Assume that X and Y are two independent normal (Gaussian) random variables with zero means and same variances. Accordingly, find the probability $\Pr(X < Y)$.

A:

1

B:

3/4

C:

1/2

D:

1/4

E:

None of them.

Let $X$ be an exponential distribution whose probability density function (PDF) is given by $f_X(x) = 2\exp(-2x)$ for $0 \le x < \infty$. Let $x_q$ denote its q-quantile. Then, find the value of the quantile $x_{3/4}$.

$2e^{-2x}$

A:
$\log(2)$

B:
$2\ \log(2)$

C:
$1 + \log(2)$

D:
$2 + \log(2)$

E:
None of them.

Let $X$ be a random variable and $c$ be a constant, then the following properties are valid:

I. $var(\mathbb{E}[c]) = 0$ +

II. $var(\mathbb{E}[X + c]) = \mathbb{E}[var(X)]$

III. $var(\mathbb{E}[cX]) = \mathbb{E}[c^2\ var(X)]$

$var(c\,E[x]))$

$var\ E[x] + \cancel{var(c)}$ $\emptyset$ $0$

A:
Only I

B:
Only II

$c^2\ \cancel{E[c}$

C:
Only III

$c^2\ var(E[x]) \rightarrow c^2\ var(E[x])$

D:
I and III

E:
I, II, and III

Is the statement given below true or false?

Estimator of a random variable is a random variable.

A:
True

B:
False

A researcher comes up a discrete function in his/her research like that

$$\Pr(X = x_i) = \begin{cases} \lambda(1 + x_i - {x_i}^2) & \text{if } x_i \in \{-2, -1, 0, 1, 2\} \\ 0 & \text{otherwise} \end{cases}$$

with a constant $\lambda \in \mathbb{R}$. This function can be a valid probability mass function (PMF) with an appropriate choice of $\lambda$

A:
True

B:
False

Is the statement given below true or false?

The sum of independent and identically distributed random variables is asymptotically following normal (Gaussian) distribution.

**A:**
True

**B:**
False

Is the statement given below true or false?

When expectation of an estimator $\hat{\theta}$ is equal to the parameter $\theta$, we say that the estimator is consistent.

A:
True

B:
False

Assume that $\widehat{X}$ estimates population parameter $X$ and is defined as

$$X = \frac{X_1+X_2+...+X_n}{n},$$

with a random sample $(X1, X2, \ldots, X_n)$ having $n$ units. What is the standard error of the estimator $\widehat{X}$?

Note: The mean and variance of $X$ is given as $\mu$ and $\sigma^2$, respectively.

A:
$n\sigma$

B:
$\sigma^2$

C:
$\frac{X}{n}$

D:
$n\mu$

E:
$\frac{\sigma}{\sqrt{n}}$

Is the statement given below true or false?

We need to use continuity correction when we approximate Gamma distribution using Normal distribution.

**A:**
True

**B:**
False

Is the statement given below true or false?

The estimate $\hat{\theta}$ of the parameter $\theta$ follows a random variable.

A:
True

B:
False

Let $X$ be any discrete or continuous random variable, then the equality $\mathbb{E}[X^2] \geq \mathbb{E}[X]^2$ is always true.

A:
True

B:
False

Is the statement given below true or false?

Simple random sampling is to collect units from the entire population independently of each other, all being equally likely to be sampled.

A:
True

B:
False

Let $X$ and $Y$ be two random variables whose joint probability density function is given by

$$f_{X,Y}(x,y) = \begin{cases} cx & \text{when } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ -cy & \text{when } -1 \le x \le 0 \text{ and } -1 \le y \le 0 \\ 0 & \text{elsewhere} \end{cases}$$

where $c \in \mathbb{R}^+$ is such a number guaranteeing that $f_{X,Y}(x,y)$ is certainly a joint probability density function. Find $f_X(x)$, the marginal PDF of $X$?

A:

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ x/2 & -1 \le x \le 0 \\ 0 & elsewhere \end{cases}$$

B:

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 1/2 & -1 \le x \le 0 \\ 0 & elsewhere \end{cases}$$

C:

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ x/2 & -1 \le x \le 0 \\ 0 & elsewhere \end{cases}$$

D:

$$f(x) = \begin{cases} x & -1 \le x \le 1 \\ 1/2 & -1 \le x \le 0 \\ 0 & elsewhere \end{cases}$$

E:

None of them

Consider the sample $S = (20, 10, 3)$. What is the sample variance?

