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ME 233 Advanced Control II

Lecture 19

Stability Analysis Using
The Hyperstability Theorem

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Adaptive Control

Basic Adaptive Control Principle

Controller parameters **are not constant,** rather, they are adjusted in an online fashion by a ***Parameter Adaptation Algorithm (PAA)***

When is adaptive control used?

- Plant parameters are unknown
- Plant parameters are time varying

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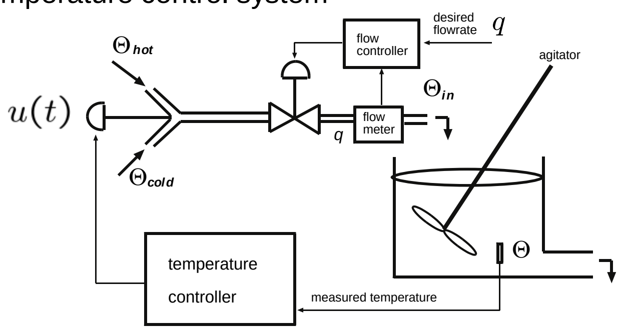
Example of a system with varying parameters

- Temperature control system



$$\frac{d}{dt}\theta(t) = -\underbrace{\frac{q}{V}}_{a(q)}\ \theta(t) + \underbrace{\frac{kq}{V}}_{b(q)}\ u(t-t_d)$$

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Adaptive Control Classification

- Continuous time VS **<u>discrete time</u>**
- Direct VS indirect
- MRAS VS **<u>STR</u>**

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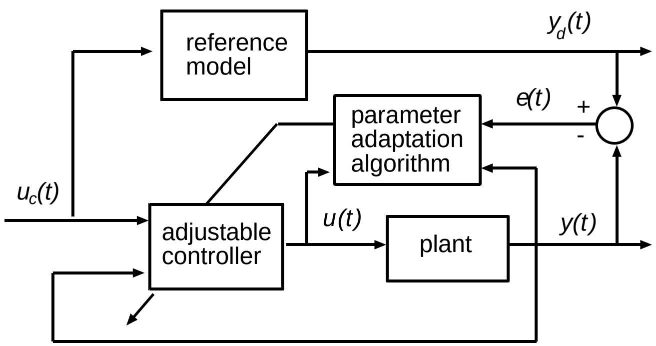
Model Reference Adaptive Systems (MRAS)



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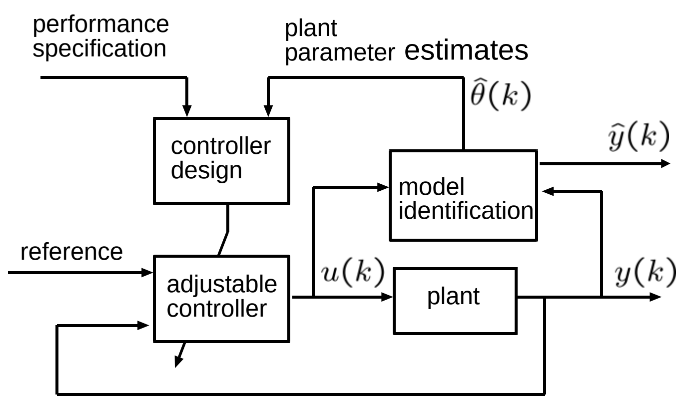
Self-Tuning Regulators (STR)



$$e(k) = y(k) - \widehat{y}(k)$$

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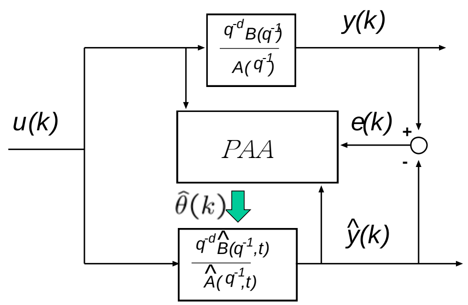
Identification of a LTI system



Parallel model

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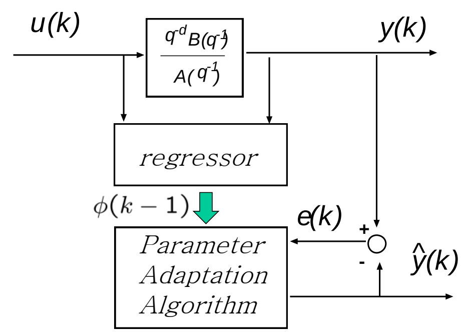
Identification of a LTI system



Series-parallel model

(We will use this model throughout this lecture)

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Plant ARMA Model

Plant model

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k)$$

where

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) &= b_o + b_1 q^{-1} + \cdots + b_m q^{-m} \end{aligned}$$

