Notes

September 3, 2019

1 Main Idea

Integer linear programming is pretty useful apparently. The problem boils down to solving a system of linear equations and inequalities. Just Google linear integer programming, and Wikipedia has a good example for the general type of problem that is being solved. The underlying problem is classically NP-hard, so there is an possibility to exponentially speed-up this entire set of problems with quantum annealing. The main idea here is to encode inequalities into a quantum annealer.

Quick overview, a quantum annealers solves a QUBO problem (which is basically Ising Model). The problem can be boiled down to the form xQx where x is a vector of qubits (so they can only take values of 0 or 1 or some super-position). We have to "program" Q to solve our problem.

Following https://arxiv.org/abs/1812.06917 we know 1) how to encode fixed-precision numbers into a set of qubits. 2) We also know how to encode higher order problems (*e.g.* $Jx^4 + Qx^2$). 3) We also learned that in general, since QA can only solve for the global minimum, we must optimize the squared-residual of a set of equations / inequalities. Anyways, read the paper if you haven't already.

To encode an inequality of the form $A < y < B$ we can introduce an auxiliary qubit y_a such that the logical qubit is $Y = (y, y_a)$. The inequality is then encoded as minimizing the following $C_1Y^4 - C_2Y^2 + C_3$ such that we get a mexican-hat potential. Basically this was inspired from spontaneous symmetry breaking. I'm sure this can be connected to solving scalar phi-4 theory if we realllly want to go there. But tuning the Cs will tune the radius of the degeneracy and therefore change the range of A and B. Also C_3 shifts the minimum to zero, which is useful if we want to keep the energy positive, and interpret it as the squared-residual. Travis suggested that this might be connected to "slack variables" (see https://arxiv.org/abs/1811.11538 that I haven't read yet).

I'm worried that y_a has nonlinear dependence with y as we walk along the degeneracy though, which in practice may be an issue. Though increasing the number of qubits used to represent a fixed-precision decimal solves this problem exponentially quickly.