Learning from Monetary and Fiscal Policy[†]

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Abstract

We present a dynamic incomplete information model where monetary and fiscal policy instruments serve as endogenous signals for the private sector. We highlight a novel information channel of policy interactions, and show the general equilibrium (GE) information feedback between policies largely shapes the economy's response to policy shocks. We document a non-monotone signaling effect of policies with respect to the policy rule parameters. Our analysis shows the GE information feedback is quantitatively significant, and the model provides a unified explanation of the various policy impacts on inflation, the dynamics of survey expectations, and the missing inflation after the Great Recession.

Keywords: Incomplete Information; Multiple Endogenous Signals; General Equilibrium; Signal Extraction; Inflation Dynamics.

JEL Classification: D83, E31, E63

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1 Introduction

Monetary and fiscal policies share the goal of stabilizing the real economy. This common objective has become ever more important after the U.S. economy witnessed the two worst fallouts since the Great Depression—the Great Recession of 2007–2009 and the ongoing COVID-19 pandemic. In response to these big economic downturns, both the Federal Reserve and the U.S. Treasury have conducted a series of large-scale expansionary policies to pull the economy out of the recession. Economists have long recognized the stabilization effects of monetary and fiscal policy. Nevertheless, most work featuring both policies assume that economic agents are endowed with full information about economic fundamentals.

In tumultuous times when policy interventions are needed, the full information assumption is a dubious one. When an exogenous policy shock is realized, the magnitude and nature of the shock are usually unclear to the private sector. Meanwhile, it induces changes in the publicly observed policy instrument (e.g., federal funds rate or government outlays) that conveys useful information about economic fundamentals. Consequently, market participants may interpret the exogenous policy shock as an endogenous policy response to changing economic conditions. While aggregate fluctuations can lead to adjustments in both policy instruments, recent studies have focused on the signaling effect of monetary policy [see Melosi (2017) and Nakamura and Steinsson (2018)].

In this paper, we study the joint signaling role of monetary and fiscal policy and its macroeconomic implications. A salient feature of these policies is that they are inherently endogenous and convey different information about the aggregate economy to the private sector. When one policy shock hits the economy, the automatic stabilizer induces an endogenous adjustment in the other policy through the general equilibrium (henceforth GE) feedback channel. In an incomplete information environment, allowing the private sector to learn from both policies introduces a novel information interaction between policies. In equilibrium, such an interaction alters both policies' information effects and will inevitably affect their transmission mechanism and ability to stabilize the economy.

We conduct the analysis within a standard new Keynesian framework featuring incomplete information and learning from policies. The information friction operates through a demand shock (i.e., discount factor shock) and a supply shock (i.e., labor disutility shock), which enter the representative household's preference and are not observed by firms. The monetary authority sets the nominal interest rate, and the fiscal authority determines the government spending. Each policy follows a simple rule that responds to the state of the economy. Assuming the policymakers' information sets are no worse than the household, both policy instruments will contain information about the demand and supply shocks. As a result, firms use private and public signals (which include the policy instruments) to infer the true state of the economy, giving rise to the information interaction between policies.

Our model provides a unified explanation of the various policy impacts on the price level documented in the vector autoregression (VAR) literature. That is, (i) the initial price level could either increase or decrease following a monetary contraction, and (ii) expansionary government spending could crowd in private consumption while dampening inflation. Without all of the bells and whistles, we capture the key model insights in a simple version of the model that permits closed-form characterization of the economy. For example, when a positive government spending shock hits the economy, the *direct signaling* effect of a counter-cyclical fiscal policy induces firms to attribute such a fiscal expansion to a negative demand shock. The perceived lousy demand condition causes firms to lower their inflation expectations and hence charge a lower price. Meanwhile, lower inflation triggers a GE *real feedback* from monetary policy, which leads to a lower real interest rate and hence encourages the household's consumption. More importantly, changes in the monetary policy signal also creates a GE *information feedback* that further shifts firms' beliefs. We find a similar equilibrium mechanism that underpins the response of inflation to the monetary policy shock.

A key result of this paper is that the GE information feedback largely depends on the policy rule coefficients, the model's information structure, as well as the conventional GE real feedback of the economy. These factors determine whether the monetary and fiscal policy signals convey a similar message about the economic condition that reinforce each other, or opposite messages that attenuate each other. We perform an explicit decomposition of the equilibrium response of inflation to quantify the GE information feedback between policies. Such a feedback effect is determined as the product of the policy signal's marginal value and its actual change. We then analyze how changes in policy shock volatilities affect the information feedback and equilibrium dynamics.

Another essential result is that varying the policy rule coefficient has a non-monotone effect on the information channel of monetary and fiscal policy. Specifically, the signaling effect first rises and then falls as the policy response to the endogenous change of the economy becomes increasingly aggressive. The intuition behind this finding is that when the initial policy response is very dovish, a larger responsiveness induces firms to place more probability weight on the endogenous component of the policy. On the other hand, when the initial policy response is very hawkish, firms will interpret most of the signal change as a result of the policy authority being overly aggressive rather than a change in the economic condition.

Our theoretical findings extend to more general settings. The numerical analysis suggests that the GE information feedback is quantitatively significant in accounting for the inflation dynamics. For example, due to the information feedback from monetary policy, even a weakly counter-cyclical fiscal policy can generate a sizable decrease in inflation following a positive government spending shock. We also quantify the information flow transmitted by each policy instrument. While monetary policy is found to be more informative, learning from fiscal policy is essential for the model to explain the VAR impulse responses to a government spending shock. Meanwhile, our numerical model is able to explain the dynamics of inflation forecast error and the lack of inflationary pressure after the Great Recession. We add the caveat that a more dovish monetary policy response to inflation or output gap may not resolve the missing inflation puzzle.

We discuss the novelty of our theoretical results by connecting them to the existing literature. First, our framework incorporates both monetary and fiscal policies as sources of information for firms. This setup extends the signaling channel of monetary policy [see Melosi (2017), Nakamura and Steinsson (2018), and Benhima and Blengini (2020)]. We uncover a novel GE information feedback between policies that is quantitatively important in determining the response of inflation to policy shocks. Compared with models that only accommodate exogenous signals [e.g., Blanchard, L'Huillier and Lorenzoni (2013), Chahrour and Jurado (2018), and Benhima and Poilly (2020)], we show that one policy shock could either amplify or dampen the signaling effect of the other policy. While we do not take a stance on which information structure characterizes the actual data, we believe that identifying and understanding this feedback channel is important both theoretically and empirically. Moreover, the insights obtained herein may be generalized to the information channel of other economic policies.

Second, this paper belongs to a burgeoning literature that explores the macroeconomic implications of information frictions [see Nimark (2008), Lorenzoni (2009), Coibion and Gorodnichenko (2012), and Angeletos and Huo (2020)]. Among those, Berkelmans (2011) shows that a monetary tightening can raise the initial price level (i.e., monetary price puzzle) using similar information frictions. On the fiscal side, Murphy (2015) studies how government spending can stimulate consumption under incomplete information, but the flip side of the fiscal price puzzle (i.e., lower inflation) remains unexplored. We introduce learning from monetary and fiscal policy that accounts for both puzzles in a unified framework.

Third, we contribute to the literature that seeks to identify sources of information frictions [see Barsky and Sims (2012), Schmitt-Grohé and Uribe (2012), and Blanchard, L'Huillier and Lorenzoni (2013)]. While the literature emphasizes the roles of news and noise of productivity shocks, in our model, the main information friction stems from firms' incomplete information about the household's demand due to the unobserved preference shocks. Our approach is consistent with Benhima and Poilly (2020) and Angeletos, Collard and Dellas (2020), both of which provide empirical evidence that demand side uncertainties contribute substantially to business cycle fluctuations.

Finally, this paper belongs to the literature on the interaction between monetary and fiscal policies in determining the price level [see Leeper (1991), Sargent and Wallace (1981), Sims (1994), and Woodford (2001)]. In our paper, we focus entirely on the conventional "active monetary and passive fiscal" regime, where monetary policy is assigned to control inflation by raising nominal interest rate aggressively with inflation and fiscal policy is assigned to stabilize debt by adjusting taxes or spending. Nevertheless, we share the spirit of Leeper and Leith (2015) that inflation is a joint monetary-fiscal phenomenon. Our approach is also similar to Eusepi and Preston (2018), who propose a theory of the fiscal foundations of inflation based on imperfect knowledge and recursive least-square learning under the same regime.

The rest of the paper is planned as follows. Section 2 describes the incomplete information model environment. Section 3 solves a simple version of the model analytically to shed light on the information channel of monetary and fiscal policy. Section 4 performs a numerical analysis that replicates the empirical policy effects on inflation and survey expectations using our full model. We then calculate the information flow of each policy instrument, and link our theory to the lack of inflationary pressure after the Great Recession. Section 5 concludes.

2 Model Environment

We consider a prototypical new Keynesian model and augment it with an endogenous fiscal rule on government spending. The model contains a representative household, a continuum of monopolistic competitive firms with Calvo (1983) pricing, and the monetary and fiscal authorities. The main information friction originates from two structural shocks that enter the household's preference but cannot be observed by firms.

2.1 Household

The representative household maximizes the utility function

$$\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t \exp(\varepsilon_t^{\beta}) \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \exp(\varepsilon_t^N) \frac{N_t^{1+\gamma}}{1+\gamma} \right], \tag{2.1}$$

where $\beta \in (0,1)$ is the deterministic discount factor, $1/\sigma > 0$ is the intertemporal elasticity of substitution, and $1/\gamma > 0$ is the Frisch elasticity of labor supply. $\mathbb{E}_t^{HH}$ is an expectation operator conditional on the household's information set $\mathcal{I}_t^{HH}$, which will be specified below. There are two preference shocks, a demand shock ε_t^{β} and a labor disutility shock ε_t^N, that affect the household's inter- and intra-temporal substitutions. These shocks have been identified as important drivers of business cycle fluctuations [see, e.g., Smets and Wouters (2003, 2007)]. For simplicity, both shocks follow *i.i.d.* Gaussian distributions, i.e., $\varepsilon_t^{\beta} \sim \mathbb{N}(0, \sigma_{\beta}^2)$ and $\varepsilon_t^N \sim \mathbb{N}(0, \sigma_N^2)$. During each period, the household earns real wage income $W_t N_t$ and receives real lump-sum dividend D_t from firms. They spend income on the final consumption C_t and have access to the one-period, risk-free bond B_t. They also need to pay lump-sum net taxes T_t. Denote R_t as the nominal interest rate and P_t as the aggregate price level. The household's budget constraint is given by

$$C_t + \frac{1}{R_t} \frac{B_t}{P_t} + T_t = W_t N_t + \frac{B_{t-1}}{P_t} + D_t. \tag{2.2}$$

Final consumption is aggregated according to $C_t = \left(\int_0^1 C_t(j)^{(\nu-1)/\nu} dj\right)^{\nu/(\nu-1)}$, where $C_t(j)$ is consumption of the good produced by firm j in period t, and $\nu > 0$ is the elasticity of substitution between consumption goods.

2.2 Firms

There are a continuum of firms indexed by $j \in [0, 1]$. Firms are endowed with a linear technology $Y_t(j) = A_t(j)N_t(j)$, where $Y_t(j)$ is the production level and $N_t(j)$ is the amount of labor hired by firm j at time t. The firm-specific level of technology $a_{t,j} = \log A_t(j)$ can be decomposed into a persistent aggregate technology a_t and an *i.i.d.* idiosyncratic component $\eta_{t,j}$,

$$a_{t,j} = a_t + \eta_{t,j}, \tag{2.3}$$

where $\eta_{t,j} \sim \mathbb{N}(0, \sigma_\eta^2)$ and $a_t = \rho_A a_{t-1} + \varepsilon_t^A$ with $\varepsilon_t^A \sim \mathbb{N}(0, \sigma_A^2)$.

Denote $\theta > 0$ the price stickiness. During each period, there is a constant probability $(1 - \theta)$ that a firm can reset its price to maximize its current market value of profit generated while that price remains effective. Using the lowercase variables to denote the log deviations from their non-stochastic steady states, the optimal price for firm j who can re-optimize at time t is given by the discounted sum of its current and expected future nominal marginal costs

$$p_{t,j}^* = (1 - \beta\theta)\mathbb{E}_{t,j}^{Firm} \sum_{k=0}^{\infty} (\beta\theta)^k (p_{t+k} + mc_{t+k,j}), \tag{2.4}$$

where $mc_{t,j}$ is firm j's real marginal cost at time t. The individual firm's expectations operator, $\mathbb{E}_{t,j}^{Firm}$, is conditional on its information set $I_{t,j}^{Firm}$, which will be specified below. Aggregate price level p_t is the weighted average between firms who cannot re-optimize and firms who re-optimize their prices, i.e., $p_t = \theta p_{t-1} + (1 - \theta) \int p_{t,j}^* dj$. Inflation is defined as $\pi_t = p_t - p_{t-1}$.

2.3 Monetary and Fiscal Policy

The policy authorities follow simple rules. The monetary authority sets the nominal interest rate R_t according to a Taylor-type rule that reacts to inflation and output gap. Let i_t denote the log-deviation of R_t from its steady state R^*. Monetary policy is given by

$$i_t = \phi_\pi \pi_t + \phi_y (y_t - y_t^n) + u_t^R, \tag{2.5}$$

where y_t^n is the log-deviation of output under flexible price (i.e., the natural output), and the term, $y_t - y_t^n$, defines the output gap at time t. The exogenous monetary shock follows an AR(1) process $u_t^R = \rho_R u_{t-1}^R + \varepsilon_t^R$ with $\varepsilon_t^R \sim \mathbb{N}(0, \sigma_R^2)$. The policy coefficients satisfy $\phi_\pi > 1$ (i.e., the Taylor principle) and $\phi_y > 0$.

The fiscal authority runs a balanced budget constraint and collects lump-sum net taxes T_t to finance its expenditure G_t. Let g_t denote the log-deviation of G_t from its steady-state level. Following Leeper, Plante and Traum (2010), the fiscal rule on government spending embeds an "automatic stabilizer" and is given by

$$g_t \quad = -\phi_g (y_t - y_t^n) + u_t^G. \tag{2.6}$$

The exogenous fiscal shock follows an AR(1) process $u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$ with $\varepsilon_t^G \sim \mathbb{N}(0, \sigma_G^2)$. In what follows, we focus on a counter-cyclical fiscal policy that requires $\phi_g > 0$.

2.4 Timing, Information, and Equilibrium

Our timing assumption follows Melosi (2017), who emphasizes the signaling effects of monetary policy but omits fiscal policy. Each period t is divided into three stages.

At stage 0, all shocks hit the economy. We deviate minimally from the convention and assume that both policy authorities are equipped with full information. After observing the realization of shocks, the central bank sets the nominal interest rate i_t according to (2.5), and the government commits to g_t according to (2.6). Notice that firms have not yet started production at stage 0. However, policymakers with full information can still set and commit to (i_t, g_t) as they have learned the mapping between output and the underlying shocks.

At stage 1, firms update their information set by observing the firm-specific technology and both policy instruments. This timing assumption thus embeds policy foresight that is inherent to the

U.S. monetary and fiscal policy [Jarociński and Karadi (2020), Leeper, Walker and Yang (2013)]. Denote the information set of firm j at time T as $I_{T,j}^{Firm}$. Previous literature has assumed that firms can either learn, as in Nimark (2008)), or do not learn, as in Melosi (2017), from past histories of output and inflation due to rational inattention. To showcase our main results hold under these alternatives, we consider three cases of $I_{T,j}^{Firm}$:

$$\begin{aligned} \text{Case I:}\ I_{T,j}^{Firm} &= \{a_{t,j}, i_t, g_t, \pi_{t-1}, y_{t-1} : t \leqslant T\}, \\ \text{Case II:}\ I_{T,j}^{Firm} &= \{a_{t,j}, i_t, g_t, y_{t-1} : t \leqslant T\}, \\ \text{Case III:}\ I_{T,j}^{Firm} &= \{a_{t,j}, i_t, g_t : t \leqslant T\}. \end{aligned} \tag{2.7}$$

Given each $I_{T,j}^{Firm}$, firms set their prices.[1]

At stage 2, the household forms expectations. Since the shocks ε_t^{β} and ε_t^{N} enter the preference directly, we include them in the household's information set. We assume the household also knows their labor productivity a_t. Define $I_T^{HH} = \{\varepsilon_t^{\beta}, \varepsilon_t^{N}, a_t, i_t, g_t, \pi_{t-1}, y_{t-1} : t \leqslant T\}$, which is equivalent to

$$I_T^{HH} = \{\varepsilon_t^{\beta}, \varepsilon_t^{N}, \varepsilon_t^{A}, \varepsilon_t^{R}, \varepsilon_t^{G} : t \leqslant T\}. \tag{2.8}$$

The household thus learns about the realization of all aggregate shocks. At stage 2, they provide labor, and make consumption and saving decisions. Firms hire labor and produce. The government fulfills its spending and imposes taxes. In equilibrium, the goods, labor, and bond markets all clear. The transversality condition on debt holds. Appendix A provides the log-linearized equilibrium system.

3 Analytical Results

In this section, we focus on a special case of the model that serves two purposes. First, it is simple enough to admit an analytical characterization of the economy, yet rich enough to highlight the general features of the information channel of monetary and fiscal policy. Such a closed-form solution

[1] The current inflation π_t and output y_t are not realized at stage 1 of time t. We omit the case where $I_{T,j}^{Firm} = \{a_{t,j}, i_t, g_t, \pi_{t-1} : t \leqslant T\}$ as it yields similar results as Case III.

is also rare in dynamic incomplete information settings with multiple endogenous signals. Second, it delivers a clean graphical illustration of our main results on the simple consumption-inflation $(c - \pi)$ plane. We relegate the detailed proofs of propositions presented herein to Appendix B.

3.1 Closed-Form Solution

To begin with, the simple model derives from the following assumption.

Assumption 1. *The household is equipped with log utility of consumption ($\sigma = 1$) and linear disutility of labor ($\gamma = 0$). There are neither labor disutility shocks ($\sigma_N = 0$) nor aggregate and idiosyncratic technology shocks ($\rho_A = 0, \sigma_A = \sigma_\eta = 0, A_t = 1$). All remaining shocks are i.i.d., i.e., $\rho_R = \rho_G = 0$. Moreover, the monetary and fiscal policy coefficients on the output gap satisfy $\phi_y = \phi_g = \phi > 0$.*[2]

Assumption 1 allows us to consider the simple information structure

$$I_T^{Firm} = \{i_t, g_t : t \leqslant T\}. \tag{3.1}$$

Several remarks about (3.1) are in order. First, since we shut down the technology shocks and all remaining shocks are *i.i.d.*, the three cases of $I_{T,j}^{Firm}$ in (2.7) all reduce to (3.1) where the individual index j can be dropped. This result is confirmed in Proposition 3.1 below as the equilibrium allocations are *i.i.d.* and all past information can be excluded from the information set. Second, there is no information heterogeneity. The current and past history of inflation $\{\pi_t : t \leqslant T\}$ becomes common knowledge since it is fully determined by the firm's homogenous conditional expectation (as evident from (3.2) below). As a result, there is no need to augment (3.1) with $\{\pi_t : t \leqslant T\}$ since it is redundant information. Third, (3.1) allows us to focus on how monetary and fiscal policy signals influence firms' beliefs about the demand-side uncertainty—one centerpiece in our study of the policy information channel.

In the simple model, inflation is determined solely by the firm's conditional expectation about the current real marginal cost and future inflation. We derive the log-linearized new Keynesian

[2] The restriction $\phi_y = \phi_g$ is nonessential to any of our results, but it substantially eases the algebra.

