İşaret İşleme
Zaman Domaini Analizi-H3CD1

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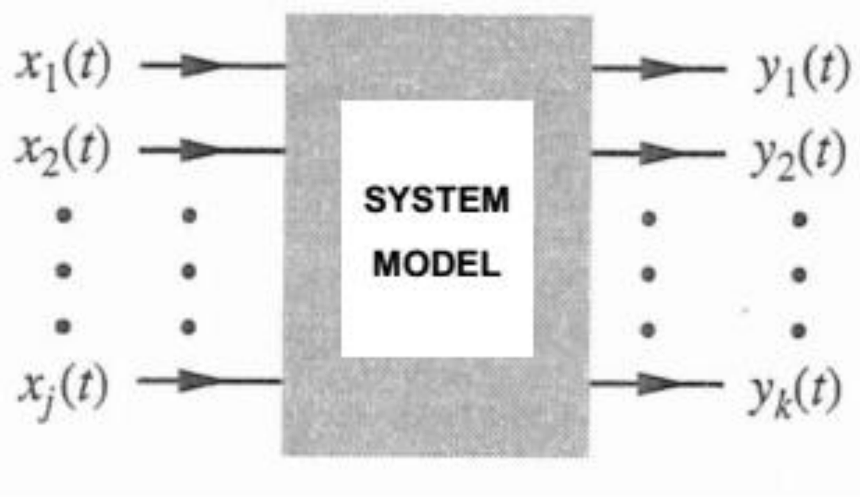
versiyon21920

Sürekli zamanlı sistemlerde zaman domaini analizi



In general, relationship between $x(t)$ and $y(t)$ in a **linear time-invariant (LTI)** differential system is given by (where all coefficients a_i and b_i are constants):

$$\frac{d^N y}{dt^N} + a_1 \frac{d^{N-1} y}{dt^{N-1}} + \cdots + a_{N-1} \frac{dy}{dt} + a_N y(t)$$

$$= b_{N-M} \frac{d^M x}{dt^M} + b_{N-M+1} \frac{d^{M-1} x}{dt^{M-1}} + \cdots + b_{N-1} \frac{dx}{dt} + b_N x(t)$$

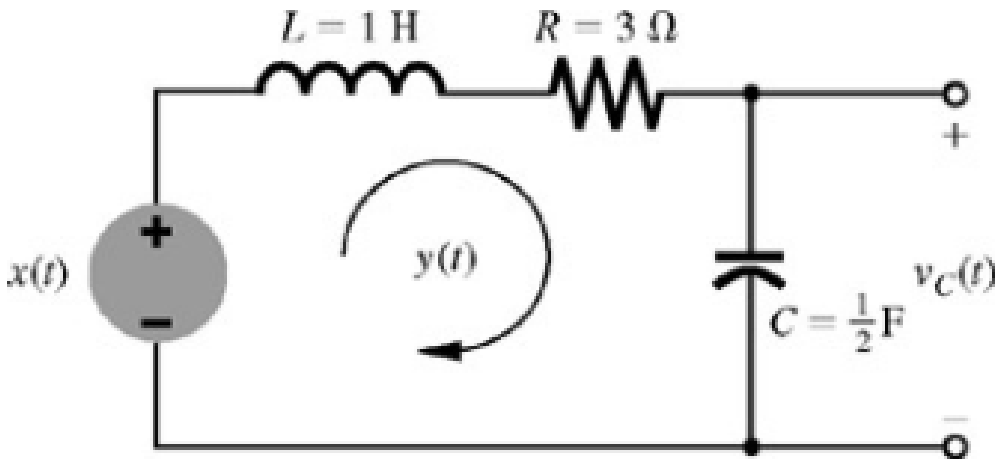
Use compact notation **D** for **operator *d/dt***, i.e $\frac{dy}{dt} \equiv Dy(t)$ and $\frac{d^2 y}{dt^2} \equiv D^2 y(t)$ etc.

