İşaret İşleme

Örnekleme Teoremi-H13CD1

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Örnekleme-Sampling

- Örnekleme;
  - sürekli zamanlı sinyalleri işlemek,
  - kaydetmek,
  - iletmek,
  - saklamak ve
  - almak için modern dijital elektroniklerin kullanılmasına izin verir.

- Neden sinyaller hakkında düşünmek isteyesiniz ki ?

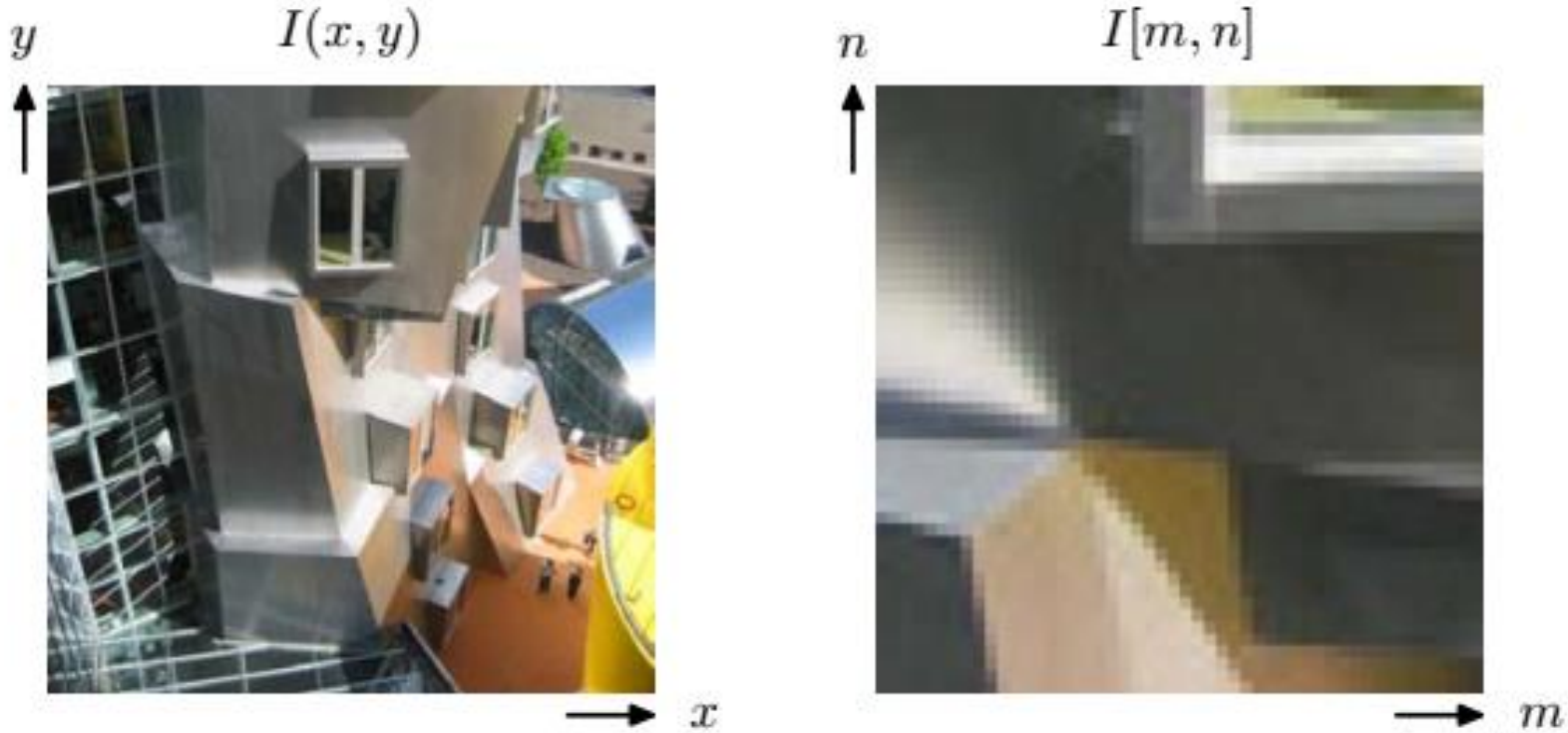
Örnekleme-Sampling

- Neden sinyaller hakkında düşünmek isteyesiniz ki ?
  - Ses: MP3, CD, hücresel telefon,..
  - Resim: dijital kamera, yazıcı,..
  - Video: DVD,..
  - Web üzerindeki her şey yüzünden

Sampling

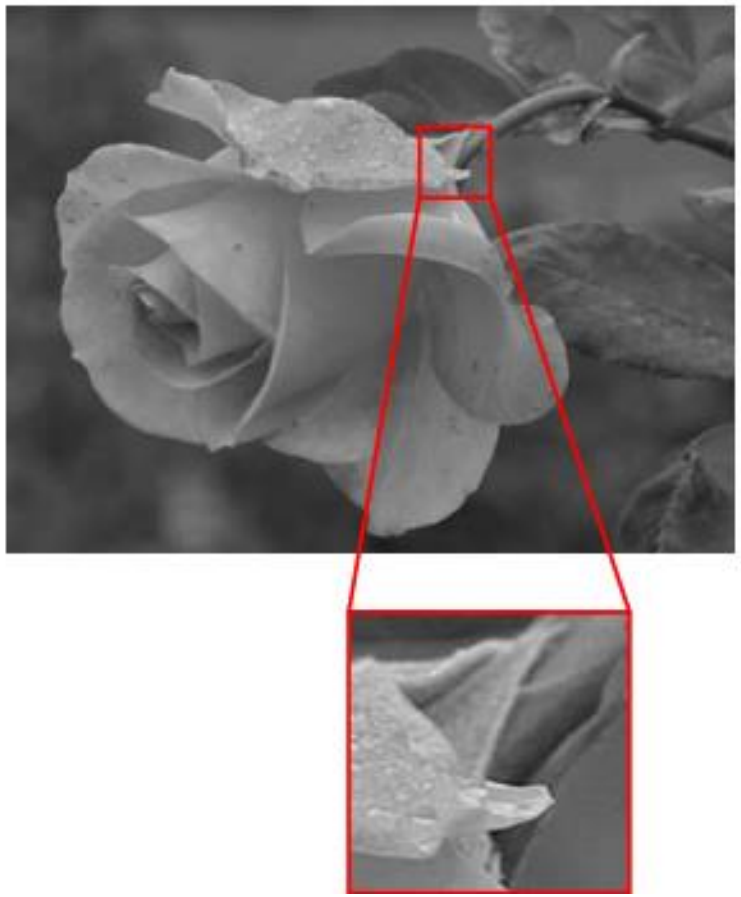
Sampling is pervasive.

