A DUAL-MATRIX APPROACH TO THE TRANSPORTATION PROBLEM

Ping JI and K. F. CHU
Department of Industrial and Systems Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
email: mfpji@polyu.edu.hk

The transportation model is a special case of linear programming models, widely used in the areas of inventory control, employment scheduling, aggregate planning, and personnel assignment, among others. Due to its special structure, the stepping-stone method is commonly adopted in order to improve the computational efficiency instead of the regular simplex method. This paper proposes a new approach, the dual-matrix approach to solve the transportation problem. The dual-matrix approach is very efficient in terms of computation. The algorithm of this approach is presented, and explained briefly as the regular simplex method and the stepping-stone method. Finally, a numerical example is described in the paper to show its efficiency.

Keywords: Transportation problem, dual-matrix approach, stepping-stone method, simplex method, linear programming models.

1. Introduction

The transportation model considers minimum-cost planning problems for shipping a product from some origins to other destinations, such as from factories to warehouses, or from warehouses to supermarkets, with the shipping cost from one location to another being a linear function of the number of units shipped. The transportation model is a special case of the linear programming models, and obviously, it can be solved by the regular simplex method. Due to its special structure of the model, the stepping-stone method (Charnes and Cooper, 1954) was developed for the efficiency reason while the simplex method is not suitable for the transportation problem, especially for those large-scale transportation problems. Other research results can be found from Ford and Fulkerson (1956), Balinski and Gomory (1964), Muller-Merbach (1966), Grigoriadis and Walker (1968), Glover *et al.* (1974), Shafaat and Goyal (1988), and Arsham and Kahn (1989). A brief review on this area was presented by Gass (1990).

This paper presents a new simple approach to solve the transportation problem, which can be an alternative to the stepping-stone method. The approach considers the dual of the transportation model instead of the primal, and obtains the optimal solution of the dual by use of matrix operations. So, it is called a dual-matrix approach. The approach will be explained as the simplex method or the stepping-stone method. The algorithm of the approach is detailed in the paper, and finally a numerical example is given to illustrate the approach.

2. The Model and Its Dual

The purpose of transportation modeling is to find the least cost means of shipping supplies from several origins to several destinations. Origins can be factories, warehouses, or any other locations from which goods are shipped. Destinations are any locations that receive goods. The transportation problem is usually presented as a matrix as shown in Figure 1. The unit transportation cost is generally indicated on the northeast corner in each cell.

	Destination 1	Destination 2	Destination ...	Destination j	Destination ...	Destination n	Supply
Origin 1	c_{11} x_{11}	c_{12} x_{12}	...	c_{1j} x_{1j}	...	c_{1n} x_{1n}	a_1
Origin 2	c_{12} x_{21}	c_{22} x_{22}	...	c_{2j} x_{2j}	...	c_{2n} x_{2n}	a_2
Origin ...	... ...	... ...	...	... ...	...	... ...	$a_{...}$
Origin i	c_{i1} x_{i1}	c_{i2} x_{i2}	...	c_{ij} x_{ij}	...	c_{in} x_{in}	a_i
Origin ...	... ...	... ...	...	... ...	...	... ...	$a_{...}$
Origin m	c_{m1} x_{m1}	c_{m2} x_{m2}	...	c_{mj} x_{mj}	...	c_{mn} x_{mn}	a_m
Demand	b_1	b_2	$b_{...}$	b_j	$b_{...}$	b_n	

Figure 1. Transportation problem matrix

This problem can be expressed as a linear programming model as follows:

$$\text{Minimize} \quad \phi = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, ..., m) \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, ..., n) \tag{2}$$

$$x_{ij} \geq 0 \qquad (i = 1, ..., m; \ j = 1, ..., n) \tag{LP1}$$

Here, all a_i and b_j are assumed to be positive, and a_i are normally called supplies and b_j are called demands, as shown in Figure 1. The cost c_{ij} are all nonnegative. If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, it is a balanced transportation problem. If this condition is not met, a dummy origin or destination is generally introduced to make the problem balanced in order to use the stepping-stone method. The dual-matrix approach presented later does not require a transportation problem to be balanced. The approach can be applied to both balanced and unbalanced problems. So the constraints set (1) is represented as "$\leq$" and constraint set (2) as "$\geq$" instead of "$=$" for both cases in a balanced problem. This is one of the advantages of this new approach over the stepping-stone method: the unbalanced problem is not required to be converted into a balanced problem, no dummy origin or destination is introduced, so some time and space are saved.

