Sampling and Estimation

Part 3: Calibration Weights

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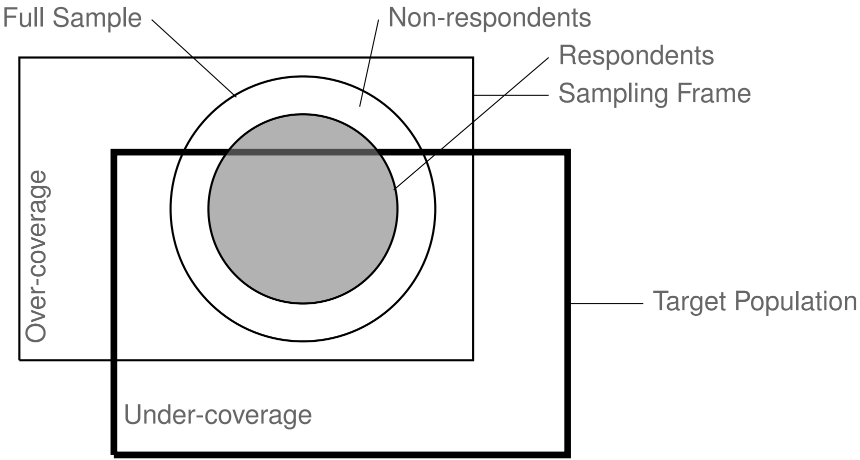
Full Sample
Non-respondents
Respondents
Sampling Frame
Over-coverage
Target Population
Under-coverage

Procedures based on the **available cases** only, i.e., only those cases that are completely recorded for the variables of interest.

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$\longrightarrow$ Methods for handling coverage errors are not so widely spread, simply because there is often no reliable auxiliary information on just the target population. However if there is, it can receive a treatment similar to that of weighting by non-response.

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MNAR missing not at random

RP depends on variables of interest $\mathcal{Y}$;
cannot be modeled, because $\mathcal{Y}$ not known for non-respondents

(see [1])

$\longrightarrow$ In multivariate analysis 30% to 40% of the data are often lost with case deletion, assuming MCAR!

Calibration approach The design weighs are calibrated to the totals of some auxiliary variables $\mathcal{X}$ (see [2]).

Sample estimates using the calibrated weights will exactly replicated those totals.

If the used auxiliary variables help to explain the response process the calibrated weight can reduce the non-response error.

Two-phase approach The response process is modeled to obtain the response propensities ψ_k for all $k \in \mathcal{s}$. The new weight of element k is $\frac{d_k}{\psi_k}$. (Two phases: 1. Sampling $\rightarrow$ 2. Responding).

In addition the new weights $\frac{d_k}{\psi_k}$ might then also be calibrated.

Often used models are:

- Response homogeneity classes, every element in a class has the same probability to respond.
- Generalized liner models (*probit*, *logit*, *log-log*), treating response as a latent variable.

The calibration approach is more direct, as the design weights are directly calibrated without considering the response propensities. Also, if the same models are used for both the modeling of the response propensities and the calibration, the two approaches can be equivalent.

Weights

Generic estimators for a total and a mean

$$\hat{\tau}_w = \sum_{k \in \mathcal{s}} w_k y_k \quad \text{and} \quad \overline{y}_w = \frac{\sum_{k \in \mathcal{s}} w_k y_k}{\sum_{k \in \mathcal{s}} w_k} \ ,$$

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$$w_k = \begin{cases} d_k g_k & \text{for } k \in \mathcal{s} \\ 0 & \text{else} \end{cases} .$$

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Sometimes called base weights or design weights, the inverse of inclusion probabilities $d_k = \pi^{-1}$ is usually the first step in weighting. If we have $g_k = 1$, the $\hat{\tau}_w$ would be the HT estimator or π-estimator. The factor g_k adjusts the design weights to reduce

- the sampling error (i.e. variance),
- the non-response error, and
- the coverage error

of estimator $\hat{\tau}_w$ or $\overline{y}_w$. Thereby the w_k's should not deviate to much from the d_k's as these weights ensure an unbiased estimation.

