UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II, Spring 2010

Homework #10

Assigned: Th., April 15
Due: Th., April 22

The second ME233 midterm will be on Thursday, April 29. The exam will be closed book and notes, but you are allowed to bring 6 8.5×11 pages of handwritten notes and a calculator. You can also bring photocopies of the Laplace and Z-transform tables in the ME232 class notes. The examination will cover material after the first midterm and up to and including this homework and the class lecture in 4/22.

Warning: This homework involves doing quite a bit of computer simulation. Please do not procrastinate until the last day.

1. In this problem we consider again the design of a compensator for the same disk file voice-coil actuator models used in Homework 9. However, this time we will use the Frequency-Shaped LQR design methodology described in Lecture 14 of the Class Notes and matlab. We will use two plant models:

   Simplified Nominal Model: This model will be used as the plant (control object) in the LQG-LTR synthesis procedure. In this case we assume that the VCM can be modeled as a second order system with transfer function

   $$G_p(s) = \frac{\omega_b^2}{s^2 + 2\zeta_b \omega_b \, s + \omega_b^2} \tag{1}$$

   where $\zeta_b = 0.707$ and $\omega_b = 10$ rad/sec, is the resonance frequency of the so-called bearing (and ribbon cable) resonance mode.

   Actual Model: This model will later be used in place of $G_p(s)$ to test the robustness of the designed feedback systems. In this case we assume that the plant includes the a low-frequency bearing (and ribbon cable) resonance mode in and $G_p(s)$ plus high-frequency VCM "butterfly" and suspension torsional resonance modes:

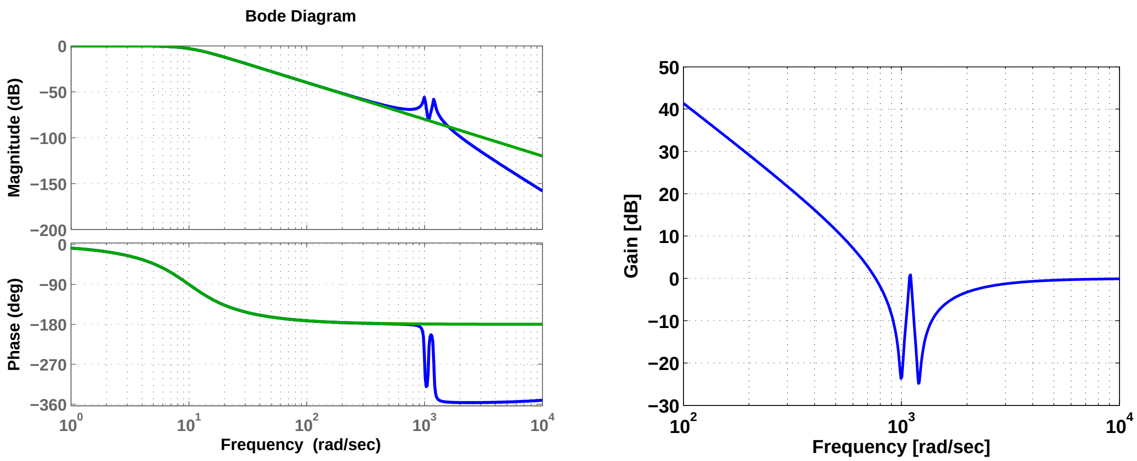
   $$G_{PA}(s) = G_p(s) \left(\frac{\omega_r^2 (\frac{\zeta_r}{\omega_r} s + 1)}{s^2 + 2\zeta_r \omega_r \, s + \omega_r^2} \right) \left(\frac{\frac{\omega_t^2}{\omega_n^2}(s^2 + 2\zeta_t \omega_n \, s + \omega_n^2)}{s^2 + 2\zeta_t \omega_t \, s + \omega_t^2} \right) \tag{2}$$

   where $\zeta_r = 0.015$, $\omega_r = 1000$ rad/sec, $\zeta_t = 0.015$, $\omega_t = 1200$ rad/sec, and $\omega_n = 0.9\,\omega_t$. Figure 1(a) shows the Bode plots of the two models.