The stepping-stone method is very popularly used to solve the transportation problem. It is a technique for moving from an initial feasible solution to an optimal solution by evaluating all non-basic cells, i.e., the empty cells. The stepping-stone method adopts the path tracing approach to evaluate an empty cell. Another way to evaluate empty cells is the modification distribution method (MODI). The modification distribution method is similar to the stepping-stone method, and Charnes and Cooper (1954) explained how these two methods are related. So both methods are referred to as the stepping-stone method later in this paper. One serious problem of the stepping-stone method is the degeneracy, that is, too few basic (occupied) cells in a feasible solution. Some researches were carried out to solve the degeneracy problem (Goyal, 1984 and Shafaat and Goyal, 1988). The dual-matrix approach presented later is similar to the idea of the stepping-stone method: find an initial feasible solution first, then get next improved solution by assessing all non-basic cells until the optimal solution is found. However, the dual-matrix approach considers the dual model of the transportation problem, and main operations are calculated on a matrix. No

path tracing is required in this approach. Besides, the degeneracy in the stepping-stone method does not exist in the dual-matrix approach due to no path tracing.

If model LP1 is considered as the primal, then its dual can be for $_{1}$-lated as follows:

$$\text{Maximize} \quad \psi = \sum_{j=1}^{n} b_j v_j - \sum_{i=1}^{m} a_i u_i$$

$$\text{Subject to} \quad v_j - u_i \leq c_{ij} \quad (i = 1, ..., m; \ j = 1, ..., n) \qquad (3)$$

$$u_i, \ v_j \geq 0 \quad (i = 1, ..., m; \ j = 1, ..., n) \qquad (LP2)$$

Here, all u_i and v_j are dual variables.

The primal problem has $m + n$ constraints and $m \times n$ variables. As pointed out by Gass (1990), the stepping-stone method "is not the method of choice for those who are serious about solving the transportation problem – such as an analyst who is concerned with solving quite large problems and may have to solve the problems repetitively". The dual has $m + n$ variables and $m \times n$ constraints. Because of fewer variables, the dual problem will be solved by the dual-matrix approach presented later. However, all $m \times n$ constraints are not necessary to be presented explicitly as in the dual LP2. All these constraints are kept as in their original matrix form, e.g., the transportation matrix, as indicated in Figure 1. This is the same as the stepping-stone method.

3. The Dual-Matrix Approach

Similar to the stepping-stone method, the occupied cells are called basic cells, and all other empty cells are called non-basic cells in the dual-matrix approach. The main idea of the dual-matrix approach is first to obtain a feasible solution to the dual problem and its corresponding matrix. Then the duality theory is used to check the optimal condition and to get the leaving cell. All non-basic cells are evaluated in order to get the entering cell. Finally, the entering cell replaces the leaving cell and the matrix is updated. The dual-matrix approach is presented as follows:

Step 0 Initialization:

      Step 0.1: Set $\boldsymbol{A} = (b_1, b_2, ..., b_n, \ -a_1, -a_2, ..., -a_m)$.

      Step 0.2: Set $u_i = 0$; $(i = 1, 2, ..., m)$ and let
            $v_j = \bar{c}_{ij} = \min\{c_{ij}, \ i = 1, 2, ..., m\}$; $j = 1, 2, ..., n$. Ties
            can be broken arbitrarily. The corresponding cells to $\bar{c}_{ij}$
            are (i_j, j) $(j = 1, 2, ..., n)$, respectively.

Step 0.3: Let the basic cell set $\Gamma = \{(i_1,1),(i_2,2),...,(i_n,n),\ (1,0),$ $(2,0),...,(m,0)\}$. The cells $(1,0),(2,0),...,(m,0)$ are called virtual cells because they do not exist in the original transportation problem matrix.

