Sheet 2

Topic: Linear Algebra

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Exercise 1: Linear Algebra

Part a)

Exercise 1:

a)

$$A = \begin{pmatrix} 0.25 & 0.1 \\ 0.2 & 0.5 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 0.25 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}$$

$A \neq A^T$ so A is not symetric and not positive definite

$$B = \begin{pmatrix} 0.25 & -0.3 \\ -0.3 & 0.5 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 0.25 & -0.3 \\ -0.3 & 0.5 \end{pmatrix}$$

$B = B^T$ symmetric

eigenvalues:

$|B - \lambda I| = 0$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$B - \lambda I = \begin{pmatrix} 0.25-\lambda & -0.3 \\ -0.3 & 0.5-\lambda \end{pmatrix}$$

$|B - \lambda I| = \lambda^2 - 0.75\lambda + 0.035$

$\lambda_1 = 0.7$ $\quad \lambda_1, \lambda_2 > 0$

$\lambda_2 = 0.05$

Positive Definite

Part b)

b)

$$C = \begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mu I = \begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix}$$

$$C + \mu I = \begin{pmatrix} -3+\mu & 0 \\ 0 & 1+\mu \end{pmatrix}$$

if eigenvalues are smaller or equal to zero, the matrix is not symmetric positive definite.

$(-3+\mu).(1+\mu) \leq 0$

$-3+\mu \leq 0$, $1+\mu \leq 0$

$\mu \leq 3$, $\mu \leq -1$

the largest value of μ is $\boxed{3}$ //

Part c and d are in the python file

Exercise 2: 2D Transformations as Affine Matrices

Part a)

Exercise 2:

a)

$$ {}^{g}T_{x_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & x_1 \\ \sin\theta_1 & \cos\theta_1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad {}^{x_1}l = \begin{bmatrix} l_x \\ l_y \\ 1 \end{bmatrix} $$

${}^{g}T_{x_1}$: is the matrix experssion in homogeneous form of pose X_1 w.r.t. the global frame.

${}^{x_1}l$: is the vector expression in homogeneous form of the landmark w.r.t. the robot reference frame. X_1

$$ \boxed{ {}^{g}l = {}^{g}T_{x_1} \cdot {}^{x_1}l } $$

Part b, c)

b) ${}^{g}l$ and ${}^{g}T_{x_1}$ are given.

$$ \boxed{ {}^{x_1}l = {}^{x_1}T_{g} \cdot {}^{g}l = \left({}^{g}T_{x_1}\right)^{-1} \cdot {}^{g}l } $$

c)

$$ {}^{g}T_{x_2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & x_2 \\ \sin\theta_2 & \cos\theta_2 & y_2 \\ 0 & 0 & 1 \end{bmatrix} $$

${}^{g}T_{x_2}$: is the matrix expression in homogeneous form of pose X_2 w.r.t. the global reference frame.

$$ {}^{x_1}T_{x_2} = {}^{x_1}T_{g} \cdot {}^{g}T_{x_2} = \left({}^{g}T_{x_1}\right)^{-1} \cdot {}^{g}T_{x_2} = T_{12} $$

Part d)

d) I computed ${}^{x_1}T_{x_2}$ previously.

$$ {}^{x_2}1 = {}^{x_2}T_{x_1} \cdot {}^{x_1}1 = \left({}^{x_1}T_{x_2}\right)^{-1} \cdot {}^{x_1}1 $$

$$ \left({}^{x_1}T_{x_2}\right)^{-1} = \left(\left({}^{g}T_{x_1}\right)^{-1} \cdot {}^{g}T_{x_2}\right)^{-1} = \left({}^{g}T_{x_2}\right)^{-1} \cdot {}^{g}T_{x_1} $$

$$ \Rightarrow \boxed{{}^{x_2}1 = \left({}^{g}T_{x_2}\right)^{-1} \cdot {}^{g}T_{x_1} \cdot {}^{x_1}1} $$

Exercise 3: Sensing is on the python file