Tutorial: Sampling, Weighting and Estimation
Day 2

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Selecting a Subgroup by logical operators:

To select a subgroup where all elements equal or do not equal a specific value, you can use `==` and `!=`

```
a <- as.vector(c("Aa","AA","Aa","Bb","AA","A","BB","Ba"))
a=="Aa"
```

```
[1]  TRUE FALSE  TRUE FALSE FALSE FALSE FALSE FALSE
```

```
a[a!="AA"]
```

```
[1] "Aa" "Aa" "Bb" "A"  "BB" "Ba"
```

```
b <- 1:length(a)
a == "AA" & b > 3
```

```
[1] FALSE FALSE FALSE FALSE  TRUE FALSE FALSE FALSE
```

```
ab <- which(a=="Aa" & b<=3)
ab
```

```
[1] 1 3
```

```
a[ab]
```

```
[1] "Aa" "Aa"
```

Does an element belong to a group?

```
a %in% c("AA","Ba")
```

```
[1] FALSE  TRUE FALSE FALSE  TRUE FALSE FALSE  TRUE
```

```
sub1 <- bm[bm$Province==3,]
head(sub1[,1:7])
```

```
         Commune   INS Province Arrondiss Men04 Women04   Tot04
182      Beernem 31003        3        31  7496    7055   14551
183 Blankenberge 31004        3        31  8591    9452   18043
184       Bruges 31005        3        31 56565   60283  116848
185        Damme 31006        3        31  5494    5482   10976
186     Jabbeke 31012        3        31  6879    6807   13686
187    Oostkamp 31022        3        31 10616   10837   21453
```

```
s <- which(bm$Commune %in% c("Brecht", "Grimbergen","As","Dinant"))
sub2 <- bm[s,]
sub2[,1:7]
```

```
       Commune   INS Province Arrondiss Men04 Women04 Tot04
7       Brecht 11009        1        11 12975   12976 25951
96  Grimbergen 23025        2        23 16002   17420 33422
464         As 71002        7        71  3701    3705  7406
556     Dinant 91034        9        91  6138    6668 12806
```

The `subset()` function can also be employed to generate subsets of a data frame

subset {base}

Subsetting Vectors, Matrices and Data Frames

Description

Return subsets of vectors, matrices or data frames which meet conditions.

Usage

```
subset(x, ...)

## Default S3 method:
subset(x, subset, ...)

## S3 method for class 'matrix'
subset(x, subset, select, drop = FALSE, ...)

## S3 method for class 'data.frame'
subset(x, subset, select, drop = FALSE, ...)
```

Arguments

`x`	object to be subsetted.
`subset`	logical expression indicating elements or rows to keep: missing values are taken as false.
`select`	expression, indicating columns to select from a data frame.
`drop`	passed on to `[` indexing operator.
`...`	further arguments to be passed to or from other methods.

```
sub3 <- subset(bm,Commune %in% c("Brecht", "Grimbergen","As","Dinant")
sub3$Commune == sub2$Commune
```

```
[1] TRUE TRUE TRUE TRUE
```

```
sub4 <- subset(bm,substr(as.character(Commune),1,1)=="B")
head(sub4[,1:7])
```

```
       Commune   INS Province Arrondiss Men04 Women04 Tot04
3     Boechout 11004        1        11  6027    5927 11954
4         Boom 11005        1        11  7640    8066 15706
5     Borsbeek 11007        1        11  4948    5328 10276
6   Brasschaat 11008        1        11 18142   18916 37058
7      Brecht 11009        1        11 12975   12976 25951
31     Berlaar 12002        1        12  5145    5206 10351
```

- Loops are convenient when conducting one task several times

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$\Rightarrow$ avoid loops if possible (esp. for large datasets)

- Loops are convenient when conducting one task several times
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- But: CPU-intensive

⇒ avoid loops if possible (esp. for large datasets)

```
 A1 <- vector()
 for(i in 1:10){
+   A1[i] <- sample(1:10,1)
+ }
 A1

 [1] 10 10  3  9  7  6  8  2  7  8
```

