An Analytical Approach to New Keynesian Models under the Fiscal Theory

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Abstract

This article illustrates a widely applicable frequency-domain methodology to solving multivariate linear rational expectations models. As an example, we solve a prototypical new Keynesian model under the assumption that primary surpluses evolve independently of government liabilities, a regime in which the fiscal theory of the price level is valid. The resulting analytical solution is useful in characterizing the cross-equation restrictions and illustrating the complex interaction between the fiscal theory and price rigidity. We also highlight some useful by-products of such method which are not easily obtainable for more sophisticated models using time-domain methods.

Keywords: solution methods; frequency domain; fiscal theory of price level.

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1 Introduction

This article builds on the seminal work, most notably of Hansen and Sargent (1980) and Whiteman (1983), that developed analytical approaches of integrating dynamic economic theories with econometric methods for the purpose of formulating and interpreting economic time series. We show that the frequency-domain methodology of Tan and Walker (2015) to solving linear rational expectations models, who generalized its predecessors to the multivariate setting, is widely applicable for solving well-known dynamic macroeconomic models. In particular, we walk the reader through the details in applying such method and highlight some useful by-products which are not easily obtainable for sophisticated models using time-domain methods.

As an example, we solve a prototypical new Keynesian model of the kind presented in Woodford (2003) and Galí (2008). This has the advantage of keeping the illustration simple and concrete, but it should be emphasized that the techniques we describe are of wide applicability in more general settings, e.g. models with a maturity structure, which we leave for future research. We derive an analytical solution to a linearized version of the model under the assumption that primary surpluses evolve independently of government liabilities, a regime in which the fiscal theory of the price level is valid [Leeper (1991), Woodford (1995), Cochrane (1998), Davig and Leeper (2006), Sims (2013)]. This solution is useful in characterizing the cross-equation restrictions and illustrating the complex interaction between the fiscal theory and price regidity. It also presents a new way of testing the validity of this theory. An equivalent derivation using time-domain methods, as well as an extensive study of the fiscal theory, can be found in Leeper and Leith (2015).

2 A Prototypical New Keynesian Model

The model's essential elements include: a representative household and a continuum of firms, each producing a differentiated good; only a fraction of firms can reset their prices each period; a cashless economy with one-period nominal bonds B_t that sell at price $1/R_t$, where R_t is the monetary policy instrument; lump-sum taxation and zero government spending so that consumption equals output, $c_t = y_t$; a monetary authority and a fiscal authority.

2.1 Linearized System

Let $\hat{x} \equiv \ln(x_t) - \ln(x^*)$ denote the log-deviation of a variable x_t from its steady state x^*. It is straightforward to show that a linear approximation to the model's equilibrium conditions leads to the following equations. First, the household's optimizing behavior, when imposed by the goods market clearing condition, implies

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1}) \tag{2.1}$$

where $\sigma > 0$ is the elasticity of intertemporal substitution, $\pi_t = P_t/P_{t-1}$ is the inflation between periods $t-1$ and t, and $\mathbb{E}_t$ represents mathematical expectation given information available at time t. The firm's optimal price-setting behavior reduces to

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t \tag{2.2}$$

where $0 < \beta < 1$ is the discount factor and $\kappa > 0$ is the slope of the so-called new Keynesian Phillips curve.

Next, the monetary authority follows an interest rate feedback rule that reacts to deviations of inflation from its steady state

$$\hat{R}_t = \alpha \hat{\pi}_t + \theta_t \tag{2.3}$$

where θ_t is an exogenous policy shock.[1] In addition, the fiscal authority sets an exogenous primary surplus process, s_t, that evolves independently of government liabilities. This profligate fiscal policy requires that the monetary policy adjust nominal interest rate only weakly to inflation deviations, i.e. $0 \leqslant \alpha < 1$ [Leeper (1991)]. We assume that $(\theta_t, \hat{s}_t)$ is jointly a white noise, normally distributed with mean zero and covariance matrix Σ.