The general idea is to exploit the relationship between auxiliary variables and the variable of interest to improve the efficiency of estimators.

Weights Calibration I

The following problem is solved with weight calibration:
For a given design $p(.)$ and a sample $\mathcal{s}$ weights w_k for all $k \in \mathcal{s}$ have to be found that minimize

$$\sum_{k \in \mathcal{s}} G_k(w_k, d_k, c_k) \ ,$$

subject to constraints

$$\sum_{k \in \mathcal{s}} w_k \mathbf{x}_k = \sum_{k \in \mathcal{U}} \mathbf{x}_k = \boldsymbol{\tau}_x$$

where $\mathbf{x}_k = (x_{k1}, x_{k2}, \ldots, x_{kQ})^\top$ is a vector of q auxiliary variables for element k. G_k is a measure of distance between w_k and d_k and c_k is a factor that can be freely chosen for additional flexibility.

To calculate the weights, the $\mathbf{x}_k$'s are only needed for the elements in the net sample (i.e. typically only for the respondents), but $\boldsymbol{\tau}_x$, their population totals, need to be known.

The auxiliary variables can be metric (e.g. income or age) or categorical (e.g. gender or age groups).

Depending on the choice of G_k, different calibration estimators can be obtained, some of the most common are:

- Post-stratification Estimator
- Raking Estimator
- Generalized Regression Estimator

Note that the w_k's typically depend on the sample $\mathcal{s}$, in contrast to the d_k, which are given by the sampling design.

Post-stratification is typically used if only categorical auxiliary variables are available. It is implemented by forming weighting cells by crossing *all* categories of the auxiliary variables. These weighting cells are the post-strata $\mathcal{U}_q$ with $q = 1, \ldots, Q$. The weight are then adjusted to replicate the counts in these cells. For $k \in \mathcal{U}_q$ we have

$$g_k = \frac{\tau_{x_q}}{\hat{\tau}_{x_q}} \ ,$$

where $\tau_{x_q} = \sum_{k \in \mathcal{U}} x_{kq}$ and

$$x_{kq} = \begin{cases} 1 & \text{if } k \in \mathcal{U}_q \\ 0 & \text{else} \end{cases} .$$

$\hat{\tau}_{x_q \, \pi} = \sum_{k \in \mathcal{s}} d_k x_{kq}$ its estimator for τ_{x_q} based on the design weights. The auxiliary variables are the post-stratum indicators, i.e. $\mathbf{x}_k = (x_{k1}, x_{k2}, \ldots, x_{kQ})^\top$. An adjustment to the totals of a metric variable within the post-strata would also be possible.

Table: Population Counts τ_{x_q} for Hair and Eye Colour

	Brown	Blue	Hazel	Green
Black	68	20	15	5
Brown	119	84	54	29
Red	26	17	14	14
Blond	7	94	10	16

TABLE: Population Counts τ_{x_q} for Hair and Eye Colour

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TABLE: Sample counts $\sum_{k\in s} x_{kq}$ in a SRS with $n = 150$

	Brown	Blue	Hazel	Green
Black	14	7	2	2
Brown	36	22	17	5
Red	7	3	1	4
Blond	1	23	1	5

TABLE: Population Counts τ_{x_q} for Hair and Eye Colour

	Brown	Blue	Hazel	Green
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TABLE: Estimated totals $\hat{\tau}_{x_q\,\pi} = \sum_{k \in s} x_{kq} d_k$

	Brown	Blue	Hazel	Green
Black	55.2533	27.6267	7.8933	7.8933
Brown	142.0800	86.8267	67.0933	19.7333
Red	27.6267	11.8400	3.9467	15.7867
Blond	3.9467	90.7733	3.9467	19.7333

Table: Population Counts τ_{x_q} for Hair and Eye Colour

	Brown	Blue	Hazel	Green
Black	68	20	15	5
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Blond	7	94	10	16

Table: Post-stratification $g_k = \frac{\tau_{x_q}}{\hat{\tau}_{x_q}}$

	Brown	Blue	Hazel	Green
Black	1.2307	0.7239	1.9003	0.6334
Brown	0.8376	0.9674	0.8048	1.4696
Red	0.9411	1.4358	3.5473	0.8868
Blond	1.7736	1.0355	2.5338	0.8108

Beware, there must be at least one element in the sample from each post-stratum, otherwise we divide by zero!

