UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II, Spring 2010

Homework #9

Assigned: Th., April 8
Due: Th., April 15

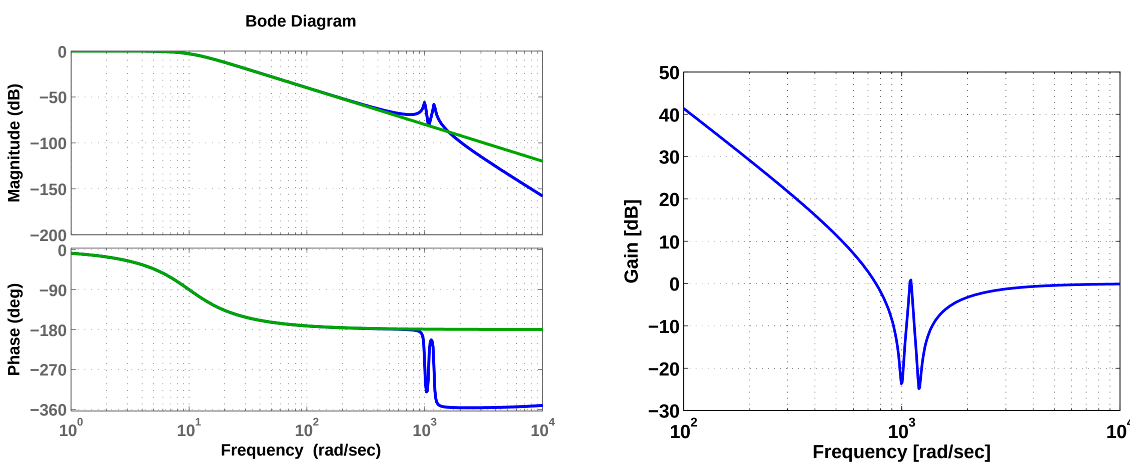
Warning: This homework involves doing quite a bit of computer simulation. Please do not procrastinate until the last day.

1. In this problem we consider the design of a compensator for a disk file voice-coil motor and head suspension using the LQG/LTR design methodology and matlab. We will refer to these components as the VCM. Most VCM have numerous high-frequency vibratory modes, which may vary from one unit to another, and are often not included in the actuator model. Instead, they are collectively treated as plant dynamic uncertainty. We will pursue this approach here, be neglecting all high-frequency vibratory VCM and suspension modes in the nominal design, but we will need to verify that the design is robust to these uncertainties. To this end, we will use two plant models:

Simplified Nominal Model: This model will be used as the plant (control object) in the LQG-LTR synthesis procedure. In this case we assume that the VCM can be modeled as a second order system with transfer function

$$G_p(s) = \frac{\omega_b^2}{s^2 + 2\zeta_b \omega_b\, s + \omega_b^2} \tag{1}$$

where $\zeta_b = 0.707$ and $\omega_b = 10$ rad/sec, is the resonance frequency of the so-called bearing (and ribbon cable) resonance mode.



(a) (b)

Figure 1: (a) Bode plots of disk drive voice-coil actuator simplified nominal model transfer function $G_p(j\omega)$ in Eqs. (1) and the "actual" model transfer function $G_{PA}(j\omega)$ in (2); (b) Magnitude Bode plot of $1/\Delta(j\omega)$

Actual Model: This model will later be used in place of $G_p(s)$ to test the robustness of the designed feedback systems. In this case we assume that the plant includes the a low-frequency bearing (and ribbon cable) resonance mode in and $G_p(s)$ plus high-frequency

VCM "butterfly" and suspension torsional resonance modes:

$$G_{PA}(s) = G_p(s)\left(\frac{\omega_r^2(\frac{\zeta_r}{\omega_r}s+1)}{s^2+2\zeta_r\omega_r\,s+\omega_r^2}\right)\left(\frac{\frac{\omega_t^2}{\omega_n^2}(s^2+2\zeta_t\omega_n\,s+\omega_n^2)}{s^2+2\zeta_t\omega_t\,s+\omega_t^2}\right) \tag{2}$$

where $\zeta_r = 0.015$, $\omega_r = 1000$ rad/sec, $\zeta_t = 0.015$, $\omega_t = 1200$ rad/sec, and $\omega_n = 0.9\,\omega_t$. Figure 1(a) shows the Bode plots of the two models.

Here we first use the LQG-LTR design procedure outlined in Lecture class notes, assuming that the plant is the Simplified Nominal Model, $G_p(s)$ in (1). In order to cancel the effect of a constant input disturbance W_d in Fig. 2, we augment the plant dynamics with an additional integrator, $G_a(s) = \frac{1}{s}$, by defining the extended system

$$G_e(s) = G_p(s)G_a(s) == C_e\Phi_e(s)B_e \tag{3}$$

We now need to design an LQG compensator for the extended system, as shown in Fig. 2, where

$$C_{LQG}(s) = K_e(sI - A_e + B_eK_e + C_eL_e)^{-1}L_e$$

using the LQG-LTR procedure.