Phillips curve (NKPC), which embeds the intermediate firm's optimal price-setting behavior, as

$$\pi_t = \beta \mathbb{E}_t^{Firm} \pi_{t+1} + \kappa \mathbb{E}_t^{Firm} mc_t,$$

where $\kappa = (1-\beta\theta)(1-\theta)/\theta > 0$ measures the positive slope of the NKPC under full information. By the *i.i.d.* assumption for all shocks, $\mathbb{E}_t^{Firm}\pi_{t+1} = 0$. The real marginal cost mc_t, which is given by the real wage, can be derived from the household's intratemporal Euler equation, $mc_t = \sigma c_t + \gamma y_t = c_t$ with $\sigma = 1, \gamma = 0$. It follows that inflation depends entirely on the firm's nowcast about the current consumption demand c_t

$$\pi_t = \kappa \mathbb{E}_t^{Firm} c_t. \tag{3.2}$$

The monetary and fiscal policy signals then affect inflation and the NKPC movement through the expectation formation in (3.2).

On the other hand, we derive the log-linearized consumption demand (i.e., the dynamic IS curve) from the household's intertemporal Euler equation

$$c_t = \mathbb{E}_t^{HH} c_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t^{HH}\pi_{t+1} - \varepsilon_t^{\beta} + \mathbb{E}_t^{HH}\varepsilon_{t+1}^{\beta}).$$

Given the *i.i.d.* assumption for all shocks and the fact that the household has full information, $\mathbb{E}_t^{HH} c_{t+1} = \mathbb{E}_t^{HH}\pi_{t+1} = \mathbb{E}_t^{HH}\varepsilon_{t+1}^{\beta} = 0$. Substituting the monetary policy rule (2.5) for i_t and rearranging, the IS curve can be expressed as

$$c_t = \frac{1}{1+\Phi}(\varepsilon_t^{\beta} - \phi_\pi \pi_t - \varepsilon_t^{R}), \tag{3.3}$$

where $\Phi = \phi\frac{1-g_y}{1+g_y\phi} > 0$ measures the policy response to consumption, and $-\phi_\pi/(1+\Phi) < 0$ captures the negative slope of the IS curve. It is clear from (3.3) that monetary policy affects the consumption demand and the IS curve movement directly through its shock ε_t^R. As will be shown below, both policies affect the consumption demand also indirectly through their signaling and GE information feedback effects.[3]

Based on the consumption-inflation ($c - \pi$) relations (3.2) and (3.3), the following proposition characterizes the unique incomplete information equilibrium in closed-form.[4]

Proposition 3.1. *Given Assumption 1 and the information structure (3.1), the model features a unique equilibrium in which inflation and consumption follow*

$$\pi_t = S_R s_t^R + S_G s_t^G, \tag{3.4}$$

$$c_t = C_\beta \varepsilon_t^\beta + C_R \varepsilon_t^R + C_G \varepsilon_t^G, \tag{3.5}$$

where (s_t^R, s_t^G) in (3.4) are informationally equivalent transformation of (i_t, g_t) in (3.1), i.e.,

$$s_t^R = \Phi c_t + \varepsilon_t^R, \tag{3.6}$$

$$s_t^G = -\Phi c_t + \varepsilon_t^G, \tag{3.7}$$

with $\Phi = \phi \frac{1-g_y}{1+g_y\phi} > 0$. In particular, the coefficients in (3.4) and (3.5) are given by

$$S_R = \frac{\kappa\sigma_G^2(\sigma_\beta^2\Phi - \sigma_R^2)}{\Delta_1}, \qquad S_G = -\frac{\kappa\Phi\sigma_\beta^2\sigma_R^2}{\Delta_1},$$
$$C_\beta = \frac{1}{\Delta_2}, \qquad C_R = -\frac{1+\phi_\pi S_R}{\Delta_2}, \qquad C_G = -\frac{\phi_\pi S_G}{\Delta_2},$$

where $\Delta_1 = (1+\kappa\phi_\pi)\sigma_G^2\sigma_R^2 + (\sigma_G^2 + \sigma_R^2)\Phi^2\sigma_\beta^2 > 0$ and $\Delta_2 = 1 + \phi_\pi(S_R - S_G)\Phi + \Phi > 0$.

The transformed information structure (3.6) and (3.7) makes clear what the firm can and cannot observe in terms of the model's fundamentals, and thus deserves some elaboration. Because inflation becomes perfectly observable in this simple model, we can deduce from the monetary policy rule (2.5) that the firm in effect observes $s_t^R \equiv i_t - \phi_\pi \pi_t$, which equals the sum of the output gap component that is linked to the consumption demand via $\phi(y_t - y_t^n) = \Phi c_t$, and the monetary shock ε_t^R. For this reason, we call s_t^R the effective monetary policy signal. Analogously, we can deduce

[3] Under full information, however, fiscal policy has no impact on consumption and inflation since $\mathbb{E}_t^{Firm} c_t = c_t$ and ε_t^G enters neither the NKPC equation (3.2) nor the IS equation (3.3).

[4] We use the frequency-domain numerical procedure of Han, Tan and Wu (2020) to verify our closed-form solution.

from the fiscal policy rule (2.6) that by observing $s_t^G \equiv g_t$, the firm in effect knows the sum of the consumption demand $-\Phi c_t$ and the fiscal shock ε_t^G. Substituting the inflation process (3.4) into the NKPC equation (3.2),

$$\mathbb{E}_t^{Firm} c_t = \frac{1}{\kappa}(S_R s_t^R + S_G s_t^G), \tag{3.8}$$

one may immediately see how the coefficients (S_R, S_G) capture the *direct signaling* effects of (s_t^R, s_t^G) on the firm's perception about the consumption demand, $\mathbb{E}_t^{Firm} c_t$. More generally, we define the direct signaling effect of a policy as the confounding between the policy's endogenous response to the economic condition and its exogenous shock. In this sense, (S_R, S_G) represent the marginal values of policy signals in predicting the endogenous policy target—they measure the change in firms' beliefs about the endogenous policy target given one unit increase in the actual policies (s_t^R, s_t^G). With only two signals available, the firm is facing a non-square signal extraction problem that prevents it from fully recovering each of the three fundamentals $(c_t, \varepsilon_t^R, \varepsilon_t^G)$, or equivalently, $(\varepsilon_t^\beta, \varepsilon_t^R, \varepsilon_t^G)$. The resulting demand-side uncertainty faced by firms generates the central friction of the simple model. It also gives rise to an important information channel by which monetary and fiscal policies affect the inflation process, as will be explained below.

The underlying signal extraction problem departs from the standard (exogenous) noisy information filtering in important ways. First, because monetary and fiscal policies share the goal of stabilizing the private consumption, changes in one policy instrument generate a general equilibrium (GE) feedback from the other policy through the endogenous policy target (i.e., c_t), thereby creating a signaling interaction between policies. Second, monetary and fiscal policy disturbances play a dual role of information shocks through the signal extraction problem and conventional structural shocks through the structural equations of the model economy.

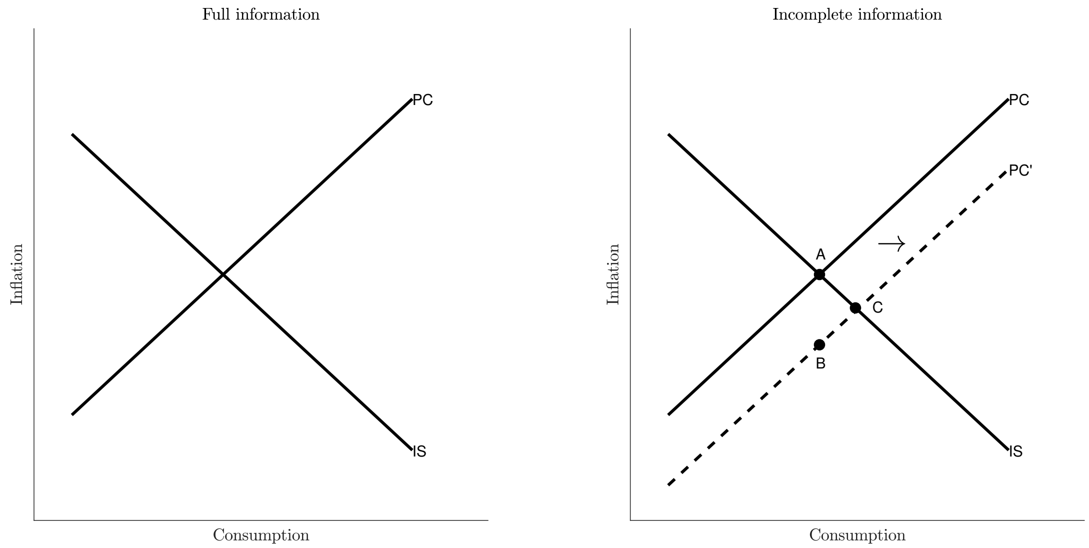

Figure 1: Effects of a positive government spending shock in the consumption-inflation space.

3.2 Key Intuitions of the Information Channel

To illustrate how the information channel works, it is useful to rewrite the solution for inflation (3.4) in the space of structural shocks

$$\pi_t = \Pi_\beta \varepsilon_t^\beta + \Pi_R \varepsilon_t^R + \Pi_G \varepsilon_t^G, \tag{3.9}$$

where the coefficients $(\Pi_\beta, \Pi_R, \Pi_G)$ are provided in Appendix B. In conjunction with the consumption process (3.5), the above solution immediately leads to the following characterization of the economy's response to policy shocks.

Proposition 3.2. *Given Assumption 1 and the information structure (3.1), the model generates the fiscal price puzzle (FPP), i.e.,* $\frac{\partial \pi_t}{\partial \varepsilon_t^G} = \Pi_G < 0$ *and* $\frac{\partial c_t}{\partial \varepsilon_t^G} = C_G > 0$*. If in addition,*

$$\frac{1}{\Phi} + \Phi \frac{\sigma_\beta^2}{\sigma_G^2} \leqslant \frac{\sigma_\beta^2}{\sigma_R^2}, \tag{3.10}$$

then the model also produces the monetary price puzzle (MPP), i.e., $\frac{\partial \pi_t}{\partial \varepsilon_t^R} = \Pi_R > 0$*.*

The FPP is a relatively new finding in the VAR literature. Among others, D'Alessandro, Fella and Melosi (2019) and Jørgensen and Ravn (2019) both find that an increase in government spending raises private consumption while reducing inflation.[5] To build intuition behind the FPP, it is helpful to look at Figure 1 that plots the impacts of a positive government spending shock $\varepsilon_t^G > 0$ in the consumption-inflation space. For comparison purposes, the left panel shows the full information case where neither the IS nor the Phillips curve moves, leaving the equilibrium point unchanged—a result of log utility and linear labor disutility. In other words, the fiscal shock only serves as an information shock in the simple model. The right panel depicts the incomplete information case in which this expansionary shock shifts the Phillips curve to the right.[6] Suppose for now that the actual consumption does not change. Although the fiscal shock does not enter the NKPC equation explicitly, it can shift the Phillips curve by affecting the firm's belief about the prevailing demand condition. In particular, this higher spending shock raises the counter-cyclical government spending $s_t^G = -\Phi c_t + \varepsilon_t^G$ (recall $-\Phi < 0$) that in turn signals a weaker consumption demand c_t, triggered by say an adverse demand shock $\varepsilon_t^\beta < 0$, to unaware firms. As a result, firms will revise their beliefs towards a lower consumption demand. Given such a pessimistic view about the state of the economy, this expansionary policy signal induces the firm to set a lower price, as evinced by $S_G < 0$ in Proposition 3.1, which leads to a fall in inflation and thus a downward movement of the equilibrium point in Figure 1 ($A \rightarrow B$).

Nevertheless, consumption as an endogenous quantity does change in equilibrium. In response to the lower inflation, monetary policy responds by reducing the nominal rate i_t more than one-for-one (i.e., $\phi_\pi > 1$). This is the GE *real feedback* channel between policies, which is operated through the structural relations that underpin conventional monetary-fiscal policy interactions. On the other hand, the household's expected future inflation $\mathbb{E}_t^{HH}\pi_{t+1}$ is identically zero in this simple model. The result is thus a net decrease in the real rate $r_t \equiv i_t - \mathbb{E}_t^{HH}\pi_{t+1}$, inducing the household to save less and consume more today.[7] Provided that monetary policy is active ($\phi_\pi > 1$), this crowding-in effect also holds in the full version of the model where the household's expected inflation falls, as will be shown in Section 4. We emphasize that the mechanism illustrated here is fundamentally

[5] These work maintain the full information assumption in their theoretical models.

[6] The slope of the NKPC under incomplete information is indeed flatter than its full information counterpart. This difference is not shown in Figure 1 and 2 for illustrative simplicity.

[7] The higher consumption results in a further GE real feedback from monetary policy that raises the nominal rate i_t, but the standard GE attenuation effect implies that such a change is dominated by the impact induced by inflation. In Appendix B, we show that in equilibrium i_t decreases.

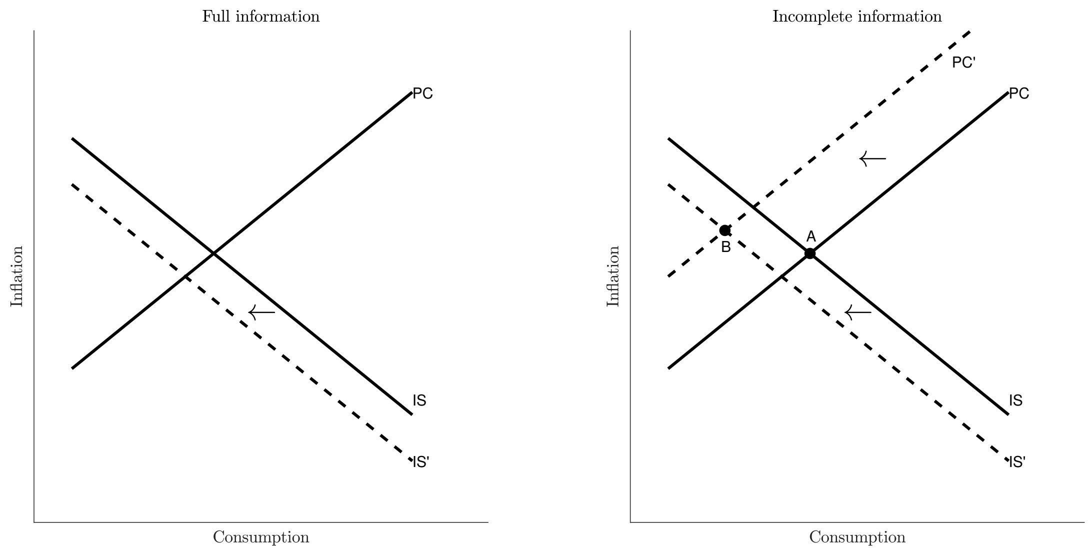

Figure 2: Effects of a positive interest rate shock in the consumption-inflation space.

different from that of Dupor and Li (2015), who require a *higher* expected inflation in conjunction with a passive monetary policy ($\phi_\pi < 1$) to reduce the real rate and hence stimulate consumption. Our theoretical results are, however, more in line with the VAR evidence of Appendix C that both actual and expected inflation *fall* in response to a positive government spending shock.

Continuing the equilibrium adjustment process, the induced change in consumption also leads to a novel GE *information feedback* that creates a signaling interaction between policies. To see this point, notice that the higher consumption mitigates the signaling effect of fiscal policy by lowering the fiscal signal $s_t^G = -\Phi c_t + \varepsilon_t^G$. More importantly, it also engenders a monetary policy feedback effect by raising the effective monetary signal $s_t^R = \Phi c_t + \varepsilon_t^R$. In this section, we assume the parameter restriction (3.10) holds, which implies $\sigma_\beta^2/\sigma_R^2 > 1/\Phi$. Proposition 3.1 then implies $S_R > 0$, and it follows from (3.8) that a higher s_t^R induces firms to adjust upward their estimates about the consumption demand. Therefore, the GE information feedback causes monetary policy to attenuate the signaling effect of fiscal policy. In this parameter case, monetary policy serves a role of partially revealing information. Such a belief change further affects the actual inflation and consumption, and the above equilibrium adjustment continues until the economy settles at the new intersection point C in Figure 1.

Turning to the MPP that holds under (3.10), Sims (1992) first identifies an initial increase in the price level following a contractionary monetary shock [see also Eichenbaum (1992), Hanson (2004), Romer and Romer (2004), Giordani (2004), Bernanke, Boivin and Eliasz (2005), and Brissimis and Magginas (2006), among others]. Like the FPP, a similar reasoning can be visualized in Figure 2 that plots the impacts of a positive interest rate shock $\varepsilon_t^R > 0$ in the consumption-inflation space. Because ε_t^R enters the IS equation (3.3) directly, this contractionary shock shifts the IS curve to the left in both panels. Under incomplete information and the condition (3.10), it shifts the Phillips curve to the left since $\sigma_\beta^2/\sigma_R^2 > 1/\Phi$ and $S_R > 0$. In this case, unaware firms interpret the resulting increase in the effective monetary signal $s_t^R = \Phi c_t + \varepsilon_t^R$ (recall $\Phi > 0$) as a sign of stronger consumption demand and hence charge a higher price, which leads to a rise in inflation.

As in the FPP case, the GE real feedback channel implies that the monetary authority responds to the higher inflation by further raising the nominal rate. Given that the household's expected future inflation is zero, the result is a net increase in the real rate, inducing the household to save more and consume less today. Moreover, the falling consumption also triggers the GE information feedback from both policy signals, which continue to affect the actual inflation and consumption through influencing the firm's belief about the consumption demand.[8] Given the initial signaling effect and the subsequent GE feedback, the resulting equilibrium is determined by the relative movement of the two curves; when the condition (3.10) holds, the Phillips curve shifts to the left more than the IS curve does in Figure 2 and the economy features the MPP ($A \rightarrow B$).

3.3 GE Information Feedback Effects

We have shown that under incomplete information, the economy's response to a policy shock is determined by the combination of the policy's direct signaling effect and the GE real and information feedback effects between policies. A key insight of this paper is that the role of such a feedback channel depends crucially on the parameters of policy rules, the information structure of private agents, and the GE real feedback of the economy. When a policy shock hits the economy, these factors determine whether monetary and fiscal policies convey a similar message that reinforce each other, or different messages that attenuate each other. The signaling interaction between policies

[8]The rise in the fiscal signal $s_t^G = -\Phi c_t + \varepsilon_t^G$ is interpreted by firms as falling consumption demand since $S_G < 0$, which attenuates the signaling effect of monetary policy. Note that in our simple model, the fiscal shock ε_t^G has no impact on c_t and π_t per se. Therefore, the fiscal signal serves the role of revealing the true consumption demand and the fiscal shock acts as a "noise".

in turn affects the economy's response under incomplete information.

We now quantify the information channel by separating the GE information feedback effect from the direct signaling effect. To this end, it is useful to decompose the overall response of inflation to policy shocks as follows

$$\Pi_G = S_G - S_G \frac{\partial s_t^G}{\partial c_t} \frac{\partial c_t}{\partial \varepsilon_t^G} + S_R \frac{\partial s_t^R}{\partial c_t} \frac{\partial c_t}{\partial \varepsilon_t^G} = \underbrace{S_G}_{\text{FP Signaling}} - \underbrace{S_G \Phi C_G}_{\text{FP Feedback}} + \underbrace{S_R \Phi C_G}_{\text{MP Feedback}}, \tag{3.11}$$

$$\Pi_R = S_R + S_R \frac{\partial s_t^R}{\partial c_t} \frac{\partial c_t}{\partial \varepsilon_t^R} - S_G \frac{\partial s_t^G}{\partial c_t} \frac{\partial c_t}{\partial \varepsilon_t^R} = \underbrace{S_R}_{\text{MP Signaling}} + \underbrace{S_R \Phi C_R}_{\text{MP Feedback}} - \underbrace{S_G \Phi C_R}_{\text{FP Feedback}}, \tag{3.12}$$

where the first terms capture the direct signaling effects, and the remaining terms represent the GE information feedback effects of both policies due to changes in the consumption demand. Since $C_G > 0$ and $C_R < 0$, the feedback from the same policy (the second terms) always attenuates the direct signaling effects. The cross-policy feedback (the last terms), however, requires some more elaboration.

