MCE 412 Term Project
Extended Kalman Filter

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Exercise 1

a)

Exercise 1 :

$P(\underbrace{x_t}_{\text{State}} | \underbrace{u_t}_{\text{Control input}}, \underbrace{x_{t-1}}_{\text{Previous state}})$. => It allows us to predict the next state from previous state and control input/vector.

$$X_t = g(u_t, x_{t-1}) \cong \underbrace{g(u_t, \mu_{t-1})}_{\text{the mean}}$$

- $P(z_t | x_t)$ => this disturibition allows us to predict the measurement using current state.

The mean is $h(\mu_t)$ where R_t is the covarience matrix

- $bel(x_t) = \eta \underbrace{P(z_t | x_t)}_{\text{measurement model}} \int \underbrace{P(x_t | u_t, x_{t-1})}_{\text{motion planned system model}} \underbrace{bel(x_{t-1})}_{\text{Prior belief}} \, dx_{t-1}$

Prediction => $\overline{bel}(x_t) = \int P(x_t | u_t, x_{t-1}) \, bel(x_{t-1}) \, dx_{t-1}$

Correction => $bel(x_t) = P(z_t | x_t) \, \overline{bel} \, x_t$

From Kalman Filter

$bel(x_t)$ is the distrubition of the state giving the current estimate

We use Gaussian as long as we start with Gaussians and perform linear transformations. Since we are dealing with EKF which is for non-linear systems we need to consider non Gaussian distrubitions. To do this, we have local linearization,

=> $P(x_t | u_t, x_{t-1})$ $P(z_t | x_t)$ are Gaussian if $g(u_t, \mu_{t-1})$ $h(\mu_t)$ are linear,

=> If $g(u_t, \mu_{t-1})$ $h(\mu_t)$ are non-linear, $g(u_t, \mu_{t-1})$ $h(\mu_t)$ are different from the mean of actual distrubitions b)

b)

$g(u,x) \Rightarrow$ the motion model of system which estimates the current state (x_t) from previous State and control vector.

$G_t \Rightarrow$ Jacobien Matrix. G_t is defined as $G_t = \left.\frac{\partial g}{\partial x}\right|_{x=\mu_{t-1}}$ In KF, we have A_t which coresponds to G_t in EKF

$Q_t \Rightarrow$ Covarience Matrix which is assumed to be known of zero-mean Gaussian Error corrupting the Prediction of state.

$h(x) \Rightarrow$ the measorment model of system estimated z_t from current state variable.

$H_t \Rightarrow$ Jacobian Matrix which defined as $H_t = \left.\frac{\partial h}{\partial x}\right|_{x=\mu_t}$ in KF, we have C_t corresponds to H_t in EK

$R_t \Rightarrow$ Covarience Matrix assumed to be known of zero-mean Gaussian Error corrupting the measurment. (measurment is given by h with some noise)

$\Sigma_t \Rightarrow$ coverience Matrix of distribution state which is for estimating Q_t

* Most of the time we do not have linearity. Linear KF are optimal filters in terms of min-varience. Whatever the initial condition is, we can reach optimal solutions. Therefore, when we do not have linearity, we use EKF. For EKF, we do not have optimaly guaranteed. If the initial position is not enough trough starting point, we may not reach optimal solution.

Kalman Filter

$$\mu_t = A_t \mu_{t-1} + B_t u_t$$

$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + Q_t$$

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + R_t)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \mu_t)$$

$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

Extended Kalman Filter

$$\mu_t = g(u_t, \mu_{t-1})$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + Q_t$$

$$K_t = \bar{\Sigma}_t H_t^T (H_t \Sigma_t H_t^T + R_t)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$$

Exercise 2

Exercise 2 :

a)

we know $g(\mu_t, x_{t-1}) = g(\mu_{t-1}, \mu_t) + G_t (x_{t-1} - \mu_{t-1})$

$G_t = \dfrac{\partial g(\mu_t, \mu_{t-1})}{\partial x_{t-1}}$ => the rate of change in 'x' / slope of the function.

Let's take state variables as $S_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$ and $\mu_t = \begin{bmatrix} \delta_1 \\ \delta_t \\ \delta_2 \end{bmatrix}$

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = g(S_{t-1}, \mu_t) = \begin{bmatrix} x_{t-1} + \delta_t \cdot \cos(\theta_{t-1} + \delta_1) \\ y_{t-1} + \delta_t \cdot \sin(\theta_{t-1} + \delta_1) \\ \theta_{t-1} + \delta_1 + \delta_2 \end{bmatrix}$$

if we do differentiation for each row according to x_{t-1}, y_{t-1}, θ_{t-1}

$$G_t = \begin{array}{l} \begin{bmatrix} 1 & 0 & -\delta_t \sin(\mu_{\theta,t-1} + \delta_1) \\ 0 & 1 & \delta_t \cos(\mu_{\theta,t-1} + \delta_1) \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} g_1 \\ g_2 \\ g_3 \end{array}$$

∵ while we are calculating $\mu_{x,t-1}$, $\mu_{y,t-1}$, $\mu_{\theta,t-1}$ we realize that $\mu_{x,t-1}$ and $\mu_{y,t-1}$ are not seen since they are linear

Exercise 3

Exercise 3 :

a)

We know that $h(x_t) \approx h(\bar{\mu}_t) + H_t(x_t - \bar{\mu}_t)$ and $H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$

Predicted measurments mean $z_t = \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2}$

We can express landmark (l) as $l = \begin{bmatrix} l_x \\ l_y \end{bmatrix}$ according to work frame

$$z_t = h(s_t, l) = \sqrt{(x_{t-1} - l_x)^2 + (y_{t-1} - l_y)^2}$$

if we make differentation

$$H_t = \begin{bmatrix} \frac{\bar{\mu}_{x,t} - l_x}{h(\bar{\mu}_t, l)} & \frac{\bar{\mu}_{y,t} - l_y}{h(\bar{\mu}_t, l)} & 0 \end{bmatrix}$$

Since motion model makes the position and orientation correlated, we will be able to know the orientation eventhough h_t does not care about the orientation.