UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2011

Midterm Examination II

Your Name:

Closed book and closed notes.
Four double-sided sheets (i.e. 8 pages) of handwritten notes on 8.5" $\times$ 11" paper are allowed.
Please answer all questions.

Problem:	1	2	3	Total
Max. Grade:	40	40	20	100
Grade:				

Problem 1

Consider the state-space system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_w w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

where

$$A = \begin{bmatrix} \alpha & 1 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad B_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and $\alpha = \sqrt{\frac{3}{4}}$. The sequences $w(k)$ and $v(k)$ are jointly Gaussian WSS random sequences that satisfy the usual assumptions:

$$\begin{aligned} E\{w(k)\} &= 0 & E\{v(k)\} &= 0 \\ E\{w(k+j)w(k)\} &= W\delta(j) & E\{v(k+j)v(k)\} &= V\delta(j) \\ E\{w(k+j)v(k)\} &= 0\ . \end{aligned}$$

1. Draw the root locus for the stationary Kalman filter closed-loop poles and their inverses for $\frac{W}{V} \in (0, \infty)$. What do the closed-loop poles converge to as $\frac{W}{V} \to 0$?
2. Find the Kalman filter for this system when $W = 1$ and $V = 4$. (Remember to include the a-posteriori state estimate.)
3. Determine whether or not there exists a unique asymptotically stabilizing output feedback controller that optimizes the cost function

$$J = E\{x^T(k)C^T C x(k) + \rho u^2(k)\}$$

where $\rho > 0$.

Problem 2

Consider the discrete-time system

$$(1 - 0.3q^{-1})y(k) = q^{-1}[(1 - 2q^{-1})u(k) + d(k)]$$

where q^{-1} is the one-step delay operator, $u(k)$ is the controlling input, $y(k)$ is the measured output, and $d(k)$ is a disturbance given by

This material will not be tested in midterm 2 this year

$$d(k) = \bar{d}\sin(\omega k + \phi)$$

where ω is known, but $\bar{d}$ and ϕ are unknown[1].

1. Explain why repetitive control is not, in general, a viable control design technique for achieving perfect disturbance rejection for this system.

2. Design a controller that achieves the following:
   - The system output tracks an arbitrary desired output $y_d(k)$, which is known two steps in advance, with zero phase error.
   - The set of closed-loop poles of the system only includes poles at the origin and any canceled zeros.

   Clearly indicate all steps of your design process, which includes solving a Diophantine (Bezout) equation. You do not have to explicitly solve this equation. However, you should write down the linear equation $Mx = b$, that must be solved in order to obtain the solution of the Diophantine equation. This includes clearly defining the elements of the unknown vector x and all of the coefficients of the matrix M and the vector b.

3. Suppose the desired output has the form

   $$y_d(k) = \bar{y}_d \sin(\omega_y k + \phi_y)$$

   where $\omega_y \in [0, \pi]$. Ignoring any transient response, determine $y(k)$. (Your expression should be explicit; it should not depend on $y_d(k)$ or other values of $y(k)$.) Find a frequency ω_y for which perfect tracking is achieved.

[1] If you do not have the appropriate annihilating polynomial in your notes, it can be derived from the trigonometric identity $\sin(\omega(k \pm 1) + \phi) = \cos(\omega)\sin(\omega k + \phi) \pm \sin(\omega)\cos(\omega k + \phi)$.

Problem 3

Consider the discrete-time system described by

$$G(z) = \frac{z^{-1}B(z^{-1})}{A(z^{-1})} = G_n(z)(1 + \Delta(z)), \qquad G_n(z) = \frac{z^{-1}B_n(z^{-1})}{A_n(z^{-1})}$$

where $G_n(z)$ is a nominal model of the system. The poles and zeros of the nominal model are all inside the unit circle, i.e. $A_n(z^{-1})$ and $B_n(z^{-1})$ are anti-Schur polynomials in z^{-1}. The control structure in Fig. 1 has been proposed for robust disturbance rejection.

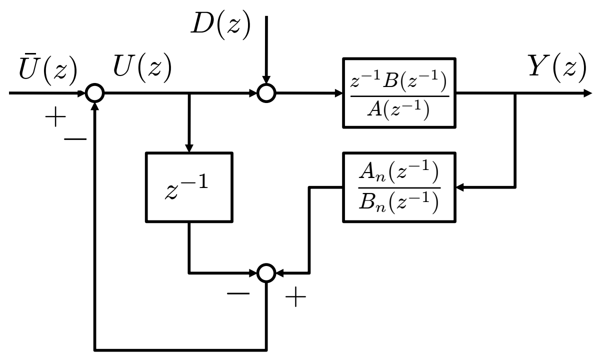

Figure 1: Control structure

1. Find the closed-loop transfer function from $D(z)$ to $Y(z)$ and the closed-loop transfer function from $\bar{U}(z)$ to $Y(z)$.
2. For $\Delta(z) = 0$ (i.e. the system is exactly given by the nominal model), explain why the proposed controller is effective in achieving disturbance rejection when $d(k) \approx d(k-1)$.