A:
48.67

B:
60.23

C:
73.00

D:
82.00

E:
90.87

$\overline{X} = \underline{11}$

$\underline{X - \overline{X}}$

$81 + 1 + 64$

$\dfrac{81 + 1 + 36}{3}$

$$\begin{array}{r} 12 \\ 27 \\ +\ 0{,}3 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ 27 \\ +\ 0{,}3 \\ \hline \boxed{59{,}3} \end{array}$$

Consider the sample $S = (20, 10, 3)$. What is the sample mean?

A:
11

B:
20

C:
3

D:
15

E:
10

Let $X$ be a non-negative random variable that follows the probability density function (PDF) that is given by $f_X(x)$ for $0 \leq x < \infty$. Find the PDF of $Y = aX$ in terms of the PDF $f_X(x)$, where $a \in \mathbb{R}^+$ is a positive constant number.

A:

$$f_Y(y) = \begin{cases} a\, f_X(y) & \text{if } 0 \leq y < \infty \\ 0 & \text{otherwise} \end{cases}$$

B:

$$f_Y(y) = \begin{cases} a\, f_X(ay) & \text{if } 0 \leq y < \infty \\ 0 & \text{otherwise} \end{cases}$$

C:

$$f_Y(y) = \begin{cases} \frac{1}{a}\, f_X\left(\frac{y}{a}\right) & \text{if } 0 \leq y < \infty \\ 0 & \text{otherwise} \end{cases}$$

D:

$$f_Y(y) = \begin{cases} f_X(ay) & \text{if } 0 \leq y < \infty \\ 0 & \text{otherwise} \end{cases}$$

E:

None of them.

Which ones of the following functions are a cumulative distribution function (CDF)?

I. $F_X(x) = \begin{cases} 1 - exp(-x^2) & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

II. $F_X(x) = \begin{cases} exp(-1/x) & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

III. $F_X(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 0 & \text{if } 0 < x \leq \frac{1}{2} \\ 1 & \text{if } x > \frac{1}{2} \end{cases}$

A:

Only I

B:

I and II

C:

I and III

D:

II and III

E:

I, II, and III

Consider the sample $S = (\cancel{20}, \cancel{10}, \cancel{5}, \cancel{1}, \cancel{1}, \cancel{12}, \cancel{13}, \cancel{16})$. What is the $(1/3)$-quantile estimate?

1 1 5 10 12 13 16, 20

A:
1

B:
5

C:
10

D:
12

E:
20

Let $X$ be a random variable whose probability density function (PDF) is given by

$$f_X(x) = \begin{cases} \frac{3}{4}\left(1 - x^2\right) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the median of the random variable $X$.

A:
$3/4$

B:
$1 - x^2$

C:
1

D:
$\sqrt{3}$

E:
$[-0.5,\ 0.5)$

A farmer is working on the irrigation task of his garden, where the time it takes to complete the task is a random variable $X$ that follows a uniform distribution between 30 minutes and 2 hours. Given the farmer has already worked on the task for 1 hour, what is the probability that it will take no more than 30 more minutes to finish the task?

A:

1

B:

1/2

C:

1/3

D:

1/4

E:

None of them.

The normal (Gaussian) random variable $X$ with a variance $\sigma$ and with a mean $\mu$ follows the probability density function (PDF) given as

$$f_X(x) = \frac{1}{\sqrt{2\pi\mu}} \exp\left(-\frac{(x-\sigma)^2}{2\mu^2}\right), \quad -\infty < x < \infty$$

A:
True

B:
False

Is the statement given below true or false?

In our statistics computations, by taking 2000 samples instead of 1000 samples, we can reduce the non-sampling errors.

A:
True

B:
False

Let $X_1, X_2, \ldots, X_n$ be such a random sample collected from the population $X$ that it has the following histogram in which

- the number of samples greater than or equal to 0 and less than 5 is 15,
- the number of samples greater than or equal to 5 and less than 10 is 25,
- the number of samples greater than or equal to 10 and less than 15 is 10.

Following from the histogram, estimate the probability $\Pr(X < 10)$.

A:
1/5

B:
2/5

C:
3/5

D:
4/5

E:
None of them

Let $X$ be a continuous random variable and $c$ be a constant number. Then, which ones of the following statements are correct.

I. $var(c) = 0$

II. $var(X + c) = var(X)$

III. $var(cX) = c^2\ var(X)$

A:

Only I

B:

Only II

C:

Only III

D:

I and III

E:

I, II and III

Let $X$ and $Y$ be two random variables. If $\mathbb{E}[XY] = \mathbb{E}[X]\ \mathbb{E}[Y]$, then $X$ and $Y$ are certainly independent.

A:
True

B:
False