Appetite for Treasuries, Debt Cycles, and Fiscal Inflation[†]

Fei Tan*

[This Version: October 5, 2022]

Abstract

Despite accelerating debt levels, the real yield on U.S. Treasuries remains low due to investors' desire for their extreme safety and liquidity services. Economic theory makes clear that the fiscal surplus as a proportion of the outstanding debt must average out to the real interest rate over time. Exploring these equilibrium relations in a change-point vector autoregression model, I estimate the state-dependent properties of U.S. inflation and its stance of fiscal policy that characterize long-term debt cycles. An archetypal debt cycle consists of alternating phases of persistent deficits and surpluses in tandem with alternating patterns of inflation and fiscal stance. In line with these key properties found in the data, I present a simple analytical model based on the fiscal theory of the price level where the household has a preference for holding government bonds. Determinacy admits a standard passive monetary policy coupled with a broad range of active fiscal policy. When the real interest rate falls below the economy's growth rate, permanent fiscal deficits can be sustained in the long run. The model explains why fiscal inflation has largely remained benign over the past two decades.

Keywords: monetary and fiscal policy; inflation dynamics; $r < g$; change points.

JEL Classification: C34, E31, E63, H62

[†]This study originated in a conversation with David Rapach. I thank SLU undergraduates enrolled in my ECON 3120 Intermediate Macroeconomics class for helpful discussions. Financial support from the Chaifetz School of Business summer research grant is also gratefully acknowledged.

*Department of Economics, Chaifetz School of Business, Saint Louis University and Center for Economic Behavior and Decision-Making, Zhejiang University of Finance and Economics. Email: econdojo@gmail.com.

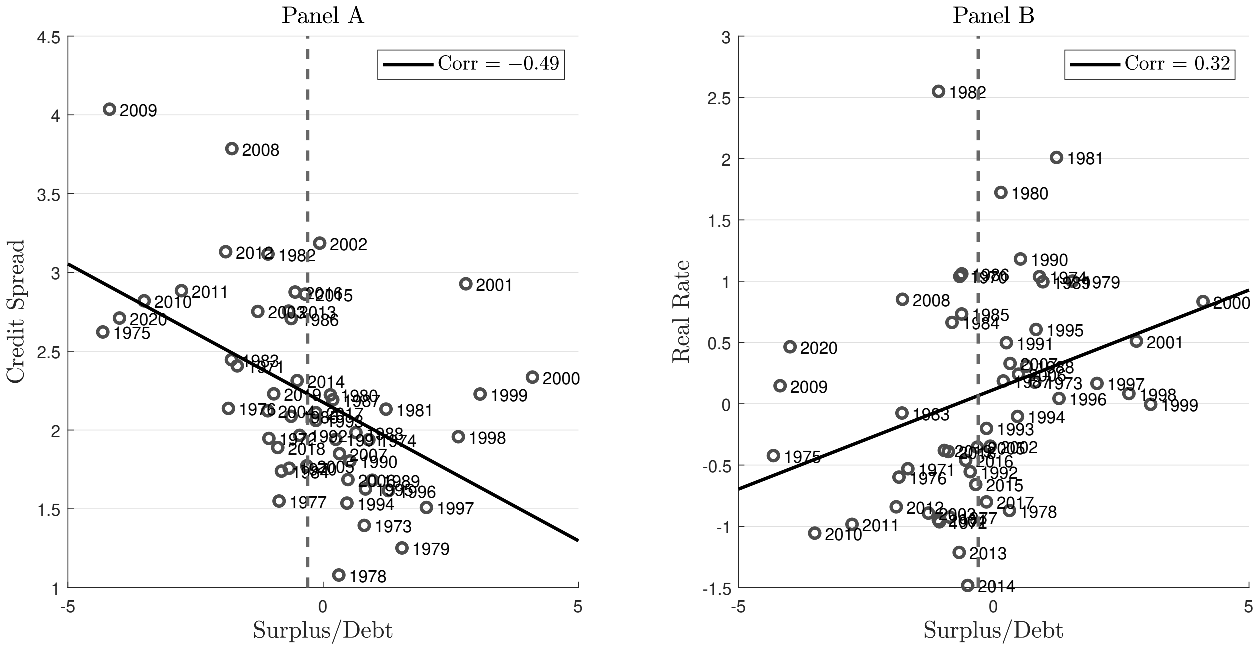

Figure 1: Scatter plots for measures of financial stress, real interest rate, and fiscal stance. Notes: Credit spread in Panel A is measured by annualized percentage spread between Baa corporate bond yield and 10-year Treasury constant maturity rate. For comparison ease, real rate in Panel B is non-annualized and adjusted for economic growth. Surplus-debt ratio is given by primary surplus as a percentage of market value of privately held debt. Solid lines denote linear least-squares regressions. Dashed lines indicate sample mean (−0.3%) of surplus-debt ratio. All data are aggregated by quarterly average to annual frequency.

1 Introduction

I begin by pointing to recent U.S. data that motivate this article. The debt-GDP ratio has nearly tripled since the new millennium. During that period, short-term interest rates have averaged well below 2%. Despite accelerating debt levels and unusually low rates, inflation by any measure has remained benign and averaged less than 2% until the post-pandemic era. Meanwhile, long-term Treasury yields have been trending down, suggesting that financial markets do not expect inflation to rise.[1] Why haven't these been inflationary over the past two decades? As Leeper (2017) puts it: "*In a phrase: bond-market pessimism. During the financial crisis, there was a world-wide flight to safety; investors have an insatiable appetite for Treasuries.*" That ensures the demand for bonds more than absorbs its expanding supply. But what happens to inflation if such an appetite is quenched? In a word: goods-market euphoria. With rising interest receipts and no anticipation of higher offsetting taxes, bond holders feel wealthier and demand more goods and services. At the time of writing, inflation soared over 9% at its peak and showed little signs of cooling off for the foreseeable future.

This article studies the state-dependent properties of inflation and the stance of fiscal policy while recognizing that investors value the extreme safety and liquidity services of U.S. Treasuries. Before I present the formal results, it is useful to look at some unstructured data summary of

[1] Japan shares a similar experience.

fiscal financing over time. Sims (2011) emphasizes the most natural single measure of fiscal stance is the primary surplus as a proportion of the outstanding debt, which must average out to the real interest rate. On the other hand, as shown in Krishnamurthy and Vissing-Jorgensen (2012), the credit spread between Treasuries and assets that do not share the same extent of safety and liquidity should reflect investors' desire for these attributes offered by Treasuries. Figure 1 displays prima facie evidence for these assertions. When the economic condition deteriorates, investors' demand for the safety and liquidity services of Treasuries is high. As a result, the Treasury yield is low relative to (say) the corporate bond yield, allowing the fiscal authority to expand deficits at a low (or even negative) real cost. This happened during the 1970s to early 1980s and the Great Recession of 2007–2009 when both the credit spread and deficit-debt ratio surged. The opposite applies when the economic condition improves, such as the 1990s. The overall fiscal stance, though, seems profligate as the surplus-debt ratio has averaged at negative 0.3%.

