Precalculus Workbook Solutions

Matrices

MATRIX DIMENSIONS AND ENTRIES

■ 1. Give the dimensions of the matrix.

$$D = \begin{bmatrix} 11 & 9 \\ -4 & 8 \end{bmatrix}$$

Solution:

We always give the dimensions of a matrix as rows × columns. Matrix D has 2 rows and 2 columns, so D is a 2×2 matrix.

■ 2. Give the dimensions of the matrix.

$$A = \begin{bmatrix} 3 & 5 & -2 & 1 & 8 \end{bmatrix}$$

Solution:

We always give the dimensions of a matrix as rows × columns. Matrix A has 1 row and 5 columns, so A is a 1×5 matrix.

■ 3. Given matrix J, find $J_{4,1}$.

$$J = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 1 \end{bmatrix}$$

Solution:

The value of $J_{4,1}$ is the entry in the fourth row, first column of matrix J, which is 1, so $J_{4,1} = 1$.

■ 4. Given matrix C, find $C_{1,2}$.

$$C = \begin{bmatrix} 3 & 12 \\ 1 & 4 \\ 9 & 5 \\ -3 & 2 \end{bmatrix}$$

Solution:

The value of $C_{1,2}$ is the entry in the first row, second column of matrix C, which is 12, so $C_{1,2} = 12$.

■ 5. Given matrix N, state the dimensions and find $N_{1,3}$.

$$N = \begin{bmatrix} 1 & 5 & 9 \\ 14 & -8 & 6 \end{bmatrix}$$

Solution:

We always give the dimensions of a matrix as rows × columns. Matrix N has 2 rows and 3 columns, so N is a 2×3 matrix.

The value of $N_{1,3}$ is the entry in the first row, third column of matrix N, which is 9, so $N_{1,3} = 9$.

■ 6. Given matrix S, state the dimensions and find $S_{3,4}$.

$$S = \begin{bmatrix} 3 & 6 & -7 & 1 & 0 \\ 0 & 9 & 15 & 3 & 4 \\ 4 & 0 & 2 & 11 & 8 \\ -5 & 8 & 7 & 9 & 2 \end{bmatrix}$$

Solution:

We always give the dimensions of a matrix as rows × columns. Matrix S has 4 rows and 5 columns, so S is a 4×5 matrix.

The value of $S_{3,4}$ is the entry in the third row, fourth column of matrix S, which is 11, so $S_{3,4} = 11$.

REPRESENTING SYSTEMS WITH MATRICES

■ 1. Represent the system with an augmented matrix called A.

$$-2x + 5y = 12$$

$$6x - 2y = 4$$

Solution:

The system contains the variables x and y along with a constant. Which means the augmented matrix will have two columns, one for each variable, plus a column for the constants, so three columns in total. Because there are two equations in the system, the matrix will have two rows. Plugging the coefficients and constants into an augmented matrix gives

$$A = \begin{bmatrix} -2 & 5 & 12 \\ 6 & -2 & 4 \end{bmatrix}$$

■ 2. Represent the system with an augmented matrix called D.

$$9y - 3x + 12 = 0$$

$$8 - 4x = 11y$$

Solution:

This system can be reorganized by putting each equation in order, with x and y on the left side, and the constant on the right side.

$$-3x + 9y = -12$$

$$4x + 11y = 8$$

The system contains the variables x and y along with a constant. Which means the augmented matrix will have two columns, one for each variable, plus a column for the constants, so three columns in total. Because there are two equations in the system, the matrix will have two rows. Plugging the coefficients and constants into an augmented matrix gives

$$D = \begin{bmatrix} -3 & 9 & -12 \\ 4 & 11 & 8 \end{bmatrix}$$

■ 3. Represent the system with an augmented matrix called H.

$$4a + 7b - 5c + 13d = 6$$

$$3a - 8b = -2c + 1$$

Solution:

The second equation can be reorganized by putting a, b, and c on the left side, and the constant on the right side. We also recognize that there is no d-term in the second equation, so we add in a 0 "filler" term.

$$4a + 7b - 5c + 13d = 6$$

$$3a - 8b + 2c + 0d = 1$$

The system contains the variables a, b, c, and d, along with a constant. Which means the augmented matrix will have four columns, one for each variable, plus a column for the constants, so five columns in total. Because there are two equations in the system, the matrix will have two rows. Plugging the coefficients and constants into an augmented matrix gives

$$H = \begin{bmatrix} 4 & 7 & -5 & 13 & 6 \\ 3 & -8 & 2 & 0 & 1 \end{bmatrix}$$

■ 4. Represent the system with an augmented matrix called M.