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Unknown plant parameters

Assume ARMA model parameters are **unknown**

$$y(k) = -\underline{a_1}\, y(k-1) \cdots - \underline{a_n}\, y(k-n) + \underline{b_o} u(k-d) \cdots + \underline{b_m}\, u(k-d-m)$$

Define:

$$\theta = \left[\ \underline{a_1 \ \cdots \ a_n} \ \ \underline{b_o \ \cdots \ b_m}\ \right]^T$$

As the *unknown* parameter vector

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Regressor vector

Collect all measurable signals in one vector

$$y(k) = -a_1\, \underline{y(k-1)} \cdots - a_n\, \underline{y(k-n)} + b_o \underline{u(k-d)} \cdots + b_m\, \underline{u(k-d-m)}$$

We define

$$\phi(k-1) = [\underline{-y(k-1) \ \cdots \ -y(k-n)} \ \ \underline{u(k-d) \ \cdots \ u(k-d-m)}]^T$$

as the ***known*** regressor vector

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Plant ARMA Model

Plant model

$$y(k) = \phi^T(k-1)\,\theta$$

where

$$\theta = \left[\ a_1 \ \cdots \ a_n \ \ b_o \ \cdots \ b_m\ \right]^T$$

$$\phi(k-1) = [-y(k-1) \ \cdots \ -y(k-n) \ \ u(k-d) \ \cdots \ u(k-d-m)]^T$$

Plant ARMA Model

Plant estimate (series-parallel)

$$\hat{y}(k) = \phi^T(k-1)\,\hat{\theta}(k)$$

where

$$\hat{\theta}(k) = \left[\begin{array}{cccccc} \hat{a}_1(k) & \cdots & \hat{a}_n(k) & \hat{b}_o(k) & \cdots & \hat{b}_m(k) \end{array}\right]^T$$

$$\phi(k-1) = [-y(k-1) \;\; \cdots \;\; -y(k-n) \;\; u(k-d) \;\; \cdots \;\; u(k-d-m)]^T$$

Plant output estimate

Plant a-posteriori estimate

$$\hat{y}(k) = \phi^T(k-1)\,\hat{\theta}(k)$$

Plant a-priori estimate

$$\hat{y}^o(k) = \phi^T(k-1)\,\hat{\theta}(k-1)$$

Plant a-posteriori error

$$y(k) = \phi^T(k-1)\,\theta$$

$$\hat{y}(k) = \phi^T(k-1)\,\hat{\theta}(k)$$

error: $e(k) = y(k) - \hat{y}(k)$

$$e(k) = \phi^T(k-1)\left[\theta - \hat{\theta}(k)\right]$$

$$= \phi^T(k-1)\tilde{\theta}(k)$$

$\tilde{\theta}(k) = \theta - \hat{\theta}(k)$

A Parameter Adaptation Algorithm

PAA $F = F^T \succ 0$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + F\,\phi(k-1)e(k)$$

Parameter error update law: $\tilde{\theta}(k) = \theta - \hat{\theta}(k)$

$$\tilde{\theta}(k) = \tilde{\theta}(k-1) - F\,\phi(k-1)e(k)$$

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Adaptation Dynamics

a-posteriori error: $e(k) = y(k) - \hat{y}(k)$

$$e(k) = \phi^T(k-1)\tilde{\theta}(k)$$

Parameter error update law: $\tilde{\theta}(k) = \theta - \hat{\theta}(k)$

$$\tilde{\theta}(k) = \tilde{\theta}(k-1) - F\,\phi(k-1)e(k)$$

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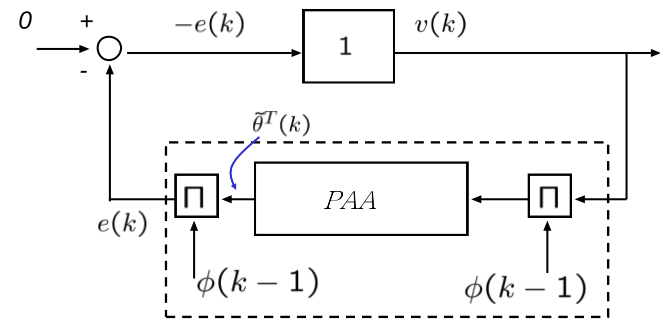
Adaptation Dynamics



PAA: $\tilde{\theta}(k) = \tilde{\theta}(k-1) + F\,\phi(k-1)v(k)$

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Convergence of Adaptive Systems

Adaptive systems are nonlinear

We need to prove that the algorithms converge:

