0.4306.

Step 5/5
e.

The average number of car stored in parking place at any time is

$$E(X) = 0\times 0.2345 + 1\times 0.3349 + 2\times 0.4306$$
$$= 1.1961$$

Hence the expected revenue will be $24\times 60\times 1.1961\times 25\times 0.5 = 21529.8$ cent.

Hence the required expected revenue is 21529.8 cent.

Chapter 7, Problem 19E
(0)

Problem
(*This exercise may require a computer or at least a calculator.*) A walk-in hairdressing saloon has two hairdressers and two extra chairs for waiting customers. We model this system as a Bernoulli queuing process with two servers, 1-minute frames, capacity limited by 4 customers, arrival rate of 3 customers per hour, and the average service time of 45 minutes, not including the waiting time.

(a) Compute the transition probability matrix for the number of customers in the saloon at any time and find the steady-state distribution.

(b) Use this steady-state distribution to compute the expected number of customers in the saloon at any time, the expected number of customers waiting for a hairdresser, and the fraction of customers who found all the available seats already occupied.

(c) Each hairdresser comes for an eight-hour working day. How does it split between their working time and their resting time?

(d) How will the performance characteristics in (b,c) change if they simply put two more chairs into the waiting area?

Step-by-step solution

Show all steps

Step 1/4
a.

This system is k = 2 servers, capacity C = 4, $\lambda_A = 3hrs^{-1} = 1/20\min^{-1}$, $\lambda_S = 1/45\min^{-1}$ and $\Delta = 1\min$

Therefore

$$P_A = \lambda_A \Delta = 0.05$$
$$P_S = \lambda_S \Delta = 0.0222$$

The transition probabilities are computed as

$$P_{i,i+1} = P_A (1-P_S)^n$$
$$P_{i,i} = P_A {}^nC_1 P_S (1-P_S)^{n-1} + (1-P_A)(1-P_S)^n$$
$$P_{i,i-1} = P_A {}^nC_2 P_S^2 (1-P_S)^{n-2} + (1-P_A) {}^nC_1 P_S (1-P_S)^{n-1}$$
$$P_{i,i-n} = (1-P_A) P_S^n$$

The transition probability matrix will be:

$$P = \begin{bmatrix} 0.95 & 0.05 & 0 & 0 & 0 \\ 0.0211 & 0.93 & 0.0489 & 0 & 0 \\ 0 & 0.0211 & 0.93 & 0.0489 & 0 \\ 0 & 0 & 0.0211 & 0.93 & 0.0489 \\ 0 & 0 & 0 & 0.0211 & 0.9789 \end{bmatrix}$$

The steady state distribution is computed by using the equations

$$\pi P = \pi$$

$$\Rightarrow [\pi_0 \quad \pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4] \times \begin{bmatrix} 0.95 & 0.05 & 0 & 0 & 0 \\ 0.0211 & 0.93 & 0.0489 & 0 & 0 \\ 0 & 0.0211 & 0.93 & 0.0489 & 0 \\ 0 & 0 & 0.0211 & 0.93 & 0.0489 \\ 0 & 0 & 0 & 0.0211 & 0.9789 \end{bmatrix} = [\pi_0 \quad \pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4]$$

Solving by using $\pi_0 + \pi_1 + \pi_2 = 1$ the steady state distribution is:

$$\pi = [0.0196 \quad 0.0464 \quad 0.1075 \quad 0.2491 \quad 0.5774]$$

Step 2/4
b.

The expected number of customers in the saloon at any time will be

$$E(X) = \sum_{x=0}^{4} x.\pi_x = 3.3183$$

The expected number of customer waiting for a hairdresser is $E(X) - k = 1.3183$.

The fraction of customers who already occupied the available seat is
$\pi_0 + \pi_1 + \pi_2 = 0.0196 + 0.0464 + 0.1075 = 0.23675$

Therefore the expected number of customers in the saloon at any time is 3.3183, the required expected number of customers waiting for a hairdresser is 1.3183 and the fraction of customers who occupied the available seat is 0.23675.

Step 3/4
c.

The fraction of time when there is no customer in the saloon is 0.0196. The fraction of time when there is one hairdresser has no service is 0.0464. Therefore the fraction of time there is at least one hairdresser getting rest will be 0.066. Since they work for eight hours so the total rest time will be 31.68 minutes.

Step 4/4
d.

Since the expected number of waiting for a hairdresser is 1.3183, so there is two chairs are sufficient for waiting. If they put two more chairs in waiting area, the performance characteristics do not change.

Chapter 7, Problem 20E
(0)

Problem
Two tellers are now on duty in a bank from Exercise 7.11, and they work as an M/M/2 queuing system with the arrival rate of 0.4 min^{-1} and the mean service time of 2 min.

(a) Compute the steady state probabilities $\pi_0, \dots, \pi_{10}$ and use them to approximate the expected number of customers in the bank as

$$\mathbf{E}(X) \approx \sum_{x=0}^{10} x\pi_x.$$

(b) Use the Little’s Law to find the expected response time.

(c) From this, derive the expected waiting time and the expected number of waiting customers.

(d) Derive the exact expected number of customers ***E***(*X*) without the approximation in (a) and use it to recalculate the answers in (b) and (c).

Hint: ***E****(X) is computed as a series; relate it to the expected number of jobs in the M/M/1 system*.

Step-by-step solution

Show all steps

Step 1/5

a.

This system is k = 2 servers, $\lambda_A = 0.4\,\text{min}^{-1}$, $\lambda_S = 1/2\,\text{min}^{-1}$. Therefore the utilization is

$$r = \frac{\lambda_A}{\lambda_S} = 0.2$$

.

Let the frame is 1 second

Therefore

$$P_A = \lambda_A \Delta = 0.007$$
$$P_S = \lambda_S \Delta = 0.008$$

The steady state distribution will be

$$\pi_x = P(X = x) = \frac{r^x}{x!}\pi_0 \qquad for \quad x \le k$$
$$\pi_x = P(X = x) = \frac{r^k}{k!}\pi_0 \left(\frac{\text{r}}{\text{k}}\right)^{\text{x-k}} \qquad for \quad x > k$$
$$\pi_0 = P(X = 0) = \frac{1}{\sum_{i=1}^{k-1} \frac{r^i}{i!} + \frac{r^k}{(1 - r/k)k!}}$$

Here

$$\pi_0 = \frac{1}{1 + \frac{0.2}{\left(1 + 0.2 + \frac{0.2^2}{0.18}\right)}} = 0.8767$$

$$\pi_1 = 0.1753$$
$$\pi_2 = 0.0175$$
$$\pi_3 = 0.0075$$
$$\vdots$$
$$\pi_{10} = 1.753^{-10}$$

Therefore

$$E(X) = \sum_{x=0}^{10} x.\pi_x = 0.2165$$

Step 2/5
Hence the approximated expected number of customers in the bank is 0.2165.

Step 3/5
b.

Using Little's Law, the expected response time will be

$$E(R) = \frac{E(X)}{\lambda_A} = \frac{0.2165}{0.4} = 0.541\,\text{min}$$

Therefore the expected response time will be 32 seconds.

Step 4/5
c.

The expected waiting time will be

$$E(W) = \frac{E(X)}{\lambda_S} = \frac{0.2165}{0.5} = 0.433\,\text{min}$$

The expected number of waiting customers will be

$$E(X_W) = \lambda_A E(W) = 0.5 \times 0.433 = 0.1732$$

Therefore the expected waiting time is 26 second and the expected number of waiting customer is 0.1732.

Step 5/5
d.

The exact expected number of customers in the bank is

$$E(X) = \frac{r}{1-r} = \frac{0.2}{1-0.2} = 0.25$$

The expected response time is

$$E(R) = \frac{E(X)}{\lambda_A} = \frac{0.25}{0.4} = 0.625\,\text{min}$$

The expected waiting time will be

$$E(W) = \frac{E(X)}{\lambda_S} = \frac{0.25}{0.5} = 0.5\,\text{min}$$

The expected number of waiting customers will be

$$E(X_W) = \lambda_A E(W) = 0.4 \times 0.5 = 0.02$$

Hence the exact expected number of customers in the bank is 0.25, the expected response time is 38 seconds, the expected waiting time is 30 seconds and expected number of waiting customers is 0.02.

Chapter 7, Problem 21E
(0)

Problem
A toll area on a highway has three toll booths and works as an M/M/3 queuing system. On the average, cars arrive at the rate of one car every 5 seconds, and it takes 12 seconds to pay the toll, not including the waiting time. Compute the fraction of time when there are ten or more cars waiting in the line.

Step-by-step solution

Show all steps

Step 1/1

This system is k = 3 servers, $\lambda_A = 12\,\text{min}^{-1}$, $\lambda_S = 5\,\text{min}^{-1}$. Therefore the utilization is

$$r = \frac{\lambda_A}{\lambda_S} = 2.4$$
.

We know that for a M/M/k system,

$$P(X \le k) = \frac{r^x}{x!}\pi_0$$

$$P(X > k) = \frac{r^k}{k!}\pi_0 \left(\frac{\mathrm{r}}{\mathrm{k}}\right)^{\mathrm{x\text{-}k}}$$

$$\pi_0 = P(X = 0) = \frac{1}{\sum_{i=1}^{k-1} \frac{r^i}{i!} + \frac{r^k}{(1 - r/k)k!}}$$

Here

$$\pi_0 = \frac{1}{1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{(1 - 2.4/3) \times 3!}} = 0.05618$$

Therefore

$$P(X \le 9) = 0.8643$$

Implies

$$P(X \geq 10) = 1 - 0.8643 = 0.1357$$

Hence the fraction of time when there are ten or more cars waiting in the line is 0.1357.

Chapter 7, Problem 22E

(0)

Problem

Sports fans tune to a local sports talk radio station according to a Poisson process with the rate of three fans every two minutes and listen it for an Exponential amount of time with the average of 20 minutes.

(a) What queuing system is the most appropriate for this situation?

(b) Compute the expected number of concurrent listeners at any time.

(b) Find the fraction of time when 40 or more fans are tuned to this station.

Step-by-step solution

Show all steps

Step 1/3

a.

For this situation $M/M/\infty$ system will be the best appropriate system.

Step 2/3

b.

The expected number of concurrent at any time will be:

$$E(X) = r$$

Here

$$r = \frac{\mu_S}{\mu_A} = \frac{20}{2/3} = 30$$

Therefore expected number of concurrent at any time is 30.

Step 3/3

c.

The fraction of time when 40 or more fans are tuned to this station will be

$$P(X \geq 40) = 1 - P(X \leq 39) = 1 - 0.9537 = 0.0463$$

Hence the fraction of time when 40 or more fans are tuned to this station is 0.0463.

Chapter 7, Problem 23E
(0)

Problem
Internet users visit a certain web site according to an M/M/∞ queuing system with the arrival rate of 2 $\min^{-1}$ and the expected time of minutes spent at the site. Compute

(a) the expected number of visitors of the web site at any time;

(b) the fraction of time when nobody is browsing this web site.

Step-by-step solution

Show all steps

Step 1/2
a.

It is given that the arrival rate $\lambda_A = 2\min^{-1}$ and the arrival rate $\lambda_S = 1/5\min^{-1}$ The expected number of visitors at any time will be:

$$E(X) = r$$

Here

$$r = \frac{\lambda_A}{\lambda_S} = \frac{2}{1/5} = 10$$

Therefore expected number of visitors at any time is 30.

Step 2/2
b.

The fraction of time when nobody is browsing this web site will be

$$P(X = 0) = e^{-r} = e^{-10}$$

Hence the fraction of time when 40 or more fans are tuned to this station is e^{-10}.

Chapter 8, Problem 1E
(0)

Problem
The numbers of blocked intrusion attempts on each day during the first two weeks of the month were

$$56, 47, 49, 37, 38, 60, 50, 43, 43, 59, 50, 56, 54, 58$$

After the change of firewall settings, the numbers of blocked intrusions during the next 20 days were

$$53, 21, 32, 49, 45, 38, 44, 33, 32, 43, 53, 46, 36, 48, 39, 35, 37, 36, 39, 45.$$

Comparing the number of blocked intrusions before and after the change,

(a) construct side-by-side stem-and-leaf plots;

(b) compute the five-point summaries and construct parallel boxplots;

(c) comment on your findings.

These data are available in data set Intrusions.

Step-by-step solution

Show all steps

Step 1/5
The following data represents the number of blocked intrusion attempts on each day during the first two weeks of the month before and after the change firewall settings:

Before

56	47	49
37	38	60
50	43	43
59	50	56
54	58	

After

53	21	32	49
45	38	44	33
32	43	53	46
36	48	39	35
37	36	39	45

Step 2/5
a)

Construct the side by side stem and leaf plots for before and after readings.

Use MINITAB to construct the Stem and leaf plot.

Step-1: Enter the data into MINITAB sheet

Step-2: Click Graph→Stem and Leaf

Step-3: Enter the data into graph variables box

Step-3: Click Ok

The obtained output is as follows:

Before		After
Leaf	Stem	Leaf
	2	1
7 8	3	2 2 3 5 6 6 7 8 9 9
3 3 7 9	4	3 4 5 5 6 8 9
0 0 4 6 6 8 9	5	3 3
0	6	

Step 3/5
b)

Calculate the five- point's summaries and construct parallel boxplots.

Use MINITAB to construct the five number statistics and box plot.

Step-1: Enter the data into MINITAB sheet

Step-2: Click Basic statistics→Descriptive statistics

Step-3: Enter the data into variables box

Step-3: Click Ok.

The obtained output is as follows:

Descriptive Statistics: Befor, After

Variable	N	N*	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Befor	14	0	50.00	7.62	37.00	43.00	50.00	56.00	60.00
After	20	0	40.20	7.96	21.00	35.25	39.00	46.00	53.00

Step 4/5
Use MINITAB to construct the Box plots for before and after data.

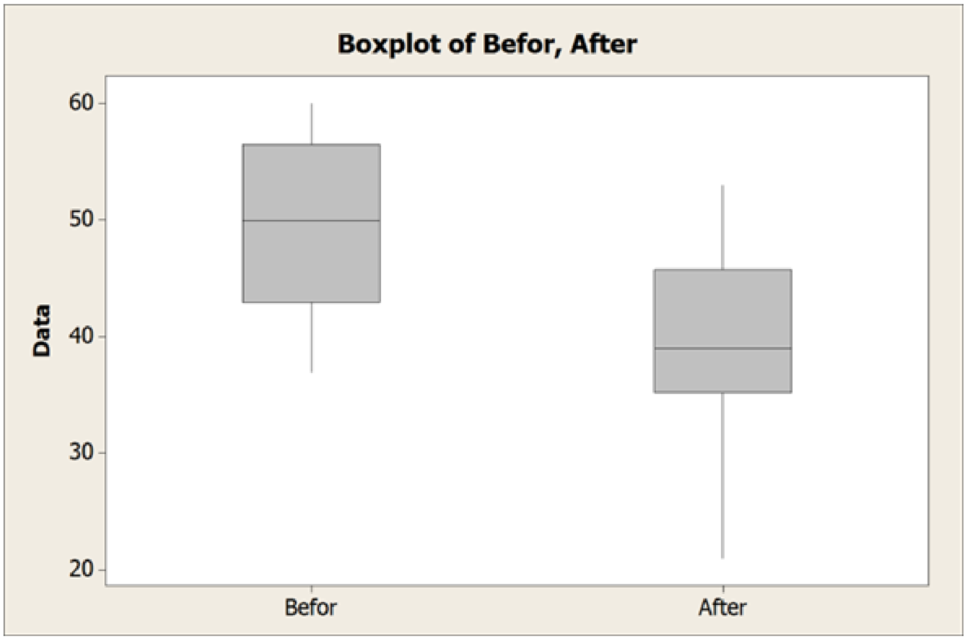
Step-1: Enter the data into MINITAB sheet

Step-2: Click Graph→Box plot→Select Multiple Simple

Step-3: Enter the data into graph variables box

Step-3: Click Ok.

