Topics in Database Theory – Homework 4

Fall, 2023

1 Descriptive Complexity

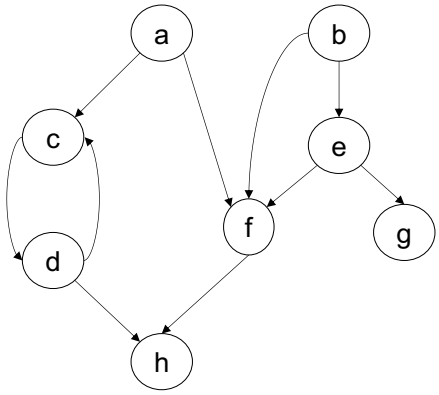
1. (0 points)

   More details about information inequalities can be found in [1].

   (a) Let $G = (V, E)$ be a finite graph, and consider the following query:

   $$q(x) = [\texttt{lfp}_{S,x}(\forall y(E(x,y) \rightarrow S(y)))](x)$$

   i. Which nodes x does the query return on the graph below?

   ii. Write an FO sentence (without fixpoints!) that is equivalent to $\forall x \neg q(x)$.

   iii. Consider these complexity classes: AC^0, $PTIME$, NP, $PSPACE$. Indicate the lowest complexity class to which q belongs. You can just indicate the lowest complexity class, no need to prove that it's not lower than that (but you are welcome to do so).

   (b) Consider the vocabulary $(<, P_a, P_b, P_c)$ of strings over the alphabet $\Sigma = \{a, b, c\}$.

   i. Write each of the regular expressions below in FO or in MSO. Use succ, $\leq$, $\min, \max$ when needed, since these are expressible using $<$.

$$E_1 = (a|b)^*.c^* \qquad E_2 = (a.b)^* \qquad E_3 = (a.a.a)^*$$

ii. Write a regular expression describing the following language:

$$\forall S(\exists x(S(x) \wedge P_a(x))) \rightarrow (\exists y(S(y) \wedge P_b(y))$$

References

[1] D. Suciu. Applications of information inequalities to database theory problems. In *LICS*, pages 1–30, 2023.