- **Output error convergence** $\boxed{e(k) \to 0}$

$$e(k) = y(k) - \hat{y}(k)$$

- **Parameter error convergence** $\boxed{\tilde{\theta}(k) \to 0}$

$$\tilde{\theta}(k) = \theta - \hat{\theta}(k)$$

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Output error Convergence

Our first goal will be to prove the asymptotic convergence of the output error:

$$e(k) \to 0$$

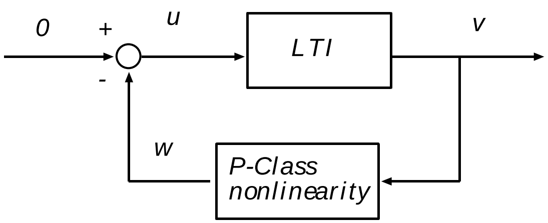
Two frequently used methods of stability analysis are**:**

- **Stability analysis using Lyapunov's direct method**
  - State space approach
- **Stability analysis using the Passivity or Hyperstability theorems**
  - Input/output approach

Hyperstability

Hyperstability Theory

- Developed by V.M. Popov to analyze the stability of a class of feedback systems (monograph published in 1973)

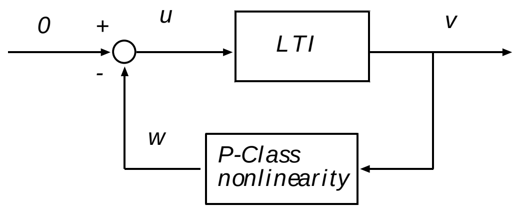


- Popularized by I.D. Landau for the analysis of adaptive systems (first book published in 1979)

Hyperstability Theory

Hyperstability Theory

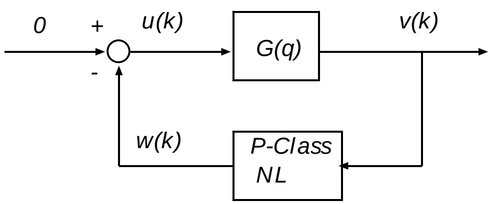
- Applies to both continuous time and discrete time systems



- **Abuse of notation:** We will denote the LTI block by its transfer function

DT Hyperstability Theory

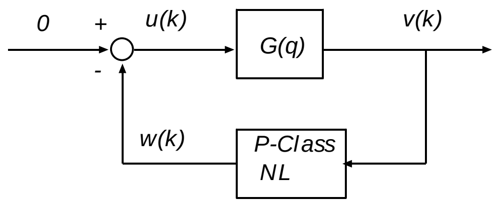
$$G(z) = C(zI - A)^{-1}B + D$$



- State space description of the LTI Block:

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) \\ v(k) &= C\,x(k) + D\,u(k) \end{aligned}$$

DT Hyperstability Theory



- P-class nonlinearity: (passive nonlinearities)

$$\boxed{\sum_{j=0}^{k} w^T(j)v(j) \geq -\gamma_o^2 \qquad \forall\, k \geq 0}$$

Where γ_o is a bounded constant.

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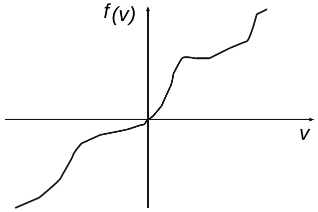
Example: Static nonlinearity:

$$v(k) \longrightarrow \boxed{\text{P-block}} \longrightarrow w(k)$$

$$w = f(v)$$

$$v\, f(v) \geq 0$$



$$\sum_{j=0}^{k} w^T(j)v(j) = \sum_{j=0}^{k} \underbrace{f(v(j))v(j)}_{\geq 0} \geq 0 > -\gamma_o^2$$

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Example: Dynamic P-class block

$$v(k) \longrightarrow \boxed{\text{P-block}} \longrightarrow w(k)$$

$$\begin{cases} \tilde{\theta}(k) = \tilde{\theta}(k-1) + F\phi(k-1)v(k) \\ w(k) = \phi^T(k-1)\tilde{\theta}(k) \end{cases}$$

$$\phi(k) \in \mathcal{R}^n$$

$$\tilde{\theta}(-1) \in \mathcal{R}^n$$

$$F = F^T \succ 0$$

$$\|\tilde{\theta}(-1)\| < \infty$$

$$\|\phi(k)\| < \infty$$

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Example: Dynamic P-class block

$$w(k) = \phi^T(k-1)\tilde{\theta}(k) \qquad \tilde{\theta}(k) = \tilde{\theta}(k-1) + F\phi(k-1)v(k)$$

$$\begin{aligned} \sum_{j=0}^{k} w(j)v(j) &= \sum_{j=0}^{k} \phi^T(j-1)\tilde{\theta}(j)v(j) \\ &= \sum_{j=0}^{k} \tilde{\theta}^T(j)\underbrace{[\phi(j-1)v(j)]}_{F^{-1}\left[\tilde{\theta}(j)-\tilde{\theta}(j-1)\right]} \\ &= \sum_{j=0}^{k} \tilde{\theta}^T(j)F^{-1}\left[\tilde{\theta}(j)-\tilde{\theta}(j-1)\right] \\ &= \sum_{j=0}^{k} \left\{\tilde{\theta}^T(j)F^{-1}\tilde{\theta}(j) - \tilde{\theta}^T(j)F^{-1}\tilde{\theta}(j-1)\right\} \end{aligned}$$