The obtained Box plots are shown in below,



Step 5/5
c)

From the above box plots, after changing the firewall number of block intrusions decreases and both box plots indicate symmetric distribution and follows a normal distribution.

Chapter 8, Problem 2E
(0)

Problem
A network provider investigates the load of its network. The number of concurrent users is recorded at fifty locations (thousands of people),

17.2	22.1	18.5	17.2	18.6	14.8	21.7	15.8	16.3	22.8
24.1	13.3	16.2	17.5	19.0	23.9	14.8	22.2	21.7	20.7
13.5	15.8	13.1	16.1	21.9	23.9	19.3	12.0	19.9	19.4
15.4	16.7	19.5	16.2	16.9	17.1	20.2	13.4	19.8	17.7
19.7	18.7	17.6	15.9	15.2	17.1	15.0	18.8	21.6	11.9

These data are available in data set ConcurrentUsers.

(a) Compute the sample mean, variance, and standard deviation of the number of concurrent users.

(b) Estimate the standard error of the sample mean.

(c) Compute the five-point summary and construct a boxplot.

(d) Compute the interquartile range. Are there any outliers?

(e) It is reported that the number of concurrent users follows approximately Normal distribution. Does the histogram support this claim?

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/5

From the information, observe that a network provider investigates the load of its network.

The number of concurrent users is recorded at fifty locations as follows:

17.2	22.1	18.5	17.2	18.6	14.8	21.7	15.8	16.3	22.8
24.1	13.3	16.2	17.5	19	23.9	14.8	22.2	21.7	20.7
13.5	15.8	13.1	16.1	21.9	23.9	19.3	12	19.9	19.4
15.4	16.7	19.5	16.2	16.9	17.1	20.2	13.4	19.8	17.7
19.7	18.7	17.6	15.9	15.2	17.1	15	18.8	21.6	11.9

a)

Calculate the mean, variance and standard deviation of the number of concurrent users.

$$\begin{aligned}\text{Mean} &= \frac{1}{n}\sum x_i \\ &= \frac{1}{50}(17.2+22.1+....+11.9) \\ &= \frac{897.7}{50} \\ &= 17.954\end{aligned}$$

$$\begin{aligned}\text{Variance} &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 \\ &= \frac{(17.2-17.954)^2+(24.1-17.954)^2+...+(11.9-17.954)^2}{50-1} \\ &= \frac{488.445}{49} \\ &= 9.968\end{aligned}$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\text{Variance}} \\ &= \sqrt{9.968} \\ &= 3.157\end{aligned}$$

Step 2/5
b)

Calculate the standard error of the sample mean.

$$\begin{aligned} SE_{(\bar{x})} &= \frac{\text{Variance}}{\sqrt{n}} \\ &= \frac{3.157}{\sqrt{50}} \\ &= \frac{3.15}{7.071} \\ &= 0.447 \end{aligned}$$

Step 3/5
c)

Calculate the five-point summary and box plot.

Use MINITAB to construct the five number statistics and box plot.

Step-1: Enter the data into MINITAB sheet

Step-2: Click Basic statistics $\rightarrow$ Descriptive statistics

Step-3: Enter the data into variables box

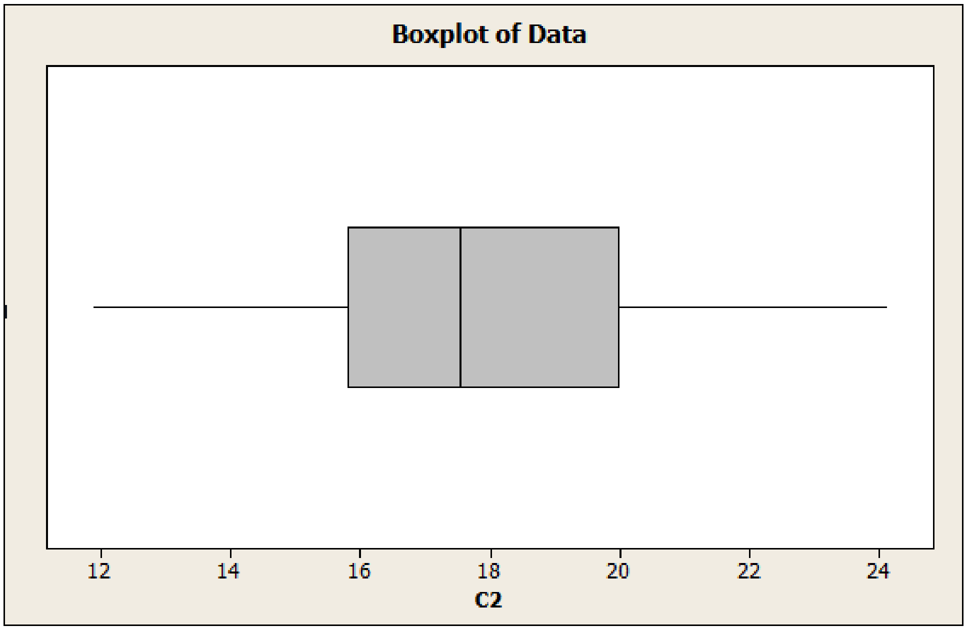
Step-4: Click Graphs $\rightarrow$ Select Box plot

Step-3: Click Ok.

The obtained output is as follows:

Descriptive Statistics: Data

Variable	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
C2	17.954	3.157	11.900	15.800	17.550	19.900	24.100



Based on the above box plot, we say that there is no outlier in the given data.

Step 4/5
d)

Calculate the interquartile range (IQR).

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 19.900 - 15.800 \\ &= 4.100 \end{aligned}$$

Calculate the lower and upper fences.

$$\begin{aligned} \text{Lower fence} &= Q_1 - 1.5(IQR) \\ &= 15.800 - 1.5(4.1) \\ &= 15.8 - 6.15 \\ &= 9.65 \end{aligned}$$

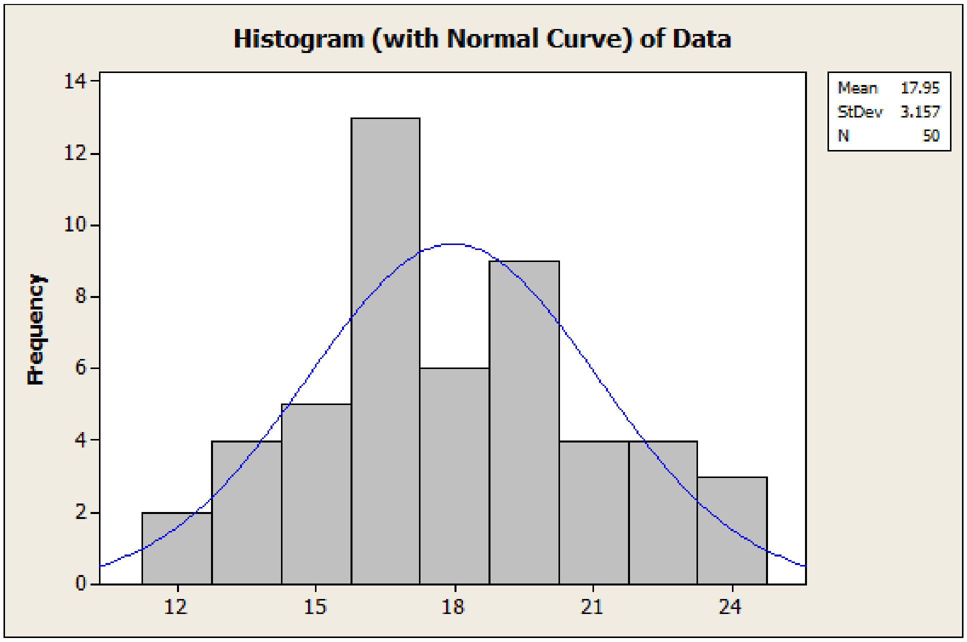
$$\begin{aligned} \text{Lower fence} &= Q_3 + 1.5(IQR) \\ &= 19.9 + 1.5(4.1) \\ &= 19.9 + 6.15 \\ &= 26.50 \end{aligned}$$

From the lower and upper fences, observe that there is no observation is lower than 9.65 and not exceeds to 26.50

Hence, there is no outlier presents in the given data.

Step 5/5
e)

Construct a histogram for the given data.



From the above histogram, the number of concurrent users does not follow normal distribution.

Chapter 8, Problem 3E
(0)

Problem
Verify the use of Chebyshev's inequality in (8.6) of Example 8.16. Show that if the population mean is indeed 48.2333 and the population standard deviation is indeed 26.5170, then at least 8/9 of all tasks require less than 127.78 seconds of CPU time.

Step-by-step solution

Show all steps

Step 1/1
The given information is as shown in below,

$\bar{X} = 48.2333$ and $s = 26.5170$

The Chebyshev's inequality is, $\bar{X} + 3s$

$$\begin{aligned}\bar{X}+3s &= 48.2333+3(26.5170)\\ &= 48.2333+79.551\\ &= 127.7843\\ &= 127.78 \qquad \text{(Round to 2 decimal place)}\end{aligned}$$

Hence, the all tasks require less than 127.78 seconds.

Chapter 8, Problem 4E
(0)

Problem
Use Table A4 to compute the probability for *any* Normal random variable to take a value within 1.5 interquartile ranges from population quartiles.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/3
The objective is to find the probability for any Normal random variable to take a value within 1.5 interquartile ranges from population quartiles.

Q_1 is first quartile and Q_3 is third quartile.

The interquartile range is written below:

$$IQR = Q_3 - Q_1$$

As the random variable *Z* is normally distributed, therefore probability of random variable *Z* greater than z_1 can be written as below:

$$P(Z > z_1) = 1 - P(Z < z_1)$$

As Q_1 contain 25% of the total area. Therefore, Quartile Q_1 of standard normal variable can be written as below:

$$P(Z < Q_1) = 0.25$$

Subtract each side from 1

$$1-P(Z<Q_1)=1-0.25$$

$$P(Z>Q_1)=0.75 \qquad \{P(Z>z_1)=1-P(Z<z_1)\}$$

$$P(Z<-Q_1)=0.75 \qquad \left\{\begin{array}{l}\text{Due to symmetry}\\ P(Z<-z_1)=P(Z>z_1)\end{array}\right\} \qquad \text{...... (1)}$$

Step 2/3

Procedure to calculate Z value from Table A4 is given below:

In the second table of Table A4, locate the nearest value of the 0.75 in the areas value (in the middle of the table) and note the z value in the corresponding row and column.

The screen shot of the Table is provided below:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

$$P(Z<0.67)\approx 0.75 \qquad \text{......(2)}$$

After compare equation (1) and (2),

$$-Q_1=0.67$$
$$Q_1=-0.67$$

Step 3/3

The third quartile Q_3 contain 75% of the total area. Therefore, probability can be written as bellow:

$$P(Z<Q_3)=0.75 \qquad \text{...... (3)}$$

After compare equation (2) and (3),

$$Q_3=0.67$$

Therefore, IQR (inter quartile range) is given below:

$$\begin{aligned}IQR&=Q_3-Q_1\\&=0.67-(-0.67)\\&=1.34\end{aligned}$$

$$
\begin{aligned}
1.5 \times IQR &= 1.5 \times 1.34 \\
&= 2.01
\end{aligned}
$$

Now, the probability for any normal random variable to take a value within 1.5 interquartile range from population quartiles is given below:

$$
\begin{aligned}
P\begin{pmatrix} (Q_1 - 1.5 \times IQR) < Z < \\ (Q_3 + 1.5 \times IQR) \end{pmatrix} &= P\left(\left(-0.67 - 2.01\right) < Z < \left(0.67 + 2.01\right)\right) \\
&= P\left(-2.68 < Z < 2.68\right) \\
&= P\left(Z < 2.68\right) - P\left(Z < -2.68\right) \quad \left\{\begin{aligned} &\text{Due to symmetry,} \\ &P\left(-z_1 < Z < z_1\right) \\ &= P\left(Z < z_1\right) - P\left(Z < -z_1\right) \end{aligned}\right\} \\
&= P\left(Z < 2.68\right) - \left[1 - P\left(Z < 2.68\right)\right] \quad \left\{\begin{aligned} &P\left(Z < -z_1\right) \\ &= 1 - P\left(Z < z_1\right) \end{aligned}\right\} \\
&= 2 \times P\left(Z < 2.68\right) - 1
\end{aligned}
$$

From the Table A4 probability can be calculated by the following procedure:

In Table A4 locate 2.6 in the first column and 0.08 in the first row, the value which is at the intersection of both will be the required value.

The screen shot of the table is given below:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

$$P(Z<2.68)=0.9963$$

The required probability can be written as below:

$$\begin{aligned}P\left(\left(Q_1-1.5\times IQR\right)<Z<\left(Q_3+1.5\times IQR\right)\right)&=2\times P(Z<2.68)-1\\&=2\times 0.9963-1\\&=0.9926\end{aligned}$$

Therefore, the probability for any normal random variable to take a value within 1.5 interquartile range from population quartiles is 0.9926

Chapter 8, Problem 5E

(0)

Problem

The following data set shows population of the United States (in million) since 1790,

Year	1790	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900
Population	3.9	5.3	7.2	9.6	12.9	17.1	23.2	31.4	38.6	50.2	63.0	76.2

Year	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
Population	92.2	106.0	123.2	132.2	151.3	179.3	203.3	226.5	248.7	281.4	308.7

Construct a time plot for the U.S. population. What kind of trend do you see? What information can be extracted from this plot?

These data are available in data set PopulationUSA.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/2

The following data describes the population of the United States (in millions):

Year	Population	Year	Population
1790	3.9	1910	92.2
1800	5.3	1920	106
1810	7.2	1930	123.2
1820	9.6	1940	132.2
1830	12.9	1950	151.3
1840	17.1	1960	179.3
1850	23.2	1970	203.3
1860	31.4	1980	226.5
1870	38.6	1990	248.7
1880	50.2	2000	281.4
1890	63	2010	308.7
1900	76.2		

Step 2/2

Construct a time series plot for the US population.

Use the following MINITAB steps to draw a time plot.

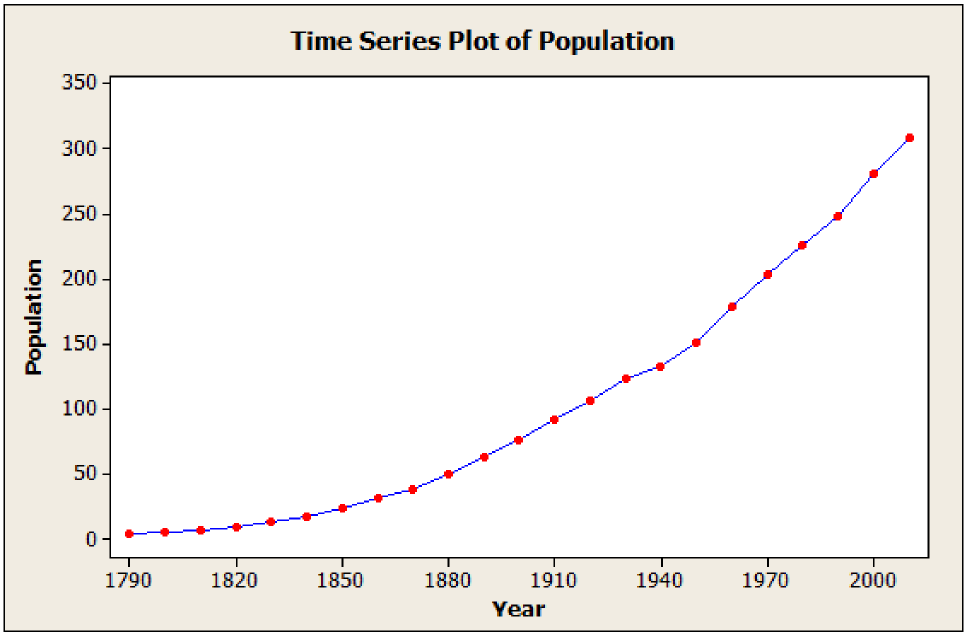
Step-1: Enter the data into MINITAB sheet

Step-2: Click Graph → Time series plot → Simple → Ok

Step-3: Enter the population variables in series box.