Lastly, any policy choice must satisfy the flow government budget constraint, $\frac{1}{R_t}\frac{B_t}{P_t} + s_t = \frac{B_{t-1}}{P_t}$, which is linearized as

$$\hat{b}_t = \hat{R}_t + \beta^{-1}(\hat{b}_{t-1} - \hat{\pi}_t) - (\beta^{-1} - 1)\hat{s}_t \tag{2.4}$$

where $b_t = B_t/P_t$ is the real debt at the end of period t. Note that the real value of outstanding debt at the beginning of period t, $\hat{b}_{t-1} - \hat{\pi}_t$, is determined in equilibrium at time t. (2.1)—(2.4) constitute a system of expectational difference equations in the variables $\{\hat{y}_t, \hat{\pi}_t, \hat{R}_t, \hat{b}_t\}$, which fully characterizes the model dynamics under the fiscal theory.

2.2 Analytical Solution

To simplify the exhibition, we substitute the monetary policy rule (2.3) into (2.1) and (2.4) and rewrite the resulting system in the following form

$$\begin{aligned}
&\left[\underbrace{\begin{pmatrix} 1 & \sigma & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\Gamma_{-1}} L^{-1} + \underbrace{\begin{pmatrix} -1 & -\alpha\sigma & 0 \\ \kappa & -1 & 0 \\ 0 & \beta^{-1} - \alpha & 1 \end{pmatrix}}_{\Gamma_0} L^{0} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\beta^{-1} \end{pmatrix}}_{\Gamma_1} L \right] \underbrace{\begin{pmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{b}_t \end{pmatrix}}_{\hat{x}_t} \\
&= \underbrace{\begin{pmatrix} \sigma & 0 \\ 0 & 0 \\ 1 & 1 - \beta^{-1} \end{pmatrix}}_{\Psi_0} L^0 \underbrace{\begin{pmatrix} \theta_t \\ \hat{s}_t \end{pmatrix}}_{\varepsilon_t} + \underbrace{\begin{pmatrix} 1 & \sigma & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\Gamma_{-1}} \underbrace{\begin{pmatrix} \eta^y_{t+1} \\ \eta^\pi_{t+1} \\ \eta^b_{t+1} \end{pmatrix}}_{\eta_{t+1}}
\end{aligned} \tag{2.5}$$

[1] For analytical clarity, we assume that the monetary authority does not respond to output deviations.

where L is the lag operator: $L^k \hat{x}_t = \hat{x}_{t-k}$, $\{\Gamma_{-1}, \Gamma_0, \Gamma_1, \Psi_0\}$ are matrix coefficients, and η_{t+1} is a vector of endogenous forecasting errors defined as $\eta_{t+1} = \hat{x}_{t+1} - \mathbb{E}_t \hat{x}_{t+1}$ so that $\mathbb{E}_t \eta_{t+1} = 0$.

Suppose a solution $\hat{x}_t = [\hat{y}_t, \hat{\pi}_t, \hat{b}_t]'$ to (2.5) is of the form

$$\hat{x}_t = \sum_{k=0}^{\infty} C_k \varepsilon_{t-k} \equiv C(L)\varepsilon_t \tag{2.6}$$

where $\varepsilon_t = [\theta_t, \hat{s}_t]'$, $\hat{x}_t$ is taken to be covariance stationary, and $C(L)$ is a polynomial in the lag operator. Note that such moving average representation of the solution is very useful because it also leads to the impulse response function—the coefficient $C_k(i,j)$ measures exactly the response of $\hat{x}_{t+k}(i)$ to a shock $\varepsilon_t(j)$. In what follows, we walk the reader through the key steps in deriving the content of $C(\cdot)$.

Step 1: transform the time-domain system (2.5) into its equivalent frequency-domain representation. To this end, we evaluate the forecasting errors $\eta_{t+1} = [\eta^y_{t+1}, \eta^\pi_{t+1}, \eta^b_{t+1}]'$ using (2.6) and the Wiener-Kolmogorov optimal prediction formula

$$\eta_{t+1} = \left\{ C(L)L^{-1} - \left[\frac{C(L)}{L} \right]_+ \right\} \varepsilon_t = C_0 L^{-1} \varepsilon_t \tag{2.7}$$

where $[\cdot]_+$ is the annihilation operator that ignores negative powers of L. An implicit assumption underlying (2.7) is that the history of monetary and fiscal shocks are perfectly observed up to period t. Define $\Gamma(L) = \Gamma_{-1}L^{-1} + \Gamma_0 + \Gamma_1 L$ and substitute (2.6) and (2.7) into (2.5)

