UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2011

Final Examination

Your Name:

Closed book and closed notes.
Eight double-sided sheets (i.e. 16 pages) of handwritten notes on 8.5” ×11” paper are allowed.
Please answer all questions.

Problem:	1	2	3	4	5	Total
Max. Grade:	20	20	15	20	25	100
Grade:						

Problem 1

Consider the state-space system

$$x(k+1) = Ax(k) + Bu(k) + B_w w(k) \tag{1}$$
$$y(k) = Cx(k) + v(k) \tag{2}$$

where the sequences $w(k)$ and $v(k)$ are jointly Gaussian WSS random sequences that satisfy the usual assumptions:

$$E\{w(k)\} = 0 \qquad E\{v(k)\} = 0$$
$$E\{w(k+j)w(k)\} = W\delta(j) \qquad E\{v(k+j)v(k)\} = V\delta(j)$$
$$E\{w(k+j)v(k)\} = 0 \ .$$

By the internal model principle, a reasonable way to reject constant disturbances is to impose the control structure

$$u(k+1) = u(k) + \bar{u}(k) \ . \tag{3}$$

where $\bar{u}(k)$ is the incremental control to be designed. We measure the performance of the closed-loop system using the cost function

$$J = E\{x^T(k)Qx(k) + u^T(k)Ru(k)\} \ .$$

1. Append the controller dynamics (3) to the system dynamics (1)–(2) and express the resulting system in the form

$$x_e(k+1) = A_e x_e(k) + B_e \bar{u}(k) + B_{we} w(k) \tag{4}$$
$$y(k) = C_e x_e(k) + v(k) \tag{5}$$

2. Show that, for the system (4)–(5), there <u>does not</u> exist a Kalman filter with asymptotically stable estimation error dynamics. Assume that the Kalman filter only has access to the measurements $y(k)$ and the incremental control $\bar{u}(k)$; do not treat $u(k)$ as measurable.

3. Suppose we now modify the control structure to instead be

$$u(k+1) = u(k) + \bar{u}(k) + \eta(k)$$

where $\eta(k)$ is a Gaussian WSS random sequence that is independent from $w(k)$ and $v(k)$ and satisfies

$$E\{\eta(k)\} = 0 \qquad\qquad E\{\eta^T(k+j)\eta(k)\} = \alpha I \delta(j)$$

where $\alpha \in \mathcal{R}$. It can be shown under some reasonable assumptions on Q, R, and the system (1)–(2) that we can construct LQG controllers that optimize J for the system (4)–(5) whenever $\alpha > 0$. (You do not need to find the corresponding conditions or prove existence of an optimal controller.) Thus, adding the noise $\eta(k)$ into the control law makes the optimal LQG control problem solvable. Is is possible, via choice of α, to design a high-performance controller (in terms of J) that rejects constant disturbances? Give a brief justification of your answer.

Problem 2

Consider the discrete-time system

$$(1 - 0.3q^{-1})y(k) = q^{-1}[(1 - 0.5q^{-1})u(k) + d_1(k) + d_2(k)]$$

where q^{-1} is the one-step delay operator, $u(k)$ is the controlling input, $y(k)$ is the measured output, and $d_1(k)$ and $d_2(k)$ are disturbances. The disturbance $d_1(k)$ is given by

$$d_1(k) = \bar{d}\sin(\omega k + \phi)$$

where ω is known, but $\bar{d}$ and ϕ are unknown[1]. It is also known that

$$d_2(k) = d_2(k + N), \quad \forall k$$

i.e. $d_2(k)$ is periodic, with period N. It is assumed here that N is large. In this problem, we will design a controller for this system in two steps.

Throughout this problem, clearly indicate all steps of your design process, which includes solving a Diophantine (Bezout) equation. You do not have to explicitly solve this equation. However, you should write down the linear equation $Mx = b$, that must be solved in order to obtain the solution of the Diophantine equation. This includes clearly defining the elements of the unknown vector x and all of the coefficients of the matrix M and the vector b.