In raking, only the marginal totals are needed, *not* the totals for all the cross-categories. Raking can be implemented as iterative post-stratification to adjust the design weights to the margins of the different auxiliary variables.

The design weights of a SRSC cluster sample of school districts are raked to variables school type (`stype`) and the accomplishment of the growth target (`sch.wide`).

```
##                   dname            name stype sch.wide
## 1 Alameda City Unified    Alameda High     H      Yes
## 2 Alameda City Unified    Encinal High     H      Yes
## 3 Alameda City Unified  Chipman Middle     M      Yes
## 4 Alameda City Unified Lum (Donald D.)     E      Yes
## 5 Alameda City Unified Edison Elementa     E      Yes
## 6 Alameda City Unified Otis (Frank) El     E      Yes
```

TABLE: Population Counts τ_{x_q} for School Type (`stype`) and School Target (`sch.wide`)

	No	Yes	SUM
E	472	3949	4421
H	334	421	755
M	266	752	1018
SUM	1072	5122	6194

```
set.seed(-57844)
#selection the SRCS
apiclus <- apipop[apipop$dnum%in%sample(unique(apipop$dnum),10),]
apiclus$fpc <- length(unique(apipop$dnum))

dclus1<- svydesign(id=~dnum, data=apiclus, fpc=~fpc)
#initial weight
w1    <- weights(dclus1)
#convergence is declared if the maximum change in a
#table entry is less than 'eps' ...
eps   <- 1
#... otherwise the process stops after 'maxit' iterations
maxit <- 100

tau_stype    <- table(apipop$stype)
tau_sch.wide <- table(apipop$sch.wide)

#Raking (i.e. iterative post-stratification) for two variables
tab_x <- tab_y <- list()
```

```r
for (i in 1:maxit) {
    ## Post-stratification to the first variable
    w1 <- split(w1, apiclus$stype)
    adj1 <- tau_stype/sapply(w1, sum)
    # new weight
    w1. <- w1 <- mapply(function(x, y) x * y, w1, adj1)
    # return to original order
    w1 <- unlist(w1.)
    names(w1) <- unlist(sapply(w1., names))
    w1 <- w1[as.character(sort(as.numeric(names(w1))))]
    tab_x[[i]] <- tapply(w1, list(apiclus$stype, apiclus$sch.wide), sum)

    ## Post-stratification to the second variable
    w2 <- split(w1, apiclus$sch.wide)
    adj2 <- tau_sch.wide/sapply(w2, sum)
    # new weight
    w2. <- w2 <- mapply(function(x, y) x * y, w2, adj2)
    # return to original order
    w2 <- unlist(w2.)
    names(w2) <- unlist(sapply(w2., names))
    w2 <- w2[as.character(sort(as.numeric(names(w2))))]
    tab_y[[i]] <- tapply(w2, list(apiclus$stype, apiclus$sch.wide), sum)

    if (i > 1) {
        tab.diff <- abs(tab_y[[i - 1]] - tab_y[[i]])

        if (max(tab.diff) < eps)
            break
    }
    w1 <- w2

}
```

TABLE: Estimated Totals $\hat{\tau}_{x_q\,\pi} = \sum_{k \in s} x_{kq} d_k$ from a SRCS of Districts (`dname`) with $n_{\text{I}} = 10$

	No	Yes	SUM
E	984.1	4087.8	5071.9
H	378.5	302.8	681.3
M	378.5	832.7	1211.2
SUM	1741.1	5223.3	6964.4

Table: Estimated Totals after Adjustment to 'stype' in the 1 Interation

	No	Yes	SUM
E	857.8	3563.2	4421.0
H	419.4	335.6	755.0
M	318.1	699.9	1018.0
SUM	1595.4	4598.6	6194.0

