ME 233 Advanced Control II

Lecture 24

Direct Adaptive Pole Placement, and Tracking Control

Direct vs. Indirect Adaptive Control

- Both use pole-placement, tracking control and deterministic disturbance rejection controller synthesis methodology.
- **Indirect adaptive control:**
  1. Plant parameters are estimated using a RLS PAA.
  2. Controller parameters are calculated using the certainty equivalence principle.
  - **Use with plants that have non-minimum phase zeros.** (Plant unstable zeros are not cancelled).
- **Direct adaptive control:**
  1. Controller parameters are updated directly using a RLS PAA.
  - **Use with plants that do not have non-minimum phase zeros.** (Plant zeros are cancelled).

Direct Adaptive Control

1. Plants with minimum phase zeros and no disturbances:
   - Controller design (review)
   1. Controller PAA
   2. Adaptive Controller
2. Plants with minimum phase zeros and constant disturbances:

- Read section: ***Direct adaptive control with integral action for plants with stable zeros*** in the ME233 class notes, part II.

Deterministic SISO ARMA models

SISO ARMA model

$$A(q^{-1})\, y(k) = q^{-d}\, B(q^{-1})\, u(k)$$

Where all inputs and outputs are scalars:

- $u(k)$ control input
- $y(k)$ output

d is the ***known*** pure time delay

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Deterministic SISO ARMA models

$$A(q^{-1})\, y(k) = q^{-\mathrm{d}}\, B(q^{-1})\, u(k)$$

Where polynomials:

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) &= b_o + b_1 q^{-1} + \cdots + b_m q^{-m} \end{aligned}$$

are co-prime and $B(q^{-1})$ is anti-Schur

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Control Objectives

1. Pole Placement: The poles of the closed-loop system must be placed at specific locations in the complex plane.

- **Closed-loop pole polynomial:**

$$\boxed{A_c(q^{-1}) = B(q^{-1})\, A_c^{'}(q^{-1})}$$

Where:

- $B(q^{-1})$ cancelable plant zeros
- $A_c^{'}(q^{-1})$ anti-Schur polynomial chosen by the designer

$$A_c^{'}(q^{-1}) = \underline{1} + a_{c1}^{'} q^{-1} + \cdots + a_{c n_c^{'}}^{'} q^{-n_c^{'}}$$

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Control Objectives

2. Tracking: The output sequence $y(k)$ must follow a ***reference*** sequence $y_d(k)$ which is known

- **Reference model:**

$$\boxed{A_c^{'}(q^{-1}) y_d(k) = q^{-\mathrm{d}}\, B_m(q^{-1})\, u_d(k)}$$

Where:

- $u_d(k)$ **known** reference input control input sequence
- $A_c^{'}(q^{-1})$ (from the previous slide)
- $B_m(q^{-1})$ zero polynomial, chosen by the designer

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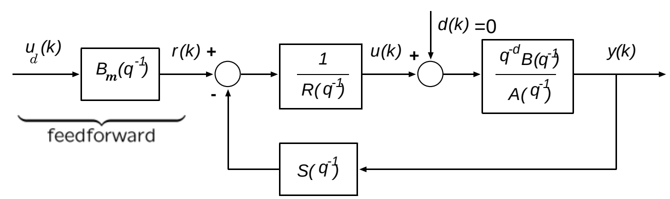
Control Law

- Feedback and feedforward actions:



$$u(k) = \frac{1}{R(q^{-1})}\left[r(k) - S(q^{-1}) y(k)\right]$$

$$r(k) = q^{+\mathrm{d}} A_c^{'}(q^{-1}) y_d(k) \; = B_m(q^{-1}) u_d(k)$$

$$\frac{q^{-\mathrm{d}} B_m(q^{-1})}{A_c^{'}(q^{-1})} u_d(k)$$

Feedforward is causal

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Feedback Controller

Diophantine equation: Obtain polynomials $\underline{R'(q^{-1})}, \underline{S(q^{-1})}$ that satisfy:

$$\boxed{A_c'(q^{-1}) = A(q^{-1})\,\underline{R'(q^{-1})} + q^{-\mathrm{d}}\,\underline{S(q^{-1})}}$$

Closed-loop poles *Plant poles* *Plant pure delays*

$$\boxed{\begin{aligned} R(q^{-1}) &= R'(q^{-1})\,B(q^{-1}) \\ A_c(q^{-1}) &= B(q^{-1})\,A_c'(q^{-1}) \end{aligned}}$$

