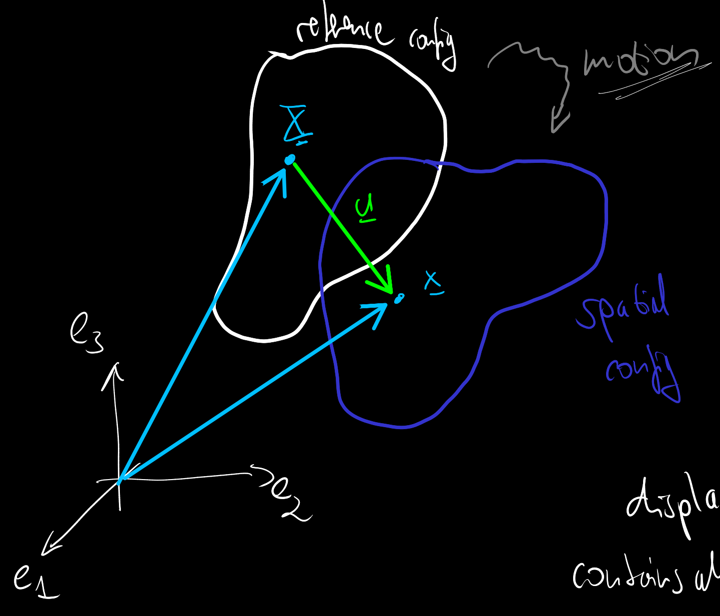
Displacement Gradient



displacement vector $\underline{u}$

contains all information of potato change

translation

rotation

shape change

shear

elongation

compression

$\Rightarrow$ Displacement gradient $\underline{\underline{H}}$

wrt reference config

$$\underline{\underline{H}} = \mathrm{Grad}\left(\underline{u}(\underline{X},t)\right) = \frac{\partial \underline{u}}{\partial \underline{X}}$$

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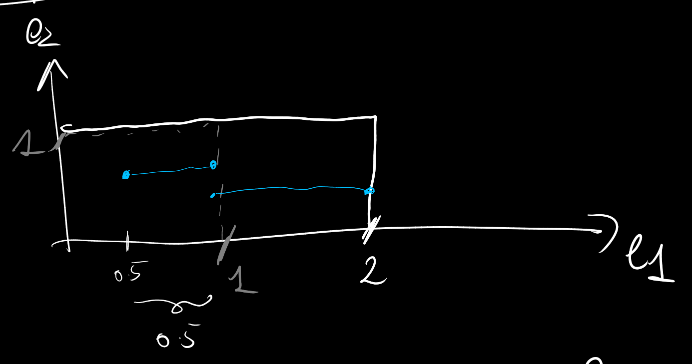
example



$$\underline{u}(\underline{X},t) \xrightarrow{\text{here}} \underline{u}(\underline{X}) = \begin{bmatrix} X_1 \\ 0 \end{bmatrix}$$

$$\underline{\underline{H}} = \frac{\partial \underline{u}}{\partial \underline{X}} = \begin{bmatrix} \frac{\partial u_1}{\partial X_1} & \frac{\partial u_1}{\partial X_2} \\ \frac{\partial u_2}{\partial X_1} & \frac{\partial u_2}{\partial X_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

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example

$$\underline{u}(\underline{X},t) = \begin{bmatrix} 3X_3^2 t - X_2 X_1 \\ X_1^4 \\ 5X_1^2 X_3^2 \end{bmatrix}$$

$$\underline{\underline{H}} = \frac{\partial \underline{u}}{\partial \underline{X}} = \begin{bmatrix} \frac{\partial u_1}{\partial X_1} & \frac{\partial u_1}{\partial X_2} & \frac{\partial u_1}{\partial X_3} \\ \frac{\partial u_2}{\partial X_1} & \frac{\partial u_2}{\partial X_2} & \frac{\partial u_2}{\partial X_3} \\ \frac{\partial u_3}{\partial X_1} & \frac{\partial u_3}{\partial X_2} & \frac{\partial u_3}{\partial X_3} \end{bmatrix} = \begin{bmatrix} -X_2 & -X_1 & 6X_3 t \\ 4X_1^3 & 0 & 0 \\ 10X_1 X_3^2 & 0 & 10X_1^2 X_3 \end{bmatrix}$$

Remarks:

1. generally $\underline{\underline{H}}$ is not symmetric
2. $\underline{\underline{H}}$ is still a function of $\underline{X}$ & t