UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II

Final Examination **Spring 2008**

Closed book and closed ME233 class notes.
Notes allowed: Lecture Notes (slides), 8 sheets of handwritten notes and Laplace and Z-transform tables from ME232 class notes.

Your Name:

Please answer all questions.

Problem:	1	2	3	4	5	Total
Max. Grade:	20	20	20	20	20	100
Grade:						

1 Problem

A set of random variables, X, Y and Z are Gaussian

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

i.e.

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = E\left\{ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \right\} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad E\left\{ \begin{bmatrix} X-\hat{x} \\ Y-\hat{y} \\ Z-\hat{z} \end{bmatrix} \begin{bmatrix} X-\hat{x} \\ Y-\hat{y} \\ Z-\hat{z} \end{bmatrix}^T \right\} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. Determine the value of the parameter $\theta \in \mathcal{R}$ that minimizes

$$J = E\{(X - \theta Y)^2\}\,.$$

2. Determine the value of the parameter $\theta \in \mathcal{R}$ that minimizes

$$J = E\{(X - \theta Y)^2 | Y = 0.5\}\,.$$

3. Determine the value of the parameter $\theta \in \mathcal{R}$ that minimizes

$$J = E\{(X - \theta Y)^2 | Y = 0.5,\ Z = -0.5\}\,.$$

2 Problem

Consider the design of a Kalman filter for a LTI stationary discrete time system that satisfies the standard assumptions, except for the fact that the input and output noises are correlated:

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) + B\,w(k) \\ y(k) &= C\,x(k) + v(k) \end{aligned}$$

where

- $x(k) \in \mathcal{R}^n$ is the state and

$$E\{x(0)\} = x_o \qquad E\{(x(0) - x_o)(x(0) - x_o)^T\} = X_o$$

- $u(k) \in \mathcal{R}^m$, $m \leq n$ is the control input.
- $y(k) \in \mathcal{R}^p$, $p \leq n$ is the measured output.
- $w(k)$ and $v(k)$ are stationary Gaussian, zero-mean, white noises that satisfy

$$E\left\{ \begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \begin{bmatrix} w^T(k+L) & v^T(k+L) \end{bmatrix} \right\} = \begin{bmatrix} W & S \\ S^T & V \end{bmatrix} \delta(L)$$

  for all integers L, where

$$\begin{bmatrix} W & S \\ S^T & V \end{bmatrix} = \begin{bmatrix} W^T & S \\ S^T & V^T \end{bmatrix} \succeq 0 \qquad V \succ 0$$

  and

$$E\{(x(0) - x_o) \begin{bmatrix} w^T(k) & v^T(k) \end{bmatrix}\} = 0$$

To design a Kalman filter for this system, we will first use the following output injection control law

$$u(k) = r(k) - Ky(k)$$

where K is a constant gain to be determined, and $r(k)$ becomes the new exogenous input to the system.

1. Show that the resulting closed loop system can be expressed as follows

$$\begin{aligned} x(k+1) &= A_c\,x(k) + B\,r(k) + B\,w_c(k) \\ y(k) &= C\,x(k) + v(k) \end{aligned}$$

   and obtain expressions for A_c and $w_c(k)$.

2. Determine K so that

$$E\{w_c(k)v^T(k)\} = 0$$

and obtain the resulting expressions for the variance

$$W_c = E\{w_c(k)w_c^T(k)\}$$

3. Write down the Kalman filter equations for estimating the state conditional mean $\hat{x}(k) = E\{x(k)|Y_k\}$, where $Y_k = \{y(0), \cdots, y(k)\}$.

4. Using prior known steady-state Kalman filter results, show that $[A, C]$ detectable guarantees the convergence of the a-priori state estimation error covariance to a positive semi definite matrix that satisfies an algebraic Riccati equation.

5. Using prior known steady-state Kalman filter results, list the conditions that guarantee the existence of a unique asymptotically stabilizing stationary Kalman filter for this system.

3 Problem

Consider the first order system

$$x(k+1) = x(k) + u(k) + w(k)$$

where

- $E\{x(0)\} = x_o = 1$
- $E\{\tilde{x}^2(0)\} = X_o = 1$, where $\tilde{x}(0) = x(0) - x_o$.
- $w(k) \sim \mathcal{N}(0,1)$ i.e. $w(k)$ is zero-mean, normal with variance 1.
- $E\{w(k)x(0)\} = 0$ for all $k \geq 0$

Assume that the state $x(k)$ is directly measurable.

