University of California, Berkeley
Department of Mechanical Engineering
ME 233 Advanced Control Systems II

Spring 2014 **Final (05/15/2014) 7-10pm**

Open book and open lecture notes.

#1	#2	#3	#4	#5	Total
12	16	22	30	20	100

Write down your name and student ID on all pages that you submit for grading.

1. [12 points] Consider a plant

$$y(k) = \frac{z^{-3}\left(0.5 + z^{-1}\right)}{\left(1 - z^{-1}\right)^2} u(k) + \frac{1 - 0.5z^{-1}}{\left(1 - z^{-1}\right)^2} n(k)$$

where $n(k)$ is a white and Gaussian random process with $\mathrm{E}[n(k)] = 0$, $\mathrm{E}\left[n(k)^2\right] = N$. Obtain the three-step-ahead best prediction $\hat{y}(k+3|k)$.

2. [16 points] Consider a state estimation problem for a servo motor. The angular position is measured by an encoder and the angular acceleration is measured by an accelerometer. In this problem we construct a Kalman Filter to estimate the angular velocity. This quantity is useful for various control designs. The kinematic model between the angular acceleration $a(t)$ and the angular position $\theta(t)$ is

$$\ddot{\theta}(t) = a(t) = u(t) + w(t)$$

where $a(t)$ is the actual acceleration, which equals the measurement from the accelerometer, $u(t)$, plus some measurement noise term $w(t)$. Let

$$y(t) = \theta(t) + v(t)$$

which is the measurement from the encoder, with $v(t)$ being the measurement noise. $w(t)$ and $v(t)$ are white Gaussian random processes with $\mathrm{E}\left[w(t)\, v(t+\tau)^T\right] = 0$

$$\mathrm{E}[w(t)] = 0,\ \mathrm{E}\left[w(t)\, w(t+\tau)^T\right] = 0.1\delta(\tau)$$
$$\mathrm{E}[v(t)] = 0,\ \mathrm{E}\left[v(t)\, v(t+\tau)^T\right] = 0.4\delta(\tau)$$

and the initial states of the system are zero mean, Gaussian, and independent from $w(t)$ and $v(t)$.

(a) [8 points] Obtain the full-order Kalman Filter equations that provide the optimally estimate of the augular velocity $\dot{\theta}(t)$. Obtain the numerical values of the steady-state Kalman Filter poles.

(b) [8 points] After you have designed the Kalman Filter in part (a), an engineer came to you with another set of encoder and accelerometer, with the same whiteness and Gaussian assumptions, but

$$\mathrm{E}\left[w\left(t\right)\right]=0,\ \mathrm{E}\left[w\left(t\right)w\left(t+\tau\right)^T\right]=1\delta\left(\tau\right)$$
$$\mathrm{E}\left[v\left(t\right)\right]=0,\ \mathrm{E}\left[v\left(t\right)v\left(t+\tau\right)^T\right]=4\delta\left(\tau\right)$$

i. Using the same Kalman Filter gain in part (a), will you still get the optimal estimation of the angular velocity? Explain your reasoning.

ii. Suppose you redesigned a Kalman Filter that is optimal with respect to the new noise properties. What are the steady-state Kalman Filter poles and the estimation error covariance?

3. [22 points] Consider a single-input single-output system whose transfer function is

$$P\left(s\right)=C\left(sI-A\right)^{-1}B=\frac{1}{s^2+bs+c},\ b>0,\ c>0$$

There is an input disturbance $w\left(t\right)$ which satisfies

$$w\left(t\right)=\alpha\cos\left(\omega_0 t+\phi\right)+d,\ d\neq 0,\ \alpha\neq 0,\ \omega_0\neq 0$$

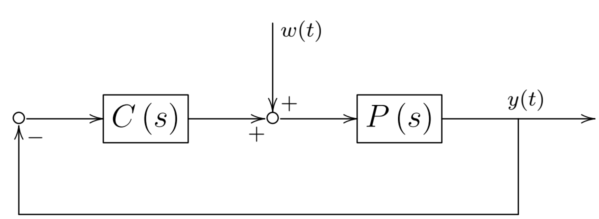
Hence we have

$$\dot{x}(t)=Ax(t)+B\left[u(t)+w\left(t\right)\right]$$
$$y\left(t\right)=Cx\left(t\right)$$

where $x\left(t\right)$ is the state of the plant, $A\in\mathbb{R}^{2\times 2}$, $B\in\mathbb{R}^{2\times 1}$, $C\in\mathbb{R}^{1\times 2}$. Additionally, assume we know the disturbance frequency ω_0 but do not know the values of α, ϕ, and d.

(a) [10 points] We can use frequency-shaped LQ algorithm for controller design. Focus on asymptotic disturbance rejection. Design the cost function and obtain the optimal control law with the associated Riccati equation.

(b) [7 points] We can also use internal model principle and design a controller $C\left(s\right)$ as shown in the block diagram below. Describe the key steps to use pole placement for designing a $C\left(s\right)$ that asymptotically rejects $w\left(t\right)$. Assume that the desired stable closed-loop characteristic polynomial is $D\left(s\right)$. What is/are the assumption(s) for the pole-placement design to have a solution?



Hint: this is a continuous-time design problem.

(c) [5 points] Obtain the transfer function from $x\left(t\right)$ to $u\left(t\right)$ in **part (a)**. And show how the internal model principle is satisfied in this controller.

4. [30 points] Consider a stable plant with input-output behavior

$$y(k+1) = \frac{2 + b_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}} [u(k) + d(k)]$$

The values of a_1, a_2, and b_1 are unknown but we know $|b_1| < 2$ and $|a_1| + |a_2| < 1$. The disturbance is periodic and satisfies $d(k) = d(k-2)$. This is the only known information about the disturbance.

(a) [14 points] We can use either direct or indirect adaptive control for the problem. Consider first the indirect adaptive control approach. You will need to identify the plant parameters. Use a non-series-parallel parameter adaptation algorithm (PAA) to identify the unknown plant parameters. Your PAA should be hyperstable, and unbiased in the presence of the disturbance $d(k)$.

(b) [16 points] Consider instead a direct adaptive control algorithm. Design a direct adaptive controller to achieve

$$\lim_{k \to \infty} \left\{1 + d_1 z^{-1} + d_2 z^{-2}\right\} [y_d(k) - y(k)] = 0$$

Hint: the periodic disturbance satisfies $d(k) = \frac{c_1 + c_2 z^{-1}}{1 - z^{-2}} \delta(k)$, namely, in a state-space representation,

$$x_d(k+1) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x_d(k), \; x_d(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$d(k) = [c_1, c_2] \, x_d(k)$$

where c_1 and c_2 are unknown.

5. [20 points] For a sinusoidal signal $y(k)$ of frequency ω_0 rad/sec, we know that

$$\left(1 - 2\cos\omega_0 z^{-1} + z^{-2}\right) y(k) = 0, \;\; k \geq 2$$

(a) [4 points] If ω_0 is known, obtain a two-step-ahead predictor for the signal, namely, predict $y(k+2)$ using $y(i)$, $i \leq k$.

(b) [9 points] Assume ω_0 is unknown. Obtain an adaptive two-step-ahead predictor using recursive least squares. Write down the PAA.

(c) [7 points] How would you obtain a 50-step-ahead predictor for the signal? Provide the design steps and key equations. You do not need to solve for the numerical coefficients of the predictor.