$$\Gamma(L)C(L)\varepsilon_t = (\Psi_0 + \Gamma_{-1}C_0 L^{-1})\varepsilon_t$$

which must hold for all realizations of ε_t. Therefore, the coefficient matrices are related by the z-transform identities

$$z\Gamma(z)C(z) = z\Psi_0 + \Gamma_{-1}C_0$$

where z is a complex variable. In solving for $C(z)$, ideally one would multiply both sides by $(z\Gamma(z))^{-1}$, but $C(z)$ needs to have only non-negative powers of z by (2.6) and be analytic inside the unit circle so that its coefficients are square-summable by covariance stationarity. This requirement can be examined by a careful decomposition of $z\Gamma(z)$ in the next step.

Step 2: apply the Smith canonical decomposition to the polynomial matrix $z\Gamma(z)$[2]

$$z\Gamma(z) = U(z)^{-1} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & z(z-\beta)(z-\lambda_-) \end{pmatrix}}_{S(z)} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & z-\lambda_+ \end{pmatrix} V(z)^{-1}}_{T(z)}$$

[2]The Smith decomposition is available in MAPLE or MATLAB's Symbolic Toolbox.

which factorizes all roots inside the unit circle from those outside and collects them in the diagonal polynomial matrix $S(z)$. Here $U(z)$ and $V(z)$ are unimodular matrices and $\lambda_\pm = (\gamma_1 \pm \sqrt{\gamma_1^2 - 4\gamma_0})/(2\gamma_0)$, where $\gamma_0 = (1+\alpha\sigma\kappa)/\beta$ and $\gamma_1 = (1+\beta+\sigma\kappa)/\beta$. It is straightforward to show that $\frac{\partial\lambda_+}{\partial\alpha} < 0$ and $\frac{\partial\lambda_-}{\partial\alpha} > 0$. Moreover, given the parameter restrictions underlying the fiscal theory, both roots are real, one inside the unit circle, $|\lambda_-| < 1$, and one outside, $|\lambda_+| > 1$

$$0 < \lambda_- < \frac{\beta}{1+\alpha\sigma\kappa} < \beta < 1, \qquad \lambda_+ > \frac{1+\sigma\kappa}{1+\alpha\sigma\kappa} > 1 \tag{2.8}$$

The zero root arises whenever the model is forward-looking, i.e. $\Gamma_{-1} \neq 0$. The root $z = \beta$ emerges as the reciprocal of the root from the government budget constraint (2.4) viewed as a difference equation in $\hat{b}$. To see where the pair of roots $z = \lambda_\pm$ comes from, combine (2.1)—(2.3) and substitute out $\hat{y}$ and $\hat{R}$ to obtain a second order expectational difference equation for inflation

$$\mathbb{E}_t\hat{\pi}_{t+2} - \frac{1+\beta+\sigma\kappa}{\beta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{1+\alpha\sigma\kappa}{\beta}\hat{\pi}_t = -\frac{\sigma\kappa}{\beta}\theta_t$$

The roots governing the dynamics of this equation, which are derived in Leeper and Leith (2015), are exactly $1/\lambda_\pm$. More generally, Tan and Walker (2015) show that roots inside (outside) the unit circle in the frequency-domain correspond to the reciprocals of unstable (stable) roots in the time-domain.

Step 3: examine the existence of solution. A covariance stationary solution exists if the elements of C_0 can be chosen to cancel those problematic roots in $S(z)$. To check that, multiply both sides of the z-transform identities by $S(z)^{-1}$

$$T(z)C(z) = \begin{pmatrix} U_{1\cdot}(z) \\ U_{2\cdot}(z) \\ \frac{1}{z(z-\beta)(z-\lambda_-)}U_{3\cdot}(z) \end{pmatrix}(z\Psi_0 + \Gamma_{-1}C_0)$$

where $U_{j\cdot}(z)$ is the jth row of $U(z)$. These identities are valid for all z on the open unit disk except for $z = 0, \beta, \lambda_-$. But since $C(z)$ must be well-defined for all $|z| < 1$, this condition places the following restrictions on the unknown matrix coefficient C_0

$$U_{3\cdot}(z)(z\Psi_0 + \Gamma_{-1}C_0)|_{z=0,\beta,\lambda_-} = 0 \tag{2.9}$$

Stacking the restrictions in (2.9) yields[3]