We get: $$(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y(t)$$
$$= (b_{N-M} D^M + b_{N-M+1} D^{M-1} + \cdots + b_{N-1} D + b_N) x(t)$$

or $$Q(D) y(t) = P(D) x(t)$$

$$Q(D) = D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N$$
$$P(D) = b_{N-M} D^M + b_{N-M+1} D^{M-1} + \cdots + b_{N-1} D + b_N$$

Önceki ders yaptığımız son örneği hatırlayalım



Application of Kirchhoff's voltage law around the loop yields

$$v_L(t) + v_R(t) + v_C(t) = x(t) \tag{1.47}$$

By using the voltage-current laws of each element (inductor, resistor, and capacitor), we can express this equation as

$$\frac{dy}{dt} + 3y(t) + 2\int_{-\infty}^{t} y(\tau)\,d\tau = x(t) \tag{1.48}$$

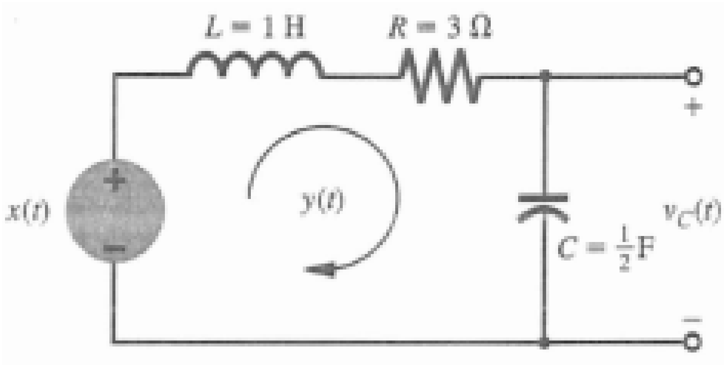
Differentiating both sides of this equation, we obtain

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = \frac{dx}{dt} \tag{1.49}$$

This differential equation is the input-output relationship between the output $y(t)$ and the input $x(t)$.

Önceki ders yaptığımız son örneği hatırlayalım

Let us consider this example again:



The system equation is:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = \frac{dx}{dt}$$

This can be re-written as:

$$\underbrace{(D^2 + 3D + 2)}_{Q(D)}y(t) = \underbrace{D}_{P(D)}x(t)$$

Also
$$\int_{-\infty}^{t} y(\tau)\,d\tau \equiv \frac{1}{D}y(t)$$
$$\frac{d}{dt}\left[\int_{-\infty}^{t} y(\tau)\,d\tau\right] = y(t)$$

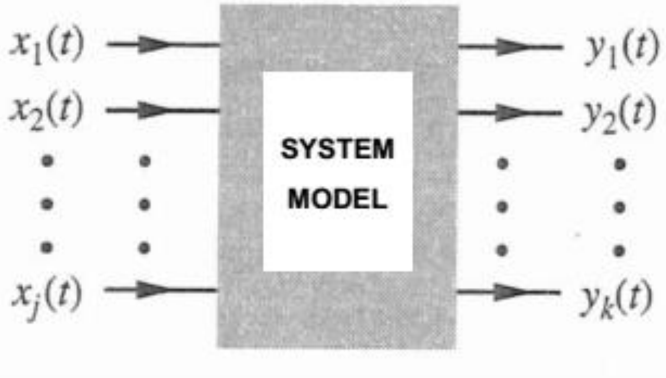
For this system, $N = 2$, $M = 1$, $a_1 = 3$, $a_2 = 2$, $b_1 = 1$, $b_2 = 0$.

For practical systems, M ≤ N. It can be shown that if M > N, a LTI differential system acts as an (M – N)th-order **differentiator**.

A differentiator is an unstable system because **bounded input** (e.g. a step input) results in an **unbounded output** (a Dirac impulse δ(t)).

