

Sayısal Sistemler-H3CD2

Boole Cebri ve Kanonik Formlar

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Bu derste öğreneceklerimiz

2 Boolean Algebra and Logic Gates

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Kanonik ve Standart Formlar

Minterm & Maxterm

Bir fonksiyonun çarpımların toplamı veya toplamların çarpımı şeklinde gösterilmesine uygun olarak minterm veya maxterm'ler ifade edilir.

İkili bir değişken, normal formunda (x) veya tümleyen formunda (x') yazılabilmektedir.

Minterm ve Maxterm'ler birbirinin değili/tümleyenidir.

Minterm & Maxterm

- **Minterm:**
- Değişkenlerin «1» değeri alanları kendisiyle, «0» değeri alanları değişkenin değili/tümleyeni ile sembolize edilerek **çarpım** formunda yazılırsa ve genel ifade **çarpımların toplamı** ile verilirse bu gösterime '**Minterm**' adı verilir. Buna 1. Kanonik Açılım da denir. Minterm'ler küçük harfle sembolize edilir ($m_0, m_1, \dots, m_7, \dots$).

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

Minterm & Maxterm

- **Maxterm:**
- Değişkenlerin «0» değeri alanları kendisiyle, «1» değeri alanları değişkenin değili/tümleyeni ile sembolize edilerek **toplama** formunda yazılırsa ve genel ifade **toplamların çarpımı** ile verilirse bu gösterime ‘**Maxterm**’ adı verilir. Buna 2. Kanonik Açılım da denir. Maxterm’ler büyük harfle sembolize edilir ($M_0, M_1, \dots, M_7, \dots$).

$$\begin{aligned} f_2 &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \\ &= M_0 M_1 M_2 M_4 \end{aligned}$$

Minterm & Maxterm

Table 2.3
Minterms and Maxterms for Three Binary Variables

x	y	z	Minterms		Maxterms	
			Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterm & Maxterm

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

$$\begin{aligned} f_1 &= (x + y + z)(x + y' + z)(x' + y + z')(x' + y' + z) \\ &= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6 \end{aligned}$$

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

$$\begin{aligned} f_2 &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \\ &= M_0 M_1 M_2 M_4 \end{aligned}$$

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

$$\begin{aligned} f_2 &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \\ &= M_0M_1M_2M_4 \end{aligned}$$

Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Minterm'lerin toplamına bir örnek

EXAMPLE 2.4

Express the Boolean function $F = A + B'C$ as a sum of minterms. The function has three variables: A , B , and C . The first term A is missing two variables; therefore,

$$A = A(B + B') = AB + AB'$$


This function is still missing one variable, so

$$\begin{aligned} A &= AB(C + C') + AB'(C + C') \\ &= ABC + ABC' + AB'C + AB'C' \end{aligned}$$

The second term $B'C$ is missing one variable; hence,

$$B'C = B'C(A + A') = AB'C + A'B'C$$

Combining all terms, we have



$$\begin{aligned} F &= A + B'C \\ &= ABC + ABC' + AB'C + AB'C' + A'B'C \end{aligned}$$

Minterm'lerin toplamına bir örnek

EXAMPLE 2.4

Table 2.5

Truth Table for $F = A + B'C$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

But $AB'C$ appears twice, and according to theorem 1 ($x + x = x$), it is possible to remove one of those occurrences. Rearranging the minterms in ascending order, we finally obtain

$$\begin{aligned}
 F &= A'B'C + AB'C + AB'C + ABC' + ABC \\
 &= m_1 + m_4 + m_5 + m_6 + m_7
 \end{aligned}$$

When a Boolean function is in its sum-of-minterms form, it is sometimes convenient to express the function in the following brief notation:

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

Maxterm'lerin arpımına bir rnek

EXAMPLE 2.5

Express the Boolean function $F = xy + x'z$ as a product of maxterms. First, convert the function into OR terms by using the distributive law:

$$\begin{aligned} F &= xy + x'z = (xy + x')(xy + z) \\ &= (x + x')(y + x')(x + z)(y + z) \\ &= (x' + y)(x + z)(y + z) \end{aligned}$$

The function has three variables: x , y , and z . Each OR term is missing one variable; therefore,

$$\begin{aligned} x' + y &= x' + y + zz' = (x' + y + z)(x' + y + z') \\ x + z &= x + z + yy' = (x + y + z)(x + y' + z) \\ y + z &= y + z + xx' = (x + y + z)(x' + y + z) \end{aligned}$$

Maxterm'lerin çarpımına bir örnek

EXAMPLE 2.5

Express the Boolean function $F = xy + x'z$

Combining all the terms and removing those which appear more than once, we finally obtain

$$\begin{aligned} F &= (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z') \\ &= M_0 M_2 M_4 M_5 \end{aligned}$$

A convenient way to express this function is as follows:

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

The product symbol, Π , denotes the ANDing of maxterms; the numbers are the indices of the maxterms of the function.