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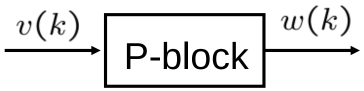
Example: Dynamic P-class block

$$\begin{aligned} \sum_{j=0}^{k} w(j)v(j) = &\sum_{j=0}^{k} \left\{\tilde{\theta}^T(j)F^{-1}\tilde{\theta}(j) - \tilde{\theta}^T(j)F^{-1}\tilde{\theta}(j-1)\right\} \\ &+ \frac{1}{2}\sum_{j=0}^{k} \tilde{\theta}^T(j-1)F^{-1}\tilde{\theta}(j-1) - \frac{1}{2}\sum_{j=0}^{k} \tilde{\theta}^T(j-1)F^{-1}\tilde{\theta}(j-1) \end{aligned}$$

$$\begin{aligned} \sum_{j=0}^{k} w(j)v(j) = &\frac{1}{2}\sum_{j=0}^{k} \left\{\tilde{\theta}^T(j)F^{-1}\tilde{\theta}(j) - \tilde{\theta}^T(j-1)F^{-1}\tilde{\theta}(j-1)\right\} \\ &+ \underbrace{\frac{1}{2}\sum_{j=0}^{k} \left[\tilde{\theta}(j)-\tilde{\theta}(j-1)\right]^T F^{-1}\left[\tilde{\theta}(j)-\tilde{\theta}(j-1)\right]}_{\geq 0} \end{aligned}$$

Example: Dynamic P-class block



$$\sum_{j=0}^{k} w(j)v(j) \geq \frac{1}{2}\sum_{j=0}^{k}\left\{\tilde{\theta}^T(j)F^{-1}\tilde{\theta}(j) - \tilde{\theta}^T(j-1)F^{-1}\tilde{\theta}(j-1)\right\}$$

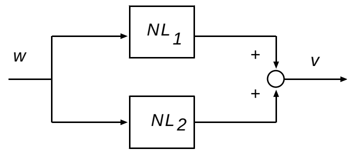
$$= \frac{1}{2}\tilde{\theta}^T(k)F^{-1}\tilde{\theta}(k) - \underbrace{\frac{1}{2}\tilde{\theta}^T(-1)F^{-1}\tilde{\theta}(-1)}_{\gamma_o^2}$$

$$\geq -\gamma_o^2$$

Examples of P-class NL

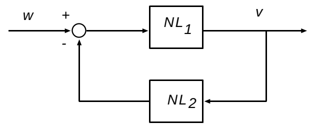
Lemma:

- The parallel combination of two P-class nonlinearities is also a P-class nonlinearity.



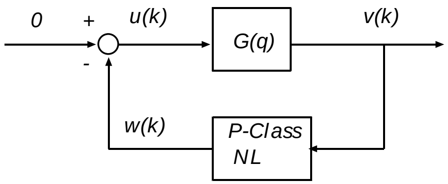
$$\sum_{j=0}^{k} w^T(j)v(j) \geq -\gamma_o^2$$

- The feedback combination of two P-class nonlinearities is also a P-class nonlinearity.



$$\sum_{j=0}^{k} w^T(j)v(j) \geq -\gamma_o^2$$

DT Hyperstability

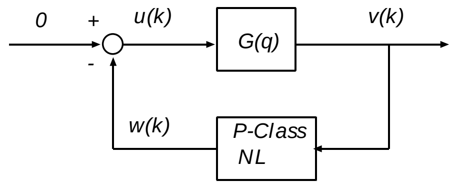


Hyperstability: The above feedback system is hyperstable if there exist positive bounded constants δ_1, δ_2 such that, for any state space realization of *G(q)*,

$$\boxed{\|x(k)\| < \delta_1\left[\|x(0)\| + \delta_2\right] \qquad \forall\, k \geq 0}$$

FOR ALL P-class nonlinearities

DT Asymptotic Hyperstability

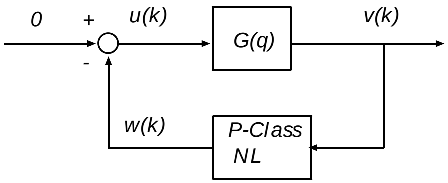


Asymptotic Hyperstability: The above feedback system is asymptotically hyperstable if

