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Reinforcement and Online Learning
Kalman Filter
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1 Synthetic second order AR process

An autoregressive time series(AR Process) of order 2 is given by the generating model as below:

$$s(n) = \sum_{k=1}^{2} a_k s(n-k) + v(n)$$

where n is index over time, $a_k$are the parameters of process and $v(n)$is Gaussian random noise. Figure 1 shows the pictures of a random excitation signal and a second order autoregressive process.

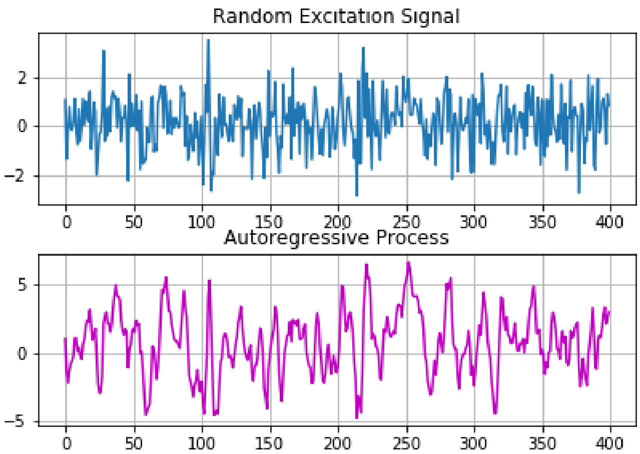

Figure 1: random excitation signal and synthetic second order AR process

2 Implement Kalman filter to estimate the parameter of a synthetic second order AR process

The state space model for sequential estimation of parameters of a time series model is as follows:

$$\theta(n) = \theta(n-1) + w(n)$$

$$y(n) = \theta^T x_n + v(n)$$

where we have assumed a random walk model on the unknown parameters θ, a univariate observation and 2 past samples are held in the vector $x(n) = [x(n-1), x(n-2)]$. At every step in time, our task is to make the best estimate θ from observation $y(n)$ and input $x(n)$. The Kalman filter equtions for the above are given below. For the random walk, our predictions of the state and uncertainty on the state (error covariance matrix) are:

$$\theta(n|n-1) = \theta(n-1|n-1)$$

$$P(n|n-1) = P(n-1|n-1) + Q$$

At time n, we predict the target signal as $\hat{y}(n) = \boldsymbol{x}(n)^T \boldsymbol{\theta}(n \mid n-1$ and innovation signal is:

$$e(n) = y(n) - \boldsymbol{x}(n)^T \boldsymbol{\theta}(n \mid n-1)$$

We now make posterior updates to the state estimates and the corresponding error covariance matrix:

$$\boldsymbol{\theta}(n \mid n) = \boldsymbol{\theta}(n \mid n-1) + \boldsymbol{k}(n) e(n)$$
$$P(n \mid n) = \left(I - \boldsymbol{k}(n)\boldsymbol{x}(n)^T\right) P(n \mid n-1)$$

where the Kalman gain $k(n)$ is given by

$$\boldsymbol{k}(n) = \frac{P(n \mid n-1)\boldsymbol{x}(n)}{R + \boldsymbol{x}(n)^T P(n \mid n-1)\boldsymbol{x}(n)}$$

Now I am going to discuss how hyperparameters process noise covariance Q and mesurement noise varianceR will effect the convergence speed of AR process.
Initially, I set the Q to be $0.01I$ and R to be 0.01 times the variance of the excitation signal of the AR process, and then discover another four groups of Q,R values. The estimated parameters of process are shown in Figure 2, the red lines stands for the true parameters of process, which are 1.2, -0.4.

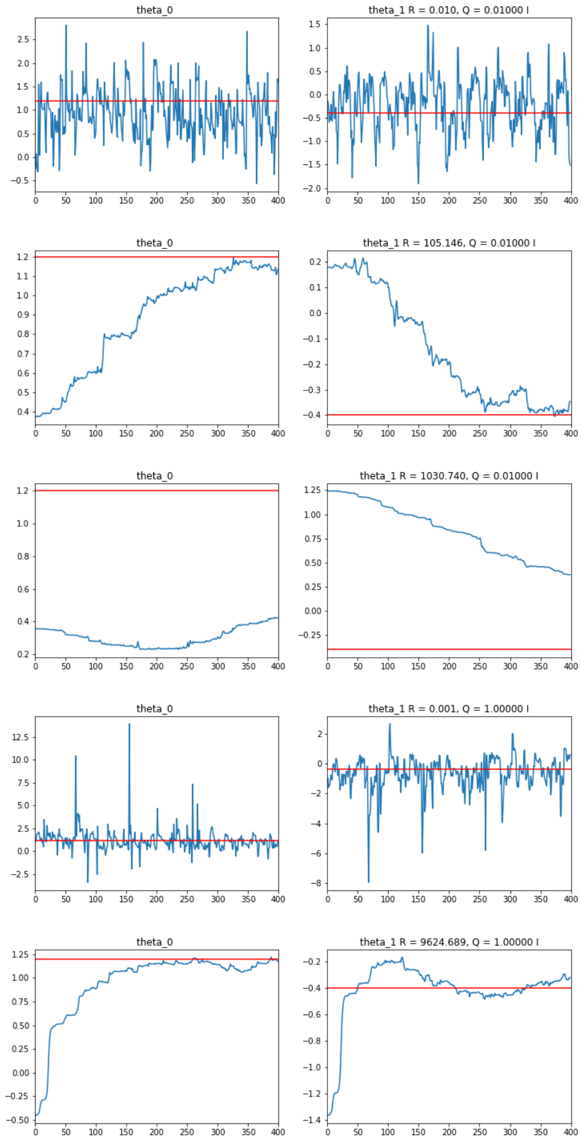

Figure 2: Estimated parameters of AR process for different Hyperparameters

From the Table 1, we can notice that when R is close to 0, the estimated parameters does not satisfy what we expected. As according to the formula of state space model, $\boldsymbol{\theta}(n|n)$ will be equal to $\frac{y(n)}{\boldsymbol{x(n)^T}}$, which means parameters would largely depends on $x(n)$, the two previous state of AR process, being more random. However, if R is very large, in the remaining three experiments, the parameters will be smoother during the whole

Table 1: Different Hyperparameters Value and convergence speed

Q	R	Convergence speed
$0.01I$	0.01	Not converged
$0.01I$	105.146	around 400 steps in time
$0.01I$	1030.740	Not converged
I	0.001	Not converged
I	9624.689	around 200 steps in time

process. In this case, the convergence speed depends on the proportion of R and Q which means if there is a large gap between them, the process needs more steps in time to get the optimal parameters.