İşaret İşleme

Örnekleme Teoremi-H13CD2

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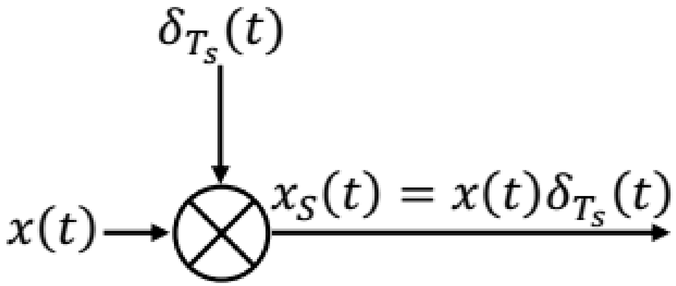
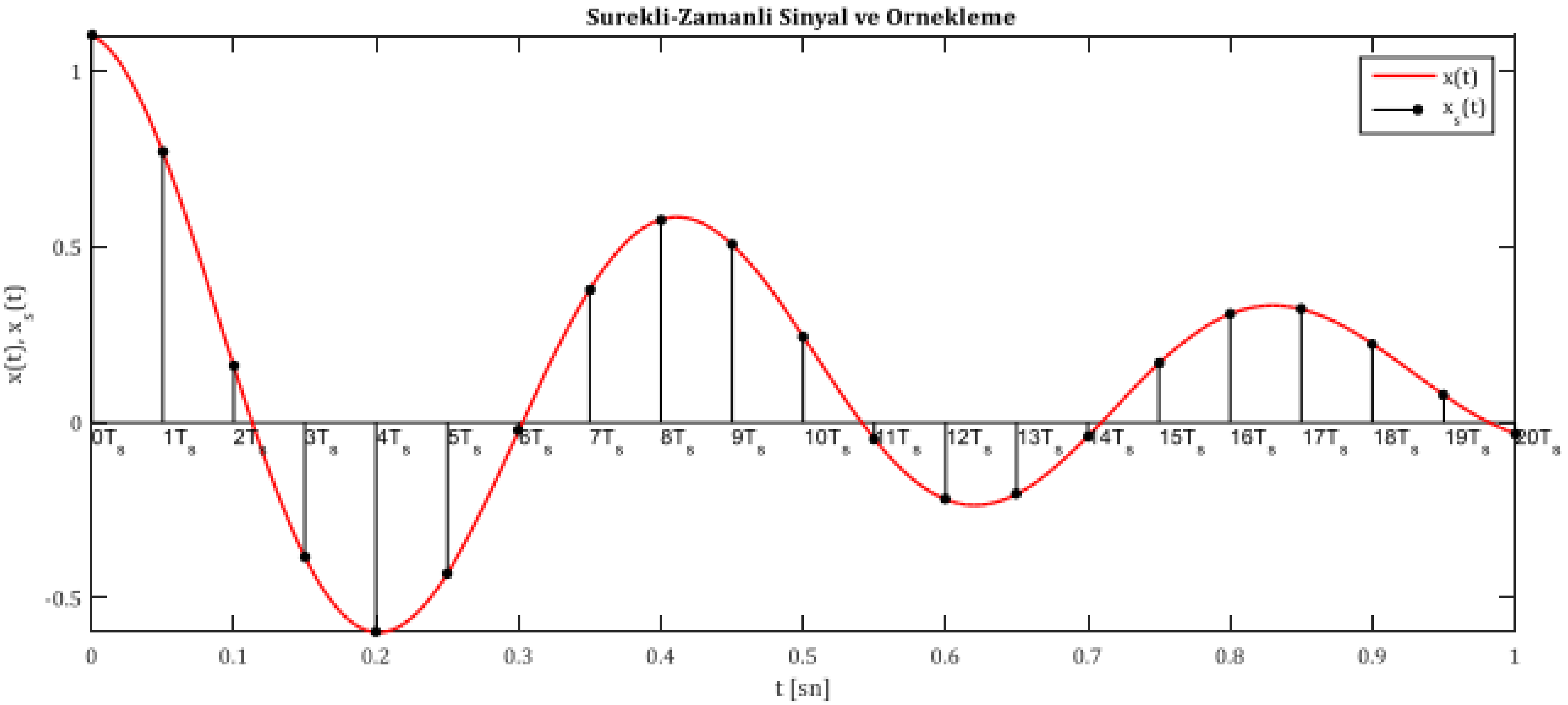
Örnekleme

Örnekleme, dijital sinyal işlemenin temelini oluşturan bir işlem olup zaman domenindeki w_B gibi sonlu bant genişlikli sürekli-zamanlı bir sinyalin T_s periyotlu $\delta_{T_s}(t)$ darbe katarı çarpılarak ayrık-zamanlı hale getirilmesini ve bu sayede dijital sinyal işlemeye uygun hale getirilmesini sağlar. Bunu aşağıdaki şekilde görmek mümkündür.