$$-2x + 4y = 9 - 6z$$

$$7y + 2z - 3 = -3t - 9x$$

Solution:

Both equations can be reorganized by putting x, y, z, and t on the left side, and the constant on the right side. We also recognize that there is no t-term in the first equation, so we add in a 0 "filler" term.

$$-2x + 4y + 6z + 0t = 9$$

$$9x + 7y + 2z + 3t = 3$$

The system contains the variables x, y, z, and t, along with a constant. Which means the augmented matrix will have four columns, one for each variable, plus a column for the constants, so five columns in total. Because there are two equations in the system, the matrix will have two rows. Plugging the coefficients and constants into an augmented matrix gives

$$M = \begin{bmatrix} -2 & 4 & 6 & 0 & 9 \\ 9 & 7 & 2 & 3 & 3 \end{bmatrix}$$

■ 5. Represent the system with an augmented matrix called A.

$$3x - 8y + z = 7$$

$$2z = 3y - 2x + 4$$

$$5y = 12 - 9x$$

Solution:

The second and third equations can be reorganized by putting x, y, and z on the left side, and the constant on the right side. We also recognize that there is no z-term in the third equation, so we add in a 0 "filler" term.

$$3x - 8y + z = 7$$

$$2x - 3y + 2z = 4$$

$$9x + 5y + 0z = 12$$

The system contains the variables x, y, and z, along with a constant. Which means the augmented matrix will have three columns, one for each variable, plus a column for the constants, so four columns in total. Because there are three equations in the system, the matrix will have three rows. Plugging the coefficients and constants into an augmented matrix gives

$$A = \begin{bmatrix} 3 & -8 & 1 & 7 \\ 2 & -3 & 2 & 4 \\ 9 & 5 & 0 & 12 \end{bmatrix}$$

■ 6. Represent the system with an augmented matrix called K.

$$-4b + 2c = 3 - 7a$$

$$9c = 4 - 2b$$

$$8a - 2c = 5b$$

Solution:

All three of these equations can be reorganized by putting a, b, and c on the left side, and the constant on the right side. We also recognize that there is no a-term in the second equation, and no constant in the third equation, so we add in 0 "filler" terms.

$$7a - 4b + 2c = 3$$

$$0a + 2b + 9c = 4$$

$$8a - 5b - 2c = 0$$

The system contains the variables a, b, and c, along with a constant. Which means the augmented matrix will have three columns, one for each variable, plus a column for the constants, so four columns in total. Because there are three equations in the system, the matrix will have three rows. Plugging the coefficients and constants into an augmented matrix gives

$$K = \begin{bmatrix} 7 & -4 & 2 & 3 \\ 0 & 2 & 9 & 4 \\ 8 & -5 & -2 & 0 \end{bmatrix}$$

SIMPLE ROW OPERATIONS

■ 1. Write the new matrix after $R_1 \leftrightarrow R_2$.

$$\begin{bmatrix} 2 & 6 & -4 & 1 \\ 8 & 2 & 1 & -5 \end{bmatrix}$$

Solution:

The operation described by $R_1 \leftrightarrow R_2$ is switching row 1 with row 2. The matrix after $R_1 \leftrightarrow R_2$ is

$$\begin{bmatrix} 8 & 2 & 1 & -5 \\ 2 & 6 & -4 & 1 \end{bmatrix}$$

■ 2. Write the new matrix after $R_2 \leftrightarrow R_4$.

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 6 & 1 & 5 & -4 \\ -7 & 7 & 0 & 3 \\ 9 & 2 & 8 & 3 \end{bmatrix}$$

Solution:

The operation described by $R_2 \leftrightarrow R_4$ is switching row 2 with row 4. Nothing will happen to rows 1 and 3. The matrix after $R_2 \leftrightarrow R_4$ is

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 9 & 2 & 8 & 3 \\ -7 & 7 & 0 & 3 \\ 6 & 1 & 5 & -4 \end{bmatrix}$$

■ 3. Write the new matrix after $R_1 \leftrightarrow 3R_2$.

$$\begin{bmatrix} 9 & 2 & -7 \\ 1 & 6 & 4 \end{bmatrix}$$

Solution:

The operation described by $R_1 \leftrightarrow 3R_2$ is multiplying row 2 by a constant of 3 and then switching those two rows. The matrix after $3R_2$ is

$$\begin{bmatrix} 9 & 2 & -7 \\ 3 & 18 & 12 \end{bmatrix}$$

The matrix after $R_1 \leftrightarrow 3R_2$ is

$$\begin{bmatrix} 3 & 18 & 12 \\ 9 & 2 & -7 \end{bmatrix}$$

■ 4. Write the new matrix after $3R_2 \leftrightarrow 3R_4$.

$$\begin{bmatrix} 0 & 11 & 6 \\ 7 & -3 & 9 \\ 8 & 8 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

Solution:

The operation described by $3R_2 \leftrightarrow 3R_4$ is multiplying row 2 by a constant of 3, multiplying row 4 by a constant of 3, and then switching those two rows. Nothing will happen to rows 1 and 3. The matrix after $3R_2$ is

$$\begin{bmatrix} 0 & 11 & 6 \\ 21 & -9 & 27 \\ 8 & 8 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

The matrix after $3R_4$ is

$$\begin{bmatrix} 0 & 11 & 6 \\ 21 & -9 & 27 \\ 8 & 8 & 1 \\ 18 & 6 & 12 \end{bmatrix}$$

The matrix after $3R_2 \leftrightarrow 3R_4$ is

$$\begin{bmatrix} 0 & 11 & 6 \\ 18 & 6 & 12 \\ 8 & 8 & 1 \\ 21 & -9 & 27 \end{bmatrix}$$

■ 5. Write the new matrix after $R_1 + 2R_2 \rightarrow R_1$.

$$\begin{bmatrix} 6 & 2 & 7 \\ 1 & -5 & 15 \end{bmatrix}$$

Solution:

The operation described by $R_1 + 2R_2 \rightarrow R_1$ is multiplying row 2 by a constant of 2, adding that resulting row to row 1, and using that result to replace row 1. $2R_2$ is

$$\begin{bmatrix} 2(1) & 2(-5) & 2(15) \end{bmatrix}$$

$$\begin{bmatrix} 2 & -10 & 30 \end{bmatrix}$$

The sum $R_1 + 2R_2$ is

$$\begin{bmatrix} 6 + 2 & 2 - 10 & 7 + 30 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -8 & 37 \end{bmatrix}$$

The matrix after $R_1 + 2R_2 \rightarrow R_1$, which is replacing row 1 with this row we just found, is

$$\begin{bmatrix} 8 & -8 & 37 \\ 1 & -5 & 15 \end{bmatrix}$$

■ 6. Write the new matrix after $4R_2 + R_3 \rightarrow R_3$.

$$\begin{bmatrix} 13 & 5 & -2 & 9 \\ 8 & 2 & 0 & 6 \\ 4 & 1 & 7 & -3 \end{bmatrix}$$

Solution:

The operation described by $4R_2 + R_3 \rightarrow R_3$ is multiplying row 2 by a constant of 4, adding that resulting row to row 3, and using that result to replace row 3. $4R_2$ is

$$\begin{bmatrix} 4(8) & 4(2) & 4(0) & 4(6) \end{bmatrix}$$

$$\begin{bmatrix} 32 & 8 & 0 & 24 \end{bmatrix}$$

The sum $4R_2 + R_3$ is

$$\begin{bmatrix} 32+4 & 8+1 & 0+7 & 24-3 \end{bmatrix}$$

$$\begin{bmatrix} 36 & 9 & 7 & 21 \end{bmatrix}$$

The matrix after $4R_2 + R_3 \rightarrow R_3$, which is replacing row 3 with this row we just found, is

$$\begin{bmatrix} 13 & 5 & -2 & 9 \\ 8 & 2 & 0 & 6 \\ 36 & 9 & 7 & 21 \end{bmatrix}$$

GAUSS-JORDAN ELIMINATION AND REDUCED ROW-ECHELON FORM

■ 1. Use Gauss-Jordan elimination to solve the system.

$$x + 2y = -2$$

$$3x + 2y = 6$$

Solution:

The augmented matrix for the system is

$$\begin{bmatrix} 1 & 2 & = & -2 \\ 3 & 2 & = & 6 \end{bmatrix}$$

After $3R_1 - R_2 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 2 & = & -2 \\ 0 & 4 & = & -12 \end{bmatrix}$$

The first column is done. After $(1/4)R_2 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 2 & = & -2 \\ 0 & 1 & = & -3 \end{bmatrix}$$

After $R_1 - 2R_2 \rightarrow R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & = & 4 \\ 0 & 1 & = & -3 \end{bmatrix}$$