The rest of the paper is planned as follows. Section 2 establishes a set of stylized patterns of inflation and fiscal stance that characterizes long-term U.S. debt cycles. Section 3 provides a structural interpretation using a simple analytical model of price level determination. Section 4 concludes.

2 Navigating Big Debt Cycles

While a particular pattern may reoccur in a subsequent episode, it is neither predestined to repeat in exactly the same manner nor to take exactly the same amount of time. To this end, I estimate a Bayesian vector autoregression (VAR) model subject to multiple change points with non-recurrent state transitions.[2] Consider the following pth-order VAR model for the $n \times 1$ observable vector y_t

$$y_t = \Phi_{0,s_t} + \Phi_{1,s_t} y_{t-1} + \cdots + \Phi_{p,s_t} y_{t-p} + u_t, \qquad u_t \sim \mathbb{N}(0, \Sigma_{s_t}) \tag{2.1}$$

where u_t is a vector of one-step-ahead forecast errors that follows a multivariate Gaussian distribution conditional on past observations of y_t, and $s_t \in \{1, \ldots, m+1\}$ is a latent discrete state variable that indicates the population from which y_t has been drawn. To produce change-point dynamics, I follow Chib (1998) and assume that s_t evolves according to a discrete-state Markov chain with its transition probability matrix constrained such that s_t can either stay at the current value or jump to the next higher value

$$P = \begin{bmatrix} p_{11} & 1-p_{11} & 0 & \cdots & 0 & 0 \\ 0 & p_{22} & 1-p_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & p_{mm} & 1-p_{mm} \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{2.2}$$

[2] I thank Siddhartha Chib for suggesting the use of change-point models.

Table 1: Posterior estimates of state-dependent population means

	State-1: 1970–1987		State-2: 1988–2001		State-3: 2002–2020	
Observables	Mean	70% HPD	Mean	70% HPD	Mean	70% HPD
GDP growth	0.29	$[0.21, 0.45]$	0.67	$[0.06, 1.02]$	0.13	$[0.05, 0.31]$
Inflation	1.45	$[1.25, 1.71]$	0.63	$[0.39, 1.15]$	0.42	$[0.36, 0.51]$
Interest rate	2.52	$[1.72, 2.49]$	1.50	$[0.55, 2.55]$	-0.04	$[-0.26, 0.31]$
Surplus/debt	-0.25	$[-1.01, -0.22]$	1.73	$[-6.61, 6.99]$	-1.89	$[-2.26, -0.86]$
Credit spread	0.52	$[0.44, 0.53]$	0.49	$[0.09, 0.80]$	0.72	$[0.62, 0.77]$

NOTES: For each VAR parameter draw, I compute the implied state-dependent population means for observables. Posterior means and 70% highest probability density (HPD) intervals are computed using 10,000 posterior draws. Interest rate is measured by the shadow federal funds rate and thus can drop into negative territory. All observables are non-annualized.

where p_{ii} is the probability of remaining at state-i from one period to the next. The chain begins in state-1 at time $t = 1$ and the terminal state is $m + 1$. Note that the state transitions identify m change points $\{\tau_1, \ldots, \tau_m\}$: the ith change occurs at τ_i if $s_{\tau_i} = i$ and $s_{\tau_i+1} = i + 1$.

I consider a parsimonious specification with $p = 2$ lags, $m = 2$ change points (or equivalently, three distinct states), and $n = 5$ quarterly observables, including GDP growth rate, inflation rate, nominal interest rate, surplus-debt ratio, and credit spread. Appendix A contains details about the data construction and estimation procedure. Table 1 reports the estimated population means for the observables conditional on each state. Panel A of Figure 2 depicts the debt-GDP ratio (left axis) against the posterior probability series for each state (right axis) over the full sample 1970–2020. It clearly identifies that the first break occurs at 1987 or so and the second at around 2001. Accordingly, the model uncovers three fiscal states that fall more broadly into two defining phases of a long-term debt cycle—phase-D and phase-S—as described below.

Phase-D ("D" for "deficit") captures the common properties of state-1 (1970–1987) and state-3 (2002–2020). It prevails when the fiscal stance becomes extravagant and the economy experiences a prolonged period of fiscal imbalance (left axis of Panel B). The resulting increases in the debt-GDP ratio will unfold over many years, typically starting small and then gaining momentum. Prominent fiscal events in this phase include: the Ford tax cut and tax rebate of 1975 when the surplus-debt ratio reached its postwar low; the Reagan Economic Recovery Plan of 1981 when the debt-GDP ratio was picking up from its postwar low in the early 1980s; the Bush Economic Growth and Tax Relief Reconciliation Act of 2001 and the Jobs and Growth Tax Relief Reconciliation Act of 2003 when the debt-GDP ratio was rising again in the early 2000s; and more recently, the Obama American Recovery and Reinvestment Act of 2009 and the Trump Coronavirus Aid, Relief, and Economic Security Act of 2020 when the debt-GDP ratio was accelerating.

Phase-D also features elevated financial distress as can be seen from several spikes in the credit spread under state-1 and state-3 (right axis of Panel B), implying a high market demand for Treasuries. As alluded to earlier, fiscal inflation did not materialize over the past two decades