1. It is hyperstable
2. for any state space realization of *G(z)*,

$$\lim_{k\to\infty} x(k) = 0$$

DT Hyperstability Theorems



Hyperstability Theorem: The above feedback system is hyperstable **iff** the transfer function $G(z)$ of the LTI block is **Positive Real.**

Asymptotical Hyperstability Theorem: The above feedback system is asymptotically hyperstable **iff** the transfer function $G(z)$ of the LTI block is **Strictly Positive Real.**

Strictly Positive Real (SPR) TF

$$G(z) = C(zI - A)^{-1}B + D$$

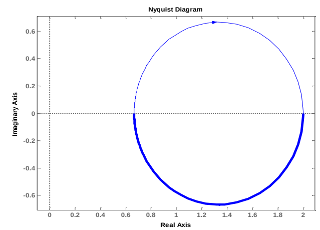
Is **Strictly Positive Real (SPR)** iff:

1. All poles of $G(z)$ are asymptotically stable.
2. $G(e^{j\omega}) + G^T(e^{-j\omega}) \succ 0$

   for all $0 \leq \omega \leq \pi$

   Example:

$$G(z) = \frac{z}{z + 0.5}$$



Strictly Positive Real (SPR) TF

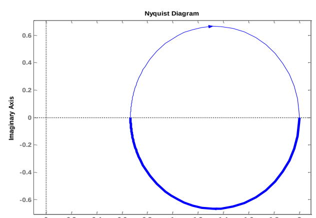
For scalar rational transfer functions

$$G(z) = \frac{B(z)}{A(z)}$$

1. All poles of $G(z)$ are asymptotically stable.
2. $\text{Re}\{G(e^{j\omega})\} > 0$ for all $\omega, \quad 0 \leq \omega \leq \pi$

Note:

A necessary (but not sufficient) condition for $G(z)$ to be SPR is that its relative degree must be 0.



Matrix Inequality Interpretation of SPR

The transfer function

$$G(z) = C(zI - A)^{-1}B + D$$

is **Strictly Positive Real (SPR)** if and only if

there exists $P \succ 0$ such that

$$\begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \prec 0$$

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SPR state-space realization fact

Theorem: If $G(z) = C(zI-A)^{-1}B + D$ is SPR, then

$$D + D^T \succ 0$$

Proof: Choose $P \succ 0$ such that

$$\begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \prec 0$$

Note that

$$B^TPB - D - D^T \prec 0$$

$$\Rightarrow D + D^T \succ B^TPB \succeq 0$$

■

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SPR TF is P-class

Let $G(z) = C(zI - A)^{-1}B + D$ be SPR

Then there exist positive definite functions

$V(x) \succ 0$ $\lambda_1(x, u) \succ 0$

Such that any input $u(k)$ output $y(k)$ pair satisfies

$$\sum_{j=0}^{k} y^T(j)u(j) = V(x(k+1)) - V(x(0)) + \sum_{j=0}^{k} \lambda_1(x(j), u(j))$$

$$\geq -\gamma_0^2 \qquad \gamma_o^2 = V(x(0))$$

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Shorthand notation

$$x(k) \to x_k$$
$$u(k) \to u_k$$
$$y(k) \to y_k$$
$$v(k) \to v_k$$
$$w(k) \to w_k$$

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Proof

Let $G(z) = C(zI - A)^{-1}B + D$ be SPR

Choose $P \succ 0$ such that

$$\begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \prec 0$$

Define the Lyapunov function

$$V(x_k) = \frac{1}{2} x_k^T P x_k \succ 0$$

and the function

$$\lambda_1(x, u) = -\frac{1}{2} \begin{bmatrix} x \\ u \end{bmatrix}^T \begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \succ 0$$

Proof

$$V(x_{k+1}) - V(x_k) = \frac{1}{2}(Ax_k + Bu_k)^T P (Ax_k + Bu_k) - \frac{1}{2} x_k^T P x_k$$

$$= \frac{1}{2}\begin{bmatrix} x_k \\ u_k \end{bmatrix}^T \begin{bmatrix} A^TPA - P & A^TPB \\ B^TPA & B^TPB \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} x_k \\ u_k \end{bmatrix}^T \begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix}$$

$$+ \frac{1}{2}\left[u_k^T C x_k + u_k^T D u_k + u_k^T D^T u_k + x_k^T C^T u_k\right]$$

$$= -\lambda_1(x_k, u_k) + (Cx_k + Du_k)^T u_k$$

$$= -\lambda_1(x_k, u_k) + y_k^T u_k$$

Proof

From the previous slide

$$V(x_{k+1}) - V(x_k) = -\lambda_1(x_k, u_k) + y_k^T u_k$$

$$\Rightarrow \quad y_k^T u_k = V(x_{k+1}) - V(x_k) + \lambda_1(x_k, u_k)$$

Summing both sides of the equation yields

$$\sum_{j=0}^{k} y_j^T u_j = V(x_{k+1}) - V(x_0) + \sum_{j=0}^{k} \lambda_1(x_j, u_j)$$

