UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2012

Homework #7

Assigned:	Apr. 3	(Tu)
Due:	Apr. 10	(Tu)

The second ME233 midterm will be held on Thursday, April 12th. The exam will be closed book and notes, but you are allowed to bring 4 double-sided sheets (i.e. 8 pages) of hand-written notes on $8.5'' \times 11''$ paper and a calculator. The midterm will focus on the material covered in Lectures 9–16, which includes the material in this assignment.

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1. Figure 1 shows the feedback interconnection for a system with a disturbance observer. When implementing the disturbance observer, we only implement the portion that

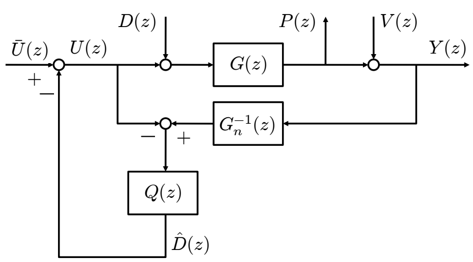

Figure 1: Disturbance Observer Structure

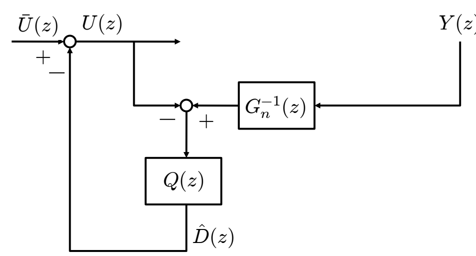

Figure 2: Disturbance Observer—Controller Only

generates $U(z)$ from $\bar{U}(z)$ and $Y(z)$, as shown in Fig. 2.

(a) Find the transfer function from $Y(z)$ and $\bar{U}(z)$ to $U(z)$ in Fig. 2.

(b) Suppose that G_n^{-1} is proper. In this case, it is valid to choose $Q(z) = \alpha \in \mathcal{R}$. Based on your answer from the previous part, note that the block diagram in Fig. 2 is not well-posed when $\alpha = 1$. Does there exist $\alpha \in \mathcal{R}$ such that the closed-loop transfer function from $D(z)$ to $P(z)$ is zero? If not, is there a limit to how small we can make the transfer function from $D(z)$ to $P(z)$?

2. Consider the feedback system in Fig. 3 where $u(k)$ and $d(k)$ are respectively the

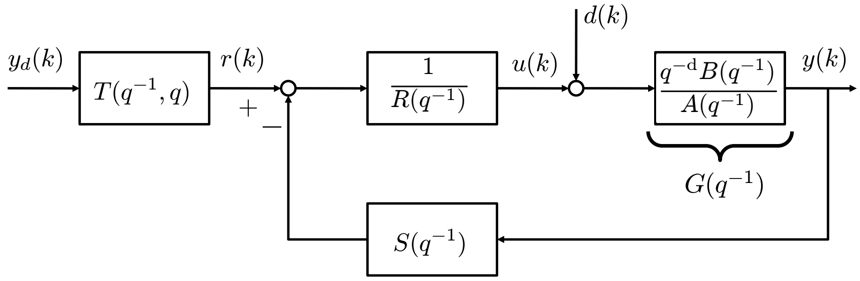

Figure 3: Feedback System

control and disturbance plant inputs, $y_d(k)$ is the reference model's output, and $r(k)$ is the reference input to the feedback block.

The control objective is to reject the persistent deterministic disturbance $d(k)$, place the feedback closed-loop poles, and track the desired output $y_d(k)$.

In order to help you verify your solutions of the Diophantine equation (also known as the Bezout equation), I have uploaded the MATLAB file bezout.m, which solves this equation. However, I advise you to solve the Diophantine equations in this problem by hand, so that you gain an understanding of what is involved in the solution of this type of equation.

(a) The plant transfer function $G(z)$ is derived from a continuous time transfer function $G(s)$ that is preceded by a zero-order hold and followed by a sampler, and is given by

$$G(z) = \frac{\bar{B}(z)}{\bar{A}(z)} = (1 - z^{-1})\, \mathcal{Z}\left\{ \mathcal{L}^{-1}\left(\frac{G(s)}{s} \right) \right\} \,,$$

where

$$G(s) = \frac{1}{s(s+1)}$$

and the sampling time is $T = 0.5$ seconds.
Calculate the plant polynomials $A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2}$, $B(q^{-1}) = b_o + b_1 q^{-1}$ and pure delay time d. You can use the MATLAB function `c2d` for this purpose.

