Interpreting Rational Expectations Econometrics via Analytic Function Approach

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Abstract

An analytic function method is applied to illustrate Geweke (2010)'s three econometric interpretations for a generic rational expectations (RE) model. This delivers an explicit characterization of the model's cross-equation restrictions imposed by the RE hypothesis under each interpretation. It is shown that the degree of identification on the deep parameters is positively related to the strength of the underlying econometric interpretation, and observationally equivalent models may arise once the cross-equation restrictions are interpreted in a minimal sense. This offers important insights into the econometric modeling and evaluation of dynamic economic models.

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1 Introduction

Rational expectations models typically produce cross-equation restrictions on the implied reduced-form models that are useful for making inferences about the deep parameters of the original behavioral models [Hansen and Sargent (1991)]. Because these RE models abstract sufficiently from their measured economic behavior, it is important to articulate the dimensions of reality that they are intended to mimic. This note applies an analytic function method to explore three alternative econometric interpretations, as proposed in Geweke (2010), for the relation between a generic univariate RE model and its measured economic behavior.

The method provides an explicit characterization of the cross-equation restrictions imposed by the RE hypothesis under each interpretation, which sheds new light on parameter identification in RE models. Early notable contributions in this area include Wallis (1980), Pesaran (1981), and Blanchard (1982), while recent progress can be found in Qu and Tkachenko (2013). It is important to note that identification in the existing work is often discussed as if its associated cross-equation restrictions are interpreted in a unique, conventional, sense. This ignorance of alternative, probably unconventional, senses in which identification is in less urgent need renders the endeavor to achieve identification absolute. As argued forcefully by Hurwicz (1962), however, both the degree of and the need for identification are *relative* notions, and they are meaningful only relative to the purpose for which the model is designed.[1] It is well known that economic models oftentimes have certain deep parameters that are left undetermined from the restrictions imposed by the models alone so that some *a priori* information is needed. If we denote by $\Theta_0(\mathcal{I})$ the identified set of parameters after imposing *a priori* information $\mathcal{I}$, and by $\Theta_0(\mathcal{E})$ the identified set of parameters relative to the purpose $\mathcal{E}$ for which the model is designed, then Hurwicz's insight on parameter identification instructs us to impose *a priori* information sufficiently to eliminate all those parameter values that are at odds with our purpose so that $\Theta_0(\mathcal{I}) \subseteq \Theta_0(\mathcal{E})$. The rest of this article is devoted to providing an explicit accounting for the content of $\Theta_0(\mathcal{E})$ by couching the modeler's purpose $\mathcal{E}$ in terms of the three econometric interpretations outlined by Geweke (2010). This explicit characterization of $\Theta_0(\mathcal{E})$ turns out to offer important insights into the econometric modeling and evaluation of dynamic economic models.

2 Generic Model

We consider the following generic univariate linear RE model

$$y_t = \frac{1}{\alpha}\mathbb{E}_t y_{t+1} + B(L)y_{t-1} + x_t, \quad E(L)x_t = F(L)\epsilon_t, \quad \epsilon_t \sim \text{i.i.d. } \mathcal{N}(0, \sigma_\epsilon^2) \tag{1}$$

in which y_t is the observed endogenous variable, x_t is the exogenous driving process, and $\mathbb{E}_t$ represents the mathematical expectation conditional on information available at time t, including the model's structure and all past and present realizations of the exogenous and

[1] In Hurwicz (1962), identification—the need for knowledge of the true old behavioral pattern—is discussed relative to the purpose of prediction. Another standard textbook paradigm is that the unidentified parameters of a linear regression model will not cause any trouble if the purpose for the model is not to give a partial effect interpretation but make prediction.

