University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2013
Midterm Examination I March 7, 2013 (Th)
Closed Books, Closed Notes, Open one summary sheet.

[1] (20 points) A discrete time first order system is described by

$$x(k+1) = 0.8x(k) + w(k)$$

The input noise $w(k)$ is zero mean and Gaussian but is not white. The auto-correlation function of $w(k)$ is

$$X_{ww}(\ell) = 0.2^{|\ell|}\left(\frac{3}{8}\right)$$

1. Obtain the spectral density of $x(k)$.

The measurement equation for $x(k)$ is

$$y(k) = x(k) + v(k)$$

where $v(k)$ is a zero mean Gaussian white noise with $E[v(k)v(j)] = V\delta_{kj}$.

2. Obtain the Riccati equation for the steady state Kalman filter to obtain $\hat{x}(k|k)$ and $\hat{w}(k|k)$. You do not have to solve the Riccati equation. Obtain the Kalman filter.
3. Show on the complex domain how the eigenvalues of the Kalman filter vary as the covariance of the measurement noise is varied from 0 to ∞.

[2] (20 points) A discrete time LQ problem is formulated as follows.

Linear discrete time system: $x(k+1) = Ax(k) + Bu(k), \qquad x(0) = x_0$

Quadratic performance index: $J = \frac{1}{2}x^T(N)Sx(N) + \frac{1}{2}\sum_{j=0}^{N-1}\left\{x^T(j)Qx(j) + 2u^T(j)Mx(j) + u^T(j)Ru(j)\right\}$

where R is positive definite and $Q - M^T R^{-1} M$ is positive semidefinite.
Note: $2u^T(j)Mx(j) = u^T(j)Mx(j) + x^T(j)M^T u(j)$

1. Find the optimal control input, $u^o(k)$, by dynamic programming. Be sure to utilize the key Dynamic Programming equation obtained by applying the principle of optimality.
2. Explain the positive semi-definiteness assumption for $Q - M^T R^{-1} M$.

[3] (10) Consider the stationary LQG solution for a linear system described by.

$$\frac{dx(t)}{dt} = -x(t) + u(t) + w(t),$$
$$y(t) = x(t) + v(t)$$

where $w(t)$ and $v(t)$ are independent Gussian random processes with

$E[w(t)] = 0, E[w(t)w(t+\tau)] = W\delta(\tau), E[v(t)] = 0$ and $E[v(t)v(t+\tau)] = V\delta(\tau)$.

The stationary LQG solution minimizes

$$J' = E[Qx^2(t) + Ru^2(t)]$$

The optimal closed loop system has two eigenvalues, one at $-\sqrt{2}$ and the other at $-\sqrt{3}$. Obtain all possible values for *W*, *V*, *Q* and *R*.