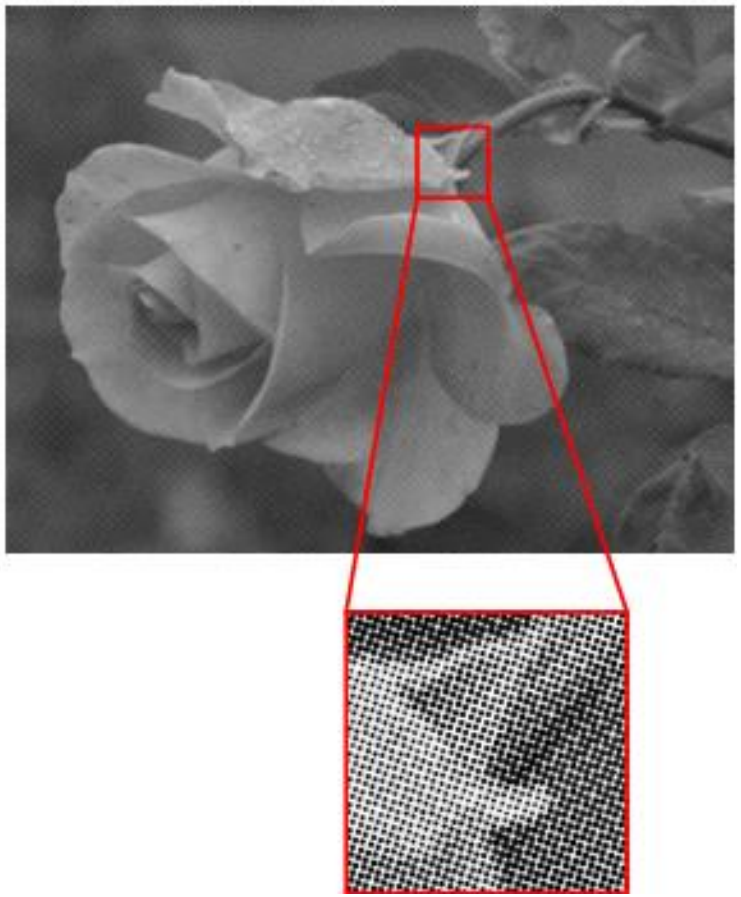
Example: digital cameras record sampled images.



Sampling

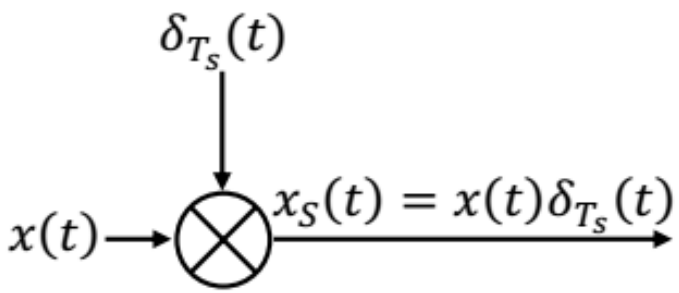
Zoom in to see the binary pattern.



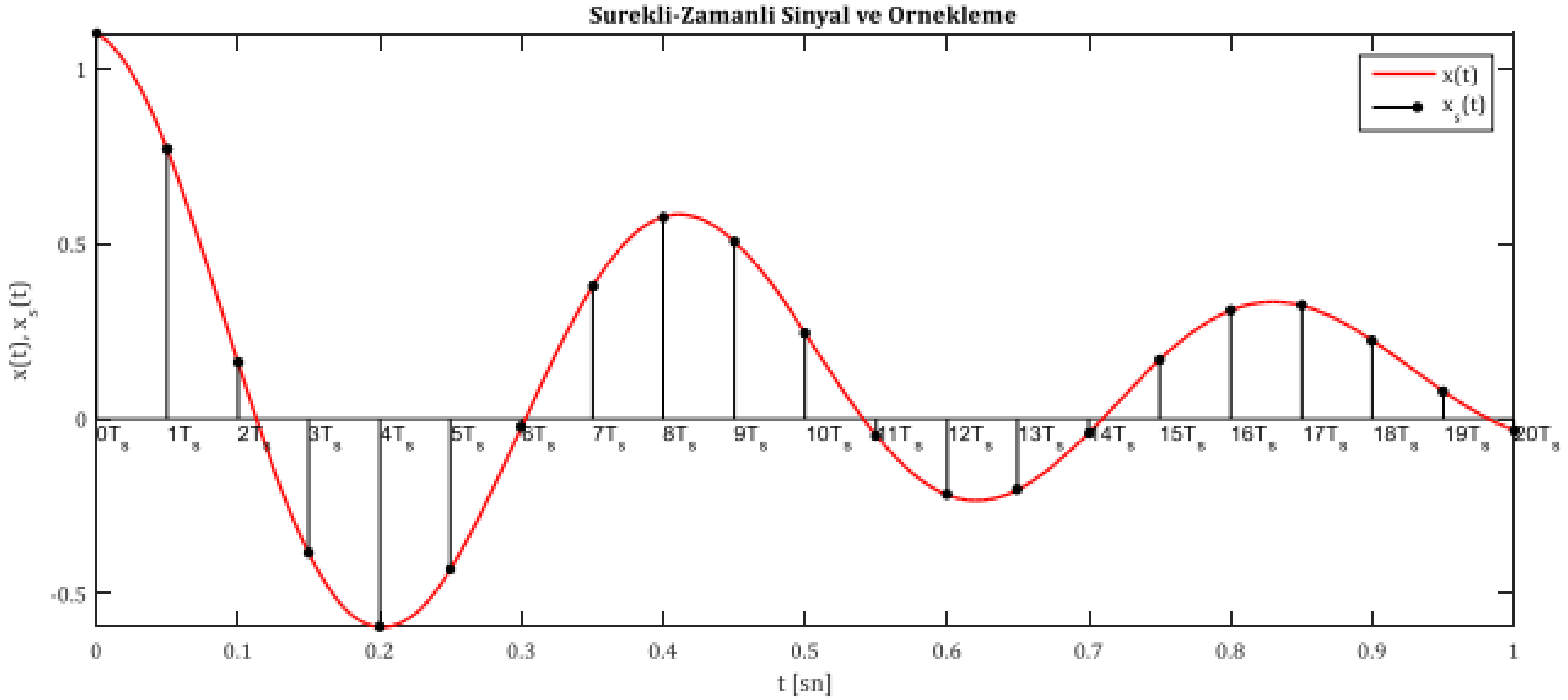


Örnekleme

Örnekleme, dijital sinyal işlemenin temelini oluşturan bir işlem olup zaman domenindeki w_B gibi sonlu bant genişlikli sürekli-zamanlı bir sinyalin T_s periyotlu $\delta_{T_s}(t)$ darbe katarı çarpılarak ayrık-zamanlı hale getirilmesini ve bu sayede dijital sinyal işlemeye uygun hale getirilmesini sağlar. Bunu aşağıdaki şekilde görmek mümkündür.



