University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2007
Midterm Examination I March 1, 2007 (Th)
Closed Books, Closed Notes, Open one summary sheet.

[1] (15 points) A discrete time second order system is described by

$$x(k+1)=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}x(k)+\begin{bmatrix}0\\ 1\end{bmatrix}u(k)$$

The quadratic performance index for this problem is

$$J=\sum_{k=0}^{\infty}\{x^T(k)\begin{bmatrix}1\\ 2\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix}x(k)+Ru^2(k)\}$$

1. Obtain the Riccati equation that you need to obtain the optimal control law. Find the solution of the Riccati equation that you need to obtain the optimal control.
2. Find the optimal feedback control law and optimal closed loop system for $R=0$. (Note: In discrete optimal LQ problem, R may often be made 0.) As implied in the performance index, the output we care is $y(k)=x_1(k)+2x_2(k)$. Find the output of the optimal closed loop system for $R=0$.
3. By utilizing the symmetric root locus method for the optimal discrete time LQ system, obtain the root locus plot of the optimal closed loop poles for varying R from 0 to ∞.

[2] (10 points) An asymptotically stable first order system is excited by a zero mean colored random input. Namely, the plan is described by

$$\frac{dx(t)}{dt}=ax(t)+bw(t)$$

where $\Phi_{ww}(\omega)=\dfrac{1}{\omega^2+\omega_0^2}$. Find $E[x^2(t)]$ at the steady state.

[3] (10 points) y is defined by

$$y=Cx+v$$

where x and v are zero mean Gaussian random vectors with

$$E[xx^T]=X,\quad E[vv^T]=V,\quad E[xv^T]=S$$

What is the least square estimate of x given y and the corresponding estimation error covariance matrix?

[4] (10 point) Consider the discrete time system described by

$$x(k+1)=Ax(k)+Bu(k)+B_w w(k)$$
$$y(k)=Cx(k)+v(k)$$

where $x(k)$ is an n-dimensional state vector, $u(k)$ is an m-dimensional input vector, $y(k)$ is an r-dimensional output vector, $w(k)$ is an s-dimensional input noise and $v(k)$ is an r-dimensional measurement noise. $x(0)$, $w(k)$ and $v(k)$ are Gaussian distributed and independent with

$E[x(0)]=x_0,\quad E[(x(0)-x_0)(x(0)-x_0)^T]=X_0,\quad E[w(k)]=0,\quad E[w(k)w^T(j)]=W\delta_{kj},\quad E[v(k)]=0,\quad E[v(k)v^T(j)]=V\delta_{kj}$

List all equations that you need to find $\hat{x}(k+3\,|\,k)$ and the associated estimation error covariance.