L2.1 p151

Sıfır Giriş Cevabı (SGC)-(Zero input response)



Remember that for a Linear System

Total response = zero-input response + zero-state response

In this lecture, we will focus on a linear system's **zero-input response**, $y_0(t)$, which is the solution of the system equation when input $x(t) = 0$.

$$(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y(t) = (b_{N-M} D^M + b_{N-M+1} D^{M-1} + \cdots + b_{N-1} D + b_N) x(t)$$

$$\Rightarrow \quad Q(D) y(t) = P(D) x(t)$$

$$\Rightarrow \quad Q(D) y_0(t) = 0$$

$$\Rightarrow \quad (D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y_0(t) = 0$$

SGC için genel çözüm-1

From maths course on differential equations, we may solve the equation:

$$(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y_0(t) = 0 \quad \ldots\ldots\ldots \quad (3.1)$$

by letting $y_0(t) = ce^{\lambda t}$, where **c and λ are constants**

Then:

$$\left.\begin{aligned} Dy_0(t) &= \frac{dy_0}{dt} = c\lambda e^{\lambda t} \\ D^2 y_0(t) &= \frac{d^2 y_0}{dt^2} = c\lambda^2 e^{\lambda t} \\ &\vdots \\ D^N y_0(t) &= \frac{d^N y_0}{dt^N} = c\lambda^N e^{\lambda t} \end{aligned}\right\} \quad \text{Substitute into (3.1)}$$

SGC için genel çözüm-2

We get:

$$c(\lambda^N + a_1\lambda^{N-1} + \cdots + a_{N-1}\lambda + a_N)e^{\lambda t} = 0$$

$$\lambda^N + a_1\lambda^{N-1} + \cdots + a_{N-1}\lambda + a_N = 0$$

This is identical to the polynomial Q(D) **with λ replacing D**, i.e.

$$Q(\lambda) = 0$$

We can now express Q(λ) in **factorized form**:

$$Q(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)\cdots(\lambda - \lambda_N) = 0 \qquad \text{..........} \quad (3.2)$$

Therefore **λ has N solutions**: λ_1, λ_2, ... , λ_N, assuming that all λ_i are distinct.

Characteristic Polynomial of a system

$Q(\lambda)$ is called the **characteristic polynomial** of the system

$Q(\lambda) = 0$ is the **characteristic equation** of the system

The **roots** to the characteristic equation $Q(\lambda) = 0$, i.e. $\lambda_1, \lambda_2, \ldots, \lambda_N$, are extremely important.

They are called by different names:

- Characteristic values
- **Eigenvalues**
- **Natural frequencies**

The exponentials $e^{\lambda_i t} (i = 1, 2, \ldots, n)$ are the **characteristic modes** (also known as **natural modes**) of the system

Characteristics modes determine the system's behaviour

$Q(\lambda)$ is called the **characteristic polynomial** of the system

The **roots** to the characteristic equation $Q(\lambda) = 0$, i.e. $\lambda_1, \lambda_2, \ldots, \lambda_N$, are extremely important.

- **NOT:**
  - **Sistemin karakteristik polinomuna ait kökler (λ) 3 farklı durumda olabilir.**
    - **Katsız (tekrar etmeyen) kök**
    - **Katlı (tekrar eden) kök**
    - **Kompleks kök**
  - **Her 3 durum için sistemin sıfır giriş cevabı farklı genel çözümlerle bulunur.**

SGC için genel çözüm-3

Therefore, equation (3.1): $(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y_0(t) = 0$

has **N possible solutions**: $c_1 e^{\lambda_1 t}, c_2 e^{\lambda_2 t}, \ldots, c_N e^{\lambda_N t}$

where $c_1, c_2, \ldots, c_N$ are arbitrary constants.

It can be shown that the **general solution** is the sum of all these terms:

$$y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \cdots + c_N e^{\lambda_N t}$$

Katsız kök olduğunda

In order to determine the N arbitrary constants, we need to have **N constraints** (i.e. initial or boundary or auxiliary conditions).

Özetle;

- **Sistemin karakteristik polinomuna ait kökler (λ_i) Katsız (tekrar etmeyen) kökler olduğunda sistemin sıfır giriş cevabına ait genel çözüm;**

$$y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \cdots + c_N e^{\lambda_N t}$$

Ödev1

Verilen sistem ve başlangıç şartları için sistemin sıfır giriş cevabını bulunuz

Süre: 10 Ekim 2021 saat 23.59

Cevaplarınızı A4 kağıdına el yazınızla yazarak çözümün fotoğrafını çekin ve EDS'de belirtilen alana bu belgeyi (jpg yada pdf) yükleyin.