Step-4: Click OK

The obtained output is as follow:



The above time plot indicates that the US population is follows an upward trend and increasing year to year.

Chapter 8, Problem 6E
(1)

Problem
Refer to Exercise 8.5 (data set PopulationUSA). Compute 10-year *increments* of the population growth $x_1 = 5.3 - 3.9$, $x_2 = 7.2 - 5.3$, etc.

(a) Compute sample mean, median, and variance of 10-year increments. Discuss how the U.S. population changes during a decade.

(b) Construct a time plot of 10-year increments and discuss the observed pattern.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/6
The following data describes the population growth of the United States.

3.9	23.2	92.2	203.3
5.3	31.4	106	226.5
7.2	38.6	123.2	248.7
9.6	50.2	132.2	281.4
12.9	63	151.3	308.7
17.1	76.2	179.3	

Step 2/6

Find the x_i observations.

Population	x_i
3.9	$--$
5.3	$x_1 = 5.3 - 3.9 = 1.4$
7.2	$x_2 = 7.2 - 5.3 = 1.9$
9.6	$x_3 = 9.6 - 7.2 = 2.4$
.	.
.	.
.	.
281.4	$x_{21} = 281.4 - 248.7 = 32.7$
308.7	$x_{22} = 308.7 - 281.4 = 27.3$

Step 3/6

a)

Calculate the mean of 10 increments.

$$\begin{aligned}\text{Mean} &= \frac{\sum x_i}{n} \\ &= \frac{1.4 + 1.9 + 2.4 + ... + 32.7 + 27.3}{22} \\ &= \frac{304.8}{22} \\ &= 13.85\end{aligned}$$

Step 4/6

Calculate the median of 10 increments.

Re arrange the x_i data in ascending order,

1.4	1.9	2.4	3.3	4.2	6.1
7.2	8.2	9	11.6	12.8	13.2
13.8	16	17.2	19.1	22.2	23.2
24	27.3	28	32.7		

$$\text{Median} = \frac{11^{\text{th}}\text{ observation} + 12^{\text{th}}\text{ observation}}{2}$$
$$= \frac{12.8 + 13.2}{2}$$
$$= \frac{26}{2}$$
$$= 13$$

Step 5/6

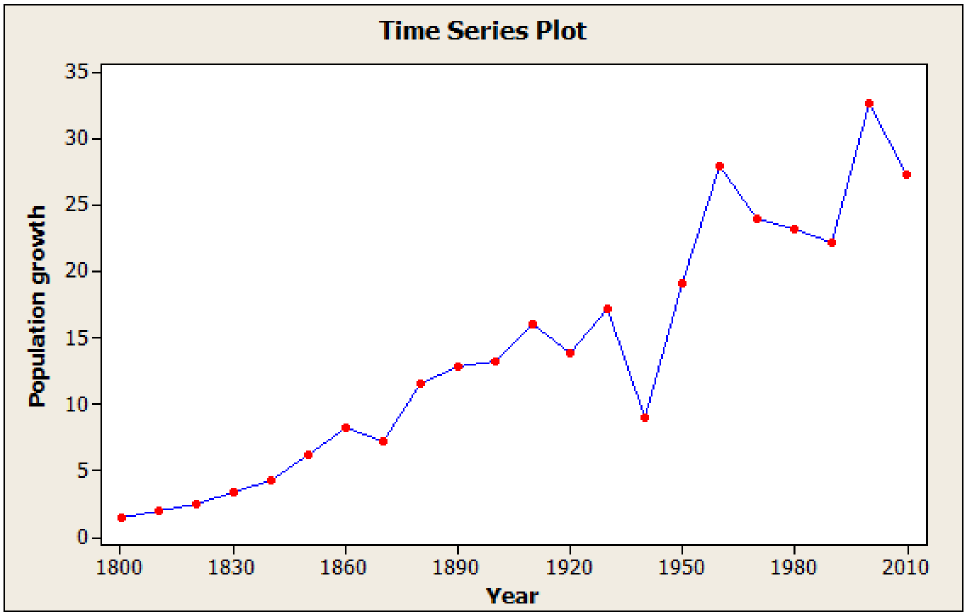
Calculate the variance of 10 increments.

$$\text{Variance} = \frac{\sum (x_i - \overline{x})^2}{n-1}$$
$$= \frac{(1.4 - 13.85)^2 + (1.9 - 13.85)^2 + \ldots + (27.3 - 13.85)^2}{22-1}$$
$$= \frac{1839.675}{21}$$
$$= 87.60$$

Step 6/6

b)

Construct a time plot of 10 years increments.



From the above time plot, the population growth is follows an upward direction and there is some fluctuations from 1930 to 2010.

Chapter 8, Problem 7E
(0)

Problem
Refer to Exercise 8.5 (data set PopulationUSA). Compute 10-year *relative population change* $y_1 = (5.3 - 3.9)/3.9$, $y_2 = (7.2 - 5.3)/5.3$, etc.

(a) Compute sample mean, median, and variance of the relative population change.

(b) Construct a time plot of the relative population change. What trend do you see now?

(c) Comparing the time plots in Exercises 8.6 and 8.7, what kind of correlation between x_i and y_i would you expect? Verify by computing the sample correlation coefficient

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})/(n-1)}{s_x s_y}.$$

What can you conclude? How would you explain this phenomenon?

Step-by-step solution

Show all steps

Step 1/7
The following data describes the population growth of the United States.

3.9	23.2	92.2	203.3
5.3	31.4	106	226.5
7.2	38.6	123.2	248.7
9.6	50.2	132.2	281.4
12.9	63	151.3	308.7
17.1	76.2	179.3	

Step 2/7

Find the y_i observations.

Population	y_i
3.9	$--$
5.3	$y_1 = \frac{5.3-3.9}{3.9} = 0.359$
7.2	$y_2 = \frac{7.2-5.3}{5.3} = 0.358$
9.6	$y_3 = \frac{9.6-7.2}{7.2} = 0.333$
. . .	. . .
281.4	$y_{21} = \frac{281.4-248.7}{248.7} = 0.131$
308.7	$y_{22} = \frac{308.7-281.4}{281.4} = 0.097$

Step 3/7
a)

Calculate the mean of 10 increments.

$$\begin{aligned}\text{Mean} &= \frac{\sum y_i}{n} \\ &= \frac{0.359+0.358+0.333+\ldots+0.131+0.097}{22} \\ &= \frac{4.924}{22} \\ &= 0.2238\end{aligned}$$

Step 4/7
Calculate the median of 10 increments.

Re arrange the x_i data in ascending order,

0.073	0.097	0.098	0.114	0.131	0.134
0.144	0.15	0.162	0.185	0.21	0.21
0.229	0.255	0.301	0.326	0.333	0.344
0.353	0.357	0.358	0.359		

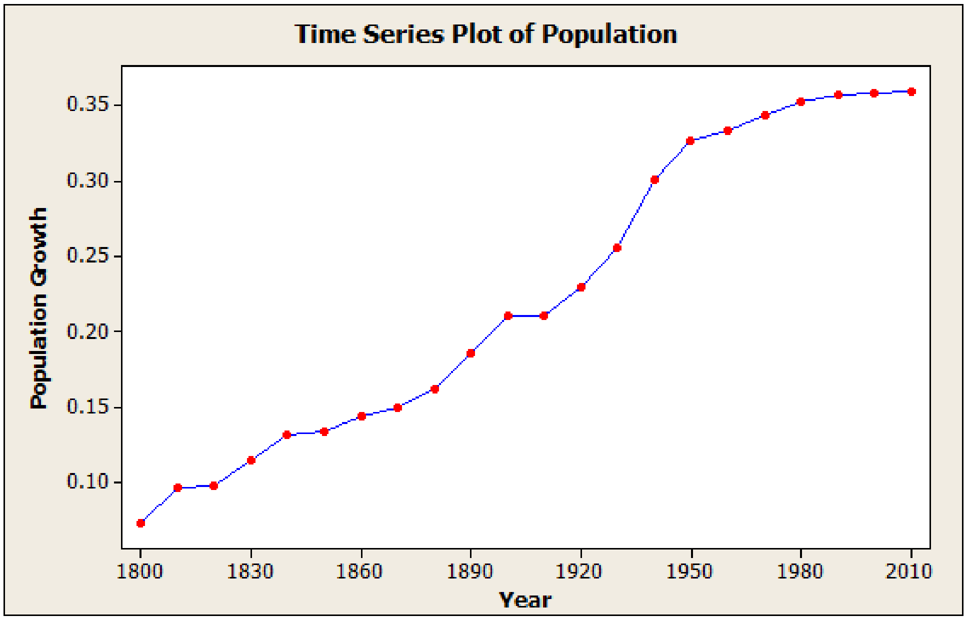
$$\begin{aligned}\text{Median} &= \frac{11^{\text{th}}\text{observation} + 12^{\text{th}}\text{observation}}{2}\\ &= \frac{0.21+0.21}{2}\\ &= \frac{42}{2}\\ &= 0.21\end{aligned}$$

Step 5/7
Calculate the variance of 10 increments.

$$\begin{aligned}\text{Variance} &= \frac{\sum(x_i - \overline{x})^2}{n-1}\\ &= \frac{(0.073-0.2238)^2 + (0.097-0.2238)^2 + \ldots + (0.359-0.2238)^2}{22-1}\\ &= \frac{0.2154}{21}\\ &= 0.010257\end{aligned}$$

Step 6/7
b)

Construct a time plot of 10 years increments.



From the above time plot, the population growth indicates the upward direction and there is some fluctuations from 1930 to 2010.

Step 7/7

c)

Calculate the correlation coefficient of (x_i, y_i).

x_i	y_i	x^2	y^2	xy
1.4	0.359	1.96	0.129	0.503
1.9	0.358	3.61	0.128	0.680
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
32.7	0.097	1069.29	0.009	3.172
$\sum x = 304.8$	$\sum y = 4.923$	$\sum x^2 = 6062.54$	$\sum y^2 = 1.317$	$\sum xy = 50.346$

$$\begin{aligned}
\text{Correlation}(\rho) &= \frac{\sum (x_i - \bar{x})(y_i - \bar{y}) / n-1}{S_x S_y} \\
&= \frac{n\sum xy - (\sum x \sum y)}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right)\left(n\sum y^2 - (\sum y)^2\right)}} \\
&= \frac{22(50.346) - (304.8 \times 4.923)}{\sqrt{22\left(6062.54 - (304.8)^2\right)\left(22(1.317) - (4.923)^2\right)}} \\
&= -0.897
\end{aligned}$$

There is a strong negative correlation occurs between the x_i and y_i variables.

Chapter 8, Problem 8E
(0)

Problem
Consider three data sets (also, in data set Symmetry).

(1) 19, 24, 12, 19, 18, 24, 8, 5, 9, 20, 13, 11, 1, 12, 11, 10, 22, 21, 7, 16, 15, 15, 26, 16, 1, 13, 21, 21, 20, 19

(2) 17, 24, 21, 22, 26, 22, 19, 21, 23, 11, 19, 14, 23, 25, 26, 15, 17, 26, 21, 18, 19, 21, 24, 18, 16, 20, 21, 20, 23, 33

(3) 56, 52, 13, 34, 33, 18, 44, 41, 48, 75, 24, 19, 35, 27, 46, 62, 71, 24, 66, 94, 40, 18, 15, 39, 53, 23, 41, 78, 15, 35

(a) For each data set, draw a histogram and determine whether the distribution is right-skewed, left-skewed, or symmetric.

(b) Compute sample means and sample medians. Do they support your findings about skewness and symmetry? How?

Step-by-step solution

Show all steps

100% (6 ratings) for this solution

Step 1/4
a)

Construct a Histogram and determine shape of the distribution for group-1.

Use MINITAB Software to draw a Histogram.

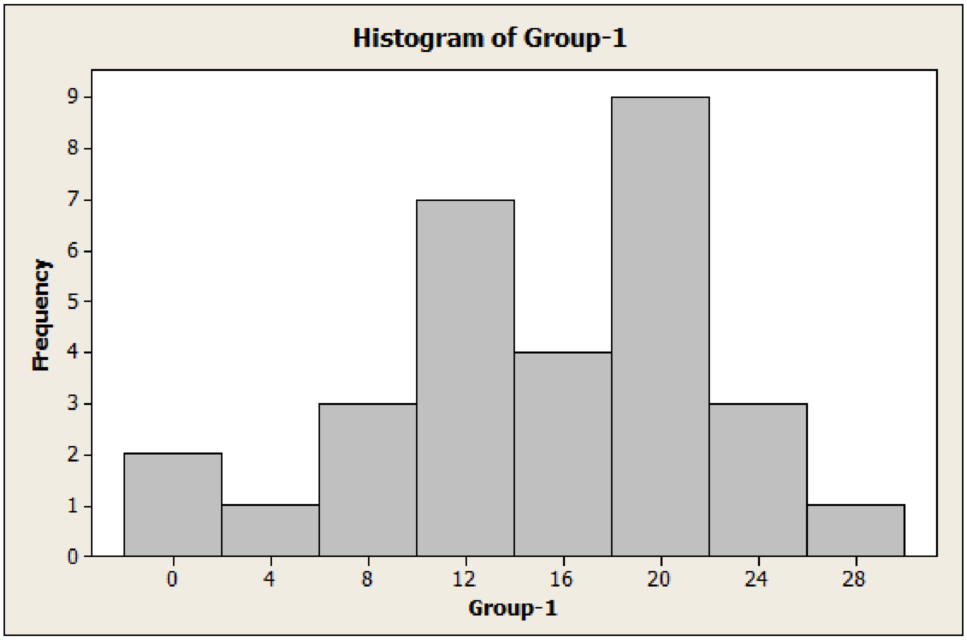
Step-1: Enter the data into MINITAB sheet

Step-2: Click Basic statistics $\rightarrow$ Descriptive statistics

Step-3: Enter the data into variables box

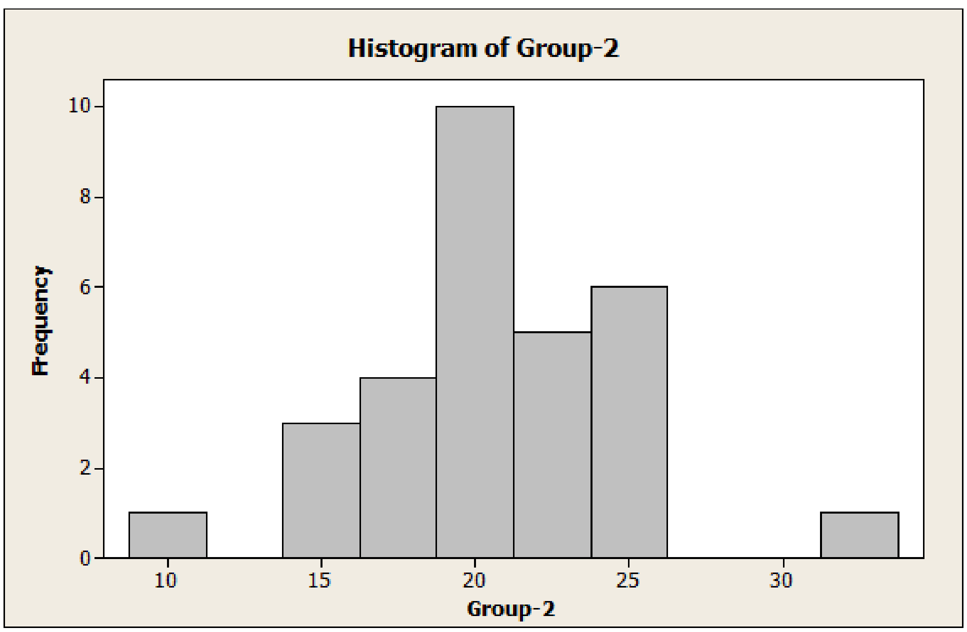
Step-4: Click Graphs $\rightarrow$ Histogram

Step-3: Click Ok.



