UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II, Spring 2010

Homework #11

Assigned: Saturday, May 1

Due: Friday, May 7. Please turn the solution to Sheila Caguiat in 5102 before 4:00 PM

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1. A discrete time plant is given by

$$A(q^{-1})y(k) = q^{-d}\,B(q^{-1})\,\{u(k) + d(k)\} \tag{1}$$

where $A(q^{-1}) = (1 - 0.8q^{-1})(1 - 0.7q^{-1})$, $d = 1$ and $B(q^{-1}) = 0.1$. $u(k)$ is the control input, $y(k)$ is the output, and $d(k)$ represent the disturbance. The disturbance is known to be periodic and the period is $N = 8$ (i.e. $d(k+8) = d(k)$).

(a) Use the fact that $A_d(q^{-1})\,d(k) = 0$, where $A_d(q^{-1}) = 1 - q^{-8}$ and design the repetitive regulator of the form

$$u(k) = -\frac{S(q^{-1})}{A_d(q^{-1})\,R'(q^{-1})}\,y(k)$$

which drives $y(k)$ to zero in finite time. (i.e. the closed loop polynomial should be $A_c(q^{-1}) = 1$. Find the polynomials $R'(q^{-1})$ and $S(q^{-1})$ as the solution of the diophantine equation

$$A_c(q^{-1}) = A_d(q^{-1})\,A(q^{-1})\,R'(q^{-1}) + q^{-d}\,B(q^{-1})\,S(q^{-1})\,.$$

(b) Simulate the repetitive control system designed above when the periodic disturbance is given by $d(0) = 0$, $d(1) = 1.5$, $d(2) = 3$, $d(3) = 0$, $d(4) = -2$, $d(5) = -2$, $d(6) = 0$, $d(7) = 0.5$ and $d(k+8) = d(k)$. Assume that the plant initial condition is zero.

(c) Simulate now the repetitive control system when the repetitive controller is given by

$$u(k) = -\frac{k_r q^{-(N-d)} A(q^{-1})}{A_d(q^{-1})\,B(q^{-1})}\,y(k)$$

under the same conditions as the previous problem. Try values of 0.5, 1 and 1.3 for k_r.

(d) Assume now that the plant is still described by Eq. (1), but the plant dynamics is now given by

$$\begin{aligned} A(q^{-1}) &= (1 - 0.2q^{-1})\,\bar{A}(q^{-1}) \\ \bar{A}(q^{-1}) &= (1 - 0.8q^{-1})(1 - 0.7q^{-1}) \\ B(q^{-1}) &= 0.08q^{-1} \qquad\qquad d = 1. \end{aligned}$$

Moreover, the repetitive controller is given by

$$u(k) = -\frac{k_r q^{-(N-d)} \bar{A}(q^{-1})}{0.1\,A_d(q^{-1})}\,y(k)\,.$$

In other words, the real plant is

$$G_A(s) = \frac{0.1q^{-1}}{(1 - 0.8q^{-1})(1 - 0.7q^{-1})}\,\frac{0.8q^{-1}}{(1 - 0.2q^{-1})}$$

but we use the simplified plant

$$G(s) = \frac{0.1q^{-1}}{(1-0.8q^{-1})(1-0.7q^{-1})}$$

in the control system design process.

Show, using the root locus technique, that the resulting repetitive control system is unstable for any positive k_r.

(e) Assume again that the plant is described by Eq. (1),with the plant dynamics given by

$$\begin{aligned} A(q^{-1}) &= (1-0.2q^{-1})\,\bar{A}(q^{-1}) \\ \bar{A}(q^{-1}) &= (1-0.8q^{-1})(1-0.7q^{-1}) \\ B(q^{-1}) &= 0.08q^{-1} \qquad\qquad d=1. \end{aligned}$$

However, we now incorporate the Q-filter modification to the repetitive compensator, in order to make the repetitive controller more robust. Thus,

$$u(k) = -\frac{k_r q^{-(N-d)}\,\bar{A}(q^{-1})}{0.1\,(1-Q(q,q^{-1})q^{-N})}\,y(k)\,.$$

where

$$Q(q,q^{-1}) = \frac{q+2+q^{-1}}{4}\,.$$

(i) Plot the root locus of the close loop poles of the resulting repetitive control system for $k_r \geq 0$ and determine, a value of k_r for which the resulting repetitive control system is asymptotically stable.

(ii) Simulate the repetitive control system designed above when the periodic disturbance is given by $d(0)=0,\ d(1)=1.5,\ d(2)=3,\ d(3)=0,\ d(4)=-2,\ d(5)=-2,\ d(6)=0,\ d(7)=0.5$ and $d(k+8)=d(k)$. Assume that the plant initial condition is zero.

