University of California
Department of Mechanical Engineering

ME233 Advanced Control Systems II Spring 2007

Final Examination May 11, 2007 (F)
Open reader, Open notes; you may refer to your own summary sheets for midterm exams.

[1] (20 points) Consider a discrete time plant described by

$$x(k+1) = Ax(k) + Bu(k) + B_w w(k)$$

where x is an n-dimensional state vector, u is an m-dimensional input vector and w is an s-dimensional zero-mean Gaussian input noise vector with $E[w(k)w^T(j)] = W\delta_{kj}$. The output equation is

$$y(k) = Cx(k) + v(k)$$

where $y(k)$ is an r-dimensional output vector and $v(k)$ is an r-dimensional Gaussian zero-mean measurement noise vector with $E[v(k)v^T(j)] = V\delta_{kj}$.

The Kalman filter has been designed for this system under a set of standard assumptions: i.e. whiteness of the input and measurement noises and the independence of the initial condition, input noise and measurement noise. The covariance of measurement noise for an assumed sensor was V_a. It was later found that a measurement-noise-free sensor is available for state estimation, i.e. the covariance of measurement noise is zero. In view that the original Kalman filter is performing good, the control engineer did not redesign the Kalman filter. Obtain the estimation error covariance $E[(x(k+1) - \hat{x}(k+1\,|\,k))(x(k+1) - \hat{x}(k+1\,|\,k)^T)]$ and $E[(x(k+1) - \hat{x}(k+1\,|\,k+1))(x(k+1) - \hat{x}(k+1\,|\,k+1)^T)]$ when the original Kalman filter ($V = V_a$) is applied to the measurement-noise-free case.

[2] (20 points) Consider a continuous time plant described by

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = x_1(t)$$

The performance index for LQ controller design is

$$J = \int_{-\infty}^{\infty} \frac{1}{\omega^2} Y(-j\omega)Y(j\omega) + RU(-j\omega)U(j\omega)d\omega$$

Draw the root locus for the optimal closed loop poles for R varying from 0 to ∞.

[3] (20 points) Consider a feedback control system sketched below.

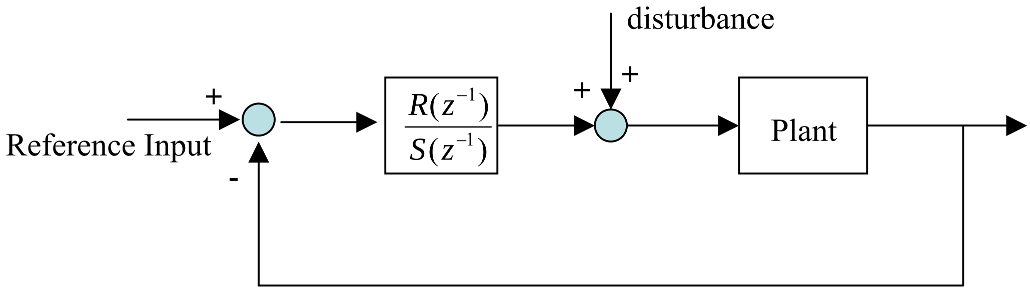


Fig. 3-1 Feedback Control System

The plant is one step delay: i.e. $G_p(z^{-1}) = z^{-1}$. The disturbance input is known to be in the following form.

$$d(k) = d_c + c\sin(\omega k + \phi)$$

where ω is known but d_c, c and ϕ are unknown constants. Design the internal model based controller that achieves asymptotic regulation. Assume that the reference input is 0. Keep the order of the controller to minimal, and assign all closed loop poles at 0.9.

[4] (20 points) Consider the discrete time system

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k-1) + b_2 u(k-2) + d(k)$$

where $d(k)$ represents the disturbance in the form $d(k) = d_0 \delta^k$, $0 < \delta < 1$. a_1, b_1 and δ are known. Other parameters are unknown. Devise a least squares algorithm with a forgetting factor $\lambda = 0.98$ for identifying a_2 and b_2.

[5] (40 points) Consider a control system sketched below.

