Partielle Ableitungen

Ableitungen bei Skalarfeldern

$$s = s(x_1, x_2) = x_1^2 + x_2^2$$

$$\frac{ds}{dx_1} = ?$$

↳ partiell (alle anderen Variablen Konstant halten)

$$\frac{\partial s}{\partial x_1} \stackrel{!}{=} \frac{d}{dx_1}(x_1^2 + a^2) = 2x_1^2$$

Beispiele: $s(x_1, x_2) = x_1^2 x_2^3$

$$\frac{\partial s}{\partial x_1} = 2x_1 x_2^3$$

$$\frac{\partial s}{\partial x_2} = x_1^2 3x_2^2$$

$$s(x_1, x_2) = x_1 \log(x_1) x_2^3$$

$$\frac{\partial s}{\partial x_1} = \dots \text{Produktregel} \dots = \log(x_1) x_2^3 + \frac{x_1}{x_1} x_2^3 = \log(x_1) x_2^3 + x_2^3$$

$$\frac{\partial s}{\partial x_2} = x_1 \log(x_1) 3x_2^2$$

Höhere partielle Ableitungen

$$s(x_1, x_2) = x_1 \log(x_1) x_2^3$$

$$\frac{\partial s}{\partial x_1} = \log(x_1) x_2^3 + x_2^3$$

$$\frac{\partial^2 s}{\partial x_2 \partial x_1} = \log(x_1) 3x_2^2 + 3x_2^2$$

$$\frac{\partial^2 s}{\partial x_1^2} = \frac{x_2^3}{x_1} + x_2^3$$

$$\frac{\partial s}{\partial x_2} = x_1 \log(x_1) 3x_2^2$$

$$\frac{\partial^2 s}{\partial x_1 \partial x_2} = \log(x_1) 3x_2^2 + \frac{x_1}{x_1} 3x_2^2$$

$$\frac{\partial^2 s}{\partial x_2^2} = x_1 \log(x_1) 6x_2$$

Satz von Schwartz

$$\frac{\partial^2 s}{\partial x_2 \partial x_1} = \frac{\partial^2 s}{\partial x_1 \partial x_2}$$

Reihenfolge der Ableitungen egal?