2. Let N_1 and N_2 be two P-class nonlinearities:

$$\begin{aligned} y_1(t) &= N_1(u_1(t)) \\ y_2(t) &= N_2(u_2(t)) \end{aligned}$$

such that each satisfies the Popov inequality:

$$\int_0^t y_i^T(\tau)u_i(\tau)d\tau \geq -\gamma_i^2 \qquad i=1,2$$

Prove the following two lemmas:

Lemma 1: The parallel combination of N_1 and N_2,

$$y(t) = N_1(u(t)) + N_2(u(t))$$

also satisfies the Popov inequality

$$\int_0^t y^T(\tau)u(\tau)d\tau \geq -\gamma^2 \qquad i=1,2 \tag{2}$$

Lemma 2: The feedback combination of N_1 and N_2,

$$\begin{aligned} y(t) &= N_1(u(t) - y_1(t)) \\ y_1(t) &= N_2(y(t)) \end{aligned}$$

also satisfies the Popov inequality (2).

3. In this problem, you will conduct the stability analysis of a first order, continuous time, model reference adaptive controller (MRAC) using the sufficiency portions of the Hyperstability and Asymptotic Hyperstability theorems.

The system that we want to control is the first order LTI system

$$Y(s) = \frac{b}{s-a}\, U(s)\,, \tag{3}$$

where the constants a and $b > 0$ are unknown, except that we know that b is positive.

The control objective is to asymptotically track the first order model

$$Y_r(s) = \frac{b_r}{s-a_r}\, R(s)\,, \tag{4}$$

where $b_r > 0$, $a_r < 0$ are known constants and $r(t)$ is a known bounded signal, specified by the designer.

The proposed control law is

$$u(t) = \phi(t)^T \hat{\theta}(t) \tag{5}$$

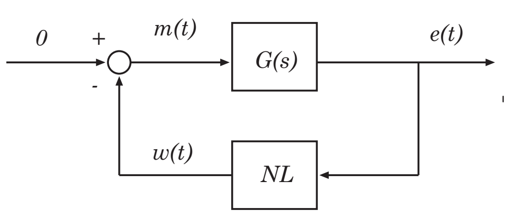
where

$$\phi(t) = \begin{bmatrix} y(t) & r(t) \end{bmatrix}^T \qquad \hat{\theta}(t) = \begin{bmatrix} \hat{\alpha}(t) & \hat{\beta}(t) \end{bmatrix}^T \tag{6}$$

and the parameter adaptation algorithm (PAA) is

$$\begin{aligned} \frac{d}{dt}\hat{\theta}(t) &= F\,\phi(t)e(t) \qquad F = F^T \succ 0 \\ e(t) &= y_r(t) - y(t) \end{aligned} \tag{7}$$

(a) Show that the adaptation algorithm error dynamics can be described by the equivalent block diagram in Fig. 1



$$G(s) = \frac{b}{s-a_r} \tag{8}$$

$$\begin{aligned} m(t) &= \tilde{\theta}^T(t)\,\phi(t) \\ \tilde{\theta}(t) &= \theta - \hat{\theta}(t) \\ \theta &= \begin{bmatrix} \alpha & \beta \end{bmatrix}^T \end{aligned} \tag{9}$$

Figure 1: PAA equivalent Feedback Block

and determine expressions for the parameters α and β in Eq. (9).

(b) Taking the time derivative of the PDF function

$$V(\tilde{\theta}(t)) = \frac{1}{2}\,\tilde{\theta}^T(t)F^{-1}\tilde{\theta}(t) \tag{10}$$

and noticing that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$, show that that the PAA (7) achieves

$$\frac{d}{dt}V(\tilde{\theta}(t)) = w(t)e(t)\,. \tag{11}$$

Utilizing (11), show that

$$\int_0^t w(\tau)e(\tau)d\tau \geq -\gamma_o^2 \qquad \forall\, t \geq 0\,.$$

Thus, the nonlinear block NL in Fig. 1 is a P-class nonlinearity. As a consequence, since $b > 0$ and $a_r < 0$, $G(s)$ in (8) is SPR and the feedback system in Fig. 1 is hyperstable.

By the sufficiency portion of the Hyperstability theorem, assuming that $|e(0)| < \infty$, it follows that $|e(t)| < \infty$.

(c) Show that $|y(t)| < \infty$ and therefore $\|\phi(t)\| < \infty$.

(d) We will now show that if $\|\tilde{\theta}(0)\| < \infty$, then $\|\tilde{\theta}(t)\|$ remains bounded. Since $G(s)$ in (8) is SPR, it follows that

$$\int_0^t m(\tau)e(\tau)d\tau \geq -\gamma_1^2 \qquad \forall\, t \geq 0\,.$$

Use this fact and integrate both sides of Eq. (11) from 0 to t to prove that

$$V(\tilde{\theta}(t)) < \infty$$

and as a consequence, that $\|\tilde{\theta}(t)\| < \infty$.

(e) Prove that $w(t)$ remains bounded.

Once $w(t)$ is shown to be bounded, since $G(s)$ is SPR and the nonlinear block NL is a P-class nonlinearity, we can use the sufficiency portion of the Asymptotic Hyperstability theorem to conclude that

$$\lim_{t\to\infty} e(t) = 0\,.$$