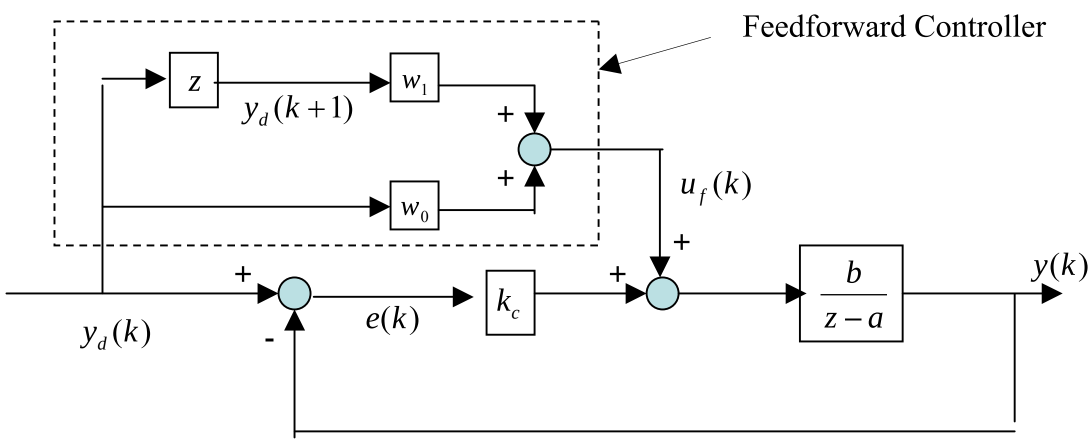


Fig. 5-1 Control System with Feedforward Control

The parameter of the controlled plant satisfy

$$0 < a < 1 \text{ and } b > 0 \qquad (1)$$

The proportional feedback control gain is selected so that

$$0 < a - bk_c < a,\ k_c > 0 \qquad (2)$$

(a) (10 points) Show that for $w_1 = 1/b$ and $w_0 = -a/b$, the tracking error $e(k)$ approaches to zero while satisfying

$$e(k+1) = (a - bk_c)e(k)$$

Assume that the plant parameters are unknown, but that conditions (1) and (2) are satisfied. The adaptive feedforward controller for this system is proposed to be

$$u_f(k) = \hat{w}_0(k) y_d(k) + \hat{w}_1(k) y_d(k+1) = \hat{\theta}^T(k)\phi_d(k)$$

where

$$\hat{\theta}^T(k) = [\hat{w}_0 k), \quad \hat{w}_1 k)] \text{ and } \phi_d(k) = [y_d(k), \quad y_d(k+1)]$$

The parameter adaptation algorithm is

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \gamma\phi_d(k-1)e(k)$$

where γ is selected so that

$$\frac{\gamma}{2}\|\phi_d(k-1)\|^2 = \frac{\gamma}{2}\phi_d^{\ T}(k-1)\phi_d(k-1) < \frac{1}{b}[1-(a-bk_c)]$$

(b) (10 points) Show that the error equation can be represented by the feedback loop sketched in Fig. 5-2.

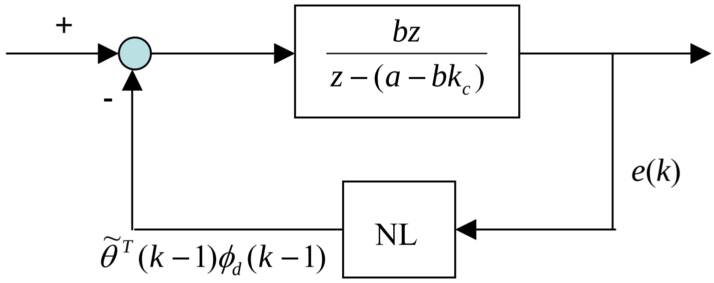
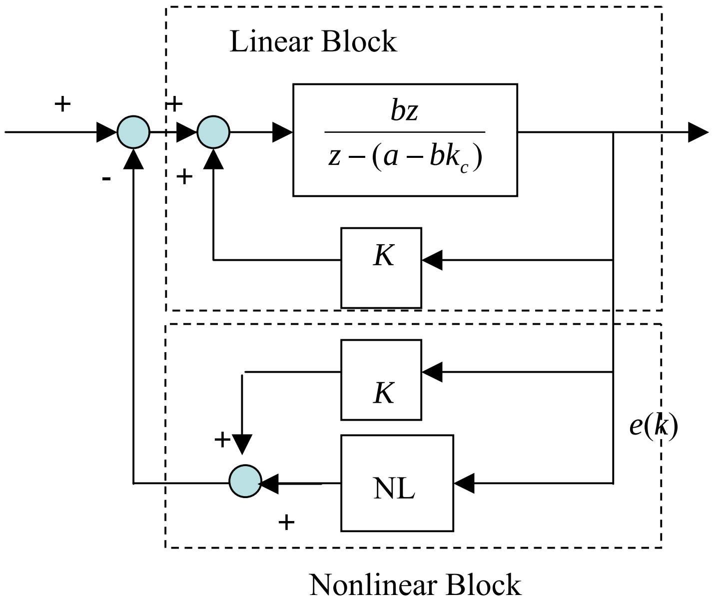


Fig. 5-2 Equivalent Feedback Loop

(c) (20 points) Show that the error signal $e(k)$ converges to zero under the proposed adaptive control scheme. (Hint: First show that the equivalent feedback loop in Fig. 5-2 may be transformed to the feedback system shown in Fig. 5-3.



K satisfies $\frac{\gamma}{2}\phi_d^T(k-1)\phi_d(k-1) < K < \frac{1}{b}[1-(a-bk_c)]$

Fig. 5-3 Feedback Loop Equivalent to the one in Fig. 5-2.