

$$\boxed{3} \quad \left. \begin{array}{l} x - 2y + 3z = 1 \\ 2x + \lambda y + 6z = 6 \\ -x + 3y + (\lambda - 3)z = 0 \end{array} \right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 2 & \lambda & 6 & 6 \\ -1 & 3 & \lambda-3 & 0 \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow R_2 - 2 \cdot R_1 \\ R_3 \rightarrow R_3 + R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & \lambda+4 & 0 & 4 \\ 0 & 1 & \lambda & 1 \end{array} \right]$$

$$\lambda \neq -4$$

$$R_2 \leftrightarrow R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & \lambda & 1 \\ 0 & \lambda+4 & 0 & 4 \end{array} \right]$$

$$R_3 \rightarrow R_3 - (\lambda+4) \cdot R_2 \quad \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & \lambda & 1 \\ 0 & 0 & -\lambda^2-4\lambda & -\lambda \end{array} \right] \quad \lambda \neq 0$$

$$R_3 \rightarrow \left(\frac{1}{-\lambda^2-4\lambda} \right) \cdot R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & \lambda & 1 \\ 0 & 0 & 1 & \frac{1}{\lambda+4} \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow R_2 - \lambda \cdot R_3 \\ R_1 \rightarrow R_1 - 3 \cdot R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & 0 & \frac{\lambda+1}{\lambda+4} \\ 0 & 1 & 0 & \frac{4}{\lambda+4} \\ 0 & 0 & 1 & \frac{1}{\lambda+4} \end{array} \right]$$

$$\boxed{3} \quad R_1 \rightarrow R_1 + 2 \cdot R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{\lambda+9}{\lambda+4} \\ 0 & 1 & 0 & \frac{4}{\lambda+4} \\ 0 & 0 & 1 & \frac{1}{\lambda+4} \end{array} \right]$$

i) Tek çözüm: $x = \frac{\lambda+9}{\lambda+4}$, $y = \frac{4}{\lambda+4}$, $z = \frac{1}{\lambda+4}$

$\therefore k = \left\{ \frac{\lambda+9}{\lambda+4}, \frac{4}{\lambda+4}, \frac{1}{\lambda+4} \right\}$ katsayılar matrisi tersinir olduğundan sistemin tek çözümü vardır.

ii) $\lambda^2 + 4\lambda = 0$ ise: $\boxed{a} \lambda = -4 \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 4 \end{array} \right]$

$4 \neq 0$ Dolayısıyla çözüm yoktur (çözüm­süzdür).

$\boxed{b} \lambda = 0 \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} 3-2=1 \\ \text{keyfi de\u011fer} \end{array}$

$y = 1$, $x - 2 + 3z = 1 \Rightarrow x = 3 - 3z$

Sonsuz çözüm vardır

$\therefore k = \{ 3 - 3z, 1, z \}$