endogenous processes. Moreover, we parameterize the polynomials in the lag operator L that appear in (1) as follows

$$B(L) = B_0 + B_1 L + \cdots + B_m L^m \tag{2}$$
$$E(L) = 1 - E_1 L - \cdots - E_p L^p \tag{3}$$
$$F(L) = 1 + F_1 L + \cdots + F_q L^q \tag{4}$$

where m, p, q are all finite nonnegative integers. Finally, we assume that both $E(z)$ and $F(z)$ do not have roots inside the unit circle ($|z| = 1, z \in \mathbb{C}$) and they have no common roots. This ensures that x_t follows a covariance stationary ARMA(p, q) process. Economic models in the form of (1) can easily arise in many contexts. With $B(L) = 0$, for example, (1) may be resulted from a simple monetary model by combining a sort of "Fisher relation" and a monetary policy rule with α understood to be the degree to which policy rate leans against inflationary winds. We collect all the model parameters in the vector

$$\theta = [\alpha, B_0, \ldots, B_m, E_1, \ldots, E_p, F_1, \ldots, F_q]'$$

where $\theta \in \Theta \subseteq \mathbb{R}^{2+m+p+q}$. For the remaining analysis we will restrict our attention to the parameter space Θ that delivers a unique solution to (1), and seek a subspace of Θ in which different parameter values generate observationally equivalent model implications under identification failure.[2]

To fully characterize the cross-equation restrictions imposed by (1), we closely follow the solution principle advocated in Whiteman (1983). In particular, we seek a solution in the space spanned by time-invariant square-summable linear combinations of the process ϵ fundamental for the driving process x, $y_t = C(L)\epsilon_t$, where $C(L)$ is assumed to be a polynomial in nonnegative powers of L with square-summable coefficients. Under the Gaussianity assumption on the innovation ϵ, the conditional expectation coincides with the linear projection and thus can be conveniently evaluated via the Wiener-Kolmogorov optimal prediction formula

$$\mathbb{E}_t y_{t+1} = \left[\frac{C(L)}{L}\right]_+ \epsilon_t = L^{-1}[C(L) - C_0]\epsilon_t$$

where $[\ ]_+$ is the annihilation operator that ignores negative powers of L. Because (1) holds for all realizations of ϵ, we can rewrite (1) in its z-transform as

$$C(z) = \frac{1}{\alpha} z^{-1}[C(z) - C_0] + zB(z)C(z) + \frac{F(z)}{E(z)}$$

and solving for $C(z)$ gives

$$C(z) = -\frac{zF(z) - \frac{C_0}{\alpha}E(z)}{\left[z^2 B(z) - z + \frac{1}{\alpha}\right] E(z)}$$

[2]The observational equivalence result between an indeterminate model driven by nonfundamental (sunspot) shocks and a determinate model driven by fundamental shocks can be found, for example, Beyer and Farmer (2007) and Leeper and Walker (2011). In addition, Lubik and Schorfheide (2006) and Leeper and Walker (2011) show that even two determinate models can be made observationally equivalent. Note that all the aforementioned models can be nested in the form of (1) in this note.

Clearly, indeterminacy of equilibrium would emerge if the undetermined coefficient C_0 were allowed to take any finite value. Nevertheless, recall that we seek a moving-average representation for y with square-summable coefficients of ϵ in the time domain. Appealing to the Riesz-Fisher Theorem, this is tantamount to the analyticity of $C(z)$ on the unit disk, a requirement which may provide additional restrictions just sufficient to pin down the value of C_0. Specifically, we assume that $z^2B(z) - z + \frac{1}{\alpha} = (z-\rho)D(z)$ where $|\rho| < 1$ and $D(z)$ has no root inside the unit disk. In this case, the free parameter C_0 can be used to remove the singularity of $C(z)$ at ρ by setting C_0 in such a way as to casue the residue of $C(z)$ at ρ to be zero

$$\lim_{z\to\rho}(z-\rho)C(z) = -\frac{\rho F(\rho) - \frac{C_0}{\alpha}E(\rho)}{D(\rho)E(\rho)} = 0$$

Solving for C_0 yields $C_0 = \alpha\rho\frac{F(\rho)}{E(\rho)}$. Therefore, the unique RE equilibrium is given by

$$y_t = -\frac{LF(L) - \rho\frac{F(\rho)}{E(\rho)}E(L)}{\left[L^2B(L) - L + \frac{1}{\alpha}\right]E(L)}\epsilon_t \tag{5}$$

Note that the solution in (5) clearly captures all cross-equation restrictions imposed by (1) that are the "hallmark of rational expectations models" [Hansen and Sargent (1980)]. We direct interested readers to Tan and Walker (2015) who generalize the analytic function method of Whiteman (1983) for solving linear RE models to the multivariate setting.

