Tutorial: Sampling, Weighting and Estimation Day 3

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First order inclusion probability under SRSWOR

$$\pi_k = \frac{n}{N}$$



First order inclusion probability under SRSWOR

$$\pi_k = \frac{n}{N}$$

First order inclusion probability under StrRS

$$\pi_{hk} = \frac{n_h}{N_h}$$

```
Nh <- table(bm$Province)
nh <- c(15,20,35,22,18,22,7,13,20)
pihk <- data.frame(nh/Nh)
names(pihk)[1] <- "Province"
pihk.long <- merge(pihk,bm,by = "Province")</pre>
```



First order inclusion probability under SRSWOR

$$\pi_k = \frac{n}{N}$$

First order inclusion probability under StrRS

$$\pi_{hk} = \frac{n_h}{N_h}$$

```
\label{eq:nh} $$Nh \leftarrow table(bm\Province)$$ nh \leftarrow c(15,20,35,22,18,22,7,13,20)$$ pihk \leftarrow data.frame(nh/Nh)$$ names(pihk)[1] \leftarrow "Province"$$ pihk.long \leftarrow merge(pihk,bm,by = "Province")$$
```

Or with the sampling package

pihk.sa <- inclusionprobastrata(bm\$Province,nh)</pre>



First order inclusion probability under π ps

$$\pi_k = \frac{x_k}{\sum_{l=1}^N x_l} * n$$

phi <- bm\$Tot04/sum(bm\$Tot04)
pik <- phi*n
head(pik)</pre>

[1] 0.2443 7.9021 0.2066 0.2714 0.1776 0.6403



First order inclusion probability under π ps

$$\pi_k = \frac{x_k}{\sum_{l=1}^N x_l} * n$$

phi <- bm\$Tot04/sum(bm\$Tot04)
pik <- phi*n
head(pik)</pre>

[1] 0.2443 7.9021 0.2066 0.2714 0.1776 0.6403

- \Rightarrow Sampling frequencies > 1
- \Rightarrow With replacement \rightarrow average frequency of element k in sample
- ⇒ Without replacement → each element can only be chosen once



πps and the sampling package

```
pik <- inclusionprobabilities(bm$Tot04,n)
head(pik)
[1] 0.2885 1.0000 0.2439 0.3205 0.2097 0.7561
table(pik == 1)
FALSE TRUE
562 27</pre>
```



π ps and the sampling package

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pik <- inclusionprobabilities(bm$Tot04,n)
head(pik)
[1] 0.2885 1.0000 0.2439 0.3205 0.2097 0.7561
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- ⇒ The command inclusionprobabilites displays the first order inclusion probability → not the average sampling frequencies
- \Rightarrow It sets every frequency > 1 to 1 and redistributes the "rest", because:



π ps and the sampling package

```
pik <- inclusionprobabilities(bm$Tot04,n)
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```

- ⇒ The command inclusionprobabilites displays the first order inclusion probability → not the average sampling frequencies
- \Rightarrow It sets every frequency > 1 to 1 and redistributes the "rest", because:
- \Rightarrow 0 < $\pi_k \leq 1$



The Horvitz-Thompson-Estimator

$$\hat{t}_{HT} = \sum_{k=1}^{n} d_k * y_k = \sum_{k=1}^{n} \frac{1}{\pi_k} y_k$$

```
s <- strata(bm,"Province",nh, "srswor")
samp <- getdata(bm,s)
dk <- 1/samp$Prob
sum(dk*samp$Men04)</pre>
```

[1] 5681894



The Horvitz-Thompson-Estimator

$$\hat{t}_{HT} = \sum_{k=1}^{n} d_k * y_k = \sum_{k=1}^{n} \frac{1}{\pi_k} y_k$$

```
s <- strata(bm,"Province",nh, "srswor")
samp <- getdata(bm,s)
dk <- 1/samp$Prob
sum(dk*samp$MenO4)</pre>
```

[1] 5681894

sum(bm\$Men04)

[1] 5097709

The sampling package also offers a function for the HT-estimator

HTstrata(y = samp\$Men04,pik = samp\$Prob,strata = samp\$Province)

[1] 5681894



Scaling design weights to sample size

- Typically the sum of design weights adds up to the population size $(\sum_{k=1}^{n} d_k = N)$
- It is common to use scaled weights (d_k^*) that add up to the sample size and have a mean of $1(\sum_{k=1}^n d_k^* = n; d_k^* = n * \frac{d_k}{\sum_{l=1}^n d_l})$
- ⇒ Advantage: you see if an element has a higher or lower probability to be included in your sample