Consider an expansionary fiscal shock that generates the FPP (recall $S_G < 0$). Suppose now the monetary shock volatility σ_R^2 is relatively large such that $\sigma_\beta^2/\sigma_R^2 < 1/\Phi$. In this case, we have $S_R < 0$ in Proposition 3.1 and it follows from (3.11) that the information feedback effect of monetary policy *reinforces* the direct signaling effect of fiscal policy by strengthening the firm's initial belief about a weaker consumption demand. The key intuition of this result relies on the dual role of the monetary shock both as an information shock and a conventional policy shock. As the actual consumption rises, firms will more likely interpret the increase in the effective monetary signal $s_t^R = \Phi c_t + \varepsilon_t^R$ as being triggered by a contractionary monetary shock that further depresses the consumption demand. In conjunction with the signal conveyed by the fiscal expansion, this perceived policy coordination failure reinforces the firm's initial, pessimistic belief about the consumption demand. Put differently, the structural interpretation of firms qualitatively changes the marginal value of the effective monetary signal. One unit increase in s_t^R, possibly triggered by an increase in the demand shock ε_t^β, induces firms to lower their estimates on the consumption demand since $S_R < 0$. The underlying analysis shows that the endogenous policy signals are not simply a device of information revelation—their interactions can also create a perception of policy (in)coordination.[9]

When the monetary shock uncertainty is large such that $\sigma_\beta^2/\sigma_R^2 < 1/\Phi$, even the direct signaling effect of monetary policy disappears with $S_R < 0$. In this case, the monetary shock causes the NKPC to (initially) shift to the opposite direction compared to the situation in Section 3.2. Now suppose that we reduce σ_R to a relatively moderate level such that

$$\frac{1}{\Phi} \leqslant \frac{\sigma_\beta^2}{\sigma_R^2} \leqslant \frac{1}{\Phi} + \Phi\frac{\sigma_\beta^2}{\sigma_G^2}. \tag{3.13}$$

In such a case, we have $S_R > 0$ in Proposition 3.1 but (3.12) suggests that the information feedback effect of fiscal policy attenuates and even outweighs the direct signaling effect of monetary policy,

$$\Pi_R = \underbrace{\overset{(+)}{S_R}}_{\text{MP signaling}} + \underbrace{\overset{(-)}{S_R\Phi C_R}}_{\text{MP feedback}} - \underbrace{\overset{(+)}{S_G\Phi C_R}}_{\text{FP feedback}} < 0. \tag{3.14}$$

An alternative interpretation of (3.13) and (3.14) is that a low level of the fiscal shock uncertainty σ_G (high $\sigma_\beta^2/\sigma_G^2$) tends to allow fiscal policy to reveal the true information. Therefore, the overall information effect of both policies is weak and dominated by the real effect of the monetary contraction.

The following proposition summarizes the equilibrium responses to changes in the policy shock uncertainty.

Proposition 3.3. *Given Assumption 1 and the information structure (3.1), the following properties hold.*

1. *When a fiscal shock hits the economy, a higher monetary shock volatility (i) strengthens the direct signaling effect of fiscal policy, i.e.,* $\frac{\partial |S_G|}{\partial \sigma_R^2} > 0$, *and (ii) decreases the marginal value of the monetary signal if* $\sigma_\beta^2/\sigma_R^2 < 1/\Phi$, *i.e.,* $\frac{\partial S_R}{\partial \sigma_R^2} < 0$ *and* $S_R < 0$; *(iii) changes in both channels amplify the responses of inflation and consumption, i.e.,* $\frac{\partial \Pi_G}{\partial \sigma_R^2} < 0$ *and* $\frac{\partial C_G}{\partial \sigma_R^2} > 0$.

2. *When a monetary shock hits the economy and the condition (3.10) holds, a higher fiscal shock volatility (i) strengthens the direct signaling effect of monetary policy, i.e.,* $\frac{\partial |S_R|}{\partial \sigma_G^2} > 0$, *and (ii)*

[9] For example, if ε_t^R is a purely exogenous noise, then S_R will have a lower bound of zero, i.e., $\lim_{\sigma_R \to \infty} S_R = 0$.

decreases the marginal value of the fiscal signal, i.e., $\frac{\partial |S_G|}{\partial \sigma_G^2} < 0$ *and* $\lim_{\sigma_G \to \infty} S_G = 0$*; (iii) changes in both channels amplify the responses of inflation and consumption, i.e.,* $\frac{\partial \Pi_R}{\partial \sigma_G^2} < 0$ *and* $\frac{\partial C_R}{\partial \sigma_G^2} > 0$.

Proposition 3.3 states that increasing one policy shock's uncertainty increases the other policy's direct signaling effect. This is a straightforward result as a higher policy shock uncertainty increases the uncertainty of the endogenous policy component (i.e., consumption demand). As such, firms put more probability weight on the consumption demand when interpreting the policy signals. This insight continues to hold in the full model. On the other hand, higher uncertainty in one policy shock decreases the marginal value of the information feedback from the other policy. [10] As (3.11) and (3.12) indicate, the total information feedback is the product of the marginal value and the actual change in signals. While changes in the marginal value hold in more general settings, changes in the actual signals are model-dependent and can convey different messages to firms. In the simple model, a higher policy shock uncertainty changes the overall information feedback in a way that reinforces the initial policy signaling.

3.4 Non-monotone Effects of the Information Channel

Policy coefficients, such as $(\phi_\pi, \phi_y, \phi_g)$, can change over time and vary across countries. For example, Clarida, Gali and Gertler (2000) find substantial differences in the estimated monetary policy rule for the postwar U.S. economy before and after Volcker's appointment as Fed Chairman in 1979. On the other hand, Kaminsky, Reinhart and Végh (2004) document distinct cyclical patterns in fiscal policy between advanced and developing economies. Motivated by these empirical observations, a natural question is how different values of policy coefficients affect the information channel of monetary and fiscal policy. In this simple environment, the signaling effect operates entirely through the perceived consumption demand. We therefore focus on the role of policy responses to consumption as captured by the coefficient Φ in (3.6) and (3.7). The following proposition characterizes a non-monotone relation between Φ and its signaling effect.

[10] One needs to be cautious about interpreting this result. First, a lower marginal value does not mean that the GE information feedback is dampened. Rather, as described before, it represents a qualitative change (from positive to negative) in the interpretation of the GE information feedback. Second, how such a change in the marginal value and interpretation affects the overall response to policy shocks depends on the information structure of the model.

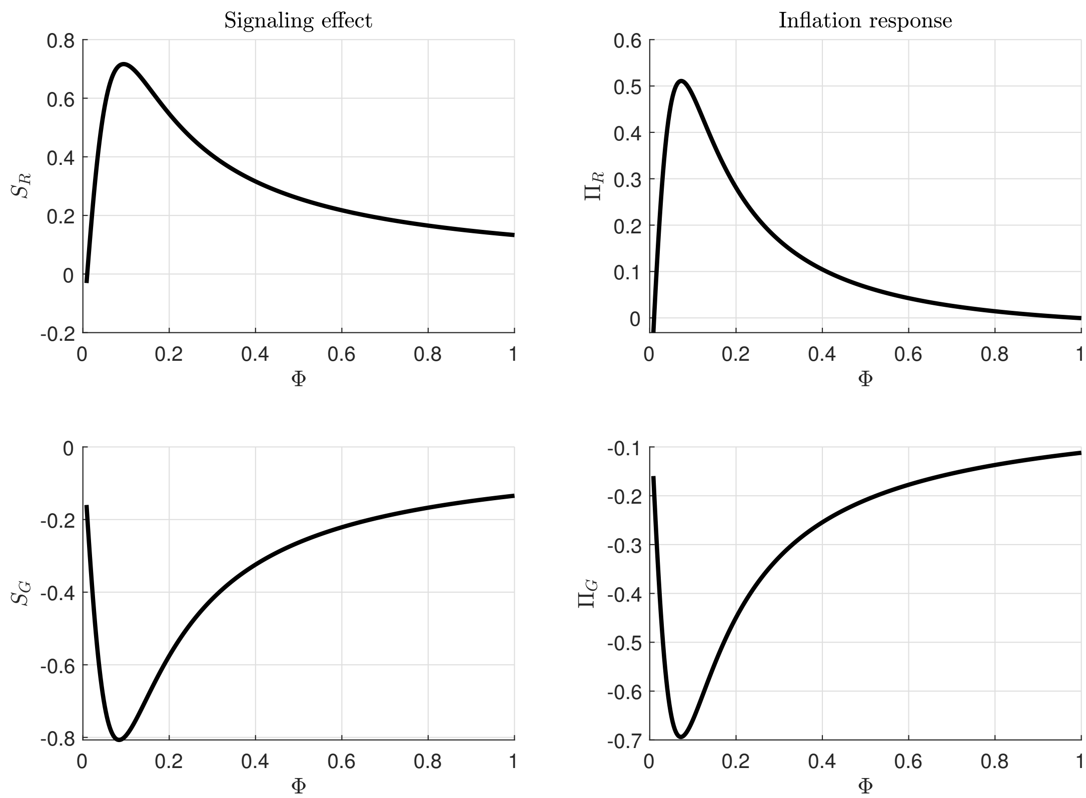

Figure 3: Non-monotone signaling effects (left panels) and implied inflation responses (right panels). Parameter values are fixed at: $\beta = 0.99$, $\theta = 0.6$, $\phi_\pi = 1.5$, $g_y = 0.16$, and $\sigma_\beta = \sigma_G = \sigma_R = 4$.

Proposition 3.4. *Given Assumption 1 and the information structure (3.1), the signaling effects* (S_R, S_G) *of monetary and fiscal policy are both non-monotonic functions of the policy response coefficient* $\Phi > 0$. *In particular, there exist threshold values* $\Phi_R, \Phi_G > 0$ *such that*

$$\begin{cases} \frac{dS_R(\Phi)}{d\Phi} > 0, & \Phi < \Phi_R \\ \frac{dS_R(\Phi)}{d\Phi} < 0, & \Phi > \Phi_R \end{cases}, \qquad \begin{cases} \frac{d|S_G(\Phi)|}{d\Phi} > 0, & \Phi < \Phi_G \\ \frac{d|S_G(\Phi)|}{d\Phi} < 0, & \Phi > \Phi_G \end{cases}.$$

The maximal signaling effects are achieved at

$$\Phi_R = \tau_{R,\beta} + \frac{\sqrt{\tau_{R,\beta}^2(1+\tau_{R,G})^2 + (1+\tau_{R,G})(1+\kappa\phi_\pi)\tau_{R,\beta}}}{1+\tau_{R,G}}, \qquad \Phi_G = \sqrt{\frac{(1+\kappa\phi_\pi)\tau_{R,\beta}}{1+\tau_{R,G}}}$$

where $\tau_{R,G} \equiv \frac{\sigma_R^2}{\sigma_G^2}$ *and* $\tau_{R,\beta} \equiv \frac{\sigma_R^2}{\sigma_\beta^2}$.

Figure 3 plots the signaling effects (S_R, S_G) and the implied inflation responses (Π_R, Π_G) as functions of Φ, holding other parameters constant. Intuitively, neither a very inactive nor a very aggressive policy stance will likely generate sizable signaling effects. For example, when the policy response is very inactive (say Φ is close to zero) so that policies become far more discretionary than rule-based, firms will place little, if any, probability mass on having a change in the consumption demand when betting on which policy component drives the signal change. On the other hand, when the policy response is very aggressive (say Φ is close to one), firms will attribute most of the signal change, if not all, to the policy authorities being overly responsive rather than to a change in the consumption demand.[11] From an empirical standpoint, however, episodes of extreme policy responses, such as those adopted during the recessions, may be rare and short-lived. In normal times, we might expect the signaling effects to be more evident under ordinary policy responses.

Admittedly, the preceding analysis is still limited in that the interest rate response to inflation, ϕ_π, does not enter the information channel as inflation is perfectly observable. In addition, we assume the same monetary and fiscal policy response to output gap, $\phi_y = \phi_g$. In the full version of the model, we further investigate the distinct role of each policy coefficient and their cross-policy impacts through the information channel of the general equilibrium feedback.

4 Numerical Analysis

The intuition developed from the simple model extends to more general settings. In this section, we relax Assumption 1 and perform a numerical analysis based on the full model, which features persistent information frictions, different policy responses to output gap, and heterogeneity in firms' beliefs.

4.1 Solution Method and Parameterization

To solve the full model in section 2, we first log-linearize its equilibrium conditions around the deterministic steady state. The incomplete information model introduces two types of conditional

[11]Note that the right panel of Figure 3 is also in contrast to its full information counterpart where increasing Φ and hence the automatic stabilizer monotonically dampens the inflation response to policy shocks.

Table 1: Parameters values and estimated government spending rule

Parameter	Value	Description
Households		
β	0.99	Quarterly discount factor
σ	2	Relative risk aversion
γ	1	Inverse of Frisch elasticity
σ_β^2	4^2	Demand shock's variance
σ_N^2	4^2	Labor disutility shock's variance
Firms		
θ	0.6	Price stickiness
ρ_A	0.9	Aggregate tech. shock's persistence
σ_A^2	1.4^2	Aggregate tech. innovation's variance
σ_η^2	2.6^2	Idiosyncratic tech. innovation's variance
Monetary policy		
ϕ_π	1.5	MP response to inflation
ϕ_y	0.5	MP response to output gap
ρ_R	0.75	MP shock's persistence
σ_R^2	0.4^2	MP innovation's variance
Fiscal policy		
g_y	0.16	Steady-state gov. spending to output ratio
ϕ_g	0.041 (0.040)	Gov. spending response to output gap
ρ_G	0.909 (0.036)	Gov. spending shock's persistence
σ_G^2	0.384^2 (0.020)	Gov. spending innovation's variance

NOTES: For estimated fiscal parameters, we report the standard errors in parentheses. See Appendix C for details of the data set.

expectations, i.e., $\mathbb{E}_t^{HH}$ and $\mathbb{E}_{t,j}^{Firm}$, resulting in a breakdown of the law of iterated expectations and the infinite regress problem of "forecasting the forecast of others" [Townsend (1983)]. Instead of using the time-domain methods that require truncation of the state space of higher-order beliefs [see Nimark (2008)], we solve the model using the approach developed by Han, Tan and Wu (2020), who provide an algorithm based on policy function iterations in the frequency domain. Appendix D contains mathematical details of our numerical computations.

We parameterize the model before simulations. Table 1 summarizes the parameter values, which are conventional for quarterly models. We follow Nakamura and Steinsson (2008) and fix $\theta = 0.6$.

The moderate price stickiness implies on average firms reset prices every 7.5 months, consistent with micro-evidence. We follow Schmitt-Grohé and Uribe (2012) and set $\sigma_\beta = 4$. The standard deviation of the labor disutility shock is often estimated to be of the same magnitude as the demand shock [see Smets and Wouters (2007) and Leeper, Plante and Traum (2010)], and we set $\sigma_N = 4$. The coefficients of the Taylor rule ($\phi_\pi = 1.5, \phi_y = 0.5$) are commonly used in the literature, and are also consistent with the estimated results of Melosi (2017) and Rudebusch (2002). We follow Rudebusch (2002) and set $\rho_R = 0.75$ and $\sigma_R = 0.4$. We follow Eusepi and Preston (2018) and set the steady-state spending-to-output ratio (g_y) to 16 percent. The standard deviations of aggregate and idiosyncratic technology shocks come from Melosi (2017).

We then estimate the fiscal rule (2.6) by running linear regressions with AR(1) error terms. The mean estimate of the output response coefficient (ϕ_g) is 0.04, with a standard error of the same size. The statistical non-significance of ϕ_g is not surprising. The literature has documented ample evidence of both counter and pro-cyclical government spending across time. Appendix F shows when g_t is pro-cyclical (i.e., $\phi_g < 0$), a positive government spending shock crowds out private consumption and increases inflation.[12] In the following analysis, we fix all fiscal parameters at their mean estimates.

4.2 Policy-Driven Inflation Dynamics

We now turn to the simulation results for inflation and consumption dynamics under alternative $I_{T,j}^{Firm}$ (i.e., Case I–III). Figure 4 plots the impulse responses of inflation and consumption to an increase in the government spending shock u_t^G. Under full information, this expansionary shock is inflationary as it increases the real marginal cost; it also crowds out the private consumption through the conventional interest rate channel. Compared with the simple model, u_t^G now affects both the IS curve and NKPC via the output gap. On the other hand, under incomplete information, the initial responses of inflation are universally negative before turning positive, generating the FPP. The hump-shaped impulse responses under Case I–III eventually converge to the full information counterpart. The more signals in $I_{T,j}^{Firm}$ (i.e., Case I), the faster the convergence rate is. The impulse

[12] In the simple model of Section 3, a positive government spending shock shifts the Phillips curve to the left when $\phi_g < 0$, resulting in lower consumption and higher inflation.

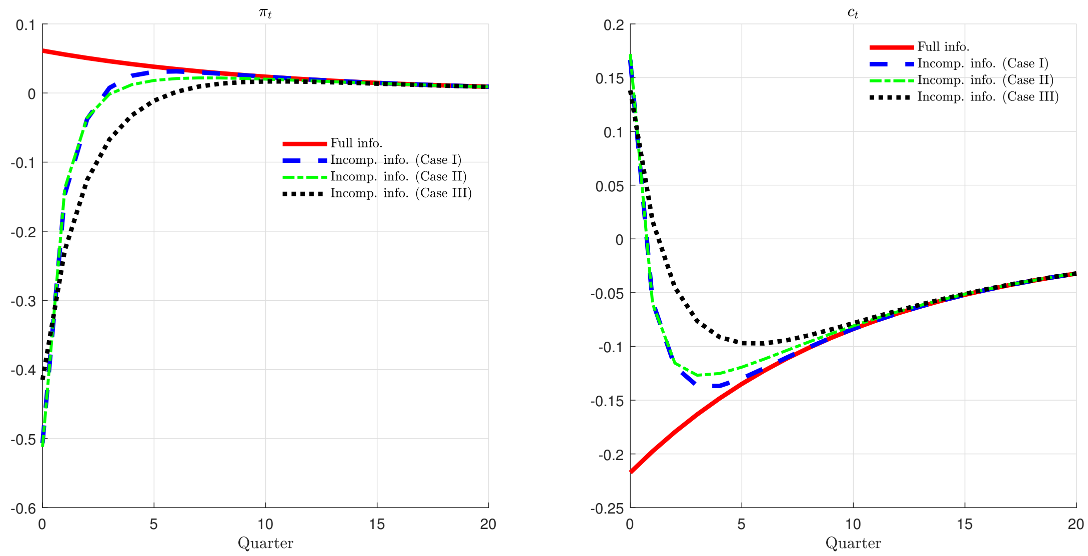

Figure 4: Impulse responses of (i) inflation (left panel) and (ii) consumption (right panel) to a one-unit expansionary government spending shock under alternative information sets $I_{T,j}^{Firm}$. All numbers are annualized and in percentage.

responses of Case II lie between those of Case I and III. While we consider a dispersed information environment in each case, in reality, firms' information sets may well be hierarchical. In this sense, Case I's and III's impulse responses provide a confidence interval for the hierarchical economy's impulse responses.