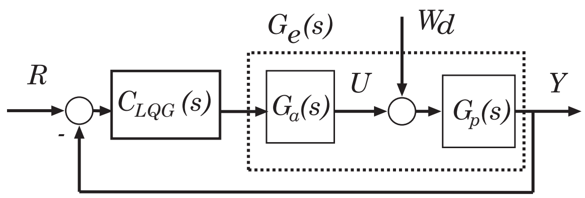


Figure 2: LQG Compensator applied on the extended systems

(a) Obtain using matlab the transfer function $\Delta(s)$ such that

$$G_{PA}(s) = (1+\Delta(s))G_p(s)$$

The magnitude Bode plot of $1/\Delta(j\omega)$ should look as in Fig. 1(b).

(b) Obtain using matlab the extended transfer function $G_e(s)$ in Eq. (3). You will also need later the actual extended system $G_{EA}(s) = G_{PA}(s)G_a(s)$.

(c) We now need to design the target fictitious Kalman filter dynamics in Fig. 3 setting the

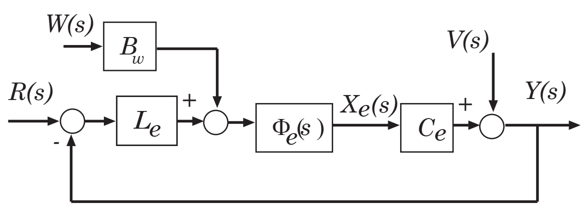


Figure 3: LQG Compensator applied on the extended systems

input noise intensity $W = 1$, the measurement noise intensity $V = \mu^2$ and using μ and the elements of the vector B_w as design parameters.

i. Set $B_w = B_e$ and plot the Bode plot of $G_w(s) = C_e\Phi_e(s)B_w$. Remember that at low frequencies where $|G_w(j\omega)|/\mu > 1$, the target open loop $G_{o_{kf}}(s) = C_e\Phi_e(s)L_e$ satisfies $|G_{o_{kf}}(j\omega)| \approx |G_w(j\omega)|/\mu$.

ii. Calculate the observer gain L_e from

$$L_e = \frac{1}{V} M\, C_e^T$$
$$A_e M + M A_e^T + B_w B_w^T - \frac{1}{V} M C_e^T C_e M = 0$$

by selecting $B_w = B_e$ and several values of $V = \mu^2$. Plot the resulting Bode plots of the open loop target transfer function

$$G_{o_{kf}}(s) = C_e\Phi_e(s)L_e$$

iii. Remembering the the gain crossover frequency of $G_{o_{kf}}(j\omega)$ is inversely proportional to μ, find the value of μ_1 so that inequality (4) is approximately satisfied.

$$|G_{o_{kf}}(j\omega)|_{db} \leq |\frac{1}{\Delta(j\omega)}|_{db} - 5db \tag{4}$$

for all $\omega > 0$.

iv. Verify that, since at high frequencies the complementary sensitivity transfer function

$$T_{kf}(s) = \frac{G_{o_{kf}}(j\omega)}{1 + G_{o_{kf}}(j\omega)}$$

aproximately satisfies $|T_{kf}(j\omega)| \approx ||G_{o_{kf}}(j\omega)|$, it should also approximatedly satisfy

$$|T_{kf}(j\omega)|_{db} \leq |\frac{1}{\Delta(j\omega)}|_{db} - 5db\,.$$

v. Plot the resulting closed loop unit step response of the target system in Fig. 3 for the value of L_e that is obtained from μ_1.

(d) Once the observer gain L_e has been determined, it is now necessary to compute the state feedback gain K_e. Compute K_e using the recovery procedure:

$$K_e = \frac{1}{\rho} N^{-1} B_e^T P$$
$$A_e^T\, P + P\, A_e + C_e^T C_e - \frac{1}{\rho} P\, B_e\, B_e^T P = 0$$

for small values of ρ. Plot the Bode plots (gain and phase) of $G_{KF}(s)$ and $G_o(s) = G_e(s)C_{LQG}(s)$ and keep decreasing ρ until $Go(j\omega) \rightarrow G_{KF}(j\omega)$ in the frequency range of interest. (You may stop when the recovery is attained pass critical frequencies such as the gain and phase crossover frequencies). Obtain the closed loop unit step response of the system in Fig. 2 for the values of L_e and K_e that you have selected.

