DM Homework

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10.1 - Five of the following statements are negations of the other five. Pair each statement with its negation.
$(a) - p \oplus q - 0110 - negates(h)$
$(b) - \neg p \wedge q - 0100 - negates(g)$
$(c) - p \Rightarrow (q \Rightarrow p) - 1111 - negates(f)$
$(d) - p \Rightarrow q - 1101 - negates(e)$
$(e) - p \wedge \neg q - 0010 - negates(d)$
$(f) - q \wedge (p \wedge \neg p) - 0000 - negates(c)$
$(g) - p \vee \neg q - 1011 - negates(b)$
$(h) - p \Leftrightarrow q - 1001 - negates(a)$
$(i) - p \wedge (q \vee \neg q) - 0011 - negates(j)$
$(j) - (p \Rightarrow q) \Rightarrow p - 1100 - negates(i)$

10.3 - For each of the following formulas, decide whether it is a tautology, satisfiable, or unsatisfiable. Justify your answers.
$(a) - (p \vee q) \vee (q \Rightarrow p) - 1111 - Tautology$
$(b) - (p \Rightarrow q) \Rightarrow p - 0011 - Satisfiable$
$(c) - p \Rightarrow (q \Rightarrow p) - 0111 - Satisfiable$
$(d) - (\neg p \wedge q) \wedge (q \Rightarrow p) - 0000 - Unsatisfiable$
$(e) - (p \Rightarrow q) \Rightarrow (\neg p \Rightarrow \neg q) - 1011 - Satisfiable$
$(f) - (\neg p \Rightarrow \neg q) \Leftrightarrow (q \Rightarrow p) - 1111 - Tautology$

10.5 - (a)Show that for any formulas a, b, and y
$(\alpha \wedge \beta) \vee \alpha \vee \gamma \equiv \alpha \vee \gamma$
Because of the OR symbols, the first expression is false, Meaning that the whole expression is false making it equal
(b)Give the corresponding rule for simplifying
$(\alpha \vee \beta) \wedge \alpha \wedge \gamma$
Using the Absorption Law the equation becomes $\alpha \wedge \gamma$
(c)Find the simplest possible disjunctive and conjunctive normal forms of the formula $(p \wedge q) \Rightarrow (p \oplus q)$

$DNF - (\neg(p \wedge q)) \vee ((p \wedge \neg q) \vee (\neg p \wedge q))$
$CNF - (\neg p \vee \neg q \vee (p \wedge \neg q) \vee (\neg p \wedge q))$

10.7 - In this problem you will show that putting a formula into conjunctive normal form may increase its length exponentially. Consider the formula
$p_1 \wedge q_1 \vee ... \vee p_n \wedge q_n$
where $n \geq 1$ and the $piandqi$ are propositional variables. This formula has length $4n - 1$, if we count each occurrence of a propositional variable or an operator as adding 1 to the length and ignore the implicit parentheses.
(a) Write out a conjunctive normal form of this formula in the case $n = 3$.
(b) How long is your conjunctive normal form of this formula, using the same conventions as above? For general n, how long is the conjunctive normal form as a function of n?
(c) Show that similarly, putting a formula into disjunctive normal form may increase its length exponentially.
(d) Consider the following algorithm for determining whether a formula is satisfiable: Put the formula into disjunctive normal form by the method of this chapter, and then check to see if all of the disjuncts are contradictions (containing both a variable and its complement). If not, the formula is satisfiable. Why is this algorithm exponentially costly?

10.9 - This problem introduces resolution theorem-proving.
(a) - Suppose that $(e1 \vee ... \vee e_m - 1 \vee f_2 \vee ... \vee f_n)$ and $(f_1 \vee ...f_n)$ are clauses of a formula α, which is in conjunctive normal form, where $m, n \geq 1$ and each ei and fi is a literal. Assume that all the ei are distinct from each other, and all the fi are distinct from each other, so the clauses are essentially sets of literals. Suppose that e_m is a propositional variable p and $f_1 = \neg p$ from the second. That is,
$\alpha \equiv \alpha \wedge (e_1 \vee ... \vee e_m - 1 \vee f_2 \vee ...f_n)$,
or to be precise, the result of dropping any duplicate literals from this clause. The new clause is derived from the other two by resolution, and is said to be their resolvent. To make sense of this expression in the case $m = n = 1$, we must construe the empty clause containing no literals as identically false—which makes sense since it is derived from the two clauses $(p) \wedge (\neg p)$. The empty clause, in other words, is another name for the identically false proposition F. It follows that a formula is unsatisfiable if the empty clause results from the process of forming resolvents, adding them to the formula, and repeating.
(b) - Show that the converse of part (a) is true; that is, that if the process of forming resolvents and adding them to the formula ends without producing the empty clause, then the original formula (and all the equivalent formulas derived from it by resolution) has a satisfying truth assignment.