3 Three Econometric Interpretations

Following Geweke (2010), this section explores three alternative econometric interpretations for the relation between model (1) and its measured economic behavior by making the cross-equation restrictions imposed by (1) explicit under each category.

To sharpen the analysis, we focus on two parameterizations of (1) that are potentially observationally equivalent, one assumed to be the true data generating process and the other being the econometrician's misspecified approximating model.[3] In particular, suppose that the true model is parameterized under θ_0 where $\alpha = \alpha_0 \neq 0$, $B(L) = B_0 \neq 0$, and $E(L) = F(L) = 1$.[4] This restricts the propagation mechanism of the exogenous disturbances to be purely endogenous. We denote the z-transform of the unique equilibrium under θ_0 by

$$C_{\theta_0}(z) = -\frac{1}{B_0 z - \frac{1+\sqrt{1-4B_0/\alpha_0}}{2}} \tag{6}$$

We further assume that the econometrician mistakenly believes that the propagation mechanism is purely exogenous and parameterizes her model under θ_1 where $\alpha = \alpha_1 \neq 0$, $B(L) = 0$, $E(L) = 1 - E_1 L$ and $F(L) = 1 + F_1 L$.[5] Accordingly, we denote the z-transform of the unique

[3]See also Lubik and Schorfheide (2006) for a similar comparison.

[4]To ensure determinacy, we need to impose $\left|\frac{1-\sqrt{1-4B_0/\alpha_0}}{2B_0}\right| < 1$ and $\left|\frac{1+\sqrt{1-4B_0/\alpha_0}}{2B_0}\right| > 1$.

[5]Again to ensure determinacy, we need to impose $|\alpha_1| > 1$. In the context of a monetary model, this means that monetary policy obeys a Taylor-type rule.

equilibrium under θ_1 by

$$C_{\theta_1}(z) = \frac{F_1 z + \frac{1+F_1/\alpha_1}{1-E_1/\alpha_1}}{1 - E_1 z} \tag{7}$$

where $\theta_1 = [\alpha_1, E_1, F_1]' \in \Theta \subseteq \mathbb{R}^3$. In what follows, we explicitly characterize the content of $\Theta_0(\mathcal{E})$—the identified set for the misspecified model under interpretation $\mathcal{E}$—in terms of the valid cross-equation restrictions. The cardinality of $\Theta_0(\mathcal{E})$ should be naturally interpreted as the degree of identification *relative* to interpretation $\mathcal{E}$.

3.1 Strong Interpretation

The strong interpretation ($\mathcal{E} = \mathcal{S}$) of RE econometrics is that the model provides a predictive distribution for the observables, leading to the likelihood-based econometrics. A natural measure that evaluates the "distance" of the probability distribution over y_t produced by the misspecified model from that implied by the true model is given by the so-called Kullback-Leibler (K-L) divergence. To make it operational here, we propose to use the frequency-domain expression of the K-L divergence[6]

$$D_{\mathrm{KL}}(\theta_0, \theta_1) = \frac{1}{4\pi} \int_{-\pi}^{\pi} 1_A(w) \left[\mathrm{tr}(S_{\theta_1}^{-1}(w) S_{\theta_0}(w)) - \log\det(S_{\theta_1}^{-1}(w) S_{\theta_0}(w)) - n_y \right] dw$$

where $S_{\theta_0}(w)$ and $S_{\theta_1}(w)$ are the spectral density matrices associated with θ_0 and θ_1, and summarize all relevant information under normality as assumed in (1). The indicator function $1_A(w)$ is symmetric about zero and specifies a subset of frequencies by which the K-L divergence is evaluated. The above expression is also utilized by Qu and Tkachenko (2013) as a computational device for checking global identification of DSGE models as well as a measure for the empirical closeness between two DSGE models. To couch the above expression in terms of the z-transformed solutions $C_{\theta_0}(z)$ and $C_{\theta_1}(z)$ derived earlier, we make the change of variable $z = e^{-iw}$ to get

