

CSE 321
INTRODUCTION TO ALGORITHM DESIGN
HOMEWORK 2

BERKE BELGİN
171044065

1)

[6, 5, 3, 11, 7, 5, 2], index = 0;

[6, 5, 3, 11, 7, 5, 2], index = 1;

(5 < 6) => set [6, 6, 3, 11, 7, 5, 2]

index == 0 => insert [5, 6, 3, 11, 7, 5, 2]

[5, 6, 3, 11, 7, 5, 2], index = 2;

(3 < 6) => set [5, 6, 6, 11, 7, 5, 2]

(3 < 5) => set [5, 5, 6, 11, 7, 5, 2]

index == 0 => insert [3, 5, 6, 11, 7, 5, 2]

[3, 5, 6, 11, 7, 5, 2], index = 3;

(11 => 6) => insert [3, 5, 6, 11, 7, 5, 2]

[3, 5, 6, 11, 7, 5, 2], index = 4;

(7 < 11) => set [3, 5, 6, 11, 11, 5, 2]

(7 => 6) => insert [3, 5, 6, 7, 11, 5, 2]

[3, 5, 6, 7, 11, 5, 2], index = 5;

(5 < 11) => set [3, 5, 6, 7, 11, 11, 2]

(5 < 7) => set [3, 5, 6, 7, 7, 11, 2]

(5 < 6) => set [3, 5, 6, 6, 7, 11, 2]

(5 >= 5) => insert [3, 5, 5, 6, 7, 11, 2]

[3, 5, 5, 6, 7, 11, 2], index = 6;

(2 < 11) => set [3, 5, 5, 6, 7, 11, 11]

(2 < 7) => set [3, 5, 5, 6, 7, 7, 11]

(2 < 6) => set [3, 5, 5, 6, 6, 7, 11]

(2 < 5) => set [3, 5, 5, 5, 6, 7, 11]

(2 < 5) => set [3, 5, 5, 6, 7, 7, 11]

(2 < 3) => set [3, 3, 5, 6, 7, 7, 11]

index == 0 => insert [2, 3, 5, 6, 7, 7, 11]

[2, 3, 5, 5, 6, 7, 11]

2)

```
function(int n){  
    if (n==1) -> Dependent on n. n time(s) when n = 1  
        return;  
    for (int i=1; i<=n; i++){ -> n times  
        for (int j=1; j<=n; j++){ -> 1 time since there is a break  
            printf("*");  
            break;  
        }  
    }  
}
```

So, the time complexity of the above program is $\Omega(n) = Q(n) = O(n)$.

```
void function(int n){  
    int count = 0;  
    for (int i=n/3; i<=n; i++) -> (n - n*1/3) = n*2/3 times  
        for (int j=1; j+n/3<=n; j = j++) -> (n - n*1/3) = n*2/3 times  
            for (int k=1; k<=n; k = k * 3) -> x times where  $3^x = n$ ,  $\log_3 n$  times  
                count++;  
    }
```

So, the time complexity of the above program is:

$(n*2/3)*(n*2/3)*\log_3 n \rightarrow \Omega(n^2 \log n) = Q(n^2 \log n) = O(n^2 \log n)$.

3)

```
PROCEDURE heapify(arr, n, i)
    largest = i
    l = 2 * i + 1
    r = 2 * i + 2

    IF l < n AND arr[i] < arr[l] THEN
        largest = l
    ENDIF

    IF r < n AND arr[largest] < arr[r] THEN
        largest = r
    ENDIF

    IF largest != i THEN
        arr[i], arr[largest] = arr[largest], arr[i]
        heapify(arr, n, largest)
    ENDIF
END

PROCEDURE heapSort(arr)
    n = len(arr)

    FOR i IN range(n // 2 - 1, -1, -1) DO
        heapify(arr, n, i)
    ENDFOR

    FOR i IN range(n-1, 0, -1) DO
        arr[i], arr[0] = arr[0], arr[i]
        heapify(arr, i, 0)
    ENDFOR
END

PROCEDURE findMultipliers(arr, num)
    indxStart = 0
    indxEnd = len(arr) - 1

    WHILE indxStart != indxEnd DO
        IF arr[indxStart] * arr[indxEnd] == num THEN
            print(arr[indxStart], arr[indxEnd])
            indxStart += 1
        ELSE IF num % arr[indxStart] == 0 THEN
            indxEnd -= 1
        ELSE
            indxStart += 1
        ENDIF
    ENDWHILE
END

arr = [1,2,3,6,5,4]
heapSort(arr)
findMultipliers(arr, 6)
```

Since Heap Sort has a complexity of $n \log n$ and finding multipliers has a complexity of n because of a single while loop, the program has a complexity of $O(n \log n + n) = O(n \log n)$.

4)

Since regardless of the data structure, best case time complexity of traversing a list of elements is n and we can traverse the binary tree to be merged easily in n steps with a simple recursive function. Then for each element, we can insert them into destination BST in $\log n$ steps since the time complexity of finding an element in BST is $\log n$ and inserting a node to a node of a tree is 1 . So, we can say that merging two BSTs by traversing one tree, finding the appropriate position for the every element in the traversed BST in destination BST and then inserting the element has a time complexity of $(n * \log n * 1)$ which is equal to $O(n * \log n)$.

5)

```
PROCEDURE getSubArray(bigArray[] , smallArray[])
    maxVal = 0
    minVal = 0
    FOR EACH i IN bigArray DO
        IF i > maxVal maxVal = i ENDIF
        IF i < minVal minVal = i ENDIF
    ENDFOR

    tempArraySize = maxVal - minVal + 1
    tempArray[tempArraySize]

    FOR int i = 0 TO bigArray.length DO
        tempArrayIndex = bigArray[i] - minVal
        tempArray[tempArrayIndex] = bigArray[i]
    ENDFOR

    resultArraySize = smallArray.length
    resultArray[resultArraySize]
    resultArrayIndex = 0

    FOR EACH i IN smallArray DO
        tempArrayIndex = i - minVal
        IF tempArray[tempArrayIndex] is null THEN
            resultArray[resultArrayIndex] = i
            resultArrayIndex++
        ENDIF
    ENDFOR

    RETURN resultArray
END
```

Since there were no restrictions in terms of memory space, I assumed we have an infinite source of memory. So, I implemented a Hash like system where it's both best and worst case is $O(n)$ but may take up as much memory space as the value gap between the smallest and the greatest value of the array.

Since the function has no nested loops and there are 2 loops for big array and 1 loop for small array in a row, we can accept that this functions complexity is:

$T(2a + b) \Rightarrow T(3n) \Rightarrow \Omega(n) = Q(n) = O(n)$.