$i_1 = j\omega (C_{gs} + C_{mil}) v_{gs}$

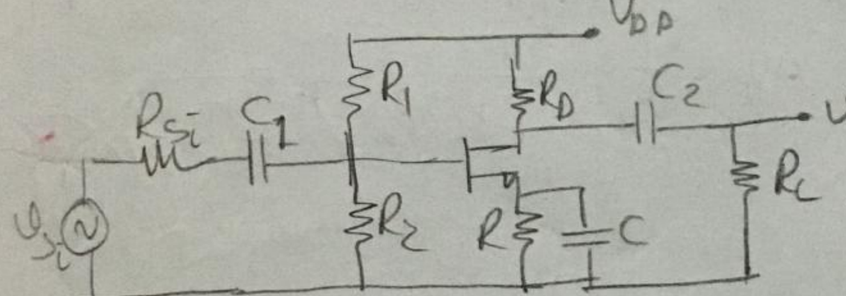
$i_d = g_m v_{gs}$

$$|A_I| = \left|\frac{i_d}{i_1}\right| = \frac{g_m}{2\pi f \cdot \underbrace{(C_{gs}+C_{mil})}_{C_G \text{ : total input capacitance}}}$$

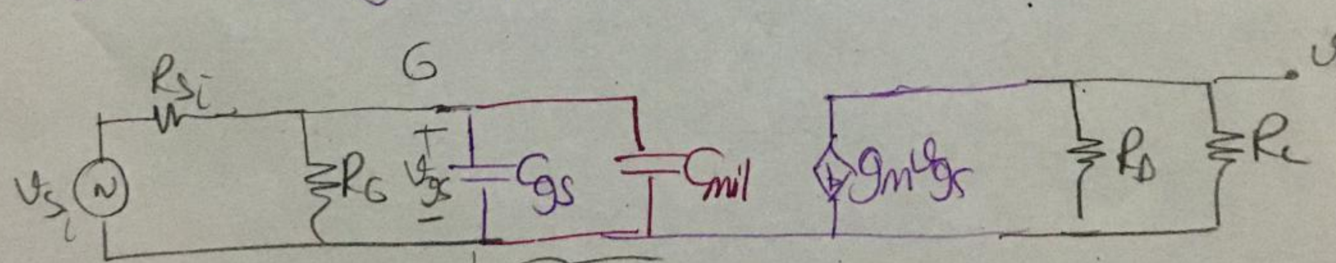
$$f_T = \frac{g_m}{2\pi (C_{gs}+C_{mil})} = \frac{g_m}{2\pi C_G}$$

5.3 High-Frequency Response of MOSFET Circuits.

5.3.1 Common Source Circuit:



high frequency equivalent circuit becomes:



$C_G = C_{gs} + C_{mil}$, $C_{mil} = C_{gd} \cdot (1 + g_m R_L')$, $R_G = R_1 // R_2$

$R_L' = R_D // R_L$

$V_o = -g_m v_{gs} \underbrace{R_D // R_L}_{R_L'}$

$$v_{gs} = \frac{Z_i}{Z_i + R_{si}} \cdot v_{si}, \quad Z_i = R_G // C_G = \frac{R_G}{1 + sC_G R_G}$$

$$\frac{v_o(s)}{v_{s_i}(s)} = A_v(s) = -g_m R_L' \frac{Z_i}{Z_i + R_{S_i}}$$

$$A_v(s) = -g_m R_L' \cdot \frac{R_G/(1+sR_G C_G)}{\frac{R_G}{1+sR_G C_G} + R_{S_i}}$$

$$A_v(s) = -g_m R_L' \frac{R_G}{R_G + R_{S_i}(1+sR_G C_G)} = -g_m R_L' \frac{R_G}{R_G+R_{S_i}} \cdot \frac{1}{1+s\underbrace{\frac{R_G R_{S_i}}{R_G+R_{S_i}}}_{R_G \parallel R_{S_i}} C_G}$$

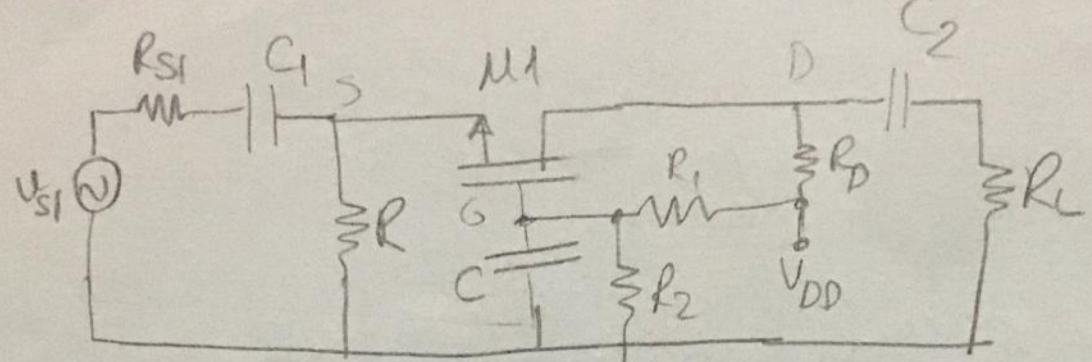
$$A_v(s) = -g_m R_L' \frac{R_G}{R_G+R_{S_i}} \cdot \frac{1}{1+s\,R_G \parallel R_{S_i} \cdot C_G} \quad \tau_H$$

