$$|E_{L-R}| \leq \frac{(b-a)^2}{2n} \cdot M \qquad f'$$

$$|E_M| \leq \frac{(b-a)^3}{24n^2} \cdot M$$

$$|E_T| \leq \frac{(b-a)^3}{12n^2} \cdot M$$

f''

$$|E_{S_{1/3}}| \leq \frac{(b-a)^5}{180n^4} \cdot M$$

$$|E_{S_{3/8}}| \leq \frac{(b-a)^5}{80n^4} M$$

$|f^{(4)}| \leq M$

$$|E_{mid-p}| \le \max_{a \le x \le b} |f''(x)| \cdot \frac{h^3}{24} \Rightarrow$$

$$|E| \le \max |f''(x)| (b-a) \frac{h^2}{24}$$

$$\le \frac{K (b-a)(b-a)^2}{24 n^2} = \frac{K (b-a)^3}{24 n^2}$$

Example: How many subintervals do you need to approximate the integral $\int_1^2 \frac{1}{x}\,dx$ to a precision of 0.0001 using Simpson's $\frac{1}{3}$ Rule.

$$f(x) = \frac{1}{x} = x^{-1}$$

$$|E_s| \leq \frac{M(b-a)^5}{180 * n^4}$$

$$f'(x) = -\frac{1}{x^2} = -x^{-2}$$

$$f''(x) = 2x^{-3}$$

$$f'''(x) = -6x^{-4}$$

$$f^{(4)}(x) = 24x^{-5} \qquad M = \max|f^{(4)}(x)| \text{ on } [1,2]$$

$$f^{(4)}(1) = 24 = M$$

$$|E_s| \leq \frac{24(2-1)^5}{180 * n^4} \leq 0.0001$$

$$\frac{24}{180 * 0.0001} \leq n^4$$

$$n^4 \geq \frac{4000}{3}$$

$$n \geq 6.043$$

$$n = 8$$

n çift olmalıdır

In estimating $\int_{-1}^{4} \cos(4x)\,dx$ using Simpson's Rule with $n=10$, we can estimate the error involved in approximation using the Error Bounds formula. For Simpson's $\frac{1}{3}$ Rule, the error will be less than ---- .

$f(x) = \cos 4x$

$f'(x) = -4\sin 4x$

$f''(x) = -16\cos 4x$

$f'''(x) = +64\sin 4x$

$f^{(4)}(x) = 256\cos 4x$

$\left[\max_{[-4\,4]} |f^{(4)}(x)|\right] = M$

maximum of $f^{(4)}(x)$ on $[-1, 4] \to \underline{\underline{256}}$

$\underline{f^{(4)}(x) = 256}$

$$|E_S| \le \frac{M(b-a)^5}{180n^4} = \frac{256(4-(-1))^5}{180 \cdot 10^4} = \frac{800\,000}{1\,800\,000} = \frac{4}{9}$$

$E \le \frac{4}{9}$

örnek

$\int_0^1 e^{x^2} dx$, simps. $\frac{1}{3}$, $n=4$

$|E_s| \leq \frac{M(b-a)^5}{180 n^4}$

$f(x) = e^{x^2}$

$f'(x) = 2x e^{x^2}$

$f''(x) = 2e^{x^2} + 4x^2 e^{x^2}$

$f'''(x) = 12x e^{x^2} + 8x^3 e^{x^2}$

$f^{(4)}(x) = 12 e^{x^2} + 48 x^2 e^{x^2} + 16 x^4 e^{x^2}$

$|E_s|$.

$[0,1]$ de $M = \max|f^{(4)}(x)|$?

$f^{(4)}(1) = 12e + 48e + 16e = 76e$

$|E_s| \leq \frac{76e(1-0)^5}{180 * 4^4} = \frac{76e}{46080} = 0.0045$

$|E_s| \leq 0.0045$ hata 0.0045 den büyük olmamalı!