   In order to facilitate the controller design procedure, I have written a couple of matlab functions:

   `fslqr`: This function sets up the extended matrices A_e, B_e, C_e and D_e in pages 47-48, computes K_e in page 50 and determines the compensators $C_r(s)$, $C_1(s)$ and $C_2(s)$ in page 52 of Lecture 14 FSLQR. [1].

[1] For more information on the input and output arguments type >> `help fslqr`.



(a) (b)

Figure 1: (a) Bode plots of disk drive voice-coil actuator simplified nominal model transfer function $G_p(j\omega)$ in Eqs. (1) and the "actual" model transfer function $G_{PA}(j\omega)$ in (2); (b) Magnitude Bode plot of $1/\Delta(j\omega)$

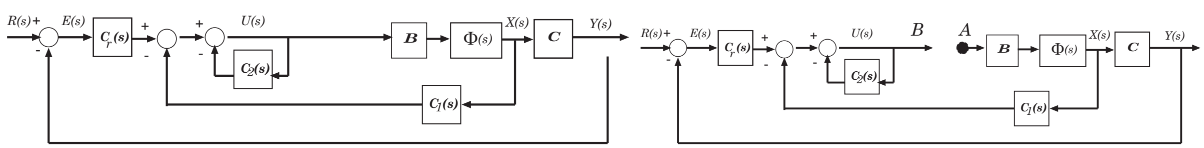
`fslqr_reg`: This function constructs the block diagram in page 52 and computes:

- The open loop transfer function $G_{oE->Y}(s)$ from $E \rightarrow Y$ in Fig 2(a) and its respective complementary and sensitivity transfer functions: $(I+G_{oE->Y}(s))^{-1}\, G_{oE->Y}(s)$ and $(I+G_{oE->Y}(s))^{-1}$.
- The open loop transfer function $G_{o(-A)->B}(s)$ from $(-A) \rightarrow B$ in Fig 2(b) and its respective complementary transfer function: $(I+G_{o(-A)->B}(s))^{-1}\, G_{o(-A)->B}(s)$ [2].

`fslqr_reg_robust_test`: This function constructs the block diagram in Fig 3 and computes:

- The open loop transfer function $G_{oE->Y}(s)$ from $E \rightarrow Y$ in Fig 2(a) and its respective complementary and sensitivity transfer functions: $(I+G_{oE->Y}(s))^{-1}\, G_{oE->Y}(s)$ and $(I+G_{oE->Y}(s))^{-1}$. [3]. Notice that it is assumed that the state of the nominal transfer function $C\Phi(s)B$ is directly available.

You can download these functions as part of the file `fslqr.zip` from the homework assignment location in the `bspace` ME233 web site.



(a) (b)

Figure 2: Frequency Shape LQR with Reference input

(a) Here we first use the FSLQR with reference input design procedure outlined in Lecture 14 class notes, assuming that the plant is the simplified nominal model, $G_p(s)$ in (1) and that the full state is available.

[2]For more information on the input and output arguments type >> `help fslqr_reg`.
[3]For more information on the input and output arguments type >> `help fslqr_reg_robust_test`.

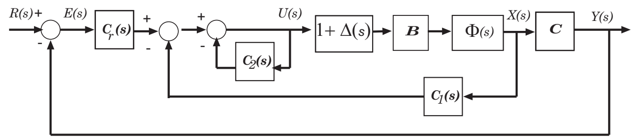


Figure 3: Frequency Shape LQR with Reference input - Robustness test

i. To achieve zero steady state error to a constant reference input, we define the reference frequency weight filter $Q_r(s)$ in page 41 of the class notes to be a pure integrator,

$$Q_r(s) = \frac{1}{s}\,, \tag{3}$$

obtain the optimal controller extended feedback gain K_e in page 36 that minimizes [4]

$$J \;=\; \frac{1}{2}\int_{-\infty}^{\infty} \left\{X^*(j\omega)C^T\, Q_r^*(j\omega)\, Q_r(j\omega)CX(j\omega) + \rho|U(j\omega)|^2\right\} d\omega$$

and assemble the feedback system represented by the block diagrams in pages 37 and 38 of lecture 15.