$$\tau_H = C_G \cdot R_G \parallel R_S \quad , \omega_H = \frac{1}{\tau_H} = \frac{1}{C_G \cdot R_G \parallel R_S} , f_H = \frac{1}{2\pi C_G\, R_G \parallel R_S}$$

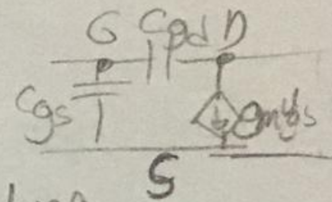
As before, we could have calculated the upper corner 3dB frequency using the time constant analysis. The effective resistance seen by the total gate capacitance C_G is $R_G \parallel R_{S_i}$ (is $\mathcal{L}_O$). Therefore

$$f_H = \frac{1}{2\pi C_G R_G \parallel R_{S_i}} \quad , \quad C_G = C_{gs} + \underbrace{C_{gd}(1+g_m R_L')}_{C_{Mil}}$$

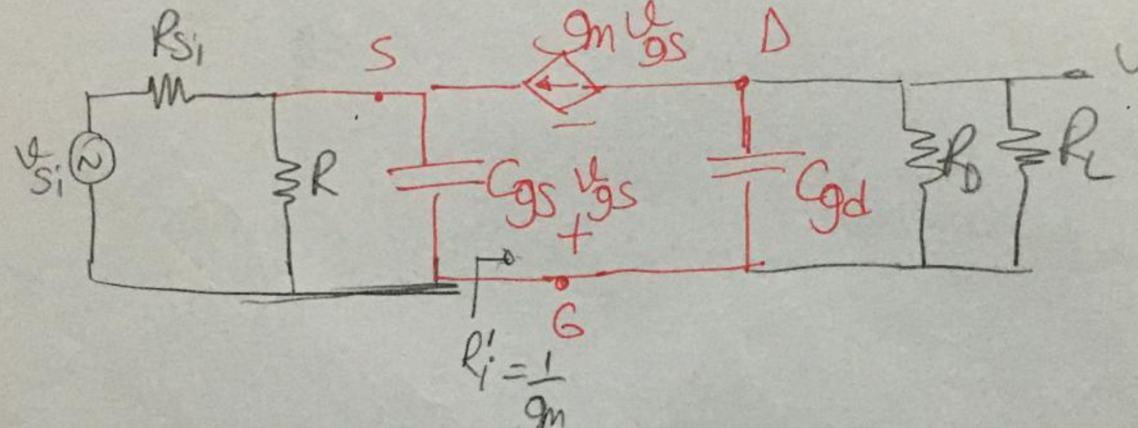
5.3.2. Common Gate Circuit



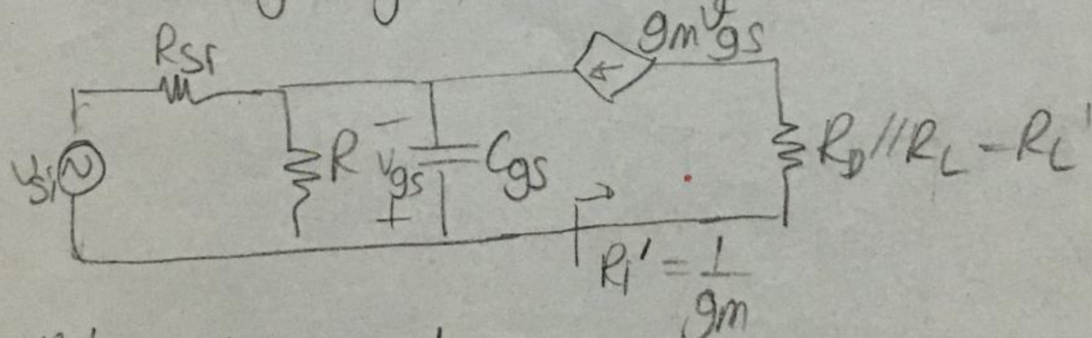
(we assume $\lambda=0$ for M1)