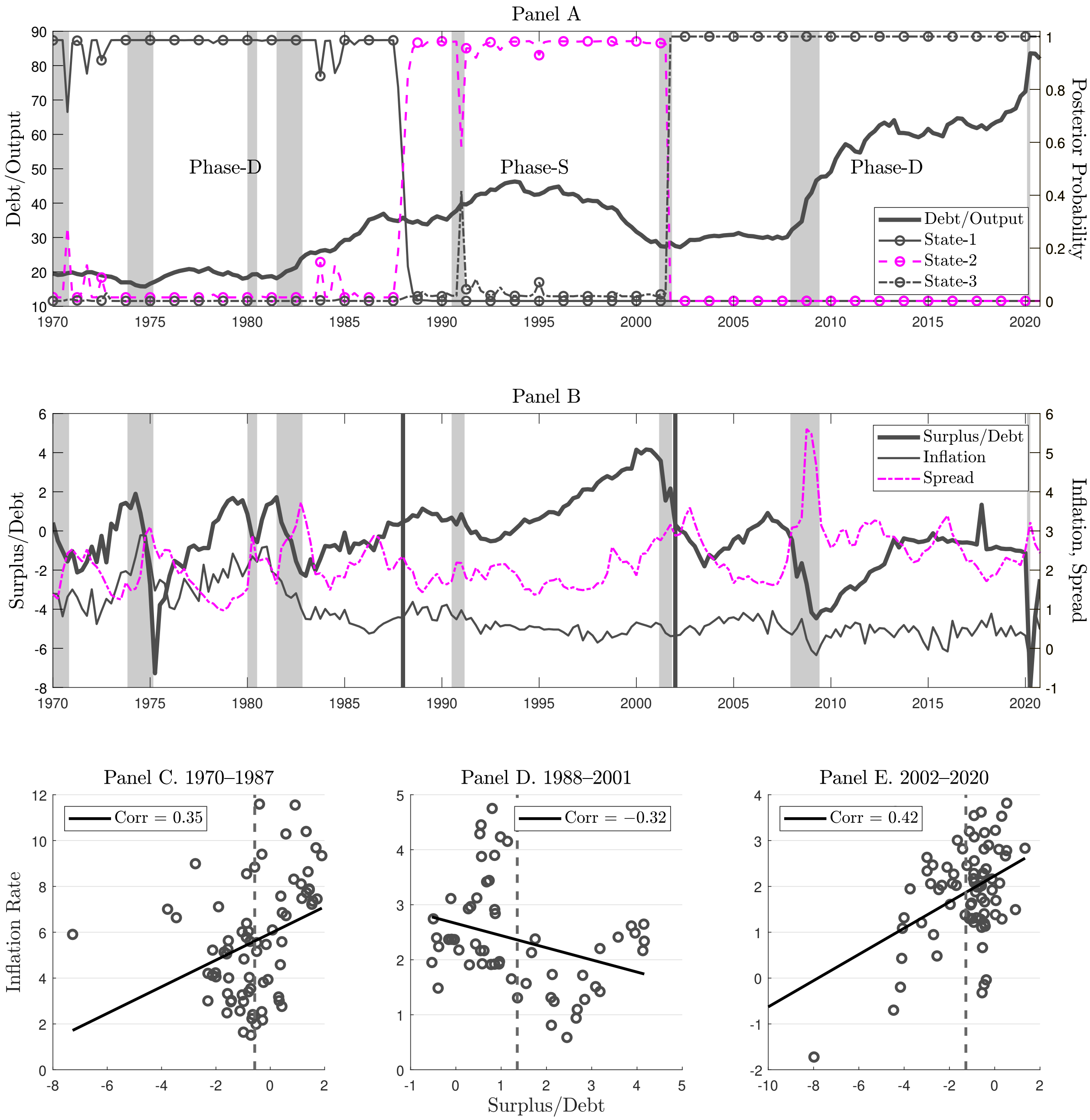

Figure 2: Debt cycles, fiscal stance, and inflation. Notes: Panel A plots debt-GDP ratio (left axis, GDP is annualized) against posterior probability for each state (right axis). For comparison ease, Panel B plots surplus-debt ratio (left axis) against non-annualized inflation and annualized credit spread (right axis). Vertical lines delineate change points. Shaded bars indicate NBER dated recessions. Panels C–E plot subsample patterns between surplus-debt ratio and annualized inflation.

because investors' unquenchable demand for Treasuries soaked up any of its excess supply. While the long-run inflation can take both high and low values (see Table 1), the short-run inflation turns out to be positively correlated with the underlying fiscal stance (Panels C and E) in phase-D, which is somewhat contrary to the conventional belief.

Phase-S ("S" for "surplus"), on the other hand, summarizes the properties of state-2 (1988–

2001). It is in place when the fiscal authority is virtuous and the economy experiences a period of sustained and growing primary surpluses, such as the latter part of the 1990s during the Clinton administration. In contrast to phase-D, it features on average higher output growth, modest inflation and interest rate, stabilized debt-GDP ratio, and lower credit spread (see Table 1), suggesting a low market demand for Treasuries. Moreover, the short-run inflation is negatively correlated with the prevailing fiscal stance (Panel D).

In a nutshell, an archetypal debt cycle consists of alternating phases of persistent deficits (phase-D) and surpluses (phase-S) *joint* with alternating patterns of inflation and fiscal stance. In line with these key features found in the data, I present a structural model that draws on the fiscal theory of the price level (FTPL) in the next section [see Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (1999) for early FTPL contributions]. This model is simple enough to admit an analytical characterization, yet rich enough to illustrate the general mechanism at work. It addresses the following positive questions: If inflation depends importantly on the fiscal behavior and financial market conditions, what coordination of monetary and fiscal policy can jointly determine inflation and stabilize debt? Can permanent fiscal deficits be sustained in the long run? And how does inflation respond to monetary and fiscal policy shocks?

3 A Structural Interpretation

The model's essential elements include: an infinitely lived representative household endowed each period with an amount of non-durable goods $Y_t = g^t y$, which grow at a constant rate g; a cashless economy with government-issued nominal one-period bonds B_t that sell at price $1/R_t$, where R_t is the monetary policy instrument; the household has a time-varying preference for the various benefits derived from the safety and liquidity services of government debt [see, e.g., Fisher (2015)]; the price level P_t is defined as the rate at which bonds exchange for goods; lump-sum taxes (net of transfer payments) T_t and zero government spending so that consumption equals output, $C_t = Y_t$, and net taxes equal primary surpluses, $T_t = S_t$; a monetary authority and a fiscal authority.

3.1 Steady-State Analysis

The household chooses a sequence of consumption and bonds $\{C_t, B_t\}$ to maximize the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, b_t)$$

subject to the period-by-period budget constraint

$$P_t C_t + \frac{B_t}{R_t} + T_t = P_t Y_t + B_{t-1}$$

taking P_t and B_{t-1} as given. Here $\beta \in (0, 1)$ is the discount factor, $c_t = C_t/g^t$ is the detrended real consumption, $b_t = B_t/(g^t R_t P_t)$ is the detrended real debt at the beginning of period t, and $\mathbb{E}_t$ represents the conditional expectation given information available at time t. Since they are inconsequential for the main result but ease the algebraic exposition, I assume that consumption and debt are additively separable in the preference, i.e., $U(c_t, b_t) = u(c_t) + v(b_t)$, where $u(c_t) = \ln c_t$, $v(b_t) = \chi_t \ln b_t$, and $\chi_t \geqslant 0$ is a scalar that determines the steady-state debt-output ratio.[3]

The private sector's optimization problem leads to the following first-order condition

$$\mathbb{E}_t[M_{t+1,t}R_t] = 1, \qquad M_{t+1,t} = \beta \frac{1}{1 - v'(b_t)/u'(c_t)} \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{g\pi_{t+1}}$$

where $M_{t+1,t}$ is the stochastic discount factor for pricing any nominal contingent claims, and $\pi_t = P_t/P_{t-1}$ is the inflation between periods $t-1$ and t. Let a variable without a time subscript denote its steady-state value. Imposing the goods market clearing condition $c = y$, the steady-state real interest rate is given by

$$r = \frac{R}{\pi} = \frac{g\lambda}{\beta}, \qquad \lambda = 1 - \frac{\chi}{b/y} \tag{3.1}$$

where b/y is the debt-output ratio, and $\lambda = R/R^* \in (0, 1]$ is the discount on the nominal interest rate R relative to its value $R^* = g\pi/\beta$ in the absence of a preference for holding risk-free government bonds (i.e., $\chi = 0$). It follows from (3.1) that a lower value of λ (or higher χ) raises the marginal benefits of saving and thus increases the household's demand for safety and liquidity. In particular, when λ is smaller than the household's time discount rate, this will drive the steady-state real return on debt below the economy's growth rate, i.e., $r < g$ if $\lambda < \beta$.