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Diophantine equation

$$A_c'(q^{-1}) = A(q^{-1})\,R'(q^{-1}) + q^{-\mathrm{d}}\,S(q^{-1})$$

Solution:

$$\begin{aligned} R'(q^{-1}) &= 1 + r_1' q^{-1} + \cdots + r_{n_r'}' q^{-n_r'} \\ S(q^{-1}) &= s_o + s_1 q^{-1} + \cdots + s_{n_s} q^{-n_s} \end{aligned}$$

$$\boxed{\begin{aligned} n_r' &= \mathrm{d} - 1 \\ n_s &= \max\{\, n-1 \,,\, n_c' - \mathrm{d} \,\} \end{aligned}}$$

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Feedback Controller

$$u(k) = \frac{1}{R(q^{-1})}\left[r(k) - S(q^{-1})y(k)\right]$$

where

$$R(q^{-1}) = R'(q^{-1})\,B(q^{-1})$$

$$\boxed{\begin{aligned} n_r' &= \mathrm{d} - 1 \\ n_s &= \max\{\, n-1 \,,\, n_c' - \mathrm{d} \,\} \\ n_r &= n_r' + m \end{aligned}}$$

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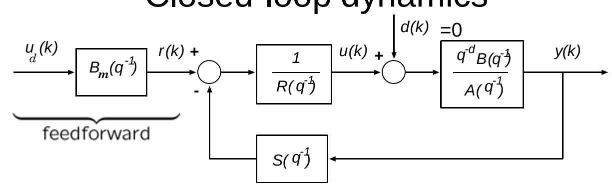
Closed-loop dynamics



$$\begin{aligned} A_c'(q^{-1})\,y(k) &= q^{-\mathrm{d}}\,r(k) \\ &= q^{-\mathrm{d}} B_m(q^{-1}) u_d(k) \\ &= A_c'(q^{-1})\,y_d(k) \end{aligned}$$

$$\boxed{A_c'(q^{-1})\,\{\,y(k) - y_d(k)\} = 0}$$

Direct Adaptive Control

1. Plants with minimum phase zeros and no disturbances:
   - Controller design
   1. Controller PAA
   2. Adaptive Controller

Controller parameters

We want to identify the controller polynomials

$$R(q^{-1}) \qquad S(q^{-1})$$

directly, where

$$R(q^{-1}) = R'(q^{-1})\, B(q^{-1})$$

$$R(q^{-1}) = \underbrace{r_o}_{=b_o} + r_1\, q^{-1} + \cdots + r_{n_r}\, q^{-n_r}$$

$$S(q^{-1}) = s_o + s_1\, q^{-1} + \cdots + s_{n_s}\, q^{-n_s}$$

Controller parameters

Start with the Diophantine equation

$$A_c'(q^{-1}) = A(q^{-1})\, R'(q^{-1}) + q^{-\mathrm{d}}\, S(q^{-1})$$

Multiply both sides by $y(k)$

$$A_c'(q^{-1})\, y(k) = R'(q^{-1})\, A(q^{-1})\, y(k) + q^{-\mathrm{d}}\, S(q^{-1})\, y(k)$$

Controller parameters

$$A_c'(q^{-1})\, y(k) = R'(q^{-1})\, \underline{A(q^{-1})\, y(k)} + q^{-\mathrm{d}}\, S(q^{-1})\, y(k)$$

Insert plant dynamics

$$A(q^{-1})\, y(k) = q^{-\mathrm{d}}\, B(q^{-1})\, u(k)$$

$$A_c'(q^{-1})\, y(k) = q^{-\mathrm{d}} \left[\underline{R'(q^{-1})\, B(q^{-1})}\, u(k) + S(q^{-1})\, y(k)\right]$$

$$\boxed{A_c'(q^{-1})\, y(k) = q^{-\mathrm{d}} \left[R(q^{-1})\, u(k) + S(q^{-1})\, y(k)\right]}$$

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PAA – version 1

$$A_c'(q^{-1})\,y(k) = q^{-\mathrm{d}}\left[R(q^{-1})\,u(k) + S(q^{-1})\,y(k)\right]$$