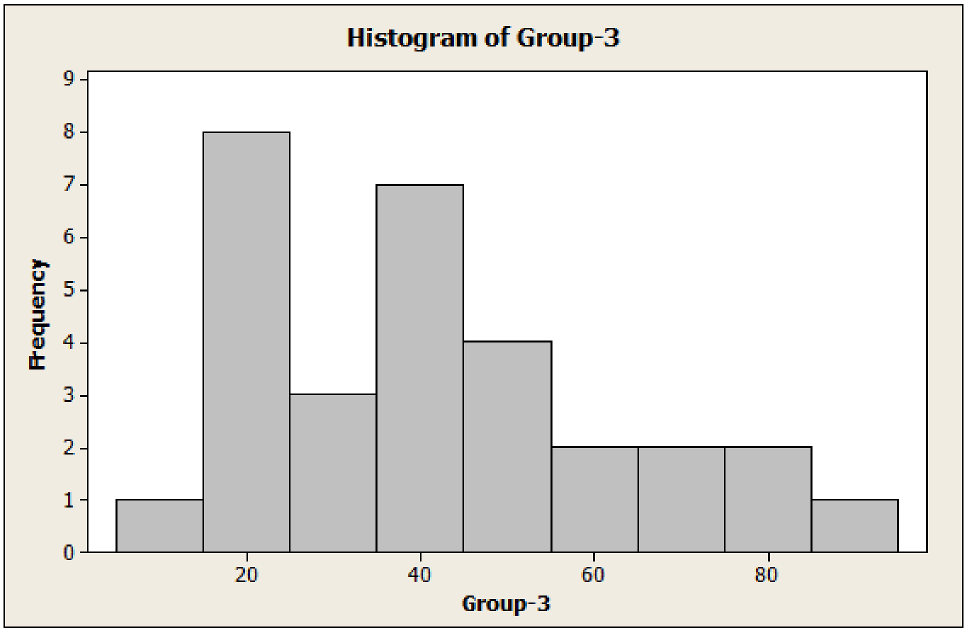
The above histogram is skewed to the right, so the group-1 data follows a left skewed.

Step 2/4
Construct a Histogram and determine shape of the distribution for group-2.



The above histogram is symmetric, so the group-2 data follows a normal distribution.

Step 3/4
Construct a Histogram and determine shape of the distribution for group-3.



The above histogram is skewed to the right, so the group-2 data follows a right skewed distribution.

Step 4/4

b)

Compute the sample means and medians for each group.

Use MINITAB Software to calculate the Mean and Median.

Step-1: Enter the data into MINITAB sheet

Step-2: Click Basic statistics → Descriptive statistics

Step-3: Enter the data into variables box

Step-3: Click Ok.

The obtained output is as follows:

Descriptive Statistics: Group-1, Group-2, Group-3

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Group-1	30	0	14.97	1.21	6.63	1.00	10.75	15.50	20.25
Group-2	30	0	20.833	0.788	4.316	11.000	18.000	21.000	23.250
Group-3	30	0	41.30	3.84	21.04	13.00	23.75	39.50	53.75

Hence, these findings are support you’re finding about skewnees and symmetric.

Chapter 8, Problem 9E
(0)

Problem
The following data set represents the number of new computer accounts registered during ten consecutive days.

$$43, 37, 50, 51, 58, 105, 52, 45, 45, 10.$$

(a) Compute the mean, median, quartiles, and standard deviation.

(b) Check for outliers using the 1.5(IQR) rule.

(c) Delete the detected outliers and compute the mean, median, quartiles, and standard deviation again.

(d) Make a conclusion about the effect of outliers on basic descriptive statistics.

These data are available in data set Accounts.

Step-by-step solution

Show all steps

100% (6 ratings) for this solution

Step 1/10
The following data describes the number of new computer accounts registered during ten consecutive days:

43	37	50	51	58
105	52	45	45	10

Here, the sample size $n = 10.$

(a)

Calculate the mean, median, quartiles and standard deviation of the given data.

Use the following formula to compute the mean:

$$\begin{aligned} \bar{x} &= \frac{1}{n}\sum x_i \\ &= \frac{10+37+...+58+105}{10} \\ &= \frac{496}{10} \\ &= \boxed{49.6} \end{aligned}$$

Therefore, the mean value is 49.6.

Step 2/10

Use the following formula to compute the median:

$$\text{Median}_{(\text{Odd})} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

$$\text{Median}_{(\text{Even})} = \frac{\left(\frac{n+2}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2}\right)^{th} \text{ observation}}{2}$$

Here, *n* is the total number of observations = 10.

$$\begin{aligned}\text{Median}_{(\text{Even})} &= \frac{\left(\frac{10+2}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{10}{2}\right)^{th} \text{ observation}}{2} \\ &= \frac{5^{th} \text{ observation} + 6^{\text{th}} \text{ observation}}{2} \\ &= \frac{45+50}{2} \\ &= 47.5 \\ &= Q_2\end{aligned}$$

Therefore, the median value is 47.5.

Step 3/10

Use the following formula to compute the first and third quartiles:

$$Q_1 = np \text{ observation}$$
$$Q_3 = n(1-p) \text{ observation}$$

Here, *n* is the sample size.

p quartile proportion value.

From the known information, the quartile proportion value is 0.25.

$$\begin{aligned}Q_1 &= n(1-p) \text{ observation} \\ &= (10)(0.25) \text{ observation} \\ &= 2.5 \text{ observation} \\ &\approx 3^{rd} \text{ observation} \\ &= \boxed{43}\end{aligned}$$

$$\begin{aligned}
Q_3 &= n(1-p) \text{ observation} \\
&= (10)(1-0.25) \text{ observation} \\
&= 7.5 \text{ observation} \\
&\approx 8^{th} \text{ observation} \\
&= \boxed{52}
\end{aligned}$$

Therefore, the values of first and third quartiles are 43 and 52 respectively.

Step 4/10

Use the following formula to compute the standard deviation:

$$\begin{aligned}
s &= \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \\
&= \sqrt{\frac{(10-49.6)^2 + (37-49.6)^2 + \text{L} \ + (105-49.6)^2}{10-1}} \\
&= \sqrt{\frac{4960.4}{9}} \\
&= \boxed{23.4767}
\end{aligned}$$

Therefore, the standard deviation value is 23.4767.

Step 5/10

(b)

Check for outlier using the 1.5(IQR) rule.

$$\begin{aligned}
\text{Lower limit} &= Q_1 - 1.5(Q_3 - Q_1) \\
&= 43 - 1.5(52-43) \\
&= 43 - 13.5 \\
&= 29.5
\end{aligned}$$

$$\begin{aligned}
\text{Upper limit} &= Q_3 + 1.5(Q_3 - Q_1) \\
&= 52 + 1.5(52-43) \\
&= 65.5
\end{aligned}$$

Now, identify the outliers in the given data,

43	37	50	51	58
105	52	45	45	10

Here, 10 and 105 are outliers which is outside of the interval $[29.5, 65.5]$.

Step 6/10
(c)

Delete the detected outliers and calculate the mean, median, quartiles and standard deviation.

The available data is,

43	37	50	51	58
	52	45	45	

Use the following formula to compute the mean:

$$\begin{aligned}\bar{x} &= \frac{1}{n}\sum x_i \\ &= \frac{37+43+...+58}{8} \\ &= \frac{381}{8} \\ &= \boxed{47.625}\end{aligned}$$

Therefore, the mean value is 47.625.

Step 7/10
Use the following formula to compute the median:

$$\text{Median}_{(\text{Odd})} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{observation}$$

$$\text{Median}_{(\text{Even})} = \frac{\left(\frac{n+2}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2}\right)^{th} \text{observation}}{2}$$

Here, *n* is the total number of observations = 8.

$$\begin{aligned}\text{Median}_{(\text{Even})} &= \frac{\left(\frac{8+2}{2}\right)^{\text{th}} \text{observation} + \left(\frac{8}{2}\right)^{th} \text{observation}}{2} \\ &= \frac{4^{th}\text{observation} + 5^{\text{th}}\text{observation}}{2} \\ &= \frac{45+50}{2} \\ &= 47.5 \\ &= Q_2\end{aligned}$$

Therefore, the median value is 47.5.

Step 8/10
Use the following formula to compute the first and third quartiles:

$$Q_1 = np \text{ observation}$$
$$Q_3 = n(1-p) \text{ observation}$$

Here, *n* is the sample size.

p quartile proportion value.

From the known information, the quartile proportion value is 0.25.

$$\begin{aligned} Q_1 &= n(1-p) \text{ observation} \\ &= (8)(0.25) \text{ observation} \\ &= 2^{nd} \text{ observation} \\ &= \boxed{43} \end{aligned}$$

$$\begin{aligned} Q_3 &= n(1-p) \text{ observation} \\ &= (8)(1-0.25) \text{ observation} \\ &= 6^{th} \text{ observation} \\ &= \boxed{51} \end{aligned}$$

Therefore, the values of first and third quartiles are 43 and 51 respectively.

Step 9/10
Use the following formula to compute the standard deviation:

$$\begin{aligned} s &= \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \\ &= \sqrt{\frac{(37-47.625)^2 + (43-47.625)^2 + \text{L} \ + (58-47.625)^2}{8-1}} \\ &= \sqrt{\frac{291.875}{7}} \\ &= \boxed{6.4573} \end{aligned}$$

Therefore, the standard deviation value is 6.4573.

Step 10/10
(d)

Conclusion: From the given data, the mean, standard deviation and quartile values are decreased after removing the outliers.

Chapter 9, Problem 1E
(2)

Problem
Estimate the unknown parameter θ from a sample

$$3, 3, 3, 3, 3, 7, 7, 7$$

drawn from a discrete distribution with the probability mass function

$$\begin{cases} P(3) & = & \theta \\ P(7) & = & 1-\theta \end{cases}$$

Compute two estimators of θ:

(a) the method of moments estimator;

(b) the maximum likelihood estimator.

Also,

(c) Estimate the standard error of each estimator of θ.

Step-by-step solution

Show all steps

100% (17 ratings) for this solution

Step 1/5
Consider a sample of numbers $3,3,3,3,3,7,7,7$ drawn from a discrete distribution with the probability mass function,

$$\begin{cases} P(3)=\theta \\ P(7)=1-\theta \end{cases}$$

Step 2/5

(a)

The estimator of θ using the method of moments estimator is required.

The first population moment is defined as,

$$\mu_1' = E(X) = \sum xP(x)$$

Substitute the probabilities and so the expected value is calculated as,

$$\begin{aligned} \mu_1' &= 3(\theta) + 7(1-\theta) \\ &= 3\theta + 7 - 7\theta \\ &= 7 - 4\theta \end{aligned}$$

The first sample moment is defined as,

$$m_1 = \overline{X}$$

Substitute the observations of the sample and the number of observations is 8. So, the first sample moment is calculated as,

$$\begin{aligned} m_1 &= \frac{3+3+3+3+3+7+7+7}{8} \\ &= \frac{36}{8} \\ &= 4.5 \end{aligned}$$

Equate the first population moment and first sample moment to calculate the estimator of θ.

$$\begin{aligned} \mu_1' &= m_1 \\ 7 - 4\theta &= 4.5 \\ \theta &= \frac{7-4.5}{4} \\ \theta &= 0.625 \end{aligned}$$

Therefore, the estimator of θ using the method of moments estimator is $\boxed{0.625}$.

Step 3/5

(b)

The estimator of θ using the method of moments estimator is required.

First find the likelihood of observing the sample $X = \{3,3,3,3,3,7,7,7\}$.

$$\begin{aligned} L(\theta) &= P(X=3)P(X=3)P(X=3)P(X=3)P(X=3)P(X=7)P(X=7)P(X=7) \\ &= (\theta)(\theta)(\theta)(\theta)(\theta)(1-\theta)(1-\theta)(1-\theta) \\ &= \theta^5(1-\theta)^3 \end{aligned}$$

Take logarithm of the obtained likelihood.

$$\begin{aligned} l(\theta) &= \log L(\theta) \\ &= 5\log\theta + 3\log(1-\theta) \end{aligned}$$

Differentiate and solve the obtained derivative equating to zero for θ.

$$l'(\theta) = \frac{5}{\theta} - \frac{3}{1-\theta} = 0$$

Simplify. So,

$$\begin{aligned} 5(1-\theta) - 3\theta &= 0 \\ 5 - 5\theta - 3\theta &= 0 \\ 8\theta &= 5 \\ \theta &= \frac{5}{8} \\ &= 0.625 \end{aligned}$$

Therefore, the estimator of θ using the maximum likelihood estimator is $\boxed{0.625}$.

Step 4/5
(c)

The standard error of each estimator of θ is required. The estimator of θ is obtained as 0.625 in both the methods.

Use the formula for standard error of estimator and substitute $\hat{p} = 0.625$.

Step 5/5
So,

$$\begin{aligned} SE_{\hat{p}} &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= \sqrt{\frac{0.625(1-0.625)}{8}} \\ &= \sqrt{0.029296875} \\ &\approx 0.171 \end{aligned}$$

Therefore, the standard error of the estimator of θ is $\boxed{0.171}$.

Chapter 9, Problem 2E
(0)

Problem
The number of times a computer code is executed until it runs without errors has a Geometric distribution with unknown parameter *p*. For 5 independent computer projects, a student records the following numbers of runs:

3 7 5 3 2

Estimate *p*

(a) by the method of moments;

(b) by the method of maximum likelihood.

Step-by-step solution

Show all steps

Step 1/4
From the information, observe that the number of times a computer code is executed until it runs without error has a Geometric distribution with unknown parameter.

The probability mass function of the geometric distribution is as follows:

$$p(x) = pq^{x-1} \quad x = 1,2,\ldots.n$$

Consider a random sample of 5 independent computer projects.

Step 2/4
a)

Estimate the p value by use method of moments.

$$p = \frac{1}{\overline{X}}$$
$$= \frac{1}{\frac{1}{n}\sum_{i=1}^{n} X_i}$$
$$= \frac{1}{\frac{1}{5}(3+7+5+3+2)}$$
$$= \frac{5}{20}$$
$$= \boxed{0.25}$$

Step 3/4

b)

Estimate the p value by use method of maximum likelihood.

$$\text{Likelihood function}(L) = p^n (1-p) \sum_{i=1}^{n} X_i$$
$$\ln(L) = n\ln(p) + \left[\sum_{i=1}^{n} X_i - n\right]\ln(1-p)$$
$$\frac{dL}{dp} = \frac{n}{p} - \frac{\left[\sum_{i=1}^{n} X_i - n\right]}{1-p}$$
$$\Rightarrow \frac{n}{p} = \frac{(n\overline{X} - n)}{(1-p)}$$

$$\Rightarrow n(1-p) = p\left(n - \overline{X}\right)$$
$$\Rightarrow n - np = n\overline{X}p - np$$
$$\Rightarrow \frac{n}{n} = p\overline{X}$$
$$\Rightarrow p = \frac{1}{\overline{X}}$$

Step 4/4

Thus, the p estimated value by using maximum likelihood hood method is equal to the method of moments.

$$\begin{aligned} p &= \frac{1}{\bar{X}} \\ &= \frac{1}{\frac{1}{n}\sum_{i=1}^{n} X_i} \\ &= \frac{1}{\frac{1}{5}(3+7+5+3+2)} \\ &= \frac{5}{20} \\ &= \boxed{0.25} \end{aligned}$$

hapter 9, Problem 4E
(1)

Problem
A sample of 3 observations ($X_1 = 0.4$, $X_2 = 0.7$, $X_3 = 0.9$) is collected from a continuous distribution with density

$$f(x) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Estimate θ by *your favorite method.*

Step-by-step solution

Show all steps

100% (7 ratings) for this solution

Step 1/3
From the information, consider a random sample of 3 observations is collected from a continuous distribution with density function is as follows:

$$f(x) = \begin{cases} \theta x^{\theta-1} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Consider $X_1, X_2 \text{ and } X_3$ are the three variables that represent the three observations 0.40, 0.70 and 0.90 respectively.