$$-\underbrace{\begin{pmatrix} \frac{\beta^2\kappa(\alpha\beta-1)}{1+\alpha\sigma\kappa} & \frac{\beta^2(\alpha\beta-1)(\beta-1+\sigma\kappa)}{1+\alpha\sigma\kappa} & 0 \\ \frac{\lambda_-^2\kappa(\alpha\beta-1)}{1+\alpha\sigma\kappa} & \frac{\lambda_-^2(\alpha\beta-1)(\sigma\kappa+\beta)}{1+\alpha\sigma\kappa} - \frac{\lambda_-\beta(\alpha\beta-1)}{1+\alpha\sigma\kappa} & 0 \end{pmatrix}}_{R} C_0 = \underbrace{\begin{pmatrix} 0 & \frac{\beta^2\sigma\kappa(1-\beta)(\alpha\beta-1)}{1+\alpha\sigma\kappa} \\ \frac{\sigma\kappa\lambda_-^3(\alpha\beta-1)}{1+\alpha\sigma\kappa} & 0 \end{pmatrix}}_{A}$$

[3]Here we omit the restriction imposed by $z = 0$ because it is unrestrictive.

Apparently, the solution exists if and only if the column space of R spans the column space of A, i.e. $\text{span}(A) \subseteq \text{span}(R)$, which is satisfied here. Solving for C_0 gives

$$\begin{pmatrix} C_0(1,1) & C_0(1,2) \\ C_0(2,1) & C_0(2,2) \end{pmatrix} = \begin{pmatrix} \frac{\sigma\lambda_-^2(\beta-1+\sigma\kappa)}{\lambda_- - \beta} & -\frac{(1-\beta)\sigma[(\sigma\kappa+\beta)\lambda_- - \beta]}{\lambda_- - \beta} \\ -\frac{\sigma\kappa\lambda_-^2}{\lambda_- - \beta} & \frac{\sigma\kappa\lambda_-(1-\beta)}{\lambda_- - \beta} \end{pmatrix}$$

where $C_0(3,1)$ and $C_0(3,2)$ are left undetermined.

Step 4: examine the uniqueness of solution. In order for the solution to be unique, we must be able to determine $\{C_k\}_{k=0}^{\infty}$ from the parameter restrictions supplied by $-RC_0 = A$. It requires that from knowledge of RC_0 one be able to pin down $U_{3\cdot}(z)\Gamma_{-1}C_0$ evaluated at the reciprocals of roots outside the unit circle. This is tantamount to verifying whether the columns of R' span the space spanned by the rows of

$$Q = U_{3\cdot}(\lambda_+^{-1})\Gamma_{-1} = \begin{pmatrix} \frac{\kappa(\alpha\beta-1)}{\lambda_+^2(1+\alpha\sigma\kappa)} & \frac{(\alpha\beta-1)[\sigma\kappa+\beta(1-\lambda_+)]}{\lambda_+^2(1+\alpha\sigma\kappa)} & 0 \end{pmatrix}$$

i.e. $\text{span}(Q') \subseteq \text{span}(R')$, which is also satisfied here. Uniqueness would fail if the government budget constraint (2.4) were dropped from the system, leading to the more familiar indeterminacy result in the new Keynesian literature. But this implicitly assumes a different fiscal behavior—the primary surplus always adjusts systematically to assure fiscal solvency. Technically, one would have insufficient restrictions in (2.9) for the determination of C_0 by losing a root inside the unit circle (i.e. $z = \beta$).

Practically, the two space spanning conditions for existence and uniqueness and the computation of C_0 can be obtained by employing the singular value decompositions of A, R, and Q. Now the unique solution can be computed as

