University of California, Berkeley
Department of Mechanical Engineering
ME 233 Advanced Control Systems II

Spring 2014 **Midterm II (April 15 2014)**

Closed book and closed lecture notes; Two 8.5 $\times$ 11 handwritten summary sheet allowed (one from midterm I, one newly prepared);

Write down your name and student ID on all pages that you submit for grading.

1. [18 points] Consider a linear quadratic optimal control problem for the plant

$$P(s) = \frac{1}{s(s+5)}$$

with the frequency-domain performance index

$$J = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{1}{\omega^2}Y(-j\omega)Y(j\omega) + R\frac{\omega^2+1}{\omega^2+4}U(-j\omega)U(j\omega)\right\}\mathrm{d}\omega,\ R>0$$

(a) [4 points] Explain the roles of the two weighting functions in the performance index. Be specific. For instance, if one of them is for robustness consideration, explain why it improves the robustness.

(b) [8 points] Obtain the optimal control law and the associated Riccati equation.

(c) [3 points] Draw the block diagram that explains the implementation of the control system. Your block diagram should clearly explain how your control signal is composed using the extended states of the system.

(d) [3 points] The system is actually subjected to a sinusoidal disturbance at frequency $\omega_0 = 8$ rad/s. Propose a new performance index for the frequency shaped linear quadratic optimal control algorithm. Explain why it achieves the performance goal.

2. [12 points] Consider a stable plant with input-output behavior:

$$u(k) \longrightarrow \boxed{\frac{2+b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}} \longrightarrow y(k+1)$$

where $|b_1| < 2$.

(a) [4 points] Obtain the equations of recursive least squares (RLS), with a forgetting factor $\lambda = 0.999$, to estimate the plant parameters.

(b) [4 points] Translate the parameter adaptation algorithm to a feedback block diagram and list the steps to apply Popov's hyperstability theory for stability analysis.

(c) [4 points] Someone proposed to look at the system from a different perspective:

$$y(k+1) \longrightarrow \boxed{\frac{1+a_1z^{-1}+a_2z^{-2}}{2+b_1z^{-1}}} \longrightarrow u(k)$$

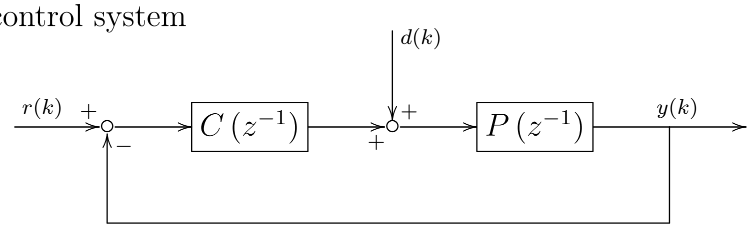
i.e., to estimate $u(k)$ instead of $y(k+1)$ and use the estimation error of $u(k)$ for the parameter adaptation algorithm. Obtain the RLS PAA with $\lambda = 0.999$.

3. [20 points] Regulation and tracking

(a) [10 points] Consider the plant

$$P\left(z^{-1}\right)=\frac{z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^{2}} \tag{1}$$

and the feedback control system



An engineer told you the disturbance satisfies $d(k)=\alpha\cos(\omega_0 k+\phi)$, where $\omega_0=\pi/10$ is known; α and ϕ are unknown but we know $\alpha\neq 0$. You are asked to design the controller $C\left(z^{-1}\right)=B_c\left(z^{-1}\right)/A_c\left(z^{-1}\right)$ to reject $d(k)$ in $y(k)$, and place all closed-loop poles at 0.

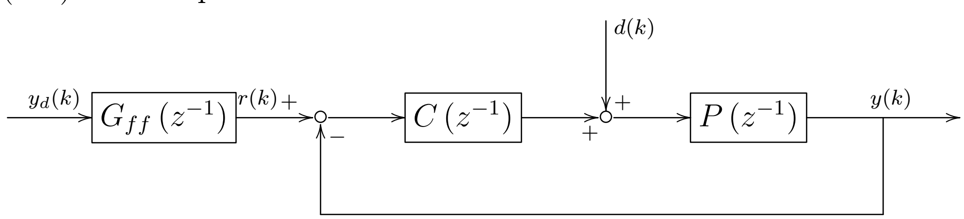
i. [6 points] List the conditions that $B_c\left(z^{-1}\right)$ and $A_c\left(z^{-1}\right)$ have to satisfy to achieve the design goals. Specify the orders of $B_c\left(z^{-1}\right)$, $A_c\left(z^{-1}\right)$, and the closed-loop system. You do not need to solve for the numerical values of the controller coefficients.

ii. [4 points] If the disturbance is instead $d(k)=\alpha\cos(\omega_0 k+\phi)+d_0$ where $d_0\neq 0$, list the conditions $B_c\left(z^{-1}\right)$ and $A_c\left(z^{-1}\right)$ have to satisfy for asymptotic disturbance rejection.

(b) [5 points] Suppose your controller is designed such that the transfer function from $r(k)$ to $y(k)$ is

$$\frac{0.1198(1+z)(z-3.376)(z-1.248)(z-0.9645)}{(z-0.9)^2\,(z-0.2)^3}$$

This is not the correct answer for part (a). But pretend it is for this part of the problem. For improving the tracking performance, design a zero-phase-error-tracking feedforward controller $G_{ff}\left(z^{-1}\right)$ to be implemented as follows:



and obtain the tracking error $e(k)\triangleq y(k)-y_d(k)$ as a function of $y_d(k)$.

(c) [5 points] Actually there are system uncertainties and (1) is only the nominal model of the plant. On top of the achieved controllers in (a) and (b), design a disturbance observer (DOB) for robustness improvement. Draw the block diagram of the entire control system with DOB. Let the sampling time be T_s and use a low-pass type of Q filter. List transfer functions of all the elements in your DOB design.
Hint: the Q filter is not unique (provide one that suits the application and specify all your design coefficients).