The Challenges in Variational Inference

ELBO $\mathcal{L}(q) = \mathbb{E}_{z \sim q(z)} \left[\log \left(\frac{p(z, x=D)}{q(z)} \right) \right]$

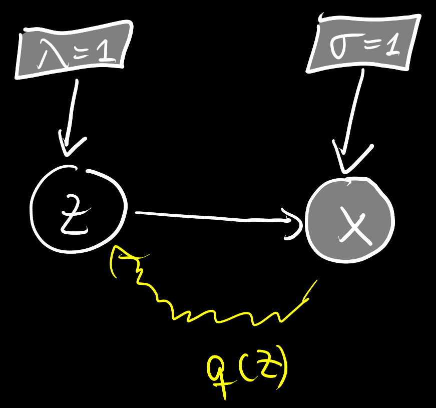
$\hookrightarrow q(z)^* = \arg\max \left(\mathcal{L}(q) \right) \implies q(z) \approx p(z \mid x = D)$

"we have the joint" ???

Agenda

1. An Example with DGM
2. What we have, what we do not have, what we want
3. Visualization

Exponential-Normal Model



$z \in \mathbb{R}_+$, $x \in \mathbb{R}$ } both 1D

$z \sim \mathrm{Exp}(z; \lambda = 1)$

$x \sim \mathcal{N}(x; \mu = z, \sigma = 1)$

What do we know?

1. The prior (on z): $p(z) = \mathrm{Exp}(z; \lambda = 1)$

$$= \begin{cases} \exp(-z), & z \geq 0 \\ 0, & \text{else} \end{cases}$$

$$= \exp(-z)\, \mathbb{I}(z \geq 0)$$

Indicator Function

2. The likelihood: $p(x \mid z) = \mathcal{N}(x; \mu = z, \sigma = 1)$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x - z)^2\right)$$

3. The joint: $p(x, z) = p(z)\, p(x \mid z)$

$$= \exp(-z)\, \mathbb{I}(z \geq 0) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x - z)^2\right)$$

What we don't know

1. The marginal: $p(x) = \int_z p(x, z)\, dz$

here $= \underbrace{\int_0^\infty \exp(-z) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-z)^2\right) dz}_{\text{intractable}}$

2. The posterior: $p(z \mid x) = \frac{p(x, z)}{p(x)}$

but we want the posterior ? :(

$\implies$ VI attempts to find a surrogate $q(z)$ by maximizing the ELBO

→ we observe some data $D = \{ x^{[0]} = 1.3 \}$

$\hookrightarrow$ fix the joint to the data

$$p(z, x = D) = \exp(-z)\, \mathbb{I}(z \geq 0) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(1.3 - z)^2\right)$$

we can query this for any arbitrary z values

$p(z = 1.5, x = 1.3) \approx 0.087$