Aşağıdaki şekilde tipik bir örneklenmiş sinyal görülmektedir.



Burada en önemli soru T_s periyodunun nasıl seçileceğidir. Şekle bakıldığında, Fourier dönüşümleri arasında bir girişim ya da örtüşmenin olmaması için

$$w_B < 2w_s$$

şartının sağlanması gerekir ki bu da örnekleme teoreminin en önemli sonuçlarından biridir. Buna göre, örnekleme frekansı, örneklenecek sinyalin bant genişliğinin en az iki katı olmalıdır.

We can hear sounds with frequency components between 20 Hz and 20 kHz.

What is the maximum sampling interval T that can be used to sample a signal without loss of audible information?

1. $100\,\mu s$
2. $50\,\mu s$
3. $25\,\mu s$
4. $100\pi\,\mu s$
5. $50\pi\,\mu s$
6. $25\pi\,\mu s$

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$$2\pi f_m = \omega_m < \frac{\omega_s}{2} = \frac{2\pi}{2T}$$

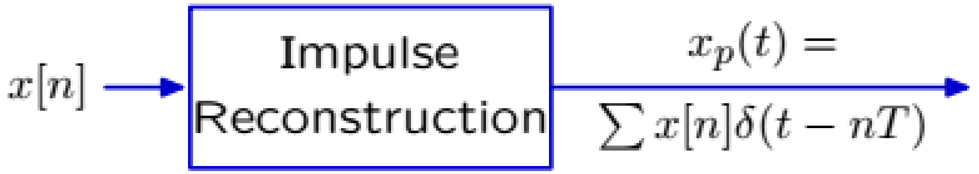
$$T < \frac{1}{2f_m} = \frac{1}{2 \times 20\ \text{kHz}} = 25\,\mu s$$

Last Time: Sampling Theory

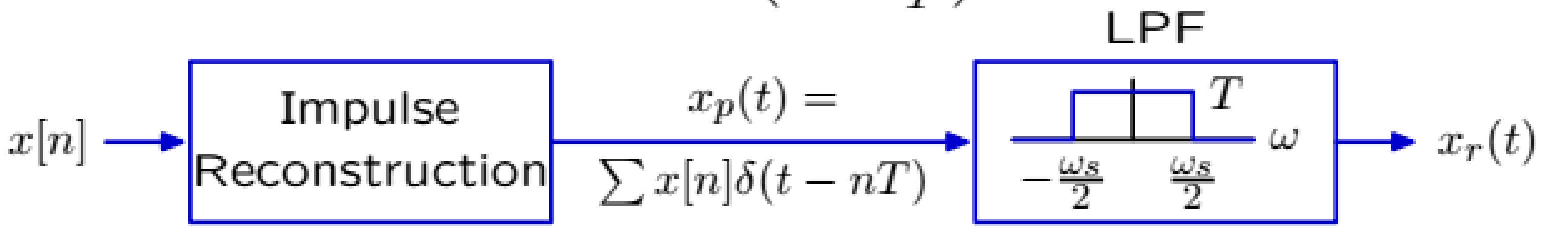
Sampling

$$x(t) \rightarrow x[n] = x(nT)$$

Impulse Reconstruction



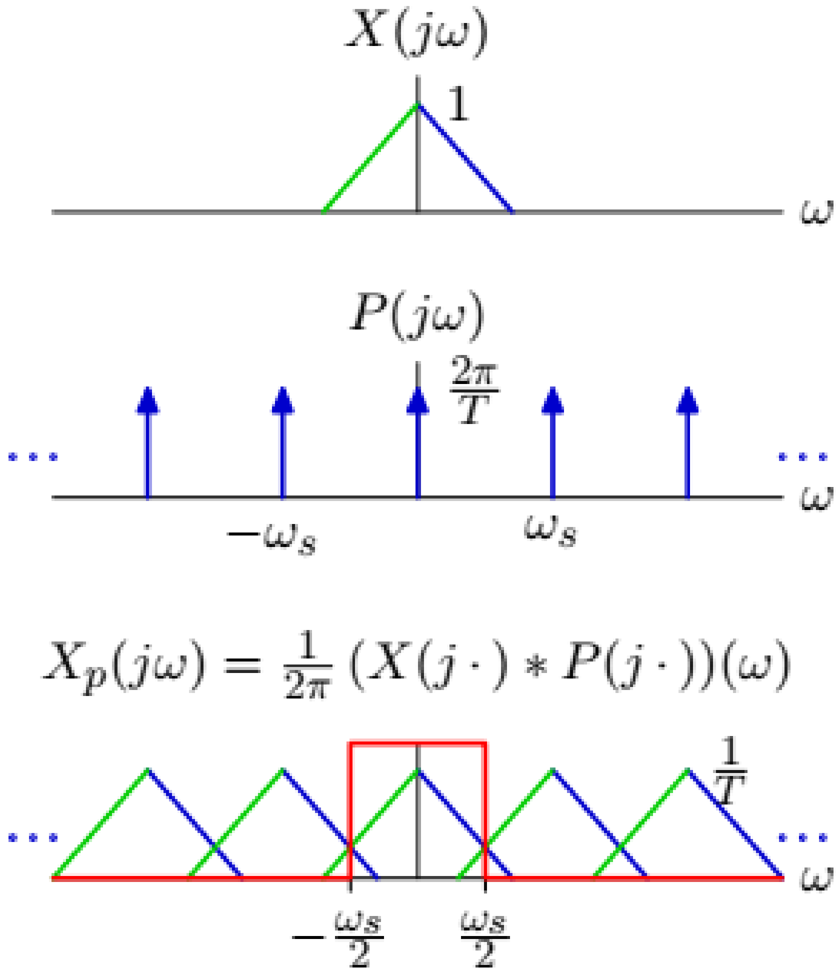
Bandlimited Reconstruction $\left(\omega_s = \frac{2\pi}{T}\right)$