Turning to the public sector, any policy choice must satisfy the flow government budget constraint

$$b_t + s_t = \frac{b_{t-1}R_{t-1}}{g\pi_t}$$

where $s_t = S_t/(g^t P_t)$ is the detrended real primary surplus. It follows that the steady-state surplus-debt ratio equals to the growth-adjusted net real interest rate

$$\frac{s}{b} = \frac{r}{g} - 1 = \frac{\lambda}{\beta} - 1 \tag{3.2}$$

where the second equality holds due to (3.1). Particularly, insofar as this real return on debt stays negative (i.e., $\lambda < \beta$), the real value of debt becomes self-stabilizing even without the fiscal authority running any budge surplus.

Some long-run implications of (3.1)–(3.2) for the observables are worth highlighting. First, the actual surplus-debt ratio must approximately average to the net real return on debt over time. Second, a high market valuation of Treasuries may push the real interest rate below the GDP growth rate, making it possible for the fiscal authority to run persistent deficits at a low real cost. Third, for a given choice of the policy instruments (R, s), a stronger preference for Treasuries can effectively alleviate the inflationary pressure of rising debt-GDP ratio even with a lack of fiscal

[3] The function $v(\cdot)$ captures the household's desire for short-term Treasuries in addition to her intertemporal substitution and risk aversion motives. The nominal debt is deflated by the price because she cares about its real value.

Table 2: Calibrated stead states and implied parameter values

Steady states	State-1: 1970–1987	State-2: 1988–2001	State-3: 2002–2020
s/b, surplus-debt ratio	−0.0025	0.0173	−0.0189
b/y, debt-GDP ratio	0.8918	1.5428	2.0040
λ, discount on interest rate	0.9776	0.9970	0.9615
χ, preference for Treasuries	0.0200	0.0047	0.0772
Credit spread	0.52%	0.49%	0.72%
γ^*, determinacy boundary	8.3333	1.1765	1.0811

NOTES: $\beta = 0.98$ is used in calibration. Steady states of surplus-debt ratio and credit spread correspond to their posterior estimates reported in Table 1. GDP is non-annualized in debt-GDP ratio.

discipline. These implications find empirical support as documented earlier. Lastly, the ranking of the calibrated values of χ aligns well with that of the credit spread estimates (see Table 2).

3.2 MODEL DYNAMICS

Let $\hat{x} \equiv (x_t - x)/x$ denote the percent deviation of a variable x_t from its steady state x. A linear approximation to the model's equilibrium conditions leads to the following bare-bones FTPL equations. The household's optimizing behavior implies a demand curve

$$(1-\lambda)(\hat{b}_t - \epsilon_t^b) = \lambda(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}) \tag{3.3}$$

linking the real debt $\hat{b}_t$ to the real rate $\hat{r}_t = \hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}$, where $\hat{\chi}_t = \epsilon_t^b$ is a bond preference shock. The monetary authority reacts to inflation

$$\hat{R}_t = \alpha\hat{\pi}_t + \epsilon_t^R \tag{3.4}$$

where $\alpha \geqslant 0$ and ϵ_t^R is a monetary policy shock. In addition, the fiscal authority responds to lagged debt

$$\hat{s}_t = \gamma\hat{b}_{t-1} + \epsilon_t^s \tag{3.5}$$

where ϵ_t^s is a fiscal policy shock. Cautions are needed when interpreting (3.5): when the steady-state budget is positive (i.e., $s > 0$ if $\lambda > \beta$), $\hat{s}_t$ refers to the percent deviation in surplus and $\gamma \geqslant 0$; otherwise, $\hat{s}_t$ represents the percent deviation in deficit and $\gamma \leqslant 0$. Lastly, the government budget constraint is linearized as

$$\hat{b}_t = (\lambda/\beta)(\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t) - (\lambda/\beta - 1)\hat{s}_t \tag{3.6}$$

where the real value of outstanding debt at the beginning of period t, $\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t$, is determined in equilibrium at time t.

Together with the steady-state equilibrium relations (3.1)–(3.2), equations (3.3)–(3.6) constitute a system of expectational difference equations in the variables $\{\hat{\pi}_t, \hat{b}_t, \hat{R}_t, \hat{s}_t\}$. In the absence of bond preference ($\lambda = 1$), the steady-state budget is always in surplus, and the monetary and fiscal authorities can coordinate to determine inflation and stabilize debt in two manners. Under the conventional monetarist paradigm, active monetary policy controls inflation by raising nominal interest rate more than one for one with inflation ($\alpha > 1$), and passive fiscal policy stabilizes government debt by sufficiently adjusts primary surpluses ($\gamma > 1$). Under the fiscal theory, policy roles are completely reversed with passive monetary policy stabilizing debt by responding weakly to inflation ($0 \leqslant \alpha < 1$), and active fiscal policy determining inflation by reacting weakly to debt ($0 \leqslant \gamma < 1$). In what follows, I focus on the model dynamics under the fiscal theory with bond preference ($0 < \lambda < 1$) because the overall fiscal stance in the U.S. seems profligate.

Assume that all shocks $(\epsilon_t^R, \epsilon_t^s, \epsilon_t^b)$ are mutually and serially uncorrelated with bounded supports. To simplify the exhibition, substitute (3.4)–(3.5) into (3.3) and (3.6) to eliminate $(\hat{R}_t, \hat{s}_t)$. The following proposition characterizes the local determinacy and closed-form solution for $(\hat{\pi}_t, \hat{b}_t)$.

Proposition 3.1. *For $\lambda \in (0, 1]$, define $\gamma^* = (1 - \beta)/|\lambda - \beta|$. Given a passive monetary policy $\alpha \in [0, 1)$, there exist two disjoint regions of the parameter space (λ, γ) within which the model admits a locally unique, bounded equilibrium:*

1. *Region-D (permanent deficits): $\lambda \in (0, \beta)$ and $\gamma \in (-\gamma^*, 0]$;*
2. *Region-S (permanent surpluses): $\lambda \in (\beta, 1]$ and $\gamma \in [0, \gamma^*)$.*

Under both regions, the unique equilibrium inflation and real debt take the form of

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} \frac{L}{1-\alpha L} & -\frac{|\beta/\lambda - 1|}{1-\alpha L} & \frac{\lambda-1}{\lambda(1+\gamma\beta-\gamma\lambda)}\frac{\beta-(\lambda+\gamma\beta-\gamma\lambda)L}{1-\alpha L} \\ 0 & 0 & \frac{1-\lambda}{1+\gamma\beta-\gamma\lambda} \end{bmatrix} \begin{bmatrix} \epsilon_t^R \\ \epsilon_t^s \\ \epsilon_t^b \end{bmatrix} \tag{3.7}$$

where L denotes the lag operator, i.e., $L^k \hat{x}_t = \hat{x}_{t-k}$, and ϵ_t^s refers to a primary surplus shock.