(e) After completing the recovery process above, plot the magnitude Bode plot of the nominal complementary transfer function

$$T_p(s) = \frac{G_{op}(j\omega)}{1 + G_{op}(j\omega)} \qquad G_{op}(j\omega) = G_p(j\omega)G_a(j\omega)G_{LQG}(j\omega)$$

and verify that it satisfies the small gain constraint

$$|T_p(j\omega)| < |\frac{1}{\Delta(j\omega)}|\,. \tag{5}$$

(f) Test the response of the LQG-LTR feedback system in Fig. 3 with the LQG compensators that were designed in the previous section, but use the actual plant $G_{PA}(s)$ in Eq. (2) in place of the simplified plant $G_p(s)$ in Eq. (1):

i. Compare the bode plots of $G_p(s)G_a(s)C_{LQG}(s)$ with that of $G_{PA}(s)G_a(s)C_{LQG}(s)$ and find their respective stability (gain and phase) margins.

ii. Compare the step responses of

$$\frac{G_p(s)G_a(s)C_{LQG}(s)}{1+G_p(s)G_a(s)C_{LQG}(s)} \qquad \text{with} \qquad \frac{G_{PA}(s)G_a(s)C_{LQG}(s)}{1+G_{PA}(s)G_a(s)C_{LQG}(s)}$$

for the LQG-LTR compensators $C_{LQG}(s)$ that were designed in sections (c) and (d).

(g) We will now explore the consequences of violating the the small gain constraint (5). Remember that this constraint is both necessary and sufficient for the closed loop system to remain stable <u>for all</u> possible uncertainties $\bar{\Delta}(s)$ with frequency response magnitude $|\bar{\Delta}(j\omega)| \leq |\Delta(j\omega)|$. However, in this case, we are considering a <u>specific</u> uncertainty $\Delta(s)$. Therefore, the robustness constraint (5) will be only a sufficient condition.

i. Consider the case when $\mu = 0.001$, for which the target "fictitious" Kalman filter complementary sensitivity transfer function, $|T_{kf}(j\omega)|$ is slightly larger than $|\frac{1}{\Delta(j\omega)}|$ for some frequencies. Obtain, the LQG compensator through the loop transfer recovery process and test whether it will stabilize the actual plant $G_{PA}(s)$.

ii. Consider now the case when $\mu_2 = 10^{-4}$. In this case, the target "fictitious" Kalman filter complementary sensitivity transfer function, $|T_{kf}(j\omega)|$ is significantly larger than $|\frac{1}{\Delta(j\omega)}|$ for some frequencies. Obtain, the LQG compensator through the loop transfer recovery process and test whether it will stabilize the actual plant $G_{PA}(s)$.

2. Consider the FS-LTR extended dynamics and cost function:

$$\dot{x}_e = A_e\, x_e + B_e\, u \tag{6}$$

$$J = \int_0^\infty \left\{ x_e^T C_e^T C e x_e + 2 x_e^T N_e u + \rho u^T D_2^T D_2 u \right\} dt \tag{7}$$

where

$$C_e = \begin{bmatrix} D_r C & C_r & 0 & 0 \\ D_1 & 0 & C_1 & 0 \\ 0 & 0 & 0 & \sqrt{\rho} C_2 \end{bmatrix} = \begin{bmatrix} & C_q & & \\ 0 & 0 & 0 & \sqrt{\rho} C_2 \end{bmatrix}$$

$$N_e^T = \begin{bmatrix} 0 & 0 & 0 & \rho D_2^T C_2 \end{bmatrix} \qquad D_2 = D_2^T \succ 0 \qquad \rho > 0$$

We will prove that when

- the pair $[A_e, B_e]$ is stabilizable and
- the pair $[A_e - B_e R_e^{-1} N_e^T, C_q]$ is detectable

the optimal control

$$u = -K_e x_e$$

$$K_e = R_e^{-1} \left[B_e^T P_e + N_e^T \right]$$

$$P_e A_e + A_e^T P_e + Q_e - \left[B_e^T P_e + N_e^T \right]^T R_e^{-1} \left[B_e^T P_e + N_e^T \right] = 0$$

yields an exponentially stable close loop system.

Step 1: Define the control law

$$u = -Lx_e + v\,, \tag{8}$$

where L is a gain to be determine and v is the new control input. Insert the control law (8) into Eqs. (6) and (7).

Step 2: Determine the required value of L so that we now obtain

$$\dot{x}_e = \bar{A}_e\, x_e + B_e\, v \tag{9}$$

$$J = \int_0^\infty \left\{ x_e^T \bar{Q}_e x_e + \rho v^T D_2^T D_2 v \right\} dt \tag{10}$$

and show that $\bar{A}_e = A_e - B_e R_e^{-1} N_e^T$ and $\bar{Q}_e = C_q^T C_q$. Finally, remember that $[A_e, B_e]$ is stabilizable iff $[A_e - B_e R_e^{-1} N_e^T, B_e]$ is stabilizable (why?).