Aşağıdaki şekilde tipik bir örneklenmiş sinyal görülmektedir.



Örnekleme işlemi sonucunda elde edilen örneklenmiş sinyalin Fourier dönüşümünün bulunmasında, Fourier dönüşümünün

$$x_1(t)x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(w) * X_2(w)$$

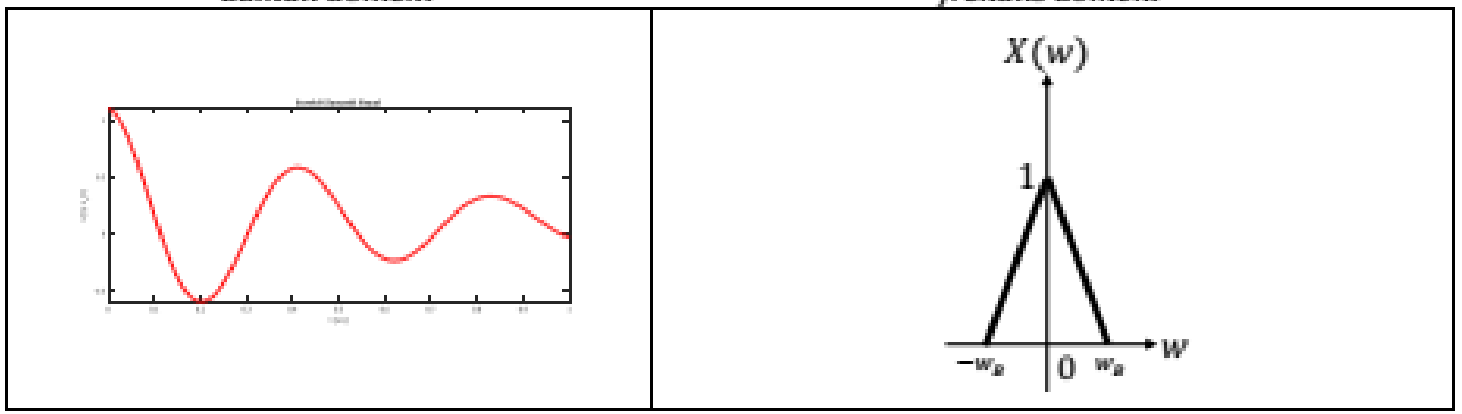
şeklindeki çarpma özelliğinden yararlanılır. Örneklenecek $x(t)$ sinyali ile T_s periyotlu $\delta_{T_s}(t)$ darbe katarı sinyalinin çarpılması ile elde edilen örneklenmiş $x_S(t)$ sinyalinin Fourier dönüşümü, çarpma özelliğine göre şu şekilde bulunur:

$$x_S(t) = x(t)\delta_{T_s}(t) \leftrightarrow \frac{1}{2\pi} X(w) * \mathcal{F}\{\delta_{T_s}(t)\} = \frac{1}{2\pi} X(w) * w_s \sum_{k=-\infty}^{\infty} \delta(w - kw_s)$$

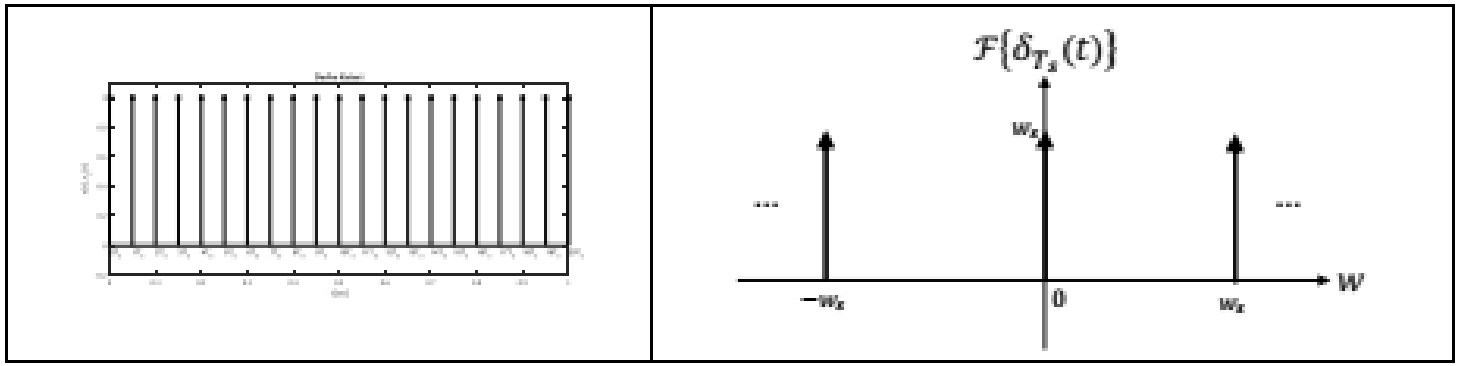
$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(w - kw_s)$$

Görüldüğü gibi örneklenmiş sinyalin Fourier dönüşümü, orjinal sinyalin Fourier dönüşümünün tüm frekans domenine yayılmış hali gibidir. Bu durum aşağıdaki şekilde görülmektedir.

