Part V: Advanced Topics

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The Road Ahead...

- DSGE models with Student-t shocks
- DSGE-VAR
- Regime-switching DSGE models
- Confronting model uncertainty

Student-t Shocks

- DSGE model solution

$$s_t = C(\theta) + G(\theta)s_{t-1} + M(\theta)\epsilon_t, \quad \epsilon_t \sim t_v(0, \Sigma_\theta)$$

  - $\epsilon_t = \lambda_t^{-1/2}\nu_t$ where $\lambda_t \sim \text{Gam}(v/2, v/2)$, $\nu_t \sim \mathbb{N}(0, \Sigma_\theta)$
  - Student-t captures what local solutions dismiss

- MCMC algorithm
  - step 1: sample $\theta | y_{1:T}, \lambda_{1:T}$ by TaRB-MH
  - step 2: sample $\nu_{1:T} | y_{1:T}, \theta, \lambda_{1:T}$ by disturbance smoother
  - step 3: sample $\lambda_{1:T} | y_{1:T}, \theta, \nu_{1:T}$ from

$$\lambda_t \sim \text{Gam}\left(\frac{v + n_\epsilon}{2}, \frac{v + \nu_t' \Sigma_\theta^{-1} \nu_t}{2}\right)$$

- Chib & Ramamurphy (2014), "*DSGE Models with Student-t Errors*", Econometric Reviews

DSGE-VAR

- VAR as likelihood

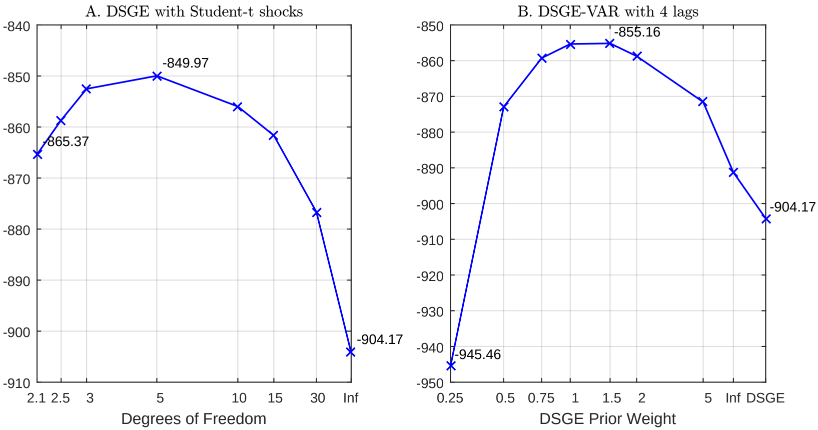
$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + u_t, \quad u_t \sim \mathbb{N}(0, \Sigma_u)$$

  - hierarchical prior $p_\lambda(\Phi, \Sigma_u, \theta) = p_\lambda(\Phi, \Sigma_u|\theta) p(\theta)$
  - both forecast well and usable for policy analysis
- MCMC algorithm
  - step 1: sample $p_\lambda(\theta|y_{1:T}) \propto p_\lambda(y_{1:T}|\theta) p(\theta)$ where

$$p_\lambda(y_{1:T}|\theta) = \frac{p(y_{1:T}|\Phi, \Sigma_u) p_\lambda(\Phi, \Sigma_u|\theta)}{p(\Phi, \Sigma_u|y_{1:T}, \theta)}$$

  - step 2: sample $p(\Phi, \Sigma_u|y_{1:T}, \theta)$ by standard methods
- Del Negro & Schorfheide (2004), "*Priors from General Equilibrium Models for VARs*", IER

Smets-Wouters Model



- 6-block 5,000 draws after 500 burn-in; original data 1966:Q1-2004:Q4
- Run time: DSGE $\approx$ 4h, VAR $\approx$ 1.5h, based on quad-core processor

