UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2012

Homework #1

Assigned: Jan. 24 (Tu)
Due: Jan. 31 (Tu)

1. A product is produced by three different factories: A, B, and C. Factories A, B, and C respectively produce 25%, 50%, and 25% of the total production. In factories A and B, 98% of the items produced are not defective, whereas in factory C, 99% are not defective. Calculate: (a) The probability that a randomly-chosen item is defective and (b) the probability that a (randomly-chosen) non-defective item comes from factory C.

2. In the "Monty Hall" three-door problem, a contestant is asked to choose one of three doors. One of the three doors conceals a prize while the other two do not. After the contestant chooses, Monty Hall (the master of ceremonies of the Let's Make a Deal television show) opens one of the two doors the player did not choose to reveal one door that does not conceal the prize. The contestant is then permitted to either stay with their original choice or switch to the other unopened door. Determine the contestant's probability of getting the prize if she switches. Assume that before Monty Hall opens one of the doors, the prize is equally likely to be hidden behind each of the three doors.

   Hint: Let the doors be called x, y, and z. Let C_x be the event that the prize is behind door x and so on. Let H_x be the event that the host opens door x and so on. Assuming that you choose door x, the probability that you win a car if you then switch your choice is given by

   $$P((H_z \cap C_y) \cup (H_y \cap C_z)) \; .$$

   Notice that, by Baye's rule, $P(H_z \cap C_y) = P(H_z|C_y)P(C_y)$.

3. A parent has two children. Given that at least one of the children is a boy born on a Tuesday, what is the probability that both children are boys? Assume that, in the absence of the given information, all $14^2 = 196$ possibilities for the genders of the children and days of the week on which the children were born are equally likely.

4. Consider three independent random variables, X_1, X_2, and X_3, each of which is uniformly distributed between 0 and 1. Obtain the probability density function (PDF) for:

   (a) $X_1 + X_2$

   (b) $X_1 + X_2 + X_3$

   Hint: Recall from lecture 3 that, if X and Y are two independent random variables and $Z = X + Y$, then the PDF of Z, $p_Z(z)$, is the convolution of the PDF of X, $p_X(x)$, and the PDF of Y, $p_Y(y)$, i.e.

   $$p_Z(z) = p_X(\cdot) * p_Y(\cdot) = \int_{-\infty}^{\infty} p_X(x) p_Y(z - x) dx \; .$$

5. Let $X \sim N(m_X, \sigma_X^2)$, i.e. let X be a Gaussian random variable with mean m_X and variance σ_X^2. Then the moment generating function of X is

$$P_X(j\omega) = \mathcal{F}\{p_X(\cdot)\} = E\left\{e^{-j\omega X}\right\} = \exp\left(j\omega m_X - \frac{\sigma_X^2 \omega^2}{2}\right) .$$

Now let X and Y be independent, where $Y \sim N(m_Y, \sigma_Y^2)$. Show that if $Z = X + Y$, then $Z \sim N(m_X + m_Y, \sigma_X^2 + \sigma_Y^2)$.