ME 233 Advance Control II

Lecture 22

Direct Adaptive Pole Placement, and Tracking Control

Direct vs. Indirect Adaptive Control

- Both use pole-placement, tracking control and deterministic disturbance rejection controller synthesis methodology.
- **Indirect adaptive control:**
  1. Plant parameters are estimated using a RLS PAA.
  2. Controller parameters are calculated using the certainty equivalence principle.
  - **Use with plants that have non-minimum phase zeros.** (Plant unstable zeros are not cancelled).
- **Direct adaptive control:**
  1. Controller parameters are updated directly using a RLS PAA.
  - **Use with plants that do not have non-minimum phase zeros.** (Plant zeros are cancelled).

Direct Adaptive Control

1. Plants with minimum phase zeros and no disturbances:
   - Controller design
   1. Controller PAA
   2. Adaptive Controller
2. Plants with minimum phase zeros and constant disturbances:

- Read section: ***Direct adaptive control with integral action for plants with stable zeros*** in the ME233 class notes, part II.

Deterministic SISO ARMA models

SISO ARMA model

$$A(q^{-1})\, y(k) = q^{-\mathrm{d}}\, B(q^{-1})\, u(k)$$

Where all inputs and outputs are scalars:

- $u(k)$ control input
- $y(k)$ output

d is the ***known*** pure time delay

Deterministic SISO ARMA models

$$A(q^{-1})\, y(k) = q^{-\mathrm{d}}\, B(q^{-1})\, u(k)$$

Where polynomials:

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) &= b_o + b_1 q^{-1} + \cdots + b_m q^{-m} \end{aligned}$$

are co-prime and $B(q^{-1})$ is Hurwitz

Control Objectives

1. Pole Placement: The poles of the closed loop system must be placed at specific locations in the complex plane.

- **Closed loop pole polynomial:**

$$\boxed{A_c(q^{-1}) = B(q^{-1})\, A_c^{'}(q^{-1})}$$

Where:

- $B(q^{-1})$ cancelable plant zeros
- $A_c^{'}(q^{-1})$ monic Hurwitz polynomial chosen by the designer

$$A_c^{'}(q^{-1}) = 1 + a_{c1}^{'} q^{-1} + \cdots + a_{c n_c^{'}}^{'} q^{-n_c^{'}}$$

Control Objectives

2. Tracking: The output sequence $y(k)$ must follow a ***reference*** sequence $y_d(k)$ which is known

- **Reference model:**

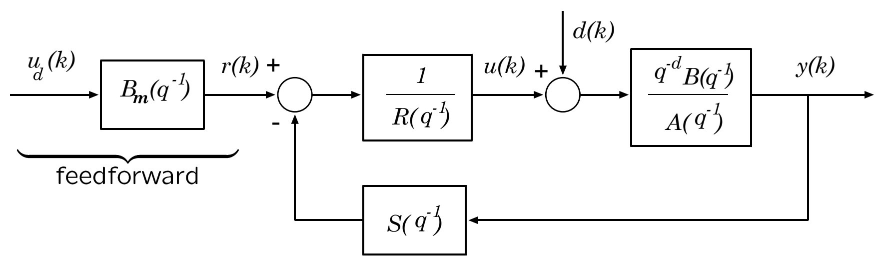
$$\boxed{A_c^{'}(q^{-1}) y_d(k) = q^{-\mathrm{d}}\, B_m(q^{-1})\, u_d(k)}$$

Where:

- $u_d(k)$ **known** reference input control input sequence
- $A_c^{'}(q^{-1})$ monic Hurwitz polynomial chosen by the designer
- $B_m(q^{-1})$ zero polynomial, chosen by the designer

Control Law

- Feedback and feedforward actions:



$$u(k) = \frac{1}{R(q^{-1})}\left[r(k) - S(q^{-1}) y(k)\right]$$

$$r(k) = B_m(q^{-1})\, u_d(k)$$ *Feedforward (causal)*

Feedback Controller

Diophantine equation: Obtain polynomials $\underline{R'(q^{-1})}, \underline{S(q^{-1})}$ which satisfy:

$$\boxed{\underbrace{A'_c(q^{-1})}_{\textit{Close loop poles}} = \underbrace{A(q^{-1})}_{\textit{Plant poles}}\, \underline{R'(q^{-1})} + \underbrace{q^{-\mathrm{d}}}_{\textit{Plant pure delays}}\, \underline{S(q^{-1})}}$$

$$\boxed{\begin{aligned} R(q^{-1}) &= R'(q^{-1})\, B(q^{-1}) \\ A_c(q^{-1}) &= B(q^{-1})\, A'_c(q^{-1}) \end{aligned}}$$

