ME 233 Spring 2011
Solution to Homework #6

1. (a) Based on the given figure in problem set, we have:

$$\begin{aligned}
U(z) &= \bar{U}(z) - \hat{D}(z) = \bar{U}(z) - Q(z)\left[G_n^{-1}(z)Y(z) - U(z)\right] \\
\Rightarrow (1 - Q(z))\,U(z) &= \bar{U}(z) - Q(z)G_n^{-1}(z)Y(z) \\
\Rightarrow U(z) &= \frac{1}{1 - Q(z)}\bar{U}(z) - \frac{Q(z)G_n^{-1}(z)}{1 - Q(z)}Y(z) \\
\Rightarrow T_{u\leftarrow y}(z) &= -\frac{Q(z)G_n^{-1}(z)}{1 - Q(z)}, \qquad T_{u\leftarrow \bar{u}}(z) = \frac{1}{1 - Q(z)}
\end{aligned}$$

(b) Based on the lecture notes, we know the closed loop transfer function from $D(z)$ to $P(z)$ is

$$T_{p\leftarrow d}(z) = -\frac{G(z)G_n(z)(1 - Q(z))}{G_n(z) + Q(z)(G(z) - G_n(z))} .$$

Obviously, in order to make the closed loop transfer function from $D(z)$ to $P(z)$ be 0, we have to choose $Q(z) = 1$. But from the result in the previous part, we know that the disturbance observer is not well-posed when $Q(z) = \alpha = 1$. Thus, we can not obtain the perfect disturbance attenuation that disturbance does not appear in the output by using the disturbance observer. But we can choose α arbitrarily closed to 1 and then we can make the effect of the disturbance on the output arbitrarily close to 0.

2. (a) To find the discretized plant, we use the MATLAB code

```
>> Gd = c2d( tf(1,[1 1 0]), 0.5 )
```

This gives that

$$\begin{aligned}
G(z) &= \frac{0.1065z + 0.0902}{z^2 - 1.607z + 0.6065} \\
\Rightarrow A(q^{-1}) &= 1 - 1.607q^{-1} + 0.6065q^{-2} \\
B(q^{-1}) &= 0.1065 + 0.0902q^{-1} \\
d &= 1.
\end{aligned}$$

(b) The continuous time poles are given by

$$\begin{aligned}
p_c &= -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \\
&= -0.707 \pm j\sqrt{1 - 0.707^2} \\
&= -0.707 \pm 0.7072j.
\end{aligned}$$

Therefore, the discrete time poles are given by

$$\begin{aligned}
p_d &= e^{p_c T} \\
&= 0.6588 \pm 0.2432j.
\end{aligned}$$

Using the MATLAB command `poly`, we find that the polynomial with these roots is given by

$$z^2 - 1.3176z + 0.4931 = 0.$$

Therefore, $A_m(q^{-1})$ is given by

$$A_m(q^{-1}) = 1 - 1.3176q^{-1} + 0.4931q^{-2}.$$

(c) We choose

$$b_{mo} = A_m(1) = 0.1755.$$

(d) As before

$$\begin{aligned} p_c &= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \\ &= -1 \pm 2j\sqrt{1-0.5^2} \\ &= -1 \pm 1.7321j \\ \Rightarrow p_d &= e^{p_c T} = 0.3929 \pm 0.462j. \end{aligned}$$

Using the MATLAB command `poly`, we find that the polynomial with these roots is given by

$$z^2 - 0.7859z + 0.3679 = 0$$

which means that we should choose

$$A_c'(q^{-1}) = 1 - 0.7859q^{-1} + 0.3679q^{-2}.$$

(e) In this case, $A_d(q^{-1}) = 1$ and $B^u(q^{-1}) = b_0 = 0.1065$. Thus, the Bezout (Diophantine) equation for this case is given by

$$\begin{aligned} A_c'(q^{-1}) &= A(q^{-1})R'(q^{-1}) + q^{-1}b_0 S(q^{-1}) \\ \Rightarrow 1 - 0.7859q^{-1} + 0.3679q^{-2} &= \left(1 - 1.607q^{-1} + 0.6065q^{-2}\right)(1) + q^{-1}0.1065\left(s_0 + s_1 q^{-1}\right). \end{aligned}$$

Equating coefficients gives

$$\begin{aligned} s_0 &= 7.7108 \\ s_1 &= -2.2404. \end{aligned}$$

Therefore

$$\begin{aligned} S(q^{-1}) &= 7.7108 - 2.2404q^{-1} \\ R(q^{-1}) &= R'(q^{-1})B^s(q^{-1}) \\ &= 1 + 0.8467q^{-1}. \end{aligned}$$