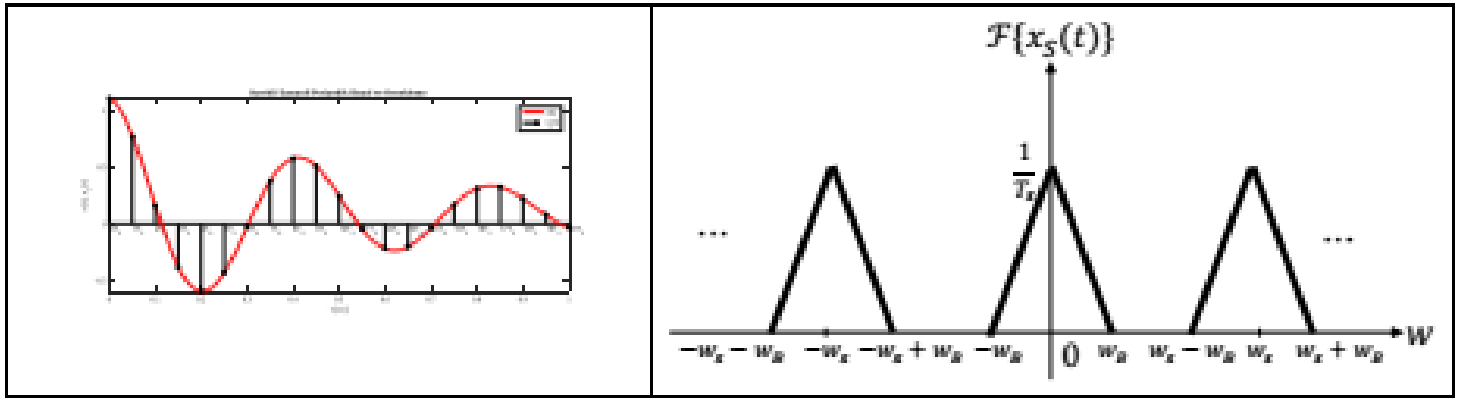
zaman domeni | *frekans domeni*



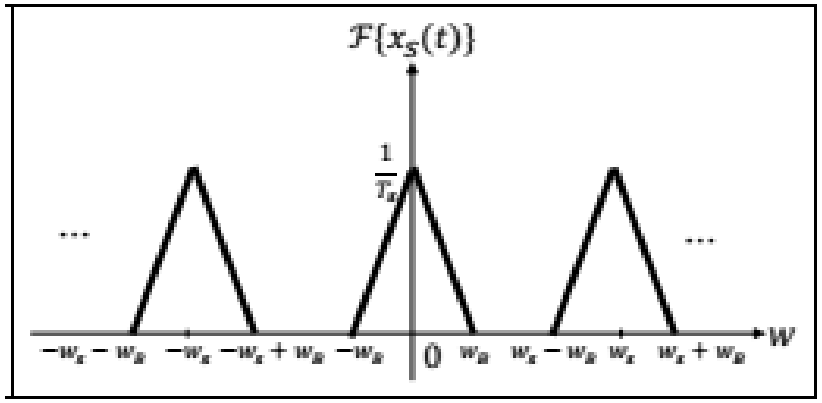
x | *****



= | **=**



Burada en önemli soru T_s periyodunun nasıl seçileceğidir. Şekle bakıldığında, Fourier dönüşümleri arasında bir girişim ya da örtüşmenin olmaması için



$$w_B < 2w_s$$

şartının sağlanması gerekir ki bu da örnekleme teoreminin en önemli sonuçlarından biridir. Buna göre, örnekleme frekansı, örneklenecek sinyalin bant genişliğinin en az iki katı olmalıdır.

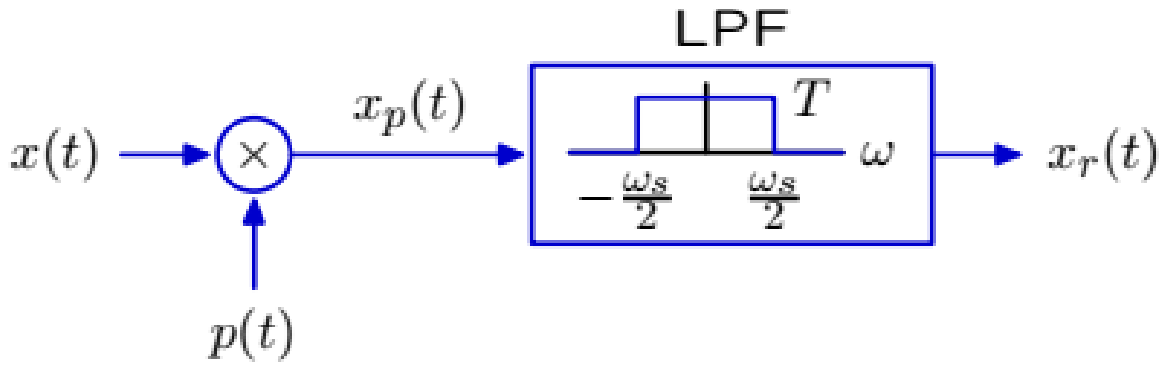
Örnekleme Teoremi (Sampling)

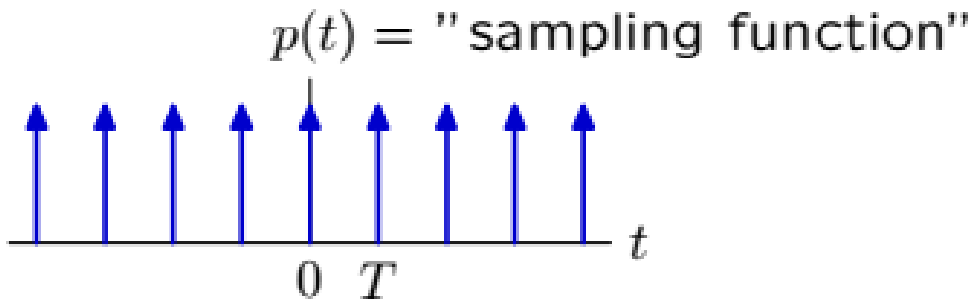
Boş bir zamanda izleyin..

https://www.youtube.com/watch?v=1El4znkRH0g

CT Model of Sampling and Reconstruction

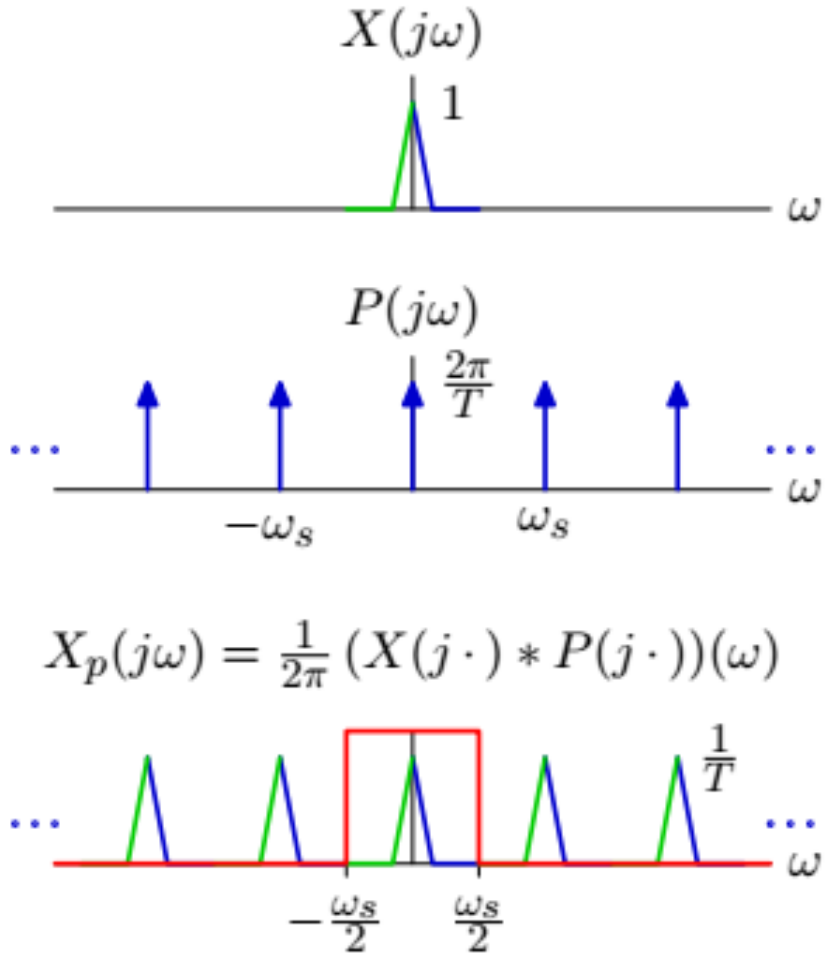
Sampling followed by bandlimited reconstruction is equivalent to multiplying by an impulse train and then low-pass filtering.