(i) The gain ρ must be obtained so that the gain crossover frequency of the transfer function $G_{oE->Y}(s)$ in Fig. 2(a) is approximately 60 rad/sec.

(ii) Plot the unit step response of the nominal feedback system.

(iii) Plot the magnitude Bode plots, of the transfer functions $G_{o(-A)->B}(s)$ and $T_{(-A)->B}(s) = \frac{G_{o(-A)->B}(s)}{1+G_{o(-A)->B}(s)}$ and determine whether the small gain condition

$$|T_{(-A)->B}(j\omega)| < |\frac{1}{\Delta(j\omega)}|\,. \tag{4}$$

is satisfied.

(iv) Check if the feedback system in Fig. 3 will be stable.

(b) In order to increase the robustness of the feedback system to the unmoderated resonance modes, it is necessary to incorporate frequency shape weight $R(j\omega) \succ 0$ in the cost function

$$J \;=\; \frac{1}{2}\int_{-\infty}^{\infty} \left\{X^*(j\omega)C^T\, Q_r^*(j\omega)\, Q_r(j\omega)CX(j\omega) + \rho R(j\omega)|U(j\omega)|^2\right\} d\omega$$

In order to satisfy the small gain condition in Eq. (4), you will need to synthesize a filter $R_f(s)$, which sharply increases in magnitude in the frequency range $(700, 1000)$ rad/sec, as outlined in the Lecture 14 class notes. (Note: your filter $R_f(s)$ may have be of a higher order than the one described in the notes).

(i) Keeping the value of the gain ρ that was obtained in the previous problem, synthesize a filter $R_f(s)$ and, determine its effect on the Bode plots of the transfer functions $G_{o(-A)->B}(s)$ and $T_{(-A)->B}(s) = \frac{G_{o(-A)->B}(s)}{1+G_{o(-A)->B}(s)}$.

[4] Notice that we are setting $Q_f = 0$ and $R_f = 1$ in this case.

Keep iterating your design until the small gain condition in Eq. (4) is satisfied and the gain crossover frequency of the transfer function $G_{oE->Y}(s)$ in Fig. 2(a) is close to 60 rad/sec (it could be a little lower).

(ii) Check if the feedback system in Fig. 3 will now stable.

(c) We now remove the assumption that the full state is measurable and instead assume that only the output $Y(s) = G_p(s)U(s)$ is measurable. In order to implement the FSLQR design in the previous section, we need to introduce a Kalman filter estimator and replace the actual state $X(s)$ with the state estimate $\hat{X}(s)$, as shown in Fig. 4, where the compensators $C_r(s)$, $C_1(s)$ and $C_2(s)$ are the same compensators described above.

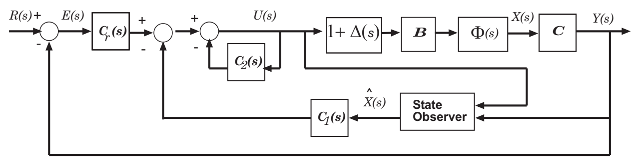
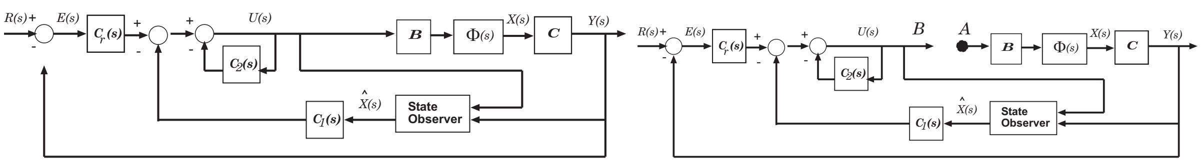


Figure 4: Frequency Shape LQR with Reference input and state estimation



(a)

(b)

