MALEK ALISMAIL / 202538833

**1]** $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 3y(t) = 2\dfrac{dx}{dt} + x(t)$  / $x(t) = u(t)$

**a)** $s^2 Y(s) + 4s Y(s) + 3Y(s) = 2sX(s) + X(s)$

$$\boxed{H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{2s+1}{s^2+4s+3}}$$

**b)** $\dfrac{2s+1}{s^2+4s+3} = \dfrac{2s+1}{(s+3)(s+1)}$

$\Rightarrow \left.\begin{array}{l} S_1 = -3 \\ S_2 = -1 \end{array}\right\}$ kökler $< 0$ olduğundan dolayı (eksen sol tarafta) $\boxed{\text{sistem kararlıdır}}$

**c)** $x(t) = u(t) \Rightarrow X(s) = \dfrac{1}{s}$

$Y(s) = H(s) \cdot X(s)$

$Y(s) = \dfrac{2s+1}{(s+3)(s+1)} \cdot \dfrac{1}{s}$

$$\boxed{Y(s) = \dfrac{1}{3} \cdot \dfrac{1}{s} + \dfrac{1}{2} \cdot \dfrac{1}{s+1} - \dfrac{5}{6} \cdot \dfrac{1}{s+3}}$$

**d)** $\boxed{y(t) = \dfrac{1}{3} \cdot u(t) + \dfrac{1}{2} \cdot e^{-t} u(t) - \dfrac{5}{6} e^{-3t} u(t)}$

**2]** $f(t) = \left[ -2e^{-5(t-3)} + 3e^{-(t-3)} \right] u(t-3)$ , $F(s) = ?$

$$= -2e^{-5(t-3)} u(t-3) + 3e^{-(t-3)} u(t-3)$$

• Laplace dönüşüm çiftleri ve Laplace dönüşümünün özellikleri tabloları kullanarak:

$\circledtimes$ $x(t-a) \Rightarrow e^{-sa} X(s)$

$\Rightarrow$ ▪ $-2 \underbrace{e^{-5(t-3)}}_{h(t) = e^{-5t}} u(t-3) = e^{-3s} \cdot \dfrac{-2}{s+5}$

▪ $3 e^{-(t-3)} u(t-3) = e^{-3s} \cdot \dfrac{3}{s+1}$

$\Rightarrow \boxed{ F(s) = \dfrac{-2 e^{-3s}}{s+5} + \dfrac{3 e^{-3s}}{s+1} }$

**3]** $k \cdot 3^{-k} u[k] * (0,2)^k u[k]$

$\Rightarrow k \cdot \left(\frac{1}{3}\right)^k u[k] * (0,2)^k u[k] = \dfrac{\frac{1}{3} \cdot \frac{1}{5}}{\left(\frac{1}{3} - \frac{1}{5}\right)^2} \left[\left(\frac{1}{5}\right)^k - \left(\frac{1}{3}\right)^k + \dfrac{\frac{1}{3} - \frac{1}{5}}{\frac{1}{5}} k \cdot \left(\frac{1}{3}\right)^k\right] u[k]$

$= \dfrac{\frac{1}{15}}{\left(\frac{2}{15}\right)^2} \left[\left(\frac{1}{5}\right)^k - \left(\frac{1}{3}\right)^k \left(1 - \frac{2}{3}k\right)\right] u[k]$

$\boxed{= \dfrac{15}{4}\left[(0,2)^k - \left(1 - \frac{2}{3}k\right) 3^{-k}\right] u[k]}$

**4]** $F[z] = \dfrac{9}{(z+2)(z-0,5)^2}$ $\qquad$ $f[k] = ?$

$\dfrac{F[z]}{z} = \dfrac{9}{z(z+2)(z-0,5)^2} = \dfrac{c_1}{z} + \dfrac{c_2}{z+2} + \dfrac{c_3}{z-0,5} + \dfrac{c_4}{(z-0,5)^2}$

$c_1 = 18 \quad / c_2 = -0,72 \quad / c_3 = -17,28 \quad / c_4 = 7,2$

$\Rightarrow \dfrac{F[z]}{z} = \dfrac{18}{z} - \dfrac{0,72}{z+2} - \dfrac{17,28}{z-0,5} + \dfrac{7,2}{(z-0,5)^2}$

$\Rightarrow F[z] = 18 - \dfrac{0,72\, z}{z+2} - \dfrac{17,28\, z}{z-0,5} + \dfrac{7,2\, z}{(z-0,5)^2}$

$\boxed{\Rightarrow f[k] = 18\,\delta[k] - \left[0,72\,(-2)^k + 17,28\,(0,5)^k - 7,2\,k\,(0,5)^k\right] u[k]}$

**5]** $y[k+2]+3y[k+1]+2y[k] = f[k+1]+3f[k]$

$y[0]=1$ / $y[1]=2$ / $f[k]=u[k]$ / $y[k]=?$

- $y[k+2]u[k] \Longleftrightarrow z^2 Y[z] - z^2 Y[0] - z Y[1] = z^2 Y[z] - z^2 - 2z$

  $y[k+1]u[k] \Longleftrightarrow z Y[z] - z Y[0] = z Y[z] - z$

  $y[k] \Longleftrightarrow Y[k]$

- $f[k]=u[k] \Rightarrow F[k] = \dfrac{z}{z-1}$

  $f[k+1]u[k] \Longleftrightarrow z F[k] - z \underset{1}{\underline{F[0]}} = \dfrac{z \cdot z}{z-1} - z$

  $f[k]u[k] \Longleftrightarrow F[z] = \dfrac{z}{z-1}$

- $z^2 Y[z] - z^2 - 2z + 3z Y[z] - 3z + 2Y[z] = \dfrac{z^2}{z-1} - z + \dfrac{3z}{z-1}$

$\Rightarrow Y[z](z^2+3z+2) - (z^2+5z) = \dfrac{4z}{z-1}$

$\Rightarrow Y[z](z^2+3z+2) = \dfrac{4z + z(z+5)(z-1)}{z-1}$

$\Rightarrow Y[z] = \dfrac{4z + z(z+5)(z-1)}{(z-1)(z+1)(z+2)}$

$\Rightarrow \dfrac{Y[z]}{z} = \dfrac{4 + (z+5)(z-1)}{(z-1)(z+1)(z+2)} = 2 \cdot \dfrac{1}{z+1} + \dfrac{2}{3} \dfrac{1}{z-1} - \dfrac{5}{3} \dfrac{1}{z+2}$

$\Rightarrow Y[z] = \dfrac{2z}{z+1} + \dfrac{2}{3} \cdot \dfrac{z}{z-1} - \dfrac{5}{3} \cdot \dfrac{z}{z+2}$

$\Rightarrow y[k] = \left[2(-1)^k + \dfrac{2}{3}(1)^k - \dfrac{5}{3}(-2)^k\right]u[k] \Rightarrow \boxed{y[k] = \left[\dfrac{2}{3} + 2(-1)^k - \dfrac{5}{3}(-2)^k\right]u[k]}$