Part I: Solving Linear Rational Expectations Models

Fei Tan

Chaifetz School of Business
Saint Louis University

Mini-Course Prepared for School of Finance
Zhejiang Gongshang University

June 21, 2018

Introduction

- Why linear solution
  - transparent mechanism
  - computationally efficient
  - facilitate likelihood-based inference

- Main references
  - Blanchard & Kahn (1980), "*The Solution of Linear Difference Models under Rational Expectations*", Econometrica
  - Klein (2000), "*Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model*", JEDC
  - Sims (2001), "*Solving Linear Rational Expectations Models*", Computational Economics

The Road Ahead...

- DSGE modeling
  - small new Keynesian DSGE
  - FRB-NY medium DSGE
  - log-linear approximation
- Sims' (2001) method
  - MATLAB program
  - analytical example (see notes)
- Impulse response functions

Small New Keynesian DSGE

- Dynamic IS relation

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} + \hat{g}_t - \mathbb{E}_t \hat{g}_{t+1} - \frac{1}{\tau}(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{z}_{t+1})$$

- New Keynesian Phillips curve

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{g}_t)$$

- Monetary policy rule

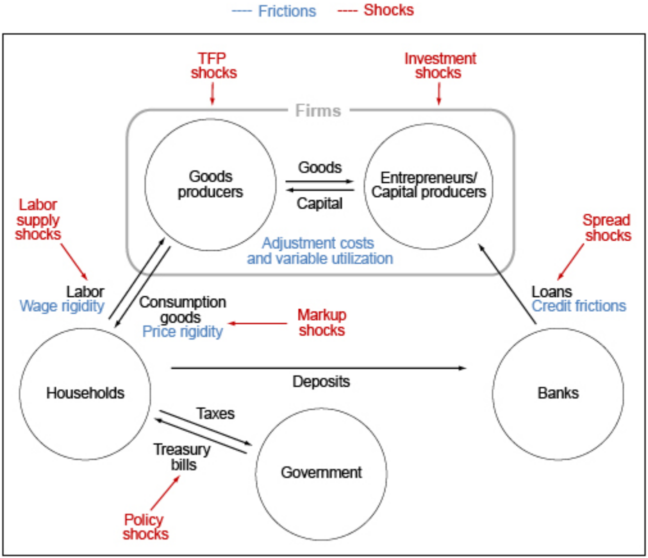
$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\psi_1 \hat{\pi}_t + (1-\rho_R)\psi_2(\hat{y}_t - \hat{g}_t) + \epsilon_{R,t}$$

- Exogenous driving processes

$$\hat{g}_t = \rho_G \hat{g}_{t-1} + \epsilon_{G,t}, \quad \hat{z}_t = \rho_Z \hat{z}_{t-1} + \epsilon_{Z,t}$$

FRB-NY Medium DSGE

A Stylized Description of the Model



Log-Linear Approximation

1st-order Taylor expansion

$$f(x_t) \equiv g(\hat{x}_t) = \underbrace{f(\bar{x})}_{g(0)} + \underbrace{f'(\bar{x})\bar{x}}_{g'(0)} \hat{x}_t + o(||\hat{x}_t||)$$

- Notations
  - $\bar{x}$: steady state of x_t
  - $\hat{x}_t \equiv \ln x_t - \ln \bar{x}$: log deviation
- Why log-linearization
  - level-linearization in economic models is not always well defined or consistent with data
  - interpretation as % change: $\hat{x}_t \approx (x_t - \bar{x})/\bar{x}$

Sims' (2001) Method

Canonical form

$$\Gamma_0(\theta)y_t = \Gamma_1(\theta)y_{t-1} + C(\theta) + \Psi(\theta)z_t + \Pi(\theta)\eta_t$$

- Notations
  - θ: structural/deep parameters
  - y_t: endogenous variables; z_t: exogenous shocks
  - $\eta_t \equiv y_t - \mathbb{E}_{t-1}y_t$: expectational errors
  - Γ_0, Γ_1, C, Ψ, Π: coefficient matrices
- MATLAB program: sims.princeton.edu/yftp/gensys

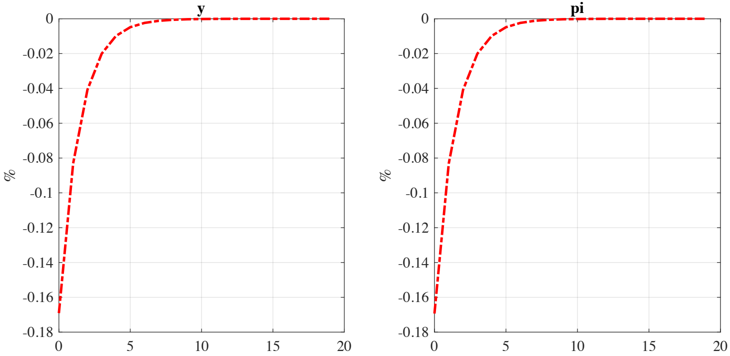
```
>>[G,C,M,fmat,fwt,ywt,gev,eu] = gensys(G0,G1,
    CC,Psi,Pi);
```

$$\Rightarrow y_t = \mathsf{G}y_{t-1} + \mathsf{CC} + \mathsf{M}z_t + \sum_{k=1}^{\infty}(\mathsf{ywt})(\mathsf{fmat})^{s-1}(\mathsf{fwt})\mathbb{E}_t z_{t+k}$$

MATLAB Pseudo-Code

```
function [C,G,M,eu] = SolveModel(para,ssp,P,V)
[...]
% Input equations
j = j+1;          % new Keynesian Phillips curve
G0(j,V.model.pi) = 1;
G0(j,V.model.E_pi) = -ssp(P.beta);
G0(j,V.model.y) = -para(P.kappa);
G0(j,V.model.g) = para(P.kappa);
j = j+1;          % trend shock
G0(j,V.model.z) = 1;
G1(j,V.model.z) = para(P.rho_Z);
Psi(j,V.shock.eps_Z) = 1;
j = j+1;          % E_pi error
G0(j,V.model.pi) = 1;
G1(j,V.model.E_pi) = 1;
Pi(j,V.fore.pi) = 1;
% Solve model
[G,C,M,~,~,~,~,eu] = gensys(G0,G1,CC,Psi,Pi);
```

Impulse Response Functions



- Response to one standard deviation monetary shock

$$y_{t+k} - \mathbb{E}_t y_{t+k} = \sum_{j=0}^{k-1} \mathsf{G}^j \mathsf{M} z_{t+k-j}$$