Figure 4 indicates that the information channel documented in Section 3 extends to the full model with dispersed information. Higher government spending induces firms to set a lower price as they interpret the rising fiscal signal as a stimulus measure to counter the weak private demand. Notice that in the full model, current inflation is not observable and is determined by firms' average expectations about current inflation, real marginal cost, and expected future price

$$\pi_t = (1-\theta)\int_0^1 \left[\mathbb{E}_{t,j}^{Firm}\pi_t + (1-\beta\theta)\mathbb{E}_{t,j}^{Firm} mc_{t,j} + \beta\theta\mathbb{E}_{t,j}^{Firm} z_{t+1,j}\right] dj. \tag{4.1}$$

Therefore, firm j's price setting decision $z_{t,j}$ is now determined by its estimate of the demand (i.e., real marginal cost) and others' beliefs about the demand. Since fiscal policy is public information, firms understand that others would also be pessimistic about the demand. Therefore, their estimates of current inflation fall due to the policy signaling effect. More formally, information dispersion implies that we can iterate (4.1) forward in terms of π_t to obtain

$$\pi_t = (1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^k \overline{\mathbb{E}}_t^{(k)} mc_{t,j} + \beta\theta\overline{\mathbb{E}}_t^{(k)} z_{t+1,j},$$

where $\overline{\mathbb{E}}_t^{(k)}[\cdot]$ denotes the k-th order expectations, and $\mathbb{E}_{t,j}^{Firm} z_{t+1,j} = \sum_{i=0}^{\infty}(\beta\theta)^i\big[\mathbb{E}_{t,j}\pi_{t+i+1,j} + (1-\beta\theta)\mathbb{E}_{t,j} mc_{t+i+1,j}\big]$. Clearly, the directly signaling effect of fiscal policy influences inflation through the formation of static and dynamic higher-order expectations about current and future real marginal costs.

Like the simple model, endogenous changes in inflation and output create the GE information feedback from monetary policy, which further affects firms' beliefs. Now changes in the monetary signal can be attributed to three possibilities: changes in inflation, output gap, or monetary shock. While the first two possibilities are different in nature, they influence firms' price setting decisions in the same direction. Therefore, the central friction faced by firms when interpreting the monetary signal remains the same as in the simple model—they only need to distinguish between changes in the endogenous policy targets and changes in the exogenous policy shock. This is indeed our general definition of the policy signaling effect given in Section 3.

One important difference compared to the simple model is that here the actual change in the monetary signal ($i_t \downarrow$ as opposed to $s_t^R \uparrow$ in the simple model) conveys a different message to firms. Since current inflation is not observable, firms interpret the lower nominal rate as either a decrease in inflation or output gap, which reinforces their original beliefs, or an expansionary monetary shock, which attenuates their original beliefs. In the latter case, firms understand the possibility of coordinated policy actions—monetary expansion and fiscal stimulus. Using Case I of $I_{T,j}^{Firm}$ as an example, the left panel of Figure 5 plots the impulse response of nominal interest rate to a positive government spending shock.

The middle panel of Figure 5 decomposes the impulse response of inflation based on the direct signaling effect of fiscal policy and the GE information feedback effect of monetary policy. Such a decomposition demonstrates the quantitative importance of the information feedback channel. It explains why inflation falls significantly in Figure 4 even though the autonomous response coefficient $\phi_g = 0.04$ is rather small. Since the monetary shock volatility is relatively small compared to those

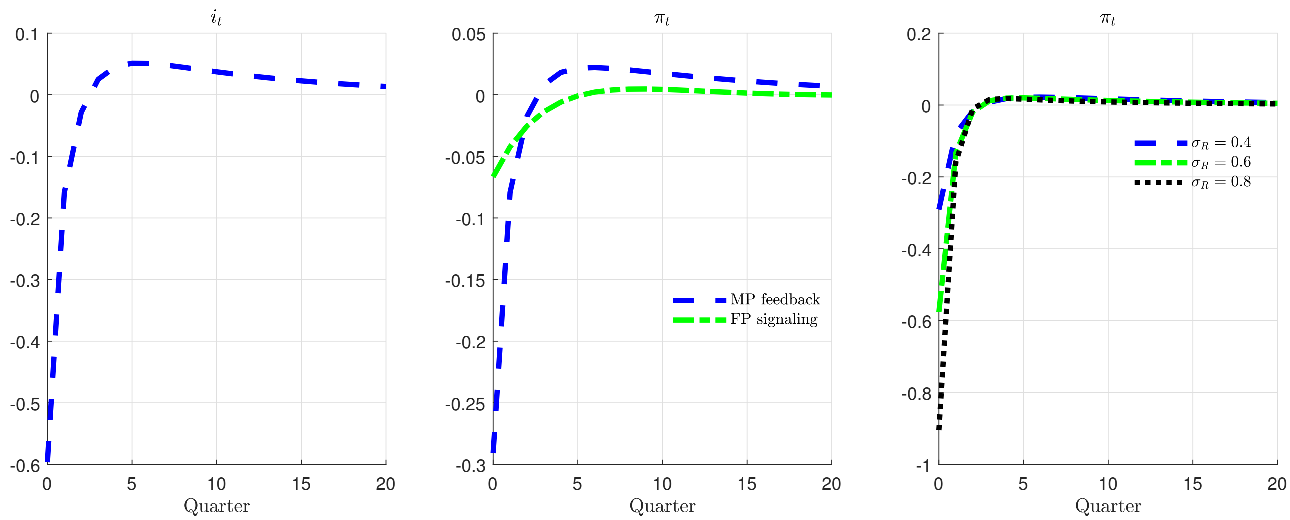

Figure 5: Decomposed impulse responses of inflation to a one-unit expansionary government spending shock under Case I of $I_{T,j}^{Firm}$. All numbers are annualized and in percentage.

of the other shocks, the GE information feedback effect of monetary policy is quite strong and amplifies the direct signaling effect of fiscal policy. Recall that the total information feedback is the product of the marginal value of signals and the actual change in signals. Here the marginal value bears the same interpretation as in the simple model, but the actual monetary signal change conveys a different message to firms, which creates the amplification effect.[13]

The right panel of Figure 5 plots the feedback effects of monetary policy under different values of the monetary shock volatility σ_R. Like the results in Proposition 3.3, increasing the monetary shock uncertainty strengthens the direct signaling effect of fiscal policy and changes its GE information feedback effect in a way that enhances its reinforcement to the direct signaling. Although the marginal value of the monetary signal decreases, the change in the actual signal (i.e., nominal interest rate) implies that the two channels work in the same direction. Our numerical results confirm this intuition.

Figure 6 plots the impulse responses of inflation to a positive nominal interest rate shock. Our analytical results (see Figure 2) suggest that the MPP can arise in the incomplete information model so long as the condition (3.10) holds. We reiterate this point by varying the standard deviation of

[13]The amplification result is robust across different information sets (i.e., Case I–III). In our full model, the decomposition is defined as $\pi_t = \Pi_G(L)\varepsilon_t^G = S_R(L)i_t + S_G(L)g_t$, where the coefficients of the lag polynomials $S_R(L)$ and $S_G(L)$ measure the marginal values of the policy signals. Here we shut down all shocks but the government spending shock. The mathematical details are contained in Appendix D.

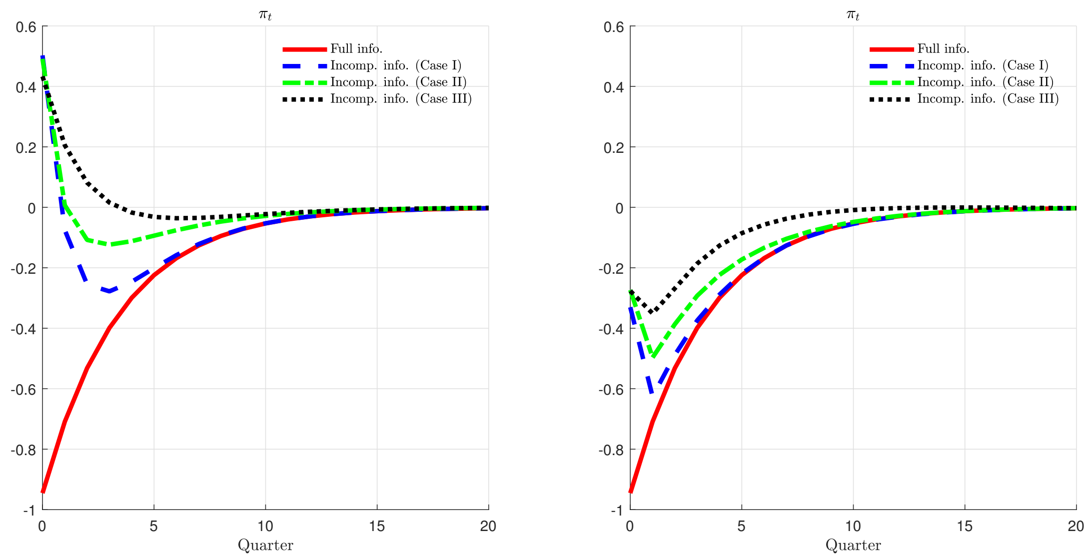

Figure 6: Impulse responses of inflation to a one-unit contractionary monetary policy shock under alternative information sets $I_{T,j}^{Firm}$ and different values of σ_R. Left panel: $\sigma_R = 0.4$; Right panel: $\sigma_R = 2$. All numbers are annualized and in percentage.

the monetary shock σ_R. Due to certainty equivalence, different values of σ_R lead to the same impulse response under full information, where inflation drops the most on impact and gradually converges to zero. The full information model therefore cannot accommodate the MPP. In contrast, when σ_R is relatively small (i.e., $\sigma_R = 0.4$), all three cases under incomplete information generate initial increases in the price level before turning negative. The hump-shaped impulse responses under Case I–III eventually converge to the full information counterpart. Moreover, the GE information feedback effect of fiscal policy is similar to the simple model, but its impact is quantitatively small compared to the direct signaling effect of monetary policy.[14] On the other hand, the right panel of Figure 6 shows that when the standard deviation of the monetary shock is large (i.e., $\sigma_R = 2$), the MPP disappears.

Comparing the theory-implied and VAR-identified impulse responses suggests that the incomplete information model explains the inflation dynamics well. As in the VAR literature, the incomplete information model captures the initial responses of inflation and generates "reversals" of inflation

[14] In the full model, the government spending shock has a stronger impact on the supply and demand sides of the economy, and is inflationary and stimulative under full information. Therefore, when σ_G is sufficiently large, the fiscal signal reinforces the signaling effect of monetary policy. In this case, an increase in the fiscal signal could be triggered by unexpected fiscal expansions, and the limiting result on the initial response of the marginal value in Proposition 3.3 no longer holds. Our numerical results confirm this intuition.

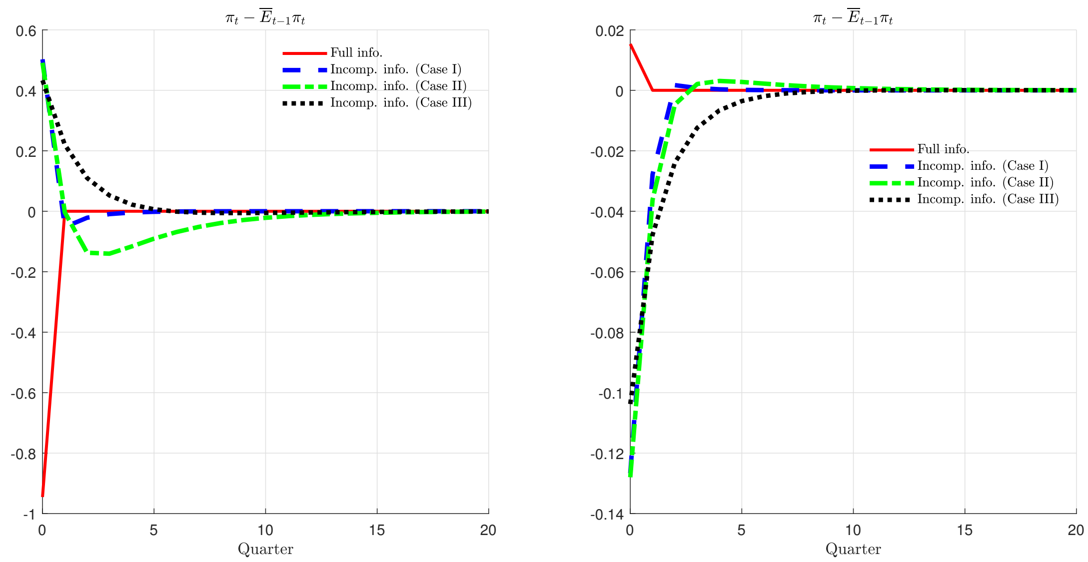

Figure 7: Impulse responses of forecast error of inflation to (i) a one-unit contractionary monetary policy shock (left panel) and (ii) a one-unit expansionary government spending shock (right panel) under alternative information sets $I_{T,j}^{Firm}$. All numbers are annualized and in percentage.

in the subsequent periods (see also Appendix C). Such reversals arise since the incomplete information model will eventually converge to the full information model. Both policy price puzzles will disappear, and a contractionary policy shock (either in i_t or g_t) will reduce inflation in the end.

4.3 Dynamics of Inflation Forecast Error

Our theory-implied impulse responses of the inflation forecast error (i.e., $\pi_t - \overline{E}_{t-1}\pi_t$) also explain the VAR evidence reasonably well. Figure 7 plots the impulse responses of $\pi_t - \overline{E}_{t-1}\pi_t$ to either a monetary (left panel) or a fiscal (right panel) shock under alternative information sets $I_{T,j}^{Firm}$. Let us consider a scenario where the single monetary shock hits the economy at $t = 0$. Given $\overline{E}_{-1}\pi_0 = 0$, while the full information model generates a decrease in the price level and thus a negative $\pi_0 - \overline{E}_{-1}\pi_0$, all incomplete information cases generate a positive $\pi_0 - \overline{E}_{-1}\pi_0$ due to the MPP.

It is well-known that forecast errors are *i.i.d.* over time under full information rational expectations. This property forces the impulse responses of the full information model to be identically zero starting from $t = 1$. In contrast, while individual firms' forecast errors are still *i.i.d.* over time under rational expectations, the average forecast errors are autocorrelated in the incomplete information model. The empirical literature has found strong evidence of persistent average forecast errors [see Coibion and Gorodnichenko (2012, 2015a)]. Indeed, Figure 7 shows that the impulse responses gradually converge to zero under Case I–III. The deviations of the impulse responses from zero are the smallest when firms' information set $I_{T,j}^{Firm}$ is the finest (i.e., Case I). When firms have Case II or III information set, they on average can either over- or under-estimate future inflation.

4.4 Monetary Signal, Fiscal Noise

Given the signaling interaction between monetary and fiscal policies, it is useful to assess how informative their instruments are with regard to key variables such as output and inflation. Following the standard practice in information theory [see, e.g., Cover and Thomas (1991) and Sims (2003)], we quantify the information flow transferred through each policy signal by an entropy-based measure. For example, the information flow that firms receive about output y_t from observing the history of policy rates $i^t \equiv \{i_t, i_{t-1}, \ldots\}$ can be measured as the (Shannon) mutual information between y_t and i^t, i.e., $I(y_t; i^t) \equiv H(y_t) - H(y_t|i^t)$, where $H(\cdot)$ and $H(\cdot|\cdot)$ denote the entropy and conditional entropy operators, respectively.[15] Intuitively, at each time t, $I(y_t; i^t)$ measures the reduction of uncertainty (i.e., entropy) about y_t due to observing i^t. For comparison ease, we normalize this quantity by the sum of the information flow about output transmitted by each signal

$$\widetilde{I}(y_t; i^t) = \frac{I(y_t; i^t)}{I(y_t; i^t) + I(y_t; g^t) + I(y_t; a_j^t) + I(y_t; \pi^{t-1}) + I(y_t; y^{t-1})}$$

so that $\widetilde{I}(y_t; i^t) + \widetilde{I}(y_t; g^t) + \widetilde{I}(y_t; a_j^t) + \widetilde{I}(y_t; \pi^{t-1}) + \widetilde{I}(y_t; y^{t-1}) = 1$.[16] Analogously, we compute the normalized information flow about inflation contained in each signal. The results under different information sets of the firm are summarized in Table 2.

[15]Because the model is linear and driven by Gaussian shocks, all endogenous variables follow normal distributions in equilibrium. As a result, the entropy associated with output, which is normally distributed with (unconditional) variance $\mathbb{V}(y_t)$, takes the form of $H(y_t) = \frac{1}{2}\log_2[2\pi e\mathbb{V}(y_t)]$. Analogously, the conditional entropy is given by $H(y_t|i^t) = \frac{1}{2}\log_2[2\pi e\mathbb{V}(y_t|i^t)]$, where $\mathbb{V}(y_t|i^t)$ denotes the variance of output conditional on firms having observed the history of policy rates. Admittedly, we adopt the mutual information $H(y_t) - H(y_t|i^t)$ as a measure of the information flow $I(y_t; i^t)$ for simplicity. This definition is, however, not innocuous in that it is symmetric and hence precludes causal relationships. An alternative and related notion is the directed information of Massey (1990), which is asymmetric and measures directional information flow.

[16]We do not normalize $I(y_t; i^t)$ by the information flow transmitted by all signals, i.e., $I(y_t; i^t, g^t, a_j^t, \pi^{t-1}, y^{t-1})$ because such normalized quantities do not add up to unity in the presence of correlated signals.

Table 2: Information flow transmitted by policy and non-policy signals

Variable	i^t	g^t	a_j^t	π^{t-1}	y^{t-1}
Case I: $I_{T,j}^{Firm} = \{a_{t,j}, i_t, g_t, \pi_{t-1}, y_{t-1} : t \leqslant T\}$					
y_t	18.0%	0.1%	33.3%	21.1%	27.5%
π_t	77.6%	0.5%	10.5%	6.3%	5.1%
Case II: $I_{T,j}^{Firm} = \{a_{t,j}, i_t, g_t, y_{t-1} : t \leqslant T\}$					
y_t	23.2%	0.1%	41.4%	n/a	35.3%
π_t	84.6%	0.6%	9.9%	n/a	4.9%
Case III: $I_{T,j}^{Firm} = \{a_{t,j}, i_t, g_t : t \leqslant T\}$					
y_t	37.0%	0.2%	62.8%	n/a	n/a
π_t	86.9%	0.7%	12.4%	n/a	n/a

Among all available signals in Case I, firms learn about output primarily from non-policy signals: 33.3% of the information flow they receive is from observing the firm-specific technology, 27.5% from the lagged output, and 21.1% from the lagged inflation. On the other hand, the nominal interest rate plays a highly informative role in signaling the monetary authority's view about inflation to firms, conveying 77.6% of the overall information firms gather about this variable. Given the small estimate of the output response coefficient $\phi_g = 0.04$, the government spending barely transmits any information flow about output and inflation to firms. The noisy nature of fiscal policy and its macroeconomic effects have also been documented in the literature [see, e.g., Fève and Pietrunti (2016)]. Case II and III further indicate that the above findings are robust to excluding the history of lagged output and inflation from the firms' information set.

That distinct policy instruments carry starkly different information also connects broadly to empirical facts about how the central bank and the government routinely act. For instance, the high informativeness of policy rate reflects the general observation that monetary policy tends to be based on a systematic analysis of ongoing economic conditions that are critical to private decision-makers. In normal times, it is not surprising that the federal funds rate has received far more attention from the private sector than other policy instruments. In contrast, government expenditure's low informativeness echos the impression that fiscal policy is driven more by political factors than economic considerations. Because of this difference, Leeper (2011) vividly labels the two policies as "monetary science, fiscal alchemy". In the same spirit, here we label them as "monetary

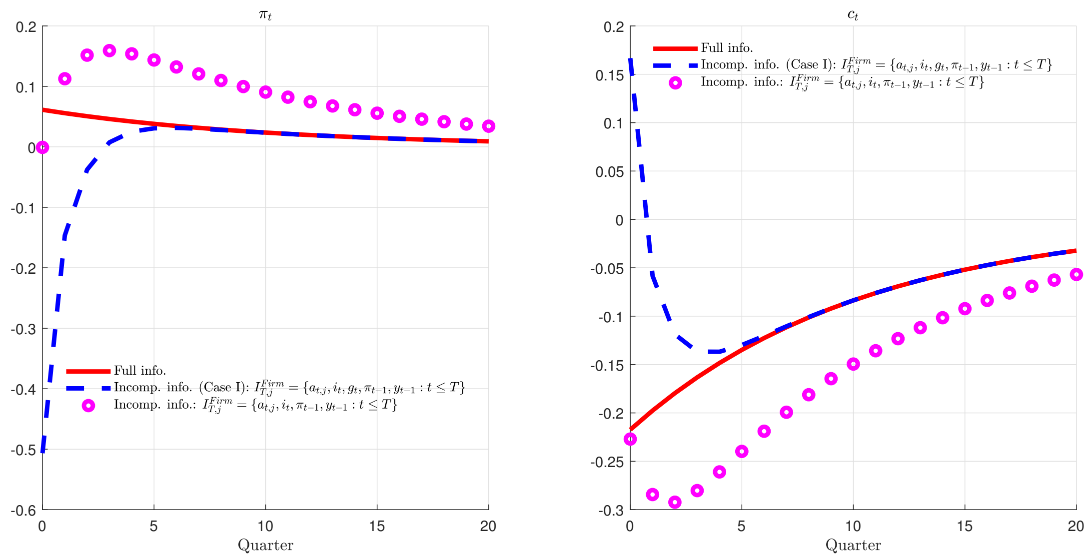

Figure 8: Impulse responses of (i) inflation (left panel) and (ii) consumption (right panel) to a one-unit expansionary government spending shock under alternative information sets $I^{Firm}_{T,j}$. The FPP disappears when $I^{Firm}_{T,j} = \{a_{t,j}, i_t, \pi_{t-1}, y_{t-1} : t \leqslant T\}$. All numbers are annualized and in percentage.

signal, fiscal noise" in the context of the information interaction between policies.

