UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II

Midterm Examination II **Spring 2008**

Closed Book and Closed Notes. Six 8.5×11 pages of handwritten notes and photocopies of the Laplace and Z-transform tables in the ME232 class notes allowed.

Your Name:

Please answer all questions.

Problem:	1	2	3	Total
Max. Grade:	35	30	35	100
Grade:				

1 Problem

Consider the following stationary stochastic system

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k) \\ y(k) &= x_1(k) + v(k) \end{aligned}$$

where $u(k)$ is the controlling input, $y(k)$ is the measured output, $w(k)$ is white, zero mean, Gaussian and stationary random input noise, $v(k)$ is white, zero mean, Gaussian and stationary random input noise. Also assume that

$$E\left\{ \begin{bmatrix} w(k) & v(k) \end{bmatrix} \begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \right\} = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix} \succ 0$$

The control objective is to design an LQG compensator

$$u(k) = -K\hat{x}(k)$$

that minimizes the following cost function

$$J = E\left\{ y^2(k) + \rho\, u^2(k) \right\} \qquad 0 < \rho < \infty$$

where $\hat{x}(k) = E\{x(k) | y(0) \cdots y(k)\}$.

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1. Determine if there exists a unique asymptotically stabilizing LQG solution to this problem.

2. Draw the root locus for the steady state Kalman filter close loop poles and their inverses for $\frac{W}{V} \in (0, \infty)$.

3. Determine the steady state Kalman filter when $W = 4$ and $V = 5/4$.

2 Problem

A continuous time LTI plant is described by the following state space realization and transfer function

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad G(s) = \frac{1}{s(s+1)}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Assuming that the plant state $x(t)$ is measurable, the optimal control that minimize the performance index

$$\begin{aligned} J &= \int_{-\infty}^{\infty} \{Y_f(-j\omega)Y_f(j\omega) + R\,U_f(-j\omega)U_f(j\omega)\}\; d\omega \\ &= \int_{-\infty}^{\infty} \left\{\frac{1}{\omega^2}Y(-j\omega)Y(j\omega) + R\,\frac{\omega^2+1}{\omega^2+4}\,U(-j\omega)U(j\omega)\right\} d\omega \qquad R > 0 \end{aligned}$$

is of the form

$$u(t) = -K_e\, x_e(t)\,.$$

where

$$\begin{aligned} &K_e = R_e^{-1}[P_e B_e^T + N_e] \\ &P_e A_e + A_e^T P_e - [P_e B_e^T + N_e] R_e^{-1} [P_e B_e^T + N_e]^T + Q_e = 0 \end{aligned}$$

1. Define the state equation for the extended state x_e and the matrices Q_e and N_e.
2. Write down the conditions that guarantee that a unique solution to the above optimal control problem exists, which asymptotically stabilizes the feedback system. (You do not need to verify that these conditions are satisfied.)

3 Problem

Consider the discrete time system

$$(1+q^{-1})\,y(k) = q^{-1}\left[(1-0.5q^{-1})(1+2q^{-1})\,u(k) + d\right]$$

where q^{-1} is the one-step delay operator, $u(k)$ is the controlling input, $y(k)$ is the measured output, d is an unknown constant disturbance.

You are required to design a control system that satisfies the following two requirements:

- The system output should track an arbitrary desired output $y_d(k)$, which is known two steps in advance, with zero phase error.
- The closed loop poles of the feedback system should only include poles at the origin and the canceled zeros.

Clearly indicate all steps of your design process, which includes solving a Diophantine (Bezout) equation. You do not have to explicitly solve this equation. However, you should write down the linear equation $Mx = b$, that must be solved in order to obtain the solution of the Diophantine equation. This includes clearly defining the elements of the unknown vector x and all of the coefficients of the matrix M and the vector b.