University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2007

Midterm Examination II May 1 (Tu)
Closed books, Closed notes; you may refer to your own summary sheet.

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[1] (30 points) Consider a plant described by

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

To set the target feedback loop, the following fictitious Kalman filtering problem is formulated.

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t), \quad E[w(t)] = 0; E[w(t)w(t+\tau)] = 1\delta(\tau)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + v(t), \quad E[v(t)] = 0; E[v(t)v(t+\tau)] = V\delta(\tau)$$

Solving the Riccati equation, the Kalman filter gain has been found to be

$$F = \begin{bmatrix} \sqrt{2}V^{-1/4} \\ V^{-1/2} \end{bmatrix}$$

Let $V = 1/4$ to fix the target feedback loop.

(a) Obtain the open loop transfer function of the target feedback loop

You apply the cheap LQ problem for LTR. Recall that the cheap LQ problem is

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \qquad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$
$$J = \int_0^\infty y^2(t) + Ru^2(t)dt, \qquad R \downarrow 0$$

The steady state equation of the Riccati equation and the feedback control gain are given by

$$P = \begin{bmatrix} \sqrt{2}R^{1/4} & R^{1/2} \\ R^{1/2} & \sqrt{2}R^{3/4} \end{bmatrix}, \quad K = \begin{bmatrix} R^{-1/2} & \sqrt{2}R^{-1/4} \end{bmatrix}$$

(b) Obtain the LQG compensator. Confirm that the overall open loop transfer function will approach to that of the target feedback loop as R is decreased. For this purpose, retain dominating terms in the numerator and denominator of the transfer function as R is decreased to 0. (Note: $R^{-1/2} >> R^{-1/4} >> 1$ for $R << 1$.)

[2] (20 points) Each axis of a two dimensional Cartesian positioning table (X-Y table) is under digital control. The zero phase error tracking controller is used for each axis and the transfer function from the desired position to actual position is

$$G_{overall}(z) = \frac{(1+bz)(1+bz^{-1})}{(1+b)^2}, \qquad b > 0$$

where b is the uncancellable zero of the closed loop system.

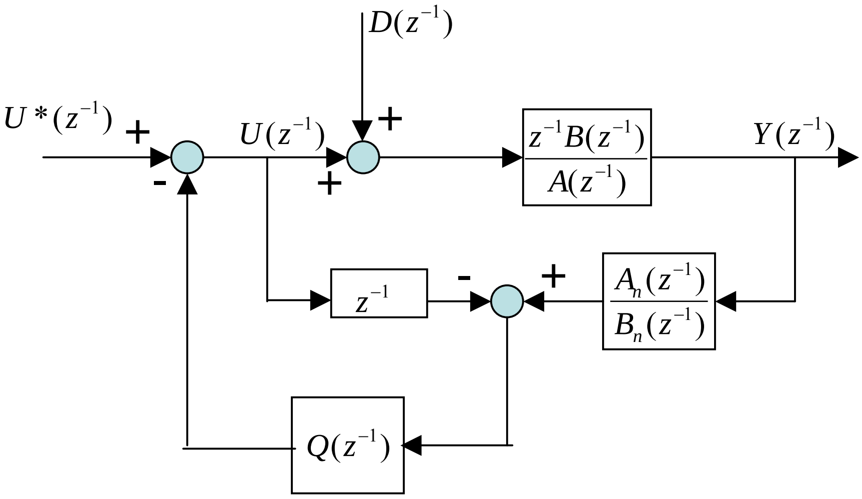
We would like to track a circular path with radius r. Letting the tangential tracking velocity and sampling time be denoted by V and Δt, respectively, the desired output for x and y axes are

$$x_d(k) = r\cos(\delta\theta k); \quad y_d(k) = r\sin(\delta\theta k) \text{ where } \delta\theta = V\Delta t / r$$

Find the contouring error defined by

Contouring error = |desired radius – actual radius|

[3] (30 points) Consider a disturbance observer structure as sketched below.



(a) Obtain the transfer function from D to Y.

(b) Show that the control structure works as an internal model control structure for sinusoidal disturbance $d(k) = c\sin(\omega k + \phi)$ if the Q filter is selected as $Q(z^{-1}) = 2\cos\omega - z^{-1}$.