```
dk.sc <- nrow(samp)*dk/sum(dk)
sum(dk.sc*samp$Men04)/nrow(samp)</pre>
```

[1] 9647



Truncating weights

- Truncating the weights within specific borders to avoid high and/ or negative weights
- MSE argument: trade-off between biased results and lower variance
- Algorithm should rescale the weights to its previous sum



```
trunc.bounds <- function(di,bound){
    n<-sum(di)
    nopt<-di
    i<-0
    s<-which(di<=0|di<bound[1]|di>bound[2])
    while(i<n){
        if(length(s)!=0){
        s1<-which(nopt<=0)
        s2<-which(nopt<bound[1])
        s3<-which(nopt>bound[2])
        nopt[s1] <-bound[1]
        nopt[s2]<-bound[1]
        nopt[s3]<-bound[2]
        su<-length(s1)*bound[1]+length(s2)*bound[1]+length(s3)*bound[2]
        ge < -(n-su)*nopt[-s]/sum(nopt[-s])
        nopt[-s]<-ge
        s <- which (nopt <= 0 | nopt < bound [1] | nopt > bound [2])}
        if(length(s)!=0){
        i<- i+1
        fi<-i}
        else f
          fi<-i+1
          i<-n
+
    cat(" number of iterations ".fi."\n".
        "number of truncated weights ",length(which(nopt%in%bound)),"\n",
    "minimal value", sum(di^2/nopt))
    return(nopt)
+ }
```



Truncating weights (example)

```
wei <- runif(5000,0.5,7)
 table(wei>=6)
FALSE
      TRUE
 4223 777
 bounds \leftarrow c(0,6)
 wei.trunc <- trunc.bounds(wei,bounds)</pre>
 number of iterations 18
 number of truncated weights 925
 minimal value 18908
 table(wei.trunc==6)
FALSE.
       TRUE.
 4075
        925
```



Variance of the HT-Estimator

$$V_{SYG}(\hat{t}_{HT}) = \sum_{\substack{k=1\\k < l}}^{N} \sum_{l=1}^{N} (\pi_k * \pi_l - \pi_{kl}) * (\frac{y_k}{\pi_k} - \frac{y_l}{\pi_l})^2$$

```
bm <- bm[-2,]
pik <- inclusionprobabilities(bm$Tot03,30)
IPkl1 <- UPsampfordpi2(pik)
IPkl2 <- UPsystematicpi2(pik)
SIGMA.samp <- IPkl1 - pik%*%t(pik)
SIGMA.syst <- IPkl2 - pik%*%t(pik)</pre>
```



Variance of the HT-Estimator



Variance of the HT-Estimator

- Same design weights
- Different sampling algorithm
- ⇒ Different Variance!



The HT-Estimator: Variance Estimation

$$\hat{V}_{SYG}(\hat{t}_{HT}) = \sum_{\substack{k=1\\k < l}}^{n} \sum_{I=1}^{n} \frac{\pi_{k} * \pi_{l} - \pi_{kl}}{\pi_{kl}} * (\frac{y_{k}}{\pi_{k}} - \frac{y_{l}}{\pi_{l}})^{2}$$

DESIGN EFFECTS



Design-based approach

$$deff = \frac{Var_c(\hat{t})}{Var_{srs}(\hat{t})}$$

var.tot.srs <- var(bm\$Tot04)/30*(1-30/nrow(bm))*nrow(bm)^2
deff <- var.HT.tot.samp/var.tot.srs
deff</pre>

- Although the sample has been drawn with unequal probabilities, the design effect is below 1
- ⇒ Highly correlated variable has been used to calculate the inclusion probabilities

DESIGN EFFECTS



Model-based approach

$$\hat{\textit{deff}} = \hat{\textit{deff}_p} * \hat{\textit{deff}_c} = n \frac{\sum_{h=1}^{I} d_h^2 n_h}{\left(\sum_{h=1}^{I} d_h n_h\right)^2} * \left(1 + \left(\overline{b} - 1\right)\rho\right)$$

 n_h is the number of units per cluster; \overline{b} is the average cluster size; ρ reflects the Intraclass Correlation Coefficient (ICC)

Design Effects



Model-based approach

$$\hat{deff} = \hat{deff_p} * \hat{deff_c} = n \frac{\sum_{h=1}^{I} d_h^2 n_h}{(\sum_{h=1}^{I} d_h n_h)^2} * (1 + (\overline{b} - 1)\rho)$$

 n_h is the number of units per cluster; \overline{b} is the average cluster size; ρ reflects the Intraclass Correlation Coefficient (ICC)

- \Rightarrow No cluster/ stratified random sample
- \Rightarrow deff_p captures the design effect due to unequal inclusion probabilities

$$\begin{split} & \text{deff_p} <- \, \text{sum}(30*(1/\text{pik}[s==1])^2)/\text{sum}(1/\text{pik}[s==1])^2 \\ & \text{deff_p} \end{split}$$

[1] 2.258

Design Effects



Model-based approach

$$\hat{\textit{deff}} = \hat{\textit{deff}}_{\textit{p}} * \hat{\textit{deff}}_{\textit{c}} = n \frac{\sum_{h=1}^{I} d_{h}^{2} n_{h}}{(\sum_{h=1}^{I} d_{h} n_{h})^{2}} * (1 + (\overline{\textit{b}} - 1)\rho)$$

 n_h is the number of units per cluster; \overline{b} is the average cluster size; ρ reflects the Intraclass Correlation Coefficient (ICC)

- \Rightarrow No cluster/ stratified random sample
- \Rightarrow deff_p captures the design effect due to unequal inclusion probabilities

[1] 2.258

Effective sample size

$$n_{eff} = \frac{n}{deff}$$

30/deff



Poststratification