Sampling Theorem: If $X(j\omega) = 0 \ \forall \ |\omega| > \frac{\omega_s}{2}$ then $x_r(t) = x(t)$.

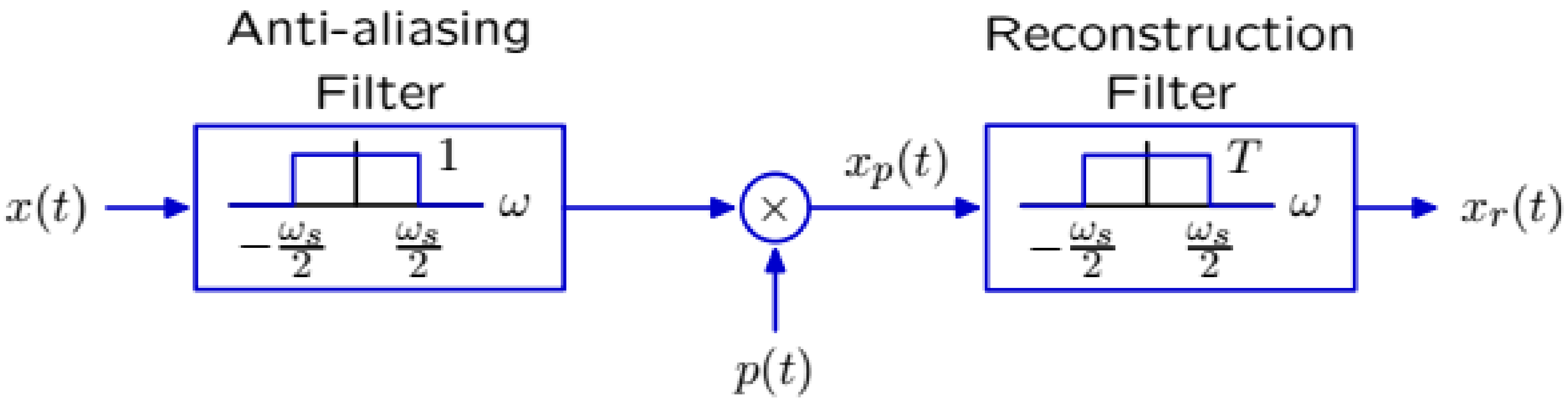
Aliasing

High frequency components of complex signals also wrap.