Aynı örnekte $|E_s| \leq 0.0001$ için $n = ?$

$\frac{76e(1-0)^5}{180 * n^4} \leq 0.0001$

$\frac{76e}{180 * 0.0001} \leq n^4$

$\sqrt[4]{\frac{76e}{0.0018}} \leq n \Rightarrow$ $n \geq 18.41$ ise $n = 19$ alt aralık olmalıdır. Fakat $n = 2k$ olacağı için $n = 20$ alınır.

(7 points)

Example: How many subdivisions should be used in the simpson's $\frac{1}{3}$ rule to approximate

$$\int_1^5 \frac{\ln x}{100}\,dx$$

with an error bound whose absolute value is less than 10^{-6}.

The error Estimate for simpson's $\frac{1}{3}$ Rule

If $f^{(4)}$ is continuous and M is upper bound for the values of $|f^{(4)}|$ on $[a,b]$, then the Error E_s is the simpson's Rule approximation of the integral of $f(x)$ from a to b. for n steps satisfies the inequality

$$|E_s| \le \frac{M(b-a)^5}{180n^4}$$

$$f(x) = \frac{\ln x}{100} \qquad f'(x) = \frac{1}{100x}$$

a) $n=22$

b) $n=12$

c) $n=26$

d) $n=38$

e) $n=32$

o Boş bırak

$$f'(x) = \frac{1}{100}x^{-1} \qquad |f^{(4)}(x)|_{max} \le M$$

$$f''(x) = -\frac{1}{100}x^{-2}$$

$$f'''(x) = \frac{2}{100}x^{-3}$$

$$f^{(4)}(x) = \frac{-3}{50}x^{-4} = -\frac{3}{50x^4}$$

$[1,5] \Rightarrow x=1$

$$f^{(4)}(1.1.) = -\frac{3}{50\cdot(1)^4} = -\frac{3}{50}$$

$$\cancel{f^{(4)}(5) = \frac{-3}{50\cdot 5^4}}$$

$$|Es| \le \frac{\frac{3}{50}\cdot(5-1)^5}{180\cdot n^4} \le 10^{-6}$$

$n \ge 24.17$

n çift olacağından $\underline{n=26}$ olmalı

Right Riemann sum

$$A_{right} = \Delta x \left[f(a+\Delta x) + f(a+2\Delta x) + \cdots + f(b) \right]$$

The error of this formula will be

$$\left| \int_a^b f(x)dx - A_{right} \right| \le \frac{M(b-a)^2}{2*n}$$

where M is the maximum value of the absolute value of $f'(x)$ on the interval $[a,b]$.

Example: Using the rectangle method with $n=4$, calculate I_1, I_2 and I_3 and compare with the exact integral of function $f(x)=e^x$ on the interval $[0,2]$. Then, find the values of h required so that the total error obtained by I_1 and that obtained by I_2 ~~error~~ are bounded by 0.001.

Solution: $n=4 \Rightarrow h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

$x_0=0$, $x_1=0.5$, $x_2=1$, $x_3=1.5$ and $x_4=2$

$f(0)=e^0=1$, $f(x_1)=e^{0.5}=1.6487$, $f(x_2)=e^1=2.7183$

$f(x_3)=e^{1.5}=4.4812$ $\quad f(x_4)=e^2=7.3891$

$$I_1 = h\sum_{i=1}^{n} f(x_{i-1}) = 0.5[1+1.6487+2.7183+4.4817]$$
$$= 4.924$$

$$I_3 = h\sum_{i=1}^{n} f(x_i) = 0.5[1.6487+2.7183+4.4812+7.3891]$$
$$= 8.119$$

For I_2 $\Rightarrow$ $f\left(\frac{x_0+x_1}{2}\right) = f(0.25) = e^{0.25} = 1.284$, $\quad f\left(\frac{x_3+x_4}{2}\right) = e^{1.75} = 5.755$