```
Using the my.pop data set of day 1
 s <- srswor(100,10000)</pre>
 samp \leftarrow my.pop[s==1,]
 genXedu.s <- data.frame(table(samp$gender,samp$education))</pre>
 genXedu.s[,3] \leftarrow genXedu.s[,3]/sum(genXedu.s[,3])
 genXedu.pop <- data.frame(table(my.pop$gender,my.pop$education))</pre>
 genXedu.pop[,3] <- genXedu.pop[,3]/sum(genXedu.pop[,3])</pre>
 adj.w <- data.frame(genXedu.pop[,3]/genXedu.s[,3])</pre>
 adj.w[,1]
[1] 0.7626 1.2745 0.8618 0.7132 2.0489 0.8720 1.8350 1.2467
 samp$to.merge <- paste(samp$gender,samp$education)</pre>
 adj.w$to.merge <- paste(genXedu.s$Var1,genXedu.s$Var2)</pre>
 adjusted <- merge(adj.w,samp,by="to.merge")
```



Raking/ Iterative Proportional Fitting

Generating the necessary data frames

```
age <- rep(as.character(1:6),times=10)</pre>
 edu <- rep(as.character(1:5),each=6,times=2)</pre>
 gender <- rep(c("m","w"),each=30)</pre>
 freq <- sample(100,60,replace=T)</pre>
 # Synthetic sample distribution
 samp <- data.frame(age,edu,gender,freq)</pre>
 samp[,4] \leftarrow samp[,4]/sum(samp[,4])
 freq2 <- sample(100,60,replace=T)</pre>
 # in population
 master <- data.frame(age,edu,gender,freq2)</pre>
 master[,4] <- master[,4]/sum(master[,4])</pre>
 masageXedu <- aggregate(master[,4],</pre>
                             list(age=master[,1],edu=master[,2]),sum)
 masageXgen <- aggregate(master[,4],</pre>
                             list(age=master[,1],gender=master[,3]),sum)
maseduXgen <- aggregate(master[,4],</pre>
                             list(edu=master[,2],gender=master[,3]),sum)
+
```



Raking/ Iterative Proportional Fitting

```
w_0 <- rep(1,times=nrow(samp))
 times <- 0
 while(times <= 1000){
    #AgexEducation
    saxe <- aggregate(samp[,4]*w_0,list(age=samp[,1],edu=samp[,2]),sum)</pre>
    w 1 <- masageXedu[.3]/saxe[.3]
    w 1 <- rep(w 1.times=(nrow(samp)/nrow(saxe)))
    #AgexGender
    saxg <- aggregate(samp[,4]*w_0*w_1,list(age=samp[,1],gen=samp[,3]),sum)</pre>
    w_2 <- masageXgen[,3]/saxg[,3]
    w_2 \leftarrow c(rep(w_2[1:(length(w_2)/2)],times=(nrow(samp)/nrow(saxg))),
           rep(w_2[(length(w_2)/2+1):length(w_2)],times=(nrow(samp)/nrow(saxg))))
    #EducationxGender
    sexg <- aggregate(samp[,4]*w_0*w_1*w_2,list(edu=samp[,2],gen=samp[,3]),sum)</pre>
    w_3 <- maseduXgen[,3]/sexg[,3]
    w 3 <- rep(w 3.each=(nrow(samp)/nrow(sexg)))
    #w4
    w_4 <- w_0*w_1*w_2*w_3
    if(max(abs(w 0-w 4))>0.05)
            √w 0<-w 4
            times<-times+1}
    else {break}
    cat("iteration".times."\n")
+ }
iteration 1
iteration 2
iteration 3
iteration 4
iteration 5
 samp$weight<-w_0*samp[,4]
```

aamnadu Vaan



Raking/ Iterative Proportional Fitting

 ${\tt sampeduXgen <- aggregate (samp[,5],list(edu=samp[,2],gen=samp[,3]),sum)}$

magaduVaan

| S | amped | uxgen | maseduxgen |
|----|-------|-----------|----------------|
| | edu | gen x | edu gender x |
| 1 | 1 | m 0.09474 | 1 1 m 0.09474 |
| 2 | 2 | m 0.09211 | 2 2 m 0.09211 |
| 3 | 3 | m 0.09836 | 3 3 m 0.09836 |
| 4 | 4 | m 0.09309 | 4 4 m 0.09309 |
| 5 | 5 | m 0.10395 | 5 5 m 0.10395 |
| 6 | 1 | w 0.10000 | 6 1 w 0.10000 |
| 7 | 2 | w 0.10724 | 7 2 w 0.10724 |
| 8 | 3 | w 0.12204 | 8 3 w 0.12204 |
| 9 | 4 | w 0.10395 | 9 4 w 0.10395 |
| 10 | 5 | w 0.08454 | 10 5 w 0.08454 |



- The survey package provides a large range of applications for complex survey samples
- Typically, the first step is to define a survey object with the svydesign() command

Simple survey object (single stage)