1. Determine the optimal control

$$U^o = \{u^o(0),\, u^o(1)\}$$

that minimizes the performance index

$$J = \sum_{k=1}^{2} E\left\{2^k\, x^2(k) + u^2(k-1)\right\}\ .$$

2. Determine the value of the optimal performance index.

4 Problem

Consider the discrete time system

$$A(q^{-1})\, y(k) = q^{-1} B(q^{-1})\ [u(k) + d(k)]$$

where q^{-1} is the one-step delay operator, $u(k)$ is the controlling input, $y(k)$ is the measured output.

$d(k)$ is a sinusoidal disturbance of the form

$$d(k) = d_o \sin(\frac{\pi}{2}\, k + \phi_o)$$

where d_o and ϕ_o are **unknown**.

Assume that the polynomials

$$\begin{aligned} A(q^{-1}) &= 1 + 0.9\, q^{-1} \\ B(q^{-1}) &= 0.5 + 0.5\, q^{-1} \end{aligned}$$

are **known**.

Design a control system that satisfies the following requirements:

- The system output should asymptotically track an **arbitrary** desired output $y_d(k)$, which is known two steps in advance, with zero phase error.
- The disturbance $d(k)$ should be rejected.
- The closed loop poles of the feedback system should be the roots of

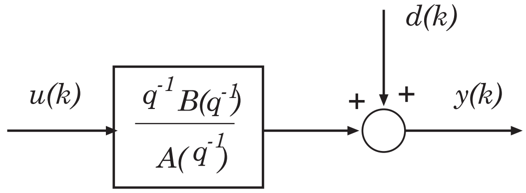
$$A_c^*(q) = q(q - 0.5)$$

Clearly indicate all steps of your design process. [1]

[1] If your answer requires the solution of a Diophantine equation, write down the linear equation $Mx = b$, that must be solved in order to obtain the solution of the Diophantine equation. This includes defining the elements of the unknown vector x and all of the coefficients of the matrix M and the vector b. You do not need to solve this linear equation.

5 Problem

Consider the identification of an open loop asymptotically stable plant with a sinusoidal output disturbance, as depicted below



where q^{-1} is the one-step delay operator, $u(k)$ is the controlling input, $y(k)$ is the measured output.

$d(k)$ is a sinusoidal disturbance of the form

$$d(k) = d_o \sin(\omega_d\, k + \phi_o)$$

where the frequency ω_d is **known** but d_o and ϕ_o are **unknown**.

The plant polynomials

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} \qquad B(q^{-1}) = b_o + b_1 q^{-1}$$

are of **known** order but their coefficients are **unknown**.

The following algorithm is proposed for identifying the plant parameters:

(i) The control input is

$$u(k) = U_1\, \sin(\omega_1 k) + U_2\, \sin(\omega_2 k)$$

where U_1 and U_2 are constants and $\omega_1 \neq \omega_2 \neq \omega_d$.

(ii) The a-priori output and error estimates are

$$\hat{y}^o(k) = \phi(k)^T \hat{\theta}(k-1) \qquad e^o(k) = y(k) - \hat{y}^o(k)$$

where the regressor $\phi(k)$ is given by

$$\phi(k) \quad = \quad \left[\begin{array}{cccccc} -y(k-1) & -y(k-2) & u(k-1) & u(k-2) & \sin(\omega_d\, k) & \cos(\omega_d\, k) \end{array} \right]^T$$

and $\hat{\theta}(k) \in \mathcal{R}^6$ is the parameter estimate, which is updated by a standard RLS PAA with forgetting factor, as shown below.

(iii) RLS PAA with forgetting factor:

$$
\begin{aligned}
e(k) &= \frac{e^o(k)}{1+\phi^T(k)F(k-1)\phi(k)} \\
\hat{\theta}(k) &= \hat{\theta}(k-1) + F(k-1)\phi(k)\, e(k) \\
F(k+1) &= \frac{1}{\lambda_1(k)}\left[F(k) - \frac{F(k)\phi(k+1)\phi^T(k+1)F(k)}{\frac{\lambda_1(k)}{\lambda_2(k)} + \phi^T(k+1)F(k)\phi(k+1)}\right] \qquad 0 < \lambda_1 \le 1 \qquad 0 \le \lambda_2 < 2
\end{aligned}
$$

Perform a stability analysis to determine if $\lim_{k\to\infty} e^o(k) = 0$.