Despite the low information flow of government spending, it is by no means that firms should not pay attention to the fiscal instrument. To underpin this point, Figure 8 compares the impulse responses of inflation and consumption to an exogenous increase in government spending when firms can and cannot learn from the history of government spending. It can be seen that the FPP disappears in the latter case when $I^{Firm}_{T,j} = \{a_{t,j}, i_t, \pi_{t-1}, y_{t-1} : t \leqslant T\}$, as inflation rises while consumption falls, which runs counter to the VAR evidence.

4.5 Implications of More Accommodative Monetary Policy

It is a widely held view that the missing inflation since the Great Recession may have contributed to a more dovish Federal Reserve thereafter. In this section, we consider the implications of a more accommodative monetary policy by varying the response coefficients (ϕ_π, ϕ_y) in the Taylor rule (2.5). Figure 9 and 10 plot the initial inflation response (i.e., π_0 in Case I) to a one-unit positive policy shock (i.e., ε^R_0 or ε^G_0) under alternative values of ϕ_π or ϕ_y, while keeping the remaining parameter values fixed.

We first reconnect Figure 9 here with Figure 3 of the simple model that imposes the assumption

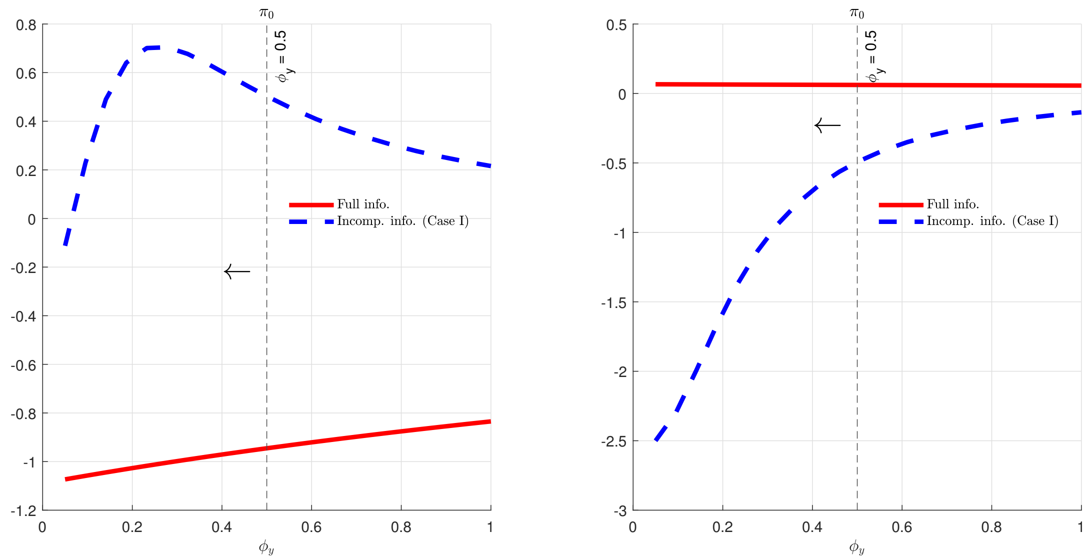

Figure 9: Impulse responses of the initial inflation π_0 to (i) a one-unit positive interest rate shock (left panel) and (ii) a one-unit positive government spending shock (right panel) under alternative values of ϕ_y. All numbers are annualized and in percentage.

that $\phi_y = \phi_g = \phi > 0$ for ease of calculation. How restrictive is such an assumption? We argue that it is not much. In particular, the left panel of Figure 9 shows that when a positive monetary shock hits the economy, the initial inflation response displays the same "inverted-U" shape as in the upper right panel of Figure 3 as we vary ϕ_y. The non-monotone relation between the signaling effect of monetary policy and the output response coefficient ϕ_y still exists in the full model. We also draw a similar conclusion to fiscal policy.[17]

We now ask the question of whether a more accommodative monetary policy can resolve the missing inflation puzzle after the Great Recession. The answer is probably no. The left panel of Figure 9 shows that when monetary policy responds very weakly to the output gap (i.e., $\phi_y \to 0$), the initial inflation will also respond weakly to monetary shocks.[18] Let us consider a scenario where $\phi_y \to 0$ and the central bank keeps shocking the economy with a sequence of expansionary monetary shocks. The small initial response indicates that if inflation is already low, it will stay

[17]Figure 17 of Appendix F shows that when a positive government spending shock hits the economy, the initial inflation response displays a similar U-shaped pattern as in the lower right panel of Figure 3 as we vary ϕ_g. The assumption $\phi > 0$ does rule out pro-cyclical government spending (i.e., $\phi_g < 0$). Figure 17 shows that when fiscal policy is pro-cyclical, the model cannot generate the FPP found in the VAR.

[18]This result is consistent with Nakamura and Steinsson (2018), who identify a small response of inflation using high-frequency data. They impose $\phi_y = 0$ in the monetary policy rule of their theoretical model.

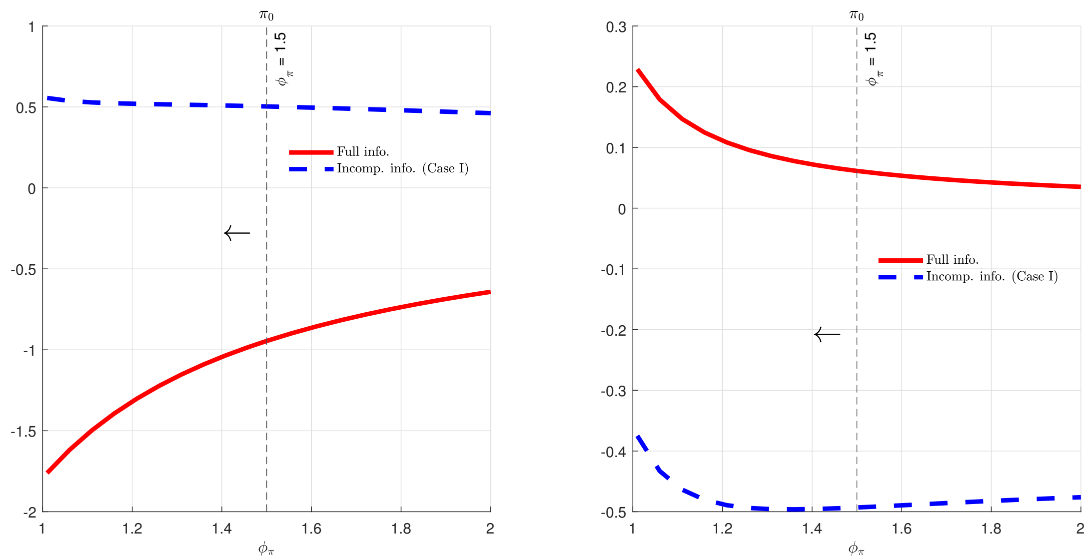

Figure 10: Impulse responses of the initial inflation π_0 to (i) a one-unit positive interest rate shock (left panel) and (ii) a one-unit positive government spending shock (right panel) under alternative values of ϕ_π. All numbers are annualized and in percentage.

low. Meanwhile, a smaller value of ϕ_y greatly amplifies the initial inflation response to fiscal shocks. The initial response is close to zero and hardly changes as we vary ϕ_y in the full information model. In the incomplete information model, a smaller value of ϕ_y implies that firms will downplay the monetary signal i_t and put more weight on the fiscal signal g_t (recall we fix $\phi_g = 0.041$) to infer the demand side uncertainty. The larger signaling effect of fiscal policy amplifies the initial inflation response. In response to the COVID-19 pandemic, it is likely that the U.S. federal government will continue to conduct further fiscal stimulus plans. The right panel of Figure 9 suggests that as ϕ_y decreases, more expansionary fiscal shocks will push inflation further down, thus exacerbating the missing inflation. The simulation experiment of Appendix E further shows that, even with the massive monetary-fiscal policy stimulus since the Great Recession, the model-implied inflation remains low under incomplete information.

On the other hand, Figure 10 shows that a more dovish monetary policy (i.e., lower ϕ_π) cannot raise inflation under current economic conditions. While in the full information model, a lower value of ϕ_π increases the sensitivity of the initial inflation response to monetary shocks, it does not cause the initial inflation to change much in the incomplete information model. Consequently, while an expansionary monetary shock leads to higher inflation in the full information model, inflation remains unresponsive to the more dovish monetary policy in the incomplete information model—the missing inflation remains.

Finally, we emphasize that our analysis is based on the assumption that the Taylor rule (2.5) models the central bank's behavior well. At the time of writing, the Federal Reserve just adopted a new strategy to allow for higher inflation. The "average inflation targeting" strategy announced in August 2020 is widely viewed as a shift to an easier monetary policy stance over time. We agree that such a strategy is not equivalent to a smaller pair of (ϕ_π, ϕ_y). However, the current analysis does raise a caveat that a more accommodative monetary policy may not achieve higher inflation if the same information friction exists. Formal modeling of this new strategy is beyond the scope of this paper, which we leave for future research.

5 Concluding Remarks

We propose a novel policy interaction that focuses on the joint signaling role of monetary and fiscal policy. Theoretically, we examine the transmission mechanism and the consequent general equilibrium feedback effects analytically. Empirically, we provide a unified explanation of the various policy impacts on inflation documented in the literature.

It is worth mentioning that our policy rule specifications are highly stylized. On the monetary side, we use a conventional Taylor rule without the zero lower bound to model the central bank's behavior. Thus, the model does not capture the effects of unconventional monetary policies such as quantitative easing and forward guidance. On the fiscal side, we focus on the role of countercyclical government spending and do not model distortionary taxes and government debt. This modeling choice helps us focus on the effects of government spending, but implies that we probably underestimate the fiscal uncertainty and its implications under incomplete information. Leaving out government debt also frees us from worrying about the issue of identifying the underlying policy regime. We leave these issues for future research.

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Appendix

Appendix A Equilibrium Conditions

Define an auxiliary variable $z_{t,j} = p^*_{t,j} - p_{t-1}$, where $p^*_{t,j}$ is firm j's optimal price at time t and p_{t-1} is the lagged aggregate price level. The auxiliary variable $z_{t,j}$ effectively removes the unit root embedded in the individual firm's optimal price $p^*_{t,j}$. The equilibrium conditions of the model are given as follows.

1. Non-fiscal sector

$$
\begin{aligned}
\text{IS curve:} \quad c_t &= \mathbb{E}^{HH}_t c_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}^{HH}_t \pi_{t+1} - \varepsilon^{\beta}_t) \\
\text{Natural output:} \quad y^n_t &= \frac{\sigma g_y}{\sigma + \gamma(1-g_y)} u^G_t + \frac{1-g_y}{\sigma + \gamma(1-g_y)} \left[(\gamma+1)a_t - \varepsilon^N_t\right] \\
\text{Resources constraint:} \quad y_t &= (1-g_y)c_t + g_y g_t \\
\text{Price aggregation:} \quad \pi_t &= (1-\theta)\int_{j\in[0,1]} z_{t,j} d_j \\
\text{Firm } j\text{'s productivity:} \quad a_{t,j} &= a_t + \eta_{t,j} \\
\text{Firm } j\text{'s real marginal cost:} \quad mc_{t,j} &= \sigma c_t + \gamma(y_t - a_t) + \varepsilon^N_t - a_{t,j} \\
\text{Firm } j\text{'s price setting:} \quad z_{t,j} &= \mathbb{E}^{Firm}_{t,j} \pi_t + (1-\beta\theta)\mathbb{E}^{Firm}_{t,j} mc_{t,j} + \beta\theta \mathbb{E}^{Firm}_{t,j} z_{t+1,j}
\end{aligned}
$$

2. Policy rules

$$
\begin{aligned}
\text{Monetary rule:} \quad i_t &= \phi_\pi \pi_t + \phi_y (y_t - y^n_t) + u^R_t \\
\text{Gov. spending rule:} \quad g_t &= -\phi_g (y_t - y^n_t) + u^G_t
\end{aligned}
$$

3. AR(1) shocks

$$\begin{aligned}\text{Technology shock:} \quad a_t &= \rho_A a_{t-1} + \varepsilon_t^A \\ \text{Monetary shock:} \quad u_t^R &= \rho_R u_{t-1}^R + \varepsilon_t^R \\ \text{Gov. spending shock:} \quad u_t^G &= \rho_G u_{t-1}^G + \varepsilon_t^G\end{aligned}$$

Appendix B Proof of Propositions

This appendix presents the proofs of propositions in the main text.

Proof of Proposition 3.1

Under flexible price and Assumption 1, we have $mc_t^n = \sigma c_t^n + \gamma y_t^n = 0$ and hence $c_t^n = 0$. Using the aggregate resource constraint and rearranging terms, the output gap is given by

$$y_t - y_t^n = \frac{1 - g_y}{1 + g_y \phi_g} c_t. \tag{B.1}$$

By Assumption 1, $\Phi = \phi \frac{1-g_y}{1+g_y\phi_g} > 0$. It can be seen from (3.2) that inflation is perfectly observable to firms in the simple model. Therefore, the signal system can be equivalently expressed as (3.6) and (3.7).

It follows that inflation can be expressed as

$$\pi_t = S_R s_t^R + S_G s_t^G = (S_R - S_G)\Phi c_t + S_R \varepsilon_t^R + S_G \varepsilon_t^G. \tag{B.2}$$

Using the *i.i.d.* assumption, the private consumption follows

$$c_t = \varepsilon_t^\beta - i_t = \varepsilon_t^\beta - \left[\phi_\pi (S_R - S_G)\Phi c_t + \phi_\pi S_R \varepsilon_t^R + \phi_\pi S_G \varepsilon_t^G + \Phi c_t + \varepsilon_t^R\right],$$

implying that

$$C_\beta = \frac{1}{\Delta}, \qquad C_R = -\frac{1+\phi_\pi S_R}{\Delta}, \qquad C_G = -\frac{\phi_\pi S_G}{\Delta}, \tag{B.3}$$

where $\Delta = 1 + \phi_\pi(S_R - S_G)\Phi + \Phi$ is defined as in the main text.

To ease analytical derivation, we rewrite inflation as

$$\pi_t = \kappa \mathbb{E}_t^{Firm} c_t = -\frac{\kappa}{\Phi} s_t^G + \frac{\kappa}{\Phi} \mathbb{E}_t^{Firm} \varepsilon_t^G. \tag{B.4}$$

Next, we express the bivariate signal process in its matrix form

$$s_t = \begin{bmatrix} s_t^R \\ s_t^G \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi C_\beta & 1 + \Phi C_R & \Phi C_G \\ -\Phi C_\beta & -\Phi C_R & 1 - \Phi C_G \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \varepsilon_t^\beta \\ \varepsilon_t^R \\ \varepsilon_t^G \end{bmatrix}}_{\varepsilon_t}.$$

The covariance matrix of s_t is given by $\Sigma_s = B\Sigma B'$, where $\Sigma = \mathrm{diag}(\sigma_\beta^2, \sigma_R^2, \sigma_G^2)$. Using the Gaussian projection formula, (B.2), and (B.4), the fixed point condition for (S_R, S_G) can be written as

$$[S_R \;\; S_G] = -\frac{\kappa}{\Phi}[0 \;\; 1] + \frac{\kappa}{\Phi}\Sigma_{gs}\Sigma_s^{-1}, \tag{B.5}$$

where $\Sigma_{gs} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \Sigma B'$.

We conjecture and later verify that the information is not fully revealed in equilibrium. Therefore, $\Sigma_s \succ 0$ has full rank. Multiplying both sides of (B.5) by Σ_s and expanding this equation, we obtain the set of nonlinear fixed point equations in (S_R, S_G), i.e.,

$$\begin{aligned}
&S_R\left[\Phi^2 C_\beta^2\sigma_\beta^2 + (1+\Phi C_R)^2\sigma_R^2 + \Phi^2 C_G^2\sigma_G^2\right] + S_G\left[-\Phi^2 C_\beta^2\sigma_\beta^2 - (1+\Phi C_R)\Phi C_R\sigma_R^2 + (1-\Phi C_G)\Phi C_G\sigma_G^2\right] \\
&\qquad = -\frac{\kappa}{\Phi}\left[-\Phi^2 C_\beta^2\sigma_\beta^2 - (1+\Phi C_R)\Phi C_R\sigma_R^2 + (1-\Phi C_G)\Phi C_G\sigma_G^2\right] + \kappa C_G\sigma_G^2
\end{aligned} \tag{B.6}$$

and

$$\begin{aligned}
&S_R\left[-\Phi^2 C_\beta^2\sigma_\beta^2 - (1+\Phi C_R)\Phi C_R\sigma_R^2 + (1-\Phi C_G)\Phi C_G\sigma_G^2\right] + S_G\left[\Phi^2 C_\beta^2\sigma_\beta^2 + \Phi^2 C_R^2\sigma_R^2 + (1-\Phi C_G)^2\sigma_G^2\right] \\
&\qquad = -\frac{\kappa}{\Phi}\left[\Phi^2 C_\beta^2\sigma_\beta^2 + \Phi^2 C_R^2\sigma_R^2 + (1-\Phi C_G)^2\sigma_G^2\right] + \frac{\kappa}{\Phi}\sigma_G^2 - \kappa C_G\sigma_G^2.
\end{aligned} \tag{B.7}$$

To analyze the cubic system (B.6) and (B.7), we start with the following observation

$$1 + \Phi C_R = 1 - \frac{\Phi(1+\phi_\pi S_R)}{\Delta} = \frac{1-\phi_\pi\Phi S_G}{\Delta}$$

and

$$1 - \Phi C_G = 1 + \frac{\Phi\phi_\pi S_G}{\Delta} = \frac{1+\Phi+\phi_\pi\Phi S_R}{\Delta}.$$

Now multiplying both sides of (B.6) by Δ^2, the left hand side (LHS) is given by

$$\text{LHS} = (S_R - S_G)\Phi^2\sigma_\beta^2 + (S_R + \Phi S_G)\sigma_R^2 - \phi_\pi\Phi S_R S_G\sigma_R^2 - \left[\phi_\pi\Phi^2\sigma_R^2 + (\Phi+\Phi^2)\phi_\pi\sigma_G^2\right]S_G^2 \tag{B.8}$$

and the right hand side (RHS) is given by

$$\text{RHS} = \kappa\Phi\sigma_\beta^2 - \kappa\left(1 - \phi_\pi\Phi S_G + \phi_\pi S_R\right)\sigma_R^2 + \kappa\phi_\pi^2\Phi S_R S_G\sigma_R^2 + \kappa\phi_\pi^2 S_G^2\Phi\sigma_G^2. \tag{B.9}$$

