Sheet 4

Topic: Bayes Rule

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Exercise 1: Bayes Rule

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Exercise 1:

g: green
b: blue

a) P(

The question asks: $P(x=b / y=b)$

$P(x=g)$ = actual color of taxi is green
$P(x=b)$ = // // // // // blue
$P(y=g)$ = the color observed by the witnes is green
$P(y=b)$ = // // // // // // // blue

$P(y=g \mid x=g) = 0.75$
$P(y=g / x=b) = 0.25$
$P(y=b / x=g) = 0.25$
$P(y=b / x=b) = 0.75$

$P(x=g) = 0.9$
$P(x=b) = 0.7$

$$P(x=b / y=b) = \frac{P(y=b \mid x=b) \cdot P(x=b)}{P(y=b \mid x=b) \cdot P(x=b) + P(y=b \mid x=g) \cdot P(x=g)}$$

lets plug the values:

$$P(x=b \mid y=b) = \frac{0.75 \cdot 0.7}{0.75 \cdot 0.7 + 0.25 \cdot 0.9} = \frac{\frac{75}{1000}}{\frac{75}{1000} + \frac{225}{1000}} = \boxed{0.25}$$

b) $P(x=g) = 0.7$ only probability of the actual color of athenian taxi changed
$P(x=b) = 0.3$ the formula and what question asked is same with previous question.

$$P(x=b \mid y=b) = \frac{0.75 \cdot 0.3}{0.75 \cdot 0.3 + 0.25 \cdot 0.7} = \frac{\frac{225}{1000}}{\frac{225}{1000} + \frac{175}{1000}} = \frac{9}{16} = \boxed{0.5625}$$

c) since the second witness is color blind. His/her additional information does not change anything. his/her has a chance of %50 of being correct about color of taxi. However, this not change anything because they are also independent things.

$$P(x=b \mid y=b, z=g) = \frac{P(x=b \mid y=b) \cdot P(z=g \mid x=b)}{P(z=g \mid x=g) P(x=g) + P(z=g \mid x=b) \cdot P(x=b)} = \frac{0.5626 \cdot 0.5}{0.5 \cdot 0.7 + 0.5 \cdot 0.3} = 0.5626$$

Exercise 2: Bayes Filter

Exercise 2:

- Let x_0 is the state of the floor on the cleaning action. Values c (clean) or d (dirty)
- Let x_1 is the state of the floor after the cleaning action. Values c (clean) d (dirty)
- Let z is the measurement of the robot after cleaning action. Values c (clean) or d (dirty)
- Let u be the command of execution of the cleaning action, Value uc (vacuum clean)

What we know:

$P(x_1=c \mid x_0=d, u=uc) = 0.7$

$P(x_1=d \mid x_0=d, u=uc) = 0.3$

$P(z=c \mid x_1=d) = 0.3$

$P(z=d \mid x_1=d) = 0.7$

$P(z=c \mid x_1=c) = 0.9$

$P(z=d \mid x_1=c) = 0.1$

We can also assume that

$P(x_1=c \mid x_0=c, u=uc) = 1$

$P(x_1=d \mid x_0=c, u=uc) = 0$

We do not know the initial state of the floor so we can say

$P(x_0=c) = q \qquad P(x_0=d) = 1-q$

The question asks $P(x_1=d \mid z=c, u=uc)$

- $P(x_1=d \mid z=c, u=uc) = \eta P(z=c \mid x_1=d) \sum_{x_0} P(x_1=d \mid x_0, u=uc) P(x_0) \Rightarrow$

$\eta P(z=c \mid x_1=d) . \left(P(x_1=d \mid x_0=c, u=uc) + P(x_1=d \mid x_0=d, u=uc) . P(x_0=d)\right) \Rightarrow$

$\eta 0.3 \left(0.q + 0.3 . (1-q)\right) = \boxed{(0.09 - 0.09q)\eta}$

- $P(x_1=c \mid z=c, u=uc) = \eta P(z=c \mid x_1=c) \sum_{x_0} P(x_1=c \mid x_0, u=uc) P(x_0) \Rightarrow$

$\eta P(z=c \mid x_1=c) \left(P(x_1=c \mid x_0=c, u=uc) . P(x_0=c) + P(x_1=c \mid x_0=d, u=uc) P(x_0=d)\right) \Rightarrow$

$\eta . 0.9 \left(1.q + 0.7 . (1-q)\right) = \boxed{(0.63 + 0.27q)\eta}$

$P(x_1=d \mid z=c, u=uc) = (0.09 - 0.09q)\eta$

$P(x_1=c \mid z=c, u=uc) = (0.63 + 0.27q)\eta$

$\Rightarrow (0.09 - 0.09q)\eta + (0.63 + 0.27q)\eta = 1$

$\boxed{\eta = \frac{1}{0.72 + 0.18q}}$

This needs to hold:

$P(x_1=d \mid z=c, u=uc) + P(x_1=c \mid z=c, u=uc) = 1$

Finally, $P(x_1=d \mid z=c, u=uc) = (0.09 - 0.09q)\eta = \frac{0.09 - 0.09q}{0.72 + 0.18q}$

- for the non informative prior of $q=0.5$: $P(x_1=d \mid z=c, u=uc) = \frac{1}{18} = 0.055$
- If we assume floor is dirty at the beginning which is $q=0$;

$P(x_1=d \mid z=c, u=uc) = \frac{1}{8} = 0.125$

b) if we take as prior ; $P(x_0=c)=1$, $P(x_0=d)=0$

we can obtain the minimum value of $P(x_1=d \mid z=c, u=uc) = 0$