1 Big picture of what we have learned

1.1 Methods related to quadratic cost functions

- dynamic programming:
  - central concept: big, difficult problem to small, easy problems
  - principle of optimality: the concept of the optimal cost to go
  - e.g. in discrete-time LQ:

$$J = x^T(N)\,Sx(N) + \sum_{i=0}^{N-1} x^T(i)\,Qx(i) + u(i)^T\,Ru(i)$$
$$\longrightarrow J_k(x(k)) = x^T(N)\,Sx(N) + \sum_{i=k}^{N-1} x^T(i)\,Qx(i) + u(i)^T\,Ru(i)$$

    which is solved backwards in time (starting from $k = N$).

- discrete-time LQ:
  - aim: $\min\left\{x^T(N)\,Sx(N) + \sum_{i=0}^{N-1} x^T(i)\,Qx(i) + u(i)^T\,Ru(i)\right\}$
  - meanings of the name:
    * Q: quadratic cost

$$\min\left\{x^T(N)\,Sx(N) + \sum_{i=0}^{N-1} x^T(i)\,Qx(i) + u(i)^T\,Ru(i)\right\} \tag{1}$$

    * L: linear system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{2}$$

  - solution concept: state feedback $u(k) = -K(k)\,x(k)$
  - stationary case: conditions and solutions

- probability and statistics:
  - why we need them
  - concepts of random variable, random process, Gaussian, white, expetation, variance, auto covariance, etc
  - joint distribution and conditional probability, such as $E(x|y)$, $X_{x|y,x|y}$ (and the special cases for Gaussian distributions)
  - spectral density and filtering a random process

- least squares:
  - aim: $\min E\left(||x-\hat{x}||_2^2|y\right)$
  - solution: $\hat{x} = E(x|y)$
  - the three properties and the pictures
  - special cases for Gaussian distributions: $E(x|y) = ...$; $X_{\tilde{x}|y,\tilde{x}|y} = ...$

- Kalman filter:
  - aim: state estimation (in the sense of least squares) for

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_w w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned} \tag{3}$$

  - conditions: whiteness, Gaussian, ...
  - solution equations and duality with LQ

- LQG:

  - aim: Q–quadratic cost

$$\min E\left\{x^T(N)\,Sx(N)+\sum_{i=0}^{N-1}x^T(i)\,Qx(i)+u(i)^T\,Ru(i)\right\}$$

    L–(3); G–Gaussian and white assumptions

  - solution concept: $u(k)=-K(k)\,\hat{x}(k)$ where $K(k)$ is from LQ solution and $\hat{x}(k)$ is from KF
  - separation theorem and steady-state conditions

- FSLQ:

  - aim: having more degrees of freedom in R and Q in LQ problems
  - tool: Parseval's theorem to connect the time domain and the frequency domain cost function
  - realization: filtering the states and the inputs
  - solution: enlarged system with filters $Q_f(s)$ and/or $R_f(s)$; meanings of $Q_f(s)$ and $R_f(s)$

- Preview control:

  - aim:

$$\min\left\{(y(N)-y_d(N))^T\,S_y\,(y(N)-y_d(N))+\sum_{j=0}^{N-1}(y(j)-y_d(j))^T\,Q_y\,(y(j)-y_d(j))+u(j)^T\,Ru(j)\right\}$$

    or

$$\min E\left\{(y(N)-y_d(N))^T\,S_y\,(y(N)-y_d(N))+\sum_{j=0}^{N-1}(y(j)-y_d(j))^T\,Q_y\,(y(j)-y_d(j))+u(j)^T\,Ru(j)\right\}$$

    with the system given by

$$\begin{aligned}x(k+1)&=Ax(k)+Bu(k)\\ y(k)&=Cx(k)\end{aligned}$$

  - origin: the optimal solution in the standard formulation requires information of the full trajectory
  - conditions: y_d available in a preview window and outside the preview window we have

$$\begin{aligned}x_d(k+1)&=A_dx_d(k)+B_dw_d(k)\\ y_d(k)&=C_dx_d(k)\end{aligned}$$

  - conversion to a solvable LQ problem

1.2 General transfer-function based control design

- basic concepts for feedback design:

  - closed-loop transfer functions: the error rejection functions and the reference following functions
  - basic loop shape
  - stability and robust stability

- LQG/LTR:

  - the target loop shape from a ficticious KF with a tunable noise ratio
  - the loop transfer recovery to approximate the target loop in a frequency range using a LQ problem with a tunable $\rho u^T(t)\,u(t)$
  - conditions: no nonminimum-phase transition zeros

- feedforward design:
  - ZPET
    * aim: make the transfer function from the reference to the output to approximate 1
    * solution: cancelable the cancelable zeros
  - (preview control to build in the information in the preview window and design both feedback and feedforward)

- pole placement:
  - the control structure
  - the result

1.3 Disturbance rejection

- FSLQ: selection of Q_f
- LQG/LTR: prefilter (e.g. integrators) in the plant
- DOB:
  - the concept of cancelling the disturbances by inverse filtering
  - the derivation of closed-loop transfer functions
  - the selection of Q filters
- Internal model principle:
  - models for different signals $A\left(q^{-1}\right)d(k)=0$
  - integration of $A(z^{-1})$ into the controller
- Repetitive control:
  - a special case of design based on internal model principle, with $\left(1-q^{-N}\right)d(k)=0$
  - the parameterization of the controller and the stability analysis
- adaptive disturbance rejection:
  - $y(k)=\frac{B(z^{-1})}{A(z^{-1})}(u(k)+d(k))$ with $d(k)=d$, $d(k)=\alpha\sin(\omega k)+\beta\cos(\omega k)$
  - see the last HW for details

1.4 System ID and adaptive control

- Big picture of system ID:

$$
\begin{aligned}
y(k+1)&=\theta^T\phi(k)\\
\hat{y}^o(k+1)&=\hat{\theta}^T(k)\,\phi(k)\\
\hat{y}(k+1)&=\hat{\theta}^T(k+1)\,\phi(k)\\
e^o(k+1)&=y(k+1)-\hat{y}^o(k+1)=-\tilde{\theta}^T(k)\,\phi(k)\\
e(k+1)&=y(k+1)-\hat{y}(k+1)=-\tilde{\theta}^T(k+1)\,\phi(k)
\end{aligned}
$$

and

$$
\hat{\theta}(k+1)=\hat{\theta}(k)+\frac{F(k)\,\psi(k)\,\nu^o(k+1)}{1+\psi^T(k)\,F(k)\,\psi(k)}
$$

with different choices of $\psi(k)$ and $\nu^o(k+1)$ (see the PAA table posted on the course website)

- stability proof for different PAAs:
  - hyperstability and Popov inequality
  - the LTI block and the nonlinear block
- adaptive control:
  - why?: unknown system or time-varying system
  - how: direct and indirect adaptive control
  - design steps:
    * deterministic controller design (assume unknown plant): e.g. pole placement ($D\left(z^{-1}\right) = ...$) and minimum variance control $\min E\left\{\left(y\left(k\right) - y_d\left(k\right)\right)^2 + ru\left(k\right)^2\right\}$
    * indirect adaptive control: identify the plant parameters and then compute the controller coefficients
    * direct adaptive control: directly update the controller parameters