Midterm Exam 2

BLM 1541: Probability and Statistics — Fall 2016

Print family (or last) name: ———————————————————————————————

Print given (or first) name: ———————————————————————————————

Print student number: ——————————————————— Group: ——— Order: ————

I have read and understand all of the instructions below, and I will obey the Academic Integrity Code of Yıldız Technical University.

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Signature and Date

- This exam has 4 pages in total, numbered 1 to 4. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book and closed-note exam. Electronic devices (e.g., cellphone, smart watch) are not allowed. Single page A4 note sheet is allowed. *Don't forget to attach your A4 note sheet into your solution; otherwise your solution will not be graded.*
- For all problems, follow these instructions:
    - Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
    - PMF stands for probability mass function; CDF stands for cumulative distribution function; $var(X)$ stands for the variance of the random variable X; $cov(X, Y)$ stands for the covariance between the random variables X and Y.
    - For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	Total
Points	20	15	20	15	15	15	100

1. **[20 points]** For the safety of YTÜ students, a speed trap (radar) were put on Yildiz Street, Davutpaşa. The speed trap recorded the speeds of vehicles as

86 74 80 72 87 60 108 72 80 74 83 102 79 74 84 72 57 65

a) Find mean and median for the speed of these vehicles.

b) Find the skewness type of the speed of these vehicles? Left, right, symmetric?

c) Find range and interquartile range (IQR) for the speed of these vehicles.

d) Find the histogram for $[0,24), [25,49), [50,74), [75,99)$ ranges of speeds, and discuss the skewness of the histogram.

2. **[20 points]** Let X and Y be random variables with means μ_X and μ_Y, variances σ_X and σ_Y, and correlation coefficient $\rho_{X,Y}$.

a) Find the value of the constant a which minimizes $E[(Y - aX)^2]$.

Hint: The value that the function $f(x)$ is minimum / maximum is found by solving $\frac{d}{dx} f(x) = 0$

b) Find the value of $E[(Y - aX)^2]$ when a is given as determined in part (a).

3. **[20 points]** The weights of a population of workers have mean 167 and standard deviation 27. If a sample of 36 workers is chosen, approximate the probability that the sample mean of their weights lies between 163 and 170.

   Hint: Notice the distribution, expectation, and variance of sample mean in this question.

4. **[20 points]** A computer lab has two printers. Printer A handles 40% of all the jobs. Its printing time is Exponential with the mean of 2 minutes. Printer B handles the remaining 60% of jobs. Its printing time is Uniform between 0 minutes and 5 minutes. A job was printed in less than 1 minute. Find the probability of that Printer A printed it?

3. **[20 points]** Let X_1 and X_2 be two random variables following the PDFs $p_{X_1}(x)$ and $p_{X_2}(x)$ with known means μ_1 and μ_2, respectively. Note that M samples from X_1 is contaminated by N samples from X_2 are mixed, i.e., they are mixed. Consider the following contaminated PDF for M and N,

$$p_Z(x) = \frac{M}{M+N} p_{X_1}(x) + \frac{N}{M+N} p_{X_2}(x)$$

Let

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