Removing More Snow in Montreal

Decision Variables:

$X_{ij} = 1$ if sector i is assigned to disposal site j where $i \in \{1,2,\dots,10\}$, $j \in \{1,2,\dots,5\}$

Objective Function:

$$\min \; 153 \cdot 1000 \cdot 0{,}10 (3.4 x_{11} + 1{,}4 x_{12} + \dots + 9{,}3 x_{15})$$
$$+ 152 \cdot 1000 \cdot 0{,}10 (2{,}4 x_{12} + 2{,}1 x_{22} + \dots + 8{,}8 x_{25})$$
$$\vdots$$
$$+ 135 \cdot 1000 \cdot 0{,}10 (3{,}2 x_{10\,2} + 6{,}5 x_{10\,2} + \dots + 8{,}3 x_{10\,5})$$

Constraints

$$\sum_{j=1}^{5} x_{ij} = 1 \quad \text{where} \quad i \in \{1,2,\dots,10\}$$

$$153 x_{11} + 152 x_{21} + \dots + 135 x_{10\,1} \leq 350$$
$$153 x_{12} + 152 x_{22} + \dots + 135 x_{10\,2} \leq 250$$
$$\vdots$$
$$153 x_{15} + 152 x_{25} + \dots + 135 x_{10\,5} \leq 200$$

where $x_{11}, x_{12}, \dots, x_{10\,5} \in \{0, 1\}$

2) Optimal Solution is:

Sector 1 goes to site 2
Sector 2 goes to site 1
Sector 3 goes to site 3
Sector 4 goes to site 3
Sector 5 goes to site 3
Sector 6 goes to site 4
Sector 7 goes to site 4
Sector 8 goes to site 5
Sector 9 goes to site 1
Sector 10 goes to site 4

3) Cost

The cost is $547000.

4)

Without capacity restrictions, every site can take every 10 sectors. Therefore, for each sector there are 5 site possibilities.

So, every possible assignments are

$$\underbrace{5 \times 5 \times \cdots \times 5}_{\text{10 times}} = 5^{10}.$$

5) I added a new variable Y_J to Gams file as a binary variable, where Y_J is 1 if an extra capacity comes to site J, 0 otherwise.

$Y_J \leq 1$ and right hand side of capacity constraints change as:

Capacity of $J + 100 \cdot Y_J$

So, capacity of <u>2nd site</u> should be increased by 100 according to GAMS.

The cost decreases to \$489680.

Therefore city should be willing to pay $547000 - 489680 = \underline{57320}$ \$ at maximum.