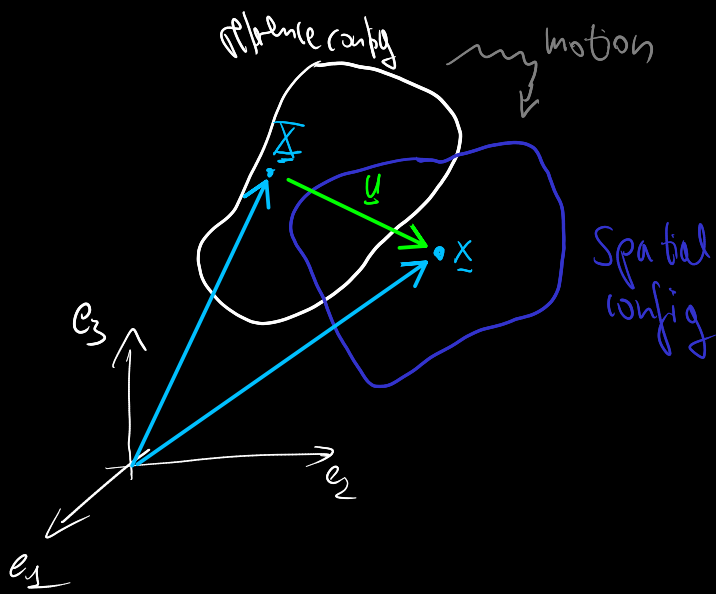


# Relation between Deformation & Displacement gradient



Motion:

$$\underline{\underline{X}} \rightarrow \underline{x}$$

assigns each material point its position in spatial config

displacement:

$$\underline{\underline{X}} \rightarrow \underline{u}$$

assigns a displacement vector to each material point reference configuration

Deformation gradient:  $\underline{\underline{F}} = \frac{\partial \underline{x}}{\partial \underline{\underline{X}}}$

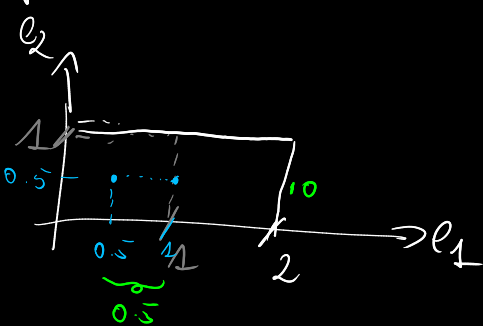
Displacement gradient:  $\underline{\underline{H}} = \frac{\partial \underline{u}}{\partial \underline{\underline{X}}}$

We know:  $\underline{x} = \underline{\underline{X}} + \underline{u}$

then  $\underline{\underline{F}} = \frac{\partial \underline{x}}{\partial \underline{\underline{X}}} = \frac{\partial \underline{\underline{X}}}{\partial \underline{\underline{X}}} + \frac{\partial \underline{u}}{\partial \underline{\underline{X}}}$

$$\underline{\underline{F}} = \underline{\underline{I}} + \underline{\underline{H}}$$

Example



$$\underline{x} = \chi(\underline{\underline{X}}) = \begin{bmatrix} 2\underline{\underline{X}}_1 \\ \underline{\underline{X}}_2 \end{bmatrix}$$

$$\underline{u} = \begin{bmatrix} \underline{\underline{X}}_1 \\ 0 \end{bmatrix}$$

$$\underline{\underline{F}} = \frac{\partial \underline{x}}{\partial \underline{\underline{X}}} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{\underline{H}} = \frac{\partial \underline{u}}{\partial \underline{\underline{X}}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{F}} = \underline{\underline{I}} + \underline{\underline{H}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \underline{\underline{\underline{\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}}}}}$$