ME 233 Spring 2016
Solution to Homework #1

1. 2014 Homework1 Problem1
2. 2014 Homework2 Problem2
3. (a) First we will define

$$\begin{aligned}
P(A) &\quad - \quad \text{probability that a randomly chosen item comes from factory A} \\
P(B) &\quad - \quad \text{probability that a randomly chosen item comes from factory B} \\
P(C) &\quad - \quad \text{probability that a randomly chosen item comes from factory C} \\
P(D) &\quad - \quad \text{probability that a randomly chosen item is defective} \\
P(N) &\quad - \quad \text{probability that a randomly chosen item is not defective}
\end{aligned}$$

With these definitions, we can state our given information as

$$\begin{aligned}
P(A) &= \frac{1}{4} \\
P(B) &= \frac{1}{2} \\
P(C) &= \frac{1}{4} \\
P(N|A) &= \frac{98}{100} \\
P(N|B) &= \frac{98}{100} \\
P(N|C) &= \frac{99}{100}
\end{aligned}$$

Using Bayes' Rule, we can say

$$\begin{aligned}
P(A \cap N) &= P(N|A)P(A) = \frac{98}{100} \times \frac{1}{4} \\
P(B \cap N) &= P(N|B)P(B) = \frac{98}{100} \times \frac{1}{2} \\
P(C \cap N) &= P(N|C)P(C) = \frac{99}{100} \times \frac{1}{4}
\end{aligned}$$

With this data, we can now construct the array for the joint probability shown in Table 1. To construct the last entry in the 'N' column, we add all of the elements above it. To construct the 'D' column, we subtract the 'N' column from the 'Marginal Probabilities' column.

Thanks to Table 1, we see that our desired result is the marginal probability of the 'D' column:

$$P(D) = \frac{7}{400}$$

	N	D	Marginal Probabilities
A	$\frac{98}{100} \times \frac{1}{4} = \frac{49}{200}$	$\frac{1}{4} - \frac{49}{200} = \frac{1}{200}$	$\frac{1}{4}$
B	$\frac{98}{100} \times \frac{1}{2} = \frac{49}{100}$	$\frac{1}{2} - \frac{49}{100} = \frac{1}{100}$	$\frac{1}{2}$
C	$\frac{99}{100} \times \frac{1}{4} = \frac{99}{400}$	$\frac{1}{4} - \frac{99}{400} = \frac{1}{400}$	$\frac{1}{4}$
Marginal Probabilities	$\frac{49}{200} + \frac{49}{100} + \frac{99}{400} = \frac{393}{400}$	$\frac{1}{200} + \frac{1}{100} + \frac{1}{400} = \frac{7}{400}$	1

Table 1: Array of joint probability

(b) Using Bayes' Rule, our desired result is given by

$$P(C|N) = \frac{P(C \cap N)}{P(N)} = \frac{99}{393}$$

4. In this problem, we will denote the three doors as x, y, and z. Without loss of generality, we will assume that the contestant originally picked door x. We now define C_i to be the event that the car is behind door i and H_j to be the event that the host opens door j. With this in mind, note that the mutually exclusive events C_x, $C_y \cap H_z$, and $C_z \cap H_y$ cover the sample space, i.e.

$$1 = P(C_x) + P(C_y \cap H_z) + P(C_z \cap H_y).$$

Given that the contestant switches her guess, the probability that she will win is given by $P((C_y \cap H_z) \cup (C_z \cap H_y))$. Since the event $C_y \cap H_z$ is disjoint from the event $C_z \cap H_y$, we can say that

$$\begin{aligned} P(\text{win}|\text{she switches}) &= P((C_y \cap H_z) \cup (C_z \cap H_y)) \\ &= P(C_y \cap H_z) + P(C_z \cap H_y) \\ &= 1 - P(C_x) = \frac{2}{3}. \end{aligned}$$

5. Upload later