Çözüm kağıdınıza ad-soyad ve numaranızı yazmayı kesinlikle unutmayınız!!

Ad-soyad ve numarası eksik kağıtlar değerlendirmeye alınmayacaktır.

BAŞARILAR..

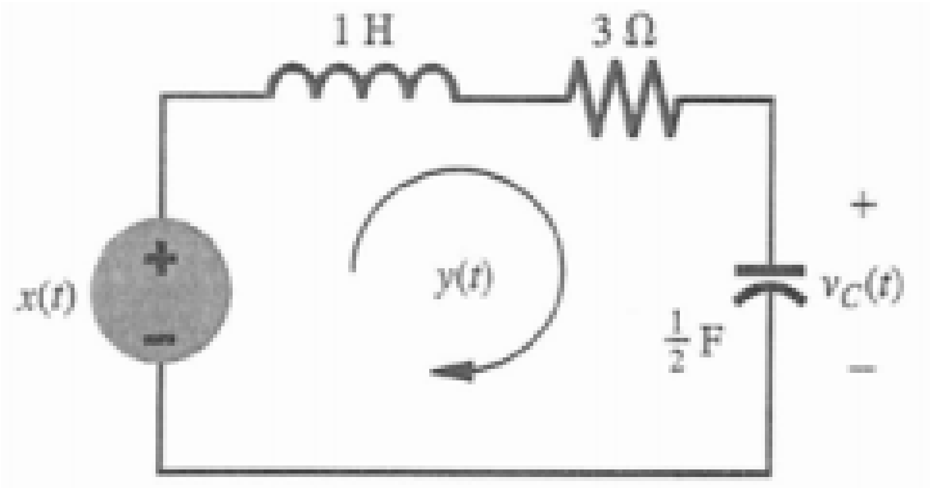
Ödev1

Find $y_0(t)$, the zero-input component of the response, for a LTI system described by the following differential equation:

$$(D^2 + 3D + 2)y(t) = Dx(t)$$

when the initial conditions are

$$y_0(0) = 0, \quad \dot{y}_0(0) = -5.$$



- Yukarıdaki sistemin sıfır giriş cevabını bulunuz.

Bu ders notu için faydalanılan kaynaklar

Lecture 3

Time-domain analysis:
Zero-input Response
(Lathi 2.1-2.2)

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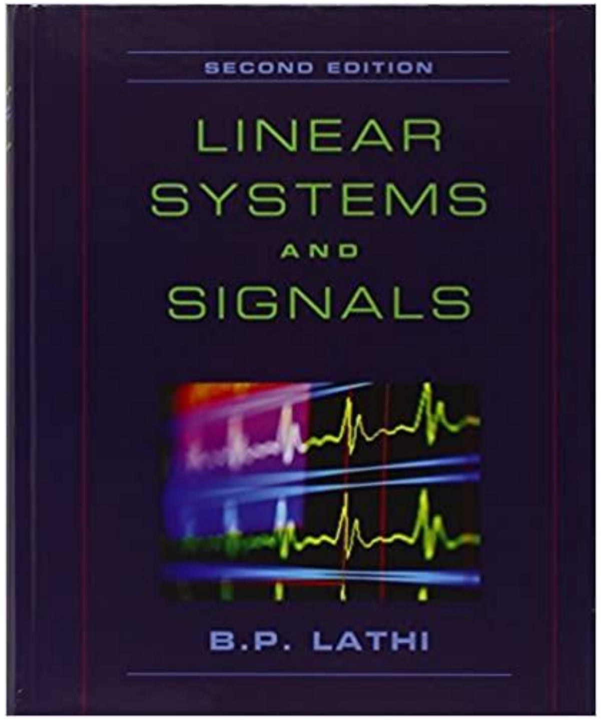

EEEN343 Sinyaller ve Sistemler

Ders Notları

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