University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II — Spring 2009

Final Examination May 19, 2009 (Tu) Six Problems.
Open reader, Open notes; you may also refer to your own summary sheets for midterm exams.

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[1] (20 points) Consider a discrete time plant described by

$$x(k+1)=\begin{bmatrix}0 & 1\\ 0 & 0.9\end{bmatrix}x(k)+\begin{bmatrix}0.2\\ 0.5\end{bmatrix}u(k)+\begin{bmatrix}0.2\\ 0.5\end{bmatrix}w(k),\qquad y(k)=\begin{bmatrix}1 & 0\end{bmatrix}x(k)$$

where, $u(k)$ is the control input, $y(k)$ is the plant output and $w(k)$ is the input noise. $w(k)$ is white, zero mean, and Gaussian with $E[w^2(k)]=W$. The state vector $x(k)$ is exactly known or directly measured.

The control input is determined to minimize

$$J=E[y^2(k)+Ru^2(k)]$$

Obtain the root locus plot for the optimal closed loop system eigenvalues.

[2] (20 points) Consider a first order discrete time system described by

$$y(k)=-a_1 y(k-1)+b_0 u(k-1)+\alpha\sin(\omega k+\varsigma)$$

where ω is known but a_1, b_0, α and ς are not known in advance. Design an adaptive controller which will achieve the following control objective asymptotically.

$$(1+d_1 z^{-1})[y_d(k+1)-y(k+1)]=0$$

[3] (20 points) Consider the following zero order hold equivalent model of a motor transfer function from input to the velocity.

$$G(z^{-1})=\frac{z^{-1}}{1-z^{-1}}$$

The output velocity is

$$Y(z^{-1})=G(z^{-1})[U(z^{-1})+D(z^{-1})]$$

where U, D and Y are the control input, disturbance input and the plant output, respectively. The disturbance input is known to take the form, $d(k)=\alpha 0.98^k$ where α is unknown.

Design the closed loop controller by pole assignment. The controller must include the internal model of the disturbance. The closed loop poles have been assigned as shown below.

$$D(z^{-1})=(1-0.8z^{-1})^3=1-2.4z^{-1}+1.92z^{-2}-0.512z^{-3}$$

[4] (20 points) A first order system is excited by a colored noise. The system output is

$$Y(s) = \frac{1}{Ts+1} W(s)$$

where T represents the time constant and $W(s)$ is a colored Gaussian noise, the spectral density of which is given by

$$\frac{1}{0.25\omega^2 + 1}$$

Find $E[y(t)y(t+\tau)]$ at the steady state.

[5] (20 points) Consider a second order continuous time system described by

$$Y(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} U(s)$$

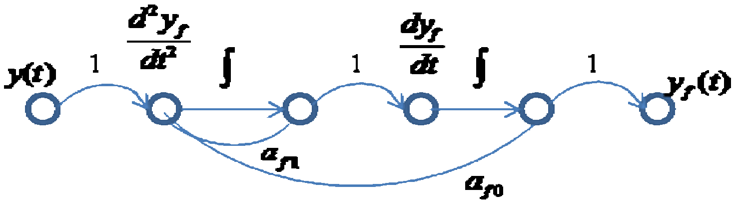
Note that $u(t)$ and $y(t)$ satisfy the differential equation

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_1 \dot{u}(t) + b_0 u(t)$$

If $\ddot{y}(t), \dot{y}(t), y(t), \dot{u}(t)$ and $u(t)$ can all be directly measurable, by sampling them every T *sec* a discrete time recursive least squares algorithm can be devised for estimation of the system parameters, a_1, a_0, b_1 and b_0. Unfortunately, $y(t)$ is measured but its derivatives are not directly measured. $u(t)$ is definitely known. To overcome the measurement problem, process $u(t)$ and $y(t)$ by a second order filter

$$G_f(s) = \frac{1}{s^2 + a_{f1}s + a_{f0}}$$

and obtain $Y_f(s) = G_f(s)Y(s)$ and $U_f(s) = G_f(s)U(s)$. The figure shown below shows a realization of the filter.



Notice that $y_f(t), \dot{y}_f(t)$ and $\ddot{y}_f(t)$ are available from this filter. The filter for $u(t)$ can give $u_f(t)$ and $\dot{u}_f(t)$.

Obtain the least squares algorithm for estimation of a_1, a_0, b_1 and b_0. It must be a discrete time algorithm working on sampled values of $y_f(t), \dot{y}_f(t)$, $\ddot{y}_f(t)$, $u_f(t)$ and $\dot{u}_f(t)$ sampled every T sec.

[6] (20 points) Parallel Model Reference Adaptive Systems (MRAS) without Compensation Block

Consider a single-input, single-output system described by

$$y(k+1) = -\sum_{i=1}^{n} a_i y(k+1-i) + \sum_{j=0}^{m} b_j u(k-j) \tag{1}$$

Consider a parallel adjustable system in Fig. PIAC-6 of the class notes. It is described by

$$\hat{y}(k+1) = \hat{\theta}^T(k+1)\phi(k) \tag{2}$$

where

$$\hat{\theta}^T(k+1) = [\hat{a}_1(k+1), \quad \cdots \quad \hat{a}_n(k+1), \quad \hat{b}_0(k+1) \quad \cdots \quad \hat{b}_m(k+1)]$$
$$\phi^T(k) = \left[-\hat{y}(k), \quad \cdots \quad -\hat{y}(k+1-n), \quad u(k), \quad \cdots \quad u(k-m)\right]$$

It is known that

$$\frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}$$

is asymptotically stable but not necessarily strictly positive real.

Prove the following theorem.

Theorem: The parallel MRAS described by (1), (2) and the following PAA is asymptotically stable.

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \frac{F\phi(k)}{1 + \phi^T(k)F\phi(k)}\varepsilon^0(k+1)$$

$$\varepsilon^0(k+1) = y(k+1) - \hat{\theta}^T(k)\phi(k)$$

where the constant adaptation gain matrix F together with the adjustable model input/output vector $\phi(k)$ satisfies

$$\frac{1}{2}\phi^T(k)F\phi(k) > K > 0$$

and K is selected so that

$$\frac{1}{1+K}\sum_{i=1}^{n} |a_i| < 1$$

Note: The condition in the theorem can be satisfied by making the adaptation gain matrix F large and/or the input signal $u(k)$ large.

Hint: After you obtain the equivalent feedback loop, apply the transformation in the figure below.