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1] $\det(\lambda I - A) = \left| \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ -2 & 1 & 4 \\ 0 & 0 & 3 \end{pmatrix} \right|$

$$= \begin{vmatrix} \lambda-3 & 0 & 0 \\ 2 & \lambda-1 & -4 \\ 0 & 0 & \lambda-3 \end{vmatrix} = (\lambda-3)^2(\lambda-1) = 0$$

$\Rightarrow \lambda_{1,2} = 3$, $\lambda_3 = 1$ A matrisinin özdeğerleridir.

Bunlara karşılık gelen özvektörleri bulalım.

$W_3 = \{ X \in \mathbb{R}^{3\times1} \mid AX = 3.X \} = \{ X \in \mathbb{R}^{3\times1} \mid (3I-A)X = 0 \}$

$(3I-A)X = 0 \Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

• $(3I-A)$ indirgenmiş matrisine dönüştürelim:

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 2 & 2 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 \to (\frac{1}{2})R_1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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1) $\begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x + y - 2z = 0$

$x = a \ / \ z = b \ / \ y = 2b - a$

$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ 2b-a \\ b \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

$\Rightarrow \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\}$ $\lambda_{1,2} = 3$ karşılık gelen özvektörlerdir.

$W_1 = \{ X \in \mathbb{R}^{3\times1} \mid AX = 1.X \} = \{ X \in \mathbb{R}^{3\times1} \mid (I-A)X = 0$

$(I-A)X = 0 \Rightarrow \begin{pmatrix} 1 & 0 & -3 \\ 2 & 0 & -4 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$(I-A)$ indirgenmiş matrisine dönüştürelim:

$\begin{pmatrix} 1 & 0 & -3 \\ 2 & 0 & -4 \\ 0 & 0 & -2 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2.R_1} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 0 & 10 \\ 0 & 0 & -2 \end{pmatrix} \xrightarrow{R_2 \to \left(\frac{1}{10}\right).R_2} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{pmatrix}$

$\xrightarrow{R_3 \to R_3 + 2R_2} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \to R_1 + 3R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

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1) $\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $x = 0$, $z = 0$

$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} = y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ $\lambda_3 = 1$ karşılık gelen özvektördür.

A'nın özvektörlerinin sütünün oluşturan $P = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ matrisi köşegenleştirme matrisidir.

$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $P^{-1} A P = D$