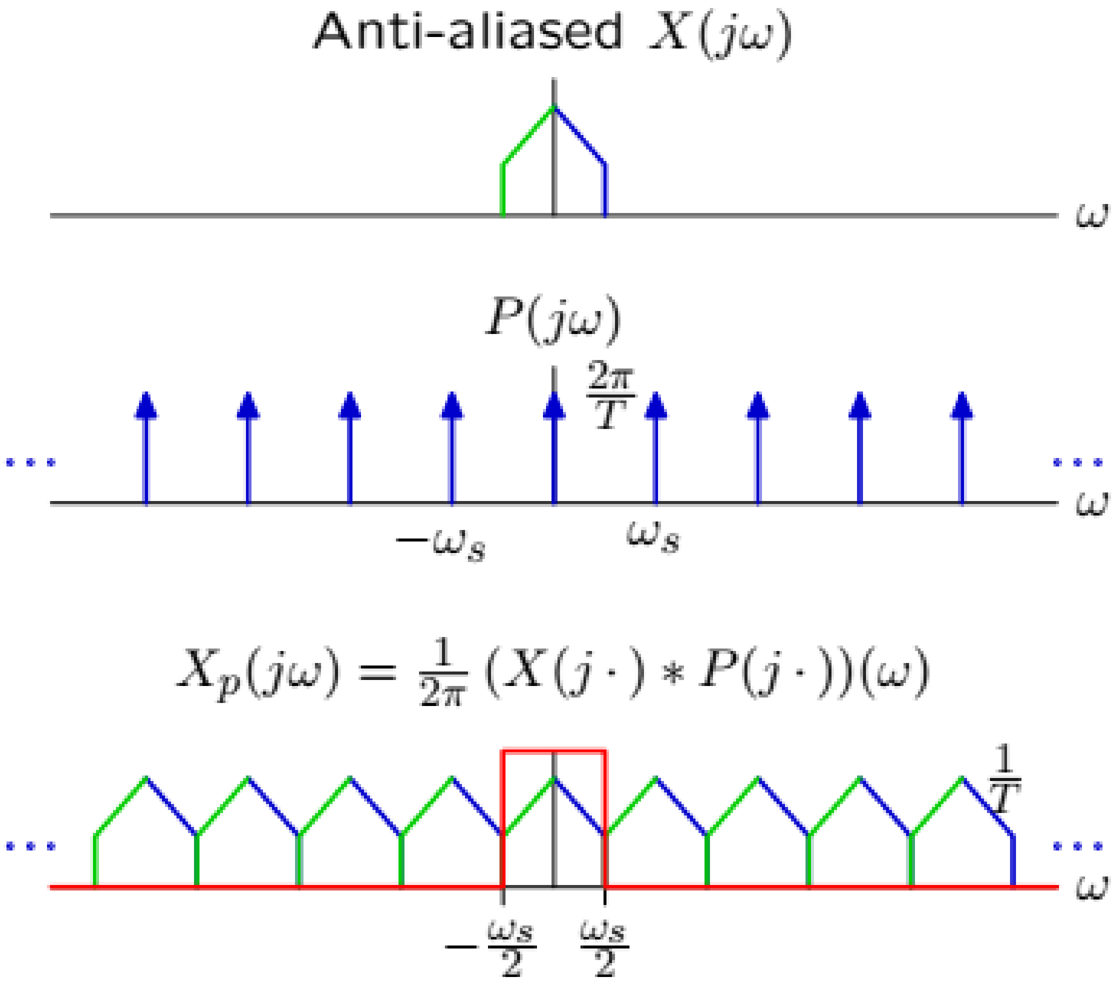
Anti-Aliasing Filter

To avoid aliasing, remove frequency components that alias before sampling.



Aliasing

Aliasing increases as the sampling rate decreases.



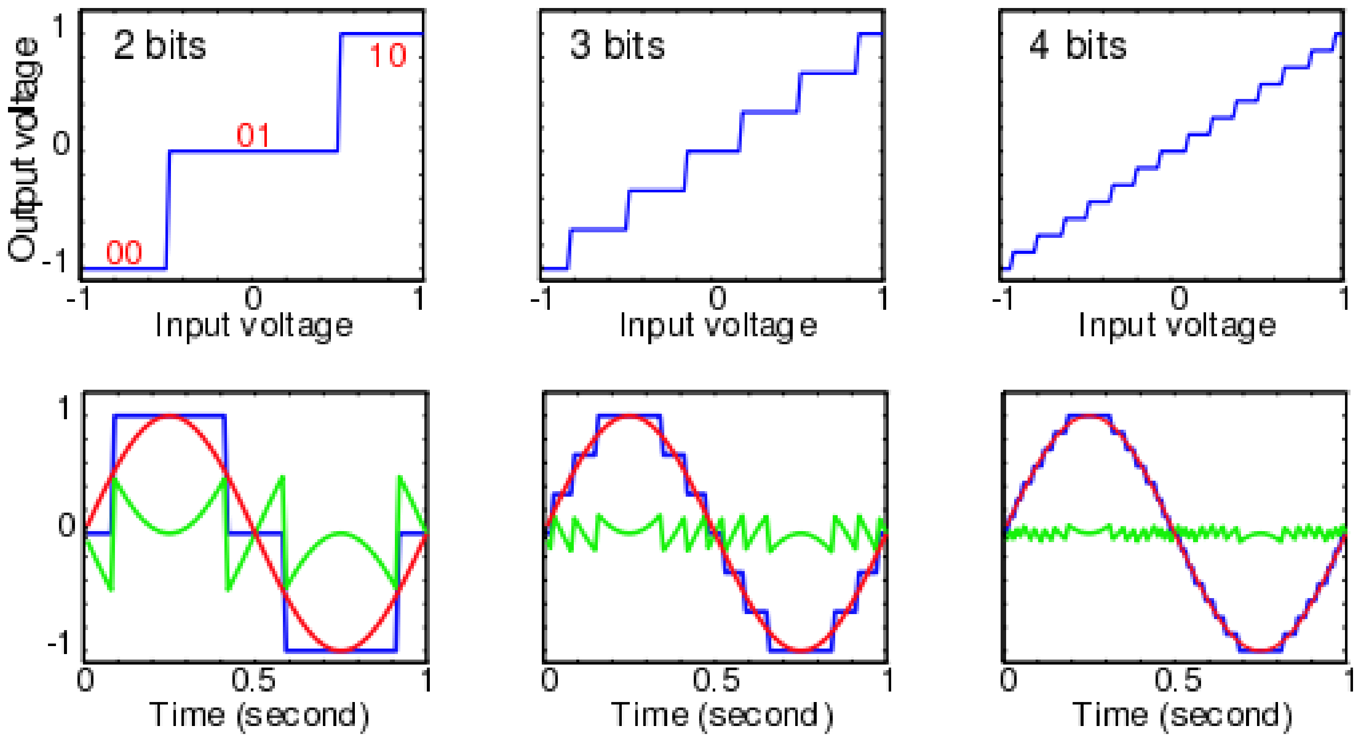
Digital recording, transmission, storage, and retrieval requires discrete representations of both time (e.g., sampling) and amplitude.

