Push forward / jvp rule of scalar multiplication

$$f(x,y) = x \cdot y = z$$

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

$$\dot{z} = \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y}$$

$$= y\dot{x} + x\dot{y}$$

$$\boxed{\mathcal{F}(\cdot, (x,y), (\dot{x},\dot{y})) = \big(\underbrace{(x \cdot y,)}_{z}, \underbrace{(y\dot{x} + x\dot{y},)}_{\dot{z}} \big)}$$