Next, we sum up (B.6) and (B.7), and multiply both sides of the resulting equation by Δ,

$$\begin{aligned}
&S_R\left[(1-\phi_\pi\Phi S_G)\sigma_R^2 - \Phi\phi_\pi S_G\sigma_G^2\right] + S_G\left[\Phi(1+\phi_\pi S_R)\sigma_R^2 + (1+\Phi+\phi_\pi\Phi S_R)\sigma_G^2\right] \\
&\quad = -\frac{\kappa}{\Phi}\left[\Phi(1+\phi_\pi S_R)\sigma_R^2 + (1+\Phi+\phi_\pi\Phi S_R)\sigma_G^2\right] + \frac{\kappa}{\Phi}\left[1 + \phi_\pi(S_R - S_G)\Phi + \Phi\right]\sigma_G^2.
\end{aligned}$$

By inspection, all second order product terms ($S_R S_G$) cancel out. Therefore, we have

$$S_R = -\underbrace{\frac{\kappa}{1+\kappa\phi_\pi}}_{\Psi} - \underbrace{\frac{\Phi\sigma_R^2 + (1+\Phi)\sigma_G^2 + \kappa\phi_\pi\sigma_G^2}{(1+\kappa\phi_\pi)\,\sigma_R^2}}_{\Theta} S_G. \tag{B.10}$$

Clearly, the composite coefficients Ψ and Θ are positive, and the signal utilization coefficients (S_R, S_G) are linearly related according to (B.10). Now substituting (B.10) into (B.8) and (B.9), we have

$$\begin{aligned}\phi_\pi\Phi\Big\{ \left[\Theta(1+\kappa\phi_\pi) - \Phi\right]\sigma_R^2 - (1+\kappa\phi_\pi+\Phi)\,\sigma_G^2 \Big\} S_G^2 + \left[\kappa - (1+\kappa\phi_\pi)\Psi\right]\sigma_R^2 - \Phi\,(\kappa+\Psi\Phi)\,\sigma_\beta^2 \\ + \Big\{ \big[\phi_\pi\Phi\Psi\,(1+\kappa\phi_\pi) - \Theta(1+\kappa\phi_\pi) + \Phi(1-\kappa\phi_\pi)\big]\sigma_R^2 - (1+\Theta)\Phi^2\sigma_\beta^2 \Big\} S_G = 0. \end{aligned} \tag{B.11}$$

Using the definition (B.10), it is easy to show that

$$\left[\Theta(1+\kappa\phi_\pi) - \Phi\right]\sigma_R^2 - (1+\kappa\phi_\pi+\Phi)\,\sigma_G^2 = 0,$$

which implies that (B.11) is indeed linear. The linear terms in (B.11) follow

$$\big[\phi_\pi\Phi\Psi\,(1+\kappa\phi_\pi) - \Theta(1+\kappa\phi_\pi) + \Phi(1-\kappa\phi_\pi)\big]\sigma_R^2 - (1+\Theta)\Phi^2\sigma_\beta^2 = -(1+\Phi+\kappa\phi_\pi)\sigma_G^2 + (1+\Theta)\Phi^2\sigma_\beta^2.$$

Finally, notice that $\left[\kappa - (1+\kappa\phi_\pi)\Psi\right]\sigma_R^2 = 0$ by definition. Using the definition (B.10), we obtain the final form of the solution to (B.11),

$$S_G = -\frac{\kappa\Phi\sigma_\beta^2\sigma_R^2}{\left[(1+\kappa\phi_\pi)\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]} < 0. \tag{B.12}$$

Note that our derivations so far hinge on the condition $\Delta \neq 0$. In the next proposition, we show that $\Delta > 0$ under Assumption 1. We also verify that $\Sigma_s \succ 0$ in this non-revealing equilibrium. Since all shocks in the simple economy are *i.i.d.*, the transversality condition holds and the equilibrium is unique. The proof is then complete.

Proof of Proposition 3.2

Substitute the solution for S_G into (B.10) to get

$$S_R - S_G = -(1+\Theta)S_G - \Psi = \frac{\kappa\left[\Phi\sigma_\beta^2(\sigma_G^2+\sigma_R^2) - \sigma_G^2\sigma_R^2\right]}{(1+\kappa\phi_\pi)\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2},$$

which leads to the expressions

$$\Delta = \frac{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}{(1+\kappa\phi_\pi)\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2}, \tag{B.13}$$

$$S_R = \frac{\kappa\left(\sigma_\beta^2\sigma_G^2\Phi - \sigma_G^2\sigma_R^2\right)}{(1+\kappa\phi_\pi)\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2}. \tag{B.14}$$

(B.13) indicates $\Delta > 0$ since κ and ϕ_π are positive, which verifies the conjecture in the proof of Proposition 3.1. Next, substituting (B.13) and (B.14) into (B.3), we obtain the solution for consumption

$$C_\beta = \frac{1}{1+\Phi+\kappa\phi_\pi} + \frac{\kappa\phi_\pi\sigma_G^2\sigma_R^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}, \tag{B.15}$$

$$C_R = -\frac{\sigma_R^2\left(\sigma_G^2 + \Phi^2\sigma_\beta^2\right) + \sigma_G^2\sigma_\beta^2\left(\Phi^2 + \Phi\kappa\phi_\pi\right)}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}, \tag{B.16}$$

$$C_G = \frac{\kappa\phi_\pi\Phi\sigma_\beta^2\sigma_R^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}. \tag{B.17}$$

Substituting the above expressions into (B.2), the solution for the inflation response to the demand shock follows

$$\Pi_\beta = (S_R - S_G)\,\Phi C_\beta = \frac{\kappa\left[\Phi^2\sigma_\beta^2(\sigma_G^2+\sigma_R^2) - \Phi\sigma_G^2\sigma_R^2\right]}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}. \tag{B.18}$$

Moreover, the inflation response to the monetary shock is given by

$$\Pi_R = \frac{\kappa\left[\Phi\sigma_G^2\sigma_\beta^2 - \sigma_R^2(\Phi^2\sigma_\beta^2 + \sigma_G^2)\right]}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}. \tag{B.19}$$

Finally, the inflation response to the fiscal shock is given by

$$\Pi_G = -\frac{\kappa\Phi(1+\Phi)\sigma_\beta^2\sigma_R^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2+(\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]}. \tag{B.20}$$

We also use our numerical code for the full model to verify the closed-form solutions (B.15)–(B.20).

Given the explicit solution, proving the statements in Proposition 3.2 is trivial. By (B.20), $\frac{\partial\pi_t}{\partial\varepsilon_t^G} < 0$ since κ, ϕ_π, and Φ are all positive. Likewise, $\frac{\partial c_t}{\partial\varepsilon_t^G} > 0$. On the other hand, by (B.19), $\frac{\partial\pi_t}{\partial\varepsilon_t^R} > 0$ if and only if

$$\Phi\sigma_G^2\sigma_\beta^2 - \sigma_R^2(\Phi^2\sigma_\beta^2+\sigma_G^2) > 0.$$

Notice that when a fiscal shock hits the economy, the equilibrium response of nominal interest rate is given by

$$\frac{\partial i_t}{\partial\varepsilon_t^G} = \phi_\pi\Pi_G + \Phi C_G = -\frac{\kappa\phi_\pi\Phi\sigma_\beta^2\sigma_R^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2+(\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]} < 0.$$

Therefore, a higher government spending lowers the nominal interest rate in the equilibrium. Our proof is now complete.

Proof of Proposition 3.3

First, notice that (B.14) can be written as

$$S_R = \frac{\kappa\sigma_\beta^2\sigma_G^2\Phi}{(1+\kappa\phi_\pi)\sigma_G^2\sigma_R^2+(\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2} - \frac{\kappa\sigma_G^2}{(1+\kappa\phi_\pi)\sigma_G^2+(\frac{\sigma_G^2}{\sigma_R^2}+1)\Phi^2\sigma_\beta^2}.$$

where the first term is monotonically decreasing in σ_R^2, and the second term is monotonically increasing in σ_R^2. Hence, $\frac{\partial S_R}{\partial\sigma_R^2} < 0$. Similarly, from (B.12) we have

$$\frac{\partial|S_G|}{\partial\sigma_R^2} = \frac{\kappa\Phi\sigma_\beta^2}{(1+\kappa\phi_\pi)\sigma_G^2+(\frac{\sigma_G^2}{\sigma_R^2}+1)\Phi^2\sigma_\beta^2} > 0.$$

In other word, a higher monetary shock volatility amplifies the signaling effect of fiscal policy. Using a similar argument for (B.17) and (B.20), we have $\frac{\partial \Pi_G}{\partial \sigma_R^2} < 0$ and $\frac{\partial C_G}{\partial \sigma_R^2} > 0$.

By symmetry, we write

$$S_R = \frac{\kappa\left(\sigma_\beta^2 \Phi - \sigma_R^2\right)}{(1+\kappa\phi_\pi)\sigma_R^2 + (1+\frac{\sigma_R^2}{\sigma_G^2})\Phi^2\sigma_\beta^2}.$$

Note that the condition (3.10) implies that $\sigma_\beta^2\Phi - \sigma_R^2 > 0$. By inspection, we see that $\frac{\partial S_R}{\partial \sigma_G^2} > 0$. From (B.12) we see that $\frac{\partial |S_G|}{\partial \sigma_G^2} < 0$.

To prove the last statement, we rewrite (B.19) as

$$\begin{aligned}
\Pi_R &= \underbrace{\frac{\kappa\Phi\sigma_\beta^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_R^2 + (1+\frac{\sigma_R^2}{\sigma_G^2})\Phi^2\sigma_\beta^2\right]}}_{V_1(\sigma_G^2)} - \frac{\kappa\sigma_R^2(\Phi^2\sigma_\beta^2+\sigma_G^2) + \kappa\Phi^2\sigma_G^2\sigma_\beta^2 - \kappa\Phi^2\sigma_G^2\sigma_\beta^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_G^2\sigma_R^2 + (\sigma_G^2+\sigma_R^2)\Phi^2\sigma_\beta^2\right]} \\
&= V_1(\sigma_G^2) - \frac{\kappa}{1+\Phi+\kappa\phi_\pi} + \underbrace{\frac{\kappa\Phi^2\sigma_\beta^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_R^2 + (1+\frac{\sigma_R^2}{\sigma_G^2})\Phi^2\sigma_\beta^2\right]}}_{V_2(\sigma_G^2)}.
\end{aligned}$$

It is easy to see that both $V_1(\sigma_G^2)$ and $V_2(\sigma_G^2)$ are monotonically increasing functions of the fiscal shock volatility. Hence, $\frac{\partial \frac{\partial \pi_t}{\partial \varepsilon_t^R}}{\partial \sigma_G^2} = \frac{\partial \Pi_R}{\partial \sigma_G^2} > 0$. Finally, rewrite (B.16) as

$$C_R = -\frac{1}{1+\Phi+\kappa\phi_\pi} - \underbrace{\frac{\kappa\phi_\pi\Phi\sigma_\beta^2}{(1+\Phi+\kappa\phi_\pi)\left[\sigma_R^2 + (1+\frac{\sigma_R^2}{\sigma_G^2})\Phi^2\sigma_\beta^2\right]}}_{V_3(\sigma_G^2)}.$$

Since $V_3'(\sigma_G^2) > 0$, we have $\frac{\partial \frac{\partial c_t}{\partial \varepsilon_t^R}}{\partial \sigma_G^2} = \frac{\partial C_R}{\partial \sigma_G^2} < 0$. This completes the proof.

Proof of Proposition 3.4

Using the definition of $\tau_{R,G}$ and $\tau_{R,\beta}$ in the proposition, we write (B.14) as

$$S_R = \frac{\kappa\left(\Phi - \tau_{R,\beta}\right)}{(1+\kappa\phi_\pi)\tau_{R,\beta} + (1+\tau_{R,G})\Phi^2},$$

which indicates that $S_R\left(\Phi\right)$ is a rational polynomial function of degree two. Taking the derivative yields

$$\frac{\partial S_R\left(\Phi\right)}{\partial \Phi} = \frac{\kappa\left[-(1+\tau_{R,G})\Phi^2 + 2(1+\tau_{R,G})\tau_{R,\beta}\Phi + (1+\kappa\phi_\pi)\tau_{R,\beta}\right]}{\left[(1+\kappa\phi_\pi) + (1+\tau_{R,G})\Phi^2\right]^2}.$$

By inspection, the property of this derivative is determined by the quadratic function

$$N_R(\Phi) \equiv -(1+\tau_{R,G})\Phi^2 + 2(1+\tau_{R,G})\tau_{R,\beta}\Phi + (1+\kappa\phi_\pi)\tau_{R,\beta},$$

which has two roots

$$\tau_{R,\beta} \pm \frac{\sqrt{\tau_{R,\beta}^2(1+\tau_{R,G})^2 + (1+\tau_{R,G})(1+\kappa\phi_\pi)\tau_{R,\beta}}}{(1+\tau_{R,G})}.$$

By our parameter restrictions, one of the roots is negative. Since $\Phi > 0$, we have one extreme point at

$$\Phi_R = \tau_{R,\beta} + \frac{\sqrt{\tau_{R,\beta}^2(1+\tau_{R,G})^2 + (1+\tau_{R,G})(1+\kappa\phi_\pi)\tau_{R,\beta}}}{(1+\tau_{R,G})},$$

which achieves the maximal signaling effect of monetary policy. Elementary property of the quadratic function $N_R(\Phi)$ immediately implies the monotonicity behavior of S_R stated in the proposition. Applying the same procedure for the function $|S_G|$ leads to the characterization of the signaling effect of fiscal policy in the proposition. This completes the proof.

Appendix C Data set and VAR

Unless otherwise noted, the following data are drawn from the National Income and Product Accounts (NIPA) released by the Bureau of Economic Analysis. The data in levels from NIPA are nominal values and divided by 4. All quantity variables are expressed as 100×logs of their per capita real terms, which are obtained by dividing these variables by the civilian noninstitutional population (series "CNP16OV", Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis) and the implicit price deflator for the gross domestic product (Table 1.1.9, line 1). The quarterly observable sequences in the VAR estimation are constructed as follows.

1. **Output.** Gross domestic product (Table 1.1.5, line 1).
2. **Consumption.** Personal consumption expenditures on nondurable goods (Table 1.1.5, line 5) + services (Table 1.1.5, line 6).
3. **Government spending.** Government consumption (Table 3.1, line 21) + government investment (Table 3.1, line 39) − consumption of fixed capital (Table 3.1, line 42).
4. **Net tax.** Current receipts (Table 3.1, line 1) − current transfer payments (Table 3.1, line 22) − interest payments (Table 3.1, line 27).
5. **Government debt.** Market value of privately held gross federal debt (Federal Reserve Bank of Dallas).
6. **Inflation.** 400×log difference of the implicit price deflator for gross domestic product.
7. **Inflation expectation.** 400×log difference of the mean forecasts about the one-quarter-ahead and the current quarter GDP implicit deflators (series "PGDP3" and "PGDP2", respectively, Survey of Professional Forecasters (SPF), Federal Reserve Bank of Philadelphia).
8. **Inflation forecast error.** Real-time inflation − inflation expectation. Real-time inflation is computed as 400×log difference of the real-time GDP deflator recorded two quarters later

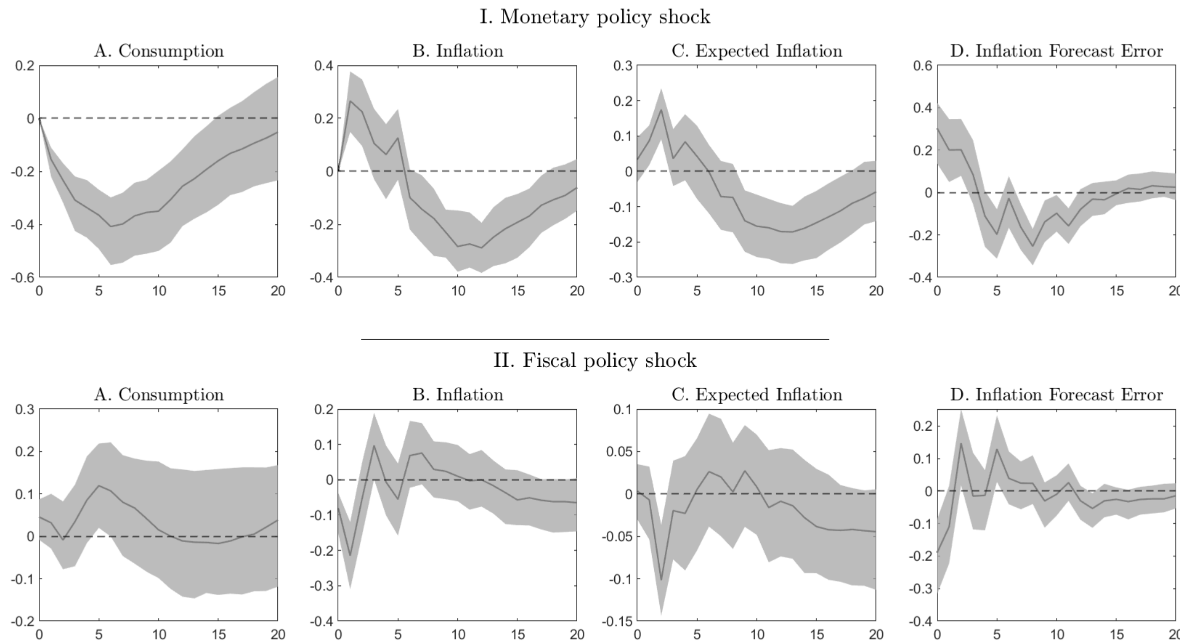

Figure 11: VAR impulse responses of key variables to a policy shock identified with the Cholesky decomposition. Notes: Responses are measured in percentage point deviations (annualized inflation, expected inflation, and inflation forecast error) and percentage deviations (consumption) from preshock levels. Solid lines denote median responses, while shaded areas delineate 70% confidence bands.

(Real-Time Data Set for Macroeconomists, Federal Reserve Bank of Philadelphia).[19]

9. **Interest rate.** Effective federal funds rate (Board of Governors of the Federal Reserve System) expressed in percentage.

There are nine observables in the VAR. They are ordered as government spending, output, net tax, consumption, inflation, interest rate, government debt, one-step-ahead inflation forecast from the SPF, and inflation forecast error. We include four lags as in most of the literature and a constant in the VAR. The results that follow are robust to alternative measures and orderings of these variables. Our sample 1968:Q4–2007:Q4 begins when the SPF forecast data first became available and ends before the federal funds rate nearly hit its effective lower bound. Both monetary and fiscal shocks are identified using the Cholesky decomposition.

Figure 11 displays the impulse response functions of key variables to 1% interest rate shock in the upper panels and 1% government spending shock in the lower panels. In addition to the puzzling

[19]For example, we use GDP deflators for 2001:Q1 and 2000:Q4 as recorded in the 2001:Q3 vintage data to calculate inflation for 2001:Q1.

behavior of *actual* inflation (panel B) that is in line with previous empirical studies, we emphasize three facts regarding the responses of *expected* inflation (panel C) and its forecast error (panel D). First, raising the interest rate may have signaled higher economic activity and hence induces inflationary expectations. Second, rising government spending may have signaled the opposite and thus trigger deflationary expectations. Third, the resulting forecast errors are autocorrelated rather than *i.i.d.* over time. The impulse responses of inflation forecast errors display under-reaction initially but delayed over-reaction as in Angeletos, Huo and Sastry (2020). As shown in the main text, our incomplete information model provides a unified explanation of these findings.

The output gap used in estimating the government spending rule is defined as the percentage deviation of real output (series "GDPC1", FRED) from the real potential output (series "GDPPOT", FRED).