Several remarks about Proposition 3.1 are in order. First, inflation follows an autoregressive moving average (ARMA) process of order $(1, 1)$, and a passive monetary policy ensures that its autoregressive component remains stable. On the other hand, real debt only responds to the bond preference shock because its real return $\hat{r}_t$ is unaffected by the policy shocks in this endowment environment. Second, consistent with phase-D (resp. phase-S) of a debt cycle, the steady-state equilibrium features a negative (resp. positive) real return on debt with permanent deficits (resp. surpluses) under the determinacy region-D (resp. region-S). Third, compared to the special case of $\lambda = 1$, the calibrated values of λ extend the determinacy boundary beyond unity (i.e., $\gamma^* > 1$), implying that a broader range of policy responses are indeed consistent with an active fiscal policy (see Table 2).[4] Although not explored here, it follows that an active monetary policy would otherwise require a more-passive-than-usual fiscal policy in place. See Appendix B for a proof of the proposition.

[4] The model solution is also determinate when $\gamma < -\gamma^*$ if $\lambda < \beta$, and when $\gamma > \gamma^*$ if $\lambda > \beta$, where $\gamma^* = (1 + \beta)/|\lambda - \beta|$ becomes unrealistically large under the calibrated values of λ.

To understand how exogenous shocks are transmitted to influence endogenous variables, it is useful to rewrite the ARMA solution (3.7) as linear combinations of all past and present shocks. Without loss of generality, I consider a special case of active fiscal policy that sets primary surpluses or deficits exogenously, i.e., $\gamma = 0$. Then inflation follows

$$\hat{\pi}_t = \sum_{k=1}^{\infty} \underbrace{\alpha^{k-1}}_{\geqslant 0} \epsilon^R_{t-k} + \sum_{k=0}^{\infty} \underbrace{-|\beta/\lambda - 1|\alpha^k}_{\leqslant 0} \epsilon^s_{t-k} + \underbrace{(\lambda - 1)(\beta/\lambda)}_{\leqslant 0} \epsilon^b_t + \sum_{k=1}^{\infty} (\lambda - 1)(\alpha\beta/\lambda - 1)\alpha^{k-1}\epsilon^b_{t-k} \tag{3.8}$$

and real debt follows

$$\hat{b}_t = \underbrace{(1 - \lambda)}_{\geqslant 0} \epsilon^b_t. \tag{3.9}$$

The economic interpretations of (3.8)–(3.9) arise from a ubiquitous relation in any dynamic macro model—the real value of government liabilities must equal to the present value of current and expected future primary surpluses

$$\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t = |1 - \beta/\lambda| \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t \hat{s}_{t+k} \tag{3.10}$$

where $\hat{b}_{t-1}$ and $\hat{R}_{t-1}$ are predetermined in period t.[5] This relation holds in any determinate bounded equilibrium, regardless of the policy rules in place.

Given an exogenous path for primary surpluses $\{\hat{s}_t\}$, (3.10) makes clear that an increase in government debt generates a positive wealth effect which in turn transmits into higher inflation. For example, consider the effects of a monetary contraction. A positive realization of ϵ^R_t lowers the price of newly-issued bonds at period t and induces the household to substitute consumption into more bond. But this substitution effect is entirely offset by the wealth effect due to the household's anticipation of no future tax increases to finance the rising government debt. As a result, the contemporaneous aggregate demand and hence inflation stay unchanged as in (3.8). In the absence of future fiscal adjustments, however, inflation must rise in the subsequent periods to devalue the nominal government debt so as to ensure its sustainability, which is evident from (3.10).

The preceding wealth effect channel triggers a similar inflationary impact under a fiscal expansion. From (3.10) it can be seen that a deficit-financed fiscal stimulus (i.e., a negative realization of ϵ^s_t) engenders higher current inflation as in (3.8). But why fiscal inflation has largely remained nil in the data over the past two decades? Recall from Figure 1 that periods of fiscal stress are oftentimes accompanied by tightened credit conditions. Now flight to safety comes into play. A shift in the risk appetite towards safety and liquidity (i.e., a positive realization of ϵ^b_t) induces investors to happily hold onto more Treasuries as in (3.9). As such the aggregate demand for goods decreases and

[5] Equation (3.10) is an *equilibrium* condition that can be obtained by substituting the consumption-Euler equation (3.3) into the government budget constraint (3.6) and iterating forward.

hence inflation falls as in (3.8). Under the determinacy region-D (i.e., with lower values of λ), this deflationary effect may outweigh fiscal inflation, thereby producing a positive correlation between inflation and fiscal stance that characterizes phase-D of a debt cycle. The opposite correlation may arise under the determinacy region-S (i.e., with higher values of λ), a pattern reminiscent of phase-S of a debt cycle.

4 Concluding Remarks

Measures of U.S. fiscal stance reveals that the federal government has on average remained profligate. Yet a maintained assumption in many existing models of monetary and fiscal policy is that the government must run budget surplus in the long run. Several recent papers have proposed to relax this assumption, but few provided ample empirical relevance for the mechanisms established therein [see, e.g., Miao and Su (2021) and Brunnermeier, Merkel and Sannikov (2021), among others]. Using a change-point VAR model, this article estimates the state-dependent properties of U.S. inflation and its fiscal stance while recognizing that investors value the extreme safety and liquidity services provided by Treasuries [see also Kliem, Kriwoluzky and Sarferaz (2016)]. A number of stylized features that characterize long-term debt cycles are obtained. Particularly, an archetypal debt cycle consists of alternating phases of persistent deficits and surpluses together with alternating patterns of inflation and fiscal stance. A related study that inspires this article is Dalio (2022), who categorizes big debt crises into two types—deflationary and inflationary—and provides a framework for understanding their mechanics.

In line with the estimated properties from the data, I provide a structural interpretation based on a simple FTPL-type model where the household has a preference for holding government bonds. A strong appetite for bonds can drive the steady-state real interest rate below the economy's growth rate, allowing the fiscal authority to run permanent deficits in the long run without posing an inflationary threat. When a standard passive monetary policy is in place, the determinacy region of this model admits a broad range of active fiscal policy that maps into different phases of a debt cycle. The closed-form solution derived herein is useful for understanding why fiscal inflation has largely remained benign over the past two decades. A quantitative assessment of the policy effects on inflation in a financially constrained environment requires a more elaborate model, such as that of Li, Pei and Tan (2021), which is deferred to a sequel to this article.

