UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2016

Final Examination

Your Name:

Open (handwritten) notes.

Problem:	1	2	3	4	5	Total
Max. Grade:	40	50	30	40	40	200
Grade:						

Problem 1

Consider the state-space system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_w w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

where

$$A = \begin{bmatrix} \sqrt{\frac{3}{4}} & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The sequences $w(k)$ and $v(k)$ are jointly Gaussian WSS random sequences that satisfy the usual assumptions:

$$\begin{aligned} E\{w(k)\} &= 0 & E\{v(k)\} &= 0 \\ E\{w(k+j)w(k)\} &= W\delta(j) & E\{v(k+j)v(k)\} &= V\delta(j) \\ E\{w(k+j)v(k)\} &= 0 \ . \end{aligned}$$

1. What do the closed-loop poles of the stationary Kalman filter converge to as $\frac{W}{V} \to 0$?
2. Find the Kalman filter for this system when $W = 1$ and $V = 4$. (Remember to include the a-posteriori state estimate.)
3. Determine whether or not there exists a unique asymptotically stabilizing output feedback controller that optimizes the cost function

$$J = E\{x^T(k)C^T Cx(k) + \rho u^2(k)\}$$

where $\rho > 0$.

Problem 2

A LTI discrete-time system is described by

$$x(k+1) = A\,x(k) + B\,u(k) + B_w\,w(k)$$

where $x(k) \in \mathcal{R}^{p_1}$ is the state, $u(k) \in \mathcal{R}^{p_2}$ is the control input and $w(k) \in \mathcal{R}^{p_3}$ is stationary colored noise given by the output of the state space model

$$\begin{aligned} x_w(k+1) &= A_w\,x_w(k) + B_n\,\eta(k) \\ w(k) &= C_w\,x_w(k) \end{aligned}$$

where $n(k) \in \mathcal{R}^{p_5}$ is stationary and white and $x_w(k) \in \mathcal{R}^{p_4}$. Assume that $x(0)$, $x_w(0)$ and $\eta(k)$ are independent and normally distributed with

$$\begin{aligned} &E\{x(0)\} = x_o, \quad E\{(x(0)-x_o)(x(0)-x_o)^T\} = X_o \\ &E\{x_w(0)\} = x_{wo}, \quad E\{(x_w(0)-x_{wo})(x_w(0)-x_{wo})^T\} = X_{wo} \\ &E\{\eta(k)\} = 0, \quad E\{\eta(k)\eta(k+l)^T\} = \Gamma\delta(l) \end{aligned}$$

The performance index to be minimized is

$$J = E\left\{ x^T(N)\,Q_f\,x(N) + \sum_{k=0}^{N-1}\left[x^T(k)\,Q\,x(k) + u^T(k)\,R\,u(k)\right]\right\}$$

where the expectation must be taken over all underlying random quantities in each of the following situations that will be described below. For each situation obtain the equations that must be solved to find the optimal control:

1. $u(k)$ is allowed to be a function of $x(0), \ldots, x(k)$ in addition to $x_w(0), \ldots, x_w(k)$
2. $u(k)$ is allowed to be a function of $y(0), \ldots, y(k)$ where $y(k) = C\,x(k) + v(k)$ and $v(k)$ is a zero-mean, white and Gaussian noise with covariance $E\{v(k)v(k+l)\} = V\,\delta(l)$ and is independent from $x(0)$, $x_w(0)$ and $\eta(k)$.
3. $x(k)$ is measurable for all k. Notice that in this case, since

$$x(k) - A\,x(k-1) - B\,u(k-1) = B_w C_w\,x_w(k-1)$$

   $x_w(k-1)$ can be determined without being contaminated by measurement noise at the instance k. Therefore, if you set the measurement covariance to zero and obtain the equations for the Kalman filter, you will obtain $\hat{x}_w(k-1|k)$.

Problem 3

1. Write down the statement of a deterministic infinite time LQR problem for a standard discrete LTI system. Write down the equation that needs to be solved to obtain the optimal state feedback controller. What is the optimal gain in terms of that solution?

2. Under what conditions does a bounded solution exist? Under what conditions is it unique?

3. How does the cost function differ in a frequency shaped LQR problem? Write a state space realization for the frequency weighted (filtered) state and input contributions to the FSLQR problem and express the solution for the optimal control as a conventional LQR problem for an augmented system.

4. Draw the block diagram for the closed loop augmented system, including internal states and filtered outputs for the realizations of the frequency shaped state cost and input cost.

Problem 4

Consider the state-space system

$$
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k)
$$

$$
y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + v(k)
$$

where $y(k)$ is the output and $u(k)$ is the control input. The signals $w(k)$ and $v(k)$ are jointly Gaussian WSS random sequences that satisfy the usual assumptions:

$$
\begin{aligned}
E\{w(k)\} &= 0 & E\{v(k)\} &= 0 \\
E\{w(k+j)w(k)\} &= W\delta(j) & E\{v(k+j)v(k)\} &= V\delta(j) \\
E\{w(k+j)v(k)\} &= 0 \,.
\end{aligned}
$$

In this problem, $W = 1$, $V = 2$.

1. Rewrite the system dynamics in the form

$$
A(q^{-1})y(k) = q^{-\mathrm{d}}B(q^{-1})u(k) + C(q^{-1})\epsilon(k) \tag{1}
$$

   where $C(q^{-1})$ is an anti-Schur polynomial of q^{-1} and $\epsilon(k)$ is a zero-mean white Gaussian random sequence.

2. Find the minimum variance regulator for (1). To receive full credit, you must explicitly find all of the controller parameters.

Problem 5

Consider the identification of the system

$$A(q^{-1})y(k) = q^{-2}B(q^{-1})u(k) + q^{-1}d(k)$$

where $u(k)$ is the control input, $y(k)$ is the system output, and $d(k)$ is the disturbance acting on the system, which satisfies

$$d(k+3) = d(k)$$

Although the plant polynomials

$$A(q^{-1}) = 1 + a_1q^{-1} + a_2q^{-2} \qquad\qquad B(q^{-1}) = b_0 + b_1q^{-1}$$

are unknown, their order is known and it is known that $b_0 \neq 0$.

1. Define θ so that the system dynamics can be expressed in the form

   $$y(k+1) = \phi^T(k)\theta$$

   where

   $$\phi(k) = \begin{bmatrix}-y(k) & -y(k-1) & u(k-1) & u(k-2) & f(k) & f(k-1) & f(k-2)\end{bmatrix}^T$$

   and $f(k)$ is the indicator function

   $$f(k) = \begin{cases} 1, & k \in \{\dots, -3, 0, 3, 6, \dots\} \\ 0, & \text{otherwise} \end{cases}$$

2. Write down a parameter adaptation algorithm that estimates θ using recursive least squares with a forgetting factor.

3. Write down a set of sufficient conditions for the a priori output estimation error $e^o(k)$ to converge to zero.

4. Describe the steps in how you would apply indirect adaptive control to close the loop on this system.

5. Describe the steps in how you would apply direct adaptive control to close the loop on this system.