Aliasing

High frequency components of complex signals also wrap.



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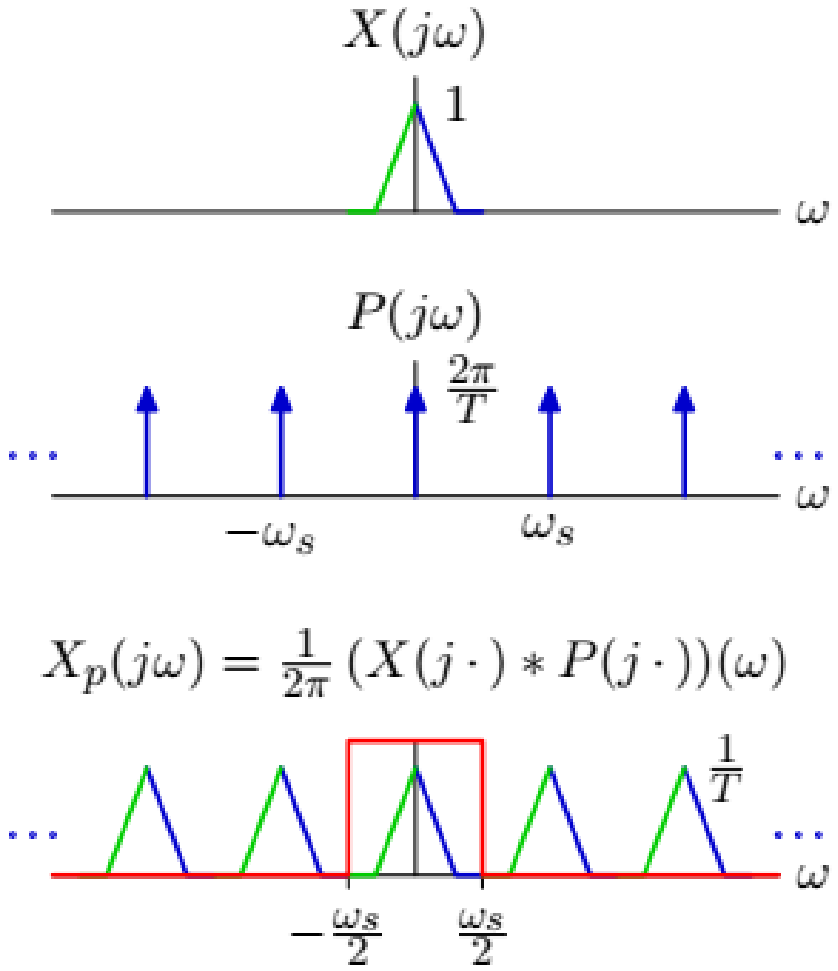

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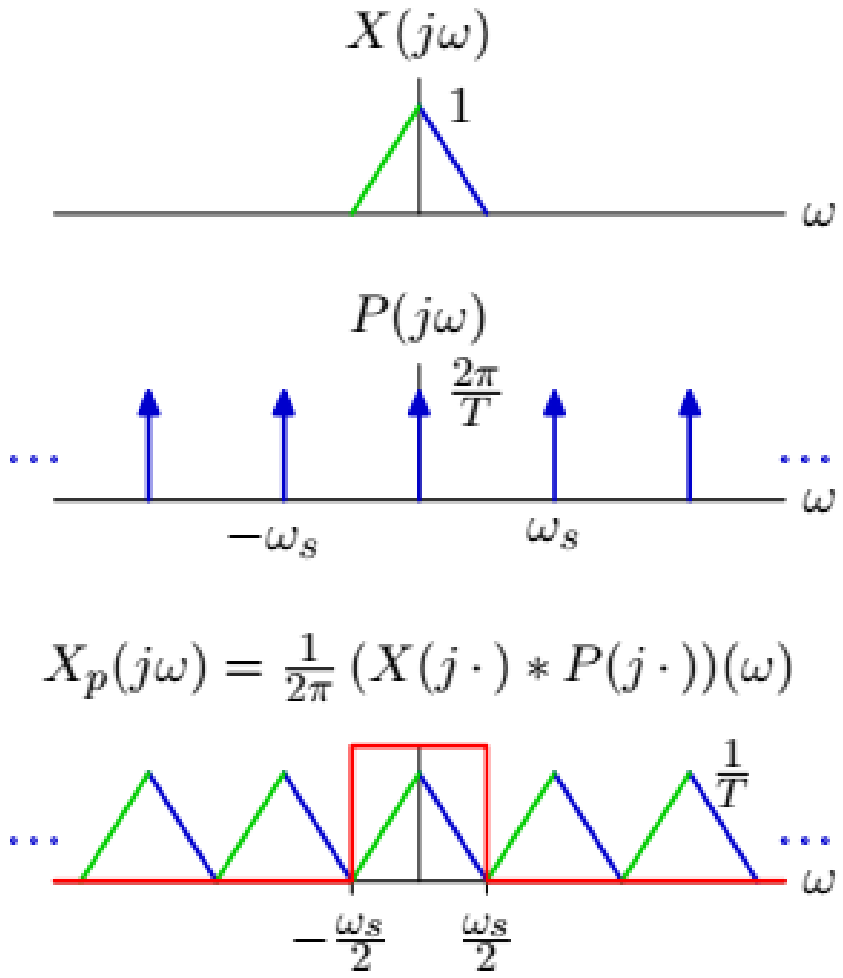

