Push forward / jvp rule for scalar addition

$$f(x,y) = x + y = z$$

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

$$\dot{z} = \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y}$$

$$= 1\dot{x} + 1\dot{y} = \dot{x} + \dot{y}$$

$$\mathcal{J}(+, (x,y), (\dot{x},\dot{y})) = (\underbrace{(x+y)}_{z}, \underbrace{(\dot{x}+\dot{y})}_{\dot{z}})$$