Appendix

A Data and Estimation

Unless otherwise stated, the following data are drawn from the National Income and Product Accounts (NIPA) released by the Bureau of Economic Analysis. All data in levels from NIPA are nominal values and divided by 4. The quarterly observable sequences in the main text are constructed for 1969:Q3–2020:Q4 as follows.

1. Per capita real output growth rate. Per capita real output is obtained by dividing gross domestic product (Table 1.1.5, line 1) by civilian non-institutional population (series "CNP16OV", Federal Reserve Economic Data, St. Louis Fed) and deflating using implicit price deflator for gross domestic product (Table 1.1.9, line 1). Growth rates are computed using the quarter-to-quarter log difference and converted into percentage by multiplying by 100.

2. Inflation rate is defined as the quarter-to-quarter log difference of implicit price deflator for gross domestic product and converted into percentage by multiplying by 100.

3. Nominal interest rate corresponds to shadow federal funds rate (Federal Reserve Bank of Atlanta) and is converted into non-annualized percentage by dividing by 4. Growth-adjusted real interest rate in Panel B of Figure 1 is computed by subtracting output growth rate and inflation rate from nominal interest rate.

4. Surplus-debt ratio. Primary surplus is defined as the sum of government's net lending or net borrowing (Table 3.2, line 49) and interest payments (Table 3.2, line 33). Government debt is market value of privately held gross federal debt (Federal Reserve Bank of Dallas). Debt-GDP ratio in Panel A of Figure 2 is given by government debt over annualized nominal output. All ratios are converted into percentage by multiplying by 100.

5. Credit spread is measured by the difference between Baa corporate bond yield and 10-year Treasury constant maturity rate (series "BAA" and "DGS10", respectively, Federal Reserve Economic Data, St. Louis Fed) and converted into non-annualized percentage by dividing by 4.

Throughout this appendix, I employ the following notation. Construct the VAR data and coefficient matrices

$$\underset{k\times 1}{x_t} = \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix}, \qquad \underset{t\times k}{X_t} = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_t' \end{bmatrix}, \qquad \underset{t\times n}{Y_t} = \begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_t' \end{bmatrix}, \qquad \underset{k\times n}{\Phi_{s_t}} = \begin{bmatrix} \Phi_{0,s_t}' \\ \Phi_{1,s_t}' \\ \vdots \\ \Phi_{p,s_t}' \end{bmatrix}$$

where $k = 1 + np$ and $t = 1, \ldots, T$. Let $S_t = (s_1, \ldots, s_t)$ and $S^{t+1} = (s_{t+1}, \ldots, s_T)$. For a given sequence of states S_T, collect the state-i observations from the full sample (X_T, Y_T) in the subsample $(\tilde{X}_i, \tilde{Y}_i)$, which has a sample size $\tilde{T}_i$, $i = 1, \ldots, m+1$. Also, let $\theta = \{\Phi_i, \Sigma_i\} \cup \{p_{ii}\}$ collect all the model parameters. In the Bayesian context, the posterior distribution $p(\theta|Y_T) \propto p(Y_T|\theta)p(\theta)$ is summarized by Markov chain Monte Carlo methods. In what follows, I augment the parameter space to include the unobserved states S_T and sample the joint posterior distribution $p(\theta, S_T|Y_T)$ with the following Gibbs steps.

1. For $i = 1, \ldots, m+1$, sample the VAR parameters (Φ_i, Σ_i) from

$$p(\Phi_i, \Sigma_i | Y_T, \{\Phi_j, \Sigma_j\}_{j \neq i}, \{p_{ii}\}, S_T) \propto p(\tilde{Y}_i | \Phi_i, \Sigma_i) \cdot p(\Phi_i, \Sigma_i). \tag{A.1}$$

   Note that some subsample $(\tilde{X}_i, \tilde{Y}_i)$ could be empty, i.e., $\tilde{T}_i = 0$. To avoid sampling from a degenerate state, I place a state-dependent training sample prior on (Φ_i, Σ_i). In my application of Section 2, I incorporate $\hat{T}_i = 20$ observations for each training sample $(\hat{Y}_i, \hat{X}_i)$: 1974:Q1–1978:Q4 for state-1, 1995:Q1–1999:Q4 for state-2, and 2008:Q1–2012:Q4 for state-3. Define the functions

$$\hat{\Phi}_i = (\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i'\hat{Y}_i,$$
$$\hat{\Sigma}_i = \frac{1}{\hat{T}_i}[\hat{Y}_i'\hat{Y}_i - \hat{Y}_i'\hat{X}_i(\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i'\hat{Y}_i].$$

   Let the prior of (Φ_i, Σ_i) be of the Normal-Inverse-Wishart form

$$\text{vec}(\Phi_i)|\Sigma_i \sim_d \mathbb{N}(\text{vec}(\hat{\Phi}_i), \Sigma_i \otimes (\hat{X}_i'\hat{X}_i)^{-1}),$$
$$\Sigma_i \sim_d \mathbb{IW}(\hat{T}_i\hat{\Sigma}_i, \nu, n)$$

   with $\nu \geqslant n$ degrees of freedom. Analogously, define the functions

$$\tilde{\Phi}_i = (\hat{X}_i'\hat{X}_i + \tilde{X}_i'\tilde{X}_i)^{-1}(\hat{X}_i'\hat{Y}_i + \tilde{X}_i'\tilde{Y}_i),$$
$$\tilde{\Sigma}_i = \frac{1}{\hat{T}_i + \tilde{T}_i}[(\hat{Y}_i'\hat{Y}_i + \tilde{Y}_i'\tilde{Y}_i) - (\hat{Y}_i'\hat{X}_i + \tilde{Y}_i'\tilde{X}_i)(\hat{X}_i'\hat{X}_i + \tilde{X}_i'\tilde{X}_i)^{-1}(\hat{X}_i'\hat{Y}_i + \tilde{X}_i'\tilde{Y}_i)].$$

   Since conditional on state-i, the prior and the likelihood are conjugate, the posterior of (Φ_i, Σ_i) is also of the Normal-Inverse-Wishart form

$$\text{vec}(\Phi_i)|\tilde{Y}_i, \Sigma_i \sim_d \mathbb{N}(\text{vec}(\tilde{\Phi}_i), \Sigma_i \otimes (\hat{X}_i'\hat{X}_i + \tilde{X}_i'\tilde{X}_i)^{-1}),$$
$$\Sigma_i|\tilde{Y}_i \sim_d \mathbb{IW}((\hat{T}_i + \tilde{T}_i)\tilde{\Sigma}_i, \nu + \tilde{T}_i, n).$$

2. For $i = 1, \ldots, m$, sample the transition probabilities p_{ii} from

$$p(p_{ii} | Y_T, \{\Phi_i, \Sigma_i\}, \{p_{jj}\}_{j \neq i}, S_T) \propto p(S_T | \{p_{jj}\}) \cdot p(p_{ii}). \tag{A.2}$$

   Let the prior of p_{ii} be Beta distributed, i.e., $\mathbb{B}(\alpha_i, \alpha_{i+1})$. In my application of Section 2, I set $\alpha_i = 8$ and $\alpha_{i+1} = 0.1$ for $i = 1, 2$, which a priori implies an expected duration of about 80 quarters for each of state-1 and state-2. Then the posterior of p_{ii} is also Beta distributed

$$p_{ii}|S_T \sim_d \mathbb{B}(\alpha_i + \beta_i, \alpha_{i+1} + 1)$$

   where β_i is the total number of one-step transitions from state-i to state-i in S_T.