Table: Estimated Totals after Adjustment to 'sch.wide' in the 1 Interation

	No	Yes	SUM
E	576.4	3968.7	4545.1
H	281.8	373.7	655.6
M	213.8	779.5	993.3
SUM	1072.0	5122.0	6194.0

TABLE: Estimated Totals after Adjustment to 'stype' in the 2 Interation

	No	Yes	SUM
E	560.7	3860.3	4421.0
H	324.6	430.4	755.0
M	219.1	798.9	1018.0
SUM	1104.3	5089.7	6194.0

TABLE: Estimated Totals after Adjustment to 'sch.wide' in the 2 Interation

	No	Yes	SUM
E	544.2	3884.9	4429.1
H	315.1	433.2	748.2
M	212.7	804.0	1016.7
SUM	1072.0	5122.0	6194.0

TABLE: Estimated Totals after Adjustment to 'stype' in the 3 Interation

	No	Yes	SUM
E	543.3	3877.7	4421.0
H	317.9	437.1	755.0
M	212.9	805.1	1018.0
SUM	1074.1	5119.9	6194.0

TABLE: Estimated Totals after Adjustment to 'sch.wide' in the 3 Interation

	No	Yes	SUM
E	542.2	3879.4	4421.5
H	317.3	437.3	754.6
M	212.5	805.4	1017.9
SUM	1072.0	5122.0	6194.0

Table: Estimated Totals after Adjustment to 'stype' in the 4 Interation

	No	Yes	SUM
E	542.1	3878.9	4421.0
H	317.5	437.5	755.0
M	212.5	805.5	1018.0
SUM	1072.1	5121.9	6194.0

TABLE: Estimated Totals after Adjustment to 'sch.wide' in the 4 Interation

	No	Yes	SUM
E	542.0	3879.0	4421.0
H	317.4	437.5	755.0
M	212.5	805.5	1018.0
SUM	1072.0	5122.0	6194.0

```
dclus1r <- rake( dclus1, list(~stype, ~sch.wide)
                ,list( table(stype=apipop$stype)
                      ,table(sch.wide=apipop$sch.wide)
                ))
svytable(~stype+sch.wide, dclus1r , round=TRUE)

##       sch.wide
## stype   No  Yes
##     E  542 3879
##     H  317  438
##     M  213  805

(w1/weights(dclus1r))[1:10]

##       863      1138      1139      1140      1141      1142      1143
## 0.9999724 1.0001319 0.9999724 0.9999724 0.9999724 0.9999724 0.9999724
##      1144      1145      1146
## 0.9999724 0.9999724 0.9999724

summary(w1/weights(dclus1r))

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       1       1       1       1       1       1
```

For the linear generalized regression estimator (GREG), the measure of distance G_k is

$$G_k(w, \pi, c) = G(w_k, d_k, c_k) = \frac{(w_k - d_k)^2}{2 d_k c_k} \ ,$$

and we have

$$\hat{\tau}_{\text{GREG}} = \hat{\tau}_{\pi} + (\boldsymbol{\tau}_x - \hat{\boldsymbol{\tau}}_{x\,\pi})^\top \widehat{\boldsymbol{\beta}},$$

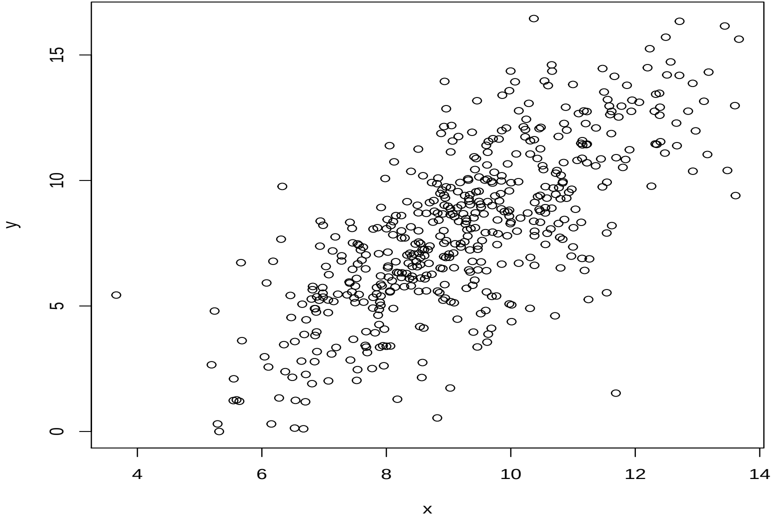
where

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{k \in \mathcal{s}} d_k c_k \mathbf{x}_k (\mathbf{x}_k)^\top \right)^{-1} \sum_{k \in \mathcal{s}} d_k c_k \mathbf{x}_k y_k \ ,$$

