UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2016

Midterm Examination 2

Your Name:

Closed book, one new sheet of handwritten notes on 8.5" × 11" paper plus one reused sheet from midterm 1 are allowed.

Problem:	1	2	3	Total
Max. Grade:	30	35	35	100
Grade:				

Problem 1

Consider the discrete-time system

$$(1 - 0.7q^{-1})y(k) = q^{-1}(1 - 0.5q^{-1})(u(k) + d(k))$$

where q^{-1} is the one-step delay operator, $u(k)$ is the controlling input, $y(k)$ is the measured output, and $d(k)$ is the disturbance acting on the system. For this problem, $d(k)$ satisfies

$$d(k) = d_1(k) + d_2(k)$$

where $d_1(k) = d_1(k + 2)$ and $d_2(k) = d_2(k + 3)$ for all k.

Design a controller that achieves the following:

- The system output perfectly tracks an arbitrary desired output $y_d(k)$, which is known at least one step in advance.
- The set of closed-loop poles of the system only includes poles at the origin and any plant zeros that are being canceled by the control scheme.

Clearly indicate all steps of your design process, which includes solving a Diophantine (Bezout) equation.

Problem 2

Consider the state space system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned}$$

where $x(0) = x_0$ is given and both $u(k)$ and $y(k)$ are scalar signals. Assume that (A, B) is stabilizable and $CB \neq 0$. In this problem, we will find the controller that minimizes

$$J = \sum_{k=0}^{\infty} y^T(k) y(k) \tag{1}$$

1. Rewrite the cost function (1) in the form

   $$J = \alpha + \sum_{k=0}^{\infty} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{2}$$

   where α does not depend on the control, $R \succ 0$, and

   $$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \succeq 0$$

2. Find the optimal control $u^o(k)$ that minimizes the cost function (2). You may assume that the state space realization $C_J(zI - A)^{-1}B + D_J$ has no transmission zeros on or outside the unit circle where

   $$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = \begin{bmatrix} C_J^T \\ D_J^T \end{bmatrix} \begin{bmatrix} C_J & D_J \end{bmatrix}$$

3. The solution of the previous part depends on the solution of a discrete algebraic Riccati equation (DARE). Relate the solution of the DARE from the previous part to the solution of the DARE

   $$\bar{P} = A^T \bar{P} A + C^T C - A^T \bar{P} B (B^T \bar{P} B)^{-1} B^T \bar{P} A$$

   and show that the optimal control from part 2 can be expressed as

   $$u^o(k) = -(B^T \bar{P} B)^{-1} B^T \bar{P} A x(k)$$

Problem 3

Consider the state space system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_w w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

where $w(k)$ and $v(k)$ are zero-mean white Gaussian random sequences satisfying

$$E\left\{ \begin{bmatrix} w(k+j) \\ v(k+j) \end{bmatrix} \begin{bmatrix} w^T(k) & v^T(k) \end{bmatrix} \right\} = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix} \delta(j)$$

$y(k)$ is the measurement signal available to the controller, and $u(k)$ is the control sequence to be designed. We are interested in finding an <u>output feedback</u> controller[1] that minimizes the cost function

$$J = \text{trace}\left[\frac{1}{2\pi} \int_{-\pi}^{\pi} Q_f(e^{j\omega}) \Phi_{XX}(\omega) Q_f^*(e^{j\omega}) d\omega \right] + \text{trace}\left[\frac{1}{2\pi} \int_{-\pi}^{\pi} \rho \Phi_{UU}(\omega) d\omega \right]$$

where $\Phi_{XX}(\omega)$ and $\Phi_{UU}(\omega)$ are respectively the power spectral densities of $x(k)$ and $u(k)$, $Q_f(z)$ has the realization $C_f(zI - A_f)^{-1}B_f + D_f$, and $\rho > 0$.

1. Define $x_f(k)$ so that cost function can be written

$$J = E\{x_f^T(k)x_f(k) + \rho u^T(k)u(k)\}$$

2. Reformulate the problem of finding the output feedback controller that minimizes J as a standard <u>output feedback</u> LQG optimal control problem, i.e. reformulate the cost function as

$$J = E\left\{x_e^T(k)Qx_e(k) + u^T(k)Ru(k)\right\}$$

where

$$\begin{aligned} x_e(k+1) &= A_e x_e(k) + B_e u(k) + B_{we}\, w(k) \\ y(k) &= C_e x_e(k) + v(k) \end{aligned}$$

3. Find the controller that optimizes this cost under the assumption that all relevant existence conditions are met. You are not asked to find the optimal cost.

[1] $u(k)$ is allowed to be a function of $y(k), y(k-1), y(k-2), \ldots$