

Homework 4 (Due October 31, 2016, 9:00 AM)

BLM 1541: Statistics and Probability — Fall 2016

Print family (or last) name: _____

Print given (or first) name: _____

Print given student number: _____

I see that this homework has 3 questions in total.

I agree that I have to submit my homework solution before the deadline (October 31, 2016, 9 AM) otherwise my homework solution will not be graded.

I accept that ***I will add the signed version of this instruction page as a first page into my homework solution***; otherwise my homework solution will not be graded.

I know that ***I have to give my solutions written on white A4-sized pages that are stapled on the left-up corner***; otherwise my homework solution will not be graded.

I will take care of the readability of my solutions, from which I may lose 10 points.

For any proofs, I am sure to provide a step-by-step argument, with justifications for every step.

I understand that, during solving this homework, it is prohibited to exchange information about solutions with any other person in any way, including by talking or ex-changing solutions / papers.

I know that the course book is “Probability and Statistics for Computer Scientists, by Michael Baron (2nd Edition) ”.

I have read, understand and accept all of the instructions above. On my honor, I pledge that I have not violated the provisions of the Academic Integrity Code of Yıldız Technical University.

Signature and Date

Problem 1 [30 Points]: Show that the following equalities are correct for any discrete random variable X ,

- a) $P(a \leq X \leq b) = P(X = a) + F_X(b) - F_X(a)$,
- b) $P(a < X < b) = F_X(b) - F_X(a) - P(X = b)$,
- c) $P(a \leq X < b) = P(X = a) + F_X(b) - F_X(a) - P(X = b)$,

where $F_X(x)$ denotes the cumulative distribution function of the random variable X .

Problem 2 [35 Points]: The way TÜBİTAK hires graduated students from Yıldız Technical University is exemplified in the following table. Each applicant is rated to a discrete “graduation point average (mezuniyet not ortalaması)” X (horizontal axis) and a discrete “personal rating index” Y (vertical axis). The top number in each cell (in bold) is the number of applicants is a given year with the associated combination. The bottom number in each cell (in italic) is the probability of being accepted. (Note that, although this is assumed the way TÜBİTAK handles applications, all numbers are fictitious).

		← Graduation Point Average X →					
		90 – 100	80 – 90	70 – 80	60 – 70	50 – 60	≤ 50
← Personal Rating Y →	10	27 1.0	32 0.9	43 0.7	30 0.5	10 0.4	6 0.3
	9	60 0.9	95 0.7	110 0.5	150 0.4	45 0.3	20 0.2
	8	86 0.7	175 0.5	305 0.4	350 0.3	78 0.2	62 0.1
	7	39 0.5	173 0.4	250 0.3	165 0.2	102 0.1	50 0.0
	6	17 0.4	54 0.3	118 0.2	151 0.1	97 0.0	68 0.0
	≤ 5	1 0.3	12 0.2	28 0.1	45 0.0	17 0.0	21 0.0

- a) Plot the marginal PMF of the two indices.
- b) Plot the conditional PMFs ($X|Y = 8$) and ($X|Y = 6$).
- c) Plot the conditional PMF of ($Y|X \leq 50$).
- d) What is the probability that an applicant with $Y = 7$ is accepted.
- e) Are X and Y independent? Why?

Problem 3 [35 Points]: Let X and Y be two discrete random variables with the joint probability mass function (PMF) given by

$$P_{XY}(x, y) = \begin{cases} k(x+1)^y, & \text{if } x \in \{-1, 0, 1\} \text{ and } y \in \{1, 2\} \\ 0, & \text{Otherwise} \end{cases}$$

- a) Find the value of k in order to make $P_{XY}(x, y)$ a joint PMF.
- b) Find the PMFs of X and Y . Are X and Y independent? Why?
- c) Find the PMF of $Z = X^2$.
- d) Find the PMF of $Z = X^Y$.