The second column is done, so we get the solution set

$$x = 4$$

$$y = -3$$

■ 2. Use Gauss-Jordan elimination to solve the system.

$$2x + 4y = 22$$

$$3x + 3y = 15$$

Solution:

The augmented matrix for the system is

$$\begin{bmatrix} 2 & 4 & = & 22 \\ 3 & 3 & = & 15 \end{bmatrix}$$

After $(1/2)R_1 \rightarrow R_1$ and $(1/3)R_2 \rightarrow R_2$ the matrix is

$$\begin{bmatrix} 1 & 2 & = & 11 \\ 1 & 1 & = & 5 \end{bmatrix}$$

After $R_1 - R_2 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 2 & = & 11 \\ 0 & 1 & = & 6 \end{bmatrix}$$

The first column is done. After $R_1 - 2R_2 \rightarrow R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & = & -1 \\ 0 & 1 & = & 6 \end{bmatrix}$$

The second column is done, so we get the solution set

$$x = -1$$

$$y = 6$$

■ 3. Use Gauss-Jordan elimination to solve the system.

$$x - 3y - 6z = 4$$

$$y + 2z = -2$$

$$-4x + 12y + 21z = -4$$

Solution:

The augmented matrix for the system is

$$\begin{bmatrix} 1 & -3 & -6 & = & 4 \\ 0 & 1 & 2 & = & -2 \\ -4 & 12 & 21 & = & -4 \end{bmatrix}$$

After $4R_1 + R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & -3 & -6 & = & 4 \\ 0 & 1 & 2 & = & -2 \\ 0 & 0 & -3 & = & 12 \end{bmatrix}$$

The first column is done. After $3R_2 + R_1 \rightarrow R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & -2 \\ 0 & 1 & 2 & = & -2 \\ 0 & 0 & -3 & = & 12 \end{bmatrix}$$

The second column is done. After $(-1/3)R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & -2 \\ 0 & 1 & 2 & = & -2 \\ 0 & 0 & 1 & = & -4 \end{bmatrix}$$

After $R_2 - 2R_3 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & -2 \\ 0 & 1 & 0 & = & 6 \\ 0 & 0 & 1 & = & -4 \end{bmatrix}$$

The third column is done, so we get the solution set

$$x = -2$$

$$y = 6$$

$$z = -4$$

■ 4. Use Gauss-Jordan elimination to solve the system.

$$2y + 4z = 4$$

$$x + 3y + 3z = 5$$

$$2x + 7y + 6z = 10$$

Solution:

The augmented matrix for the system is

$$\left[\begin{array}{ccccc} 0 & 2 & 4 & = & 4 \\ 1 & 3 & 3 & = & 5 \\ 2 & 7 & 6 & = & 10 \end{array}\right]$$

After $(1/2)R_1 \rightarrow R_1$, the matrix is

$$\left[\begin{array}{ccccc} 0 & 1 & 2 & = & 2 \\ 1 & 3 & 3 & = & 5 \\ 2 & 7 & 6 & = & 10 \end{array}\right]$$

Because the first entry in the first row is 0, swap it with the second row to get

$$\left[\begin{array}{ccccc} 1 & 3 & 3 & = & 5 \\ 0 & 1 & 2 & = & 2 \\ 2 & 7 & 6 & = & 10 \end{array}\right]$$

After $R_3 - 2R_1 \rightarrow R_3$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 3 & 3 & = & 5 \\ 0 & 1 & 2 & = & 2 \\ 0 & 1 & 0 & = & 0 \end{array}\right]$$

The first column is done. After $R_1 - 3R_2 \rightarrow R_1$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 0 & -3 & = & -1 \\ 0 & 1 & 2 & = & 2 \\ 0 & 1 & 0 & = & 0 \end{array}\right]$$

After $R_2 - R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & -3 & = & -1 \\ 0 & 1 & 2 & = & 2 \\ 0 & 0 & 2 & = & 2 \end{bmatrix}$$

The second column is done. After $(1/2)R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & -3 & = & -1 \\ 0 & 1 & 2 & = & 2 \\ 0 & 0 & 1 & = & 1 \end{bmatrix}$$

After $R_1 + 3R_3 \rightarrow R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & 2 \\ 0 & 1 & 2 & = & 2 \\ 0 & 0 & 1 & = & 1 \end{bmatrix}$$

After $R_2 - 2R_3 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & 2 \\ 0 & 1 & 0 & = & 0 \\ 0 & 0 & 1 & = & 1 \end{bmatrix}$$