3. Following Chib (1998), sample all the states S_T in reverse order from

$$p(S_T|Y_T,\theta) = p(s_T|Y_T,\theta)\cdots p(s_t|Y_T,S^{t+1},\theta)\cdots p(s_1|Y_T,S^2,\theta) \tag{A.3}$$

where $s_1 = 1$ and $s_T = m+1$. For $t = 1,\ldots,T-1$, the typical term in the above density is given by

$$p(s_t|Y_T,S^{t+1},\theta)\propto p(s_t|Y_t,\theta)\cdot p(s_{t+1}|s_t,\{p_{ii}\})$$

where s_t takes only two values conditional on s_{t+1}. The first density $p(s_t|Y_t,\theta)$ can be obtained using the recursive procedure below:

(a) Initialization. Set $p(s_1|Y_0,\theta)$ to be the mass distribution at $s_1 = 1$.

(b) Recursion. For $t = 1,\ldots,T$, first apply the updating step to obtain

$$p(s_t = j|Y_t,\theta)\propto p(y_t|Y_{t-1},\Phi_j,\Sigma_j)\cdot p(s_t = j|Y_{t-1},\theta)$$

for $j = 1,\ldots,m+1$. Then apply the forecasting step to obtain

$$p(s_{t+1} = j|Y_t,\theta) = \sum_{i=j-1}^{j} p_{ij}\cdot p(s_t = i|Y_t,\theta)$$

for $j = 1,\ldots,m+1$.

The above cycle is initialized at some starting values of (θ, S_T) and then repeated a large number of times, say $10,000$ iterations beyond a transient stage of $1,000$ iterations. Based on the draws from the joint posterior distribution, one can compute summary statistics such as posterior means and probability intervals. For example, the posterior probabilities $\mathbb{P}(s_t = i|Y_T)$ for $i = 1,2,3$ in Panel A of Figure 2 can be estimated by taking an average of $p(s_t = i|Y_{t-1},\theta)$ over the posterior draws of θ.

B Proof of Proposition

The solution procedure closely follows Tan (2017) where I derive an analytical solution to new Keynesian models under the fiscal theory. To simplify the exhibition, we substitute the policy rules

(3.4)–(3.5) for $(\hat{R}_t, \hat{s}_t)$ and rewrite the resulting system in the following form

$$\left(\underbrace{\begin{bmatrix}\lambda & 0\\ 0 & 0\end{bmatrix}}_{\Gamma_{-1}} L^{-1} + \underbrace{\begin{bmatrix}-\lambda\alpha & 1-\lambda\\ \lambda/\beta & 1\end{bmatrix}}_{\Gamma_0} L^0 + \underbrace{\begin{bmatrix}0 & 0\\ -\lambda\alpha/\beta & \gamma(\lambda/\beta-1)-\lambda/\beta\end{bmatrix}}_{\Gamma_1} L\right)\underbrace{\begin{bmatrix}\hat{\pi}_t\\ \hat{b}_t\end{bmatrix}}_{\hat{x}_t}$$
$$= \left(\underbrace{\begin{bmatrix}\lambda & 0 & 1-\lambda\\ 0 & 1-\lambda/\beta & 0\end{bmatrix}}_{\Psi_0} L^0 + \underbrace{\begin{bmatrix}0 & 0 & 0\\ \lambda/\beta & 0 & 0\end{bmatrix}}_{\Psi_1} L\right)\underbrace{\begin{bmatrix}\epsilon_t^R\\ \epsilon_t^s\\ \epsilon_t^b\end{bmatrix}}_{\epsilon_t} + \underbrace{\begin{bmatrix}\lambda & 0\\ 0 & 0\end{bmatrix}}_{\Gamma_{-1}}\underbrace{\begin{bmatrix}\eta_{t+1}^{\pi}\\ \eta_{t+1}^{b}\end{bmatrix}}_{\eta_{t+1}} \tag{B.1}$$

where $\{\Gamma_{-1}, \Gamma_0, \Gamma_1, \Psi_0, \Psi_1\}$ are matrix coefficients, and η_{t+1} is a vector of endogenous forecasting errors defined as $\eta_{t+1} = \hat{x}_{t+1} - \mathbb{E}_t\hat{x}_{t+1}$ so that $\mathbb{E}_t\eta_{t+1} = 0$.

Suppose a solution $\hat{x}_t = [\hat{\pi}_t, \hat{b}_t]'$ to (B.1) is of the form

$$\hat{x}_t = \sum_{k=0}^{\infty} C_k\epsilon_{t-k} \equiv C(L)\epsilon_t \tag{B.2}$$

where $\epsilon_t = [\epsilon_t^R, \epsilon_t^s, \epsilon_t^b]'$, $\hat{x}_t$ is taken to be covariance stationary, and $C(L)$ is a polynomial in the lag operator. Note that such moving average representation of the solution is very useful because it also leads to the impulse response function—the coefficient $C_k(i,j)$ measures exactly the response of $\hat{x}_{t+k}(i)$ to a shock $\epsilon_t(j)$.

Step 1: transform the time-domain system (B.1) into its equivalent frequency-domain representation. To this end, we evaluate the forecasting errors $\eta_{t+1} = [\eta_{t+1}^{\pi}, \eta_{t+1}^{b}]'$ using (B.2) and the Wiener-Kolmogorov optimal prediction formula

$$\eta_{t+1} = \left\{C(L)L^{-1} - \left[\frac{C(L)}{L}\right]_+\right\}\epsilon_t = C_0L^{-1}\epsilon_t \tag{B.3}$$

where $[\cdot]_+$ is the annihilation operator that ignores negative powers of L. Define $\Gamma(L) = \Gamma_{-1}L^{-1} + \Gamma_0 + \Gamma_1 L$ and substitute (B.2) and (B.3) into (B.1)

$$\Gamma(L)C(L)\varepsilon_t = (\Psi_0 + \Psi_1 L + \Gamma_{-1}C_0L^{-1})\varepsilon_t$$

which must hold for all realizations of ε_t. Therefore, the coefficient matrices are related by the z-transform identities

$$z\Gamma(z)C(z) = z\Psi_0 + z^2\Psi_1 + \Gamma_{-1}C_0$$

where z is a complex variable. In solving for $C(z)$, ideally one would multiply both sides by $(z\Gamma(z))^{-1}$, but $C(z)$ needs to have only non-negative powers of z by (B.2) and be analytic inside the unit circle so that its coefficients are square-summable by covariance stationarity. This requirement can be examined by a careful decomposition of $z\Gamma(z)$ in the next step.

