fundamental inequality.

$$V_n \geqslant V_{n+1} = \frac{\mathcal{U}_{n+1}V_n}{2} \geqslant \sqrt{\mathcal{U}_nV_n} = \mathcal{U}_{n+1} \geqslant \mathcal{U}_n.$$

Since $u_1 = \sqrt{u_0 v_0} \le \frac{u_0 + v_0}{2} = v_0$, hence we have

$$V_1 \ge V_2 \ge \cdots \ge V_n \ge U_n \ge \cdots \ge U_2 \ge V_1$$
, $\forall n \in M_+$

fun] 13 decreasing and un > lo, un. lim un exists.

12.

Pf:
$$u_1 = (\frac{1}{n} - \frac{1}{n+1}) \frac{1}{n+2} = \frac{1}{2}(\frac{1}{n} - \frac{1}{n+2}) - (\frac{1}{n+1} - \frac{1}{n+2}) = \frac{\frac{1}{2}}{n} - \frac{1}{n+1} + \frac{\frac{1}{2}}{n+2}$$
 i.e. $a = \frac{1}{2}, b = -1, c = \frac{1}{2}$.
$$= \frac{1}{2}(\frac{1}{n} - \frac{1}{n+1}) - \frac{1}{2}(\frac{1}{n+1} - \frac{1}{n+2})$$

$$V_n = \sum_{k=1}^{n} u_k = \frac{1}{2} \left(\sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right) - \sum_{k=1}^{n} \left(\frac{1}{k+1} - \frac{1}{k+2} \right) \right)$$

$$= \frac{1}{2}(1 - \frac{1}{n+1} - \frac{1}{2} + \frac{1}{n+2}) = \frac{1}{4} + \frac{1}{2}(-\frac{1}{n+1} + \frac{1}{n+2})$$

14.

Pf: Let {ak3k=1 be a sequence which has no upper bound.

 $\forall n \in \mathbb{N}$, $\exists k \in \mathbb{N}$ S.t. $\exists k \in \mathbb{N}$, otherwise $\exists k \in \mathbb{N}$, $\forall k \in \mathbb{N}$, which is a contradiction $\Rightarrow \{a_{kn}\}_{n=1}^{\infty}$ is a subsequence which diverges to $+\infty$.

lb

$$=\frac{1}{(n+1)!}+\frac{1}{(n+1)!(n+1)!}-\frac{1}{n\times n!}$$

$$= \frac{n(n+1) + n - (n+1)^2}{n(n+1) \cdot (n+1)!} = -\frac{1}{n(n+1) \cdot (n+1)!} \le 0$$

7.....y 3....y

(2) Let
$$L = \lim_{n \to \infty} U_n = \lim_{n \to \infty} V_n$$
. Suppose that $L = \frac{P}{q} \in \mathbb{R}$ S.T. P. q are coprime integers.

$$\Rightarrow \mathcal{U}_{q-1} + \frac{1}{q!} < \frac{p}{q} < \mathcal{U}_{q+1} + \frac{1}{q!} + \frac{1}{q!q!}$$

$$\Rightarrow \frac{q \cdot q! \cdot \mathcal{N}q - 1 + q}{q \cdot q!} < \frac{q \cdot q! \cdot \mathcal{N}q - 1}{q \cdot q!} < \frac{q \cdot q! \cdot \mathcal{N}q - 1}{q \cdot q!} \Rightarrow p \cdot q! \in (q \cdot q! \mathcal{N}q - 1 + q, q \cdot q! \cdot \mathcal{N}q - 1 + q + q)$$

Since 9.9! - 29-1 is a positive integer by the def of 29-1,

(3)
$$\sum_{k=0}^{n} \frac{ak+b}{k!} = a\sum_{k=1}^{n} \frac{1}{(k-1)!} + b\sum_{k=0}^{n} \frac{1}{k!}$$

$$= a\sum_{k=0}^{n-1} \frac{1}{k!} + b\sum_{k=0}^{n} \frac{1}{k!} = aU_{n-1} + bU_{n-1} \rightarrow (a+b)U \otimes n + ca$$

18.

$$|\mathcal{U}_{n}| = (5|\sin\frac{1}{n^{2}}| + \frac{1}{5}|\cos n|)^{n}$$

$$\angle (\frac{5}{n^{2}} + \frac{1}{5})^{n}$$

$$n > 5$$

$$\angle (\frac{2}{5})^{n}$$

20.

