UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2016

Midterm Examination I

Your Name:

Closed book, one sheet of notes on 8.5" × 11" paper are allowed.

Problem:	1	2	3	Total
Max. Grade:	40	30	30	100
Grade:				

Problem 1

Consider the discrete-time linear time-invariant system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

where D is square, i.e. the system has the same number of inputs as outputs. D is not necessarily symmetric, but assume here that $D + D^T \succ 0$. We would like to design an optimal control policy to minimize

$$\sum_{k=0}^{N-1} 2u^T(k)y(k).$$

Solve this problem by dynamic programming, introducing a cost-to-go function

$$J_m^o[x_m, N] = \min_{u(m),\dots,u(N-1)} \sum_{k=m}^{N-1} 2u^T(k)y(k) \quad \text{s.t.} \quad x(m) = x_m$$

for $m = 0, 1, \dots, N$. The solution can be expressed using a Riccati recurrence that depends on A, B, C, and D.

Extra credit: Prove that $J_0^o[x_0, N] \leq 0$ for all x_0, N.

Problem 2

Two scalar random variables X and Y are independent and identically distributed. Their sum $Z = X + Y$ is uniformly distributed between 0 and 1.

1. State the conditions that the pdf of X and Y must satisfy, in the continuous case and the discrete case (where Z takes on any of n possible values evenly spaced between 0 and 1). You do not have to solve for the exact pdf.

2. Sketch the approximate shapes of the pdf and cdf of X and Y, particularly the boundary points.

3. Sketch the pdf of $X + Y + W$ where W is a third random variable that is independent to and has the same distribution as X and Y.

4. What does the central limit theorem state about the result of continuing this process?

Problem 3

In this problem, we revisit problem 4 of homework 2.

A random variable X is repeatedly measured, but the measurements are noisy. Assume that the measurement process can be described by

$$Y(k) = X + V(k)$$

where $X, V(0), V(1), V(2), \ldots$ are jointly Gaussian random variables with

$$\begin{aligned} E\{X\} &= 0 & E\{X^2\} &= X_0 \\ E\{V(k)\} &= 0 & E\{V(k+j)V(k)\} &= \Sigma_V \delta(j) \\ E\{XV(k)\} &= 0\ . \end{aligned}$$

Let $y(k)$ be the k-th measurement (i.e. outcome of $Y(k)$) and let $\bar{y}(k) = \{y(0), \ldots, y(k)\}$. Using a Kalman filter, obtain the least squares estimate of X given the $k+1$ measurements $y(0), \ldots, y(k)$ and the corresponding estimation error covariance, i.e. find $\hat{x}_{|\bar{y}(k)}$ and $\Lambda_{\tilde{X}_{|\bar{y}(k)}\tilde{X}_{|\bar{y}(k)}}$. You may leave your expressions for $\hat{x}_{|\bar{y}(k)}$ and $\Lambda_{\tilde{X}_{|\bar{y}(k)}\tilde{X}_{|\bar{y}(k)}}$ in a recursive form.