Diophantine equation

$$A'_c(q^{-1}) = A(q^{-1})\, R'(q^{-1}) + q^{-\mathrm{d}}\, S(q^{-1})$$

Solution:

$$\begin{aligned} R'(q^{-1}) &= 1 + r'_1 q^{-1} + \cdots + r_{n'_r}\, q^{-n'_r} \\ S(q^{-1}) &= s_o + s_1 q^{-1} + \cdots + s_{n_s} q^{-n_s} \end{aligned}$$

$$\boxed{\begin{aligned} n'_r &= \mathrm{d} - 1 \\ n_s &= \max\{\, n-1\,,\, n'_c - \mathrm{d}\,\} \end{aligned}}$$

Feedback Controller

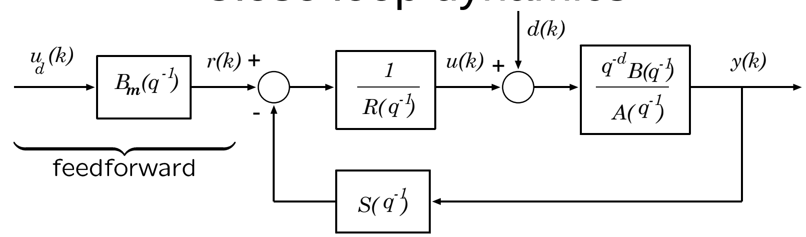
$$u(k) = \frac{1}{R(q^{-1})}\left[r(k) - S(q^{-1})y(k)\right]$$

where

$$R(q^{-1}) = R'(q^{-1})\, B(q^{-1})$$

$$\boxed{\begin{aligned} n'_r &= \mathrm{d} - 1 \\ n_s &= \max\{\, n-1\,,\, n'_c - \mathrm{d}\,\} \\ n_r &= n'_r + m \end{aligned}}$$

Close loop dynamics



$$\begin{aligned} A'_c(q^{-1})\, y(k) &= q^{-\mathrm{d}}\, r(k) \\ &= q^{-\mathrm{d}} B_m(q^{-1}) u_d(k) \\ &= A'_c(q^{-1})\, y_d(k) \end{aligned}$$

$$\boxed{A'_c(q^{-1})\,\{\, y(k) - y_d(k)\} = 0}$$

Direct Adaptive Control

1. Plants with minimum phase zeros and no disturbances:
   - Controller design
   1. Controller PAA
   2. Adaptive Controller

Controller parameters

We want to identify the controller polynomials

$$R(q^{-1}) \quad S(q^{-1})$$

directly, where

$$R(q^{-1}) = R^{'}(q^{-1})\, B(q^{-1})$$

$$R(q^{-1}) = \underbrace{r_o}_{=b_o} + r_1\, q^{-1} + \cdots + r_{n_r}\, q^{-n_r}$$

$$S(q^{-1}) = s_o + s_1\, q^{-1} + \cdots + s_{n_s}\, q^{-n_s}$$

Controller parameters

Start with the Diophantine equation

$$A_c^{'}(q^{-1}) = A(q^{-1})\, R^{'}(q^{-1}) + q^{-\mathrm{d}}\, S(q^{-1})$$

Multiply both sides by *y(k)*

$$A_c^{'}(q^{-1})\, y(k) = R^{'}(q^{-1})\, A(q^{-1})\, y(k) + q^{-\mathrm{d}}\, S(q^{-1})\, y(k)$$

Controller parameters

$$A_c^{'}(q^{-1})\, y(k) = R^{'}(q^{-1})\, \underline{A(q^{-1})\, y(k)} + q^{-\mathrm{d}}\, S(q^{-1})\, y(k)$$

Insert plant dynamics

$$A(q^{-1})\, y(k) = q^{-\mathrm{d}}\, B(q^{-1})\, u(k)$$

$$A_c^{'}(q^{-1})\, y(k) = q^{-\mathrm{d}}\left[\underline{R^{'}(q^{-1})\, B(q^{-1})}\, u(k) + S(q^{-1})\, y(k)\right]$$

$$\boxed{A_c^{'}(q^{-1})\, y(k) = q^{-\mathrm{d}}\left[R(q^{-1})\, u(k) + S(q^{-1})\, y(k)\right]}$$

PAA – version 1

$$A_c'(q^{-1})\,y(k) = q^{-\mathsf{d}}\left[R(q^{-1})\,u(k) + S(q^{-1})\,y(k)\right]$$

