DM Homework

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2.1. Is -1 an odd integer, as we have defined the term? Why or why not?

yes, because it cant be divided by 2 and is a whole number $2k + 1 = -1$, $k = -1$

2.3. Prove that the product of two odd numbers is an odd number.
$m * n = 2k + 1$ and $m = 2a + 1$ and $n = 2b + 1$ so $(2a + 1)(2b + 1) = 2k + 1$ -¿ $4ab + 2a + 2b + 1 = 2k + 1$ -¿ $2(2ab + a + b) + 1 = 2k + 1$

2.5. Prove that $\sqrt[3]{2}$ is irrational.

only factor of two is 1 and 2 making it prime and you cant cube root a prime, any rational number can be a fraction, where one is odd $\sqrt[3]{2} = a/b$ so $b^(3) = 4k^(3)$

2.7. Show that there is a fair seven-sided die; that is, a polyhedron with seven faces that is equally likely to fall on any one of its faces.

if all the sides are the same, then they all have an equal chance on landing on any side, all the faces are exactly the same.

2.9.

(a)Prove or provide a counterexample: if c and d are perfect squares, then cd is a perfect square.

$c = a^2 - > a * a$ and $d = b^2 - > b * b$ then $cd = a * a * b * b - > a * b * a * b$

(b)Prove or provide a counterexample: if cd is a perfect square and $c! = d$, then c and d are perfect squares.

If $cd = 6^2 - > 36 - > 18 * 2$ and $18! = 2$ then $c = 2$ and $d = 18$ but they are not perfect squares

(c)Prove or provide a counterexample: if c and d are perfect squares such that $c > d$, and $x^2 = c$ and $y^2 = d$, then $x > y$. (Assume x,y are integers.)

If you multiply a larger number more times it will still be larger that a small number multiplied less

2.11 Critique the following 'proof':

$$\begin{aligned} x &> y \\ x^2 &> y^2 \\ x^2 - y^2 &> 0 \\ (x-y)(x+y) &> 0 \\ x+y &> 0 \\ x &> -y \end{aligned}$$

the proof is missing $x - y > 0$ wich leads to $x > y$ as an additional answer.

2.13. Write the following statements in terms of quantifiers and implications. You may assume that the terms 'positive', 'real', and 'prime' are understood, but 'even' and 'distinct', need to be expressed in statements.

(a)Every positive real number has two distinct square roots.

For any X in set Z that is positive and real, exists n and m of set Z that are distinct such that X = square root of n and m

(b)Every positive even number can be expressed as the sum of two prime numbers.

For every X in set Z that is divisible by two, there exists n and m of Z that are prime such that $X = n * m$

2.15. Using concepts developed in Chapter 1, explain the step in the proof of Example 2.9 stating that once one individual X has been singled out, 'of the remaining 5 people, there must be at least 3 whom X knows, or at least 3 whom X does not know.'

Someone can either know noone, one, two, three, four, or five people, and if they know two or less people, they don't know 3 or more people, so it is true. And if they know 3 or more, it is also true since the max they can have without having 3 friends or not friends is 4 other people 1 extra pigeon + (2 max without going over limit * 2 groups, friends or not friends) = 5 + yourself = 6