```
library(Matrix)
library(survey)
bm$pik1 <- inclusionprobabilities(bm$Tot03,100)
s <- UPmaxentropy(bm$pik1)
samp <- getdata(bm,s)
IPk1 <- UPmaxentropypi2(bm$pik1)
surv.obj <- svydesign(id=~1,fpc = samp$pik1,
+ data = samp,pps = ppsmat(IPk1[s==1,s==1]),
+ variance = "YG")</pre>
```



```
surv.obj <- svydesign(id=~1,fpc = samp$pik1,
+ data = samp,pps = ppsmat(IPkl[s==1,s==1]),
+ variance = "YG")</pre>
```

- id specifies the identifier of PSU and SSU;id= ~0 or id=~1 stipulates a single stage sampling
- For multi-stage samples the id argument should always specify a formula with the cluster-identifier at each stage
- fpc should be used for the finite population correction
- ⇒ Either as the total population size of each stratum or as a fraction of the total population that has been sampled



```
surv.obj <- svydesign(id=~1,fpc = samp$pik,
+ data = samp,pps = ppsmat(IPk1[s==1,s==1]),
+ variance = "YG")</pre>
```

- data reflects the data set for which the design object should be defined
- pps should be used to define the design information that should be used;
 usually the second order probability of inclusion
- ⇒ ppsmat() is a wrapper for the joint inclusion probabilities of the HT-Estimator
 - With variance you can specify whether you use the Yates-Grundy- or the HT-Estimator
- ⇒ For fixed sample sizes, "YG" should be used

svytotal



Important commands

returns the estimated total of a variable

and its standard error (+deff)

svymean returns the estimated mean of a variable
and its standard error (+deff)

svyquantile Computes quantiles for data from complex surveys

veights Computes variances for data from complex surveys

Returns the (design) weights of a survey object

calibrate Calibration of a data set (uses the GREG-Estimator)
...

svytotal(~Tot04,surv.obj)

total SE Tot04 9957554 3770



Example: changing the "YG" argument with fixed sample sizes



Example: changing the "YG" argument with fixed sample sizes

⇒ The variance estimator under HT varies more than the ont of the YG estimator



Example: changing the "YG" argument with fixed sample sizes

- ⇒ The variance estimator under HT varies more than the ont of the YG estimator
- ⇒ In this case the estimator is negative



Calibrating the sample

```
The GREG-Estimator within the survey package svymean(~averageincome, surv.obj)

mean SE
averageincome 25515 455

mean(bm$averageincome)

[1] 25095
```

- 1] 25095
 - Using the variable Men03 and Arrondiss as auxiliary information
 - Calculating the population Total

```
lm1 <- lm(averageincome ~MenO3+Arrondiss, data=bm)
pop.tot <- colSums(model.matrix(lm1))</pre>
```



Calibrating the sample

Calculation the calibration weights with the calibrate() command

```
surv.obj3 <- svydesign(id=~0,fpc = samp$pik1,
+ data = samp,pps = "brewer")</pre>
```

- The calibrate function cannot be applied to the previous object of class "ppsmat"
- ⇒ Therefore we use the brewer approximation for joint inclusion probabilities



Calibrating the sample

Calculation the calibration weights with the calibrate() command