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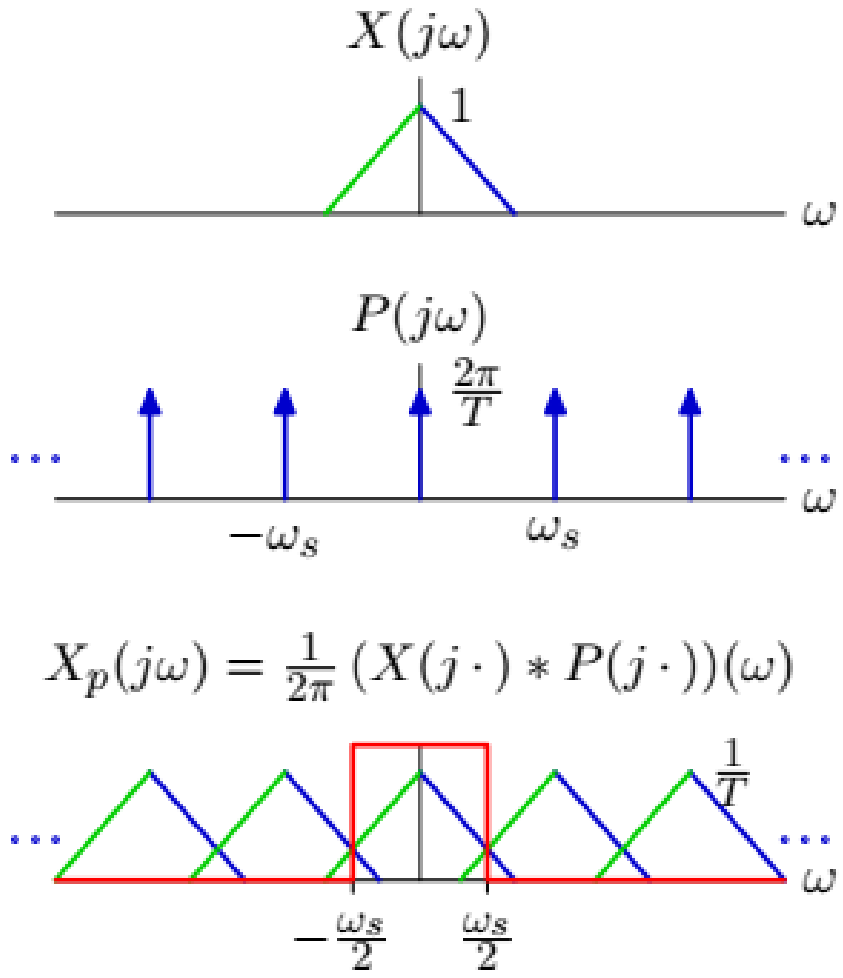
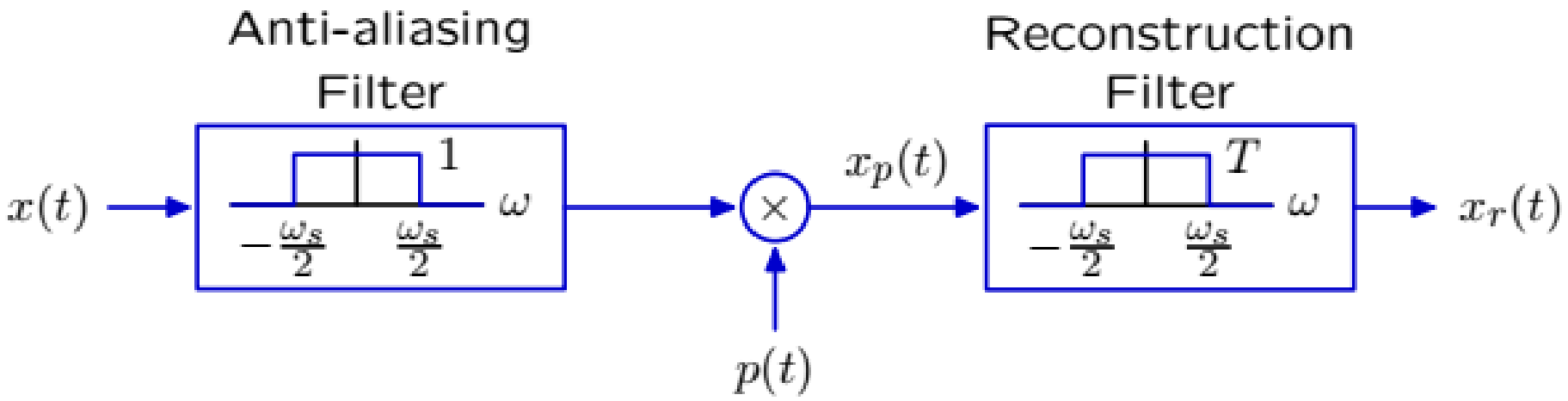