Step 0.4: Let the matrix $\boldsymbol{D} = [d_{ij}];\ i,j = 1,2,...,m+n;$

$$\text{where } d_{ij} = \begin{cases} 1 & i,j = 1,2,...,n; \\ -1 & i = 1,2,...,n;\ j = n+i_1, n+i_2,...,n+i_n; \\ -1 & i,j = n+1, n+2,...,n+m; \\ 0 & \text{otherwise.} \end{cases}$$

and compute the objective: $\psi = \sum_{j=1}^{n} b_j v_j - \sum_{i=1}^{m} a_i u_i$.

Step 1 Determination of the leaving cell:

Step 1.1: Compute $\boldsymbol{Y} = \boldsymbol{AD}$.

Step 1.2: Find the smallest value y_k in the elements of $\boldsymbol{Y}$, that is, the value of the kth element in $\boldsymbol{Y}$ is the smallest. Ties can be broken arbitrarily.

Step 1.3: If $y_k \geq 0$, the solution is optimal (both the dual and primal), stop. Otherwise, the leaving cell is the kth cell in Γ, that is, (i_k, j_k).

Step 2 Determination of the entering cell:

Step 2.1: Let

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_j \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} d_{1,k} \\ d_{2,k} \\ \vdots \\ d_{j,k} \\ \vdots \\ d_{n,k} \end{bmatrix} \text{ and } P = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_i \\ \vdots \\ p_m \end{bmatrix} = \begin{bmatrix} d_{n+1,k} \\ d_{n+2,k} \\ \vdots \\ d_{n+j,k} \\ \vdots \\ d_{n+m,k} \end{bmatrix}$$

Step 2.2: For all non-basic cells, if $p_i - q_j \leq 0$, then the dual problem is not bounded, and the original primal problem has no feasible solution, and stop. Otherwise, compute

$$\theta_{ij} = c_{ij} + u_i - v_j \quad \text{if} \quad p_i - q_j > 0$$

Step 2.3: Find the smallest value θ_{st} in all θ_{ij}, and the cell (s,t) is the entering cell. Ties can be broken arbitrarily.

Step 3 Updating:

Step 3.1: Update the matrix $\boldsymbol{D}$.

Step 3.1.1: For the elements of column k in $\boldsymbol{D}$:

$$\hat{d}_{lk} = -d_{lk} \quad l = 1, 2, ..., m + n;$$

Step 3.1.2: For the elements of other columns in $\boldsymbol{D}$:

$$\hat{d}_{lr} = d_{lr} + (d_{s-n,r} - d_{tr})\hat{d}_{lk} \begin{cases} r = 1, 2, ..., k - 1, \ k + 1, ..., m + n; \\ l = 1, 2, ..., m + n. \end{cases}$$

Step 3.2: Update the basic cell set Γ: replace the kth cell (i_k, j_k) in Γ with the entering cell (s, t).

Step 3.3: Update the objective value:
Compute:

$$\hat{u}_i = u_i - \theta_{st} p_i \quad i = 1, 2, ..., m;$$
$$\hat{v}_j = v_j - \theta_{st} q_j \quad j = 1, 2, ..., n.$$

and the objective:

$$\psi = \sum_{j=1}^{n} b_j v_j - \sum_{i=1}^{m} a_i u_i$$

Go to Step 1.

The initialization procedure (Step 0) is to obtain an initial feasible solution. By setting $u_i = 0$ and v_j being the smallest cost in the column j, obviously, they meet the constraint set (3) in the dual problem. The matrix $\boldsymbol{D}$ is an $(m - n) \times (m + n)$ matrix, which can be divided into four submatrices as follows:

1. The upper left submatrix is an $n \times n$ identity matrix.
2. The upper right submatrix is an $n \times m$ matrix: If the cell (i, j) is a basic cell (corresponding to $\bar{c}_{ij}$), then the element (j, i) in this submatrix is -1. All other elements in this submatrix are 0.
3. The lower left submatrix is an $m \times n$ zero matrix.
4. The lower right submatrix is an $m \times m$ negative identity matrix.

