University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2009

Midterm Examination II April 16 (Th)
Closed books, Closed notes; you may refer to your own summary sheet.

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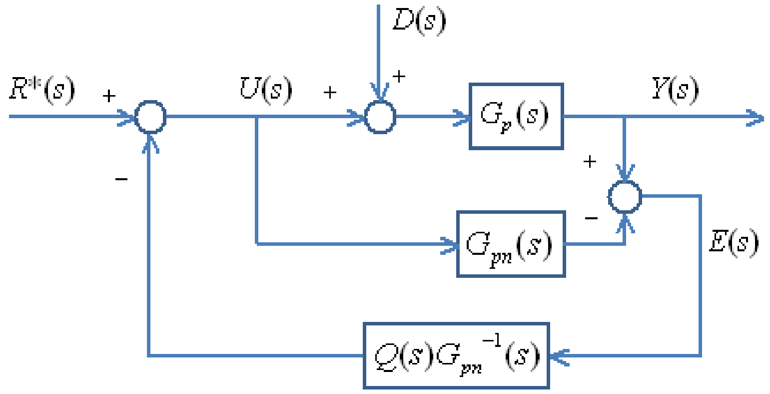
[1] (30 points) The figure shown below is a disturbance observer structured a little different from the one discussed in the class.



The transfer function of the actual system and the nominal transfer function are related by

$$G_p(s) = G_{pn}(s)(1+\Delta(s))$$

where $\Delta(s)$ represents the multiplicative uncertainty term.

a. Obtain the closed loop transfer function from $R^*(s)$ to $Y(s)$ and the one from $D(s)$ to $Y(s)$. Namely express $Y(s)$ in the form

$$Y(s) = G_{r^*y}(s)R^*(s) + G_{dy}(s)D(s).$$

As we did in the class, you can first derive

$$U(s) = G_{r^*u}(s)R^*(s) + G_{du}(s)D(s)$$

b. Show that at frequencies where $Q(j\omega)=1$,

$$Y(j\omega) = G_{pn}(j\omega)R^*(j\omega).$$

[2] (20 points) Consider a discrete time system described by

$$x(k+1) = x(k) + u(k) + w(k)$$

where x, u and w are scalar state variable, input variable and input noise, respectively. The input noise is a zero mean, Gaussian and white random sequence with $E[w^2(k)] = W$. The state variable x is directly measured without measurement noise: i.e. the state is exactly known.

The control input is determined to minimize the following performance index at the steady state.

$$J = E[x^2(k) + u^2(k)]$$

Find the minimum value J. (Apply the known results as much as you can, but you need to justify your approach.)

[3] (30 points) Consider a controllable and observable SISO plant described by

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Cx$$

The frequency shaped cost function is

$$J = \int_{-\infty}^{\infty} \left\{ Y(-j\omega) \frac{1}{(-j\omega)^2 + (\omega_0)^2} \cdot \frac{1}{(j\omega)^2 + (\omega_0)^2} Y(j\omega) + \rho U(-j\omega) R U(j\omega) \right\} d\omega$$

or

$$J = \int_{-\infty}^{\infty} \left\{ Y(-j\omega) \frac{1}{(\omega)^4 - 2(\omega_0)^2 (\omega)^2 + (\omega_0)^4} Y(j\omega) + \rho U(-j\omega) R U(j\omega) \right\} d\omega$$

where $R > 0$ and $\rho > 0$.

a. Obtain the cost function in the time domain. If you are introducing additional state variables, you need to include state equation that will generate the variables you need.
b. Draw the diagram showing the structure of the optimal system. Note that the structure may be interpreted as one coming from the internal model principle. If you would like to let $y(t)$ follow the reference input $y_d(t) = c\sin(\omega_0 t + \phi)$ where and how do you inject the reference signal to the closed loop block diagram. Show it in the diagram.