- audio: MP3, CD, cell phone
- pictures: digital camera, printer
- video: DVD
- everything on the web

Quantization: discrete representations for amplitudes

Quantization

We measure discrete amplitudes in bits.



Bit rate = (# bits/sample)×(# samples/sec)

We hear sounds that range in amplitude from 1,000,000 to 1.

How many bits are needed to represent this range?

1. 5 bits
2. 10 bits
3. 20 bits
4. 30 bits
5. 40 bits

bits	range
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
11	2,048
12	4,096
13	8,192
14	16,384
15	32,768
16	65,536
17	131,072
18	262,144
19	524,288
20	1,048,576

Quantizing Images

Converting an image from a continuous representation to a discrete representation involves the same sort of issues.

This image has 280×280 pixels, with brightness quantized to 8 bits.

Quantizing Images

8 bit image

7 bit image

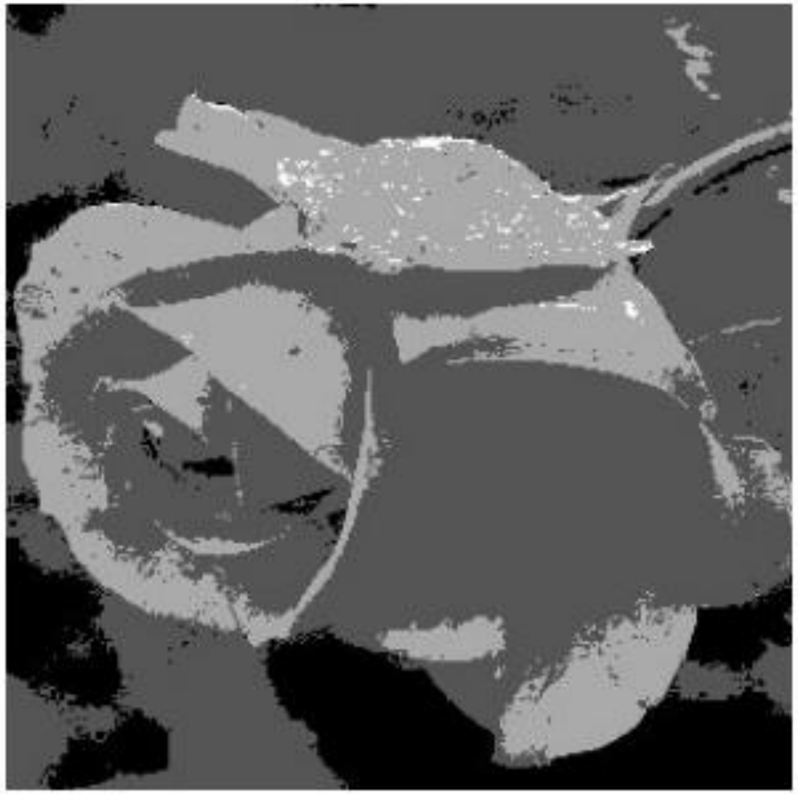
Quantizing Images

8 bit image

4 bit image

Quantizing Images

8 bit image



2 bit image

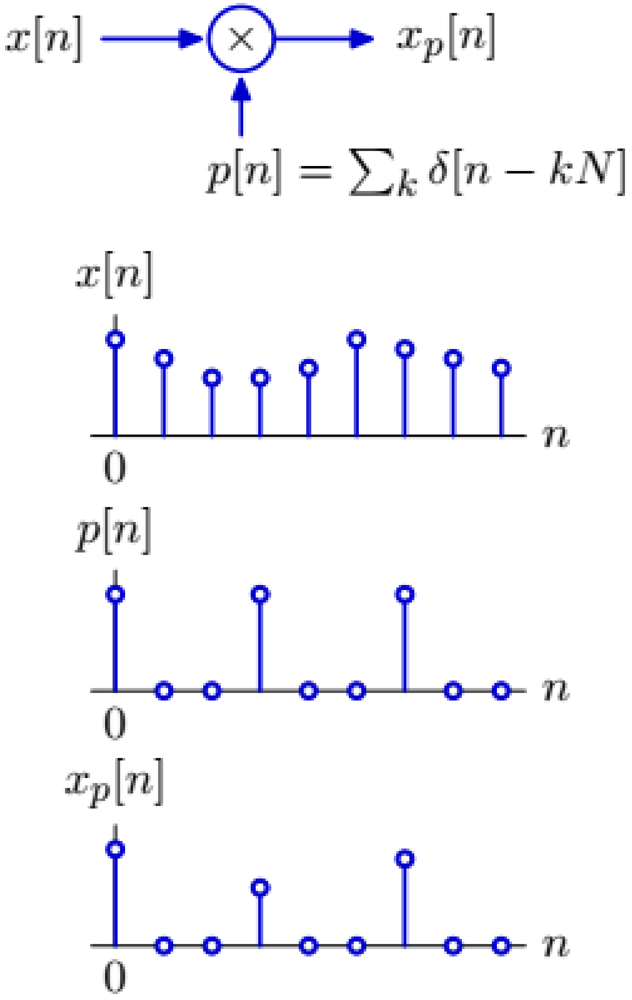
Quantizing Images

8 bit image

1 bit image

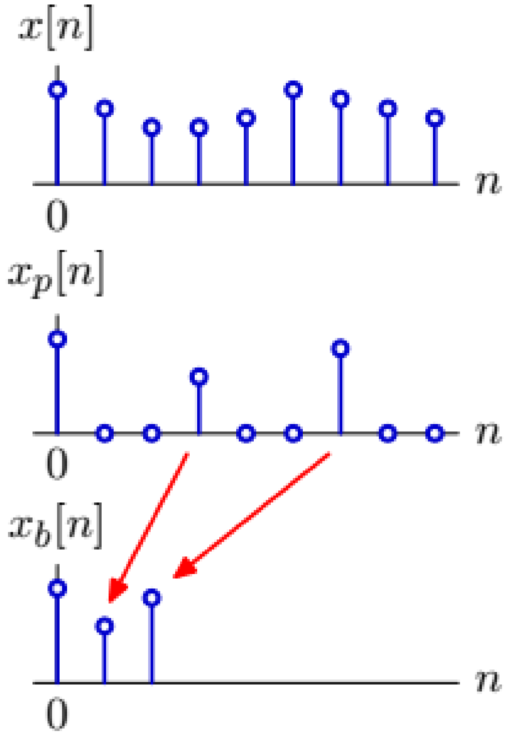
Discrete-Time Sampling (Resampling)

DT sampling is much like CT sampling.