Figure 5: Frequency Shape LQR with Reference input and state estimation

In order to design a robust estimator, we use a Kalman filter of the form

$$\frac{d}{dt}\hat{x} = Ax + Bu + L(y - C\hat{x})$$

where the estimator gain L, is computed using a *sensitivity recovery* procedure discussed in the LQG-LTR Lecture notes 13 (LQG-LTR method 2) and the FSLQR lecture notes 14. In the case the gain L is computed as follows:

$$L = \frac{1}{\mu} M\, C^T$$

$$AM + MA^T + BB^T - \frac{1}{\mu^2}\, MC^TCM \; = 0$$

for an increasingly small fictitious measurement noise standard deviation μ, and where

$$G_p(s) = C(sI - A)^{-1}B$$

To facilitate the controller design procedure, I have written the matlab function `fslqr_reg_est` [5], which constructs the block diagram in Fig. 4, and computes:

- The open loop transfer function $G_{oE->Y}(s)$ from $E \rightarrow Y$ in Fig 5(a) and its respective complementary and sensitivity transfer functions: $(I+G_{oE->Y}(s))^{-1}\, G_{oE->Y}(s)$ and $(I + G_{oE->Y}(s))^{-1}$.

[5] For more information on the input and output arguments type >> `help fslqr_reg_est`.

- The open loop transfer function $G_{o(-A)->B}(s)$ from $(-A) \rightarrow B$ in Fig 5(b)and its respective complementary transfer function: $(I + G_{o(-A)->B}(s))^{-1}G_{o(-A)->B}(s)$

i. Through and iterative process, decrease the value of the fictitious noise standard deviation μ until the Bode plot of the transfer function $G_{o(-A)->B}(s)$ in Fig 5(b) is sufficiently close to the Bode plot of the target transfer function $G_{o(-A)->B}(s)$ in Fig 2(b).

ii. Verify that the complementary sensitivity transfer function $T_{(-A)->B}(s) = \frac{G_{o(-A)->B}(s)}{1+G_{o(-A)->B}(s)}$ in Fig. 5(b) satisfies the the small gain condition in Eq. (4) and the gain crossover frequency of the transfer function $G_{oE->Y}(s)$ in Fig. 5(a) is close to 60 rad/sec (it could be a little lower).

iii. Plot the open loop bode plot, determine the gain and phase margins and plot the resulting unit step close loop response for the block diagram in Fig. 4, for the actual plant $G_{PA}(s)$ and compensators $C_r(s)$, $C_1(s)$ and $C_2(s)$ that were computed in part (b).

(d) Discuss your results. You are encourage to test other combinations of frequency filters $Q_r(s)$, $Q_f(s)$ and $R_f(s)$.

2. Consider the feedback system in Fig. 6 where $u(k)$ and $d(k)$ are respectively the control and

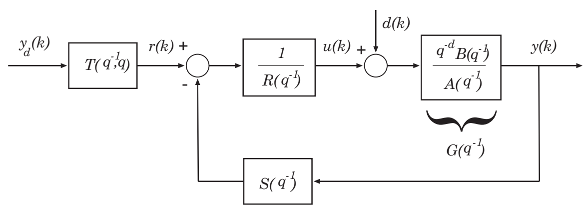


Figure 6: Feedback System

disturbance plant inputs, $y_d(k)$ is the reference model's output, and $r(k)$ is the reference input to the feedback block.

The control objective is to reject the persistent deterministic disturbance $d(k)$, place the feedback closed-loop poles, and track the desired output $y_d(k)$.

In order to help you verify your solutions of the Diophantine equation (also known as the Bezout equation), I have uploaded the matlab file bezout.m, which solves this equation. However, I advise you to at least solve the Diophantine equation in part 2e of this problem by hand, so that you gain an understanding of what is involved in the solution of this equation.

(a) The plant transfer function $G(z)$ is derived from a continuous time transfer function $G(s)$ that is preceded by a zero-order hold and followed by a sampler, and is given by

$$G(z) = \frac{B^*(z)}{A^*(z)} = (1 - z^{-1})\,\mathcal{Z}\left\{\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right\},$$

where

$$G(s) = \frac{1}{s(s+1)}$$

and the sampling time is $T = 0.5$ seconds.

Calculate the plant polynomials $A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2}$, $B(q^{-1}) = b_o + b_1 q^{-1}$ and pure delay time d. You can use the matlab `c2d` function for this purpose.