Appendix D Numerical Solution

To compute the approximate model solution, we first derive the equilibrium fixed point condition in the frequency domain. We postulate that in causal stationary equilibrium, inflation is given by the VMA(∞) representation

$$\pi_t = \underbrace{\begin{bmatrix} \Pi_\beta(L) & \Pi_N(L) & \Pi_A(L) & \Pi_R(L) & \Pi_G(L) \end{bmatrix}}_{\boldsymbol{\Pi}(L)} \underbrace{\begin{bmatrix} \varepsilon_t^\beta \\ \varepsilon_t^N \\ \varepsilon_t^A \\ \varepsilon_t^R \\ \varepsilon_t^G \end{bmatrix}}_{\boldsymbol{\varepsilon}_t}, \tag{D.1}$$

where $\boldsymbol{\Pi}(L)$ is a 1×5 vector of one-sided, square-summable polynomial in the lag operator L. Using the z-transform, $\boldsymbol{\Pi}(L)$ is equivalent to a 1×5 vector of analytic functions in the open unit disk $\mathbb{D}$ that is square-integrable on the boundary. We denote the Hilbert space of such functions as $\mathbf{H}^2_{1\times 5}(\mathbb{D})$.[20] In what follows, we will use the analytic function and lag polynomial interchangeably by virtue of the Riesz-Fischer theorem. Given the conjecture (D.1), we solve other endogenous variables using the structural relations in Appendix A.

Step 1: Solve for consumption, output, and real marginal cost given $\boldsymbol{\Pi}(L)$. The output gap can be written as

$$y_t - y_t^n = (1-g_y)c_t + g_y g_t - \frac{\sigma g_y}{\sigma + \gamma(1-g_y)} \frac{1}{1-\rho_G L} \varepsilon_t^G - \frac{(1-g_y)(1+r)}{\sigma + \gamma(1-g_y)} \frac{1}{1-\rho_A L} \varepsilon_t^A + \frac{1-g_y}{\sigma + \gamma(1-g_y)} \varepsilon_t^N.$$

[20] See Han, Tan and Wu (2020) for mathematical details on the frequency-domain methods.

Substitute the above expression into the fiscal policy rule to get

$$
\begin{aligned}
g_t = &-\phi_g(1-g_y)c_t - \phi_g g_y g_t + \phi_g \frac{\sigma g_y}{\sigma+\gamma(1-g_y)}\frac{1}{1-\rho_G L}\varepsilon_t^G + \phi_g \frac{(1-g_y)(1+r)}{\sigma+\gamma(1-g_y)}\frac{1}{1-\rho_A L}\varepsilon_t^A \\
&- \phi_g \frac{1-g_y}{\sigma+\gamma(1-g_y)}\varepsilon_t^N + \frac{1}{1-\rho_G L}\varepsilon_t^G.
\end{aligned}
$$

The government spending then follows

$$
\begin{aligned}
g_t = &-\frac{\phi_g(1-g_y)}{1+\phi_g g_y}c_t + \frac{\phi_g(1-g_y)(1+r)}{[\sigma+\gamma(1-g_y)](1+\phi_g g_y)(1-\rho_A L)}\varepsilon_t^A - \frac{\phi_g(1-g_y)}{[\sigma+\gamma(1-g_y)](1+\phi_g g_y)}\varepsilon_t^N \\
&+ \frac{(1+\phi_g g_y)\sigma+\gamma(1-g_y)}{[\sigma+\gamma(1-g_y)](1+\phi_g g_y)(1-\rho_G L)}\varepsilon_t^G.
\end{aligned}
$$

which resembles (3.7) in the simple model.

Similarly, the output gap follows

$$
\begin{aligned}
y_t - y_t^n = &\underbrace{\frac{1-g_y}{1+\phi_g g_y}}_{K_c} c_t + \underbrace{\frac{g_y\gamma(1-g_y)}{[\sigma+\gamma(1-g_y)](1+\phi_g g_y)(1-\rho_G L)}}_{K_G(L)}\varepsilon_t^G - \underbrace{\frac{(1-g_y)(1+r)}{[\sigma+\gamma(1-g_y)](1+\phi_g g_y)(1-\rho_A L)}}_{K_A(L)}\varepsilon_t^A \\
&+ \underbrace{\frac{1-g_y}{[\sigma+\gamma(1-g_y)](1+\phi_g g_y)}}_{K_N}\varepsilon_t^N, \qquad \text{(D.2)}
\end{aligned}
$$

where K_c, $K_G(L)$, $K_A(L)$, and K_N are pre-specified lag polynomials. Then the nominal interest rate can be derived as

$$
i_t = \phi_\pi \pi_t + \phi_y\left(K_c c_t + K_G(L)\varepsilon_t^G - K_A(L)\varepsilon_t^A + K_N\varepsilon_t^N\right) + \frac{1}{1-\rho_R L}\varepsilon_t^R.
$$

Substituting the above expression into the dynamic IS equation yields

$$
c_t = \frac{1}{\sigma+\phi_y K_c}\left[\sigma\mathbb{E}_t^{HH}c_{t+1} + \mathbb{E}_t^{HH}\pi_{t+1} + \underbrace{\varepsilon_t^\beta - \phi_\pi\pi_t - \phi_y\left(K_G(L)\varepsilon_t^G - K_A(L)\varepsilon_t^A + K_N\varepsilon_t^N\right) - \frac{1}{(1-\rho_R L)}\varepsilon_t^R}_{\equiv \mathbf{K}(L)\boldsymbol{\varepsilon}_\mathbf{t}}\right].
$$

Given the equilibrium solution for inflation $\mathbf{\Pi}(L)$, we define the vector of lag polynomials $\mathbf{K}(L)$ as

$$\mathbf{K}(L)^T = \begin{bmatrix} 1-\phi_\pi \Pi_\beta(L) \\ \\ -\phi_\pi \Pi_N(L) - \phi_y K_N \\ \\ -\phi_\pi \Pi_A(L) + \phi_y K_A(L) \\ \\ -\phi_\pi \Pi_R(L) - \frac{1}{(1-\rho_R L)} \\ \\ -\phi_\pi \Pi_G(L) - \phi_y K_G(L) \end{bmatrix}. \tag{D.3}$$

Therefore,

$$c_t = \frac{1}{\sigma + \phi_y K_c}\left(\sigma \mathbb{E}_t^{HH} c_{t+1} + \mathbb{E}_t^{HH}\pi_{t+1} + \mathbf{K}(L)\varepsilon_{\mathbf{t}}\right).$$

Since the household has full information, we apply the Wiener-Kolmogorov prediction formula to get

$$c_t = \frac{\sigma}{\sigma+\phi_y K_c}\frac{\mathbf{C}(L)-\mathbf{C}(0)}{L} + \frac{1}{\sigma+\phi_y K_c}\frac{\mathbf{\Pi}(L)-\mathbf{\Pi}(0)}{L} + \frac{1}{\sigma+\phi_y K_c}\mathbf{K}(L)\varepsilon_{\mathbf{t}},$$

where $\mathbf{C}(L)$ is 1×5 vector of lag polynomials that characterizes the equilibrium solution for consumption.

Now apply the z-transform to solve the above equation in the frequency domain,

$$\left[(\sigma + \phi_y K_c)z - \sigma\right]\mathbf{C}(z) = -\sigma\mathbf{C}(0) + \mathbf{\Pi}(z) - \mathbf{\Pi}(0) + \mathbf{K}(z)z,$$

or equivalently,

$$\mathbf{C}(z) = \frac{\frac{1}{\sigma+\phi_y K_c}\left(\mathbf{K}(z)z + \mathbf{\Pi}(z) - \mathbf{\Pi}(0) - \sigma\mathbf{C}(0)\right)}{z - \underbrace{\frac{\sigma}{\sigma + \phi_y K_c}}_{\equiv\lambda}}.$$

By construction, $\lambda \in (0,1)$. Hence, causal stationarity of the equilibrium consumption process requires that

$$\mathbf{K}(\lambda)\lambda + \mathbf{\Pi}(\lambda) - \mathbf{\Pi}(0) - \sigma\mathbf{C}(0) = 0,$$

which pins down the functional constant $\mathbf{C}(0)$. Therefore, the consumption is given by

$$\mathbf{C}(z) = \frac{\frac{1}{\sigma+\phi_y K_c}\left(\mathbf{K}(z)z + \mathbf{\Pi}(z) - \mathbf{K}(\lambda)\lambda - \mathbf{\Pi}(\lambda)\right)}{z - \lambda}. \tag{D.4}$$

Then other equilibrium variables are derived as

$$y_t - y_t^n = K_c\mathbf{C}(L)\varepsilon_{\mathbf{t}} + \mathbf{O}(L)\varepsilon_t, \tag{D.5}$$

where $\mathbf{O}(L) = [0, K_N, -K_A(L), 0, K_G(L)]$. The government spending is given by

$$g_t = \underbrace{\left[-\phi_g\left(K_c\mathbf{C}(L) + \mathbf{O}(L)\right) + \frac{1}{1-\rho_G L}\mathbf{e}_5^T\right]}_{\mathbf{H_G}(L)}\varepsilon_t, \tag{D.6}$$

where $\mathbf{e}_5^T = [0, 0, 0, 0, 1]$. The nominal interest rate is given by

$$i_t = \underbrace{\left[\phi_\pi\mathbf{\Pi}(L) + \phi_y\left(K_c\mathbf{C}(L) + \mathbf{O}(L)\right) + \frac{1}{1-\rho_R L}\mathbf{e}_4^T\right]}_{\mathbf{H_R}(L)}\varepsilon_t. \tag{D.7}$$

Similarly, the aggregate output is given by

$$y_t = \underbrace{\left[(1-g_y)\mathbf{C}(L) + g_y \mathbf{H_G}(L)\right]}_{\mathbf{H_Y}(L)} \varepsilon_\mathbf{t}. \tag{D.8}$$

Finally, the real marginal cost is given by

$$mc_{t,j} = \underbrace{\left(\sigma \mathbf{C}(L) + \gamma \mathbf{H_Y}(L) + \mathbf{e}_2^T - \frac{1+\gamma}{1-\rho_A L}\mathbf{e}_3^T\right)}_{\mathbf{M}(L)} \varepsilon_\mathbf{t} - \gamma \eta_{t,j}. \tag{D.9}$$

Step 2: Compute the optimal price setting decision. In Appendix A, we derive the optimal pricing decision for firm j as

$$z_{t,j} = \mathbb{E}_{t,j}^{Firm} \pi_t + (1-\beta\theta)\mathbb{E}_{t,j}^{Firm} mc_{t,j} + \beta\theta \mathbb{E}_{t,j}^{Firm} z_{t+1,j}. \tag{D.10}$$

Since (D.10) is purely expectational, we conjecture that

$$z_{t,j} = S_A(L)a_{t,j} + S_R(L)i_t + S_G(L)g_t + S_\pi(L)\mathbf{1}_{\{I\}}\pi_{t-1} + S_Y(L)\mathbf{1}_{\{I,II\}}y_{t-1} = \mathbf{S}(L)s_{t,j},$$

where $\mathbf{1}_{\{I\}}$ and $\mathbf{1}_{\{I,II\}}$ are indicator functions of the cases for different information sets. The inflation is then given by

$$\begin{aligned} \pi_t &= (1-\theta)\int_{j\in[0,1]} z_{t,j} d_j \\ &= (1-\theta)\left[S_A(L)a_t + S_R(L)i_t + S_G(L)g_t + S_\pi(L)\mathbf{1}_{\{I\}}\pi_{t-1} + S_Y(L)\mathbf{1}_{\{I,II\}}y_{t-1}\right], \end{aligned} \tag{D.11}$$

which generalizes (3.4).

The information set for firm j is defined as

$$s_{t,j} = \underbrace{\begin{bmatrix} \frac{1}{1-\rho_A L}\mathbf{e}_3^T & 1 \\ \mathbf{H}_{\mathbf{R}}(L) & 0 \\ \mathbf{H}_{\mathbf{G}}(L) & 0 \\ L\mathbf{\Pi}(L)\mathbf{1}_{\{I\}} & 0 \\ L\mathbf{H}_{\mathbf{Y}}(L)\mathbf{1}_{\{I,II\}} & 0 \end{bmatrix}}_{\mathbf{H}(L)} \underbrace{\begin{bmatrix} \varepsilon_{\mathbf{t}} \\ \eta_{t,j} \end{bmatrix}}_{v_{t,j}}. \tag{D.12}$$

In all three cases of the information set, firms face a non-square signal extraction problem in the sense that the number of signals is strictly smaller than the dimension of the innovation space $v_{t,j}$. As such, firms need to solve for a non-trivial infinite-dimensional filtering problem. In the frequency domain, it amounts to finding the Wold fundamental representation for the signal process. Specifically, we need to solve for the following spectral factorization problem,

$$f_s(z) = \frac{1}{2\pi}\mathbf{H}(z)\Sigma_v\mathbf{H}^*(z^{-1}) = \frac{1}{2\pi}\mathbf{\Gamma}(z)\mathbf{\Gamma}^*(z^{-1}), \quad z = e^{-i\omega}, \quad \omega \in [-\pi, \pi], \tag{D.13}$$

where Σ_v is the covariance matrix of $v_{t,j}$, $\mathbf{\Gamma}(z)$ is a square matrix of analytic functions in $\mathbf{H}^2(\mathbb{D})$ that has the same dimension as $s_{t,j}$, and $*$ denotes the complex conjugate.[21] $\mathbf{\Gamma}(z)$ is the Wold fundamental representation in the frequency domain, and the associated innovation process spans the same subspace as the signal process. In stochastic system theory and complex analysis, $\mathbf{\Gamma}(z)$ is termed as the "outer function" or "outer spectral factor", which possesses certain mathematical properties. Huo and Takayama (2018) and Miao, Wu and Young (2019) extensively studied the mathematical foundations of the above problem, and provided methods to compute the factorization. For more details, we refer interested readers to these papers and the references contained therein. In general, the mapping $\mathbf{H}(z) \longmapsto \mathbf{\Gamma}(z)$ is highly nonlinear.

Given the Wold representation, we compute the conditional expectations via the Wiener-Hopf

[21] In our model, the signal process has full rank, i.e., $f_s(z)$ has full rank *a.e.* in $\mathbb{D}$.

prediction formula,

$$\mathbb{E}_{t,j}^{Firm}\pi_t = \underbrace{\left[\widetilde{\mathbf{\Pi}}(L)\Sigma_v\mathbf{H}(L^{-1})^T\left(\mathbf{\Gamma}\left(L^{-1}\right)^T\right)^{-1}\right]_+\mathbf{\Gamma}(L)^{-1}}_{\mathbf{W}_\pi(L)}\mathbf{H}(L)v_{t,j} = \mathbf{W}_\pi(L)s_{t,j}, \tag{D.14}$$

where $\widetilde{\mathbf{\Pi}}(L) = [\mathbf{\Pi}(L) \quad 0]$. By definition, $\mathbf{W}_\pi(z)$ is a matrix-valued analytic function inside the unit disk $\mathbb{D}$, and $[\,\cdot\,]_+$ is the (linear) annihilation operator that ensures analyticity. Similarly, using (D.9),

$$\mathbb{E}_{t,j}^{Firm}mc_{t,j} = \underbrace{\left[\widetilde{\mathbf{M}}(L)\Sigma_v\mathbf{H}(L^{-1})^T\left(\mathbf{\Gamma}\left(L^{-1}\right)^T\right)^{-1}\right]_+\mathbf{\Gamma}(L)^{-1}}_{\mathbf{W}_M(L)}\mathbf{H}(L)v_{t,j} = \mathbf{W}_M(L)s_{t,j}, \tag{D.15}$$

where $\widetilde{\mathbf{M}}(L) = [\mathbf{M}(L) \quad -\gamma]$. Note that firms' price setting decision $z_{t,j}$ is a linear function of the current and past signals. Thus, its expectation is given by

$$\mathbb{E}_{t,j}^{Firm}z_{t+1,j} = \underbrace{\left[L^{-1}\mathbf{S}(L)\mathbf{H}(L)\Sigma_v\mathbf{H}(L^{-1})^T\left(\mathbf{\Gamma}\left(L^{-1}\right)^T\right)^{-1}\right]_+\mathbf{\Gamma}(L)^{-1}}_{\mathbf{W}_z(L)}\mathbf{H}(L)v_{t,j} = \mathbf{W}_z(L)s_{t,j}. \tag{D.16}$$

The above equation can be simplified using (D.13) as

$$\mathbf{W}_z(z) = \left[z^{-1}\mathbf{S}(z)\mathbf{\Gamma}\left(z\right)\right]_+\mathbf{\Gamma}(z)^{-1}.$$

Now substitute (D.14)–(D.16) into the price setting equation (D.10) and match the coefficients in terms of the signal representation in the frequency domain,

$$\mathbf{S}(z) = \mathbf{W}_\pi(z) + (1-\beta\theta)\mathbf{W}_M(z) + \beta\theta\mathbf{W}_z(z). \tag{D.17}$$

It is easy to see from (D.17) that in any causal stationary equilibrium, $\mathbf{S}(z)$ is analytic inside the unit disk. This fact then implies

$$\mathbf{S}(z) = \mathbf{W}_\pi(z) + (1-\beta\theta)\mathbf{W}_M(z) + \beta\theta\frac{\mathbf{S}(z)\boldsymbol{\Gamma}(z) - \mathbf{S}(0)\boldsymbol{\Gamma}(0)}{z}\boldsymbol{\Gamma}(z)^{-1},$$

which leads to

$$\mathbf{S}(z) = \frac{z\left[\mathbf{W}_\pi(z) + (1-\beta\theta)\mathbf{W}_M(z)\right] - \beta\theta\mathbf{S}(0)\boldsymbol{\Gamma}(0)\boldsymbol{\Gamma}(z)^{-1}}{z-\beta\theta}. \tag{D.18}$$

Note that $\boldsymbol{\Gamma}(z)^{-1}$ is analytic inside the unit disk by the Wold fundamentality. Hence the numerator of (D.18) is analytic. Since $\beta\theta < 1$, we need to remove this inside pole by setting

$$\beta\theta\left[\mathbf{W}_\pi(\beta\theta) + (1-\beta\theta)\mathbf{W}_M(\beta\theta)\right] = \beta\theta\mathbf{S}(0)\boldsymbol{\Gamma}(0)\boldsymbol{\Gamma}(\beta\theta)^{-1},$$

which pins down the functional constants $\mathbf{S}(0)$. The signal representation is then given by

$$\mathbf{S}(z) = \frac{z\left[\mathbf{W}_\pi(z) + (1-\beta\theta)\mathbf{W}_M(z)\right] - \beta\theta\left[\mathbf{W}_\pi(\beta\theta) + (1-\beta\theta)\mathbf{W}_M(\beta\theta)\right]\boldsymbol{\Gamma}(\beta\theta)\boldsymbol{\Gamma}(z)^{-1}}{z-\beta\theta}. \tag{D.19}$$

Step 3: Equilibrium fixed point of inflation. Given the signal representation for the individual price setting decision, it is easy to derive the equilibrium fixed point for inflation. Specifically, aggregating $z_{t,j}$ according to (D.11),

$$\pi_t = (1-\theta)\int_{j\in[0,1]}\mathbf{S}(L)\mathbf{H}(L)v_{t,j}dj = (1-\theta)\mathbf{S}(L)\mathbf{H}(L)\mathbf{T}\varepsilon_t, \quad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $\mathbf{T}$ is an auxiliary matrix used to eliminate the idiosyncratic innovation. Now come back to our original conjecture about inflation,

$$\mathbf{\Pi}(z) = (1-\theta)\frac{z\Big[\mathbf{W}_\pi(z) + (1-\beta\theta)\mathbf{W}_M(z)\Big] - \beta\theta\Big[\mathbf{W}_\pi(\beta\theta) + (1-\beta\theta)\mathbf{W}_M(\beta\theta)\Big]\mathbf{\Gamma}(\beta\theta)\mathbf{\Gamma}(z)^{-1}}{z-\beta\theta}\mathbf{H}(z)\mathbf{T}. \tag{D.20}$$

(D.20) is indeed our equilibrium fixed point condition. Given (D.2), (D.3), (D.4), (D.5)–(D.9), and (D.12)–(D.15), it is easy to see that with the endogenous information set $\mathbf{H}(L)$, (D.20) is a nonlinear functional equation in the space of analytic functions $\mathbf{H}^2_{1\times 5}(\mathbb{D})$. We apply the numerical toolbox developed by Han, Tan and Wu (2020) to solve the model. Once $\mathbf{\Pi}(z)$ is obtained, we compute the other equilibrium variables using the structural equations we derived in this section.