```
surv.obj3 <- svydesign(id=~0,fpc = samp$pik1,
+ data = samp,pps = "brewer")</pre>
```

- The calibrate function cannot be applied to the previous object of class "ppsmat"
- ⇒ Therefore we use the brewer approximation for joint inclusion probabilities
- ⇒ Seems odd, since only first order inclusion probabilities are used for calibration



Calibrating the sample

```
g_i <- calibrate(surv.obj3,formula =~MenO3+Arrondiss
+ ,population=pop.tot,calfun="linear")</pre>
```

- The names of the variables that are used for the calibration have to be identical for your survey object and your population
- formula specifies the calibration model
- With calfun you can choose between a linear model (GREG) or a raking approach
- For only one calibration variable, calibrate produces the same weights as a poststratification



Calibration results

SAMPLE svymean(~averageincome,g_i) mean SE averageincome 25185 391 svytotal(~Men03,g_i) total SE Men03 4853501 0 svytotal(~Arrondiss,g_i) total SE Arrondiss 27744 0

```
POPULATION

mean(bm$averageincome)

[1] 25095

sum(bm$Men03)

[1] 4853501

sum(bm$Arrondiss)

[1] 27744
```



Multi-stage samples

Loading the api data set and generating a multi-stage data frame

```
data(api)
score <- by(apiclus1,apiclus1$cds,
+ function(x)rnorm(x$api.stu,mean = x$api00, sd = sqrt(x$api00)</pre>
```

 the by command is similar to tapply and creates a list of normal distributed test scores for each school

```
1 <- 50
nh <- 60
apiclus1$fpc <- inclusionprobabilities(apiclus1$enroll,1)</pre>
```

 We draw a sample of I = 50 schools (PSUs) proportional to the number of enrolled pupils



Sampling the cluster

```
cs <- UPmaxentropy(apiclus1$fpc)
cs.dat <- apiclus1[cs==1,]</pre>
```

Sampling within a school



Merging the data sets/ Second stage inclusion probabilities

```
DATA.s <- merge(cs.dat,data.s,by="cds")
DATA.s$id <- 1:nrow(DATA.s)
DATA.s$fpc2 <- nh/DATA.s$enroll
```

- \Rightarrow Full sample of 50 PSUs with 60 SSUs each
- $\Rightarrow \textit{ Self-weighting } \mathsf{approach}$



Merging the data sets/ Second stage inclusion probabilities

```
DATA.s <- merge(cs.dat,data.s,by="cds")
DATA.s$id <- 1:nrow(DATA.s)
DATA.s$fpc2 <- nh/DATA.s$enrol1
```

- $\Rightarrow\,$ Full sample of 50 PSUs with 60 SSUs each
- ⇒ Self-weighting approach

Specifying the survey object

36.35

60

Exercise 4



DESIGN EFFECTS/ ONE-STAGE CLUSTER SAMPLE

- 1 Draw a sample of I = 20 FULL arondissments proportional to the population of 2003
 - ⇒ Use the maximum entropy algorithm
- 2 Calculate the design effect by the model based approach

Model Based Approach

$$\begin{split} \hat{deff} &= \hat{deff_p} * \hat{deff_c} = n \frac{\sum_{h=1}^{I} d_h^2 n_h}{(\sum_{h=1}^{I} d_h n_h)^2} * (1 + (\overline{b} - 1)\rho) \\ \hat{\rho}^{AOV} &= \frac{MSB - MSW}{MSB + (K - 1)MSW} \end{split}$$

$$MSB = \frac{SSB}{l-1};$$
 $MSW = \frac{SSW}{n-l};$ $K = \frac{1}{l-1}(n - \sum_{h=1}^{l} \frac{n_h^2}{n})$

Exercise 4



DESIGN EFFECTS/ ONE-STAGE CLUSTER SAMPLE

R-CODE MSW/ MSB

```
### MSW and MSB
```

SS <- anova(lm(DATA.c\$Tot04~DATA.c\$Arrondiss))</pre>

MSB <- SS * Mean Sq [1]

MSW <- SS\$`Mean Sq`[2]

- 3 Calculate deff under the design based approach
- 4 Calculate the effective sample size

Exercise 5



The survey package/ Multi-stage sampling

- 1 Create the normal distributed income for each individual in 2004 within each commune of the belgianmunicipalities data set (use the square root of the average income as the standard deviation)
- 2 Draw a sample of I = 50 communes (use the INS as identifier) proportional to the population of 2003
- 3 Draw a sample of $n_h = 80$ individuals within each commune
- 4 Estimate the mean income of your sample (design weighted)
- 5 Poststratify your data set to the number of men in 2003