$C_1, C_2, C \to \infty$ for the high frequency analysis,



With only C_{gs}

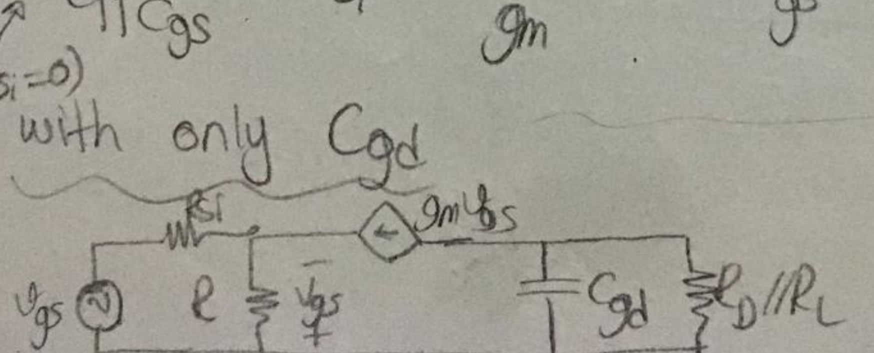


$$R_i' = \frac{-v_{gs}}{-g_m v_{gs}} = \frac{1}{g_m}$$

$$R_{eff\,C_{gs}} = R_{Si} // R // \frac{1}{g_m} \Rightarrow \omega_{C_{gs}} = \frac{1}{C_{gs} \cdot R_{eff\,C_{gs}}} = \frac{1}{C_{gs}\left(R_{Si} // R // \frac{1}{g_m}\right)}$$

($v_{Si}=0$)

with only C_{gd}



with $v_{Si}=0$

$g_m v_{gs} R_{Si} // R = -v_{gs}$

$(1+g_m R_{Si} // R)\, v_{gs} = 0$

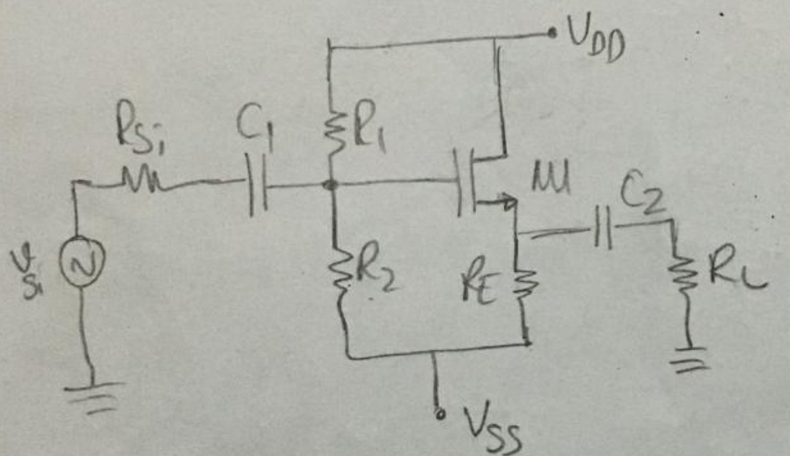
$v_{gs} = 0$

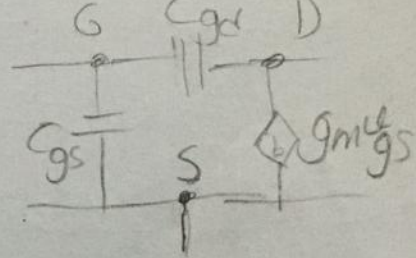
$$R_{eff\,C_{gd}} = R_L' = R_D // R_L$$

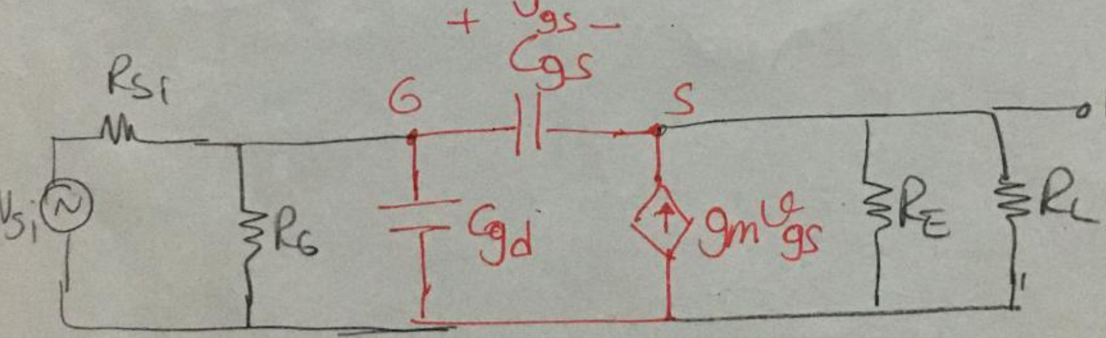
$$\omega_{C_{gd}} = \frac{1}{C_{gd} \cdot R_{eff\,C_{gd}}} = \frac{1}{C_{gd} \cdot R_D // R_L}, \quad f_{C_{gd}} = \frac{1}{2\pi C_{gd} (R_D // R_L)}$$

$$\omega_H = \min\{\omega_{C_{gd}}, \omega_{C_{gs}}\}, \quad f_H = \min\{f_{C_{gd}}, f_{C_{gs}}\}$$

