Sheet 6

Topic: Sensor Models

Name: Berkay

Surname: Gülen

No: 20170613017

Exercise 1: Distance-Only Sensor

Part a and c

Q1-)

$m \rightarrow$ being at location

$z \rightarrow$ Sensor Measurement

a) $P(m/z) = \dfrac{P(z/m)\,P(m)}{P(z)}$ (Bayes Rule)

- We don't have a information about m to assume uniform prior $P(m)$
- $P(z)$ is not depend on m $\quad P(m/z) \propto P(z/m)$
- Assume both measurements independent of each other

$$P(z/m) = P(d_0, d_1/m) = P(d_0/m) \cdot P(d_1/m)$$

- Probability density of normal distribution =>

$$P(d/m) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\left(-\frac{(d-\hat{d})^2}{2\sigma^2}\right)}$$

$\hat{d}$ = true distance between towers and query locations

- At University:

Tower 0 => $\hat{d}_0 = \sqrt{(12-10)^2 + (4-8)^2} = \sqrt{20}$ $\quad p(d_0/m_0) = \dfrac{1}{\sqrt{2\pi \cdot 1}} \cdot e^{\left(-\frac{(3.9-\sqrt{20})^2}{2\times 1}\right)}$

Tower 1 => $\hat{d}_1 = \sqrt{(5-10)^2 + (7-8)^2} = \sqrt{26}$ $\quad P(d_1/m_0) = \dfrac{1}{\sqrt{2\pi \cdot 1.5}} \cdot e^{-\left(\frac{(4.5-\sqrt{26})^2}{2\times 1.5}\right)}$

$$\boxed{P(d_0, d_1/m) = 0.0979}$$

- At Home

Tower 0 => $\hat{d}_0 = \sqrt{(2-6)^2 + (4-3)^2} = \sqrt{37}$ $\quad P(d_0/m_1) = \dfrac{1}{\sqrt{2\pi \cdot 1}} \cdot e^{-\frac{(3.9-\sqrt{37})^2}{2\times 1}}$

Tower 1 => $\hat{d}_1 = \sqrt{(5-6)^2 + (7-3)^2} = \sqrt{17}$ $\quad P(d_1/m_1) = \dfrac{1}{\sqrt{2\pi \times 1.5}} \cdot e^{-\frac{4.5\times\sqrt{17}^2}{2\times 1.5}}$

$$P(d_0, d_1/m_1) = 0.0114$$

It is more likely to obtain given measurement if the friend is at uni.

c)

$P(z) = \sum P(z/m_i)\, P(m_i)$ law of total probability

$P(z) = P(d_0/d_1) = P(d_0, d_1/m_0)\, P(m_0) + P(d_0, d_1/m_1)\, P(m_1) \Rightarrow$

$0.0979 \times 0.3 + 0.0114 \times 0.7 = \underline{\underline{0.0374}}$

University $\Rightarrow P(m_0/d_0, d_1) = \dfrac{P(d_0, d_1/m_0)\, P(m_0)}{P(d_0, d_1)} = \dfrac{0.0979 \times 0.3}{0,0374} = 0,785$

Home $\Rightarrow P(m_1, d_0, d_1) = \dfrac{P(d_0, d_1/m_1)\, P(m_1)}{P(d_0/d_1)} = \dfrac{0,0114 \times 0.7}{0.0374} = 0.213$

Exercise 2: Sensor Model

Q2)

$x \to$ given pose $\quad z_r \to$ distance $\quad$ Variances $\to \sigma_1^2, \sigma_2^2$

$l \Rightarrow$ landmark Pose $\quad z_a \to$ angle $\quad$ Sensor measurement $z = (z_r, z_a)$

$\to$ robot pose $\Rightarrow x = (x_x, x_y, x_a)$ $\quad P(z_1, x_1, l) = P(z_r, z_a \mid x, l)$

landmark pose $\Rightarrow l.\ (l_x, l_y)$ $\quad = P(z_r \mid x, l)\, P(z_a \mid x, l)$

$\to z_i \to$ measurment $\quad z_{exp,r} \Rightarrow$ expected measurment

$$P(z_i \mid x, l) = \frac{1}{\sqrt{2\pi \cdot \sigma_i^2}} = e^{-\frac{(z_i - z_{exp,i})^2}{2\sigma_i^2}} = \frac{1}{\sqrt{2\pi \cdot \sigma_i^2}} \cdot e^{-\frac{(\Delta z_i)^2}{2\sigma_i^2}}$$

$\to$ range measurment $\quad z_{exp,r} = \sqrt{(l_x - x_x)^2 + (l_y - x_y)^2}$

$$\Delta z_r = z_r - z_{exp,r} \qquad P(z_r \mid x, l) = \frac{1}{\sqrt{2\pi, \sigma_r^2}} \cdot e^{-\frac{(\Delta z_r)^2}{2\sigma_r^2}}$$

$\to$ Bearing measurment $\quad z_{exp,a} = \operatorname{atan2}\left((l_y - x_y), (l_x - x_x)\right) - x_a$

$\vec{V}_z = (\cos(z_a), \sin(z_a))^T$

unit vectors

$\vec{V}_{z,exp} = (\cos(z_{exp,a}), \sin(z_{exp,a}))^T$

$\vec{N}_z \cdot \vec{V}_{z,exp} = |\vec{V}_z| \cdot |\vec{V}_{z,exp}| \cdot \cos(\Delta z_a)$

$\Delta z_a = \arccos(z_a) \cos(z_{exp,a}) + \sin(z_a) \sin(z_{exp,a})$

$$P(z_a \mid x, l) = \frac{1}{\sqrt{2\pi \cdot \sigma_a^2}} \cdot e^{-\frac{(\Delta z_a)^2}{2\sigma_a^2}}$$