- To save the results, it is useful to create a container prior to the loop

```
 A2 <- matrix(nrow = 5,ncol = 2)
 for(i in 1:nrow(A2)){
+   A <- sample(1:50,30)
+   A2[i,1] <- mean(A)
+   A2[i,2] <- var(A)
+ }
 A2
```

```
      [,1]  [,2]
[1,] 27.40 230.7
[2,] 27.50 195.6
[3,] 25.33 221.7
[4,] 29.67 198.0
[5,] 28.40 251.1
```

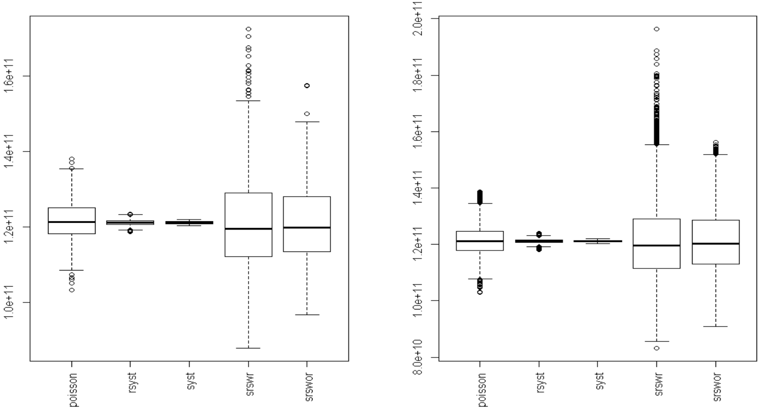
```
data(belgianmunicipalities)
pik <- inclusionprobabilities(belgianmunicipalities$Tot04,200)
# Computes the inclusion probabilities
N <- length(pik)
# population size
n <- sum(pik)
# sample size
sim <- 1000
ss <- array(0, c(sim, 5))
# sim2 <- 10000   #second simulation
# ss2 <- array(0, c(sim2, 5))
# number of simulations
y <- belgianmunicipalities$TaxableIncome
# variable of interest
ht <- numeric(5)
# Horvitz-Thompson estimator for the simulation
```

```
 for (i in 1:sim) {
+      cat("Step ", i, "\n")
+      s <- UPpoisson(pik)
+      ht[1] <- HTestimator(y[s == 1], pik[s == 1])
+      s <- UPrandomsystematic(pik)
+      ht[2] <- HTestimator(y[s == 1], pik[s == 1])
+      s <- UPsystematic(pik)
+      ht[3] <- HTestimator(y[s == 1], pik[s == 1])
+      s <- sample(y, n,replace = T)
+      ht[4] <- HTestimator(s, rep(n/N,n))
+      s <- srswor(n, N)
+      ht[5] <- HTestimator(y[s == 1], rep(n/N, n))
+      ss[i, ] <- ss[i, ] + ht
+    }
```

- `cat()` can be used to display the simulation step that is recently proceeded

Example based on: Tille,Y and Matai, A. (2010). TEACHING SURVEY SAMPLING WITH THE SAMPLING R PACKAGE. *ICOTS8 (2010) Invited Paper Refereed*

```
# boxplots of the estimators
  par(mfrow=c(1,2))
  boxplot(data.frame(ss), las = 3)
  boxplot(data.frame(ss2), las = 3)
```



apply {base} R Documentation

Apply Functions Over Array Margins

Description

Returns a vector or array or list of values obtained by applying a function to margins of an array or matrix.

Usage

```
apply(X, MARGIN, FUN, ...)
```

Arguments

`X`	an array, including a matrix.
`MARGIN`	a vector giving the subscripts which the function will be applied over. E.g., for a matrix `1` indicates rows, `2` indicates columns, `c(1, 2)` indicates rows and columns. Where `X` has named dimnames, it can be a character vector selecting dimension names.
`FUN`	the function to be applied: see 'Details'. In the case of functions like `+`, `%*%`, etc., the function name must be backquoted or quoted.
`...`	optional arguments to `FUN`.

The apply function

- If `margin=1`, the function will be applied to the rows of an array
- If `margin=2`, the function will be applied to the columns of an array

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- If `margin=1`, the function will be applied to the rows of an array
- If `margin=2`, the function will be applied to the columns of an array

```
Applydat <- matrix(1:25, nrow = 5, ncol = 5, byrow = F)
apply(Applydat,1,mean)
```

```
[1] 11 12 13 14 15
```

```
apply(Applydat,2,mean)
```

```
[1]  3  8 13 18 23
```

tapply {base} R Documentation

Apply a Function Over a Ragged Array

Description

Apply a function to each cell of a ragged array, that is to each (non-empty) group of values given by a unique combination of the levels of certain factors.