With this choice of $R(q^{-1})$ and $S(q^{-1})$, the dynamics from $r(z)$ to $y(z)$ are given by

$$\begin{aligned} y(z) &= \frac{z^{-1}b_0 B^s(z^{-1})}{B^s(z^{-1})A_c'(z^{-1})} r(z) \\ &= \frac{b_0}{zA_c'(z^{-1})} r(z). \end{aligned}$$

In order to obtain the dynamics $y(z) = y_d(z)$, we see that we should choose

$$\begin{aligned} r(z) &= \left(zA_c'(z^{-1})\right) y_d(z) \\ \Rightarrow r(k) &= \left(qb_0^{-1}A_c'(q^{-1})\right) y_d(k) \\ \Rightarrow T(q^{-1}, q) &= qb_0^{-1}A_c'(q^{-1}) \\ &= 9.3897q - 7.3793 + 3.4545q^{-1}. \end{aligned}$$

(f) See Fig. 1 for simulation results.

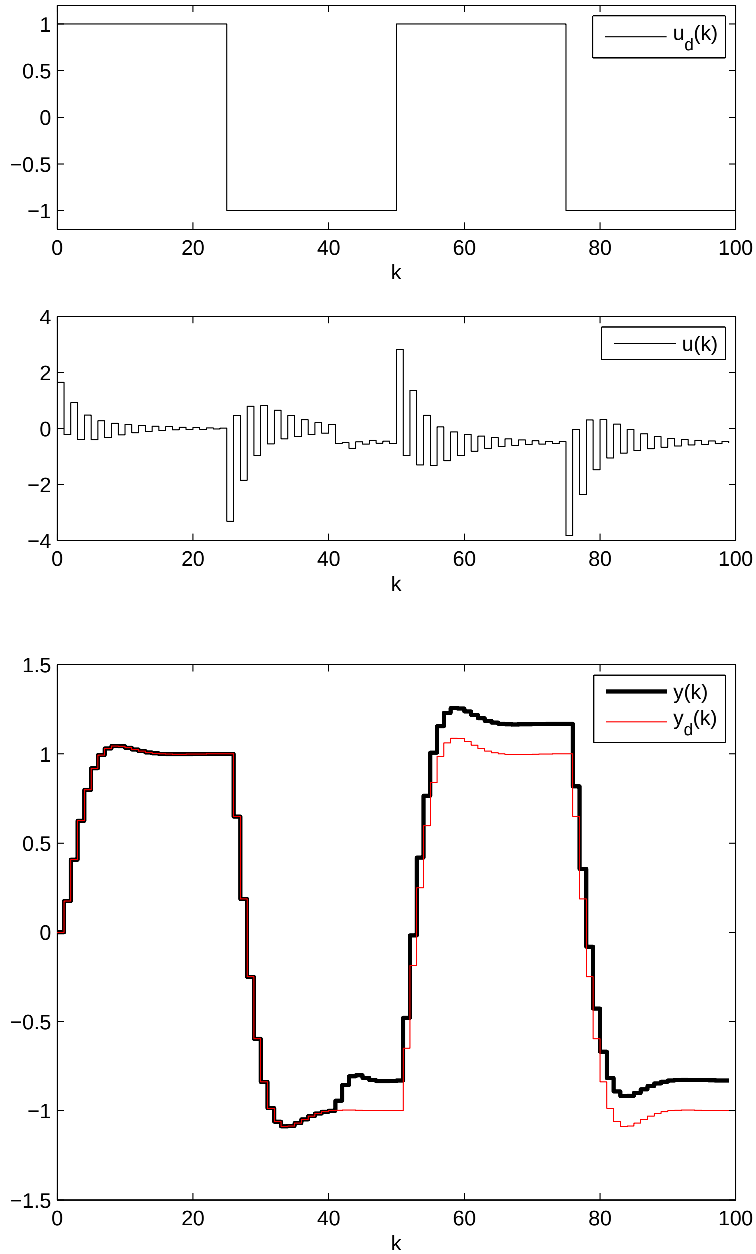

Figure 1: Closed loop simulation for $(A_d, B^u) = (1, b_0)$

(g) In this case, $A_d = 1 - q^{-1}$ and $B^u = b_0 = 0.1065$. Thus, the Bezout (Diophantine) equation for this case is given by

$$\begin{aligned} A_c' &= A_d A R' + q^{-1} b_0 S \\ \Rightarrow 1 - 0.7859q^{-1} + 0.3679q^{-2} &= \left(1 - q^{-1}\right)\left(1 - 1.607q^{-1} + 0.6065q^{-2}\right)(1) \\ &\quad + q^{-1} b_0 \left(s_0 + s_1 q^{-1} + s_2 q^{-2}\right). \end{aligned}$$

