# Multinomial Distribution - Intro

Weather (W) → Cloudy $\theta_0$, Rainy $\theta_1$, Sunny $\theta_2$ ; $\underline{\theta}$

$W \in \{C, R, S\}$

observe the weather

$D = \{C, S, S, R, C, S\}$

$N = 7$ days

$\Rightarrow$ 2 out of 7 cloudy
1 out of 7 rainy
4 out of 7 sunny

what is the probability for that?

just use a Categorial

$W \sim Cat(\underline{\theta})$

e.g. $\underline{\theta} = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}$

likelihood

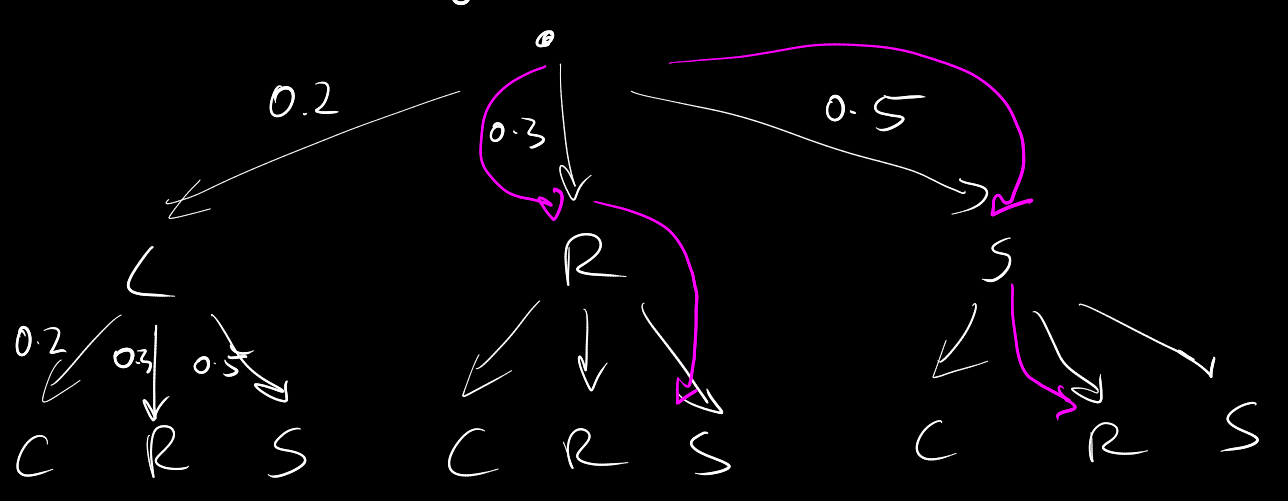
$p(D) = 0.2^2 \cdot 0.3^1 \cdot 0.5^4$

$= 0.0075$

↳ wrong (it's too low)

$\Rightarrow$ there are multiple paths (similar to Binomial)

e.g. $n = 2$ days




e.g. prob 1 out of 2 rainy, 1 out of 2 sunny

$\Rightarrow$ 2 paths

$p(\dots) = 0.3 \cdot 0.5 + 0.5 \cdot 0.3$

$= \underbrace{2} \cdot \underbrace{0.3 \cdot 0.5}$

# of paths → Multinomial coefficient

prob for any of paths → Similar to the Categorial

(why the Categorial is sometimes called Multinomial)

↳ has to be a composition of one-hot Categorials

$x = [0, 1, 1]^T = \underbrace{[0, 1, 0]^T}_{\text{rainy}} + \underbrace{[0, 0, 1]^T}_{\text{sunny}}$

(but also $x = [2, 0, 1]$, $n = 3$)

$$p(x) = \frac{n!}{\prod_{d=0}^{D-1} x_d!} \prod_{d=0}^{D-1} \theta_d^{x_d} = \text{Multinomial}(\underline{\theta}, n)$$

Multinomial coefficient — prob of one path

## To save

- $\underline{\theta}$ ... prob for each state (for each weather type)
- $n$ ... number of observations (number of days)

## restrictions

$\underline{\theta} \in [0, 1]^D$ , $\sum_{d=0}^{D-1} \theta_d = 1$ , $\sum_{d=0}^{D-1} x_d = n$

## Dataset (e.g. $n = 7$ days)

$D = \{[2, 3, 2], [1, 1, 5], [2, 1, 4], \dots\}$