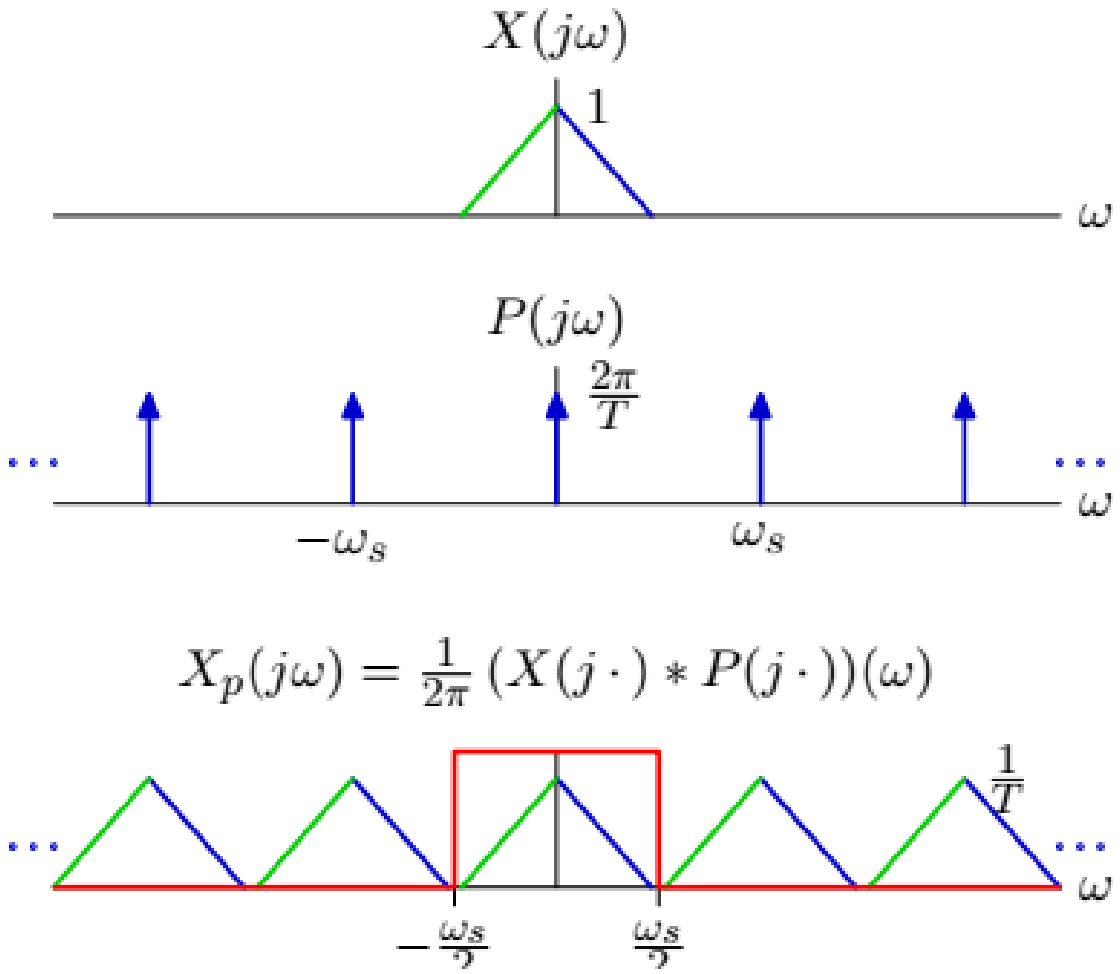
Anti-Aliasing Filter

To avoid aliasing, remove frequency components that alias before sampling.



Aliasing

Aliasing increases as the sampling rate decreases.



Aliasing

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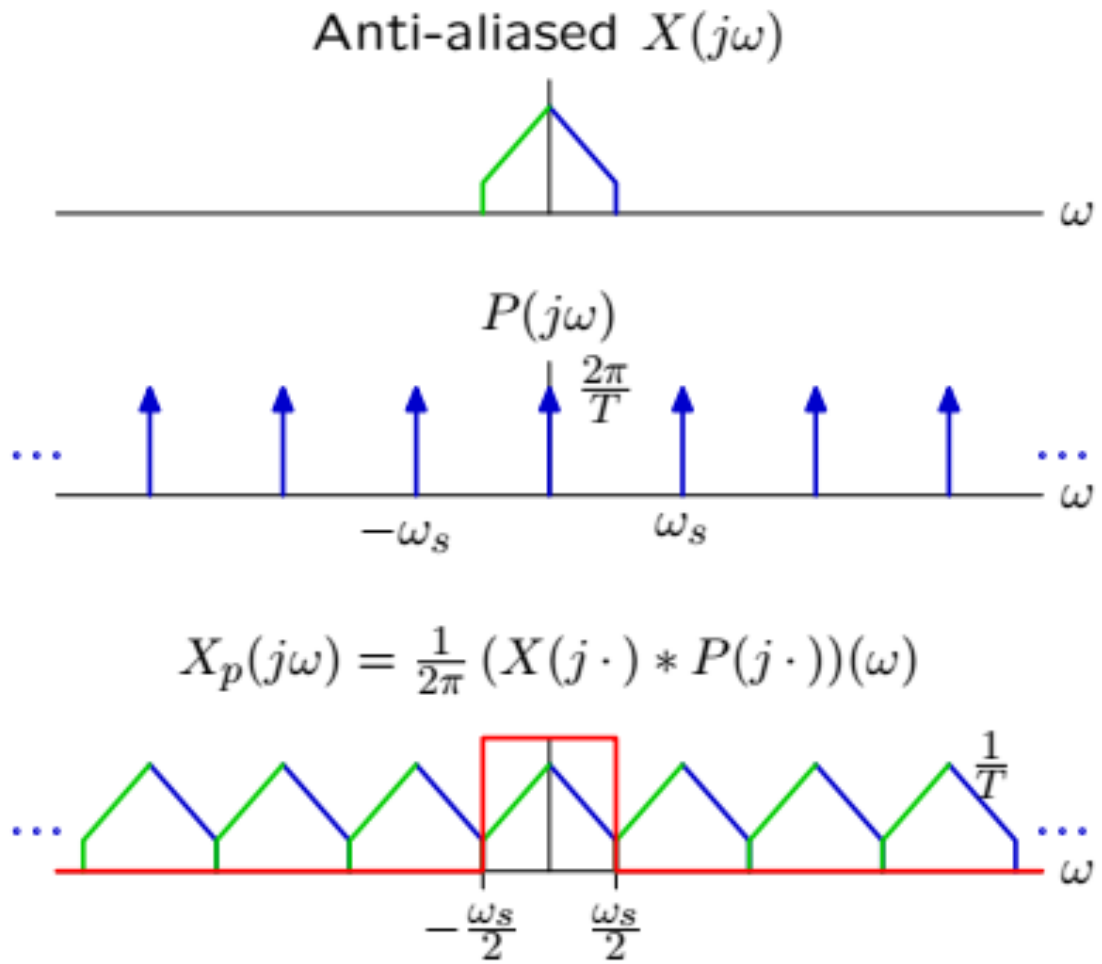