Step 2/3
Find the method of moment estimator.

$$\hat{\theta} = \frac{\overline{x}}{1-\overline{x}}$$
$$= \frac{\left(\frac{0.4+0.7+0.9}{3}\right)}{1-\left(\frac{0.4+0.7+0.9}{3}\right)}$$
$$= \frac{2/3}{1/3}$$
$$= \boxed{2}$$

Step 3/3

Find the maximum likelihood estimator.

$$L(\theta) = \prod_{i=1}^{3} \theta x_i^{\theta-1}$$
$$= \theta^3 \left(\prod_{i=1}^{3} x_i\right)^{\theta-1}$$

$$l(\theta) = 3\log\theta + (\theta-1)\sum_{i=1}^{3} \log x_i$$

$$\frac{\partial l\theta}{\partial\theta} = \frac{3}{\theta} + \sum_{i=1}^{3} \log x_i$$
$$\hat{\theta} = \frac{-3}{\sum_{i=1}^{3} \log x_i}$$
$$= \frac{-3}{\log(0.4)+\log(0.7)+\log(0.90)}$$

$$= \frac{-3}{[-0.9163-0.3567-0.1054]}$$
$$= \frac{-3}{-1.378326}$$
$$\cong \boxed{2.1766}$$

Chapter 9, Problem 5E

(0)

Problem

A sample $(X_1, \ldots, X_{10})$ is drawn from a distribution with a probability density function

$$\frac{1}{2}\left(\frac{1}{\theta}e^{-x/\theta} + \frac{1}{10}e^{-x/10}\right),\ 0 < x < \infty$$

The sum of all 10 observations equals 150.

(a) Estimate θ by the method of moments.

(b) Estimate the standard error of your estimator in (a).

Step-by-step solution

Show all steps

Step 1/4

Consider a sample $(X_1, X_2, \ldots, X_{10})$ which is taken from a distribution with a probability density function,

$$\frac{1}{2}\left(\frac{1}{\theta}e^{-x/\theta} + \frac{1}{10}e^{-x/10}\right),\ 0 < x < \infty$$

Step 2/4

The sum of all 10 observations is equal to 150.

Step 3/4

(a)

The estimator of θ using the method of moments is required.

The first population moment is defined as,

$$\mu_1' = E(X) = \int xf(x)dx$$

Substitute the probability density function and so the expected value is calculated as,

$$\begin{aligned}
\mu_1' &= \int xf(x)dx \\
&= \int_0^\infty x \cdot \frac{1}{2}\left(\frac{1}{\theta}e^{-x/\theta} + \frac{1}{10}e^{-x/10}\right)dx \\
&= \frac{1}{2}\int_0^\infty x\frac{1}{\theta}e^{-x/\theta}dx + \frac{1}{2}\int_0^\infty x\frac{1}{10}e^{-x/10}dx \\
&= \frac{1}{2\theta}\left[-\frac{1}{\theta}xe^{-x/\theta} + e^{-x/\theta}\right]_0^\infty + \frac{1}{20}\left[-\frac{1}{10}xe^{-x/10} + e^{-x/10}\right]_0^\infty
\end{aligned}$$

Substitute the limits and simplify. So,

$$\begin{aligned}\mu_1' &= \frac{1}{2\theta}\left[\left(\frac{1}{\theta}(\infty)e^{-\infty/\theta}+e^{-\infty/\theta}\right)-\left(-\frac{1}{\theta}(0)e^{-0/\theta}+e^{-0/\theta}\right)\right]+\\&\quad\frac{1}{20}\left[\left(-\frac{1}{10}(\infty)e^{-\infty/10}+e^{-\infty/10}\right)-\left(-\frac{1}{10}(0)e^{-0/10}+e^{-0/10}\right)\right]\\&=-\frac{1}{2\theta}(1)-\frac{1}{20}(1)\\&=\frac{20-2\theta}{40\theta}\end{aligned}$$

The first sample moment is defined as,

$$m_1 = \overline{X}$$

Substitute the observations of the sample and the number of observations are 10. The sum of observations is given as 150. So, the first sample moment is calculated as,

$$\begin{aligned}m_1 &= \frac{150}{10}\\&=15\end{aligned}$$

Equate the first population moment and first sample moment $\mu_1' = m_1$ to calculate the estimator of θ.

$$\begin{aligned}\frac{20-2\theta}{40\theta}&=15\\20-2\theta&=600\theta\\602\theta&=20\\\theta&\approx 0.033\end{aligned}$$

Therefore, the estimator of θ using the method of moments estimator is $\boxed{0.033}$.

Step 4/4
(b)

The standard error of each estimator of θ is required. The estimator of θ is obtained as 0.625 in both the methods.

Use the formula for standard error of estimator and substitute $\hat{p} = 0.033$. So,

$$\begin{aligned} SE_{\hat{p}} &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= \sqrt{\frac{0.033(1-0.033)}{10}} \\ &= \sqrt{0.0031911} \\ &\approx 0.057 \end{aligned}$$

Therefore, the standard error of the estimator of θ is $\boxed{0.057}$.

Problem

In order to ensure efficient usage of a server, it is necessary to estimate the mean number of concurrent users. According to records, the average number of concurrent users at 100 randomly selected times is 37.7, with a standard deviation $\sigma = 9.2$.

(a) Construct a 90% confidence interval for the expectation of the number of concurrent users.

(b) At the 1% significance level, do these data provide significant evidence that the mean number of concurrent users is greater than 35?

Step-by-step solution

Show all steps

100% (12 ratings) for this solution

Step 1/3

The number of concurrent users, $n = 100.$

The average of concurrent users, $\bar{x} = 37.7.$

The Standard deviation of the concurrent users, $\sigma = 9.2.$

Here, population standard deviation is known, so apply one sample *Z*-test.

Step 2/3

(a)

Find the 90% of confidence interval expectation of the number of concurrent users.

The formula for $(1-\alpha)\%$ confidence interval for population mean is,

$$C.I = \bar{X} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Here, $\bar{X}$ is the sample mean, σ is the population standard deviation, $Z_{\alpha/2}$ is the critical value and n is the sample size.

From standard normal tables, the critical value at 0.10 level for two tailed is 1.645.

Thus,

$$\begin{aligned} C.I &= \bar{X} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \\ &= 37.7 \pm 1.645\left(\frac{9.2}{\sqrt{100}}\right) \\ &= 37.7 \pm 1.5134 \\ &= (36.19, 39.21) \end{aligned}$$

Hence, the required confidence interval is $\boxed{(36.19, 39.21)}$.

Step 3/3

b)

The objective of the study is to test and determine whether the mean number of concurrent users is greater than 35.

Let μ is the mean number of concurrent users.

The null and alternative hypotheses can be defined as,

$$H_0 : \mu = 35$$

vs

$$H_A : \mu > 35$$

The alternative hypothesis is right tailed.

The level of the significance is 0.01.

From the standard normal distribution table, the critical value at 0.01 level for right tailed is 2.326.

Rejection region: Reject the null hypothesis if $Z > 2.326$.

The Z-statistics value is,

$$\begin{aligned} Z &= \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \\ &= \frac{37.7 - 35}{9.2 / \sqrt{100}} \\ &= 2.935 \end{aligned}$$

Here, the *Z*-statistics (2.935) is greater than the critical value (2.326). So, reject the null hypothesis.

Therefore, it can be concluded that there is significant evidence that the mean number of concurrent users is greater than 35.

Problem
Installation of a certain hardware takes random time with a standard deviation of 5 minutes.

(a) A computer technician installs this hardware on 64 different computers, with the average installation time of 42 minutes. Compute a 95% confidence interval for the population mean installation time.

(b) Suppose that the population mean installation time is 40 minutes. A technician installs the hardware on your PC. What is the probability that the installation time will be within the interval computed in (a)?

Step-by-step solution

Show all steps

100% (11 ratings) for this solution

Step 1/3
The number of computers users is, $n = 64.$

The average time of computer installation is, $\overline{x} = 42.$

The standard deviation of the computer installation is, $\sigma = 5.$

Step 2/3
a)

Find the 95% of confidence interval for the population mean installation time.

From $Z-$tabulated values, the $Z-$critical value at the 95% level is, 1.96.

$$
\begin{aligned}
95\% \text{ Confidence interval} &= \bar{x} \pm Z_{\text{critical}}\left(\frac{\sigma}{\sqrt{n}}\right) \\
&= 42 \pm 1.96\left(\frac{5}{\sqrt{64}}\right) \\
&= 42 \pm 1.225 \\
&= (40.775, 43.225)
\end{aligned}
$$

Hence, the required confidence interval is, $\boxed{(40.775, 43.225)}$.

Step 3/3

b)

Find the probability that the installation time will be within the interval computed in (a).

$$
\begin{aligned}
P(40.775 < X < 43.225) &= P\left(\frac{40.755-40}{5} < Z < \frac{43.225-40}{5}\right) \\
&= P(0.151 < Z < 0.645) \\
&= P(Z < 0.645) - P(Z < 0.151)
\end{aligned}
$$

$$
\begin{aligned}
&= 0.7405 - 0.5600 \\
&= 0.1805 \\
&= 0.181 \qquad \text{(Round to 3 decimal place)}
\end{aligned}
$$

Hence, the required probability is, $\boxed{0.181}$.

Problem

Salaries of entry-level computer engineers have Normal distribution with unknown mean and variance. Three randomly selected computer engineers have salaries (in $1000s):

$$30, 50, 70$$

(a) Construct a 90% confidence interval for the average salary of an entry-level computer engineer.

(b) Does this sample provide a significant evidence, at a 10% level of significance, that the average salary of all entry-level computer engineers is different from $80,000? Explain.

(c) Looking at this sample, one may think that the starting salaries have a great deal of variability. Construct a 90% confidence interval for the standard deviation of entry-level salaries.

Step-by-step solution

Show all steps

Step 1/5

From the given information, the salaries of entry-level computer engineers have Normal distribution with unknown mean and variance.

Three randomly selected computer engineers have salaries 30,50,70 (in $1000).

Step 2/5

(a)

Construct 90% confidence interval for the average salary of an entry-level computer engineer.

The sample mean of the entry level salaries of computer engineers is,

$$\begin{aligned}\bar{X} &= \frac{\sum x}{n} \\ &= \frac{30+50+70}{3} \\ &= \frac{150}{3} \\ &= 50\end{aligned}$$

The sample standard deviation of the entry level salaries of computer engineers is,

$$\begin{aligned}s &= \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} \\ &= \sqrt{\frac{(30-50)^2+(50-50)^2+(70-50)^2}{3-1}} \\ &= \sqrt{\frac{400+0+400}{2}} \\ &= 20\end{aligned}$$

Step 3/5

Let the level of significance be $\alpha = 0.10$.

The degrees of freedom is,

$$
\begin{aligned}
df &= n-1 \\
&= 3-1 \\
&= 2
\end{aligned}
$$
2.898

The critical value of *t*-distribution with $n-1=2$ degrees of freedom is $t_{\alpha/2}=t_{0.05}=2.920$

Then, the 90% confidence interval for the average salary of an entry-level computer engineer is:

$$
\begin{aligned}
\bar{X} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) &= 50 \pm 2.92\left(\frac{20}{\sqrt{3}}\right) \\
&= 50 \pm 33.7173 \\
&= (16.283,\ 83.717)
\end{aligned}
$$

Therefore, the 90% confidence interval for the average salary of entry level computer engineers is between 16.283 and 83.717.

Step 4/5
(b)

As the confidence interval for the average salary of entry level computer engineers is between 16.283 and 83.717 at 10% level of significance.

$80,000 lies between $16.283 and $83.717, it can be stated that the average salary of all entry-level computer engineers is different from $80,000.

Step 5/5
(c)

90% confidence interval for the standard deviation is calculated as follows:

Sample standard deviation = 20

Sample size = 3

Significance level = α = 1-0.90 = 0.10

The chi-square critical value at the $\alpha/2=0.05$ level of significance with 2 degrees of freedom is 5.991.

The chi-square critical value at the $(1-\alpha/2)=0.95$ level of significance with 2 degrees of freedom is 0.103.

The 90% confidence interval estimate of the standard deviation is:

$$\left(\sqrt{\frac{(n-1)s^2}{\chi^2_{(0.025,9)}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{(0.975,9)}}}\right) = \sqrt{\frac{(3-1)(20)^2}{5.991}} \le \sigma \le \sqrt{\frac{(3-1)(20)^2}{0.103}}$$

$$= (11.5556 < \sigma < 89.4427)$$

$$= (11.5556 < \sigma < 89.4427)$$

Hence, we are 90% confident that the above interval contains true value of population standard deviation.

Problem

We have to accept or reject a large shipment of items. For quality control purposes, we collect a sample of 200 items and find 24 defective items in it.

(a) Construct a 96% confidence interval for the proportion of defective items in the whole shipment.

(b) The manufacturer claims that at most one in 10 items in the shipment is defective. At the 4% level of significance, do we have sufficient evidence to disprove this claim? Do we have it at the 15% level?

Step-by-step solution

Show all steps

100% (12 ratings) for this solution

Step 1/4

The given information describes that the sample of 200 items and found that there are 24 defective items.

The sample proportion of defective items is,

$$\hat{p} = \frac{x}{n}$$

$$= \frac{24}{200}$$

$$= 0.12$$

From standard normal tables, the $Z-$critical value at the 0.04 level of significance is 2.054.

Step 2/4

(a)

Calculate the 96% of confidence interval for the proportion of defective items.

The formula for $(1-\alpha)\%$ confidence level for single proportion is,

$$\hat{p} \pm Z_{critical}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Here, $\hat{p}$ is the sample proportion and n is the sample size.

$$\begin{aligned}
96\%\text{Confidence interval} &= \hat{p} \pm Z_{critical}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\
&= 0.12 \pm 2.054\sqrt{\frac{0.12(1-0.12)}{200}} \\
&= 0.12 \pm 0.047 \\
&= (0.073, 0.167)
\end{aligned}$$

Therefore, the 96% confidence interval for single proportion is lies between is $\boxed{0.073 \text{ and } 0.167}$.

Step 3/4
(b)

The objective of the study is at most one in 10 items in the shipment is defective.

The null and alternative hypotheses can be defined as,

$$H_0 : p \le 0.1$$

$$H_1 : p > 0.1$$

The test statistic is,

$$\begin{aligned}
Z &= \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \\
&= \frac{0.12 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{200}}} \\
&= \frac{0.02}{0.0212} \\
&= 0.943
\end{aligned}$$

It is given that the level of the significance, $\alpha = 0.04$

From the standard normal distribution table, the critical value at 0.04 level is 1.751.

Since, the calculated value 0.943 is less than the level of the significance 1.751. So, the null hypothesis is fails to be rejected.

Therefore, it can be concluded that at most one in 10 items in the shipment is defective.

Step 4/4

It is given that the level of the significance, $\alpha = 0.15$

From the standard normal distribution table, the critical value at 0.15 level is 1.0364.

Since, the calculated value 0.943 is less than the level of the significance 1.0364. So, the null hypothesis is fails to be rejected.

Therefore, it can be concluded that at most one in 10 items in the shipment is defective.

Problem

Refer to Exercise 9.10. Having looked at the collected sample, we consider an alternative supplier. A sample of 150 items produced by the new supplier contains 13 defective items.