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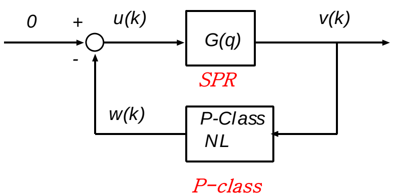
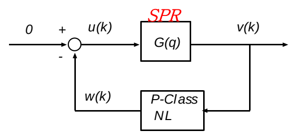
Proof of the sufficiency part of the Asymptotic Hyperstability Theorem - Discrete Time



- Since the nonlinearity is P-class, $\sum_{j=0}^{k} w_j^T v_j \geq -\gamma_1^2$
- Since LTI block is SPR, we can use the choose $P \succ 0$ such that

$$\begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \prec 0$$



Hyperstability

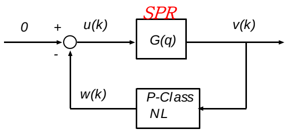
From the previous proof (SPR TF is P-class), we have

$$\sum_{j=0}^{k} v_j^T u_j = V(x_{k+1}) - V(x_0) + \sum_{j=0}^{k} \lambda_1(x_j, u_j)$$

where

$$V(x_k) = \frac{1}{2} x_k^T P x_k \succ 0$$

$$\lambda_1(x, u) = -\frac{1}{2}\begin{bmatrix} x \\ u \end{bmatrix}^T \begin{bmatrix} A^TPA - P & A^TPB - C^T \\ B^TPA - C & B^TPB - D - D^T \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \succ 0$$



Hyperstability

$$G(z) = C(zI - A)^{-1}B + D$$

Rearranging terms,

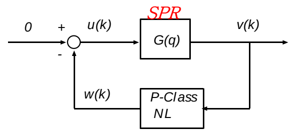
$$V(x_{k+1}) = V(x_0) + \sum_{j=0}^{k} v_j^T u_j - \sum_{j=0}^{k} \lambda_1(x_j, u_j)$$

From the P-class nonlinearity:

$$\sum_{j=0}^{k} w_j^T v_j \geq -\gamma_1^2 \quad \Rightarrow \quad \sum_{j=0}^{k} v_j^T u_j \leq \gamma_1^2$$

Therefore,

$$V(x_{k+1}) \leq V(x_0) + \gamma_1^2 - \underbrace{\sum_{j=0}^{k} \lambda_1(x_j, u_j)}_{\geq 0} \leq V(x_0) + \gamma_1^2$$



Hyperstability

$$G(z) = C(zI - A)^{-1}B + D$$

From the previous slide

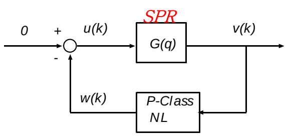
$$V(x_k) \leq V(x_0) + \gamma_1^2$$

$$\Rightarrow \quad \frac{1}{2} x_k^T P x_k \leq \frac{1}{2} x_0^T P x_0 + \gamma_1^2$$

$$\Rightarrow \quad \lambda_{min}(P)\|x_k\|^2 \leq \lambda_{max}(P)\|x_0\|^2 + 2\gamma_1^2$$

$$\Rightarrow \quad \|x_k\|^2 \leq \frac{\lambda_{max}(P)}{\lambda_{min}(P)} \left(\|x_0\|^2 + \frac{2}{\lambda_{max}(P)} \gamma_1^2 \right)$$

Therefore, the feedback system is hyperstable



Asymptotic Hyperstability

$$G(z) = C(zI - A)^{-1}B + D$$

$$0 \leq \quad V(x_{k+1}) \leq V(x_0) + \gamma_1^2 - \sum_{j=0}^{k} \lambda_1(x_j, u_j)$$

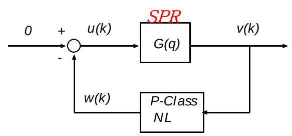
$$\Rightarrow \quad \underbrace{\sum_{j=0}^{k} \lambda_1(x_j, u_j)} \leq V(x_0) + \gamma_1^2$$