$$D_{\mathrm{KL}}(\theta_0, \theta_1) = \frac{1}{4\pi i} \oint 1_B(z) \left[\frac{|C_{\theta_0}(z)|^2}{|C_{\theta_1}(z)|^2} - \log \frac{|C_{\theta_0}(z)|^2}{|C_{\theta_1}(z)|^2} - 1 \right] \frac{dz}{z} \tag{8}$$

where $\oint$ denotes the contour integral on the positively oriented unit circle and $n_y = 1$ in our case. The indicator function $1_B(z)$ is again symmetric about the origin with B being the image of A under the mapping $z = e^{-iw}$. For the remaining analysis we will focus on the full spectrum and therefore, the identified set of parameters for the misspecified model under the strong interpretation can be characterized as

$$\Theta_0(\mathcal{S}) = \{\theta_1 \in \Theta : D_{\mathrm{KL}}(\theta_0, \theta_1) = 0\} \tag{9}$$

To see the content of $\Theta_0(\mathcal{S})$, we minimize $D_{\mathrm{KL}}(\theta_0, \theta_1)$ over Θ by taking the first-order conditions with respect to θ_1

$$\frac{1}{2\pi i} \oint \nabla C_{\theta_1}(z^{-1}) C_{\theta_1}(z) \left[\frac{1}{|C_{\theta_1}(z)|^2} - \frac{|C_{\theta_0}(z)|^2}{|C_{\theta_1}(z)|^4} \right] \frac{dz}{z} = 0 \tag{10}$$

[6]This expression was first obtained by Pinsker (1964) as the entropy rate of one stationary vector Gaussian process with respect to another.

where

$$\nabla C_{\theta_1}(z^{-1}) = \begin{pmatrix} \frac{\partial C_{\theta_1}(z^{-1})}{\partial \alpha_1} \\ \frac{\partial C_{\theta_1}(z^{-1})}{\partial E_1} \\ \frac{\partial C_{\theta_1}(z^{-1})}{\partial F_1} \end{pmatrix} = \begin{pmatrix} -\frac{\frac{E_1+F_1}{(\alpha_1-E_1)^2} z}{z-E_1} \\ \frac{\frac{\alpha_1+F_1}{(\alpha_1-E_1)^2} z^2 + \frac{(\alpha_1+F_1)(\alpha_1-2E_1)}{(\alpha_1-E_1)^2} z + F_1}{(z-E_1)^2} \\ \frac{\frac{1}{\alpha_1-E_1} z + 1}{z-E_1} \end{pmatrix}$$

Because each component of $\nabla C_{\theta_1}(z^{-1})$ contains a pole at $z = E_1$ that cannot be removed from the integrand in (10), it must be that

$$\frac{1}{|C_{\theta_1}(z)|^2} - \frac{|C_{\theta_0}(z)|^2}{|C_{\theta_1}(z)|^4} = 0$$

and hence $C_{\theta_1}(z)C_{\theta_1}(z^{-1}) = C_{\theta_0}(z)C_{\theta_0}(z^{-1})$. In the univariate case, this implies that $C_{\theta_1}(z) = C_{\theta_0}(z)$. That is, $\Theta_0(\mathcal{S})$ contains all those values of $\theta_1 \in \Theta$ such that

$$\frac{F_1 z + \frac{1+F_1/\alpha_1}{1-E_1/\alpha_1}}{1 - E_1 z} = -\frac{1}{B_0 z - \frac{1+\sqrt{1-4B_0/\alpha_0}}{2}}$$

which has a unique solution given by

$$\alpha_1 = \alpha_0, \quad E_1 = \frac{2B_0}{1+\sqrt{1-4B_0/\alpha_0}}, \quad F_1 = 0 \tag{11}$$

In this case θ_1 is just identified.[7]