The third column is done, so we get the solution set

$$x = 2$$

$$y = 0$$

$$z = 1$$

■ 5. Use Gauss-Jordan elimination to solve the system.

$$3x + 12y + 42z = -27$$

$$x + 2y + 8z = -5$$

$$2x + 5y + 16z = -6$$

Solution:

The augmented matrix for the system is

$$\left[\begin{array}{ccccc} 3 & 12 & 42 & = & -27 \\ 1 & 2 & 8 & = & -5 \\ 2 & 5 & 16 & = & -6 \end{array}\right]$$

After $(1/3)R_1 \rightarrow R_1$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 4 & 14 & = & -9 \\ 1 & 2 & 8 & = & -5 \\ 2 & 5 & 16 & = & -6 \end{array}\right]$$

After $R_1 - R_2 \rightarrow R_2$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 4 & 14 & = & -9 \\ 0 & 2 & 6 & = & -4 \\ 2 & 5 & 16 & = & -6 \end{array}\right]$$

After $2R_1 - R_3 \rightarrow R_3$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 4 & 14 & = & -9 \\ 0 & 2 & 6 & = & -4 \\ 0 & 3 & 12 & = & -12 \end{array}\right]$$

The first column is done. After $(1/2)R_2 \rightarrow R_2$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 4 & 14 & = & -9 \\ 0 & 1 & 3 & = & -2 \\ 0 & 3 & 12 & = & -12 \end{array}\right]$$

After $R_1 - 4R_2 \rightarrow R_1$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 0 & 2 & = & -1 \\ 0 & 1 & 3 & = & -2 \\ 0 & 3 & 12 & = & -12 \end{array}\right]$$

After $R_3 - 3R_2 \rightarrow R_3$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 0 & 2 & = & -1 \\ 0 & 1 & 3 & = & -2 \\ 0 & 0 & 3 & = & -6 \end{array}\right]$$

The second column is done. After $(1/3)R_3 \rightarrow R_3$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 0 & 2 & = & -1 \\ 0 & 1 & 3 & = & -2 \\ 0 & 0 & 1 & = & -2 \end{array}\right]$$

After $R_1 - 2R_3 \rightarrow R_1$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & = & 3 \\ 0 & 1 & 3 & = & -2 \\ 0 & 0 & 1 & = & -2 \end{array}\right]$$

After $R_2 - 3R_3 \rightarrow R_2$, the matrix is

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & = & 3 \\ 0 & 1 & 0 & = & 4 \\ 0 & 0 & 1 & = & -2 \end{array}\right]$$

The third column is done, so we get the solution set

$$x = 3$$

$$y = 4$$

$$z = -2$$

■ 6. Use Gauss-Jordan elimination to solve the system.

$$4x + 8y + 4z = 20$$

$$4x + 6y = 4$$

$$3x + 3y - z = 1$$

Solution:

The augmented matrix for the system is

$$\begin{bmatrix} 4 & 8 & 4 & = & 20 \\ 4 & 6 & 0 & = & 4 \\ 3 & 3 & -1 & = & 1 \end{bmatrix}$$

After $(1/4)R_1 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 2 & 1 & = & 5 \\ 4 & 6 & 0 & = & 4 \\ 3 & 3 & -1 & = & 1 \end{bmatrix}$$

After $4R_1 - R_2 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 2 & 1 & = & 5 \\ 0 & 2 & 4 & = & 16 \\ 3 & 3 & -1 & = & 1 \end{bmatrix}$$

After $3R_1 - R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & 2 & 1 & = & 5 \\ 0 & 2 & 4 & = & 16 \\ 0 & 3 & 4 & = & 14 \end{bmatrix}$$

The first column is done. After $(1/2)R_2 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 2 & 1 & = & 5 \\ 0 & 1 & 2 & = & 8 \\ 0 & 3 & 4 & = & 14 \end{bmatrix}$$

After $R_1 - 2R_2 \rightarrow R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & -3 & = & -11 \\ 0 & 1 & 2 & = & 8 \\ 0 & 3 & 4 & = & 14 \end{bmatrix}$$

After $3R_2 - R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & -3 & = & -11 \\ 0 & 1 & 2 & = & 8 \\ 0 & 0 & 2 & = & 10 \end{bmatrix}$$

The second column is done. After $(1/2)R_3 \rightarrow R_3$, the matrix is

$$\begin{bmatrix} 1 & 0 & -3 & = & -11 \\ 0 & 1 & 2 & = & 8 \\ 0 & 0 & 1 & = & 5 \end{bmatrix}$$