and $\hat{\boldsymbol{\tau}}_{x\,\pi} = (\hat{\tau}_{x_1\,\pi}, \ldots, \hat{\tau}_{x_Q\,\pi})^\top$.
The adjustment to the design weight g_k can be written as:

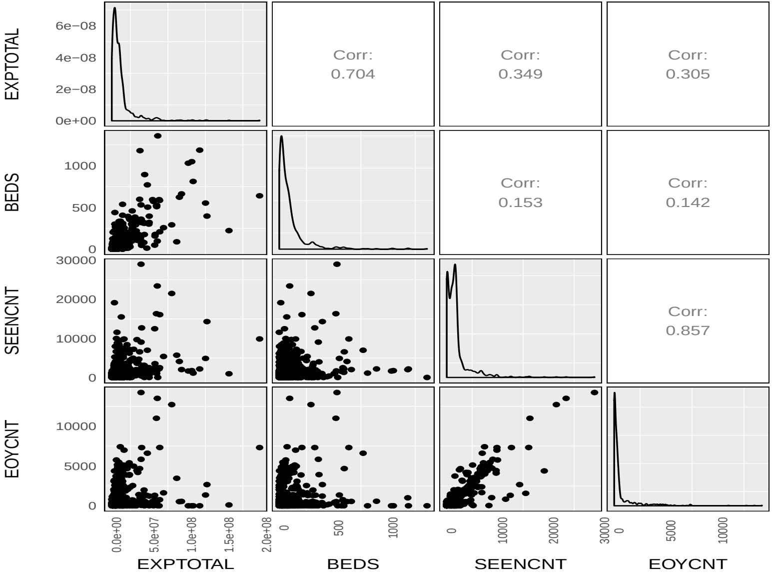
$$g_k = 1 + \left(\left(\sum_{k \in \mathcal{U}} \mathbf{x}_k - \sum_{k \in \mathcal{s}} d_k \mathbf{x}_k \right)^\top \left(\sum_{k \in \mathcal{s}} d_k c_k \mathbf{x}_k (\mathbf{x}_k)^\top \right)^{-1} \right) c_k \mathbf{x}_k$$

Graphical presentation of π- and GREG Estimator



We want to estimate total expenditures of hospitals. To improve a possible estimate we use data from survey in 1998 to explore, if there are any useful predictors for our variable of interest.

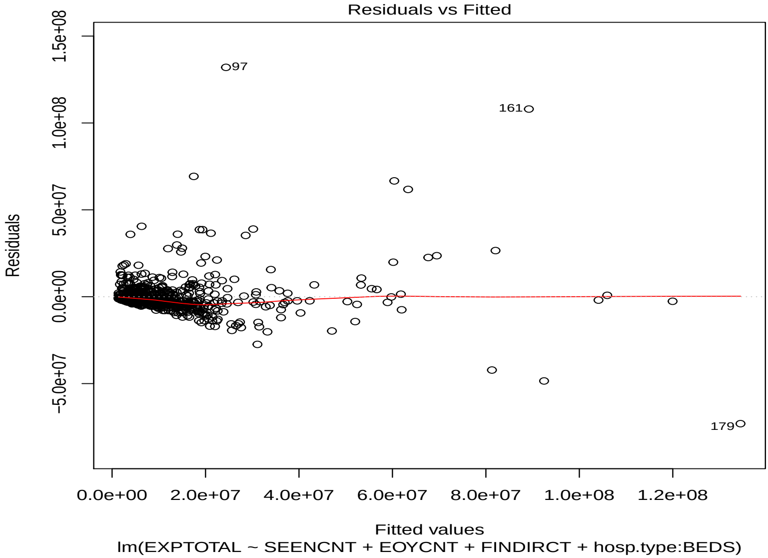
```
library(PracTools) #load the package
data(smho.N874)    #load the data set
head(smho.N874)

##   EXPTOTAL BEDS SEENCNT EOYCNT FINDIRCT hosp.type
## 1  9066430   81    1791    184        2         1
## 2  9853392   80    1870    244        2         1
## 3  3906074   26    1273      0        2         1
## 4  9853392   90    1781    154        2         1
## 5  9853392   71    1839    206        2         1
## 6  9853392   81    1823    196        2         1

#?smho.N874          #for a description of the variables

#only hospitals other than 'type 4' are considered
smho <- smho.N874[smho.N874$hosp.type != 4, ]
```