Equating coefficients gives

$$\begin{aligned} s_0 &= 17.1005 \\ s_1 &= -17.3296 \\ s_2 &= 5.6948 \end{aligned}$$

Therefore

$$\begin{aligned} S &= 17.1005 - 17.3296q^{-1} + 5.6948q^{-2} \\ R &= R' A_d B^s \\ &= (1)\left(1 - q^{-1}\right)\left(1 + 0.8467q^{-1}\right) \\ &= 1 - 0.1531q^{-1} - 0.8467q^{-2} \end{aligned}$$

As before, since the dynamics from $r(k)$ to $y(k)$ are now given by

$$b_0^{-1} A_c' y(k) = q^{-1} r(k)$$

we still set

$$T = q b_0^{-1} Ac' = 9.3897q - 7.3793 + 3.4545q^{-1}.$$

to get the desired dynamics.

(h) See Figure 2 for simulation results.

(i) In this case, $A_d = 1 - q^{-1}$, $B^s = 1$, and $B^u(q^{-1}) = 0.1065 + 0.0902q^{-1}$. Thus, the Bezout (Diophantine) equation for this case is given by

$$\begin{aligned} A_c' &= A_d A R' + q^{-1} B^u S \\ \Rightarrow 1 - 0.7859q^{-1} + 0.3679q^{-2} &= \left(1 - q^{-1}\right)\left(1 - 1.607q^{-1} + 0.6065q^{-2}\right)\left(1 + r_1 q^{-1}\right) \\ &\quad + q^{-1}\left(0.1065 + 0.0902q^{-1}\right)\left(s_0 + s_1 q^{-1} + s_2 q^{-2}\right). \end{aligned}$$

Equating coefficients gives

$$\begin{aligned} r_1 &= 0.5937 \\ s_0 &= 11.5259 \\ s_1 &= -12.5585 \\ s_2 &= 3.9919. \end{aligned}$$

Therefore

$$\begin{aligned} S &= 11.5259 - 12.5585q^{-1} + 3.9919q^{-2} \\ R &= R' A_d B^s \\ &= \left(1 + 0.5937q^{-1}\right)\left(1 - q^{-1}\right) \\ &= 1 - 0.4063q^{-1} - 0.5937^{-2}. \end{aligned}$$

The zero-phase feedforward compensator is then given by

$$\begin{aligned} T &= \frac{q A_c' B_u(q)}{[B_u(1)]^2} = \frac{\left(q - 0.7859 + 0.3679q^{-1}\right)(0.1065 + 0.0902q)}{0.1967^2} \\ &= 2.3313q^2 + 0.9204q - 1.3056 + 1.0127q^{-1}. \end{aligned}$$

