Notes on Magnetic Mirrors

June 2025

Milena Barrios, mibarriosg@unal.edu.co

When a charged particle moves through a magnetic field, the Lorentz force makes it spiral around the field lines in a helical pattern. THe distance from the center of this spiral to the particle's path is known as the gyroradius.

As the particle drifts into an area where the field lines become more tightly packed, the combination of the field's radial component and the particle's circular motion generates a force pushing it back toward regions of weaker field. This opposing force drains the particle's forward momentum until it slows, stops, and ultimately reverses direction, effectively reflecting it. [Scientific Library (nd)]

For instance, a magnetic mirror arises when a charged particle travels along an axial field $B_z(z)$ that varies with position z. Assuming a symmetric field around the z-axis $\rightarrow$ ($\frac{\partial}{\partial\theta} = 0$ and $B_\theta = 0$) we get from Gauss's law for magnetism,

$$\nabla \cdot \vec{B} = \frac{1}{r}\frac{\partial(r\,B_r)}{\partial r} + \frac{\partial B_z}{\partial z} = 0,$$

finding near the axis ($r \rightarrow 0$)

$$B_r(r,z) \approx -\frac{r}{2}\,\frac{\partial B_z}{\partial z}.$$

When a particle executes Larmor gyration with azimuthal speed v_θ, this small radial component produces a parallel force

$$F_z = q\big(v_r B_\theta - v_\theta B_r\big) = \frac{1}{2} q v_\theta r \frac{\partial B_z}{\partial z}; \longrightarrow \; F_z = -\,\mu\,\frac{\partial B}{\partial z},$$

after averaging over one gyration cycle and using the magnetic moment of the particle

$$\mu = \frac{m\,v_\perp^2}{2B}.$$

As the particle's energy must be conserved, $\mu = \textit{constant}$, is an invariant! So, considering the expression for the total energy

$$\mathcal{E} = \tfrac{1}{2}\,m\,v_\parallel^2 + \mu\,B$$

When the particle goes into regions of larger B, conservation of μ forces $v_\perp$ to increase, draining the parallel kinetic energy, because the total energy must remain constant. When $\mathcal{E} = \mu B \longrightarrow v_\parallel = 0$, the particle reverses its motion. That reversal point is what we call a mirror point.

If two such regions of intensified magnetic field lie opposite each other along the same field line (magnetic bottle), particles with initial pitch-angle θ satisfying [Islam (2020)]

$$\sin^2\theta \; > \; \frac{B_0}{B_m}$$

are trapped, bouncing between the two mirror points, while those inside this cone escape (see Fig. 1). If this inequality is satisfied, the particle cannot escape because its parallel velocity becomes zero before reaching the region with stronger magnetic field (B_m), and it reflects.

Where:

- B_0 is the minimum magnetic field strength along the field line, usually located at the center of the magnetic mirror or bottle. It's where the particle starts or where the field is weakest.

- B_m is the maximum magnetic field strength, located at the mirror points (ends of the magnetic bottle), where the particle reflects.

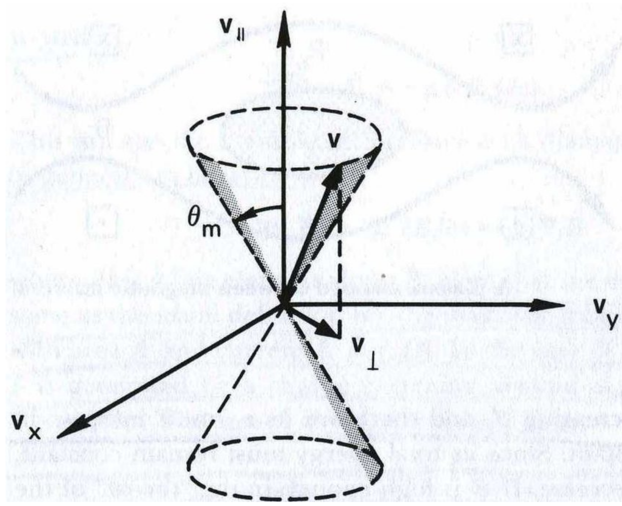

Figure 1: Velocity-space diagram of the magnetic-mirror loss cone. The narrow conical region around the $\pm v_{\parallel}$ axis defines the set of velocity vectors whose pitch angle falls below the critical angle θ_c; particles within this loss cone escape along the field lines, while those outside are reflected and remain trapped between mirror points. Taken from [Galatà (2015)].

References

Galatà, A. (2015). Physics and Technology of the SPES Charge Breeder. Research thesis, Istituto Nazionale di Fisica Nucleare, Legnaro National Laboratories. Figure 6: "The loss cone in velocity space".

Islam, R. (2020). Plasma Physics (Phys 403) Single Particle Motion: Magnetic Mirrors. https://www.scribd.com/document/478668893/Lecture-6-Plasma-Physics.

Scientific Library (n.d.). Magnetic mirror. https://www.scientificlib.com/en/Physics/LX/MagneticMirror.html.