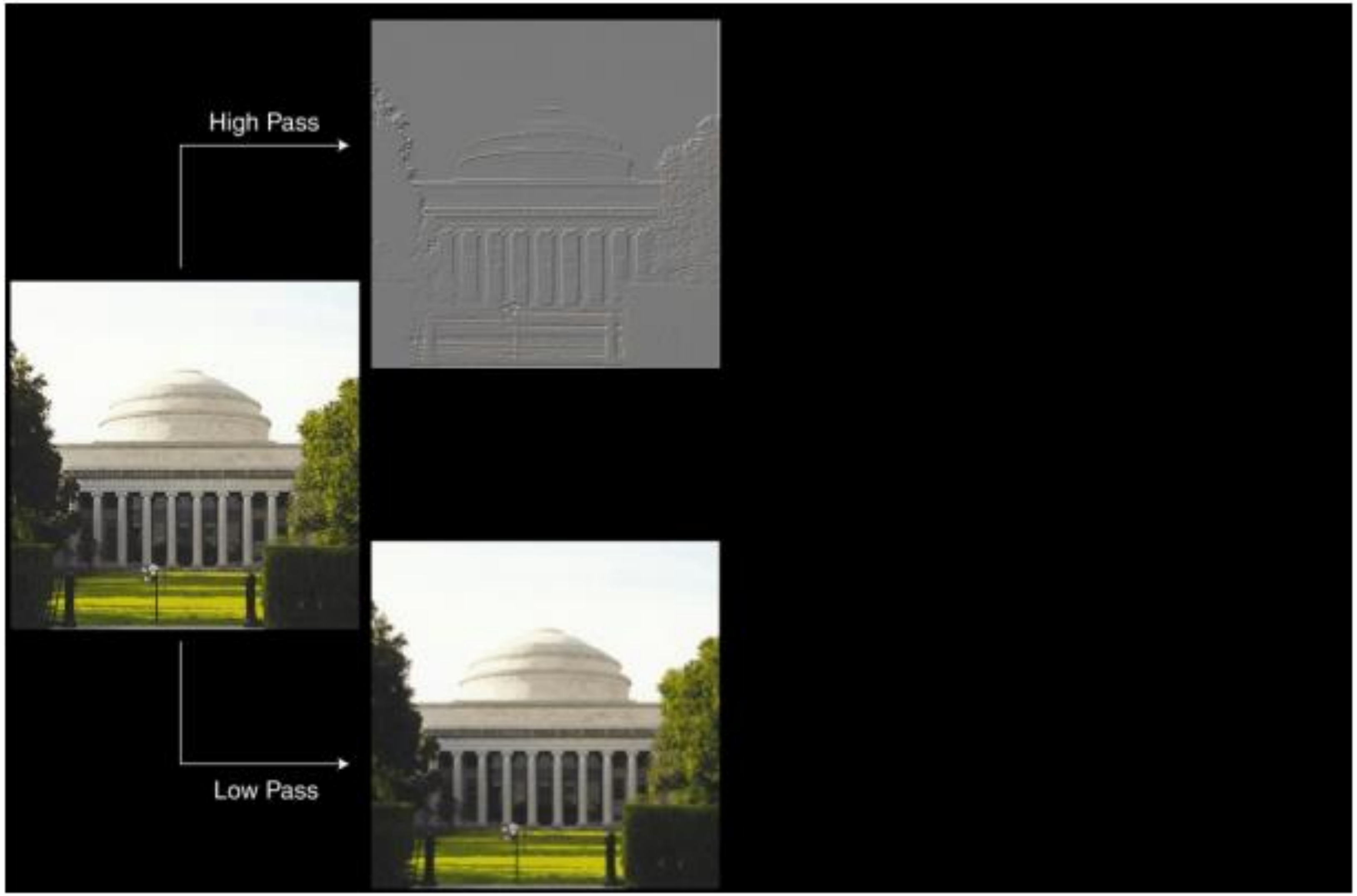
Discrete-Time Sampling

Sampling a finite sequence gives rise to a shorter sequence.