Regime-Switching DSGE Models

Switching in state space form

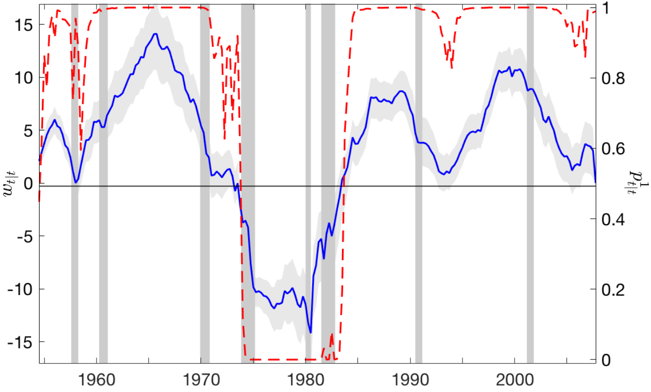
$$
\begin{aligned}
y_t &= D(s_t) + Z(s_t)x_t + \Sigma_u(s_t)^{1/2}u_t, \quad u_t \sim \mathbb{N}(0, \mathsf{I}_l) \\
x_t &= G(s_t)x_{t-1} + M(s_t)\Sigma_\epsilon(s_t)^{1/2}\epsilon_t, \quad \epsilon_t \sim \mathbb{N}(0, \mathsf{I}_n)
\end{aligned}
$$

- New regime switching
  - state process s_t driven by $s_t = 1\{w_t \geq \tau\}$
  - latent factor $w_t = \alpha w_{t-1} + v_t$ and

$$
\begin{pmatrix} \epsilon_t \\ v_{t+1} \end{pmatrix} \sim \mathbb{N}\left(\begin{pmatrix} 0_n \\ 0 \end{pmatrix}, \begin{pmatrix} \mathsf{I}_n & \rho \\ \rho' & 1 \end{pmatrix} \right)
$$

- Chang & Tan (2018), "*State Space Models with Endogenous Regime Switching*", manuscript

Extracted Regime Factor



- Regime factor (blue solid) and regime-1 probability (red dashed)
- Sluggish switching b/w more and less active regimes
- Timing and nature are consistent with narrative record

Confronting Model Uncertainty

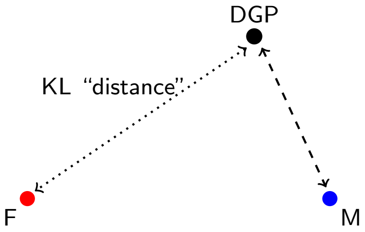


Fig. 1: Bayesian Model Averaging

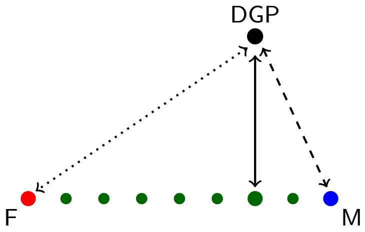


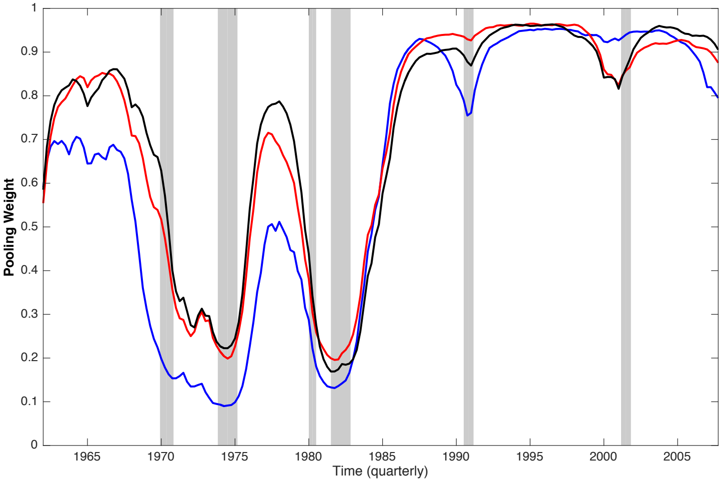
Fig. 2: Linear Prediction Pool

- Assumptions underlying model space
  - complete: either M or F is true, e.g. BEA
  - incomplete: neither M nor F is true, e.g. LPP

$$p(y_t|\lambda_t, y_{1:t-1}) = \lambda_t p_{\mathsf{M}}(y_t|y_{1:t-1}) + (1-\lambda_t)p_{\mathsf{F}}(y_t|y_{1:t-1})$$

- Del Negro Hasegawa & Schorfheide (2016), "*Dynamic Prediction Pools*", JoE

Regime-M Weight



- Pooling weight: $\lambda_t = \Phi(x_t)$, $x_t = (1-\rho)\mu + \rho x_{t-1} + \sqrt{1-\rho^2} v_t$
- Li & Tan (2018), "*Testing Monetary-Fiscal Regime: Some Caveats*", manuscript