During the main procedure of the dual-matrix approach, Step 1 is to get the leaving cell, similar to getting a leaving variable in the simplex method. As a matter of fact, the initial feasible solution in the dual-matrix approach is a very good starting point. From the objective function in the dual LP2, it is obvious that u_i should be the smaller, the better. The smallest is 0 for all u_i.

On the other hand, v_j should be the larger, the better. However, due to the constraint set (2), a v_j can only be the minimum value of c_{ij} in the column j. Step 2 is to obtain the entering cell by evaluating all non-basic cells, which is similar to the stepping-stone method. The equation in Step 2.2 is the same as the one in the stepping-stone method (MODI, precisely) except the sign of u_i. Finally, the matrix D and other relevant data are updated accordingly. To explain this approach mathematically, a cell (i, j) can be represented as an equation, that is, $v_j - u_i = c_{ij}$. The mathematical background of the dual-matrix approach is to find $m \div n$ equations, i.e., $m + n$ basic cells from constraint set (3) in the dual. If these equations cannot maximize the objective of the dual, that is, the solution is not optimal, find another cell (equation), i.e., the entering cell, to replace one equation (the leaving cell) in the equation set until an optimal solution is found. So, the algorithm of this approach is very simple, as shown previously.

4. Numerical Example

The following 3×2 problem is from Davis *et al.* (1986). The problem was used to demonstrate the degeneracy of the stepping-stone method (Davis *et al.*, 1986). This problem was also used by Arsham and Kahn (1989) to show the efficiency of their simplex-type algorithm. There are three purposes for adoption of this problem: (1) the dual-matrix approach does not have the degeneracy as the stepping-stone method has; (2) the approach is very efficient; (3) this approach is very concise compared with the Arsham and Kahn's simplex-type algorithm. The Arsham and Kahn's simplex-type algorithm needs to set up a simplex tableau (Arsham and Kahn, 1989), which is a heavy waste in space due to the special structure of the transportation problem. Their algorithm can be concerned as a variant of the simplex method, and they compared their results with a commercial linear programming package, LINDO, which treats the transportation problem as a standard linear programming model. So it is not very good to solve the transportation problem by their algorithm, as pointed out by Gass (1990).

The example is shown in Figure 2. This is an unbalanced problem. A dummy destination can be added to make the problem balanced. However, this is not required in the dual-matrix approach.

	Destination 1	Destination 2	Supply
Origin 1	x_{11} 3	x_{12} 6	400
Origin 2	x_{21} 4	x_{22} 5	300
Origin 3	x_{31} 7	x_{32} 3	400
Demand	450	350	

Figure 2. A numerical example

Step 0 Initialization:

$$A = (450, 350, -400, -300, -400)$$

$$u_1 = u_2 = u_3 = 0 \text{ and } v_1 = 3, v_2 = 3$$

$$\Gamma = \{(1,1), (3,2), (1,0), (2,0), (3,0)\}$$

$$D = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and

$$\psi = \sum_{j=1}^{n} b_j v_j - \sum_{i=1}^{m} a_i u_i = 2400$$

Step 1 Determination of the leaving cell:

$$Y = AD = (450, 350, -50, 300, 50)$$

So, $k = 3$ and the leaving cell is $(1, 0)$ in Γ.

Step 2 Determination of the entering cell:

$$Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} d_{13} \\ d_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \text{ and } P = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} d_{33} \\ d_{43} \\ d_{53} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

Among the all non-basic cells, the cells $(1, 2)$ and $(2, 2)$ have non-positive $(p_i - q_j)$'s, while the cells $(2, 1)$ and $(3, 1)$ have positive $(p_i - q_j)$'s, so

$$\theta_{st} = \min \{\theta_{21}, \theta_{31}\} = \min \{4 + 0 - 3, 7 + 0 - 3\} = \min \{1, 4\} = 1$$

Now the entering cells (s, t) is $(2, 1)$.