1. Design a controller that achieves the following:
   - The set of closed-loop poles of the system only includes poles at the origin and any canceled zeros.
   - The disturbance $d_1(k)$ is rejected, but the disturbance $d_2(k)$ is <u>not</u> rejected.
2. Using the controller designed in the previous part as a minor loop, design an outer loop controller that achieves the following:
   - The disturbance $d_2(k)$ is rejected.
   - The system output tracks a desired output $y_d(k)$, which satisfies

     $$y_d(k) = y_d(k + N), \quad \forall k$$

     i.e. $y_d(k)$ is also periodic, with period N.

[1] If you do not have the appropriate annihilating polynomial in your notes, it can be derived from the trigonometric identity $\sin(\omega(k \pm 1) + \phi) = \cos(\omega)\sin(\omega k + \phi) \pm \sin(\omega)\cos(\omega k + \phi)$.

Problem 3

Consider the state-space system

$$\begin{aligned} x(k+1) &= x(k) + u(k) + w(k) \\ y(k) &= x(k) + v(k) \end{aligned}$$

where $y(k)$ is the output and $u(k)$ is the control input. The signals $w(k)$ and $v(k)$ are jointly Gaussian WSS random sequences that satisfy the usual assumptions:

$$\begin{aligned} E\{w(k)\} &= 0 & E\{v(k)\} &= 0 \\ E\{w(k+j)w(k)\} &= W\delta(j) & E\{v(k+j)v(k)\} &= V\delta(j) \\ E\{w(k+j)v(k)\} &= 0 \,. \end{aligned}$$

In this problem, $W = 1$, $V = 2$, and all signals are scalar.

Design a controller that minimizes the cost function

$$J = E\{y^2(k)\} \,.$$

To receive full credit, you must explicitly find all of the controller parameters.

Problem 4

Consider a system of the form

$$y(k+1) = \phi^T(k)\theta$$

where $\phi(k)$ is a regressor vector whose value is known at time step k and θ is a vector of unknown coefficients. To estimate the value of θ, the following parameter adaptation algorithm (PAA) is proposed:

$$\begin{aligned}
e(k+1) &= y(k+1) - \phi^T(k)\hat{\theta}(k+1) \\
\hat{\theta}(k+1) &= \hat{\theta}(k) + F(k)\phi(k)e(k+1) \\
F(k+1) &= \frac{1}{\lambda_1}\left[F(k) - \lambda_2 \frac{F(k)\phi(k)\phi^T(k)F(k)}{\lambda_1 + \lambda_2\phi^T(k)F(k)\phi(k)}\right]
\end{aligned}$$

where $0 \leq \dfrac{\lambda_2}{2} < \lambda_1 \leq 1$. Note in particular that this is not the PAA presented in Lecture 20 of Spring 2012.

1. Show that $e(k)$ converges to zero.

   Hint: If you treat $v(k) = \lambda_1 e(k)$ as the input to the PAA, you can avoid directly checking the Popov inequality for any relevant nonlinearities by instead relating the relevant nonlinearities to ones presented in lecture.

Problem 5

Consider the ARMAX system

$$A(q^{-1})y(k) = q^{-\mathrm{d}}B(q^{-1})u(k) + C(q^{-1})w(k)$$

where $y(k)$ is the output and $u(k)$ is the control input. The signal $w(k)$ is a WSS Gaussian random sequence that satisfies

$$E\{w(k)\} = 0 \qquad\qquad E\{w(k+j)w(k)\} = \delta(j)$$

Assume that the order of the polynomials in the model is known and d is known, i.e.

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} \\ C(q^{-1}) &= 1 + c_1 q^{-1} \\ d &= 2\ . \end{aligned}$$

Although, the values of a_1, b_0, b_1, and c_1 are not known, it is known that $b_0 \geq \bar{b}_0 > 0$. It is also known that the polynomial $B(q^{-1})$ is anti-Schur.