(b) The tracking control objective is to follow the reference model $y_d(k)$ given by

$$A_m(q^{-1}) y_d(k) = q^{-d} B_m(q^{-1}) u_d(k)\,. \tag{5}$$

Select the coefficients of the second order polynomial $A_m(q^{-1}) = 1 + a_{m1}\, q^{-1} + a_{m2}\, q^{-2}$, so that the reference model has a natural frequency of 1 rad/sec and a damping ratio of 0.707.

Hint: Remember that, since $z = e^{sT}$, we can calculate the discrete time poles by $p_d = e^{p_c T}$, where p_d is the discrete time pole, p_c is the continuous time pole and T is the sampling time.

(c) Let $B_m(q^{-1}) = b_{mo}$, select b_{mo} so that the reference model's static gain is unity [6].

(d) The coefficients of the closed loop system characteristic polynomial

$$A_c^{'}(q^{-1}) = 1 + a_{c1}^{'}\, q^{-1} + a_{c2}^{'}\, q^{-2}$$

should be selected so that the close loop feedback dynamics from $r(k)$ to $y(k)$ behaves as a second order system with a natural frequency of 2 rad/sec and a damping ratio of 0.5. [7]

(e) Design the control system under the following specifications and assumptions:

(i) The closed loop system characteristic polynomial satisfies

$$A_c(q^{-1}) \quad = \quad A_c^{'}(q^{-1}) B^s(q^{-1})$$

where

$$B^s(q^{-1}) = \frac{1}{b_o} B(q^{-1}) \qquad B^u(q^{-1}) = b_o\,,$$

and b_o is the leading coefficient of $B(q^{-1})$. This means that all plant zeros will be canceled by the feedback system.

(ii) Assume that $d(k) = 0$. This means that the disturbance annihilating polynomial is selected to be $A_d(q^{-1}) = 1$.

(iii) The feedforward compensator $T(q^{-1}, q)$ must be selected so that perfect tracking is achieved under a zero initial state for both the plant and the reference model.

(f) Do a computer simulation of the control system designed in part 2e when $y_d(-1) = y_d(0) = y(-1) = y(0) = 0$ and

$$u_d(k) \quad = \quad [u_s(k) - 2\, u_s(k-25)] + [2\, u_s(k-50) - 2\, u_s(k-75)] \tag{6}$$

$$d(k) \quad = \quad 0.5\, u_s(k-40) \tag{7}$$

where

$$u_s(j) = \begin{cases} 0 & j < 0 \\ 1 & j \geq 0 \end{cases}$$

Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

[6] i.e. if $\lim_{k\to\infty} u_d(k) = u_{ss}$ then $\lim_{k\to\infty} y_d(k) = u_{ss}$.

[7] Remember that the close loop polynomial $A_c(q^{-1})$ satisfies

$$A_c(q^{-1}) = A_c^{'}(q^{-1}) B^s(q^{-1})$$

where the monic polynomial $B^s(q^{-1})$ includes the cancelable plant zeros.

(g) Design the control system under the same specifications in part 2e, except that assume now that

- $d(k) = d(k-1)$.

(h) Do a computer simulation of the control system designed in part 2g under the conditions described in part 2f.

(i) Design the control system under the following specifications and assumptions:

(i) The closed loop system characteristic polynomial satisfies

$$A_c(q^{-1}) \;=\; A_c'(q^{-1})B^s(q^{-1})$$

where

$$B^s(q^{-1}) = 1 \qquad B^u(q^{-1}) = B(q^{-1})\,,$$

This means that all plant zeros will not be canceled by the feedback system.

(ii) Assume that $d(k) = d(k-1)$.

(iii) The feedforward compensator $T(q^{-1}, q)$ is designed using the zero-phase error tracking control approach.

(j) Do a computer simulation of the control system designed in part 2i under the conditions described in part 2f. Plot $u_d(k)$, $y_d(k)$ (as defined in part 2b), $y(k)$ and $u(k)$.

(k) Discuss the outcome of the simulation results. In particular

- Comment on the effectiveness of the zero-phase feedforward control technique.
- Compare the control effort $u(k)$ when the zeros are canceled vs when the zeros are not canceled.