Filter by $1/A_c{}'(q^{-1})$ (*normally a low-pass filter*)

$$\boxed{y(k) = R(q^{-1})\,u_f(k-\mathsf{d}) + S(q^{-1})\,y_f(k-\mathsf{d})}$$

$$y_f(k) = \frac{1}{A_c'(q^{-1})}\,y(k)$$

$$u_f(k) = \frac{1}{A_c'(q^{-1})}\,u(k)$$

PAA – version 1

$$y(k) = R(q^{-1})\,u_f(k-\mathsf{d}) + S(q^{-1})\,y_f(k-\mathsf{d})$$

Is linear in the controller parameters:

$$\boxed{y(k) = \phi_f^T(k-d)\theta_c}$$

$$\theta_c = \left[\; s_o \cdots s_{n_s}\, r_o \;\cdots\; r_{n_r} \;\right]^T \in \mathcal{R}^{N_c}$$

$$N_c = n_s + n_r + 2$$

PAA – version 1

Plant dynamics:

$$\boxed{y(k) = \phi_f^T(k-d)\theta_c}$$

$$\theta_c = \left[\; s_o \cdots s_{n_s}\, r_o \;\cdots\; r_{n_r} \;\right]^T \in \mathcal{R}^{N_c}$$

$$\phi_f(k) = \frac{1}{A_c'(q^{-1})}\,\phi(k)$$

$$\phi(k) = \left[\; y(k) \;\cdots\; y(k-n_s) \;\; u(k) \;\cdots\; u(k-n_r) \;\right]^T$$

$$N_c = n_s + n_r + 2$$

PAA – version 1

Plant dynamics:

$$\boxed{y(k) = \phi_f^T(k-d)\theta_c}$$

RLS PAA:

$$\boxed{e^o(k) = y(k) - \phi_f^T(k-d)\hat{\theta}_c(k-1)}$$

$$e(k+1) = \frac{e^o(k+1)}{1+\phi_f^T(k-d+1)F(k)\phi_f(k-d+1)}$$

$$\hat{\theta}_c^o(k+1) = \hat{\theta}_c(k) + F(k)\phi_f(k-d+1)\,e(k+1)$$

$$F(k+1) = \frac{1}{\lambda_1(k)}\left[F(k) - \frac{F(k)\phi_f(k-d+1)\phi_f^T(k-d+1)F(k)}{\frac{\lambda_1(k)}{\lambda_2(k)} + \phi_f^T(k-d+1)F(k)\phi_f(k-d+1)}\right]$$

PAA – version 2

$$A_c'(q^{-1})\,y(k) = q^{-\mathrm{d}}\left[R(q^{-1})\,u(k) + S(q^{-1})\,y(k)\right]$$

$$\boxed{\eta(k) = A_c'(q^{-1})\,y(k)}$$

filtered output signal

$$\boxed{\eta(k) \;=\; \phi^T(k-d)\theta_c}$$

$$\theta_c = \left[\; s_o \cdots s_{n_s}\, r_o \;\cdots\; r_{n_r} \;\right]^T \in \mathcal{R}^{N_c}$$

$$\phi(k) \;=\; \left[\; y(k) \;\; \cdots \;\; y(k-n_s) \;\; u(k) \;\; \cdots \;\; u(k-n_r) \;\right]^T$$

PAA – version 2

Plant dynamics:

$$\boxed{\eta(k) \;=\; \phi^T(k-d)\theta_c}$$

RLS PAA:

$$\boxed{e^o(k) \;=\; \eta(k) - \phi^T(k-d)\hat{\theta}_c(k-1)}$$

$$e(k+1) \;=\; \frac{e^o(k+1)}{1+\phi^T(k-d+1)F(k)\phi(k-d+1)}$$

$$\hat{\theta}_c^o(k+1) \;=\; \hat{\theta}_c(k) + F(k)\phi(k-d+1)\; e(k+1)$$

$$F(k+1) \;=\; \frac{1}{\lambda_1(k)}\left[F(k) - \frac{F(k)\phi(k-d+1)\phi^T(k-d+1)F(k)}{\frac{\lambda_1(k)}{\lambda_2(k)} + \phi^T(k-d+1)F(k)\phi(k-d+1)}\right]$$

PAA – version 1 Vs version 2

- $A_c'(q^{-1})$ is normally a ***high-pass*** filter
- $1/A_c'(q^{-1})$ is normally a ***low-pass*** filter

