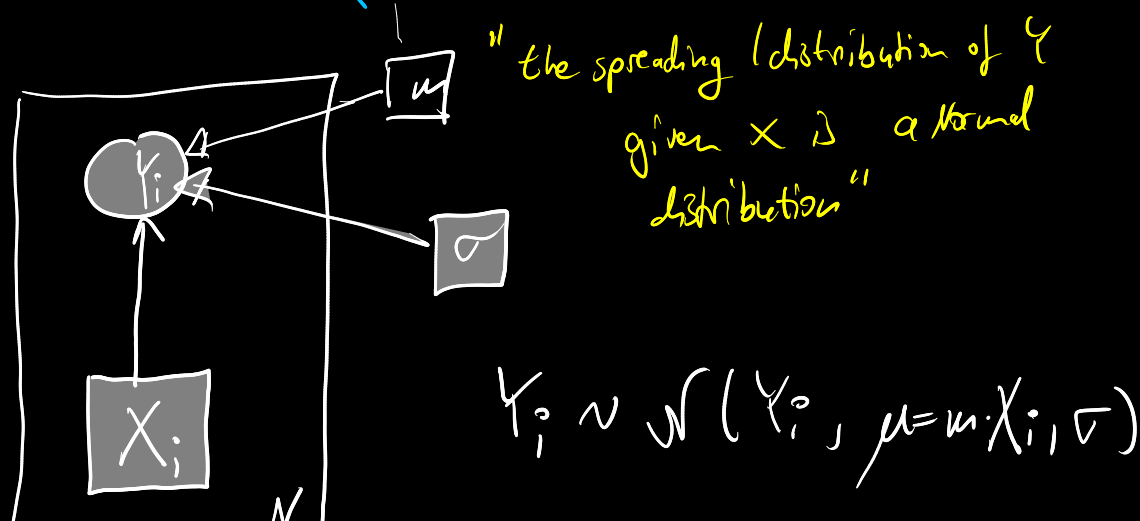
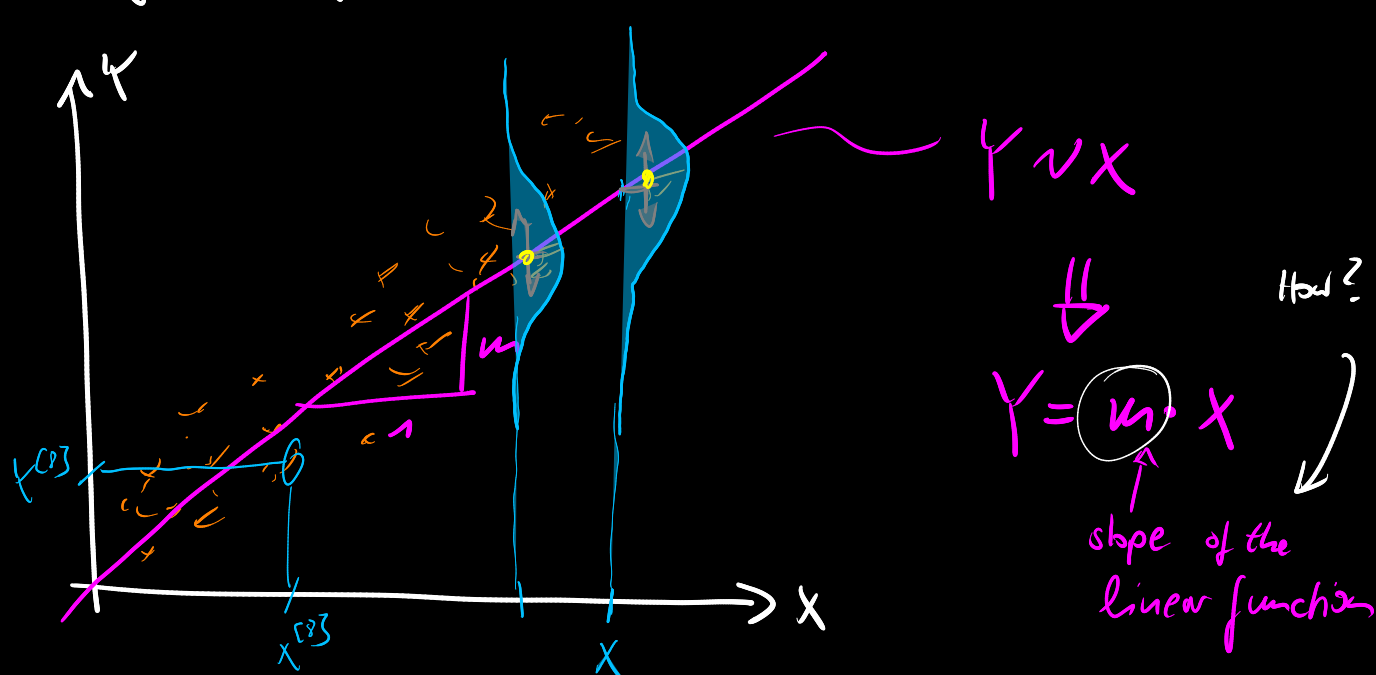


