University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2009
Midterm Examination I March 5, 2007 (Th)
Closed Books, Closed Notes, Open one summary sheet.
Three problems.

[1] (20 points) A discrete time system is described by

$$x(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(k), \qquad y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(k) = x_1(k)$$

The quadratic performance index for this problem is

$$J = \sum_{k=0}^{\infty} \{ y^2(k) + Ru^2(k) \}$$

1. Show that the identity matrix, i.e. I_4, is the positive definite solution of the algebraic Riccati equation. Obtain the optimal control law. Explain why the optimal control is as obtained.
   HINT: The response of the open loop system is
   $y(0) = x_1(0), \quad y(1) = x_2(0), \quad y(2) = x_3(0), \quad y(3) = x_4(0), \quad y(4) = 0, \ldots$
2. By applying the symmetric root locus technique, obtain the root locus plot of the optimal closed loop poles for varying R from 0 to ∞. To do this part right, it is suggested that you consider the system as a limiting case ($\varepsilon \to 0$) of

$$x(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \varepsilon & 0 & 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(k), \qquad y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(k) = x_1(k)$$

   Notice that that characteristic equation of this system is $z^4 - \varepsilon = 0$.

[2] (20 points) Consider an asymptotically stable system excited by a zero mean Gaussian colored noise: i.e.

$$x(k+1) = Ax(k) + Bw(k)$$

where $w(k)$ is a scalar colored noises modeled as the output of a shaping filter excited by a scalar zero mean Gaussian white noise.

$$x_w(k+1) = A_w x_w(k) + B_w n(k), \quad w(k) = C_w x_w(k)$$

The shaping filter is asymptotically stable. The following quantities are given:

$$E[x(k_0)] = 0, \quad E[x(k_0)x^T(k_0)] = X_o, \quad E[x_w(k_0)] = 0, \quad E[x_w(k_0)x_w^T(k_0)] = X_{wo}, \quad E[x(k_0)\, x_w^T(k_0)] = 0$$
$$E[n(k)] = 0, \quad E[n(k)n(j)] = W\delta_{kj}, \quad E[x(k_0)n(k)] = 0, \quad E[x_w(k_0)n(k)] = 0$$

1. Obtain the equation to study the propagation of the covariance matrix of the system state vector, $E[x(k)x^T(k)]$. Note: $E[x_w(k)] = 0$ and $E[x(k)] = 0$ for all k.
2. Obtain the equation for the spectral density of x at the steady state.

[3] (20 points) Consider an asymptotically stable discrete time system described by

$$x(k+1) = Ax(k) + w(k), \qquad y(k) = x(k) + v(k)$$

where $x(k_0), w(k)$ and $v(k)$ are independent zero mean Gaussian random scalars with

$$E[x^2(k_0)] = X_0, \quad E[w(k)w(j)] = W\delta_{kj}, \quad E[v(k)v(j)] = V\delta_{kj}$$

Consider the statistical steady state: i.e. $k_0 \to -\infty$. It is desired to estimate the value of $x(k+2) - x(k+1)$ using the output measurements up to time k. Write all equations that you need to find the best estimate and the estimation error variance for $x(k+2) - x(k+1)$.