EXPTOTAL
BEDS
SEENCNT
EOYCNT
6e−08
4e−08
2e−08
0e+00
Corr:
0.704
Corr:
0.349
Corr:
0.305
1000
500
0
Corr:
0.153
Corr:
0.142
30000
20000
10000
0
Corr:
0.857
10000
5000
0
0.0e+00
5.0e+07
1.0e+08
1.5e+08
2.0e+08
0
500
1000
0
10000
20000
30000
0
5000
10000
EXPTOTAL
BEDS
SEENCNT
EOYCNT

Fitting a linear model for `EXPTOTAL` with common slopes for `SEENCNT` and `EOYCNT` but a different slope for `BEDS` in each hospital type.

Table: Model Summary

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1318589.11	912432.21	1.45	0.15
SEENCNT	1033.94	310.63	3.33	0.00
EOYCNT	2036.15	603.58	3.37	0.00
FINDIRCT2	78026.06	965237.62	0.08	0.94
hosp.type1:BEDS	98139.28	3318.84	29.57	0.00
hosp.type2:BEDS	39489.35	5644.51	7.00	0.00
hosp.type3:BEDS	77578.37	15082.20	5.14	0.00
hosp.type5:BEDS	36855.78	8650.48	4.26	0.00


Residuals vs Fitted
Residuals
1.5e+08
1.0e+08
5.0e+07
0.0e+00
−5.0e+07
97
161
179
0.0e+00
2.0e+07
4.0e+07
6.0e+07
8.0e+07
1.0e+08
1.2e+08
Fitted values
lm(EXPTOTAL ~ SEENCNT + EOYCNT + FINDIRCT + hosp.type:BEDS)

We select a sample of hospitals with probability proportional to the square root of `BEDS` using a systematic sample.

```
######################################
### Select a pps to sqrt(BEDS) sample
######################################
library(sampling)    #load the 'sample' package
                     #for the 'UPsystematic' function
smho. <-    # before sampling order the data set by hospital type
  smho.[order(smho.$hosp.type),]

x <- smho.[,"BEDS"]
x[x <= 5] <- 5       # recode small hospitals to have a minimum size
x <- sqrt(x)

n <- 80              #sample size
IP  <- n*x/sum(x)

set.seed(428274453)
sam <- UPsystematic(IP)

sam.dat <- smho.[sam==1, ]
sam.dat$IP <- IP[sam==1]   #the design weight
sam.dat$d <- 1/IP[sam==1]   #the design weight
```

Now we use the `survey` package to calibrate the weights.

```
library(survey) #load the 'survey' package
#1. build a 'design' object
sam.dsgn <-
  svydesign( ids = ~1           # no clusters
            ,data = sam.dat    # the sample data
            ,fpc = ~IP         # incl. prob
            ,pps= "brewer")     # variance approx. method
    #the model we use for the GREG
lmod2 <- lm(EXPTOTAL ~ SEENCNT + EOYCNT + hosp.type:BEDS, data=smho.)
#2. compute pop totals of auxiliaries
pop.tots <- colSums(model.matrix(lmod2)) #Inefficient but convenient!

#3. use 'calibrate' to compute the new weights
sam.cal <-
  calibrate(design = sam.dsgn,
            formula = ~ SEENCNT + EOYCNT + hosp.type:BEDS,
            population = pop.tots,
            calfun='linear' )
```

Setting argument `calfun='linear'` in `'calibrate'` results in the GREG weights, other calibration function are possible, already built-in are `'raking'` and `'logit'`.