Örnek

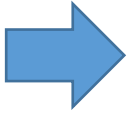
- Üç anahtar içeren bir sayısal sistemde en az iki anahtar kapatıldığında lambayı yakan lojik devreyi tasarlayın.
- Çözümlerinizi minterm ve maxtermler cinsinden ifade ediniz

Girişler			Çıkış
X	Y	Z	f

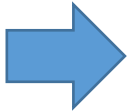
Örnek

- Üç anahtar içeren bir sayısal sistemde en az iki anahtar kapatıldığında lambayı yakan lojik devreyi tasarlayın.

- 1.kanonik form:



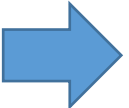
- 2.kanonik form:



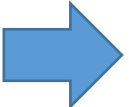
Girişler			Çıkış
X	Y	Z	f
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Örnek

- Üç anahtar içeren bir sayısal sistemde en az iki anahtar kapatıldığında lambayı yakan lojik devreyi tasarlayın.
- 1.kanonik form:


$$f = m_3 + m_5 + m_6 + m_7 = \sum (3,5,6,7)$$

- 2.kanonik form:


$$f = M_0 M_1 M_2 M_4 = \prod (0,1,2,4)$$

Girişler			Çıkış
X	Y	Z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Kanonik Formlar Arasında Dönüşüm

The complement of a function expressed as the sum of minterms equals the sum of minterms missing from the original function. This is because the original function is expressed by those minterms which make the function equal to 1, whereas its complement is a 1 for those minterms for which the function is a 0. As an example, consider the function

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

This function has a complement that can be expressed as

$$F'(A, B, C) = \Sigma(0, 2, 3) = m_0 + m_2 + m_3$$

Now, if we take the complement of F' by DeMorgan's theorem, we obtain F in a different form:

$$F = (m_0 + m_2 + m_3)' = m_0' \cdot m_2' \cdot m_3' = M_0 M_2 M_3 = \Pi(0, 2, 3)$$

The last conversion follows from the definition of minterms and maxterms as shown in Table 2.3. From the table, it is clear that the following relation holds:

$$m_j' = M_j$$

That is, the **maxterm with subscript j is a complement of the minterm with the same subscript j and vice versa.**

Kanonik Formlar Arasında Dönüşüm

$$F = xy + x'z$$

First, we derive the truth table of the function, as shown in Table 2.6. The 1's under F in the table are determined from the combination of the variables for which $xy = 11$ or $xz = 01$. The minterms of the function are read from the truth table to be 1, 3, 6, and 7. The function expressed as a sum of minterms is

$$F(x, y, z) = \Sigma(1, 3, 6, 7)$$

Kanonik Formlar Arasında Dönüşüm

Table 2.6

Truth Table for $F = xy + x'z$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Since there is a total of eight minterms or maxterms in a function of three variables, we determine the missing terms to be 0, 2, 4, and 5. The function expressed as a product of maxterms is

