[1]

(a) When V=1/4, the Kalman filter gain becomes

$$F = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$

The open loop transfer function of the target feedback loop is given by

$$G_F(s) = C(sI - A)^{-1} F = \frac{2s+2}{s^2}.$$

(b) The LQG compensator is given by

$$G_c(s) = K(sI - A + BK + FC)^{-1} F = \frac{2(R^{-1/2} + \sqrt{2}R^{-1/4})s + 2R^{-1/2}}{s^2 + (\sqrt{2}R^{-1/4} + 2)s + 2\sqrt{2}R^{-1/4} + R^{-1/2} + 2}.$$

As R is decreased to 0, $R^{-1/2} >> R^{-1/4} >> 1$. Then $G_c(s)$ converges pointwise to

$$\frac{2R^{-1/2}s + 2R^{-1/2}}{R^{-1/2}} = 2s + 2.$$

Therefore, the overall open loop transfer function approaches to $\frac{2s+2}{s^2}$.

[2]

$$G_{overall}(z) = \frac{bz + bz^{-1} + (1+b^2)}{(1+b)^2}.$$

Consider z^{-1} as the one-sample-delay operator. Then the actual X and Y positions can be expressed as

$$x(k) = G_{overall}(z)x_d(k) = \frac{r}{(1+b)^2}\left\{b\cos[\delta\theta(k+1)] + b\cos[\delta\theta(k-1)] + (1+b^2)\cos(\delta\theta k)\right\}$$
$$y(k) = G_{overall}(z)y_d(k) = \frac{r}{(1+b)^2}\left\{b\sin[\delta\theta(k+1)] + b\sin[\delta\theta(k-1)] + (1+b^2)\sin(\delta\theta k)\right\}$$

From the internal model of sinusoidal signals, we have

$$b\cos[\delta\theta(k+1)] + b\cos[\delta\theta(k-1)] = 2b\cos(\delta\theta)\cos(\delta\theta k)$$
$$b\sin[\delta\theta(k+1)] + b\sin[\delta\theta(k-1)] = 2b\cos(\delta\theta)\sin(\delta\theta k)$$

Thus,

$$x(k) = \frac{r}{(1+b)^2}\left\{2b\cos(\delta\theta)\cos(\delta\theta k) + (1+b^2)\cos(\delta\theta k)\right\} = \frac{1+b^2+2b\cos(\delta\theta)}{(1+b)^2} r\cos(\delta\theta k)$$
$$y(k) = \frac{r}{(1+b)^2}\left\{2b\cos(\delta\theta)\sin(\delta\theta k) + (1+b^2)\sin(\delta\theta k)\right\} = \frac{1+b^2+2b\cos(\delta\theta)}{(1+b)^2} r\sin(\delta\theta k)$$

which implies that the actual contour is also a circle with radius $\frac{1+b^2+2b\cos(\delta\theta)}{(1+b)^2}r$. Therefore, the contouring error is equal to $\left| r - \frac{1+b^2+2b\cos(\delta\theta)}{(1+b)^2}r \right| = \frac{2br[1-\cos(\delta\theta)]}{(1+b)^2}$.

[3]

(a) When we compute the transfer function from D to Y, make U^* zero. From the block diagram, we have the following relationship among signals:

$$Y(z^{-1}) = \frac{z^{-1}B(z^{-1})}{A(z^{-1})}[D(z^{-1}) + U(z^{-1})] \tag{1}$$

$$U(z^{-1}) = -Q(z^{-1})[\frac{A_n(z^{-1})}{B_n(z^{-1})}Y(z^{-1}) - z^{-1}U(z^{-1})] \tag{2}$$

From (2), we get

$$U(z^{-1}) = -\frac{Q(z^{-1})A_n(z^{-1})}{[1-z^{-1}Q(z^{-1})]B_n(z^{-1})}Y(z^{-1}). \tag{3}$$

Plugging (3) into (1) and rearranging terms, we can get

$$Y(z^{-1}) = \frac{z^{-1}B_n(z^{-1})B(z^{-1})[1-z^{-1}Q(z^{-1})]}{B_n(z^{-1})A(z^{-1}) + z^{-1}Q(z^{-1})[A_n(z^{-1})B(z^{-1}) - A(z^{-1})B_n(z^{-1})]}D(z^{-1}). \tag{4}$$

Therefore, the transfer function from D to Y is given by

$$G_{DY}(z^{-1}) = \frac{z^{-1}B_n(z^{-1})B(z^{-1})[1-z^{-1}Q(z^{-1})]}{B_n(z^{-1})A(z^{-1}) + z^{-1}Q(z^{-1})[A_n(z^{-1})B(z^{-1}) - A(z^{-1})B_n(z^{-1})]}. \tag{5}$$

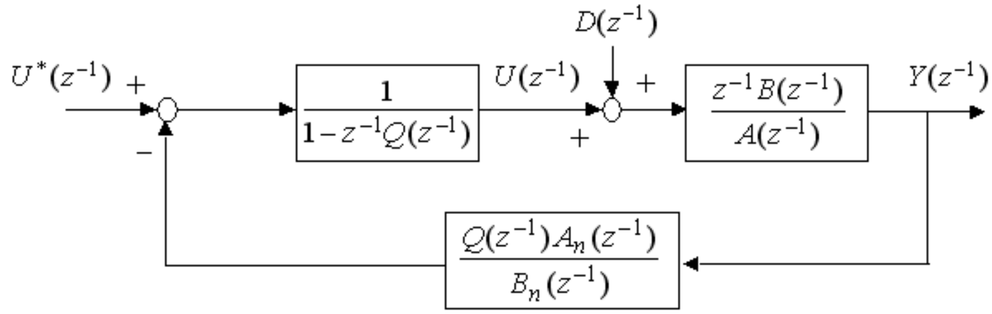
(b) If $Q(z^{-1}) = 2\cos(\omega) - z^{-1}$,

$$1 - z^{-1}Q(z^{-1}) = 1 - 2\cos(\omega)z^{-1} + z^{-2}.$$

So $[1-z^{-1}Q(z^{-1})]d(k) = 0$, when $d(k) = c\sin(\omega k + \phi)$. This implies that

$$\lim_{k\to\infty}[G_{DY}(z^{-1})d(k)] = 0,$$

when $G_{DY}(z^{-1})$ is asymptotically stable. Thus, the control structure in this problem works as an internal model control structure. We can also see this point by noticing that the given block diagram is equivalent to



The internal model of $d(k)$, $1-z^{-1}Q(z^{-1})=1-2\cos(\omega)z^{-1}+z^{-2}$, appears in the denominator of the feedback controller. So it works as an internal model controller.