$$\begin{aligned} \begin{pmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{b}_t \end{pmatrix} &= \underbrace{[L\Gamma(L)]^{-1}(L\Psi_0 + \Gamma_{-1}C_0)}_{C(L)} \begin{pmatrix} \theta_t \\ \hat{s}_t \end{pmatrix} \\ &= \begin{pmatrix} C_0(1,1)\frac{1-\frac{\beta-\lambda_-}{\beta\lambda_-(\beta-1+\sigma\kappa)}L}{1-\frac{1}{\lambda_+}L} & C_0(1,2)\frac{1}{1-\frac{1}{\lambda_+}L} \\ C_0(2,1)\frac{1-\frac{\lambda_- -\beta}{\beta\lambda_-}L}{1-\frac{1}{\lambda_+}L} & C_0(2,2)\frac{1}{1-\frac{1}{\lambda_+}L} \\ C_0(3,1)\frac{1}{1-\frac{1}{\lambda_+}L} & C_0(3,2)\frac{1}{1-\frac{1}{\lambda_+}L} \end{pmatrix} \begin{pmatrix} \theta_t \\ \hat{s}_t \end{pmatrix} \end{aligned} \tag{2.10}$$

where

$$C_0(3,1) = \frac{\beta+\sigma\kappa}{(1+\alpha\sigma\kappa)\lambda_+}, \qquad C_0(3,2) = \frac{\beta-1}{\lambda_+}$$

Evidently, $[\hat{y}_t, \hat{\pi}_t, \hat{b}_t]'$ follows a vector autoregressive moving average (ARMA) process of order $(1,1)$, where $\hat{b}_t$ only consists of an AR component. All variables share the common AR root,

λ_+, which corresponds to the root outside the unit circle from the Smith decomposition. This ensures that the AR component remains stable. The MA components in $\hat{y}_t$ and $\hat{\pi}_t$, on the other hand, stem from the surviving terms in the restriction system (2.9) after the removal of all roots inside the unit circle.

We highlight several useful by-products of the solution which are not easily obtainable for more sophisticated models using time-domain methods. First, (2.10) clearly captures all cross-equation restrictions imposed by the hypothesis of rational expectations, which are the "hallmark of rational expectations models" [Hansen and Sargent (1980)]. Second, it provides the basis for constructing the spectral density of $\hat{x}_t$ and the associated frequency-domain likelihood function [Harvey (1990)]. For diagnostic purposes, we can estimate and test the model based on various frequencies by setting certain frequencies to zero [Christiano and Vigfusson (2003), Qu and Tkachenko (2012)]. This presents a new way of testing the validity of the fiscal theory. Third, from (2.10) we can easily write output, inflation, and real debt as linear functions of all past and present policy shocks with unambiguously signed coefficients.[4] In particular, output follows

$$\begin{aligned}\hat{y}_t = &\underbrace{C_0(1,1)}_{<0}\theta_t + \sum_{k=1}^{\infty}\underbrace{C_0(1,1)\left[\frac{1}{\lambda_+} - \frac{\beta-\lambda_-}{\beta\lambda_-(\beta-1+\sigma\kappa)}\right]\left(\frac{1}{\lambda_+}\right)^{k-1}}_{>0}\theta_{t-k} \\ &+ \underbrace{C_0(1,2)}_{<0}\hat{s}_t + \sum_{k=1}^{\infty}\underbrace{C_0(1,2)\left(\frac{1}{\lambda_+}\right)^{k}}_{<0}\hat{s}_{t-k} \end{aligned} \tag{2.11}$$

inflation follows

$$\begin{aligned}\hat{\pi}_t = &\underbrace{C_0(2,1)}_{>0}\theta_t + \sum_{k=1}^{\infty}\underbrace{C_0(2,1)\left[\frac{1}{\lambda_+} - \frac{\lambda_- - \beta}{\beta\lambda_-}\right]\left(\frac{1}{\lambda_+}\right)^{k-1}}_{>0}\theta_{t-k} \\ &+ \underbrace{C_0(2,2)}_{<0}\hat{s}_t + \sum_{k=1}^{\infty}\underbrace{C_0(2,2)\left(\frac{1}{\lambda_+}\right)^{k}}_{<0}\hat{s}_{t-k} \end{aligned} \tag{2.12}$$

and real debt follows

$$\hat{b}_t = \underbrace{C_0(3,1)}_{>0}\theta_t + \sum_{k=1}^{\infty}\underbrace{C_0(3,1)\left(\frac{1}{\lambda_+}\right)^{k}}_{>0}\theta_{t-k} + \underbrace{C_0(3,2)}_{<0}\hat{s}_t + \sum_{k=1}^{\infty}\underbrace{C_0(3,2)\left(\frac{1}{\lambda_+}\right)^{k}}_{<0}\hat{s}_{t-k} \tag{2.13}$$

where we have separated shocks in the current period from those in the past.