After $R_1 + 3R_3 \rightarrow R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & 4 \\ 0 & 1 & 2 & = & 8 \\ 0 & 0 & 1 & = & 5 \end{bmatrix}$$

After $R_2 - 2R_3 \rightarrow R_2$, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & = & 4 \\ 0 & 1 & 0 & = & -2 \\ 0 & 0 & 1 & = & 5 \end{bmatrix}$$

The third column is done, so we get the solution set

$$x = 4$$

$$y = -2$$

$$z = 5$$

MATRIX ADDITION AND SUBTRACTION

■ 1. Add the matrices.

$$\begin{vmatrix} 7 & 6 \\ 17 & 9 \end{vmatrix} + \begin{vmatrix} 0 & 8 \\ -2 & 5 \end{vmatrix}$$

Solution:

To add matrices, you simply add together entries from corresponding positions in each matrix.

$$\begin{vmatrix} 7 & 6 \\ 17 & 9 \end{vmatrix} + \begin{vmatrix} 0 & 8 \\ -2 & 5 \end{vmatrix}$$

$$\begin{vmatrix} 7+0 & 6+8 \\ 17+(-2) & 9+5 \end{vmatrix}$$

$$\begin{vmatrix} 7 & 14 \\ 15 & 14 \end{vmatrix}$$

■ 2. Add the matrices.

$$\begin{vmatrix} 8 & 3 \\ -4 & 7 \\ 6 & 0 \\ 1 & 13 \end{vmatrix} + \begin{vmatrix} 6 & 7 \\ 2 & -3 \\ 9 & 11 \\ 7 & -2 \end{vmatrix}$$

Solution:

To add matrices, you simply add together entries from corresponding positions in each matrix.

$$\begin{vmatrix} 8 & 3 \\ -4 & 7 \\ 6 & 0 \\ 1 & 13 \end{vmatrix} + \begin{vmatrix} 6 & 7 \\ 2 & -3 \\ 9 & 11 \\ 7 & -2 \end{vmatrix}$$

$$\begin{vmatrix} 8+6 & 3+7 \\ -4+2 & 7+(-3) \\ 6+9 & 0+11 \\ 1+7 & 13+(-2) \end{vmatrix}$$

$$\begin{vmatrix} 14 & 10 \\ -2 & 4 \\ 15 & 11 \\ 8 & 11 \end{vmatrix}$$

■ 3. Subtract the matrices.

$$\begin{vmatrix} 7 & 9 \\ 4 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 8 \\ 12 & -3 \end{vmatrix}$$

Solution:

To subtract matrices, you simply subtract entries from corresponding positions in each matrix.

$$\begin{vmatrix} 7 & 9 \\ 4 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 8 \\ 12 & -3 \end{vmatrix}$$

$$\begin{vmatrix} 7-3 & 9-8 \\ 4-12 & -1-(-3) \end{vmatrix}$$

$$\begin{vmatrix} 4 & 1 \\ -8 & 2 \end{vmatrix}$$

■ 4. Subtract the matrices.

$$\begin{vmatrix} 8 & 11 & 2 & 9 \\ 6 & 3 & 16 & 8 \end{vmatrix} - \begin{vmatrix} 6 & 11 & 7 & -4 \\ 5 & 8 & 1 & 15 \end{vmatrix}$$

Solution:

To subtract matrices, you simply subtract entries from corresponding positions in each matrix.

$$\begin{vmatrix} 8 & 11 & 2 & 9 \\ 6 & 3 & 16 & 8 \end{vmatrix} - \begin{vmatrix} 6 & 11 & 7 & -4 \\ 5 & 8 & 1 & 15 \end{vmatrix}$$

$$\begin{vmatrix} 8-6 & 11-11 & 2-7 & 9-(-4) \\ 6-5 & 3-8 & 16-1 & 8-15 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 0 & -5 & 13 \\ 1 & -5 & 15 & -7 \end{vmatrix}$$

■ 5. Solve for m.

$$\begin{vmatrix} 6 & 5 \\ 9 & -9 \end{vmatrix} + \begin{vmatrix} 3 & 7 \\ 1 & 6 \end{vmatrix} = m + \begin{vmatrix} 7 & 12 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 8 \\ 4 & -7 \end{vmatrix}$$

Solution:

Let's start with the matrix addition on the left side of the equation and the matrix subtraction on the right side of the equation.