Is there significant evidence that the quality of items produced by the new supplier is higher than the quality of items in Exercise 9.10? What is the P-value?

Step-by-step solution

Show all steps

100% (6 ratings) for this solution

Step 1/3

The given information is as shown below,

	x	n	Proportion
Old Supplier	24	200	$\hat{p}_1 = \frac{24}{200} = 0.12$
New Supplier	13	150	$\hat{p}_2 = \frac{13}{150} = 0.087$

Find the population proportion.

$$\begin{aligned}\hat{p} &= \frac{x_1 + x_2}{n_1 + n_2} \\ &= \frac{24+13}{200+150} \\ &= 0.1057\end{aligned}$$

Step 2/3

Null hypothesis, $H_0 : p_1 \le p_2$

Alternative hypothesis, $H_0 : p_1 > p_2$

Level of significance, $\alpha = 0.05$

Test statistic is,

$$
\begin{aligned}
Z &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \\
&= \frac{0.12-0.087}{\sqrt{0.1057(1-0.1057)\left(\frac{1}{200}+\frac{1}{150}\right)}} \\
&= \frac{0.033}{0.033} \\
&= 1
\end{aligned}
$$

Step 3/3

Find the $p-$value.

$$
\begin{aligned}
p-\text{value} &= P(Z > Z_0) \\
&= 1 - P(Z \le 1) \\
&= 1 - 0.841345 \qquad \text{(From standard normal table)} \\
&= 0.158655
\end{aligned}
$$

Conclusion:

The $p-$value is greater than the given significance level, so we fail to reject the null hypothesis and conclude that there is no significance evidence to say that the quality of items produced by new suppliers higher than the quality of items.

Chapter 9, Problem 12E

(0)

Problem

An electronic parts factory produces resistors. Statistical analysis of the output suggests that resistances follow an approximately Normal distribution with a standard deviation of 0.2 ohms. A sample of 52 resistors has the average resistance of 0.62 ohms.

(a) Based on these data, construct a 95% confidence interval for the population mean resistance.

(b) If the actual population mean resistance is exactly 0.6 ohms, what is the probability that an average of 52 resistances is 0.62 ohms or higher?

Step-by-step solution

Show all steps

100% (5 ratings) for this solution

Step 1/3

The given information is as shown below,

$\bar{x} = 0.62$	$\sigma = 0.2$	$n = 52$

From standard normal tables, the $Z-$critical value at the 95% level is 1.96.

Step 2/3

a)

Calculate the 95% confidence interval for the population mean resistance.

$$\begin{aligned}\text{95\% Confidence interval} &= \bar{x} \pm Z_{\text{critical}}\left(\frac{\sigma}{\sqrt{n}}\right)\\ &= 0.62 \pm 1.96\left(\frac{0.2}{\sqrt{52}}\right)\\ &= 0.62 \pm 0.05436\\ &= \boxed{(0.56564 < \mu < 0.67436)}\end{aligned}$$

Step 3/3

b)

Find the probability that resistance is 0.62 ohms or higher.

$$\begin{aligned}P(X > 0.62) &= 1 - P(Z \le 0.62)\\ &= 1 - P\left(Z \le \frac{0.62 - 0.6}{0.2/\sqrt{52}}\right)\\ &= 1 - P(Z \le 0.721)\end{aligned}$$

$$\begin{aligned}&= 1 - 0.764545 \qquad \text{(From standard normal tables)}\\ &= 0.235455\\ &\cong \boxed{0.235} \qquad \text{(Round to 3 decimal place)}\end{aligned}$$

Chapter 9, Problem 13E
(0)

Problem
Compute a P-value for the right-tail test in Example 9.25 on p. 279 and state your conclusion about a significant increase in the number of concurrent users.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/2
Refer to the example 9.25, the given information is,

Null hypothesis, $H_0 : \mu \le 5000$

Alternative hypothesis, $H_1 : \mu > 5000$

Level of significance, $\alpha = 0.05$

The test statistic is, $Z = 2.5.$

Step 2/2
Calculate the $p-$value for the right tail test.

$$\begin{aligned} p-\text{value} &= P\left(Z > Z_0\right) \\ &= 1 - P\left(Z \le 2.5\right) \\ &= 1 - 0.99379 \qquad \left(\text{From standard normal tables}\right) \\ &= 0.00621 \end{aligned}$$

Conclusion:

The $p-$value is less than the given significance level 0.05, so we reject the null hypothesis and conclude that the mean number of users has increased.

Chapter 9, Problem 14E
(1)

Problem
Is there significant difference in speed between the two servers in Example 9.21 on p. 271?

(a) Use the confidence interval in Example 9.21 to conduct a two-sided test at the 5% level of significance.

(b) Compute a P-value of the two-sided test in (a).

(c) Is server A really faster? How strong is the evidence? Formulate the suitable hypothesis and alternative and compute the corresponding P-value.

State your conclusions in (a), (b), and (c).

Step-by-step solution

Show all steps

Step 1/9
The given information is as shown below,

	Server – A	Server – B
Mean	6.7	7.5
SD	0.6	1.2
Sample size	30	20

Step 2/9
a)

The objective is to conduct a two-sided test at 5% level of significance.

The Null hypothesis is,

H_0 : There is no significant differnce between server A and server B

Symbolically, $H_0 : \mu_1 = \mu_2$

The Alternative hypothesis is,

H_A : There is a significant differnce between server A and server B

Symbolically, $H_1 : \mu_1 \neq \mu_2$

Let the Level of significance be $\alpha = 0.05$

Step 3/9
From the given data,

Server -*A*:

Sample mean, $\mu_1 = 6.7$

Sample standard deviation, $s_1 = 0.6$

Sample size, $n_1 = 30$

Server -*B*:

Sample mean, $\mu_2 = 7.5$

Sample standard deviation, $s_2 = 1.2$

Sample size, $n_2 = 20$

The test statistic is,

$$\begin{aligned}
t &= \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\
&= \frac{6.7 - 7.5 - (0)}{\sqrt{\frac{(0.6)^2}{30} + \frac{(1.2)^2}{20}}} \\
&= \frac{-0.8}{0.2898} \\
&= \boxed{-2.76}
\end{aligned}$$

Step 4/9

Find the degrees of freedom.

$$\begin{aligned}
df &= \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^4}{n_1^2(n_1 - 1)}\right) + \left(\frac{s_2^4}{n_2^2(n_2 - 1)}\right)} \\
&= \frac{\left(\frac{(0.6)^2}{30} + \frac{(1.2)^2}{20}\right)^2}{\left(\frac{(0.6)^4}{30^2(30 - 1)}\right) + \left(\frac{(1.2)^4}{20^2(20 - 1)}\right)} \\
&= 25.4 \\
&\cong 25
\end{aligned}$$

Critical value:

From the $t-$tabulated values, the $t-$critical value at the 5% level of significance with 25 degrees of freedom is ± 2.060.

Decision rule:

Reject the null hypothesis if the absolute value of the test statistic is greater than the critical value.

Conclusion:

Here, the absolute value of the test statistic is 2.76, which is greater than the critical value 2.060.

So, reject the null hypothesis at 5% level of significance.

Therefore, there is a significant difference between the server-*A* and server-*B.*

Step 5/9
b)

Calculate the $p-$value of the two-sided test in part (a).

Excel function:

$$\begin{aligned} p-\text{value} &= 2P(T \le t)(\text{since the test is two-tailed test}) \\ &= 2P(T \le 2.76) \\ &= (=\text{TDIST}(2.76,25,2)) \quad \begin{pmatrix} \text{Use MS Excel function,} \\ \text{"=TDIST}(t, df, tails)\text{"} \end{pmatrix} \\ &= 0.010663 \end{aligned}$$

Step 6/9
Using T-table:

From *t* table, at 25 degrees of freedom, the value of the test statistic 2.76 lies between 2.485 and 2.787 corresponding to the level of significance 0.01 and 0.005.

That is, $0.005 < p-value < 0.01$.

Decision rule:

Reject the null hypothesis if $p-value < 0.05$.

Conclusion:

The $p-$value of the test statistic lies between 0.005 and 0.01 which is less than the given significance level 0.05.

So, reject the null hypothesis and conclude that there a difference between the mean execution time on server-A and server-$B.$

Step 7/9
c)

The objective is to conduct a two-sided test at 5% level of significance.

The Null hypothesis is,

H_0 : There is no significant differnce between server A and server B

Symbolically, $H_0 : \mu_1 = \mu_2$

The Alternative hypothesis is,

H_A : There is a significant differnce between server A and server B

Symbolically, $H_1 : \mu_1 > \mu_2$

Let the Level of significance be $\alpha = 0.05$

Step 8/9

The value of the test statistic is, $t = -2.76 \qquad \text{(From part}(a))$

From the alternative hypothesis, it is clear that the test is one-sided right tail test.

The $p-$value for the test is,

$$\begin{aligned} p-\text{value} &= P(t > t_0) \\ &= 1 - P(t \le 2.76) \\ &= 1 - 0.005331 \qquad \text{(From tabulated values)} \\ &= \boxed{0.994669} \end{aligned}$$

Step 9/9
Decision rule:

Reject the null hypothesis if $p-value < 0.05$.

Conclusion:

The $p-$value of the test statistic is greater than the given significance level.

Hence, fail to reject the null hypothesis and conclude that the server –A is not faster than the server –B.

Chapter 9, Problem 15E
(0)

Problem

According to Example 9.17 on p. 265, there is no significant difference, at the 5% level, between towns A and B in their support for the candidate. However, the level $\alpha = 0.05$ was chosen rather arbitrarily, and the candidate still does not know if he can trust the results when planning his campaign. Can we compare the two towns at *all* reasonable levels of significance? Compute the P-value of this test and state conclusions.

Step-by-step solution

Show all steps

100% (3 ratings) for this solution

Step 1/3

The given information is as shown in below,

	x	n	Proportion
Town $-A$	42	70	$p_A = \frac{42}{70} = 0.6$
Town $-B$	59	100	$p_B = \frac{59}{100} = 0.59$

Step 2/3

Null hypothesis, $H_0 : \hat{p}_1 - \hat{p}_2 = 0$

Alternative hypothesis, $H_1 : \hat{p}_1 - \hat{p}_2 \neq 0$

Level of significance, $\alpha = 0.05$

Test statistic is,

$$\begin{aligned} Z &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} \\ &= \frac{0.6 - 0.59}{\sqrt{\frac{0.6(1-0.6)}{70} + \frac{0.59(1-0.59)}{100}}} \\ &= \boxed{0.13} \end{aligned}$$

Step 3/3

Find the $p-$value for the test statistic.

$$\begin{aligned} p - \text{value} &= 2P(Z > Z_0) \\ &= 2(1 - P(Z \leq 0.13)) \end{aligned}$$

$$= 2(1 - 0.551717) \quad \text{(From normal tables)}$$
$$= 2(0.448283)$$
$$= 0.896566$$

Conclusion:

The $p-$value is greater than the given significance level 0.05, so we fail to reject the null hypothesis and conclude that there is a no significant difference between town-A and town-B at the 5% level of significance.

Chapter 9, Problem 16E
(0)

Problem
A sample of 250 items from lot A contains 10 defective items, and a sample of 300 items from lot B is found to contain 18 defective items.

(a) Construct a 98% confidence interval for the difference of proportions of defective items.

(b) At a significance level *α* = 0.02, is there a significant difference between the quality of the two lots?

Step-by-step solution

Show all steps

100% (7 ratings) for this solution

Step 1/4
From the information, consider *A* and *B* are the two variables that represents the two lots.

Consider a random sample of 250 items from lot *A* contains 10 defective items.

Consider a random sample of 300 items from lot *B* contains 18 defective items.

Calculate the sample proportion for lot *A*.

$$p_1 = \frac{x_1}{n_1}$$
$$= \frac{10}{250}$$
$$= 0.04$$

Calculate the sample proportion for lot *B*.

$$p_2 = \frac{x_2}{n_2}$$
$$= \frac{18}{300}$$
$$= 0.06$$

Step 2/4

a)

Construct a 98% confidence interval for the difference of proportions of defective items.

The level of significance is, $\alpha = 0.02$

From the standard z table values, observe that the critical value of z for the two tail test at the 2% level of significance is 2.33

That is, $z_{\alpha/2} = 2.33$

Therefore,

$$\begin{aligned}
98\%\text{Confidence interval} &= \left(\hat{p}_A - \hat{p}_B\right) \pm Z_{\text{critical}} \sqrt{\frac{\hat{p}_A\left(1-\hat{p}_A\right)}{n_A} + \frac{\hat{p}_B\left(1-\hat{p}_B\right)}{n_B}} \\
&= \left(0.04 - 0.06\right) \pm 2.326 \sqrt{\frac{0.04\left(1-0.04\right)}{250} + \frac{0.06\left(1-0.06\right)}{300}} \\
&= -0.02 \pm 0.043 \\
&= \boxed{\left(-0.063, 0.023\right)}
\end{aligned}$$

Step 3/4

b)

Consider Null and Alternative hypothesis.

Null hypothesis, $H_0:$ There is no significant difference between the qualities of two lots.

Alternative hypothesis, $H_1:$ There is significant difference between the qualities of two lots.

Level of significance, $\alpha = 0.02$

Calculate the test statistic value.

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$
$$= \frac{0.04 - 0.06}{\sqrt{\frac{0.04(1-0.04)}{250} + \frac{0.06(1-0.06)}{300}}}$$
$$= \boxed{-1.06}$$

Therefore, the absolute test statistic value is, $|Z| = 1.06$

Step 4/4
From the standard *z* table values, observe that the critical value of *z* for the two tail test at the 2% level of significance is 2.33

That is, $z_{\alpha/2} = 2.33$

Compare the absolute calculated test statistic value with the critical value.

Here, the absolute test statistic value lesser than the critical value, fail to reject the Null hypothesis.

Hence, conclude that there is no sufficient evidence that there is a significant difference between the quality of the two lots.

Chapter 9, Problem 17E
(2)

Problem
A news agency publishes results of a recent poll. It reports that a certain candidate has a 10-point stronger support in town A than in town B because 45% of the poll participants in town A and 35% of the poll participants in town B supported the candidate. What margin of error should the news agency report for each of the listed estimates, 45%, 35%, and 10%? Notice that 900 randomly selected registered voters participated in the poll in each town, and the reported margins of error correspond to 95% confidence intervals.

Step-by-step solution

Show all steps

Step 1/3

Find the margin of error of the $(\hat{p}_1)$.

The $Z-$critical value at the 95% confidence level is, 1.96.

Here, $\hat{p}_1 = 45\% \Rightarrow 0.45$ and $n = 900$.

$$ME = Z_{\text{critical}}\sqrt{\frac{p(1-p)}{n}}$$
$$= 1.96\sqrt{\frac{0.45(1-0.45)}{900}}$$
$$= 1.96(0.016583123)$$
$$= 0.0325$$
$$= \boxed{3.25\%}$$

Step 2/3

Find the margin of error of the $(\hat{p}_2)$.

Here, $\hat{p}_2 = 35\% \Rightarrow 0.35$ and $n = 900$.

$$ME = Z_{\text{critical}}\sqrt{\frac{p(1-p)}{n}}$$
$$= 1.96\sqrt{\frac{0.35(1-0.35)}{900}}$$
$$= 1.96(0.015898986)$$
$$= 0.0311620$$
$$= 0.0312 \qquad (\text{Round to 4 decimal place})$$
$$= \boxed{3.12\%}$$

Step 3/3

Find the margin of error of the $(\hat{p}_1 - \hat{p}_2)$.

The $Z-$critical value at the 95% confidence level is, 1.96.

$$ME = Z_{\text{critical}} SE(\hat{p}_1 - \hat{p}_2)$$
$$= 1.96\sqrt{\frac{0.45(1-0.45)}{900} + \frac{0.35(1-0.35)}{900}}$$
$$= 1.96(0.0229734)$$

$$= 0.045027892$$
$$= 0.0450 \qquad \text{(Round to 4 decimal place)}$$
$$= \boxed{4.50\%}$$

Chapter 9, Problem 18E
(1)

Problem
Consider the data about the number of blocked intrusions in Exercise 8.1, p. 240.