Usage

```
tapply(X, INDEX, FUN = NULL, ..., simplify = TRUE)
```

Arguments

`X`	an atomic object, typically a vector.
`INDEX`	list of one or more factors, each of same length as `X`. The elements are coerced to factors by `as.factor`.
`FUN`	the function to be applied, or `NULL`. In the case of functions like `+`, `%*%`, etc., the function name must be backquoted or quoted. If `FUN` is `NULL`, tapply returns a vector which can be used to subscript the multi-way array `tapply` normally produces.
`...`	optional arguments to `FUN`: the Note section.

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```
 Tapplydat <- data.frame(Income = rnorm(6,1400,200),
+                    Gender = sample(c("Male","Female"),6,replace = T))
 Tapplydat
```

```
  Income Gender
1   1703   Male
2   1452   Male
3   1418   Male
4   1376   Male
5   1161 Female
6   1522 Female
```

```
 tapply(Tapplydat$Income, Tapplydat$Gender, mean)
```

```
Female   Male
  1342   1487
```

The finite population correction...

$$1 - f = \frac{N - n}{N} = 1 - \frac{n}{N}$$

...can also be turned into a R function

```
fpc <- function(N,n){(N-n)/N}
fpc(100,8)
```

```
[1] 0.92
```

Generating a telephone sample with the approach of Gabler and Häder:

Constructing a synthetic frame:

```
fra <-data.frame(pre = sample(c(30,40,89,221,621),10000,replace = T),
+                  bank = sample(100:99999,10000,replace = T))
fra[1:4,]
```

```
  pre  bank
1 221  5728
2 221 32661
3 621 64698
4 621 35754
```

```
fra <- fra[order(fra[,1]),]
fra[1:4,]
```

```
   pre  bank
11  30 87896
13  30  7164
15  30 63673
16  30 17772
```

```
 tel.samp <- function(fra,n){
+   len <- nrow(fra)*100
+   s <- sort(sample(len,n))
+   row <- ceiling(s/100)
+   app <- s%%100
+   ts <- fra[row,]
+   num <- data.frame(prefix = paste("0",ts[,1],sep = ""),
+                     number = paste(ts[,2],app,sep = ""))
+   return(num)
+ }
```

- `fra` is the sampling frame
- `n` is the sample size
- with `return()`, one can determine the results that the function should display

Writing "advanced" functions

```
my.first.ts <- tel.samp(fra,10)
head(my.first.ts)
```

```
  prefix  number
1    030 4965129
2    030 2690436
3    040 8495419
4    040 7116060
5    040 3043965
6    089 3369754
```

- `sort()` sorts an atomic vector in an ascending or descending order and returns the values
- `order()` sorts an atomic vector or a data frame in an ascending or descending order and returns the row number

Using a loop to draw stratified random samples

```
 str.bm <- split(bm,bm$Province)
 nh <- c(2,3,7,3,2,6,7,2,9)
 res <- list()
 for(i in 1:length(str.bm)){
+   ID <- str.bm[[i]]$INS
+   res[[i]] <- sample(ID,nh[i],replace=F)
+ }
 s <- unlist(res)
 result<-bm[bm$INS %in% s,]
 table(result$Province)
```

```
1 2 3 4 5 6 7 8 9
2 3 7 3 2 6 7 2 9
```

You can also use the `strata()` command from the `sampling` package

⇒ the function returns the unit's identifier, stratum and its inclusion probability

```
s <- strata(bm,"Province",nh,method = "srswor")
result1 <- getdata(bm,s)
head(result1[,c(1:3,ncol(result)-1,ncol(result))])
```

```
                 Commune   INS Arrondiss medianincome Province
48                Dessel 13006        13        20212        1
51             Herentals 13011        13        19141        1
89   Woluwe-Saint-Pierre 21019        21        22051        2
151               Linter 24133        24        21053        2
161        Grez-Doiceau 25037        25        21029        2
182              Beernem 31003        31        20268        3
```

⇒ `getdata()` merges the sample-IDs with your original dataset and returns your sample as a data frame

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Proportional Allocation:

$$\gamma_h = \frac{N_h}{N}$$

$$n_h = n * \gamma_h$$