Filter by $1/A_c{}'(q^{-1})$ (*normally a low-pass filter*)

$$\boxed{y(k) = R(q^{-1})\,u_f(k-\mathrm{d}) + S(q^{-1})\,y_f(k-\mathrm{d})}$$

$$\begin{aligned} y_f(k) &= \frac{1}{A_c'(q^{-1})}\,y(k) \\ u_f(k) &= \frac{1}{A_c'(q^{-1})}\,u(k) \end{aligned}$$

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PAA – version 1

$$y(k) = R(q^{-1})\,u_f(k-\mathrm{d}) + S(q^{-1})\,y_f(k-\mathrm{d})$$

Is linear in the controller parameters:

$$\boxed{y(k) = \phi_f^T(k-\mathrm{d})\theta_c}$$

$$\theta_c = \begin{bmatrix} s_o & \cdots & s_{n_s} & r_o & \cdots & r_{n_r} \end{bmatrix}^T \in \mathcal{R}^{N_c}$$

$$N_c = n_s + n_r + 2$$

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PAA – version 1

Plant dynamics:

$$\boxed{y(k) = \phi_f^T(k-\mathrm{d})\theta_c}$$

$$\theta_c = \begin{bmatrix} s_o & \cdots & s_{n_s} & r_o & \cdots & r_{n_r} \end{bmatrix}^T \in \mathcal{R}^{N_c}$$

$$\phi_f(k) = \frac{1}{A_c'(q^{-1})}\,\phi(k)$$

$$\phi(k) = \begin{bmatrix} y(k) & \cdots & y(k-n_s) & u(k) & \cdots & u(k-n_r) \end{bmatrix}^T$$

$$N_c = n_s + n_r + 2$$

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PAA – version 1

Plant dynamics:

$$\boxed{y(k) = \phi_f^T(k-\mathrm{d})\theta_c}$$

RLS PAA:

$$\begin{aligned} e^o(k+1) &= y(k+1) - \phi_f^T(k-\mathrm{d}+1)\hat{\theta}_c(k) \\ e(k+1) &= \frac{\lambda_1(k)}{\lambda_1(k) + \phi_f^T(k-\mathrm{d}+1)F(k)\phi_f(k-\mathrm{d}+1)}e^o(k+1) \\ \hat{\theta}_c^o(k+1) &= \hat{\theta}_c(k) + \frac{1}{\lambda_1(k)}F(k)\phi_f(k-\mathrm{d}+1)e(k+1) \\ F(k+1) &= \frac{1}{\lambda_1(k)}\left[F(k) - \lambda_2(k)\frac{F(k)\phi_f(k-\mathrm{d}+1)\phi_f^T(k-\mathrm{d}+1)F(k)}{\lambda_1(k) + \lambda_2(k)\phi_f^T(k-\mathrm{d}+1)F(k)\phi_f(k-\mathrm{d}+1)}\right] \end{aligned}$$

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PAA – version 2

$$A_c'(q^{-1})\,y(k) = q^{-\mathrm{d}}\left[R(q^{-1})\,u(k) + S(q^{-1})\,y(k)\right]$$

$$\boxed{\eta(k) = A_c'(q^{-1})\,y(k)}$$

filtered output signal

$$\boxed{\eta(k) = \phi^T(k-\mathrm{d})\theta_c}$$

$$\theta_c = \begin{bmatrix} s_o & \cdots & s_{n_s} & r_o & \cdots & r_{n_r}\end{bmatrix}^T \in \mathcal{R}^{N_c}$$

$$\phi(k) = \begin{bmatrix} y(k) & \cdots & y(k-n_s) & u(k) & \cdots & u(k-n_r)\end{bmatrix}^T$$

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PAA – version 2

Plant dynamics:

$$\boxed{\eta(k) = \phi^T(k-\mathrm{d})\theta_c}$$

RLS PAA:

$$e^o(k+1) = \eta(k+1) - \phi^T(k-\mathrm{d}+1)\hat{\theta}_c(k)$$

$$e(k+1) = \frac{\lambda_1(k)}{\lambda_1(k) + \phi^T(k-\mathrm{d}+1)F(k)\phi(k-\mathrm{d}+1)}e^o(k+1)$$

$$\hat{\theta}_c^o(k+1) = \hat{\theta}_c(k) + \frac{1}{\lambda_1(k)}F(k)\phi(k-\mathrm{d}+1)e(k+1)$$

$$F(k+1) = \frac{1}{\lambda_1(k)}\left[F(k) - \lambda_2(k)\frac{F(k)\phi(k-\mathrm{d}+1)\phi^T(k-\mathrm{d}+1)F(k)}{\lambda_1(k) + \lambda_2(k)\phi^T(k-\mathrm{d}+1)F(k)\phi(k-\mathrm{d}+1)}\right]$$