The above result also extends to the more general specification of $B(L)$ in (2), which will produce $m+1$ roots outside the unit circle in the denominator of (6), and these roots can be fully replicated by specifying an autoregressive process of order $m+1$ for x.[8] Two important implications are in order. If the econometrician's misspecified model featured an autoregressive component of order smaller than $m+1$ for x, then the requirement that $D_{\mathrm{KL}}(\theta_0, \theta_1) = 0$ would turn out to be overly restrictive, forcing $\Theta_0(\mathcal{S})$ to be an empty set. This underlies the essence of strong econometric interpretation of RE models—it requires an explicit accounting for all the dimensions of variation observed in the data that can hardly be accounted for in the model—and partly explains why many "good" models are easily rejected under this category. On the contrary, if the autoregressive component has order larger than $m+1$ for x, then there would be possibly more than one set of values for θ_1 satisfying $D_{\mathrm{KL}}(\theta_0, \theta_1) = 0$, making $\Theta_0(\mathcal{S})$ strictly larger than a singleton. This poses a serious challenge for the identification of key policy parameters in models where policy choices have substantial impacts on agents' welfare. As emphasized by Leeper and Walker (2011), important identifying restrictions might be imposed on the model through the specification of the exogenous driving processes.

[7] Note that the true value α_0 can be accurately inferred even though the econometrician's model is largely misspecified. In the case θ_1 is unidentifiable, (9) also makes clear whether or not a particular subvector of θ_1 can be partially and/or conditionally identified.

[8] In our case $m = 0$ and this makes the inverse of E_1 match exactly the only root outside the unit circle, $\frac{1+\sqrt{1-4B_0/\alpha_0}}{2B_0}$, that appear in the denominator of (6).

3.2 Weak Interpretation

The interpretation which many macroeconomists work with is indeed the weak one ($\mathcal{E} = \mathcal{W}$)—it treats a model as providing a predictive distribution for a set of selected sample moments of the observables that the model is intended to mimic.[9] For our purposes we assume that the econometrician's misspecified model is designed to provide a predictive distribution only for the sample mean of the observables

$$\bar{y}_T = \frac{1}{T}(1 + L + L^2 + \cdots + L^{T-1})C_{\theta_1}(L)\epsilon_T \tag{12}$$

where the sample ranges from $t = 1$ to T, and its counterpart implied by the true model has the same form with $C_{\theta_1}(L)$ in (12) replaced by $C_{\theta_0}(L)$. Again a natural measure that evaluates the divergence between the two probability distributions over $\bar{y}_T$ is given by the K-L divergence in its z-transformed expression. One can easily verify that this is the same as (8), thereby the same first-order conditions with respect to θ_1. This fact implies that the identified set of parameters for the misspecified model under the weak interpretation can be characterized as $\Theta_0(\mathcal{W}) = \Theta_0(\mathcal{S})$, echoing Geweke (2010)'s insights—the weak econometric interpretation in fact makes assumptions no weaker than those underlying the strong one. Models interpreted under this category typically impose cross-equation restrictions to the same extent as the strong one, thereby implying the same degree of identification on the deep parameters.

3.3 Minimal Interpretation

To avoid the stringent assumptions inherent in the strong and weak interpretations, this section characterizes the identified set of θ_1 by considering a more modest claim for RE models. Here the econometrician's misspecified model accounts for a set of selected population moments of the observables; following Geweke (2010) and earlier work by DeJong and Whiteman (1996), the model is interpreted under the so-called minimal econometric interpretation ($\mathcal{E} = \mathcal{M}$).

Because the true and misspecified models both generate a time series for y_t with zero population mean, we assume that the econometrician's misspecified model is designed to account for the volatility of the observables characterized by the population second moment, $m = \mathbb{E}_{\theta_1}[y_t^2]$, where expectation is taken with respect to the likelihood functions associated with θ_1. Then the identified set of parameters for the misspecified model under the minimal interpretation can be characterized as

$$\Theta_0(\mathcal{M}) = \{\theta_1 \in \Theta : \mathbb{E}_{\theta_0}[y_t^2] = \mathbb{E}_{\theta_1}[y_t^2]\} \tag{13}$$

To see the content of $\Theta_0(\mathcal{M})$, we use the following inversion formula to compute the population second moments of the observables implied by both models[10]

$$\begin{aligned} \mathbb{E}_\theta[y_t^2] &= \frac{\sigma_\epsilon^2}{2\pi i}\oint C_\theta(z)C_\theta(z^{-1})\frac{dz}{z} \\ &= \sigma_\epsilon^2 \times \text{sum of residues of } C_\theta(z)C_\theta(z^{-1})z^{-1} \text{ at poles inside unit circle} \end{aligned} \tag{14}$$

[9]For example, conventional calibration exercise falls under this category and can indeed be viewed as a special case of the prior predictive analysis.

