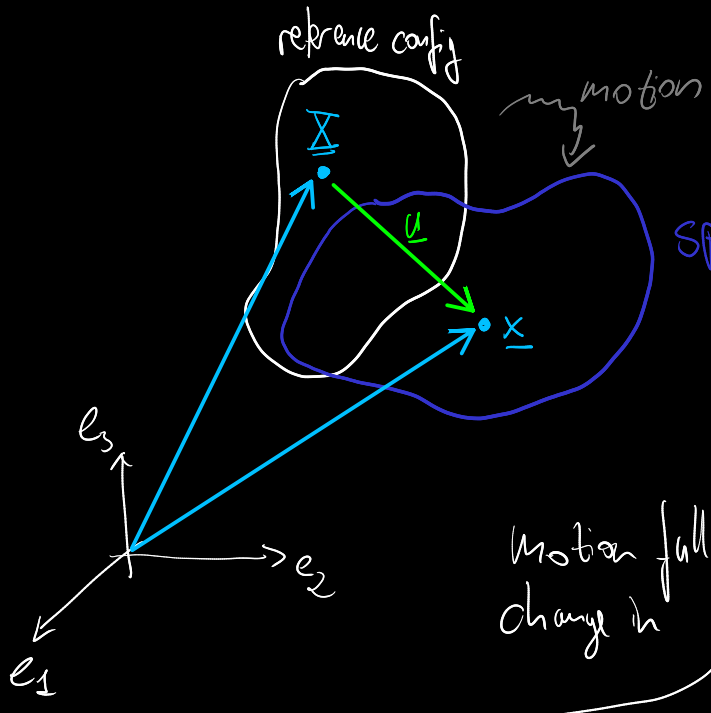
Deformation Gradient : $\underline{\underline{F}}$



motion fully describes change in configuration

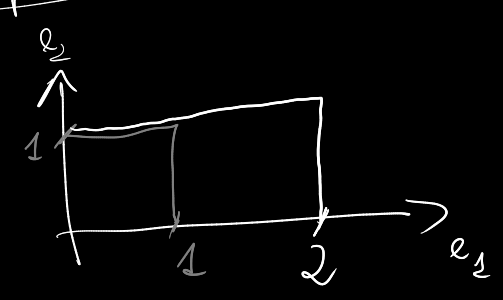
- translation
- rotation
- Shape change
  - Shear?
  - elongation?
  - bending?

gradient in reference config

$\Rightarrow$ Deformation gradient $\underline{\underline{F}}$

$$\underline{\underline{F}} = \mathrm{Grad}\left(\underline{\chi}(\underline{X}, t)\right) = \frac{\partial \underline{x}}{\partial \underline{X}}$$

Example



$$\underline{x} = \underline{\chi}(\underline{X}, t) \overset{\text{here}}{=\!=} \underline{\chi}(\underline{X}) = \begin{bmatrix} 2\underline{X}_1 \\ \underline{X}_2 \end{bmatrix}$$

$$\underline{\underline{F}} = \frac{\partial \underline{x}}{\partial \underline{X}} = \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Example 2

$$\underline{\chi}(\underline{X}, t) = \begin{bmatrix} 3\underline{X}_1^2 \underline{X}_2 + t\underline{X}_3^3 \\ \underline{X}_3^2 - t\underline{X}_1 \\ 5\underline{X}_2 \underline{X}_3 \end{bmatrix}$$

$$\underline{\underline{F}} = \frac{\partial \underline{x}}{\partial \underline{X}} = \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix} = \begin{bmatrix} 6\underline{X}_1 \underline{X}_2 & 3\underline{X}_1^2 & 3t\underline{X}_3^2 \\ -t & 0 & 2\underline{X}_3 \\ 0 & 5\underline{X}_3 & 5\underline{X}_2 \end{bmatrix}$$

① $\underline{\underline{F}}$ not necessarily symmetric

② $\underline{\underline{F}}$ is still a function of $\underline{X}$ & t