$$\begin{vmatrix} 6 & 5 \\ 9 & -9 \end{vmatrix} + \begin{vmatrix} 3 & 7 \\ 1 & 6 \end{vmatrix} = m + \begin{vmatrix} 7 & 12 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 8 \\ 4 & -7 \end{vmatrix}$$

$$\begin{vmatrix} 6+3 & 5+7 \\ 9+1 & -9+6 \end{vmatrix} = m + \begin{vmatrix} 7-1 & 12-8 \\ -3-4 & -1-(-7) \end{vmatrix}$$

$$\begin{vmatrix} 9 & 12 \\ 10 & -3 \end{vmatrix} = m + \begin{vmatrix} 6 & 4 \\ -7 & 6 \end{vmatrix}$$

To isolate m, we'll subtract the matrix on the right from both sides in order to move it to the left.

$$\begin{vmatrix} 9 & 12 \\ 10 & -3 \end{vmatrix} - \begin{vmatrix} 6 & 4 \\ -7 & 6 \end{vmatrix} = m$$

$$\begin{vmatrix} 9-6 & 12-4 \\ 10-(-7) & -3-6 \end{vmatrix} = m$$

$$\begin{vmatrix} 3 & 8 \\ 17 & -9 \end{vmatrix} = m$$

The conclusion is that the value of m that makes the equation true is this matrix:

$$m = \begin{vmatrix} 3 & 8 \\ 17 & -9 \end{vmatrix}$$

■ **6. Solve for n.**

$$\begin{vmatrix} 4 & 12 \\ 9 & 8 \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ 9 & 9 \end{vmatrix} = n - \begin{vmatrix} 6 & 3 \\ 5 & 11 \end{vmatrix} + \begin{vmatrix} 7 & -4 \\ -18 & 1 \end{vmatrix}$$

Solution:

Let's start with the matrix subtraction on the left side of the equation and the matrix addition on the right side of the equation.

$$\begin{vmatrix} 4 & 12 \\ 9 & 8 \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ 9 & 9 \end{vmatrix} = n - \begin{vmatrix} 6 & 3 \\ 5 & 11 \end{vmatrix} + \begin{vmatrix} 7 & -4 \\ -18 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 4-0 & 12-3 \\ 9-9 & 8-9 \end{vmatrix} = n - \begin{vmatrix} 6+7 & 3+(-4) \\ 5+(-18) & 11+1 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 9 \\ 0 & -1 \end{vmatrix} = n - \begin{vmatrix} 13 & -1 \\ -13 & 12 \end{vmatrix}$$

To isolate n, we'll add the matrix on the right to both sides in order to move it to the left.

$$\begin{vmatrix} 4 & 9 \\ 0 & -1 \end{vmatrix} + \begin{vmatrix} 13 & -1 \\ -13 & 12 \end{vmatrix} = n$$

$$\begin{vmatrix} 4+13 & 9+(-1) \\ 0+(-13) & -1+12 \end{vmatrix} = n$$

$$\begin{vmatrix} 17 & 8 \\ -13 & 11 \end{vmatrix} = n$$

The conclusion is that the value of n that makes the equation true is this matrix:

$$n = \begin{vmatrix} 17 & 8 \\ -13 & 11 \end{vmatrix}$$

SCALAR MULTIPLICATION AND ZERO MATRICES

■ 1. Use scalar multiplication to simplify the expression.

$$\frac{1}{4} \begin{vmatrix} 12 & 8 & 3 \\ 2 & -16 & 0 \\ 1 & 5 & 7 \end{vmatrix}$$

Solution:

The scalar $1/4$ is being multiplied by the matrix. Distribute the scalar across every entry in the matrix.

$$\frac{1}{4} \begin{vmatrix} 12 & 8 & 3 \\ 2 & -16 & 0 \\ 1 & 5 & 7 \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{4}(12) & \frac{1}{4}(8) & \frac{1}{4}(3) \\ \frac{1}{4}(2) & \frac{1}{4}(-16) & \frac{1}{4}(0) \\ \frac{1}{4}(1) & \frac{1}{4}(5) & \frac{1}{4}(7) \end{vmatrix}$$

$$\begin{vmatrix} 3 & 2 & \frac{3}{4} \\ \frac{1}{2} & -4 & 0 \\ \frac{1}{4} & \frac{5}{4} & \frac{7}{4} \end{vmatrix}$$

■ 2. Solve for y.

$$4 \begin{vmatrix} 2 & 9 \\ -5 & 0 \end{vmatrix} + y = 5 \begin{vmatrix} 1 & -3 \\ 6 & 8 \end{vmatrix}$$