```
n <- 30
gamma <- prop.table(table(bm$Province))
nh <- n*gamma
t(nh)
```

```
         1     2     3     4     5     6     7     8     9
 [1,] 3.565 5.654 3.260 3.311 3.514 4.278 2.241 2.241 1.935
```

```
s <- strata(bm,"Province",nh,"srswor")
result.p <- getdata(bm,s)
nrow(result.p)
```

```
[1] 26
```

⇒ the `strata()` command generally rounds down

⇒ use `round()` or the cox-algorithm

Optimal Allocation:

$$n_h = n * \frac{\gamma_h \sigma_h}{\sum_{g=1}^{L} \gamma_g \sigma_g} = n * \frac{N_h \sigma_h}{\sum_{g=1}^{L} N_g \sigma_g} \qquad h = 1, \ldots, L$$

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Optimal Allocation:

$$n_h = n * \frac{\gamma_h \sigma_h}{\sum_{g=1}^{L} \gamma_g \sigma_g} = n * \frac{N_h \sigma_h}{\sum_{g=1}^{L} N_g \sigma_g} \qquad h = 1, \ldots, L$$

Step 1: getting $Var(\overline{y}_{StrRS,opt})$

$$Var(\overline{y}_{StrRS,opt}) = \frac{1}{n} \sum_{h=1}^{L} (\gamma_h * \sigma_h)^2 = \frac{1}{n} \sum_{h=1}^{L} (\frac{N_h}{N} \sigma_h)^2 = \frac{1}{N^2} \sum_{h=1}^{L} \frac{N_h^2}{n_h} \sigma_h^2$$

```
 GetStratVar <- function(Y, sind, nh) {
+    Nh <- tapply(sind,sind,length)
+    N <- length(sind)
+    sum(Nh^2*tapply(Y,sind,function(x)
+      var(x)*(length(x)-1)/length(x))/nh)/N^2
+ }
 GetStratVar(bm$Tot04,bm$Province,rep(5,length(unique(bm$Province))))
```

```
[1] 18941644
```

Step 2: calculating n_h

```
 GetOptAlloc <- function(Y, sind, n){
+   L <- length(unique(sind))
+   nh <- rep(2,L)
+   Nh <- tapply(sind,sind,length)
+   v <- numeric(L)
+   M <- diag(rep(1,L))
+   while (sum(nh) < n) {
+     for (i in 1:L) {
+       if (nh[i] == Nh[i]) {
+         v[i] <- Inf
+       } else {
+         v[i] <- GetStratVar(Y, sind, nh + M[,i])
+       }
+     }
+     nh <- nh + M[,which.min(v)]
+   }
+   nh
+ }
```

Stratified Random Sampling

```
nh <- GetOptAlloc(bm$Tot04,bm$Province,50)
t(nh)
```

```
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,]   13    9    4    6    6    6    2    2    2
```

```
nh2 <- GetOptAlloc(bm$averageincome,bm$Province,50)
t(nh2)
```

```
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,]    6   14    4    5    6    6    2    4    3
```

```
nh <- GetOptAlloc(bm$Tot04,bm$Province,50)
t(nh)
```

```
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,]   13    9    4    6    6    6    2    2    2
```

```
nh2 <- GetOptAlloc(bm$averageincome,bm$Province,50)
t(nh2)
```

```
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,]    6   14    4    5    6    6    2    4    3
```

$\Rightarrow$ Allocation differs depending on the variable of interest

Simple Method:

⇒ Sample cluster proportional to size; sample all units within a cluster

```
l <- 4
gamma <- prop.table(table(bm$Province))
clus <- sample(unique(bm$Province),l, prob = gamma, replace = F)
res.clus <- bm[bm$Province %in% clus,]
nrow(res.clus)
```

```
[1] 303
```

Simple Method:

⇒ Sample cluster proportional to size; sample all units within a cluster

```
l <- 4
gamma <- prop.table(table(bm$Province))
clus <- sample(unique(bm$Province),l, prob = gamma, replace = F)
res.clus <- bm[bm$Province %in% clus,]
nrow(res.clus)
```

```
[1] 303
```

⇒ Sample size varies

Fixed Sample Size:

⇒ Sample cluster proportional to size; sample the same number of units within each cluster