[4] $C_0(1,1) < 0$ holds by (2.8) and the fact that $\beta - 1 + \sigma\kappa > 0$ for most plausible values of $\{\beta, \sigma, \kappa\}$. $\frac{1}{\lambda_+} - \frac{\beta-\lambda_-}{\beta\lambda_-(\beta-1+\sigma\kappa)} < 0$ holds by (2.8) and the property that $\lambda_+\lambda_- = \frac{\beta}{1+\alpha\sigma\kappa}$. $C_0(1,2) < 0$ holds by (2.8) and the fact that $\frac{\partial\lambda_-}{\partial\alpha} > 0$. A similar argument can be used to sign the coefficients in $\hat{\pi}$ and $\hat{b}$.

2.3 Economic Interpretations The closed-form solution (2.10), or (2.11)—(2.13), is useful in understanding how monetary and fiscal disturbances are transmitted to influence the endogenous variables under the fiscal theory.[5] Its economic interpretations hinge on a ubiquitous relation in any dynamic macro model, that government liabilities derive its real value from the present value of current and expected future primary surpluses

$$\hat{b}_{t-1} - \hat{\pi}_t = -\beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t \hat{r}_{t+k} + (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t \hat{s}_{t+k}, \qquad \forall t \tag{2.14}$$

where $\hat{b}_{t-1}$ is predetermined in period t and $\hat{r}_{t+k} = \hat{R}_{t+k} - \mathbb{E}_{t+k}\hat{\pi}_{t+k+1}$ denotes the ex-ante real interest rate. The above *equilibrium* condition can be obtained by substituting the consumption-Euler equation (2.1) into the government budget constraint (2.4) and iterating forward. Given an exogenous path for primary surpluses, (2.14) already hints at a violation of "Ricardian equivalence"—an increase in government debt, due to either monetary or fiscal shock, will generate a positive wealth effect which in turn transmits into higher inflation and, in the presence of nominal rigidities, higher real activity. A higher inflation is also needed to revalue the nominal government debt so as to ensure its sustainability. In what follows, we highlight three examples of this non-Ricardian property and the role of inflation in stabilizing government debt.

First, we consider the effects of a monetary contraction. Because prices are sticky, a higher nominal interest rate raises the real interest rate. Given exogenous primary surpluses, this leads to more rapidly growing real debt services and hence raises the real debt in (2.13). The higher real interest rate also increases private saving by inducing households to convert consumption goods into bonds in the current period. Thus, output falls initially in (2.11). However, because the higher real debt is backed up less than sufficiently by the present value of primary surpluses in the next period, households will become wealthier and convert bonds back into consumption goods. From (2.11) and (2.12), this increase in aggregate demand pushes up both output and inflation in the next period so that (2.14) is restored. Inflation must also rise in the current as well as future periods to guarantee debt sustainability. Therefore, Ricardian equivalence breaks down.

Second, we examine the impacts of a fiscal expansion. (2.14) suggests that a deficit-financed tax cut shows up as a mix of higher current inflation and a lower path for real interest rates, which in turn leads to higher output. Again, Ricardian equivalence breaks down. This is because given exogenous primary surpluses, households have no anticipation of higher future taxation so that the lower present value of primary surpluses makes them wealthier and substitute bonds into consumption goods. Through revaluation, the higher inflation also ensures that government debt remains sustainable.

Lastly, it is straightforward to show that both the extent, $|C_0(2,1)|$ and $|C_0(2,2)|$, and the decay factor, $1/\lambda_+$, of the policy effects on inflation are increasing in α—a more aggressive monetary stance not only amplifies the inflationary impacts from higher debt but makes these

[5] We follow Woodford (1998) and Leeper and Leith (2015), but see also Kim (2003) for an empirical analysis.

impacts more persistent as well, leading to an enhancement of the fiscal theory mechanism.

3 Concluding Remarks

This article illustrates a widely applicable frequency-domain solution method in the context of a prototypical new Keynesian model under the fiscal theory. The closed-form solution derived herein is useful in characterizing the cross-equation restrictions and understanding the policy transmission mechanisms. We conclude by pointing out that our approach also provides a natural framework for estimating and testing dynamic macroeconomic models along various frequencies, and defer these applications to a sequel to this paper.

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