(b) The tracking control objective is to follow the reference signal $y_d(k)$, which is the output of the reference model

$$A_m(q^{-1}) y_d(k) = q^{-\mathrm{d}} B_m(q^{-1}) u_d(k) \,. \tag{1}$$

Select the coefficients of the second order polynomial $A_m(q^{-1}) = 1 + a_{m1}\, q^{-1} + a_{m2}\, q^{-2}$, so that the reference model has a natural frequency of 1 rad/sec and a damping ratio of 0.707.

Hint: Remember that, since $z = e^{sT}$, we can calculate the discrete time poles by $p_d = e^{p_c T}$, where p_d is the discrete time pole, p_c is the continuous time pole and T is the sampling time.

(c) Letting $B_m(q^{-1}) = b_{mo}$, select b_{mo} so that the reference model has unity static gain [1].

(d) Choose the coefficients of the closed-loop system characteristic polynomial (after pole-zero cancelation)

$$A_c^{'}(q^{-1}) = 1 + a_{c1}^{'}\, q^{-1} + a_{c2}^{'}\, q^{-2}$$

so that the closed-loop feedback dynamics from $r(k)$ to $y(k)$ behaves as a second-order system with a natural frequency of 2 rad/sec and a damping ratio of 0.5.

(e) Design the control system under the following specifications and assumptions:

i. The closed-loop system characteristic polynomial (before pole-zero cancelation) is given by

$$A_c(q^{-1}) = A_c^{'}(q^{-1}) B^s(q^{-1})$$

where

$$B^s(q^{-1}) = \frac{1}{b_o} B(q^{-1}), \qquad\qquad B^u(q^{-1}) = b_o\,,$$

and b_o is the leading coefficient of $B(q^{-1})$. This means that all of the plant zeros will be canceled by the feedback system.

ii. Assume that $d(k) = 0$. This means that the disturbance annihilating polynomial is selected to be $A_d(q^{-1}) = 1$.

iii. The feedforward compensator $T(q^{-1}, q)$ must be selected so that perfect tracking is achieved under a zero initial state for both the plant and the reference model.

(f) Do a computer simulation of the control system designed in problem 2e when $y_d(-1) = y_d(0) = y(-1) = y(0) = 0$ and

$$u_d(k) = [u_s(k) - 2\, u_s(k-25)] + [2\, u_s(k-50) - 2\, u_s(k-75)] \tag{2}$$
$$d(k) = 0.5 u_s(k-40) \tag{3}$$

where $u_s(j)$ is the unit step function, i.e.

$$u_s(j) = \begin{cases} 0 & j < 0 \\ 1 & j \geq 0 \end{cases}$$

Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

(g) Design the control system under the same specifications in problem 2e, except that assume now that $d(k) = d(k-1)$.

[1] i.e. if $\lim_{k\to\infty} u_d(k) = u_{ss}$ then $\lim_{k\to\infty} y_d(k) = u_{ss}$.

(h) Do a computer simulation of the control system designed in problem 2g under the conditions described in problem 2f. Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

(i) Design the control system under the following specifications and assumptions:

i. The closed loop system characteristic polynomial satisfies

$$A_c(q^{-1}) = A_c^{'}(q^{-1})B^s(q^{-1})$$

where

$$B^s(q^{-1}) = 1, \qquad B^u(q^{-1}) = B(q^{-1}),$$

This means that none of the plant zeros will be canceled by the feedback system.

ii. Assume that $d(k) = d(k-1)$.

iii. The feedforward compensator $T(q^{-1}, q)$ is designed using the zero-phase error tracking control approach.

(j) Do a computer simulation of the control system designed in problem 2i under the conditions described in problem 2f. Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

(k) Discuss the outcome of the simulation results. In particular

- Comment on the effectiveness of the zero-phase feedforward control technique.
- Compare the control effort $u(k)$ when the zeros are canceled vs when the zeros are not canceled.