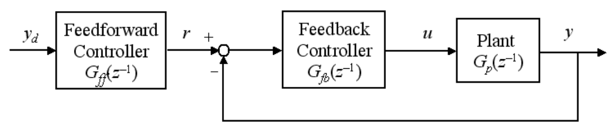
1 Midterm review

2 ZPET summary



step 1: obtain the closed-loop complementary sensitivity function, and separate the uncancellable part of the numerator (zeros on or outside the unit circle) $B^-(z^{-1})$

$$G_{closed}(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}\overbrace{B^+(z^{-1})}^{cancellable}\overbrace{B^-(z^{-1})}^{uncancellable}}{A(z^{-1})} \tag{1}$$

step 2: cancel the poles and the cancellable part of the zeros. Add a mirror block $B^-(z)$. This will make $B^-(z^{-1})B^-(z)$ zero-phase in the overall transfer function from y_d to y.

$$G_{ZPET}(z^{-1}) = \frac{A(z^{-1})}{z^{-d}B^+(z^{-1})}\frac{B^-(z)}{B^-(1)^2} \tag{2}$$

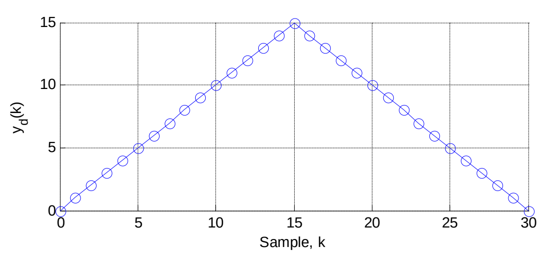
step 3: make the ZPET controller realizable by adding delays to it. As we add delays to the controller, the reference has to be shifted several steps forward, using the picture below

$$y_d(k+d+s) \rightarrow \boxed{\frac{z^{-s}A(z^{-1})}{B^+(z^{-1})}\frac{B^-(z)}{B^-(1)^2}} \rightarrow \boxed{G_{closed}(z^{-1})} \rightarrow y(k) \tag{3}$$

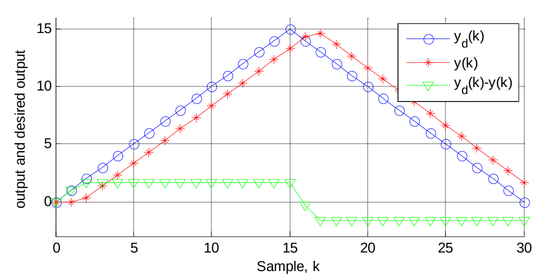
Example: suppose we have a plant-controller pair that gives us the following closed-loop transfer function:

$$G_{closed}(z^{-1}) = \frac{z^{-1}(1+2z^{-1})}{3}$$

Consider that case where we want the output to track a reference signal shown in the following figure



Case 1 (without feedforward control): If we directly add the reference to G_{closed}, the output $y(k)$ is shown in the following figure. Although $y(k)$ shows the same pattern as $y_d(k)$, the tracking error is large.

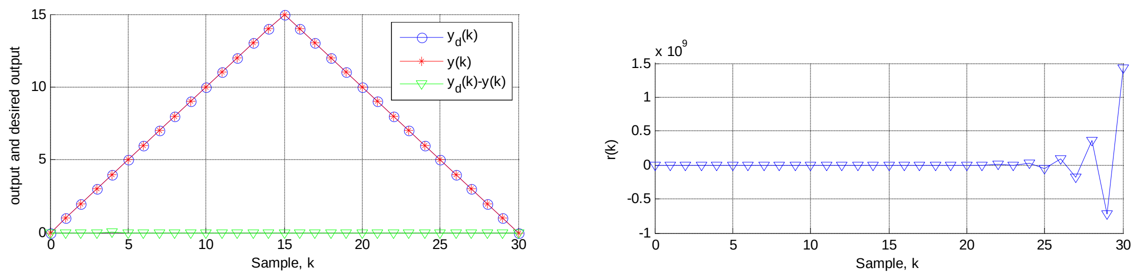


Case 2 (unrealistic perfect tracking): In order to achieve good tracking performance, we should make the closed loop transfer function from $y_d(k)$ to $y(k)$ as close to 1 as possible. The perfect tracking performance can be achieved, if we choose

$$G_{ff}(z^{-1}) = \frac{3}{z^{-1}(1+2z^{-1})}$$

which cancels all the zeros and poles of $G_{closed}(z^{-1})$. This feedforward controller is not causal, but we can implement it, if we know $y_d(k+1)$ at time k. The causal part of $G_{ff}(z^{-1})$, $\frac{3}{1+2z^{-1}}$, is used as the feedforward controller in Fig. 1 and $y_d(k+1)$ is used as the input to the system at time k.

The performance of this design is shown in Fig. 4. It looks good that there is no tracking error. However, since $G_{ff}(z^{-1})$ is unstable, the signal, $r(k)$, blows up, which makes this design unrealistic.



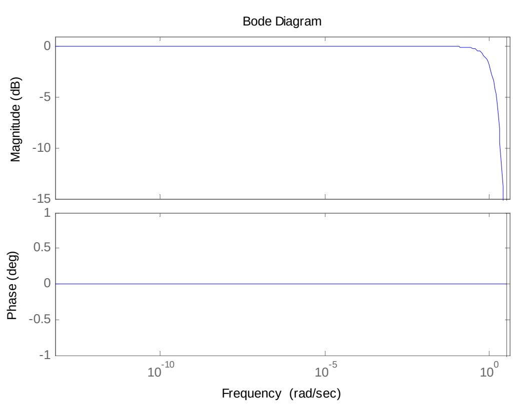
Case 3 (ZPET control): It is clear from the result of case 2 that we should avoid unstable pole-zero cancellation, when we design $G_{ff}(z^{-1})$. So we should make $G_{closed}(z^{-1})G_{ff}(z^{-1})$ close to 1 without canceling the unstable zero of $G_{closed}(z^{-1})$. This can be done by using the zero phase error tracking controller given by

$$G_{ff}(z^{-1}) = \frac{3(1+2z)}{9z^{-1}} = \frac{3(2+z^{-1})}{9z^{-2}}$$

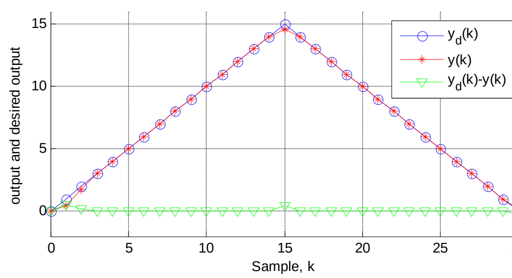
Then the transfer function from $y_d(k)$ to $y(k)$ becomes

$$G_{ff}(z^{-1})G_{closed}(z^{-1}) = \frac{(1+2z)(1+2z^{-1})}{9} = \frac{2z+5+2z^{-1}}{9}$$

which has the frequency response shown below



We can see that the magnitude is 1 at low frequencies and the phase is always zero. The tracking result is much better than the result in case 1 and every signal in the closed loop system is stable.



3 Preview Control

see the notes posted on bSpace.