Appendix E Missing Inflation Puzzle

Figure 12 plots the U.S. time series of inflation and monetary-fiscal policy instruments over the period from 2000:Q1 to 2017:Q4. We adopt the shadow rate of Wu and Xia (2016) as a measure of the monetary instrument because it conveniently summarizes the effects of unconventional monetary policy when the economy operates near the zero lower bound (ZLB) for interest rates.[22] We also detrend the government spending using the bandpass filter of Christiano and Fitzgerald (2003) to obtain a measure of the fiscal instrument. Figure 12 reveals that, even with the massive monetary-fiscal policy stimulus since the start of the Great Recession, inflation remains low throughout the economic recovery and averages about one percent lower than its pre-crisis level. This lack of inflationary pressure is referred to as the "missing inflation puzzle", which has led to a breakdown of the relation between real activity and inflation—also known as the Phillips Curve.

While the missing inflation puzzle is difficult to explain in standard full information models, it can be reconciled in our numerical model with the then-accommodative policies that had signaled an economic deterioration to the private sector. To mimic the real economy during the ZLB period from 2009:Q3 to 2015:Q4, we calculate the implied series of shock innovations $\{\varepsilon_t^R, \varepsilon_t^G\}$ from the

[22] The shadow rate is the federal funds rate when the ZLB is not binding; otherwise, it is negative to account for unconventional monetary policy tools.

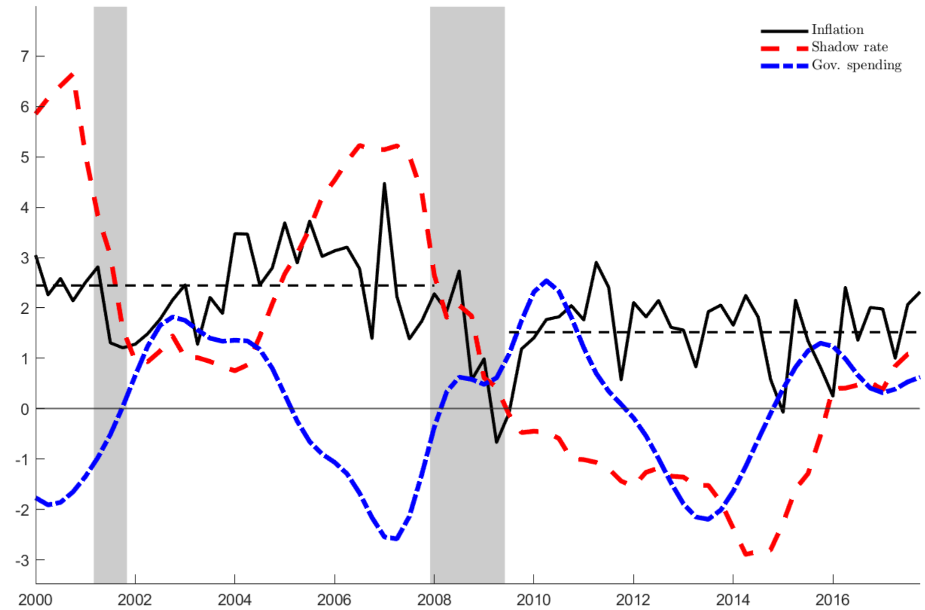

Figure 12: Inflation and policy instruments. Notes: Inflation is expressed in the annualized term. All numbers are in percentage. Horizontal dashed lines delineate the average inflation rates before and after the 2007-2009 Great Recession. Vertical bars indicate recessions as designated by the National Bureau of Economic Research.

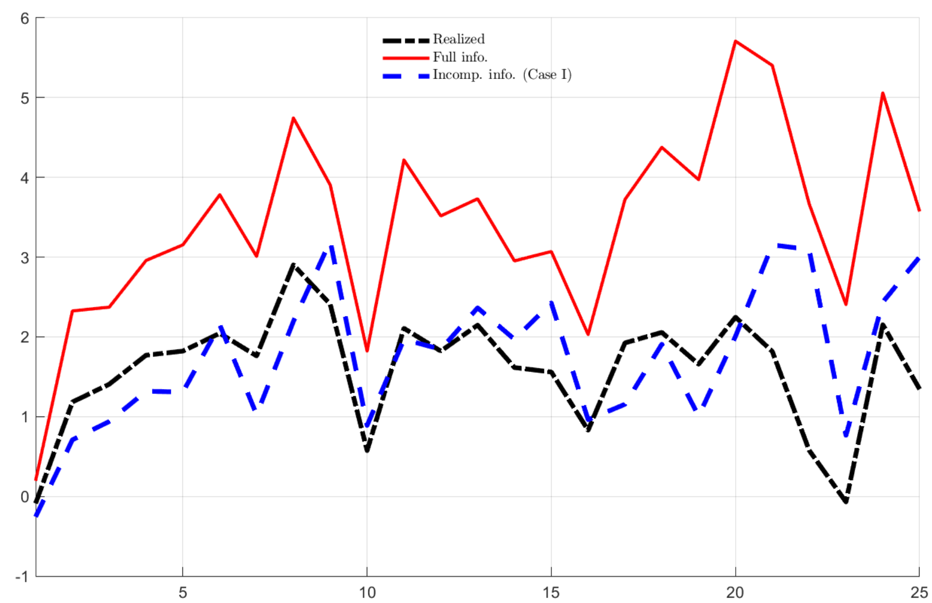

Figure 13: Realized and model-implied inflation at the ZLB. Notes: To initialize the simulation, the inflation rates just prior to the ZLB period are set to their actual values. The steady state inflation rate is set to 2%. All numbers are annualized and in percentage.

policy rules (2.5)–(2.6) and simulate the model economy based on these innovations. We also hit the economy with a series of adverse demand shocks $\varepsilon_t^\beta = -5\%$ and set $\varepsilon_t^N = \varepsilon_t^A = 0$ for all t. Figure 13 compares the path of actual inflation with those of model-implied inflation under different information assumptions. When firms cannot directly observe the policy shocks, both expansionary monetary and fiscal policies signal a weakening demand-side condition to firms, thereby lowering their inflation expectations. This signaling effect partly offsets the inflationary pressure that arises solely due to the stimulative policy shocks. As a result, the implied inflation does not rise as much under incomplete information (Case I), and its level stays largely in line with that of the actual inflation.[23] In the absence of the signaling effect, the full information model generates a counterfactually high inflation path during the policy stimulus period.

Appendix F Robustness Analysis

For algebraic simplicity, the analytical Section 3 imposes a non-essential restriction $\phi_y = \phi_g$. This section relaxes the assumption by considering various pairs of (ϕ_y, ϕ_g). We allow $\phi_y = [0.125, 0.5, 1]$. These values represent week, medium, and strong responses of monetary policy to the output gap. The selection of $\phi_y = [0.125, 0.5, 1]$ covers almost all estimated ϕ_y in the literature. For the government spending-output response coefficient ϕ_g, we set $\phi_g = [-0.04, 0.04, 0.12]$. These values correspond to the mean and the ± 2 standard errors from the mean estimates of ϕ_g in the government spending rule. The following three figures plot the impulse responses of inflation and consumption to a positive monetary policy and government spending shock when the firm's information set $I_{T,j}^{Firm} = \{a_{t,j}, i_t, g_t, \pi_{t-1}, y_{t-1} : t \leqslant T\}$(i.e., Case I). The impulse responses under Case II and III are similar and thus, are omitted.

For all the (ϕ_y, ϕ_g) pairs considered, Figure 14 shows the monetary price puzzle still arises. The incomplete information impulse responses eventually converge to their full information counterparts. The larger the ϕ_y is, the slower the convergence is.

For the fiscal price puzzle to hold, Figure 15 and 16 show that the government spending needs to

[23]The results are qualitatively similar across Case I–III in this and the next sections. For this reason, we focus on Case I to conserve space.

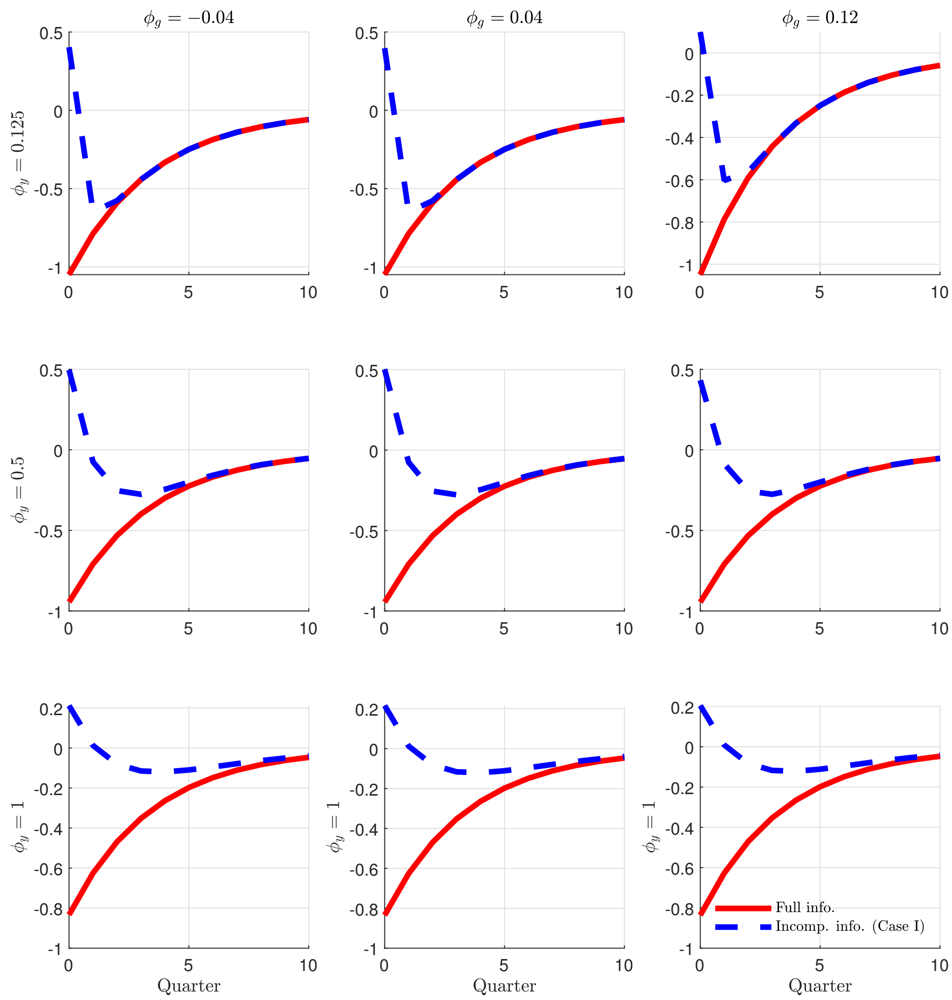

Figure 14: Impulse responses of inflation to a one-unit contractionary monetary policy shock for various (ϕ_y, ϕ_g). All numbers are annualized and in percentage.

be counter-cyclical (i.e, $\phi_g > 0$). When government spending is pro-cyclical (i.e, $\phi_g > 0$), a positive government spending shock crowds out consumption while increasing inflation, contradicting the VAR evidence presented in Figure 1.

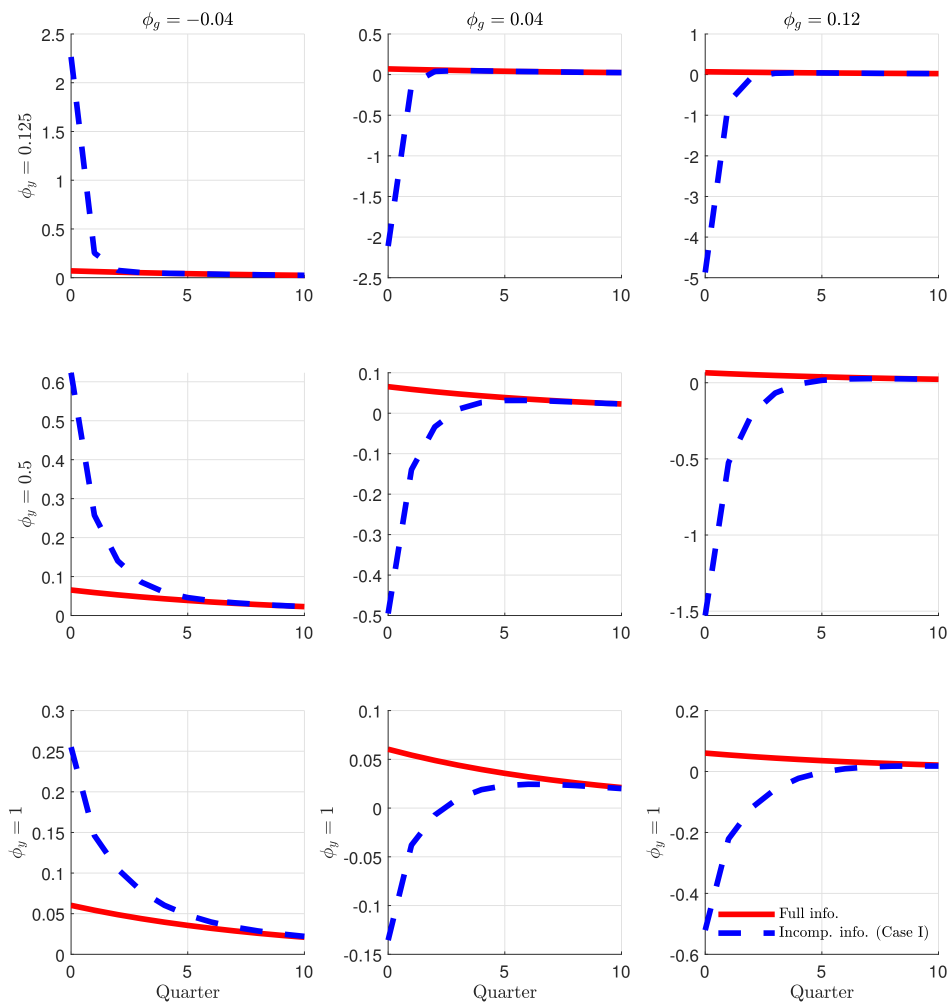

Figure 15: Impulse responses of inflation to a one-unit expansionary government spending shock for various (ϕ_y, ϕ_g). All numbers are annualized and in percentage.

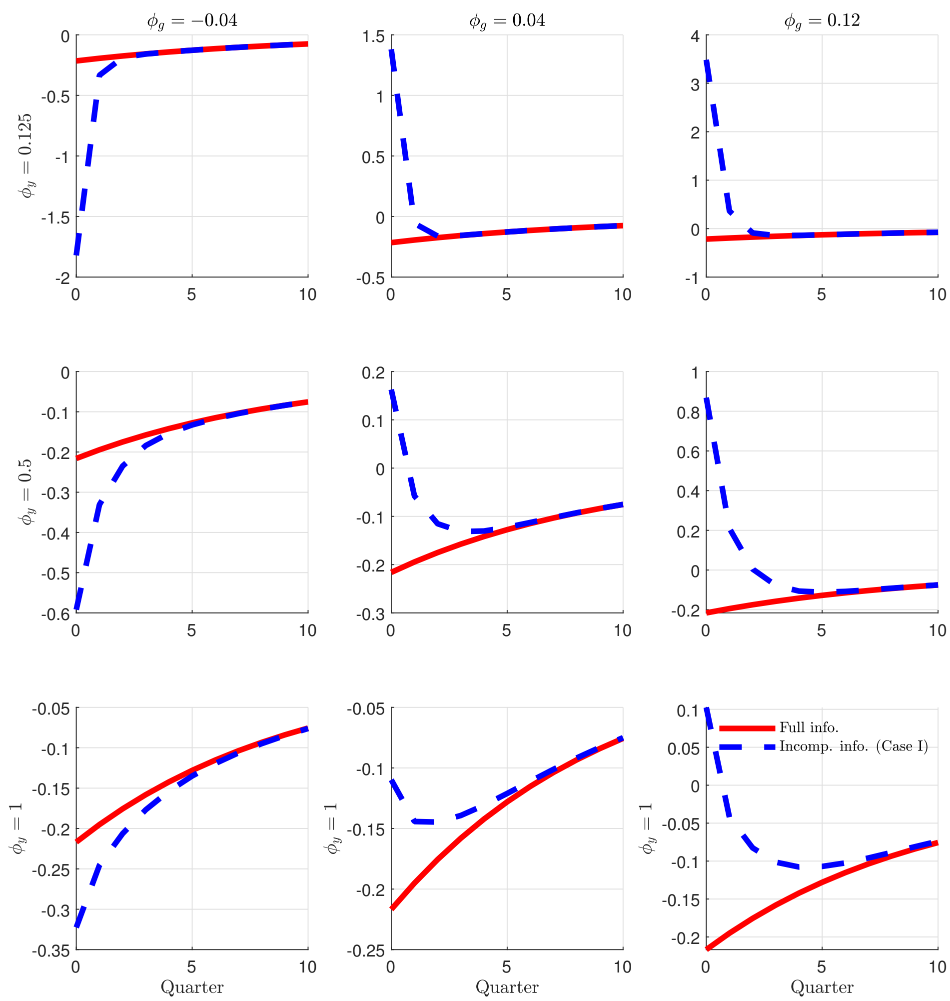

Figure 16: Impulse responses of consumption to a one-unit expansionary government spending shock for various (ϕ_y, ϕ_g). All numbers are annualized and in percentage.

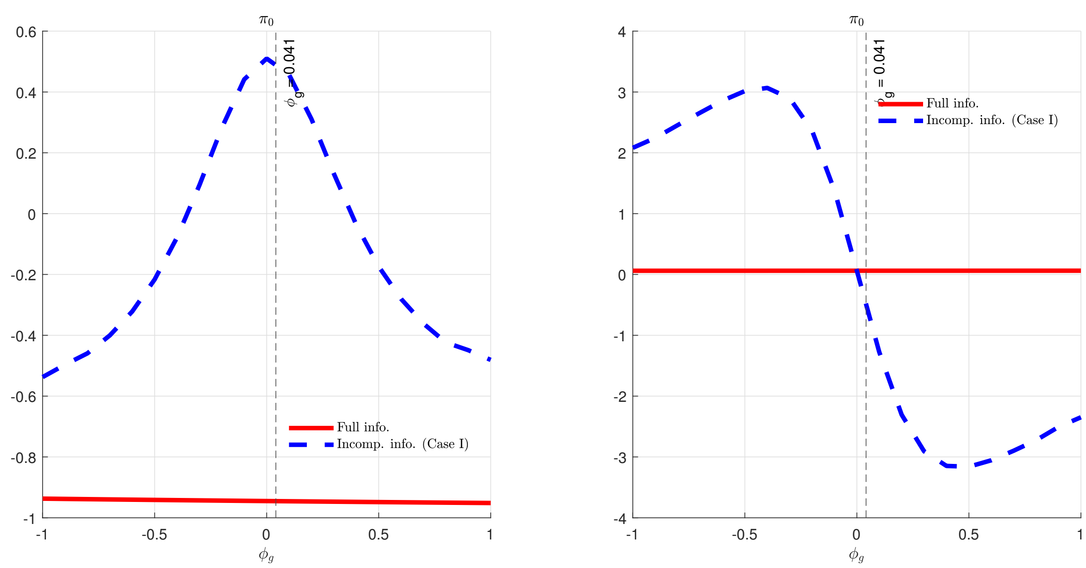

Figure 17: Impulse responses of the initial inflation π_0 to (i) a one-unit positive monetary policy shock (left panel) and (ii) a one-unit positive government spending shock (right panel) under alternative ϕ_g. The single shock hits the economy at $t = 0$. All numbers are annualized and in percentage.