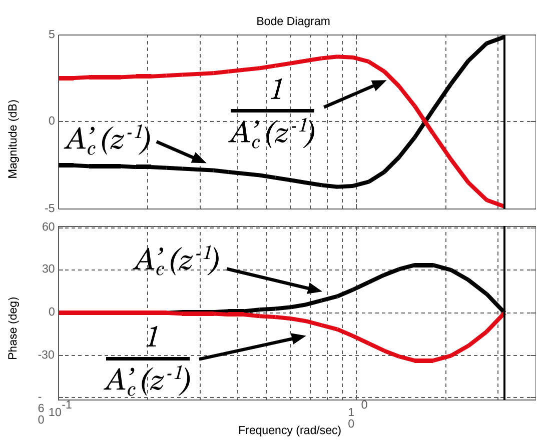
Example

$$A_c'(q^{-1}) \;=\; (1-.5q^{-1})^2$$

Version 1 is preferable

$$\phi_f(k) \;=\; \frac{1}{A_c'(q^{-1})}\phi(k)$$

filters high frequency noise



PAA projection

PAA: Projection

$$\hat{r}_o(k)$$

$$\hat{\theta}_c(k) \;=\; \begin{cases} \hat{\theta}_c^o(k) & \text{if } \;\hat{r}_o^o(k) \geq b_{mino} \\ \left[\; \hat{s}_o^o(k)\cdots\hat{s}_{n_s}^o(k)\; b_{mino} \;\cdots\; \hat{r}_{n_r}^o(k) \;\right]^T & \text{if } \;\hat{r}_o^o(k) < b_{mino} \end{cases}$$

Replace $\hat{r}_o^o(k)$ by b_{mino} if it becomes too small.

Control law will divide by $\hat{r}_o(k)$. Thus, the projection algorithm prevents the control action from becoming too large.

PAA Gain matrix

Gain matrix:

$$F(k+1) = \frac{1}{\lambda_1(k)}\left[F(k) - \frac{F(k)\phi_f(k-d+1)\phi_f^T(k-d+1)F(k)}{\frac{\lambda_1(k)}{\lambda_2(k)} + \phi_f^T(k-d+1)F(k)\phi_f(k-d+1)}\right]$$

$$0 < \lambda_1(k) \leq 1$$
$$0 \leq \lambda_2(k) < 2$$

are adjusted so that the maximum singular value of $F(k)$ is uniformly bounded, and

$$0 < K_{\min} \leq \lambda_{\min}\{F(k)\} \leq \lambda_{\max}\{F(k)\} < K_{max} < \infty\,.$$

Direct Adaptive Control

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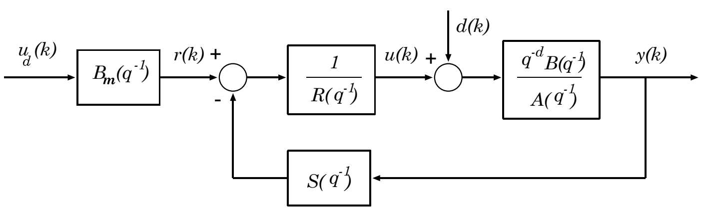
   2. Adaptive Controller

Fixed Controller

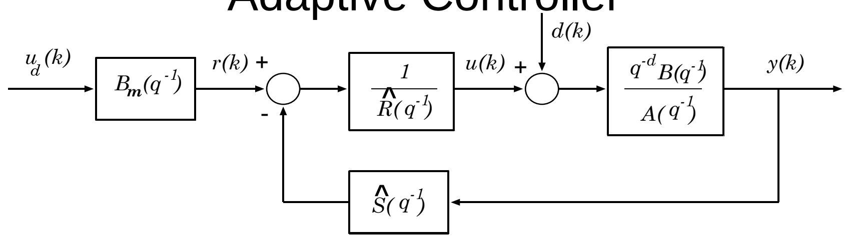


$$R(q^{-1})\,u(k) = B_m(q^{-1})\,u_d(k) - S(q^{-1})y(k)$$

$$S(q^{-1})y(k) + R(q^{-1})\,u(k) = B_m(q^{-1})\,u_d(k)$$

$$\boxed{\phi^T(k)\theta_c = B_m(q^{-1})u_d(k)}$$

Adaptive Controller



$$\hat{R}(q^{-1},k)\,u(k) = B_m(q^{-1})\,u_d(k) - \hat{S}(q^{-1},k)y(k)$$

$$\boxed{\phi^T(k)\hat{\theta}_c(k) = B_m(q^{-1})u_d(k)}$$

Direct Adaptive Control

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2. Plants with minimum phase zeros and constant disturbances:

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