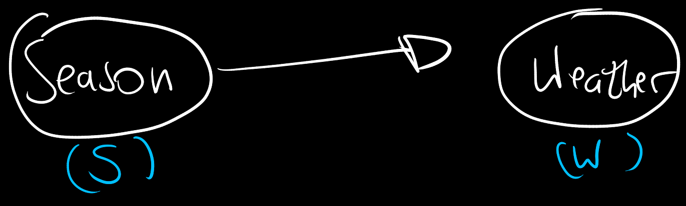
# Mixture Distributions - Intro

$S \sim$ Categorical
$W \sim$ Bernoulli




$S$... class
$W$... component

$S \in \{$ Spring, Summer, Fall, Winter $\}$ (0, 1, 2, 3)

$W \in \{$ Bad, Good $\}$ (0, 1)

special case DGM

class ... categorical

component ... any distribution (either continuous or discrete)

factorize the joint $p(S,W) = p(S) \cdot p(W|S)$ — can be any distribution *

$$= Cat(\underline{\theta}_S) \cdot Ben(\underline{\theta}_W[S])$$

$$= \left( \prod_{d=0}^{D-1} \theta_{S,d}^{\mathbb{I}(S=d)} \right) \cdot \theta_{W,S}^{W} \cdot (1-\theta_{W,S})^{1-W}$$

* even different kinds of distribution

to save:

- D-dim vector for the class probs (→ cat)
- all parameters of all component distributions (here D Bernoullis)

$$\underline{\theta}_S = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} \qquad \underline{\theta}_W = \begin{bmatrix} 0.4 \\ 0.9 \\ 0.5 \\ 0.3 \end{bmatrix}$$

assume that both RV are observed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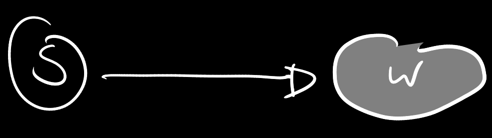
Commonly: only component is observed
↳ class is latent




Marginal:
$$p(W) = \sum_{c=0}^{D-1} p(S=c, W)$$

$$= \sum_{c=0}^{D-1} \left( \underbrace{\left( \prod_{d=0}^{D-1} \theta_{S,d}^{\mathbb{I}(c=d)} \right)}_{\theta_{S,c}} \cdot p(W|S=c) \right)$$

$$= \sum_{c=0}^{D-1} \theta_{S,c} \cdot p(W|S=c)$$

$p(W=w)$?
→ evaluate $p(W=w|S=c)$ for all possible $c$
→ then weigh the results by the respective $\theta_{S,c}$ and sum up

example:

$$p(W=1) = 0.25 \cdot 0.4 + 0.25 \cdot 0.9 + 0.25 \cdot 0.5 + 0.25 \cdot 0.3$$
$$= \underline{\underline{0.525}}$$