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3] a) $u \in \mathbb{R}^3$, $u = (7, -6, 7)$

$$u = x \cdot a_1 + y \cdot a_2 + z \cdot a_3$$

$$(7, -6, 7) = x(2, -1, 0) + y(0, 1, 2) + z(1, -1, 3)$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ -1 & 1 & -1 & -6 \\ 0 & 2 & 3 & 7 \end{array}\right) \xrightarrow{R_1 \to (\frac{1}{2}) \cdot R_1} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{7}{2} \\ -1 & 1 & -1 & -6 \\ 0 & 2 & 3 & 7 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{7}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 2 & 3 & 7 \end{array}\right) \xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{7}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & 4 & 12 \end{array}\right)$$

$$\xrightarrow{R_3 \to (\frac{1}{4}) R_3} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{7}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2 \to R_2 + \frac{1}{2} R_3} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{7}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{2} \cdot R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{l} x = 2 \\ y = -1 \\ z = 3 \end{array}$$

$$\Rightarrow (7, -6, 7) = 2(2, -1, 0) - (0, 1, 2) + 3(1, -1, 3)$$

Lineer birleşimi olarak yazılabilir.

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3] b) ilk olarak : $c_1 \cdot a_1 + c_2 \cdot a_2 + c_3 \cdot a_3 = (0,0,0)$

$\rightarrow c_1 = c_2 = c_3 = 0$ olduğunu göstermeliyiz.

- lineer bağımsız mı?

$$\begin{pmatrix} 2 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a şıkkında birim matrisine dönüşmesi gösterdik

$\{a_1, a_2, a_3\}$ lineer bağımsızdır

boy $\beta = \{a_1, a_2, a_3\} = 3$.

boy $R^3 = 3$ olduğunu bazdır.

- $R^3 = \langle S \rangle$ $\quad S = \{(2,-1,0),(0,1,2),(1,-1,3)\}$

$\forall (\alpha_1, \alpha_2, \alpha_3) \in R^3$ için

$$(\alpha_1, \alpha_2, \alpha_3) = x(2,-1,0) + y(0,1,2) + z(1,-1,3)$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & \alpha_1 \\ -1 & 1 & -1 & \alpha_2 \\ 0 & 2 & 3 & \alpha_3 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{2} \cdot R_1} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{\alpha_1}{2} \\ -1 & 1 & -1 & \alpha_2 \\ 0 & 2 & 3 & \alpha_3 \end{array}\right)$$

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3] b)

$$\xrightarrow{R_2 \to R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{\alpha_1}{2} \\ 0 & 1 & \frac{-1}{2} & \frac{\alpha_1 + 2\alpha_2}{2} \\ 0 & 2 & 3 & \alpha_3 \end{array}\right) \xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{\alpha_1}{2} \\ 0 & 1 & \frac{-1}{2} & \frac{\alpha_1 + 2\alpha_2}{2} \\ 0 & 0 & 4 & -\alpha_1 - 2\alpha_2 + \alpha_3 \end{array}\right)$$

$$\xrightarrow{R_3 \to \frac{1}{4} \cdot R_3} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{\alpha_1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{\alpha_1 + 2\alpha_2}{2} \\ 0 & 0 & 1 & \frac{-\alpha_1 - 2\alpha_2 + \alpha_3}{4} \end{array}\right)$$

$$\begin{array}{c} \xrightarrow{R_1 \to R_1 - \frac{1}{2} R_3} \\ R_2 \to R_2 + \frac{1}{2} R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{5\alpha_1 + 2\alpha_2 - \alpha_3}{8} \\ 0 & 1 & 0 & \frac{3\alpha_1 + 6\alpha_2 + \alpha_3}{8} \\ 0 & 0 & 1 & \frac{-\alpha_1 - 2\alpha_2 + \alpha_3}{4} \end{array}\right)$$

$$x = \frac{5\alpha_1 + 2\alpha_2 - \alpha_3}{8} \Big/ y = \frac{3\alpha_1 + 6\alpha_2 + \alpha_3}{8} \Big/ z = \frac{-\alpha_1 - 2\alpha_2 + \alpha_3}{4}$$

o halde $\{a_1, a_2\ a_3\}$, R^3'ün bir bazıdır. $R^3 = \langle S \rangle$