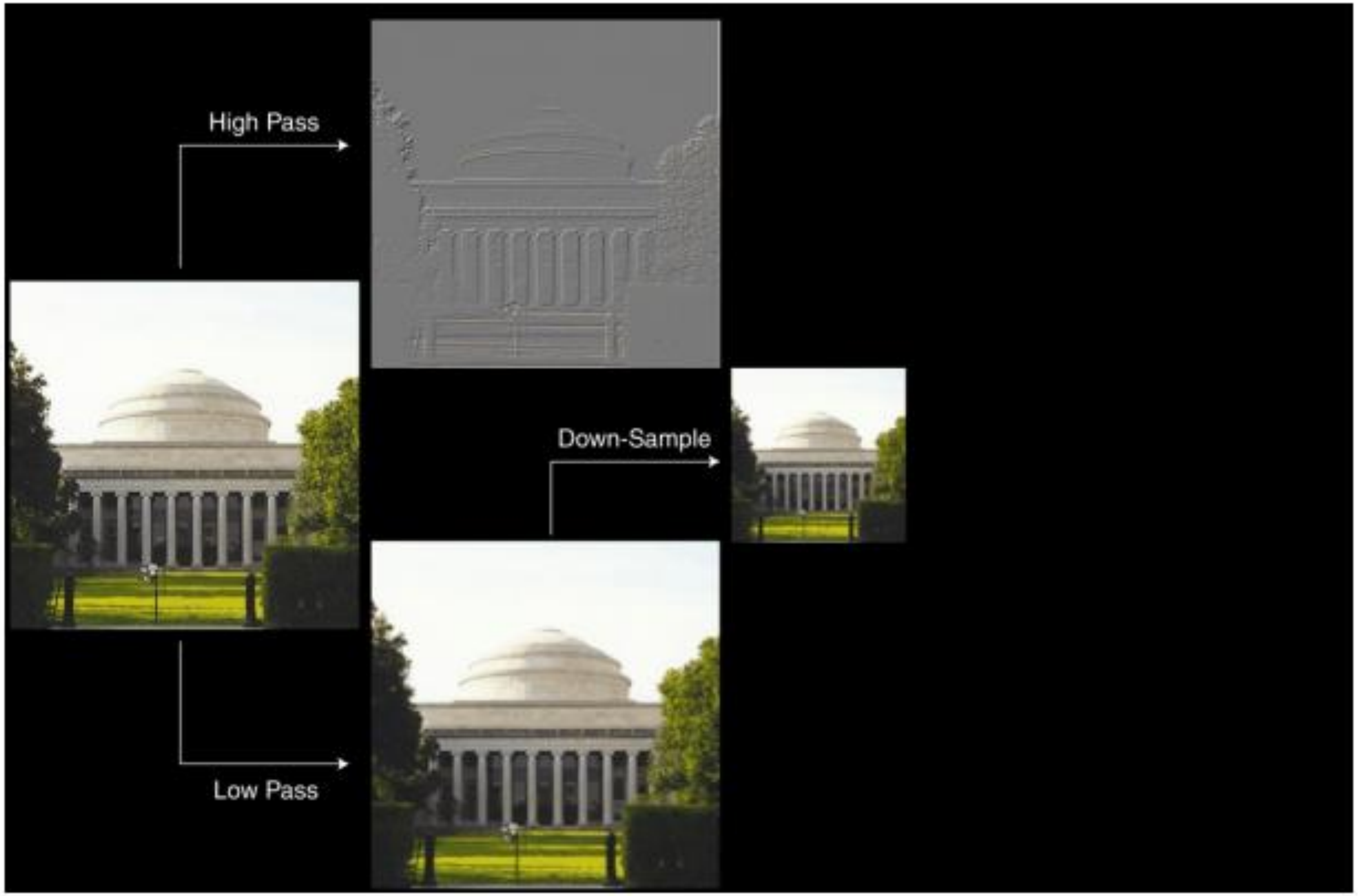


$$X_b(e^{j\Omega}) = \sum_n x_b[n]e^{-j\Omega n} = \sum_n x_p[3n]e^{-j\Omega n} = \sum_k x_p[k]e^{-j\Omega k/3} = X_p(e^{j\Omega/3})$$

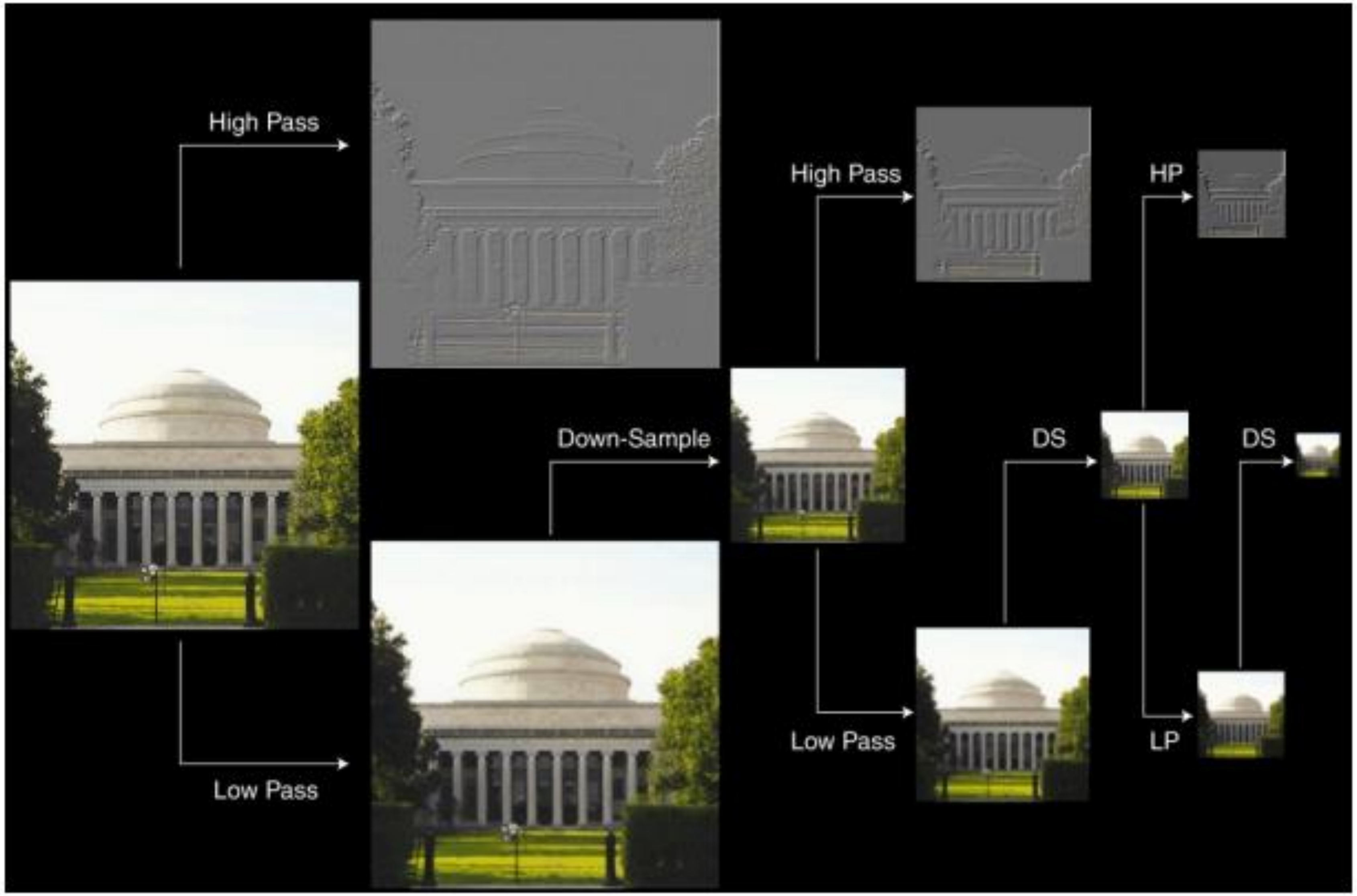
Discrete-Time Sampling



Discrete-Time Sampling



Discrete-Time Sampling



Bu ders notu için faydalanılan kaynaklar

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6.003 Signals and Systems
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