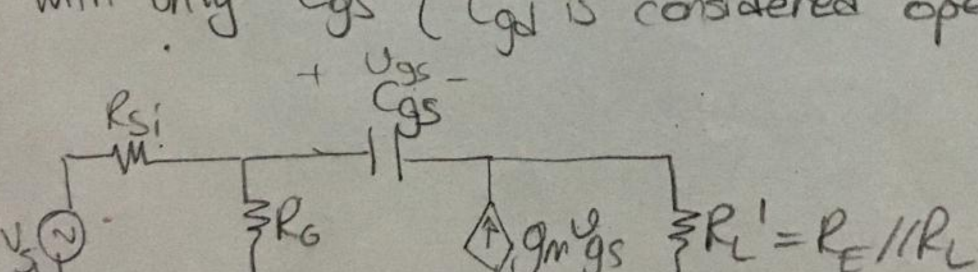
5.3.3 Common Drain Circuit (a.k.a. the Source Follower):

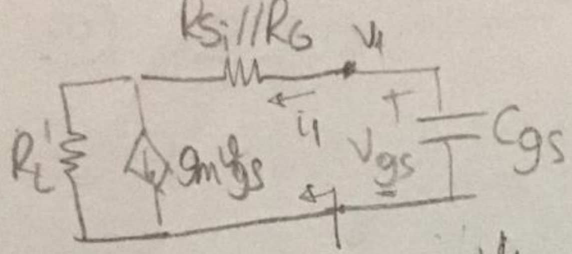






With only C_{gs} (C_{gd} is considered open circ.):





$$R_{eff_{C_{gs}}} = \frac{v_1}{i_1} = \frac{v_{gs}}{i_1}$$

$$v_1 = R_{si}//R_G \cdot i_1 + (i_1 - g_m v_{gs}) \cdot R_L' = v_{gs}$$

$$(R_{si}//R_G + R_L')\, i_1 = (1 + g_m R_L')\, v_{gs}$$

$$R_{eff_{C_{gs}}} = \frac{v_{gs}}{i_1} = \frac{R_{si}//R_G + R_L'}{1 + g_m R_L'}$$

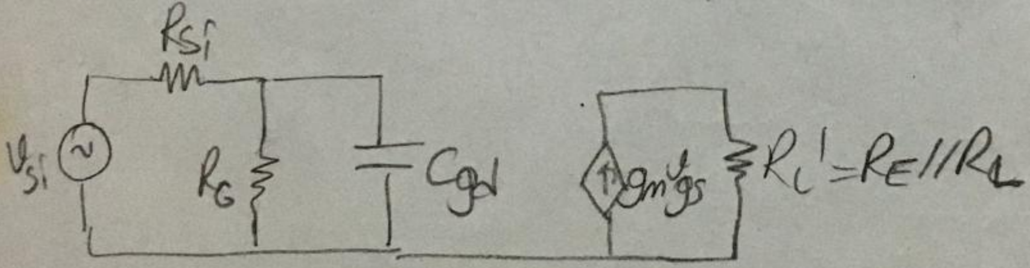
$$\omega_{C_{gs}} = \frac{1}{\tau_{C_{gs}}} = \frac{1}{C_{gs} \cdot \left[\frac{R_{si}//R_G + R_L'}{1 + g_m R_L'}\right]}$$

$, R_L' = R_E//R_L$

$R_G = R_1//R_2$

$f_{C_{gs}} = \frac{\omega_{C_{gs}}}{2\pi}$

with only C_{gd} (C_{gs} is considered open circuit):



$$R_{eff_{C_{gd}}} = R_{si}//R_G \quad , \quad \omega_{C_{gd}} = \frac{1}{\tau_{C_{gd}}} = \frac{1}{C_{gd} \cdot R_{si}//R_G} \quad f_{C_{gd}} = \frac{\omega_{C_{gd}}}{2\pi}$$

$$\omega_H = \min\{\omega_{C_{gd}}, \omega_{C_{gs}}\} \quad , \quad f_H = \min\{f_{C_{gd}}, f_{C_{gs}}\}$$