- monotonic nondecreasing sequence in k
- bounded above

$$\Rightarrow \quad \lim_{k\to\infty} \lambda_1(x_k, u_k) = 0 \quad \Rightarrow \quad \lim_{k\to\infty} x_k = 0, \quad \lim_{k\to\infty} u_k = 0$$

Therefore, the feedback system is asymtotically hyperstable

■



Additional Result

$$G(z) = C(zI - A)^{-1}B + D$$

We have already shown that

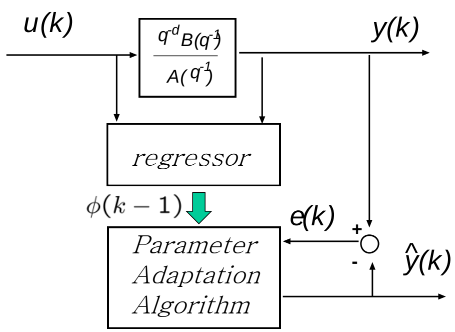
$$\lim_{k\to\infty} x_k = 0,$$
$$\lim_{k\to\infty} u_k = 0$$

From this we see that

$$\lim_{k\to\infty} v_k = \lim_{k\to\infty} (Cx_k + Du_k) = 0$$
$$\lim_{k\to\infty} w_k = \lim_{k\to\infty} (-u_k) = 0$$

Therefore, *x*(*k*), *u*(*k*), *v*(*k*), and *w*(*k*) converge to 0

Stability analysis of Series-parallel ID



Series-Parallel ID Dynamics (review)

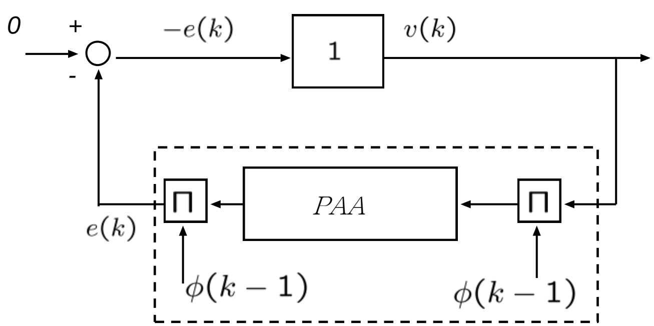
a-posteriori error: $e(k) = y(k) - \hat{y}(k)$

$$e(k) = \phi^T(k-1)\tilde{\theta}(k)$$

Parameter error update law: $\tilde{\theta}(k) = \theta - \hat{\theta}(k)$

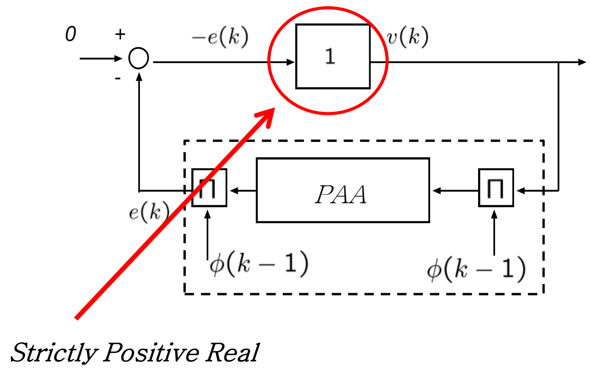
$$\tilde{\theta}(k) = \tilde{\theta}(k-1) - F\,\phi(k-1)e(k)$$

Series-Parallel ID Dynamics (review)

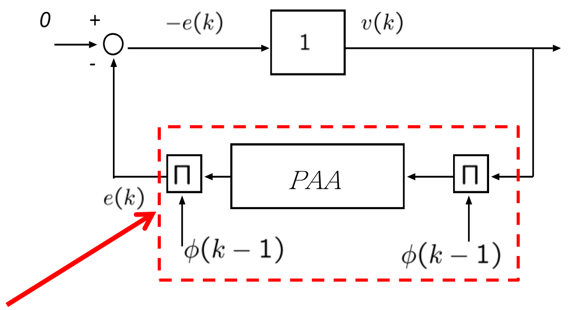


PAA: $\tilde{\theta}(k) = \tilde{\theta}(k-1) + F\,\phi(k-1)v(k)$

Stability analysis of Series-parallel ID

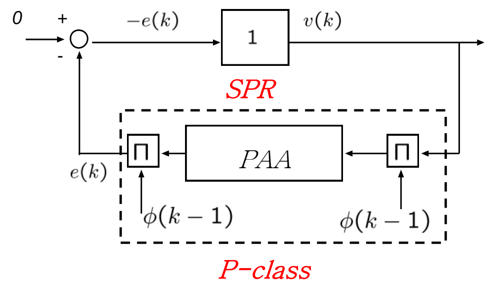


Stability analysis of Series-parallel ID



$$\left\{\begin{aligned}\tilde{\theta}(k) &= \tilde{\theta}(k-1) + F\,\phi(k-1)v(k)\\ e(k) &= \phi^T(k-1)\tilde{\theta}(k)\end{aligned}\right. \Rightarrow \sum_{j=0}^{k} e^T(j)v(j) \geq -\gamma_o^2$$