Step 2: apply the Smith canonical decomposition to the polynomial matrix $z\Gamma(z)$

$$z\Gamma(z) = \underbrace{U(z)^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & z(z-z_-)\end{bmatrix}}_{S(z)}\underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & z-z_+\end{bmatrix}V(z)^{-1}}_{T(z)}$$

which factorizes all roots inside the unit circle from those outside and collects them in the diagonal polynomial matrix $S(z)$. Here $U(z)$ and $V(z)$ are unimodular matrices, $z_- = \beta/(1+\gamma\beta-\gamma\lambda)$, and $z_+ = 1/\alpha$. Given the parameter constraints for determinacy in Proposition 3.1, both roots are real, one inside the unit circle, $|z_-| < 1$, and one outside, $|z_+| > 1$.

Step 3: examine the existence of solution. A covariance stationary solution exists if the elements of C_0 can be chosen to cancel those problematic roots in $S(z)$. To check that, multiply both sides of the z-transform identities by $S(z)^{-1}$

$$T(z)C(z) = \begin{bmatrix} U_{1\cdot}(z) \\ \frac{1}{z(z-z_-)}U_{2\cdot}(z)\end{bmatrix}(z\Psi_0 + z^2\Psi_1 + \Gamma_{-1}C_0)$$

where $U_{j\cdot}(z)$ is the jth row of $U(z)$. These identities are valid for all z on the open unit disk except for $z = 0, z_-$. But since $C(z)$ must be well-defined for all $|z| < 1$, this condition places the following restrictions on the unknown matrix coefficient C_0

$$U_{2\cdot}(z)(z\Psi_0 + z^2\Psi_1 + \Gamma_{-1}C_0)|_{z=0,z_-} = 0. \tag{B.4}$$

Stacking the restrictions in (B.4) yields[6]

$$-\underbrace{\begin{bmatrix}\lambda z_-^2/\beta & 0\end{bmatrix}}_{R}C_0 = \underbrace{\begin{bmatrix}0 & -(1-\lambda/\beta)z_-^2 & (1-\lambda)z_-^3/\beta\end{bmatrix}}_{A}.$$

Apparently, the solution exists if and only if the column space of R spans the column space of A, i.e. $\text{span}(A) \subseteq \text{span}(R)$, which is satisfied here. Solving for C_0 gives

$$\begin{bmatrix}C_0(1,1) & C_0(1,2) & C_0(1,3)\end{bmatrix} = \begin{bmatrix}0 & \beta/\lambda - 1 & (1-1/\lambda)z_-\end{bmatrix}$$

where $C_0(2,1)$ and $C_0(2,2)$ are left undetermined.

Step 4: examine the uniqueness of solution. In order for the solution to be unique, we must be able to determine $\{C_k\}_{k=0}^{\infty}$ from the parameter restrictions supplied by $-RC_0 = A$. It requires that from knowledge of RC_0 one be able to pin down $U_{2\cdot}(z)\Gamma_{-1}C_0$ evaluated at the reciprocals of roots

[6]Here we omit the restriction imposed by $z = 0$ because it is unrestrictive.

outside the unit circle. This is tantamount to verifying whether the columns of R' span the space spanned by the rows of

$$Q = U_{2\cdot}(z_+^{-1})\Gamma_{-1} = \begin{bmatrix} \frac{\alpha\lambda[(1-\lambda+\alpha\beta)z_- - \alpha^2\beta]}{\beta(1-\lambda)} & 0 \end{bmatrix}$$

i.e., $\text{span}(Q') \subseteq \text{span}(R')$, which is also satisfied here.

Now the unique solution can be computed as

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{b}_t \end{bmatrix} = \underbrace{[L\Gamma(L)]^{-1}(L\Psi_0 + L^2\Psi_1 + \Gamma_{-1}C_0)}_{C(L)} \begin{bmatrix} \epsilon_t^R \\ \epsilon_t^s \\ \epsilon_t^b \end{bmatrix} = \begin{bmatrix} \frac{L}{1-\alpha L} & \frac{\beta/\lambda - 1}{1-\alpha L} & \frac{\lambda-1}{\lambda(1+\gamma\beta-\gamma\lambda)} & \frac{\beta-(\lambda+\gamma\beta-\gamma\lambda)L}{1-\alpha L} \\ 0 & 0 & & \frac{1-\lambda}{1+\gamma\beta-\gamma\lambda} \end{bmatrix} \begin{bmatrix} \epsilon_t^R \\ \epsilon_t^s \\ \epsilon_t^b \end{bmatrix}$$

where ϵ_t^s refers to a primary surplus shock if $\lambda > \beta$; otherwise, ϵ_t^s represents a primary deficit shock. This completes the proof of Proposition 3.1.

References

Brunnermeier, M., S. Merkel, and Y. Sannikov (2021): "The Fiscal Theory of the Price Level with a Bubble," Working Paper.

Chib, S. (1998): "Estimation and comparison of multiple change-point models," *Journal of Econometrics*, 86(2), 221–241.

Cochrane, J. H. (1999): "A Frictionless View of U.S. Inflation," in *NBER Macroeconomics Annual 1998*, ed. by B. S. Bernanke, and J. J. Rotemberg, vol. 14, pp. 323–384. MIT Press, Cambridge, MA.

Dalio, R. T. (2022): *Principles for Navigating Big Debt Crises*. Avid Reader Press / Simon & Schuster.

Fisher, J. D. (2015): "On the Structural Interpretation of the Smets?Wouters "Risk Premium" Shock," *Journal of Money, Credit and Banking*, 47(2/3), 511–516.

Kliem, M., A. Kriwoluzky, and S. Sarferaz (2016): "On the Low-Frequency Relationship Between Public Deficits and Inflation," *Journal of Applied Econometrics*, 31(3), 566–583.

Krishnamurthy, A., and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120(2), 233–267.

Leeper, E. M. (1991): "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27(1), 129–147.

——— (2017): "Monetary-Fiscal Policy Interactions," Testimony before the Subcommittee on Monetary Policy and Trade, Committee on Financial Services, U.S. House of Representatives, Washington, D.C., July 20.

Li, B., P. Pei, and F. Tan (2021): "Financial distress and fiscal inflation," *Journal of Macroeconomics*, 70, 103353.

Miao, J., and D. Su (2021): "Fiscal and Monetary Policy Interactions in a Model with Low Interest Rates," Working Paper.

Sims, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4(3), 381–399.

——— (2011): "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970's," *European Economic Review*, 55(1), 48–56.

Tan, F. (2017): "An analytical approach to new Keynesian models under the fiscal theory," *Economics Letters*, 156, 133–137.

Woodford, M. (1995): "Price-Level Determinacy Without Control of a Monetary Aggregate," *Carneige-Rochester Conference Series on Public Policy*, 43, 1–46.