[10]Also see Sargent (1987) for a more thorough treatment on these formulae.

where $\theta \in \{\theta_0, \theta_1\}$, and from the theory of residue and the z-transformed solutions derived earlier we can obtain

$$\begin{aligned} \mathbb{E}_{\theta_0}[y_t^2] &= \lim_{z \to \frac{2B_0}{1+\sqrt{1-4B_0/\alpha_0}}} \left(z - \frac{2B_0}{1+\sqrt{1-4B_0/\alpha_0}} \right) C_{\theta_0}(z) C_{\theta_0}(z^{-1}) z^{-1} \sigma_\epsilon^2 \\ &= \frac{2}{1+\sqrt{1-4B_0/\alpha_0} - 2B_0/\alpha_0 - 2B_0^2} \sigma_\epsilon^2 \qquad (15) \\ \mathbb{E}_{\theta_1}[y_t^2] &= \lim_{z \to E_1} (z - E_1) C_{\theta_1}(z) C_{\theta_1}(z^{-1}) z^{-1} \sigma_\epsilon^2 \\ &= \frac{\left(E_1 F_1 + \frac{1+F_1/\alpha_1}{1-E_1/\alpha_1} \right) \left(F_1/E_1 + \frac{1+F_1/\alpha_1}{1-E_1/\alpha_1} \right)}{1 - E_1^2} \sigma_\epsilon^2 \qquad (16) \end{aligned}$$

That is, $\Theta_0(\mathcal{M})$ contains all those values of $\theta_1 \in \Theta$ such that (15) equals (16), one of which is given by (11). Indeed there is a continuum of observationally equivalent parameter values for the misspecified model under the minimal interpretation. In this case, θ_1 has a weak degree of identification.[11]

Because the minimal interpretation imposes much looser cross-equation restrictions than the strong/weak one, the set $\Theta_0(\mathcal{M})$ is in general much larger than $\Theta_0(\mathcal{S})$ and $\Theta_0(\mathcal{W})$, rendering more parameter points compatible with the modeler's purpose. This, to a large extent, explains why model fit under the minimal interpretation improves substantially relative to the strong/weak one [Del Negro and Schorfheide (2004), DeJong and Whiteman (1993), Ingram and Whiteman (1994)]. Consequently, observationally equivalent models may arise once the cross-equation restrictions are interpreted in the minimal sense. For example, Geweke (2010) shows that the equity premium puzzle disappears when m consists of only population means for the risk free rate and the equity premium.

4 Concluding Remarks

This note applies an analytic function approach to characterize the cross-equation restrictions imposed by the RE hypothesis under three econometric interpretations outlined in Geweke (2010) for a generic univariate RE model. The approach provides an explicit characterization of these cross-equation restrictions, and it also sheds new light on parameter identification in RE models. In particular, it is shown that the degree of identification on deep parameters is positively related to the strength of the underlying econometric interpretation, and observationally equivalent models may arise once the cross-equation restrictions are interpreted in the minimal sense. This explains why model fit under the so-called minimal econometric interpretation can improve substantially, thereby offering important insights into the econometric modeling and evaluation of dynamic economic models. We conclude by pointing out that an extension of the method used in this note to the multivariate setting is also straightforward. We address this point in Leeper, Tan and Walker (2014) in the context of alternative monetary and fiscal policy interactions that are potentially observationally equivalent.

[11] The equality of (15) and (16) also indicates that α_1, though unidentifiable, becomes conditionally identifiable once a particular driving process x parameterized by (E_1, F_1) is selected.

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