UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2012

Homework #6

Assigned: Mar. 15 (Th)
Due: Mar. 22 (Th)

1. The model of a hard disk drive with dual-stage actuation is given by

$$x(k+1) = Ax(k) + Bu(k) + B_w w(k)$$
$$y(k) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(k) + v(k)$$

where $w(k)$ and $v(k)$ are jointly Gaussian WSS zero-mean random vector sequences that satisfy

$$E\left\{ \begin{bmatrix} w(k+j) \\ v(k+j) \end{bmatrix} \begin{bmatrix} w^T(k) & v^T(k) \end{bmatrix} \right\} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \delta(j) \; .$$

The relevant state-space matrices are included in the file `hw6p1_model.mat`, which is located in the "MATLAB code" folder in the Resources section of bSpace.

The signals u and y respectively represent the control action and the measurements available to the controller. Two other signals of interest are

$$p_1(k) = C_1 x(k)$$
$$p_2(k) = C_2 x(k)$$

The signal $p_1(k)$ represents the position error of the read/write head and the signal $p_2(k)$ represents the displacement of the secondary actuator. Our control design goal is to make the variance of $p_1(k)$ as small as possible while maintaining

$$3\sqrt{E\{p_2^2(k)\}} \leq 500, \qquad 3\sqrt{E\{u_1^2(k)\}} \leq 5, \qquad 3\sqrt{E\{u_2^2(k)\}} \leq 20 \; . \tag{1}$$

Find positive values of α_1, α_2, and α_3 so that the controller that optimizes the infinite-horizon LQG cost function

$$J = E\{p_1^2(k) + \alpha_1 p_2^2(k) + \alpha_2 u_1^2(k) + \alpha_3 u_2^2(k)\}$$

meets the constraints in (1) and makes the variance of $p_1^2(k)$ "small." (Try to achieve the performance $3\sqrt{E\{p_1^2(k)\}} \leq 21.9$.)

Hints and comments:

- This controller design will require a trial-and-error approach. For each chosen α_1, α_2, and α_3, you will need to do the following in MATLAB:

  (a) Design an LQR using the `lqr` function.

(b) Design a Kalman filter using the `kalman` function.

(c) Connect the LQR and the Kalman filter together using the `lqgreg` function with the `'current'` option.

(d) Form the closed-loop system `syscl` from $\begin{bmatrix} w \\ v \end{bmatrix}$ to $\begin{bmatrix} p_1 \\ p_2 \\ u \end{bmatrix}$.

(e) For $i = 1, 2, 3, 4$, use the command the `norm(syscl(i,:))`. This computes $\sqrt{E\{p_1^2(k)\}}$, $\sqrt{E\{p_2^2(k)\}}$, $\sqrt{E\{u_1^2(k)\}}$, and $\sqrt{E\{u_2^2(k)\}}$ for the closed-loop system. Note that this will require 4 separate calls to the `norm` command.

Alternatively, steps (a)–(c) could be performed by directly solving the relevant discrete algebraic Riccati equations and forming the state space controller using the formulas given in lecture.

- Do not use the function `lqg` in MATLAB to find the optimal LQG controller for chosen values of α_1, α_2, and α_3; you will get the wrong answer if you do! The `lqg` function uses the a-priori state estimate in the control scheme rather than the a-posteriori state estimate, which results in suboptimal closed-loop performance.
- Note that the functions `lqr` and `kalman` take different systems as their input; be very careful when setting up the inputs into these functions.
- To verify that your algorithms are working correctly, verify that the closed-loop performance achieved when $\alpha_1 = \alpha_2 = \alpha_3 = 1$ is

$$3\sqrt{E\{p_1^2(k)\}} = 29.80 \qquad 3\sqrt{E\{p_2^2(k)\}} = 42.26,$$
$$3\sqrt{E\{u_1^2(k)\}} = 23.00, \qquad 3\sqrt{E\{u_2^2(k)\}} = 12.78\ .$$

2. Define the matrices

$$A_e := \begin{bmatrix} A & 0 & 0 \\ B_1 & A_1 & 0 \\ 0 & 0 & A_2 \end{bmatrix} \qquad B_e := \begin{bmatrix} B \\ 0 \\ B_2 \end{bmatrix}$$
$$C_e := \begin{bmatrix} D_1 & C_1 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \qquad D_e := \begin{bmatrix} 0 \\ D_2 \end{bmatrix}$$

For the frequency-shaped linear quadratic regulator (FSLQR) problem considered in the first half of Lecture 14, we derived the following set of conditions that guarantee the existence of the optimal FSLQR:

A1: (A_e, B_e) is stabilizable
A2: The state space realization $C_e(zI - A_e)^{-1}B_e + D_e$ has no transmission zeros on the unit circle.

It was subsequently stated (but not proved) that the following conditions imply that (A1)–(A2) hold:

B1: (A, B) is stabilizable
B2: A_1 and A_2 are Schur
B3: nullity $\begin{bmatrix} A_2 - \lambda I & B_2 \\ C_2 & D_2 \end{bmatrix} = 0$ whenever $|\lambda| = 1$
B4: nullity $\begin{bmatrix} A_1 - \lambda I & B_1 \\ C_1 & D_1 \end{bmatrix} = 0$ whenever λ is a unit circle eigenvalue of A

Prove that conditions (B1)–(B4) imply conditions (A1)–(A2).

Hint: You will find it useful to use the following characterizations:

- (A, B) is stabilizable if and only if nullity $\begin{bmatrix} A^T - \lambda I \\ B^T \end{bmatrix} = 0$ whenever $|\lambda| \geq 1$
- λ is a transmission zero of the realization $C_e(zI - A_e)^{-1}B_e + D_e$ if and only if nullity $\begin{bmatrix} A_e - \lambda I & B_e \\ C_e & D_e \end{bmatrix} > 0$

The second characterization is a result of $D_e^T D_e = D_2^T D_2 \succ 0$ holding for the FSLQR problem.