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PAA – version 1 Vs version 2

- $A_c'(q^{-1})$ is normally a ***high-pass*** filter
- $1/A_c'(q^{-1})$ is normally a ***low-pass*** filter

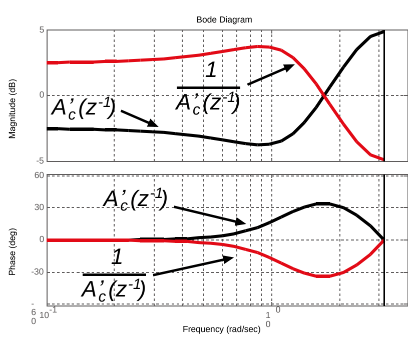
Example

$$A_c'(q^{-1}) = (1-.5q^{-1})^2$$

Version 1 is preferable

$$\phi_f(k) = \frac{1}{A_c'(q^{-1})}\phi(k)$$

filters high frequency noise



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PAA projection

PAA: Projection

$$\hat{r}_o(k)$$

$$\hat{\theta}_c(k) = \begin{cases} \hat{\theta}_c^o(k) & \text{if } \hat{r}_o^o(k) \geq b_{mino} \\ \left[\ \hat{s}_o^o(k)\cdots\hat{s}_{n_s}^o(k)\ b_{mino}\ \cdots\ \hat{r}_{n_r}^o(k)\ \right]^T & \text{if } \hat{r}_o^o(k) < b_{mino} \end{cases}$$

Replace $\hat{r}_o^o(k)$ by b_{mino} if it becomes too small.

Control law will divide by $\hat{r}_o(k)$. Thus, the projection algorithm prevents the control action from becoming too large.

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PAA Gain matrix

Gain matrix:

$$F(k+1) = \frac{1}{\lambda_1(k)}\left[F(k) - \lambda_2(k)\frac{F(k)\phi_f(k-\mathrm{d}+1)\phi_f^T(k-\mathrm{d}+1)F(k)}{\lambda_1(k)+\lambda_2(k)\phi_f^T(k-\mathrm{d}+1)F(k)\phi_f(k-\mathrm{d}+1)}\right]$$

$$\begin{aligned} 0 &< \lambda_1(k) \le 1 \\ 0 &\le \lambda_2(k) < 2 \end{aligned}$$

are adjusted so that the maximum singular value of $F(k)$ is uniformly bounded, and

$$0 < K_{\min} \le \lambda_{\min}\{F(k)\} \le \lambda_{\max}\{F(k)\} < K_{max} < \infty\,.$$

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Direct Adaptive Control

1. Plants with minimum phase zeros and no disturbances:
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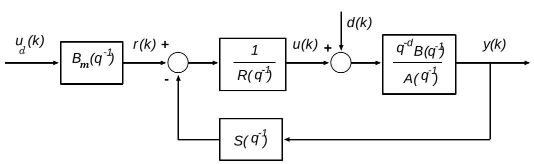
Fixed Controller



$$R(q^{-1})\,u(k) = B_m(q^{-1})\,u_d(k) - S(q^{-1})y(k)$$

Use this equation to solve for u(k)

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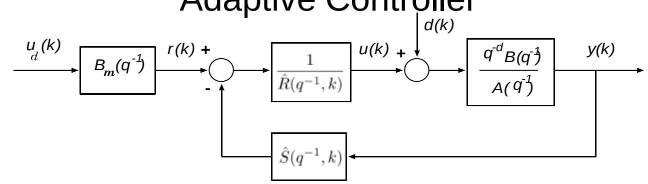
Adaptive Controller



$$\hat{R}(q^{-1},k)\,u(k) = B_m(q^{-1})\,u_d(k) - \hat{S}(q^{-1},k)y(k)$$

Use this equation to solve for u(k)

Direct Adaptive Control

1. Plants with minimum phase zeros and no disturbances:

   - Controller design
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   2. Adaptive Controller

2. Plants with minimum phase zeros and constant disturbances:

- Read section: ***Direct adaptive control with integral action for plants with stable zeros*** in the ME233 class notes, part II.