In this problem, we will consider the control scheme

$$\hat{R}(q^{-1}, k)u(k) = -\hat{S}(q^{-1}, k)y(k)$$

where

$$\begin{aligned} \hat{R}(q^{-1}, k) &= \hat{r}_0(k) + \hat{r}_1(k)q^{-1} + \hat{r}_2(k)q^{-2} \\ \hat{S}(q^{-1}, k) &= \hat{s}_0(k)\ . \end{aligned}$$

Define the following:

- Unknown parameter vector $\quad \theta = \begin{bmatrix} r_0 & r_1 & r_2 & s_0 & c_1 \end{bmatrix}^T$
- Parameter estimate vector $\quad \hat{\theta}(k) = \begin{bmatrix} \hat{r}_0(k) & \hat{r}_1(k) & \hat{r}_2(k) & \hat{s}_0(k) & \hat{c}_1(k) \end{bmatrix}^T$
- Parameter error vector $\quad \tilde{\theta}(k) = \theta - \hat{\theta}(k)$

1. Show that, using the solution of a Diophantine equation to define the polynomials $\bar{R}(q^{-1})$ and $S(q^{-1})$, the plant dynamics can be parameterized by

$$C(q^{-1})y(k) = R(q^{-1})u(k-d) + S(q^{-1})y(k-d) + C(q^{-1})v(k) \tag{6}$$

   where $R(q^{-1}) = \bar{R}(q^{-1})B(q^{-1})$ and $v(k) = \bar{R}(q^{-1})w(k)$. Also write down the corresponding Diophantine equation.

2. Let the polynomials $R(q^{-1})$ and $S(q^{-1})$ respectively have the form

$$\begin{aligned} R(q^{-1}) &= r_0 + r_1 q^{-1} + r_2 q^{-2} \\ S(q^{-1}) &= s_0\ . \end{aligned}$$

   Choose the vector $\phi(k)$ such that Eq. (6) can be expressed as

$$y(k+1) = \phi^T(k)\theta + C(q^{-1})v(k+1)\ .$$

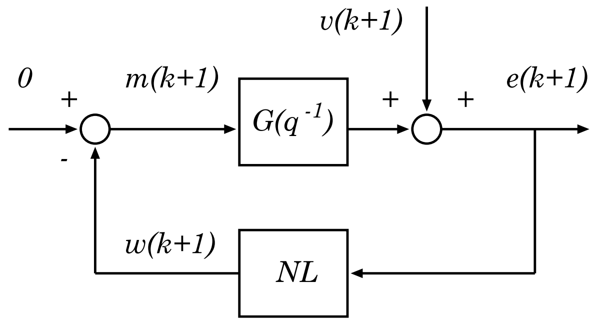

Figure 1: PAA Equivalent Feedback Loop

3. The following PAA is proposed[2]:

$$\begin{aligned}
\hat{y}(k+1) &= \hat{\phi}^T(k)\hat{\theta}(k+1) \\
e(k+1) &= y(k+1) - \hat{y}(k+1) \\
\hat{\theta}(k+1) &= \hat{\theta}(k) + \frac{1}{\lambda_1(k)} F(k)\hat{\phi}(k)e(k+1) \\
F(k+1) &= \frac{1}{\lambda_1(k)}\left[F(k) - \lambda_2(k)\frac{F(k)\hat{\phi}(k)\hat{\phi}^T(k)F(k)}{\lambda_1(k) + \lambda_2(k)\hat{\phi}^T(k)F(k)\hat{\phi}(k)}\right] .
\end{aligned}$$

where

$$\hat{\phi}(k) = \begin{bmatrix} u(k-1) & u(k-2) & u(k-3) & y(k-1) & -\hat{y}(k) \end{bmatrix}^T$$

$0 < \lambda_1(k) \leq 1$ and $0 \leq \lambda_2(k) < 2$.

Show that the PAA dynamics can be described by the equivalent block diagram in Fig. 1 where

$$w(k+1) = -\hat{\phi}^T(k)\tilde{\theta}(k+1) \ .$$

Also determine an expression for the LTI block $G(q^{-1})$ in Fig. 1.

4. Assume that $v(k) = 0$. What are sufficient conditions for $e(k)$ to converge to zero?

[2]The PAA should also include several projection algorithms, as discussed in class. However, for the sake of simplicity, we will not include them in this problem.