$f\left(\frac{x_1+x_2}{2}\right) = e^{0.75} = 2.117$

$f\left(\frac{x_2+x_3}{2}\right) = e^{1.25} = 3.49$ $\qquad I_2 = h\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right) = \underline{6.323}$

Therefore

$$I_2 = h \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)$$

$$I_2 = 0.5(1.284 + 2.112 + 3.49 + 5.755) = 6.323$$

$$I = \int_0^2 e^x dx = e^2 - e^0 = 6.38906$$

obviously, I_2 provides a good approximation with only 4 rectangles.

$$\text{Error} = |I - I_1| = |6.38906 - 4.924| = 1.4651$$

and $|I - I_3| = |6.3896 - 8.119| = 1.73$

$$|E_f| \leq \frac{M(b-a)^2}{2 * n} \qquad M = \max|f'(x)| \text{ on } [a,b]$$

$f'(x) = e^x$

$f'(2) = e^2$

$$|E_f| \leq \frac{e^2(2-0)^2}{2 \cdot n} \leq 0.001 \Rightarrow n \geq 14778.1$$

$n = 14779$

$$|E_f| \leq \frac{e^2(2-0)^2}{2 * 4} = 3.69453 = 3.695$$

$1.4651 \leq 3.695$

$1.73 \leq 3.695$

$|I - I_2| = |6.38906 - 6.3231| = 0.06606$

$|f''(x)| \le M$

$|E_M| \le \dfrac{M(b-a)^3}{24 * n^2}$

$M = \max |f''(x)|$ on $[0, 2]$

$f(x) = e^x \Rightarrow f''(x) = e^x$

$f(2) = e^2$

$|E_M| \le \dfrac{e^2(2-0)^3}{24 * 4} = 0.153939$

$0.06606 \le 0.153939$

$|E_M| \le \dfrac{e^2(2-0)^3}{24 * n^2} \le 0.001 \Rightarrow n \ge 49.6288$

$\underline{\underline{n = 50}}$ alt aralık sayısı

$I_2 = 6.38863$

$|E| = |6.38863 - 6.38906| = 0.000426$

<u>örnek:</u> $\int_1^3 e^x dx = e^3 - e = 17.3673$ (Tam sonuç)

a)

b)

c)

d)

e)

f)

g)

h)

$$y' - 2xy = 1$$

$$y(0) = \frac{1}{2}$$

$$i(x) = e^{\int -2x\,dx} = e^{-x^2}$$

$$y\,e^{-x^2} = \int e^{-x^2} \cdot 1\,dx + c$$

$$y(x) = e^{x^2}\left[\int_0^x e^{-x^2}\,dx + c\right]$$

$$y(0) = \frac{1}{2}$$

$$y(0) = e^{0}\left[\int_0^0 e^{-x^2}\,dx + c\right] =$$

$$\frac{1}{2} = c$$

$$\cancel{y=}$$

$$y(x) = e^{x^2}\left[\int_0^x e^{-x^2}\,dx + \frac{1}{2}\right]$$

$$y(2) = e^{4}\left[\int_0^2 e^{-x^2}\,dx + \frac{1}{2}\right]$$

$$y(2) = 75.45909$$

% mkdenk29.m

```
clear all
%n=input('n sayisini giriniz=  ');
n=600;
a=0;
b=2;
h=(b-a)/n;
tplm=f(a)+f(b);
for  k=1:(n-1)
   x=a+k*h;
   %if rem(k,2)==0
    %  cf=2;
   %else
    %  cf=4;
    % end
    cf=3 +(-1)^(k+1);
   tplm=tplm+ cf*f(x);
end
ytplm=h*tplm/3;
snc=exp(4)*(ytplm+0.5);
fprintf('%12.5f\n',snc)
```

% f.m

```
function yx=f(x)
yx= exp(-x^2);
```

```
>> mdenk29
75.45909
```