KALMAN FILTER

Let (x, y) be the image coordinates and (dx, dy) be the velocities in x and y directions, respectively. Also let x_k, y_k be the measured position in x and y directions at time k, respectively. Define the following matrices.

$$\boldsymbol{X}: [\boldsymbol{x\ y\ dx\ dy}]^T\ \boldsymbol{state\ vector}$$

$$\boldsymbol{X_{pred_k}}: \boldsymbol{predicted\ vector\ at\ time\ k}$$

$$\boldsymbol{X_k}: \boldsymbol{corrected\ vector\ at\ time\ k}$$

$$\boldsymbol{P_{pred}}: \boldsymbol{predicted\ covariance}$$

$$\boldsymbol{P}: \boldsymbol{corrected\ covariance}$$

$$\boldsymbol{Z_k}: [\boldsymbol{x_k\ y_k}]^T\ \boldsymbol{measured\ position\ vector}$$

Let the initial prediction and covariance matrices to be as follows:

$$\boldsymbol{X_{pred_{initial}}} = \begin{bmatrix} \boldsymbol{x_{center}} \\ \boldsymbol{y_{center}} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{P_{initial}} = \begin{bmatrix} \boldsymbol{100} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{100} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{100} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{100} \end{bmatrix}$$

State prediction can be updated by using the equation (1).

$$\boldsymbol{X_{pred_k}} = \boldsymbol{AX_{k-1}} \quad (\boldsymbol{1})$$

Error covariance prediction can be calculated by using the equation (2).

$$\boldsymbol{P_{pred_k}} = \boldsymbol{AP_{k-1}A^T} + \boldsymbol{Q} \quad (\boldsymbol{2})$$

In equation (1) and (2), Q and A are constant matrices, and can be taken as follows:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{1} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}$$

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{0.01} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0.01} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0.01} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0.01} \end{bmatrix}$$

Kalman Gain can be calculated by using the equation (3).

$$\boldsymbol{K_k = P_{pred_k} H^T \left(H P_{pred_k} H^T + R\right)^{-1}} \quad (3)$$

In equation (3), R and H are constant matrices, and can be taken as follows:

$$\boldsymbol{H = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}$$

$$\boldsymbol{R = \begin{bmatrix} 0.2845 & 0.0045 \\ 0.0045 & 0.0455 \end{bmatrix}}$$

State update can be made by using equation (4).

$$\boldsymbol{X_k = X_{pred_k} + K_k (Z_k - H X_{pred_k})} \quad (4)$$

Finally, error covariance can be updated by using equation (5).

$$\boldsymbol{P_k = (I - K_k H) P_{pred_k}} \quad (5)$$

In equation (5), I is the identity matrix with appropriate size.

Note that $X_k, X_{pred_k}, P, P_{pred_k}$ are different.