Stability analysis of Series-parallel ID



By the sufficiency portion of Asymptotic Hyperstability Theorem:

$$|v(k)| \to 0$$
$$|e(k)| \to 0$$

■

How to we implement the PAA?

a-posteriori estimate & PAA:

$$\left.\begin{aligned}e(k) &= y(k) - \phi^T(k-1)\hat{\theta}(k)\\ \hat{\theta}(k) &= \hat{\theta}(k-1) + F\phi(k-1)e(k)\end{aligned}\right\} \textit{Static coupling}$$

Solution: Use the a-priori error

$$e^o(k) = y(k) - \phi^T(k-1)\hat{\theta}(\underline{k-1})$$
$$= \phi^T(k-1)\tilde{\theta}(\underline{k-1})$$

How to we implement the PAA?

$$\tilde{\theta}(k) = \tilde{\theta}(k-1) - F\,\phi(k-1)e(k)$$

Multiply by $\phi^T(k-1) = \phi_{k-1}^T$

$$\underbrace{\phi_{k-1}^T\tilde{\theta}(k)}_{e(k)} = \underbrace{\phi_{k-1}^T\tilde{\theta}(k-1)}_{e^o(k)} - \phi_{k-1}^T F\,\phi_{k-1}e(k)$$

$$e(k) = e^o(k) - \phi_{k-1}^T F\,\phi_{k-1}e(k)$$

Therefore,

$$e(k) = \frac{e^o(k)}{1 + \phi^T(k-1)F\,\phi(k-1)}$$

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How we implement the PAA

1. $$e^o(k) = y(k) - \phi^T(k-1)\hat{\theta}(k-1)$$

2. $$e(k) = \frac{e^o(k)}{1 + \phi^T(k-1)F\,\phi(k-1)}$$

3. $$\hat{\theta}(k) = \hat{\theta}(k-1) + F\phi(k-1)e(k)$$

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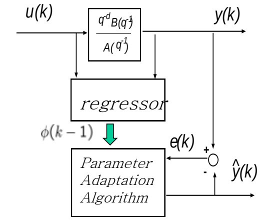
Stability analysis of Series-parallel ID



We have shown that

$e(k) \to 0$

Now we will show that

$e^o(k) \to 0$

Under the following assumptions:

$|u(k)| < \infty$ $\qquad A(q^{-1})$ is anti-Schur

Since $\quad y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) \quad \Rightarrow \quad |y(k)| < \infty$

Since $\quad \phi(k-1) = \begin{bmatrix} y(k-1) \\ \vdots \\ u(k-d) \\ \vdots \end{bmatrix} \quad \Rightarrow \quad \|\phi(k-1)\| < \infty$

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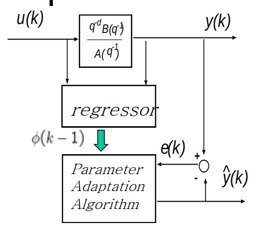
Stability analysis of Series-parallel ID



Thus, we know that

$e(k) \to 0$

$\|\phi(k-1)\| < \infty$

Remember that

$$e(k) = \frac{e^o(k)}{1 + \phi^T(k-1)F\,\phi(k-1)}$$

$$\Rightarrow \quad e^o(k) = \underbrace{e(k)}_{\to 0}\underbrace{\{1 + \phi^T(k-1)F\,\phi(k-1)\}}_{<\infty}$$

$$\Rightarrow \quad e^o(k) \to 0$$

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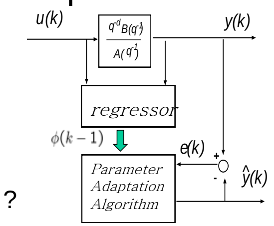
Stability analysis of Series-parallel ID



We have shown that

$e(k) \to 0 \quad e^o(k) \to 0$

$\|\phi(k-1)\| < \infty$

What about the parameter error $\tilde{\theta}(k)$?

since

$$\underbrace{e^o(k)}_{\to 0} = \phi^T(k-1)\tilde{\theta}(k-1) \quad \Rightarrow \quad |\phi^T(k)\tilde{\theta}(k)| \to 0$$

However, this **does not imply** that the parameter error goes to zero

We need to impose another condition on $u(k)$ to guarantee that the parameter error goes to zero. (**persistence of excitation**)