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

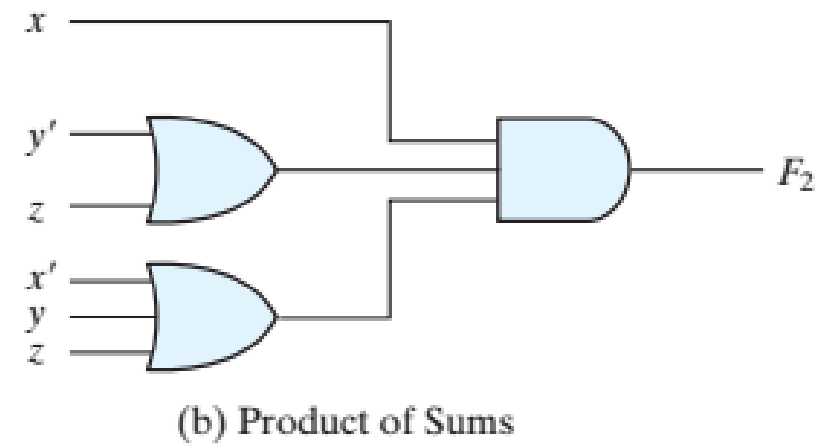
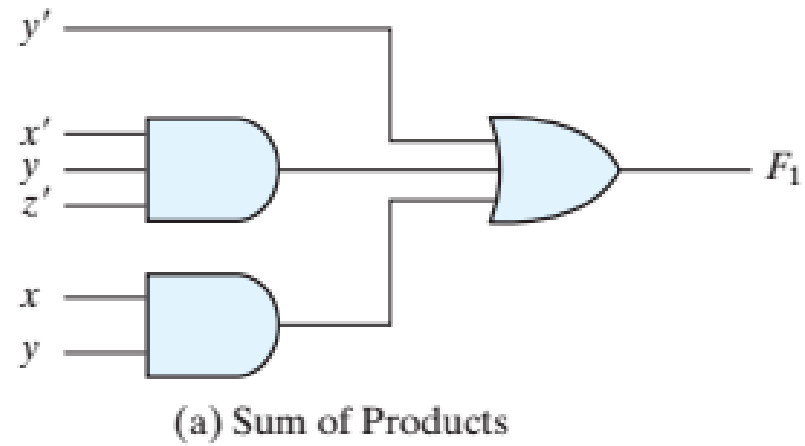


FIGURE 2.3
Two-level implementation

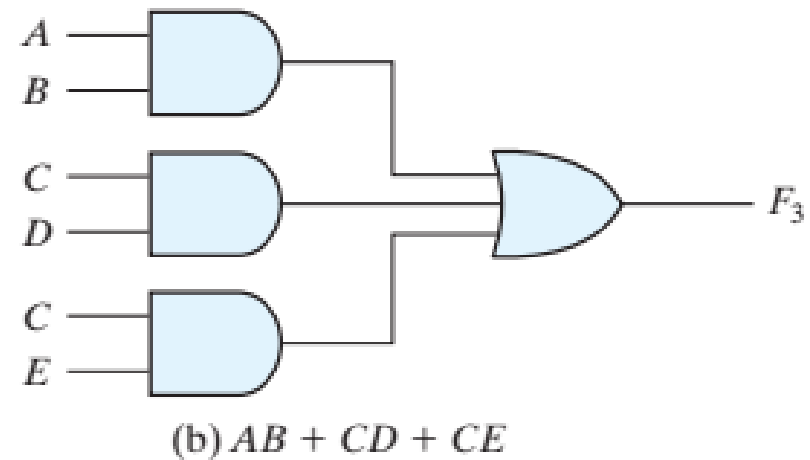
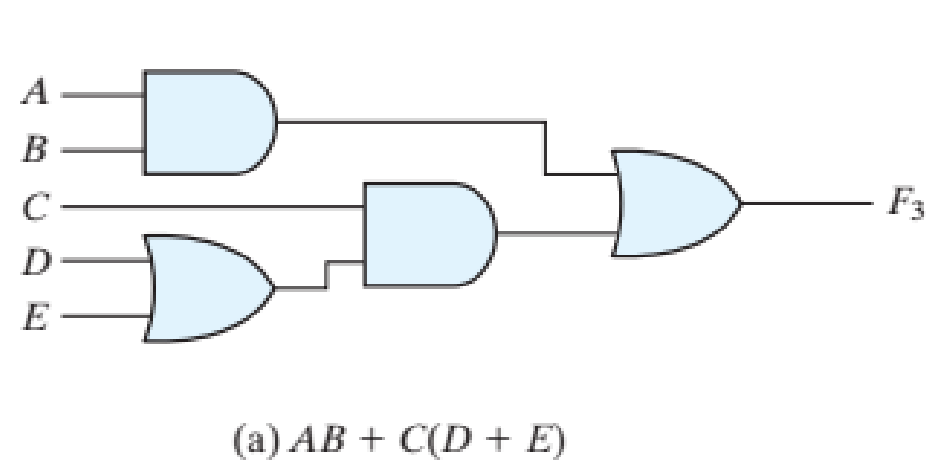


FIGURE 2.4
Three- and two-level implementation