```
l <- 4
gamma <- prop.table(table(bm$Province))
clus <- sample(unique(bm$Province),l, prob = gamma, replace = F)
fixed.res.clus <-list()
for(i in 1:l){
+   nh <- 30
+   bm.cl <- bm[bm$Province == clus[i],]
+   fixed.res.clus[[i]] <- sample(bm.cl$INS,nh, replace = F)
+ }
ID <- unlist(fixed.res.clus)
fixed.clus <- bm[bm$INS %in% ID,]
nrow(fixed.clus)

[1] 120
```

The sample package also offers a cluster function

```
l <- 4
sam.clus <- cluster(bm,"Province",4,method = "srswor")
res.clus.samp <- getdata(bm,sam.clus)
nrow(res.clus.samp)
```

```
[1] 216
```

⇒ Samples all units within a cluster

Mean: Simple Random Sampling (SRS)

$$\overline{y}_{SRS} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

```
SRS.mean <- function(Y,S){return(mean(Y[S]))}
```

Variance: Simple Random Sampling (SRS)

$$V(\overline{y}_{SRS}) = \frac{\sigma^2}{n}; \qquad V(\overline{y}_{SRSWOR}) = \frac{N-n}{N-1}\frac{\sigma^2}{n} = (1-\frac{n}{N})\frac{S^2}{n}$$

$$\hat{V}(\overline{y}_{SRSWOR}) = (1-\frac{n}{N})\frac{s^2}{n}$$

```
SRS.evar <- function(Y,S){return(var(Y[S])/length(S))}
SRSWOR.evar <- function(Y,S)
+   {return(fpc(nrow(Y),length(S))*var(Y[S])/length(S))}
```

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Mean: Stratified Random Sampling (StrRS)

$$\hat{\bar{y}}_{StrRS} = \sum_{h=1}^{L} \gamma_h \frac{1}{n_h} \sum_{i=1}^{n_h} y_i$$

```
 Strat.mean <- function(Y,sind,S){
+   Nh <- tapply(Y,sind,length)
+   Str.mean <- sum(Nh*tapply(Y[S], sind[S], mean) / sum(Nh))
+   return(Str.mean)
+ }
 S <- as.numeric(row.names(result))
 Strat.mean(bm$averageincome,bm$Province,S)
```

```
[1] 24000
```

- `Y` is the variable of interest
- `sind` is the identifier of the strata (`length(Y)`)
- `S` are the row names of the sample

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Variance: Stratified Random Sampling (StrRS)

$$V(\bar{y}_{StrRS}) = \sum_{h=1}^{L} \gamma_h^2 \frac{\sigma_h^2}{n_h}; \qquad V(\bar{y}_{StrRSWOR}) = \sum_{h=1}^{L} \gamma_h^2 \frac{\sigma_h^2}{n_h} * \frac{N_h - n_h}{N_h - 1}$$

$$\hat{V}(\bar{y}_{StrRS}) = \sum_{h=1}^{L} \gamma_h^2 \frac{s_h^2}{n_h}; \qquad \hat{V}(\bar{y}_{StrRSWOR}) = \sum_{h=1}^{L} \gamma_h^2 \frac{s_h^2}{n_h} * \frac{N_h - n_h}{N_h}$$

```
 Strat.evar<- function(Y, sind, S) {
+   Nh <- tapply(sind,sind,length)
+   nh <- tapply(sind[S], sind[S], length)
+   ssh <- tapply(Y[S], sind[S], var)
+   res <- sum((Nh/sum(Nh))^2*ssh/nh*(Nh-nh)/Nh)
+   return(res)
+ }
 S <- as.numeric(row.names(result))
 Strat.evar(bm$averageincome,bm$Province,S)
```

[1] 256082

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Mean: One-Stage Cluster Sampling (SRCS)

$$\hat{\bar{y}}_{SRCS} = \frac{L}{l}\sum_{h=1}^{l}\gamma_h^a * \bar{y}_h^a = \frac{L}{l}\sum_{h=1}^{l} * \frac{N_h^a}{\sum_{g=1}^{L} N_g^a} * \frac{1}{N_h^a}\sum_{i=1}^{N_h^a} Y_{ih}$$