Linear Regression from a Probabilistic Perspective



$$p(y_i | x_i, m, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2} (y_i - \underbrace{m \cdot x_i}_{\mu})^2\right)$$

$$\mathcal{L}(\mathcal{D}) \stackrel{\text{i.i.d.}}{=} \prod_{i=0}^{N-1} p(y_i = y^{(j)} | x_i = x^{(j)}, m, \sigma)$$

$$\mathcal{D} = \underbrace{\{(x^{(0)}, y^{(0)}), (x^{(1)}, y^{(1)}), \dots, (x^{(N-1)}, y^{(N-1)})\}}_{N \text{ samples}}$$

$$\begin{aligned} \mathcal{L}(\mathcal{D}) &= \prod_{i=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2} (y^{(j)} - m x^{(j)})^2\right) \\ &= \left(\prod_{i=0}^{N-1} \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)\right) \cdot \left(\prod_{i=0}^{N-1} \exp\left(-\frac{1}{2\sigma^2} (y^{(j)} - m x^{(j)})^2\right)\right) \\ &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N \cdot \exp\left(-\frac{1}{2\sigma^2} \sum_{i=0}^{N-1} (y^{(j)} - m x^{(j)})^2\right) \end{aligned}$$

Log-Likelihood

$$\begin{aligned} \ell(\mathcal{D}) &= \log \mathcal{L}(\mathcal{D}) \\ &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N - \frac{1}{2\sigma^2} \sum_{i=0}^{N-1} (y^{(j)} - m x^{(j)})^2 \\ &\stackrel{+}{=} -\frac{1}{2\sigma^2} \sum_{i=0}^{N-1} (y^{(j)} - m x^{(j)})^2 \\ &\sim -\frac{1}{2} \sum_{i=0}^{N-1} (y^{(j)} - m x^{(j)})^2 \end{aligned}$$

$$\text{Goal: } m^* = \arg \max_m \ell(\mathcal{D})$$

$$m^* = \arg \max_m \left(-\frac{1}{2} \sum_{i=0}^{N-1} (y^{(j)} - m x^{(j)})^2\right)$$

$$= \boxed{\arg \min_m \left(\frac{1}{2} \sum_{i=0}^{N-1} (y^{(j)} - m x^{(j)})^2\right)}$$

→ MLE under "a Gaussian error distribution"

This can be solved analytically 😊

$$\frac{\partial \ell}{\partial m} = \frac{1}{2} \sum_{i=0}^{N-1} 2(y^{(j)} - m x^{(j)}) \cdot (-x^{(j)}) \stackrel{!}{=} 0$$

$$= \sum_{i=0}^{N-1} -y^{(j)} x^{(j)} + m x^{(j)2} \stackrel{!}{=} 0$$

$$m \sum_{i=0}^{N-1} x^{(j)2} = \sum_{i=0}^{N-1} y^{(j)} x^{(j)}$$

$$m^* = \frac{\sum_{i=0}^{N-1} y^{(j)} x^{(j)}}{\sum_{i=0}^{N-1} x^{(j)2}}$$

→ for the univariate case, $x^{(j)} \in \mathbb{R}$

→ no regularization