(a) Construct a 95% confidence interval for the difference between the average number of intrusion attempts per day before and after the change of firewall settings (assume equal variances).

(b) Can we claim a significant reduction in the rate of intrusion attempts? The number of intrusion attempts each day has approximately Normal distribution. Compute P-values and state your conclusions under the assumption of equal variances and without it. Does this assumption make a difference?

Step-by-step solution

Show all steps

100% (4 ratings) for this solution

Step 1/7
The given information can be summarized below.

	Mean	SD	Sample size
Before fire wall change	50.00	7.62	14
After fire wall change	40.20	7.96	20

The degrees of freedom for variances are assumed to be equal is,

$$\begin{aligned} df &= n_1 + n_2 - 2 \\ &= 14 + 20 - 2 \\ &= 32 \end{aligned}$$

From t-table, the critical value at 95% confidence level with 32 degrees of freedom is, $t_{\alpha/2} = 2.037$.

Step 2/7
a)

Compute 95% confidence interval for the difference between the average number of intrusion attempts before and after the change of fire wall settings (assume equal variances).

The formula for computing 95% confidence interval is,

$$95\%\text{Confidence interval} = \left(\bar{X}_1 - \bar{X}_2\right) \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Where s_p is the pooled standard deviation.

From the available information,

$$n_1 = 14, \quad n_2 = 20$$
$$\bar{X}_1 = 50, \quad \bar{X}_2 = 40.20$$
$$s_1 = 7.62, \; s_2 = 7.96$$

Compute pooled standard deviation is,

$$\begin{aligned} s_p &= \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \\ &= \sqrt{\frac{(14-1)(7.62)^2 + (20-1)(7.96)^2}{14 + 20 - 2}} \\ &= \sqrt{\frac{1958.708}{32}} \\ &= \sqrt{61.2096} \end{aligned}$$

$$= 7.8236$$

Compute 95% confidence interval.

$$\begin{aligned} 95\%\text{Confidence interval} &= \left(\bar{X}_1 - \bar{X}_2\right) \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \\ &= (50 - 40.20) \pm (2.037)(7.8236)\sqrt{\frac{1}{14} + \frac{1}{20}} \\ &= 9.8 \pm (15.93667)\sqrt{0.121429} \\ &= 9.8 \pm (15.93667)(0.348466) \end{aligned}$$

$$\begin{aligned} &= 9.8 \pm 5.5534 \\ &= (9.8 - 5.5534, 9.8 + 5.5534) \\ &= \boxed{(4.2466, 15.3534)} \end{aligned}$$

Therefore, the 95% confidence interval for the difference between the average number of intrusion attempts per day before and after the change of firewall settings is, $(4.2466,\ 15.3534)$.

Step 3/7
b)

Our claim is to test whether there is a significant in the rate of intrusion attempts.

Null hypothesis:

$H_0:$ There is no significant reduction in the rate of intrusion attempts.

Alternative hypothesis:

$H_1:$ there is a significant reduction in the rate of intrusion attempts.

Level of significance:

Let $\alpha = 0.05$.

Decision rule:

Reject H_0, if $p-value < 0.05$

Step 4/7
The test statistic when variances are assumed to be equal is,

$$\begin{aligned}
t &= \frac{\overline{X}_1 - \overline{X}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{(equal variances)} \\
&= \frac{(50 - 40.20)}{7.8236\sqrt{\frac{1}{14} + \frac{1}{20}}} \quad (\text{from part(a), } s_p = 7.8236) \\
&= \frac{9.8}{2.7262} \\
&= \boxed{3.595} (\text{rounded to three decimals})
\end{aligned}$$

Find the $p-$value.

$$p - \text{value} = (=\text{TDIST}(x, \text{degrees of freedom, tails}))$$

$$\begin{aligned}
p - \text{value} &= (=\text{TDIST}(3.595,32,2)) \quad (\text{use excel function}) \\
&= \boxed{0.001091}
\end{aligned}$$

Since, the *p*-value is less than 0.05, reject the null hypothesis and conclude that there is a significant reduction in the rate of intrusion attempts.

Step 5/7
The test statistic when variances are assumed to be unequal is,

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{(s_1^2)}{n_1} + \frac{(s_2^2)}{n_2}}} \text{ (unequal variances)}$$

$$= \frac{50.00 - 40.20}{\sqrt{\frac{(7.62)^2}{14} + \frac{(7.96)^2}{20}}}$$

$$= \frac{9.8}{2.70}$$

$$= \boxed{3.629} \text{ (rounded to three decimals)}$$

Step 6/7
The degrees of freedom for variances are assumed to be equal is,

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^4}{n_1^2(n_1-1)} + \frac{s_2^4}{n_2^2(n_2-1)}\right)}$$

$$= \frac{\left(\frac{(7.62)^2}{14} + \frac{(7.96)}{20}\right)^2}{\left(\frac{(7.62)^4}{14^2(14-1)} + \frac{(7.96)^4}{20^2(20-1)}\right)}$$

$$= \frac{(7.3155)^2}{(1.3232 + 0.5282)}$$

$$= \frac{53.5165}{1.8514}$$

$$= 28.90$$

$$\simeq 28$$

Find the *p*-value.

$$p - \text{value} = (=\text{TDIST}(x, \text{degrees of freedom, tails}))$$

$$p - \text{value} = (=\text{TDIST}(3.629, 28, 2)) \qquad (\text{Use MS Excel})$$
$$= \boxed{0.001125}$$

Since, the *p*-value is less than 0.05, reject the null hypothesis and conclude that there is a significant reduction in the rate of intrusion attempts.

Step 7/7

In both the cases when variances are assumed to be equal and variances are assumed to be unequal we reject H_0.

Therefore, the assumption does not make a difference.

Hence, there is a significant reduction in the rate of intrusion attempts.

Chapter 9, Problem 19E

(0)

Problem

Consider two populations (*X*'s and *Y*'s) with different means but the same variance. Two independent samples, sizes *n* and *m*, are collected. Show that the pooled variance estimator

[illegible]

estimates their common variance unbiasedly.

Step-by-step solution

Show all steps

100% (2 ratings) for this solution

Step 1/1

There are two populations X and Y with different mean and same variance.

Show that the pooled variance estimator estimates their common variance unbiased.

$$
\begin{aligned}
E\left(S_p^2\right) &= E\left(\frac{(n-1)s_1^2+(m-1)s_2^2}{n+m-2}\right) \\
&= \frac{(n-1)}{n+m-2}E\left(s_1^2\right)+\frac{(m-1)}{n+m-2}E\left(s_2^2\right) \\
&= \frac{(n-1)}{n+m-2}\sigma^2+\frac{(m-1)}{n+m-2}\sigma^2 \\
&= \left(\frac{(n-1)+(m-1)}{n+m-2}\right)\sigma^2
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\frac{n-1+m-1}{n+m-2}\right)\sigma^2 \\
&= \left(\frac{n+m-2}{n+m-2}\right)\sigma^2 \\
&= (1)\sigma^2 \\
&= \sigma^2
\end{aligned}
$$

Hence, the pooled variance estimator estimates their common variance unbiased.

Chapter 9, Problem 20E
(1)

Problem
A manager questions the assumptions of Exercise 9.8. Her pilot sample of 40 installation times has a sample standard deviation of *s* = 6.2 min, and she says that it is significantly different from the assumed value of *σ* = 5 min. Do you agree with the manager? Conduct the suitable test of a standard deviation.

Step-by-step solution

Show all steps

100% (1 rating) for this solution

Step 1/3
The given information is as shown below:

Sample size (n)	Sample standard deviation (s)
$n=40$	$s=6.2$

Step 2/3
Test whether it is significantly different from the assumed value $\sigma=5$

To make sure that the actual variability of population standard deviation, derive a level α test based on the Chi-square distribution.

Compete the hypotheses:

Null hypothesis: The sample standard deviation is not significantly different from the assumed value.

Alternative hypothesis: The sample standard deviation is significantly different from the assumed value.

That is:

$$H_0 : \sigma = 5$$
$$H_A : \sigma \neq 5$$

Level of significance, $\alpha = 0.05 \quad (\text{Assume})$

Then the test statistic is:

$$\begin{aligned}\chi^2_{obs} &= \frac{(n-1)s^2}{\sigma_0^2} \\ &= \frac{(40-1)(6.2)^2}{(5)^2} \\ &= \frac{1499.16}{25} \\ &= \boxed{59.9664}\end{aligned}$$

Hence, the required test statistic is, $\boxed{\chi^2_{\text{Statistic}} = 59.9664}$.

Step 3/3

The degrees of freedom is,

$$\begin{aligned}df &= n-1 \\ &= 40-1 \\ &= 39\end{aligned}$$

Rejection region:

For two tail test, $\chi^2_{obs} \geq \chi^2_{\alpha/2}$ or $\chi^2_{obs} \leq \chi^2_{1-\alpha/2}$

The critical value for 39 degrees of freedom is as follows:

$$\begin{aligned}\chi^2_{\alpha/2} &= \chi^2_{0.05/2} \\ &= \chi^2_{0.025,39} \qquad \begin{pmatrix}\text{From table of} \\ \text{Chi-Square Distribution}\end{pmatrix} \\ &= 58.1\end{aligned}$$

Conclusion:

Since $\chi^2_{obs} \geq \chi^2_{\alpha/2}$ reject the null hypothesis.

Therefore, there is a sufficient evidence that the sample standard deviation is significantly different from the assumed value.

Hence, we agree with the manager.

Chapter 9, Problem 21E
(0)

Problem
If [*a, b*] is a (1 − *α*)100% confidence interval for the population variance (with *a* ≥ 0), prove that $[\sqrt{a}, \sqrt{b}]$ is a (1 − *α*)100% confidence interval for the population standard deviation.

Step-by-step solution

Show all steps

Step 1/5
The given information is as shown below,

	Server − A	Server − B
Mean	6.7	7.5
SD	0.6	1.2
Sample size	30	20

Step 2/5

Null hypothesis, $H_0 : \sigma_X = \sigma_Y$

Alternative hypothesis, $H_1 : \sigma_X \neq \sigma_Y$

Level of significance, $\alpha = 0.05$

Test statistic is, $F-$test for unequal variances.

$$\begin{aligned} F &= \frac{S_Y^2}{S_X^2} \qquad \left(\text{For}, S_Y^2 > S_X^2\right) \\ &= \frac{(1.2)^2}{(0.6)^2} \\ &= \frac{1.44}{0.36} \\ &= 4 \end{aligned}$$

Step 3/5

The degrees of freedom is,

$$\begin{aligned} v_1 &= n_1 - 1 \\ &= 30 - 1 \\ &= 29 \end{aligned}$$

$$\begin{aligned} v_2 &= n_2 - 1 \\ &= 20 - 1 \\ &= 19 \end{aligned}$$

The $p-$value is,

$$\begin{aligned} p-\text{value} &= \left(=\text{FDIST}(4,29,19)\right) \qquad (\text{Use MS Excel}) \\ &= 0.0013 \end{aligned}$$

Conclusion:

The $p-$value is less than the given significance level 0.05, so we reject the null hypothesis and conclude that there is significant evidence to say that the two population variances are unequal $\left(\sigma_X^2 \neq \sigma_Y^2\right)$.

Step 4/5

b)

Find the 95% of confidence interval the ratio of the two population variances.

The $F-$critical values are,

$$
\begin{aligned}
F_{\text{Left}} &= F_{\left(\frac{\alpha}{2}, n_1-1, n_2-1\right)} \\
&= F_{\left(\frac{0.05}{2}, 30-1, 20-1\right)} \\
&= F_{(0.025, 29, 19)} \\
&= 0.448 \qquad (\text{From}, F - \text{tabulated}, \text{values})
\end{aligned}
$$

$$
\begin{aligned}
F_{\text{Right}} &= F_{\left(1-\frac{\alpha}{2}, n_1-1, n_2-1\right)} \\
&= F_{\left(1-\frac{0.05}{2}, 30-1, 20-1\right)} \\
&= F_{(0.975, 29, 19)} \\
&= 2.402 \qquad (\text{From}, F - \text{tabulated}, \text{values})
\end{aligned}
$$

Step 5/5

Now, find the confidence interval.

$$
\begin{aligned}
95\%\text{Confidence interval} &= F_{\text{Left}}\left(\frac{S_X^2}{S_Y^2}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < F_{\text{Right}}\left(\frac{S_X^2}{S_Y^2}\right) \\
&= 0.448\left(\frac{(0.6)^2}{(1.2)^2}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < 2.402\left(\frac{(0.6)^2}{(1.2)^2}\right) \\
&= 0.448\left(\frac{0.36}{1.44}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < 2.402\left(\frac{0.36}{1.44}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 0.448(0.25) < \frac{\sigma_X^2}{\sigma_Y^2} < 2.402(0.25) \\
&= 0.112 < \frac{\sigma_X^2}{\sigma_Y^2} < 0.6005 \\
&= \left(0.11 < \frac{\sigma_X^2}{\sigma_Y^2} < 0.60\right) \quad (\text{Round to 2 decimal place})
\end{aligned}
$$

Hence, the required confidence interval is, $\boxed{\left(0.11 < \frac{\sigma_X^2}{\sigma_Y^2} < 0.60\right)}$.

Chapter 9, Problem 22E

(1)

Problem

Recall Example 9.21 on p. 271, in which samples of 30 and 20 observations produced standard deviations of 0.6 min and 1.2 min, respectively. In this Example, we assumed unequal variances and used the suitable method only because the reported sample standard deviations *seemed* too different.

(a) Argue for or against the chosen method by testing equality of the population variances.

(b) Also, construct a 95% confidence interval for the ratio of the two population variances.

Step-by-step solution

Show all steps

Step 1/5

The given information is as shown below,

	Server $-A$	Server $-B$
Mean	6.7	7.5
SD	0.6	1.2
Sample size	30	20

Step 2/5

Null hypothesis, $H_0 : \sigma_X = \sigma_Y$

Alternative hypothesis, $H_1 : \sigma_X \neq \sigma_Y$

Level of significance, $\alpha = 0.05$

Test statistic is, $F-$test for unequal variances.

$$\begin{aligned} F &= \frac{S_Y^2}{S_X^2} \qquad \left(\text{For}, S_Y^2 > S_X^2\right) \\ &= \frac{(1.2)^2}{(0.6)^2} \\ &= \frac{1.44}{0.36} \\ &= 4 \end{aligned}$$

Step 3/5

The degrees of freedom is,

$$\begin{aligned} v_1 &= n_1 - 1 \\ &= 30 - 1 \\ &= 29 \end{aligned}$$

$$\begin{aligned} v_2 &= n_2 - 1 \\ &= 20 - 1 \\ &= 19 \end{aligned}$$

The $p-$value is,

$$\begin{aligned} p-\text{value} &= (=\text{FDIST}(4,29,19)) \qquad (\text{Use MS Excel}) \\ &= 0.0013 \end{aligned}$$

Conclusion:

The $p-$value is less than the given significance level 0.05, so we reject the null hypothesis and conclude that there is significant evidence to say that the two population variances are unequal $\left(\sigma_X^2 \neq \sigma_Y^2\right)$.

Step 4/5

b)

Find the 95% of confidence interval the ratio of the two population variances.