```
 SRCS.mean <-function(Y,sind,S){
+   L <- length(unique(sind))
+   l <- length(unique(sind[S]))
+   N <- length(Y)
+   N_h_a <- tapply(Y[S],sind[S],length)
+   mu_h_a <- tapply(Y[S],sind[S],mean)
+   return(L/l*sum(N_h_a/N*mu_h_a))
+ }
 Sc <- as.numeric(row.names(res.clus))
 SRCS.mean(bm$averageincome,bm$Province,Sc)
```

```
[1] 30483
```

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Mean: One-Stage Cluster Sampling (SRCS)

$$V(\overline{y}_{SRCS}) = \frac{L^2}{N^2} * \frac{\sigma_e^2}{l} * \frac{L-l}{L-1}$$

$$\hat{V}(\overline{y}_{SRCS}) = \frac{L^2}{N^2} * \frac{s_e^2}{l} * \frac{L-l}{L};$$

$$s_e^2 = \frac{1}{l-1}\sum_{r=1}^{l}(N_r^a \overline{y}_r^a - \frac{N * \hat{\overline{y}}_{SRCS}}{L})^2$$

Calculating s_e^2

```
 se.sq <- function(Y,sind,S){
+   L <- length(unique(sind))
+   l <- length(unique(sind[S]))
+   N <- length(Y)
+   mu.SRCS <- SRCS.mean(Y,sind,S)
+   c <- N*mu.SRCS/L
+   mu_h_a <- tapply(Y[S],sind[S],mean)
+   N_h_a <- tapply(Y[S],sind[S],length)
+  return ( 1/(l-1)*sum((N_h_a*mu_h_a-c)^2))
+ }
```

Calculating $\hat{V}(\overline{y}_{SRCS})$

```
SRCS.evar <- function(Y,sind,S){
+   L <- length(unique(sind))
+   l <- length(unique(sind[S]))
+   N <- length(Y)
+   part1 <- L^2/N^2
+   part2 <- se.sq(Y,sind,S)/l
+   part3 <- (L-l)/L
+   return(part1*part2*part3)
+ }
Sc <- as.numeric(row.names(res.clus))
SRCS.evar(bm$averageincome,bm$Province,Sc)
```

```
[1] 28973878
```

Sampling and Estimation

1. Load the `belgianmunicipalities` data and allocate the sample size $n = 180$ to the provinces
   - equal allocation
   - proportional to the number of communes
   - proportional to the average income
   - proportional to the taxable income
   - under optimal allocation for the variable `Tot04`
2. Furthermore, draw the following two cluster samples
   - two full provinces
   - six full provinces
3. Draw $R = 1,000$ samples for each of the allocations and calculate the sample mean of variable `Tot04` and its variance for each simulation step
4. Calculate the average mean and variance over the 1,000 simulation steps
5. Calculate the deviation and compare it over the different sampling designs
6. Compare the average variance for each sampling design

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Variance equal allocation

$$\hat{V}(\overline{y}_{StrRS,eq}) = \frac{1}{n}\sum_{h=1}^{L}\gamma_h^2 * s_h^2 * \frac{N_h * L - n}{N_h - 1}$$

```
 Strat.eq.evar <- function(Y,sind,S){
+   L <- length(unique(sind))
+   Nh <- tapply(sind,sind,length)
+   nh <- tapply(sind[S], sind[S], length)
+   ssh <- tapply(Y[S], sind[S], var)
+   res <- 1/sum(nh)*sum((Nh/sum(Nh))^2*
+               ssh*((Nh*L-sum(nh))/(Nh-1)))
+   return(res)
+ }
```

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Variance optimal allocation

$$\hat{V}(\overline{y}_{StrRS,opt}) = \frac{1}{n}(\sum_{h=1}^{L} \gamma_h * s_h)^2 - \frac{1}{N}\sum_{h=1}^{L} \gamma_h s_h^2$$

```
 Strat.opt.evar <- function(Y, sind, S){
+   Nh <- tapply(sind,sind,length)
+   nh <- tapply(sind[S], sind[S], length)
+   ssh <- tapply(Y[S], sind[S], var)
+   part1 <- 1/sum(nh)*sum(Nh/sum(Nh)*sqrt(ssh))^2
+   part2 <- 1/sum(Nh)*sum(Nh/sum(Nh)*ssh)
+   return(part1-part2)
+ }
```