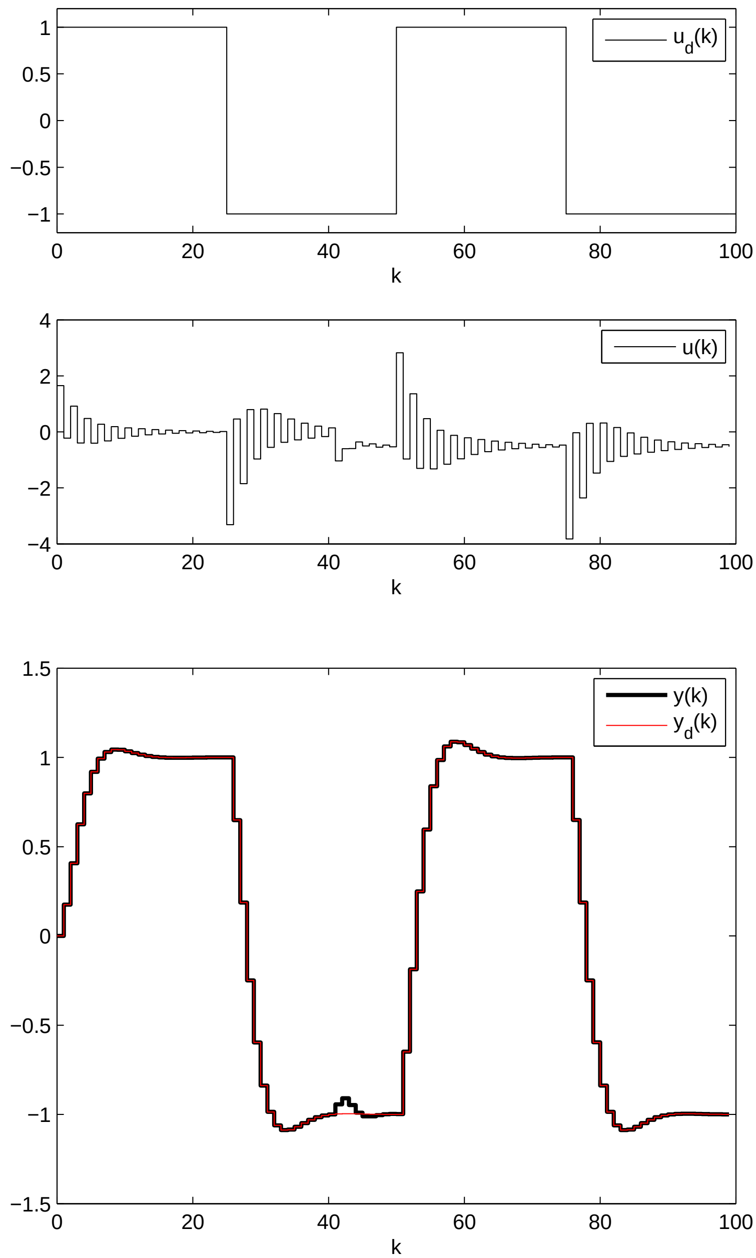

Figure 2: Closed loop simulation for $(A_d, B^u) = (1 - q^{-1}, b_0)$

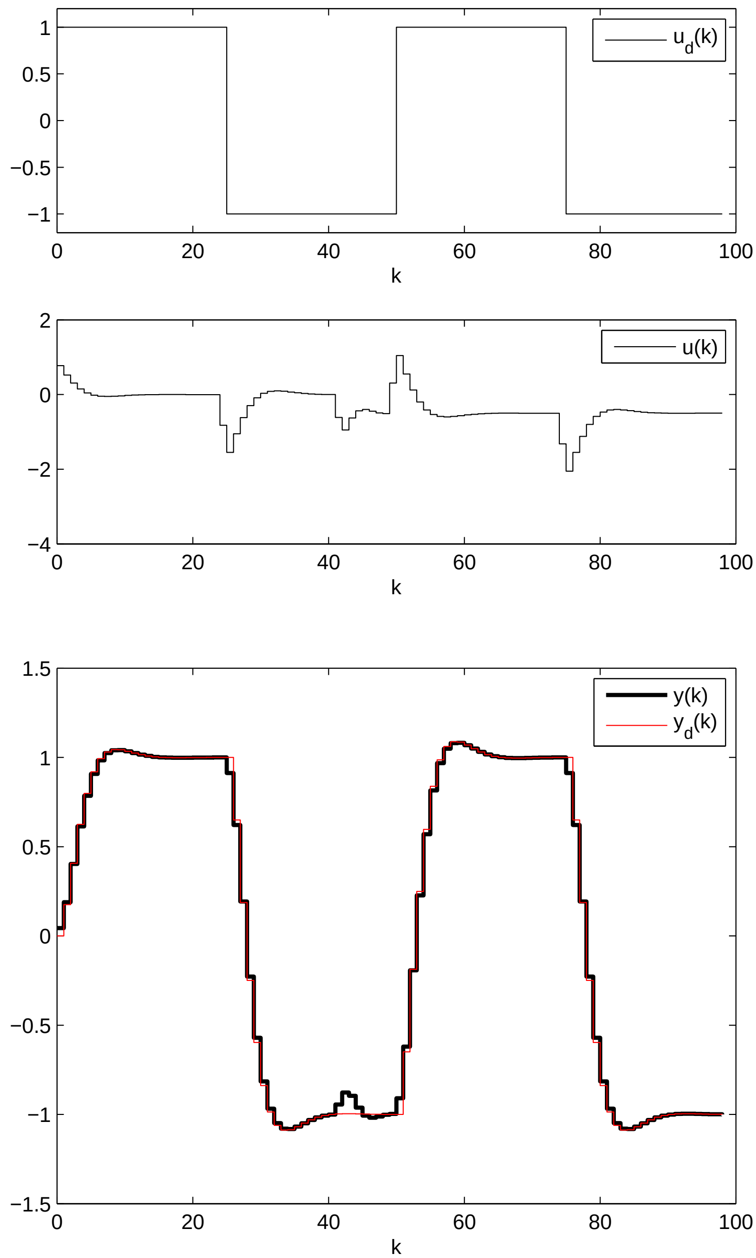

Figure 3: Closed loop simulation for $(A_d, B^u) = (1 - q^{-1}, 0.1065 + 0.0902q^{-1})$

(j) See Figure 3 for simulation results.

(k) If the disturbance is not considered in the controller design, there will be steady state error, as shown in Figure 1. When the disturbance model is considered in the controller design process, the steady state error is removed. If the stable pole-zero cancellation is performed, the tracking performance is better than the one in which pole-zero cancellation is not done, but the control input oscillates a little bit due to the lightly damped pole in the controller. With the zero-phase error tracking controller, the same performance can be achieved with smaller control effort, as can be seen by comparing Figure 2 with Figure 3.