The $F-$critical values are,

$$\begin{aligned} F_{\text{Left}} &= F_{\left(1-\frac{\alpha}{2}, n_1-1, n_2-1\right)} \\ &= F_{\left(1-\frac{0.05}{2}, 30-1, 20-1\right)} \\ &= F_{(0.975,29,19)} \\ &= 0.448 \qquad (\text{From}, F-\text{tabulated}, \text{values}) \end{aligned}$$

$$\begin{aligned} F_{\text{Right}} &= F_{\left(\frac{\alpha}{2}, n_1-1, n_2-1\right)} \\ &= F_{\left(\frac{0.05}{2}, 30-1, 20-1\right)} \\ &= F_{(0.025,29,19)} \\ &= 2.402 \qquad (\text{From}, F-\text{tabulated}, \text{values}) \end{aligned}$$

Step 5/5

Now, find the confidence interval.

$$95\%\text{Confidence interval} = F_{\text{Left}}\left(\frac{S_X^2}{S_Y^2}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < F_{\text{Right}}\left(\frac{S_X^2}{S_Y^2}\right)$$

$$= 0.448\left(\frac{(0.6)^2}{(1.2)^2}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < 2.402\left(\frac{(0.6)^2}{(1.2)^2}\right)$$

$$= 0.448\left(\frac{0.36}{1.44}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < 2.402\left(\frac{0.36}{1.44}\right)$$

$$= 0.448(0.25) < \frac{\sigma_X^2}{\sigma_Y^2} < 2.402(0.25)$$

$$= 0.112 < \frac{\sigma_X^2}{\sigma_Y^2} < 0.6005$$

$$= \left(0.11 < \frac{\sigma_X^2}{\sigma_Y^2} < 0.60\right) \quad (\text{Round to 2 decimal place})$$

Hence, the required confidence interval is, $\boxed{\left(0.11 < \frac{\sigma_X^2}{\sigma_Y^2} < 0.60\right)}$.

Chapter 9, Problem 23E

(0)

Problem

Anthony says to Eric that he is a stronger student because his average grade for the first six quizzes is higher. However, Eric replies that he is more stable because the variance of his grades is lower. The actual scores of the two friends (presumably, independent and normally distributed) are in the table.

	Quiz 1	Quiz 2	Quiz 3	Quiz 4	Quiz 5	Quiz 6
Anthony	85	92	97	65	75	96
Eric	81	79	76	84	83	77

(a) Is there significant evidence to support Anthony's claim? State H_0 and H_A. Test equality of variances and choose a suitable two-sample t-test. Then conduct the test and state conclusions.

(b) Is there significant evidence to support Eric's claim? State H_0 and H_A and conduct the test.

For each test, use the 5% level of significance.

Step-by-step solution

Show all steps

100% (5 ratings) for this solution

Step 1/5

The given data represents that the scores of two students in six quizzes.

Calculate the mean and standard deviation of the Anthony scores.

$$\begin{aligned}\text{Mean}\left(\bar{x}_A\right) &= \frac{\sum x}{n} \\ &= \frac{85+92+97+65+75+96}{6} \\ &= \frac{510}{6} \\ &= 85\end{aligned}$$

$$\begin{aligned}\text{Standard deviation}\left(s_A\right) &= \sqrt{\frac{1}{n-1}\sum\left(x_i-\bar{x}\right)^2} \\ &= \sqrt{\frac{(85-85)^2+(92-85)^2+\ldots+(96-85)^2}{6-1}} \\ &= \sqrt{\frac{814}{5}} \\ &= 12.759\end{aligned}$$

Calculate the mean and standard deviation of the Eric.

$$\begin{aligned}\text{Mean}\left(\bar{x}_E\right) &= \frac{\sum x}{n} \\ &= \frac{81+79+76+84+83+77}{6} \\ &= \frac{480}{6} \\ &= 80\end{aligned}$$

$$\text{Standard deviation}\left(s_E\right)=\sqrt{\frac{1}{n-1}\sum\left(x_i-\overline{x}\right)^2}$$
$$=\sqrt{\frac{(81-80)^2+(79-80)^2+...+(77-80)^2}{6-1}}$$
$$=\sqrt{\frac{52}{5}}$$
$$=3.225$$

Step 2/5
a)

Anthony claims that, he is a stronger student because his average grade for the first six quizzes is higher.

To test the claim, first test the equality of the population variances.

Under this test the null and alternative hypotheses are,

$$H_0:\sigma_A^2=\sigma_E^2$$
$$H_1:\sigma_A^2\neq\sigma_E^2$$

Given level of significance is $\alpha=0.05$

To test the hypotheses, the test statistic is given by,

$$F=\frac{S_A^2}{S_B^2}$$

Substitute the known values,

$$F=\frac{12.759^2}{3.225^2}$$
$$=\boxed{15.6521}$$

For two tailed test, using Excel function, $2\times(=FDIST(15.65,5,5))$, the corresponding *P*-value is 0.009.

Since, the calculated *P*-value is less than the given level of significance 0.05, we reject the null hypothesis and conclude that there is no enough evidence to conclude that the population variances are equal.

Step 3/5
As the population variances are not equal, test the Anthony’s claim using two-sample t-test with unequal variances.

Under this test the null and alternative hypotheses are,

$$H_0 : \mu_A \le \mu_E$$
$$H_1 : \mu_A > \mu_E$$

Given level of significance is $\alpha = 0.05$

Step 4/5

To test the hypotheses, the test statistic is given by,

$$\begin{aligned} t &= \frac{\overline{x}_A - \overline{x}_E}{\sqrt{\left(\frac{s_A^2}{n_A} + \frac{s_E^2}{n_E}\right)}} \\ &= \frac{85 - 80}{\sqrt{\left(\frac{12.759^2}{6} + \frac{3.225^2}{6}\right)}} \\ &= \boxed{0.9306} \end{aligned}$$

The degrees of freedom is,

$$\begin{aligned} df &= \frac{\left(\frac{s_A^2}{n_A} + \frac{s_E^2}{n_E}\right)^2}{\frac{s_A^4}{n_A^2\left(n_A - 1\right)} + \frac{s_E^4}{n_E^2\left(n_E - 1\right)}} \\ &= \frac{\left(\frac{12.759^2}{6} + \frac{3.225^2}{6}\right)^2}{\frac{12.759^4}{6^2\left(6-1\right)} + \frac{3.225^4}{6^2\left(6-1\right)}} \\ &= 5.6362 \\ &\approx 5 \qquad \left(\text{Rounded below}\right) \end{aligned}$$

The *P*-value is,

$$\begin{aligned} P - \text{value} &= \left(=\text{TDIST}(0.9306,5,1)\right) \qquad \left(\text{Use MS Excel}\right) \\ &= \boxed{0.1974} \end{aligned}$$

Since, the calculated *P*-value is greater than the given significance level, we fail to reject the null hypothesis and conclude that there is no significant evidence to support Anthony’s claim that he is a stronger student because his average grade for the first six quizzes is higher.

Step 5/5
b)

Test the Eric's claim that he is more stable because the variance of this grades is lower.

To test the claim, first test the equality of the population variances.

Under this test the null and alternative hypotheses are,

$$H_0 : \sigma_A^2 \le \sigma_E^2$$
$$H_1 : \sigma_A^2 > \sigma_E^2$$

Given level of significance is $\alpha = 0.05$

To test the hypotheses, the test statistic is given by,

$$F = \frac{S_A^2}{S_B^2}$$

Substitute the known values,

$$F = \frac{12.759^2}{3.225^2}$$
$$= \boxed{15.6521}$$

For one tailed test, using Excel function, $(= FDIST(15.65, 5, 5))$, the corresponding *P*-value is 0.004.

Since, the calculated *P*-value is less than the given level of significance 0.05, we reject the null hypothesis and conclude that there is enough evidence to conclude that the Eric's claim that he is more stable because the variance of this grades is lower.

Chapter 9, Problem 24E
(0)

Problem
Recall Exercise 9.23. Results essentially show that a sample of six quizzes was too small for Anthony to claim that he is a stronger student. We realize that each student has his own population of grades, with his own mean μ_i and variance σ_i^2. The observed quiz grades are sampled from this population, and they are different due to all the uncertainty and random factors involved when taking a quiz. Let us estimate the population parameters with some confidence.

(a) Construct a 90% confidence interval for the population mean score for each student.

(b) Construct a 90% confidence interval for the difference of population means. If you have not completed Exercise 9.23(a), start by testing equality of variances and choosing the appropriate method.

(c) Construct a 90% confidence interval for the population variance of scores for each student.

(d) Construct a 90% confidence interval for the ratio of population variances.

Step-by-step solution

Show all steps

100% (3 ratings) for this solution

Step 1/7

The calculated mean and standard deviation for the two students' scores are,

	Mean	*SD*	*n*
Anthony's	85.00	12.76	6
Eric	80.00	3.22	6

a)

Construct the 90% confidence interval for the population mean scores for each student.

Compute the 90% of confidence interval for the population mean score for Anthony.

$$\mathrm{CI} = \bar{x}_{\text{Anthony}} \pm t_{critical}\left(\frac{s_{\text{Anthony}}}{\sqrt{n_{\text{Anthony}}}}\right)$$

Degrees of freedom:

$$\begin{aligned} df &= n-1 \\ &= 6-1 \\ &= 5 \end{aligned}$$

From the *t*-table values, at 0.10 level of significance, with 5 degrees of freedom the critical value is 2.015.

Substitute the values,

$$\begin{aligned} \mathrm{CI} &= 85.00 \pm 2.015\left(\frac{12.76}{\sqrt{6}}\right) \\ &= 85.00 \pm 10.497 \\ &= \boxed{(74.503, 95.497)} \end{aligned}$$

Therefore, the required 90% confidence interval is **(74.503, 95.497)**.

Step 2/7
Compute the 90% of confidence interval for the population mean score for Eric.

$$\text{CI} = \overline{x}_{\text{Eric}} \pm t_{critical}\left(\frac{s_{\text{Eric}}}{\sqrt{n_{\text{Eric}}}}\right)$$

Degrees of freedom:

$$\begin{aligned} df &= n - 1 \\ &= 6 - 1 \\ &= 5 \end{aligned}$$

From the *t*-table values, at 0.10 level of significance, with 5 degrees of freedom the critical value is 2.015.

Substitute the values,

$$\begin{aligned} \text{CI} &= 80.00 \pm 2.015\left(\frac{3.22}{\sqrt{6}}\right) \\ &= 80.00 \pm 2.649 \\ &= \boxed{(77.351, 82.649)} \end{aligned}$$

Therefore, the required 90% confidence interval is **(77.351, 82.649)**.

Step 3/7
b)

Construct the 90% confidence interval for difference of population means.

$$\text{CI} = \left(\left(\overline{x}_{\text{Anthony}} - \overline{x}_{\text{Eric}}\right) \pm t_{\text{Critical}}\sqrt{\frac{s^2_{\text{Anthony}}}{n_{\text{Anthony}}} + \frac{s^2_{\text{Eric}}}{n_{\text{Eric}}}}\right)$$

As the population variances are not equal, test the Anthony’s claim using two-sample *t*-test with unequal variances.

The degrees of freedom is,

$$
\begin{aligned}
df &= \frac{\left(\frac{s_A^2}{n_A}+\frac{s_E^2}{n_E}\right)^2}{\frac{s_A^4}{n_A^2\left(n_A-1\right)}+\frac{s_E^4}{n_E^2\left(n_E-1\right)}} \\
&= \frac{\left(\frac{12.759^2}{6}+\frac{3.225^2}{6}\right)^2}{\frac{12.759^4}{6^2\left(6-1\right)}+\frac{3.225^4}{6^2\left(6-1\right)}} \\
&= 5.6362 \\
&\approx 5 \qquad \left(\text{Rounded below}\right)
\end{aligned}
$$

Step 4/7
From the *t*-table values, at 0.10 level of significance, with 5 degrees of freedom the critical value is 2.015.

$$
\begin{aligned}
\text{CI} &= \left(85.00-80.00\right)\pm 2.0150\sqrt{\frac{\left(12.76\right)^2}{6}+\frac{\left(3.22\right)^2}{6}} \\
&= 5 \pm 10.8257 \\
&= \boxed{\left(-5.8257, 15.8257\right)}
\end{aligned}
$$

Therefore, the required 90% confidence interval is **(–5.8257, 15.8257)**.

Step 5/7
c)

Construct the 90% confidence interval for the population variance of scores for each student.

Construct the 90% confidence interval for the population variance score for Anthony.

$$
\text{CI} = \frac{\left(n-1\right)s_{\text{Anthony}}^2}{\chi_{\text{Left}}^2} < \sigma^2 < \frac{\left(n-1\right)s_{\text{Anthony}}^2}{\chi_{\text{Right}}^2}
$$

The critical values are,

$$
\begin{aligned}
\chi_{\text{Left}}^2 &= \chi_{\alpha/2,n-1}^2 \\
&= \chi_{0.05,5}^2 \\
&= 1.145 \qquad \left(\text{From chi-sqare tables}\right)
\end{aligned}
$$

$$\begin{aligned}\chi^2_{\text{Right}} &= \chi^2_{1-\alpha/2,n-1}\\ &= \chi^2_{0.95,5}\\ &= 11.07 \qquad \text{(From chi-sqare tables)}\end{aligned}$$

$$\begin{aligned}\text{CI} &= \left(\frac{(6-1)(12.76)^2}{1.145} < \sigma^2 < \frac{(6-1)(12.76)^2}{11.07}\right)\\ &= \left(73.540018 < \sigma^2 < 710.9938865\right)\\ &= \boxed{\left(73.540 < \sigma^2 < 710.994\right)}\end{aligned}$$

Therefore, the required 90% confidence interval is **(73.540, 710.994)**.

Step 6/7
Construct the 90% confidence interval for the population variance score for Eric.

$$\begin{aligned}90\% C.I &= \left(\frac{(n-1)s^2_{\text{Eric}}}{\chi^2_{\text{Left}}} < \sigma^2 < \frac{(n-1)s^2_{\text{Eric}}}{\chi^2_{\text{Right}}}\right)\\ &= \left(\frac{(6-1)(3.22)^2}{1.145} < \sigma^2 < \frac{(6-1)(3.22)^2}{11.07}\right)\\ &= \left(4.683107 < \sigma^2 < 45.276855\right)\\ &= \boxed{\left(4.683 < \sigma^2 < 45.277\right)}\end{aligned}$$

Therefore, the required 90% confidence interval is **(4.683, 45.277)**.

Step 7/7
d)

Construct the 90% confidence interval for the ratio of population variances.

$$\text{CI} = F_{\text{Left}}\left(\frac{S_X^2}{S_Y^2}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < F_{\text{Right}}\left(\frac{S_X^2}{S_Y^2}\right)$$

The $F-$critical values are,

$$\begin{aligned}F_{\text{Left}} &= F_{\left(1-\frac{\alpha}{2},n_1-1,n_2-1\right)}\\ &= F_{\left(1-\frac{0.10}{2},6-1,6-1\right)}\\ &= F_{(0.95,5,5)}\\ &= 0.198 \qquad (\text{From}, F-\text{tabulated}, \text{values})\end{aligned}$$

$$
\begin{aligned}
F_{\text{Right}} &= F_{\left(\frac{\alpha}{2}, n_1-1, n_2-1\right)} \\
&= F_{\left(\frac{0.10}{2}, 6-1, 6-1\right)} \\
&= F_{(0.05,5,5)} \\
&= 5.0503 \qquad (\text{From}, F-\text{tabulated}, \text{values})
\end{aligned}
$$

Substitute the values,

$$
\begin{aligned}
\text{CI} &= \left(0.198\left(\frac{(12.76)^2}{(3.22)^2}\right) < \frac{\sigma_X^2}{\sigma_Y^2} < 5.0503\left(\frac{(12.76)^2}{(3.22)^2}\right)\right) \\
&= \left(0.198(15.703) < \frac{\sigma_X^2}{\sigma_Y^2} < 5.0503(15.703)\right) \\
&= \boxed{\left(3.109244 < \frac{\sigma_X^2}{\sigma_Y^2} < 79.30613\right)}
\end{aligned}
$$

Therefore, the required confidence interval is, $\boxed{\left(3.109 < \frac{\sigma_X^2}{\sigma_Y^2} < 79.306\right)}$.