Pf:
$$\forall \xi > 0$$
, $\exists P$ $\exists t. \alpha_P < \frac{\xi}{2}$, take $N = \begin{bmatrix} \frac{2P}{\xi} - 1 \end{bmatrix} + I$ i.e. $\frac{f}{N+1} \le \frac{\xi}{2}$, then
$$|\mathcal{U}_n| \le \alpha_P + \frac{P}{n+1}$$

$$|\mathcal{L}_{\frac{\xi}{2}} + \frac{P}{N+1} \le \xi$$
, $\forall n \ge N$

U.

$$u_n - \frac{\eta^2 + n}{2n^2} \cdot l = \frac{(u_1 - l) + 2(u_1 - l) + \cdots + n(u_n - l)}{n^2}$$

$$\forall \xi > 0$$
, $\exists N \in \mathbb{N}$ 3.t. $\forall n > N$, $|u_n - V| \leq \frac{\xi}{2}$, then

$$\frac{M(UM-L)+\cdots+N(UM-L)}{n^2}$$

$$\leq \frac{M+\cdots+n}{n^2} \cdot \frac{\mathcal{E}}{\mathcal{Z}} \leq \frac{n^2+n}{n^2} \cdot \frac{\mathcal{E}}{\mathcal{Z}} = \left(\frac{1}{2} + \frac{1}{2n}\right) \cdot \frac{\mathcal{E}}{\mathcal{Z}} \leq \frac{\mathcal{E}}{\mathcal{Z}}, \forall n \geq M$$

Since
$$(M-L)+2(U_2-U)+\cdots+(M_1-1)(U_{M_1-1}-U)$$
 is finite, $\exists N_2 \text{ s.t. } \forall n \geq N_2$

$$\frac{|U_1-U|+2(U_2-U)+\cdots+(M_1-1)(U_{M_1-1}-U)|}{n^2} \leq \frac{2}{2}$$

$$\Rightarrow \lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{n^2 + n}{2n^2} \cdot l = \frac{L}{2}$$

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24.

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26.

Pf. (1)
$$u_1 = \sqrt{nt_2} - \sqrt{nt_1} = \sqrt{nt_2} + \sqrt{nt_1} \sim 0$$

(3)
$$W_n = \sqrt{1 + \frac{1}{\ln(n+1)}} - 1 = \frac{1}{\ln(n+1)} \sim 0$$

)+ SIt in (n+1)

28.

$$|Y|: \qquad (1) \qquad \sqrt{RH} - \sqrt{R} = \frac{RH - R}{\sqrt{RH} + \sqrt{R}} = \frac{1}{\sqrt{RH} + \sqrt{R}}$$

$$\Rightarrow \frac{1}{2\sqrt{RH}} \leq \sqrt{RH} - \sqrt{R} \leq \frac{1}{2\sqrt{R}}$$

12).

$$=) \qquad \lim_{n\to\infty} \frac{\ln n}{2\sqrt{n}} = 1$$

 \Box

30.

$$\frac{\sum_{k=1}^{n} k!}{n!} = |+ \frac{1}{n} + \frac{\sum_{k=1}^{n-2} k!}{n!} \\
\leq |+ \frac{1}{n} + \frac{\sum_{k=1}^{n-2} (n-2)!}{n!} \\
= |+ \frac{1}{n} + \frac{n-2}{n(n-1)} \\
\leq |+ \frac{1}{n} + \frac{n-2}{n(n-1)} \\
\leq |+ \frac{1}{n} + \frac{n}{n} = |+ \frac{2}{n} \\
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