Now we check if the calibration constrains are satisfied:

```
#BEDS by hospital type
svyby(~BEDS, by=~hosp.type, design=sam.cal, FUN=svytotal)

##   hosp.type  BEDS           se
## 1         1 37978 4.100694e-12
## 2         2 13066 1.122603e-12
## 3         3  9573 3.028628e-13
## 5         5 10077 7.135892e-13

#SEENCNT and EOYCNT
svytotal(~SEENCNT+EOYCNT, sam.cal)

##           total SE
## SEENCNT 1349241  0
## EOYCNT   505345  0

pop.tots

##     (Intercept)         SEENCNT          EOYCNT hosp.type1:BEDS
##             725         1349241          505345           37978
## hosp.type2:BEDS hosp.type3:BEDS hosp.type5:BEDS
##           13066            9573           10077
```

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However the effects might be catastrophic of domain estimation, in case of estimation domains that where not considered in the calibration.

In general, it is advisable to only use calibrated weights to infer to the whole population or sub-populations that are found in the marginal totals used for the calibration!

Design weights can always be used to do unbiased domain estimation, although the precision of these estimates can be very poor.

The Variance of the Generalized Regression Estimator

Calibrated weights are **not** independent of the selected sample, i.e. they are random variables. Thus, the variance of the GREG estimator cannot be estimated as straightforwardly as for the π-estimator. We can write its approximate variance as

$$\mathrm{AV}\left(\hat{\tau}_{\mathrm{GREG}}\right) = \sum_{k \in \mathcal{U}} \sum_{l \in \mathcal{U}} \left(\pi_{kl} - \pi_k \pi_l\right) \frac{E_k}{\pi_k} \frac{E_l}{\pi_l} ,$$

where $E_k = y_k - y_k^0$, $y_k^0 = \mathbf{x}_x^\top \boldsymbol{\beta}$ and

$$\boldsymbol{\beta} = \left(\sum_{k \in \mathcal{U}} c_k \mathbf{x}_k (\mathbf{x}_k)^\top \right)^{-1} \sum_{k \in \mathcal{U}} c_k \mathbf{x}_k y_k .$$

The Variance of the Generalized Regression Estimator

Calibrated weights are **not** independent of the selected sample, i.e. they are random variables. Thus, the variance of the GREG estimator cannot be estimated as straightforwardly as for the π-estimator. A variance estimator for $\hat{\tau}_{\text{GREG}}$ is given by

$$\widehat{\mathrm{V}}\left(\hat{\tau}_{\text{GREG}}\right) = \sum_{k \in s} \sum_{l \in s} \frac{\left(\pi_{kl} - \pi_k \pi_l\right)}{\pi_{kl}} g_k \frac{e_k}{\pi_k} g_l \frac{e_l}{\pi_l} \ ,$$

where $e_k = y_k - \hat{y}_k$ and $\hat{y}_k = \mathbf{x}_x^{\top} \widehat{\beta}$.

The Variance of the Generalized Regression Estimator

Calibrated weights are **not** independent of the selected sample, i.e. they are random variables. Thus, the variance of the GREG estimator cannot be estimated as straightforwardly as for the π-estimator. Compare the estimates for the total expenditure with calibrated and design weights.

```
svytotal(~EXPTOTAL, sam.dsgn)

##                total       SE
## EXPTOTAL 9.55e+09 7.72e+08

svytotal(~EXPTOTAL, sam.cal)

##                total       SE
## EXPTOTAL 9.03e+09 5.92e+08
```

We find that $\hat{\tau}_{\mathrm{GREG}}/\hat{\tau}_{\pi} = 1.0581$, but $\widehat{\mathrm{V}}\left(\hat{\tau}_{\mathrm{GREG}}\right)/\widehat{\mathrm{V}}\left(\hat{\tau}_{\pi}\right) = 1.7006$.

Literature I

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