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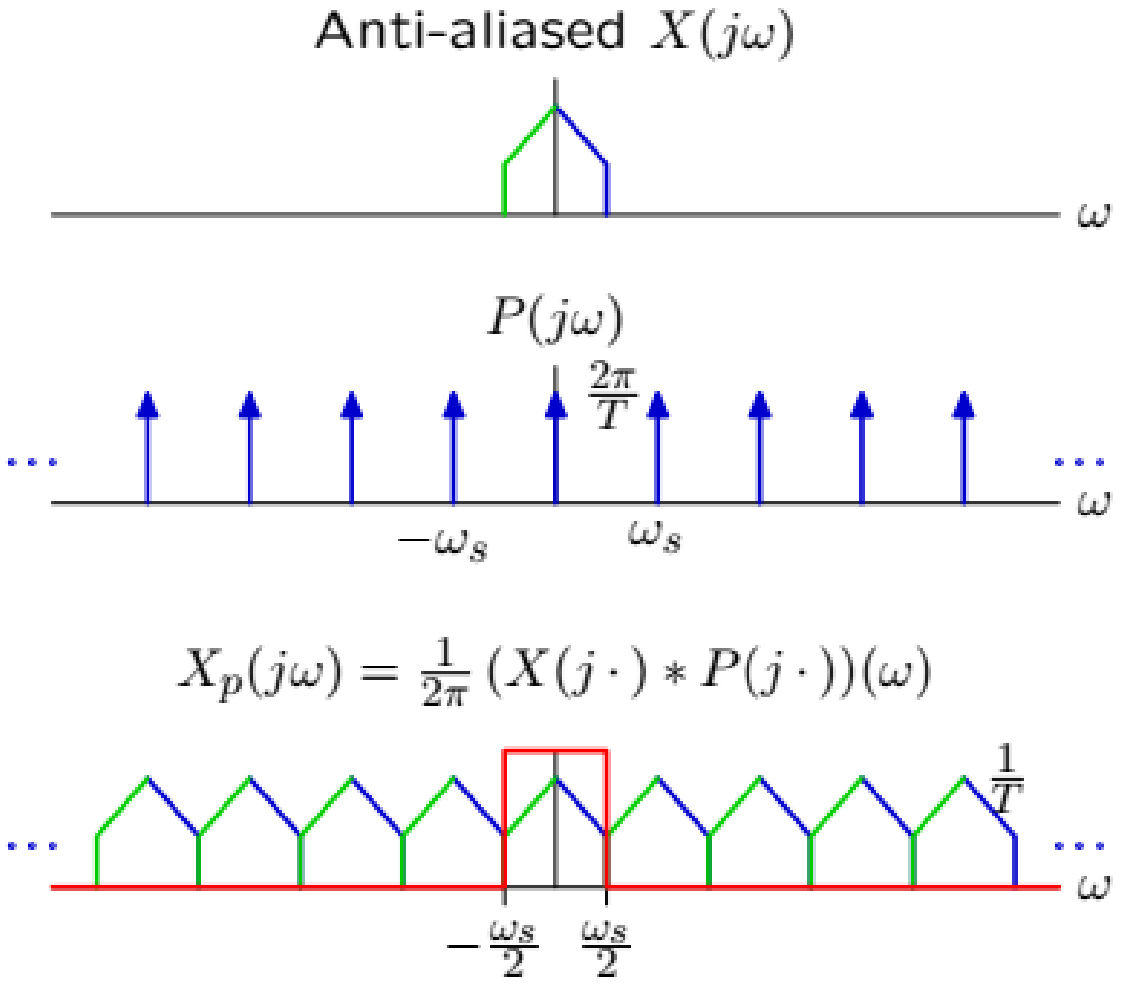
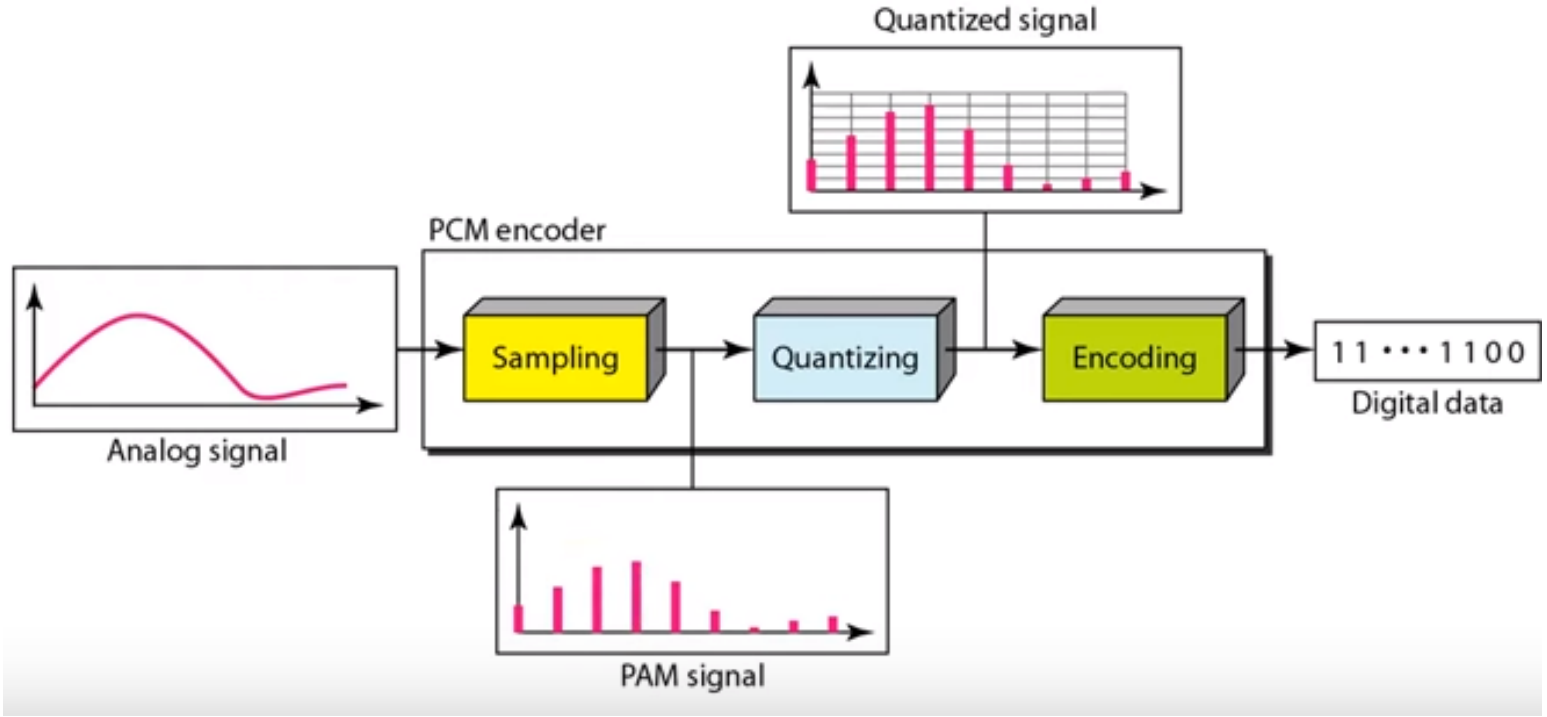


Aliasing

- https://www.youtube.com/watch?v=yWqrx08UeUs -- (6.30dk)
- https://www.youtube.com/watch?v=v7qjeUFxVwQ – (1dk)

Quantization



- https://www.youtube.com/watch?v=YJmUkNTBa8s

Bu ders notu için faydalanılan kaynaklar

EEEN343 Sinyaller ve Sistemler

Ders Notları

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Mühendislik Fakültesi
Elektrik-Elektronik Mühendisliği

MIT OpenCourseWare
http://ocw.mit.edu

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