Pideonhole Principle

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The pidgeonhole principle states that if pigeons are put into holes and the amount of pidgeons is greater than the amount of holes, there must be at least one hole with more than one pidgeon inside. This is because there is more objects than places/categories to put them in. It can be represented by

if $f : X \to Y$ and $|X| > |Y|$, then there are elements $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1) = f(x_2)$.

This means that for every member of X there is a coresponding member in set Y, and there is more elements in X than in Y. It also says that there are at least two seperate members in X that equal the same element in Y.

The pidgeonhole principle can be extended to deal with multiple groups using the extended pidgeonhole principle. This principle states that

$f : X \to Y$ then there is some $y \in Y$ such that $f(x) = y$ for at least $\lceil \frac{|X|}{|Y|} \rceil$

This means that there is a member of Y called y that has individual members if set X with the total ammount of x's equaling the ceiling of the cardinality of X divided by the cardinality of Y.
This can be further simplified by stating there is at least some number of pidgeons in the same hole, this number can be found by tating the cadrinality of X and dividing by the cardinality of Y and rounding that number up.