Step 3 Updating:

$$\begin{bmatrix} \hat{d}_{13} \\ \hat{d}_{23} \\ \hat{d}_{33} \\ \hat{d}_{43} \\ \hat{d}_{53} \end{bmatrix} = - \begin{bmatrix} d_{13} \\ d_{23} \\ d_{33} \\ d_{43} \\ d_{53} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{d}_{11} \\ \hat{d}_{21} \\ \hat{d}_{31} \\ \hat{d}_{41} \\ \hat{d}_{51} \end{bmatrix} = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \\ d_{51} \end{bmatrix} + (d_{41} - d_{11}) \begin{bmatrix} \hat{d}_{13} \\ \hat{d}_{23} \\ \hat{d}_{33} \\ \hat{d}_{43} \\ \hat{d}_{53} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (0 - 1) \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Similarly, other elements in D can be updated and

$$D = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\Gamma = \{(1,1), (3,2), (2,1), (2,0), (3,0)\}$$

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} - \theta_{st} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 1 \times \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \theta_{st} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - 1 \times \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

and

$$\psi = \sum_{j=1}^{n} b_j v_j - \sum_{i=1}^{m} a_i u_i = 2450$$

Now

$$Y = AD = (400, 350, 50, 250, 50)$$

so the optimal solution is obtained with the objective $\phi = \psi = 2450$, with $x_{11} = 400$, $x_{32} = 350$, $x_{21} = 50$, $x_{20} = 250$, and $x_{30} = 50$.

If a dummy destination is introduced to make the problem balanced with the costs are 0's for those dummy cells, the objective will be the same as above, with $x_{11} = 400$, $x_{32} = 350$, $x_{21} = 50$, $x_{23} = 250$, $x_{33} = 50$, and $x_{30} = 0$. Here, there are 6 basic cells since now it is a 3×3 transportation problem, and x_{23} and x_{33} are the dummy cells in this newly-created balanced

problem. Two virtual cells $x_{20} = 250$, and $x_{30} = 50$ in the original problem can be explained as the dummy cells in the balanced problem. However, the virtual cell $(3, 0)$ in the solution of the balanced problem is really a virtual cell because it really does not exist.

5. Conclusions

A new approach, the dual-matrix approach, to the transportation problem was addressed in this paper. The approach considers the dual of the transportation model, starts from a good feasible solution, and uses a matrix to get next better solution until an optimal solution is obtained. The approach adopts the linear algebra to solve the transportation problem. A new concept, virtual cells, is introduced in this approach.

The dual-matrix approach can be applied to both balanced and unbalanced transportation problems. An unbalanced transportation problem is not required to be converted into a balanced problem, unlike the stepping-stone method. Another advantage over the stepping-stone method is that the dual-matrix method does not have the degeneracy problem. The third feature of the approach is no path tracing. The disadvantage of the dual-matrix approach is that the approach needs an $(m+n) \times (m+n)$ matrix. This problem is not serious to today's advanced computers, even for a large-scale transportation problem.

In order to test the efficiency of the dual-matrix method, several numerical examples were carried out. By comparing with the stepping-stone method, the numerical examples show that the dual-matrix approach is always better than the stepping-stone method. For a large-scale transportation problem, the path tracing and degeneracy in the stepping-stone method may be more serious. Since the dual-matrix approach does not have these two problems, it may be a good choice for the transportation problem. So the conclusion of this paper is that the dual-matrix approach may be a good alternative to a transportation problem, and perhaps, for a large-scale transportation problem.

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Dr P. JI is an Associate Professor in the Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University. He got his Ph.D. in Industrial Engineering from West Virginia University, U.S.A. His research interests are in production management, operations research, and CAD/CAM. He has published several papers in the European Journal of Operational Research, Computers & Operations Research, International Journal of Production Research, and Transactions of ASME: Journal of Manufacturing Science and Engineering.

Dr K. F. CHU is an Assistant Professor in the Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University. He got his M.Sc. and MBA degree from the University of Bradford, U.K. Later he obtained his Advanced Diploma in Education from the Hong Kong University and Ph.D. from the Loughborough University, U.K. His areas of teaching and research interests are in engineering management, operations planning and control, productivity enhancement and human factors in industry. He has published and presented a number of papers related to the fields in various international conferences.