UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2012

Final Examination

Your Name:

Closed book and closed notes.
Eight double-sided sheets (i.e. 16 pages) of handwritten notes on 8.5" $\times$11" paper are allowed.
Please answer all questions.

Problem:	1	2	3	4	5	6	Total
Max. Grade:	30	30	30	30	40	40	200
Grade:							

Problem 1

Consider the discrete-time linear time-invariant system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k), \qquad x(0) = x_0$$

$$y(k) = Cx(k)$$

and the optimal linear quadratic regulator problem

$$J = \min_{u(0),u(1),\dots} \left\{ \sum_{k=0}^{\infty} \left[x^T(k) C^T C x(k) + R u^2(k) \right] \right\}$$

where $R > 0$. For this problem, both $u(k)$ and $y(k)$ are scalar. The control design objective for this problem is to find a controller that minimizes J (for some choice of C and R) and achieves $|p_i| < 0.5$ for $i = 1, 2$ where p_1 and p_2 are the closed-loop eigenvalues of the system.

1. Show that if C is chosen as

$$C = \begin{bmatrix} -1.25 & 1 \end{bmatrix}$$

it is not possible to choose R to meet the control design objective.

2. Choose a value of C such that it is possible to meet the control design objective for some value of R. You do not need to explicitly find a corresponding value of R that meets the control design objective.

Problem 2

Consider the discrete-time linear time-invariant system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_w w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

where $w(k)$ and $v(k)$ are independent zero-mean white Gaussian sequences, $u(k)$ is the control sequence to be designed, and $y(k)$ is the measurement sequence available to the controller. The sequences $u(k)$ and $y(k)$ are both scalar. Also, (A, B) is stabilizable and (C, A) is detectable.

In this problem, we investigate two control design problems. The first control design problem is a standard optimal linear quadratic Gaussian control problem

$$J(\rho) = \min_{K \in \mathcal{K}} E\{x^T(k)Qx(k) + \rho u^2(k)\} \tag{1}$$

where $\mathcal{K}$ is the set of causal output feedback controllers for the system. The second control design problem is an optimal linear quadratic Gaussian control problem with a variance constraint on the control sequence

$$\bar{J} = \min_{K \in \mathcal{K}} E\{x^T(k)Qx(k)\} \qquad \text{subject to} \qquad E\{u^2(k)\} \leq \alpha \tag{2}$$

The control design parameters satisfy $Q \succeq 0$ and $\rho > 0$.

1. Suppose that, for a particular value of $\rho > 0$, we have determined the value of $J(\rho)$. Prove that for any causal output feedback controller that achieves

$$E\{u^2(k)\} \leq \alpha$$

it must hold that

$$E\{x^T(k)Qx(k)\} \geq J(\rho) - \alpha\rho$$

2. Suppose that, for $\rho = \rho_* > 0$, the controller that solves (1) satisfies

$$E\{u^2(k)\} = \alpha$$

Show that this controller solves (2) and $\bar{J} = J(\rho_*) - \alpha\rho_*$.

Problem 3

Consider the system

$$A(q^{-1})y(k) = q^{-\mathrm{d}}B(q^{-1})u(k)$$

where $A(q^{-1})$ is an anti-Schur polynomial of q^{-1} and $B(q^{-1})$ is a Schur polynomial of q^{-1}. We would like the system output $y(k)$ to track a desired trajectory $y_d(k)$, which is known at least $\mathrm{d} + m$ steps in advance, where m is the order of $B(q^{-1})$.

1. Design a feedforward controller of the form

$$u(k) = T(q, q^{-1})y_d(k)$$

   such that $y(k)$ follows $y_d(k)$ with zero phase error when $y_d(k)$ is a discrete-time sinusoid.

2. Suppose now that $y_d(k)$ has the form

$$y_d(k) = \sum_{i=1}^{r} c_i \sin(\omega_i k + \phi_i)$$

   where $0 \leq \omega_1 < \ldots < \omega_r < \pi$. Also suppose that we now use the feedforward control scheme

$$u(k) = T(q, q^{-1})u_d(k)$$

   where $T(q, q^{-1})$ is the feedforward controller designed in the previous part.

   Choose $u_d(k)$ so that $y(k)$ perfectly tracks $y_d(k)$.

Problem 4

Consider the system

$$\begin{aligned} (1-0.8q^{-1})y(k) &= q^{-2}u(k) + q^{-1}d(k) \\ e(k) &= r(k) - y(k) \end{aligned}$$

where

- $u(k)$ is the controlling input
- $y(k)$ is the measured output
- $d(k)$ is the periodic disturbance, which satisfies $d(k) = d(k-5)$
- $r(k)$ is the periodic reference, which satisfies $r(k) = r(k-7)$
- $e(k)$ is the tracking error

Design a repetitive controller that will make the tracking error asymptotically converge to zero.

Problem 5

Consider the state-space system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k)$$
$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + v(k)$$

where $y(k)$ is the output and $u(k)$ is the control input. The signals $w(k)$ and $v(k)$ are jointly Gaussian WSS random sequences that satisfy the usual assumptions:

$$\begin{aligned} E\{w(k)\} &= 0 & E\{v(k)\} &= 0 \\ E\{w(k+j)w(k)\} &= W\delta(j) & E\{v(k+j)v(k)\} &= V\delta(j) \\ E\{w(k+j)v(k)\} &= 0\ . \end{aligned}$$

In this problem, $W = 1$, $V = 2$.

1. Rewrite the system dynamics in the form

$$A(q^{-1})y(k) = q^{-\mathrm{d}}B(q^{-1})u(k) + C(q^{-1})\epsilon(k) \tag{3}$$

   where $C(q^{-1})$ is an anti-Schur polynomial of q^{-1} and $\epsilon(k)$ is a zero-mean white Gaussian random sequence.

2. Find the minimum variance regulator for (3). To receive full credit, you must explicitly find all of the controller parameters.

Problem 6

Consider the identification of the system

$$A(q^{-1})y(k) = q^{-2}B(q^{-1})u(k) + q^{-1}d(k)$$

where $u(k)$ is the control input, $y(k)$ is the system output, and $d(k)$ is the disturbance acting on the system, which satisfies

$$d(k+3) = d(k)$$

Although the plant polynomials

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} \qquad\qquad B(q^{-1}) = b_0 + b_1 q^{-1}$$

are unknown, their order is known and it is known that $b_0 \neq 0$.

1. Define θ so that the system dynamics can be expressed in the form

   $$y(k+1) = \phi^T(k)\theta$$

   where

   $$\phi(k) = \begin{bmatrix} -y(k) & -y(k-1) & u(k-1) & u(k-2) & f(k) & f(k-1) & f(k-2) \end{bmatrix}^T$$

   and $f(k)$ is the indicator function

   $$f(k) = \begin{cases} 1, & k \in \{\ldots, -3, 0, 3, 6, \ldots\} \\ 0, & \text{otherwise} \end{cases}$$

2. Write down an parameter adaptation algorithm that estimates θ using recursive least squares with a forgetting factor.

3. Write down a set of sufficient conditions for the a priori output estimation error $e^o(k)$ to converge to zero.