University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2013

Midterm Examination II April 11 (Th)
Closed books, Closed notes; you may refer to your own summary sheets. (one new sheet plus the one you used for Midterm Examination I)

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[1] (30 points) We are given a single-input single-output controllable and observable linear time invariant plant described by

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

where $x \in \mathbb{R}^n$. The transfer function of the plant is

$$G_p(s) = C(sI - A)^{-1} B = \frac{B_p(s)}{A_p(s)}$$

Though the disturbance is not explicitly included in the model, we know that the plant is subjected to a sinusoidal disturbance input with known frequency ω_d. Suppose we focus only on rejecting this sinusoidal disturbance.

a. Provide a FSLQ (Frequency Shaped Linear Quadratic) formulation to achieve the objective. Write down the cost function, formulate the symbolic state-space matrices for the enlarged system, and obtain the final control law including the Riccati equation. Use A_e, B_e, C_e and Q_e as the notations of the matrices. Assume that the plant is a second-order system with $y = x_1$. Write down a realization for A_e B_e C_e and Q_e. Your A_e B_e C_e and Q_e matrices should be composed of A, B, C, ω_d, and constant numbers.

b. Assume that the transfer function is $G_p(s) = \dfrac{s+0.5}{s^2 + 1.414s + 1}$ and $\omega_d = 2$. Apply the internal model principle and obtain a controller to achieve the objective. The controller should be of second order and the closed-loop system must be asymptotically stable. Verify that the frequency response of the sensitivity function is zero at $\omega_d = 2$.

[2] (15 points) Consider a motion control system described by the pure inertia equation,

$$m\frac{d^2 y(t)}{dt^2} = u(t) \;;\quad m > 0$$

where $u(t)$ and $y(t)$ are the force input and the position output, respectively. The objective is to let the output follow the desired output $y_d(t)$. The computed force (torque) control law for this purpose is

$$u(t) = m\frac{d^2 y_d(t)}{dt^2} + k_d\left(\frac{dy_d(t)}{dt} - \frac{dy(t)}{dt}\right) + k_p\left(y_d(t) - y(t)\right)$$

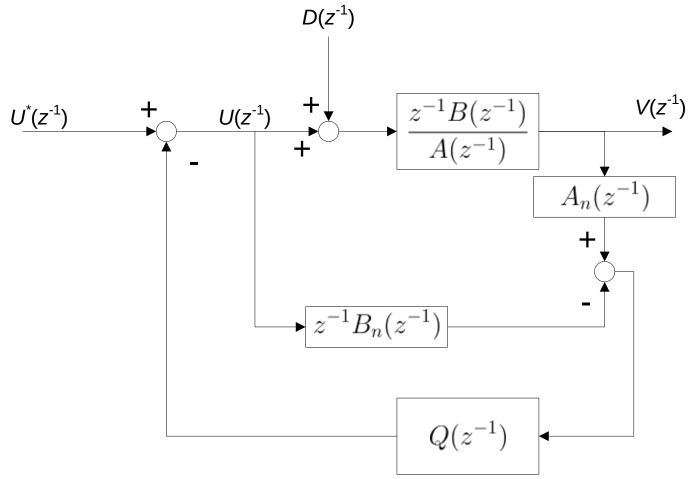
where k_d and k_p are real numbers. Define the tracking error by $e(t) = y_d(t) - y(t)$.

a. Find the condition(s) such that the error equation is asymptotically stable.
b. In the control law, all terms related to the desired output may be interpreted to define the feedforward control input, i.e.

$$u_{ff}(t) = m\frac{d^2 y_d(t)}{dt^2} + k_d \frac{dy_d(t)}{dt} + k_p y_d(t) \tag{2-1}$$

The control law is then PD feedback control with feedforward control. Obtain the (closed-loop) transfer function from $u_{ff}(t)$ to $y(t)$. Show that (2-1) is an inverse-based feedforward control law.

[3] (20 points) Consider the following block diagram which has several similarities to the disturbance-observer structure.



(a) Obtain the transfer functions from D and U^* to V

For the remaining parts of the problem, assume that $A(z^{-1}) = A_n(z^{-1})$ and $B(z^{-1}) = B_n(z^{-1})$.

(b) What is the condition for the closed loop to be stable? What is the (simplified) transfer function from the disturbance D to the output V? If $z^{-1}B(z^{-1})/A(z^{-1})$ is a minimum-phase system, how should $Q(z^{-1})$ be selected such that the above system behaves like a disturbance observer that we discussed in class?

(c) Recall that a sinusoidal signal $d(k) = c\sin(\omega_d k + \phi)$ satisfies $(1 - 2\cos\omega_d q^{-1} + q^{-2})d(k) = 0$ where q^{-1} is the one-step-delay operator and it corresponds to z^{-1} in the transform domain. Based on the minimum-phase assumption of $z^{-1}B(z^{-1})/A(z^{-1})$, propose a $Q(z^{-1})$ to perfectly reject such a disturbance and explain why it will work.

(d) Consider a simplified case where $B(z^{-1})/A(z^{-1})=1$ and $\omega_d = \pi/3$. In this case, if $Q(z^{-1}) = 0$, the transfer function from D to V is simply z^{-1}, namely, all disturbance components are transmitted to the output. For your $Q(z^{-1})$ design in (c), compute the DC gain (i.e., when z=1) and the gain at Nyquist frequency (i.e., when z=-1) of the transfer function you obtained in (c). Do you find one or both of the gains larger than 1 in your design? Explain the consequences from the perspective of disturbance amplification.