UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2012

Homework #8

Assigned:	Apr. 12	(Th)
Due:	Apr. 19	(Th)

1. A discrete-time plant is given by

$$A(q^{-1})y(k) = q^{-\mathrm{d}} B(q^{-1}) \{u(k) + d(k)\} \tag{1}$$

where $A(q^{-1}) = (1 - 0.8q^{-1})(1 - 0.7q^{-1})$, $\mathrm{d} = 1$ and $B(q^{-1}) = 0.1$. Here, $u(k)$ is the control input, $y(k)$ is the output, and $d(k)$ represent the disturbance. The disturbance is known to be periodic and the period is $N = 8$ (i.e. $d(k + 8) = d(k)$).

(a) Use the fact that $A_d(q^{-1})\, d(k) = 0$, where $A_d(q^{-1}) = 1 - q^{-8}$ and design the regulator of the form

$$u(k) = -\frac{S(q^{-1})}{A_d(q^{-1})\, R'(q^{-1})}\, y(k)$$

which drives $y(k)$ to zero in finite time. (i.e. the closed-loop polynomial should be $A_c(q^{-1}) = 1$. Find the polynomials $R'(q^{-1})$ and $S(q^{-1})$ as the solution of the Diophantine equation

$$A_c(q^{-1}) = A_d(q^{-1})\, A(q^{-1})\, R'(q^{-1}) + q^{-\mathrm{d}}\, B(q^{-1})\, S(q^{-1})\,.$$

(b) Simulate the control system designed above when the periodic disturbance is given by

$$(d(0), d(1), \ldots, d(7)) = (0,\ 1.5,\ 3,\ 0,\ -2,\ -2,\ 0,\ 0.5)$$

and $d(k + 8) = d(k)$. Assume that the plant initial condition is zero.

(c) Now consider a repetitive controller of the form

$$u(k) = -\frac{k_r q^{-(N-\mathrm{d})} A(q^{-1})}{A_d(q^{-1})\, B(q^{-1})}\, y(k)\ .$$

Simulate this repetitive controller under the same conditions described in problem 1b. Try values of 0.5, 1 and 1.3 for k_r.

(d) Assume now that the plant model form is still described by (1), but the plant dynamics are now given by

$$\begin{aligned} A(q^{-1}) &= (1 - 0.2q^{-1})\, \bar{A}(q^{-1}) \\ \bar{A}(q^{-1}) &= (1 - 0.8q^{-1})(1 - 0.7q^{-1}) \\ B(q^{-1}) &= 0.08 \\ \mathrm{d} &= 2. \end{aligned}$$

Moreover, we use the repetitive controller from the previous part, which is given by

$$u(k) = -\frac{k_r q^{-(N-1)} \bar{A}(q^{-1})}{0.1\, A_d(q^{-1})}\, y(k)\,.$$

In other words, the real plant is

$$G_{_A}(q) = \frac{0.1q^{-1}}{(1-0.8q^{-1})(1-0.7q^{-1})}\, \frac{0.8q^{-1}}{(1-0.2q^{-1})}$$

but we use the simplified plant

$$G(q) = \frac{0.1q^{-1}}{(1-0.8q^{-1})(1-0.7q^{-1})}$$

in the control system design process.

Show, using the root locus technique, that the resulting repetitive control system is unstable for any positive k_r.

(e) Assume again that the plant model form is still described by (1), but the plant dynamics are now given by

$$\begin{aligned}
A(q^{-1}) &= (1-0.2q^{-1})\, \bar{A}(q^{-1}) \\
\bar{A}(q^{-1}) &= (1-0.8q^{-1})(1-0.7q^{-1}) \\
B(q^{-1}) &= 0.08 \\
\mathrm{d} &= 2.
\end{aligned}$$

(These are the same dynamics considered in the previous part.) However, we now incorporate the Q-filter modification to the repetitive compensator, in order to make the repetitive controller more robust. Thus,

$$u(k) = -\frac{k_r q^{-(N-1)} \bar{A}(q^{-1})}{0.1\,(1 - Q(q,q^{-1})q^{-N})}\, y(k)\,.$$

where

$$Q(q,q^{-1}) = \frac{q + 2 + q^{-1}}{4}\,.$$

Do the following:

i. Plot the root locus of the closed-loop poles of the resulting repetitive control system for $k_r \geq 0$ and determine, a value of k_r for which the resulting repetitive control system is asymptotically stable.

ii. Simulate this repetitive controller under the same conditions described in problem 1b.

2. Consider the following stationary stochastic system

$$\begin{aligned}\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0.8 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k) \\ y(k) &= x_1(k) + v(k)\end{aligned} \tag{2}$$

where $u(k)$ is a deterministic (known) input, $y(k)$ is the measured output, $w(k)$ and $v(k)$ are zero-mean, jointly Gaussian WSS random sequences with

$$E\left\{ \begin{bmatrix} w(k+j) \\ v(k+j) \end{bmatrix} \begin{bmatrix} w(k) & v(k) \end{bmatrix} \right\} = \begin{bmatrix} 0.225 & 0 \\ 0 & 0.625 \end{bmatrix} \delta(j)$$

Design a minimum variance regulator for this system.

3. In this problem we conduct a minimum variance, model reference stochastic control design exercise. Consider the following ARMAX system

$$A(q^{-1})y(k) = q^{-d}\,B(q^{-1})u(k) + C(q^{-1})\epsilon(k) \tag{3}$$

where

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ C(q^{-1}) &= 1 + c_1 q^{-1} + \cdots + c_L q^{-L} \\ B(q^{-1}) &= b_o + b_1 q^{-1} + \cdots + b_m q^{-m} \end{aligned}$$

and $\epsilon(k)$ is the steady state Kalman filter residual, which satisfies

$$E\{\epsilon(k)\} = 0, \qquad \begin{aligned} &E\{\epsilon(k+j)\epsilon(k)\} = \sigma\delta(j), \\ &E\{\epsilon(k+j)y(k)\} = 0, \quad \forall j > 0\ . \end{aligned}$$

Assume that $B(q^{-1})$ and $C(q^{-1})$ are anti-Schur polynomials (i.e. the roots of the polynomials $q^m B(q^{-1})$ and $q^L C(q^{-1})$ are all strictly inside the unit circle). Define the model reference

$$A_m(q^{-1})y_m(k) = q^{-d}\,B_m(q^{-1})r_m(k) \tag{4}$$

where $A_m(q^{-1})$ is an anti-Schur polynomial of the form

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + \cdots + a_{mt}q^{-t} \tag{5}$$

$y_m(k)$ is the reference model output, and $r_m(k)$ is the deterministic reference model input. The goal is to obtain the optimal control $u^o(k)$ that minimizes the cost

$$J = E\left\{ \left[A_m(q^{-1})\,(y(k) - y_m(k)\,)\right]^2 \right\}\ . \tag{6}$$

Show that $u^o(k)$ satisfies

$$\beta(q^{-1})u^o(k) = C(q^{-1})B_m(q^{-1})r_m(k) - S(q^{-1})y(k) \tag{7}$$

where

$$\begin{aligned} &C(q^{-1})\,A_m(q^{-1}) = A(q^{-1})\,R(q^{-1}) + q^{-d}\,S(q^{-1}) \\ &R(q^{-1}) = 1 + r_1 q^{-1} + \cdots + r_{d-1}q^{-d+1} \\ &\beta(q^{-1}) = B(q^{-1})\,R(q^{-1}) \\ &S(q^{-1}) = s_o + s_1\,q^{-1} + \cdots + s_p q^{-p} \qquad\qquad p = \max(L + t - d, n-1)\,. \end{aligned}$$