Solution:

Apply the scalars to the matrices.

$$\begin{vmatrix} 4(2) & 4(9) \\ 4(-5) & 4(0) \end{vmatrix} + y = \begin{vmatrix} 5(1) & 5(-3) \\ 5(6) & 5(8) \end{vmatrix}$$

$$\begin{vmatrix} 8 & 36 \\ -20 & 0 \end{vmatrix} + y = \begin{vmatrix} 5 & -15 \\ 30 & 40 \end{vmatrix}$$

Subtract the matrix on the left from both sides of the equation in order to isolate y.

$$y = \begin{vmatrix} 5 & -15 \\ 30 & 40 \end{vmatrix} - \begin{vmatrix} 8 & 36 \\ -20 & 0 \end{vmatrix}$$

$$y = \begin{vmatrix} 5 - 8 & -15 - 36 \\ 30 - (-20) & 40 - 0 \end{vmatrix}$$

$$y = \begin{vmatrix} -3 & -51 \\ 50 & 40 \end{vmatrix}$$

■ 3. Solve for n.

$$-2 \begin{vmatrix} 6 & 5 \\ 0 & 11 \end{vmatrix} = n - 4 \begin{vmatrix} 2 & 4 \\ -1 & 9 \end{vmatrix}$$

Solution:

Apply the scalars to the matrices.

$$\begin{vmatrix} -2(6) & -2(5) \\ -2(0) & -2(11) \end{vmatrix} = n - \begin{vmatrix} 4(2) & 4(4) \\ 4(-1) & 4(9) \end{vmatrix}$$

$$\begin{vmatrix} -12 & -10 \\ 0 & -22 \end{vmatrix} = n - \begin{vmatrix} 8 & 16 \\ -4 & 36 \end{vmatrix}$$

Add the matrix on the right to both sides of the equation in order to isolate n.

$$\begin{vmatrix} -12 & -10 \\ 0 & -22 \end{vmatrix} + \begin{vmatrix} 8 & 16 \\ -4 & 36 \end{vmatrix} = n$$

$$\begin{vmatrix} -12+8 & -10+16 \\ 0+(-4) & -22+36 \end{vmatrix} = n$$

$$\begin{vmatrix} -4 & 6 \\ -4 & 14 \end{vmatrix} = n$$

$$n = \begin{vmatrix} -4 & 6 \\ -4 & 14 \end{vmatrix}$$

■ 4. Add the zero matrix to the given matrix.

$$\begin{vmatrix} 8 & 17 \\ -6 & 0 \end{vmatrix}$$

Solution:

Adding the zero matrix to any other matrix doesn't change the value of the matrix, so

$$\begin{vmatrix} 8 & 17 \\ -6 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 8 & 17 \\ -6 & 0 \end{vmatrix}$$

■ 5. Find the opposite matrix.

$$\begin{vmatrix} 6 & 8 & 0 \\ 2 & -3 & 11 \\ 4 & 12 & 9 \end{vmatrix}$$

Solution:

To get the opposite of a matrix, multiply it by a scalar of -1. Then the opposite of the given matrix is

$$(-1)\begin{vmatrix} (-1)6 & (-1)8 & (-1)0 \\ (-1)2 & (-1)(-3) & (-1)11 \\ (-1)4 & (-1)12 & (-1)9 \end{vmatrix}$$

$$\begin{vmatrix} -6 & -8 & 0 \\ -2 & 3 & -11 \\ -4 & -12 & -9 \end{vmatrix}$$

■ 6. Multiply the matrix by a scalar of 0.

$$\begin{vmatrix} 14 & -1 & 7 & 5 \\ 3 & 7 & 18 & -4 \end{vmatrix}$$

Solution:

Multiplying any matrix by a scalar of 0 results in a zero matrix.

$$(0)\begin{vmatrix} 14(0) & -1(0) & 7(0) & 5(0) \\ 3(0) & 7(0) & 18(0) & -4(0) \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

MATRIX MULTIPLICATION

■ 1. If matrix A is 3×3 and matrix B is 3×4, say whether AB or BA is defined, and give the dimensions of any product that's defined.

Solution:

Line up the dimensions for the products AB and BA, and compare the middle terms, which represent the columns from the first matrix and the rows from the second matrix.

AB: $3 \times$ 3   3 $\times 4$

BA: $3 \times$ 4   3 $\times 3$

The middle numbers match for AB, so that product is defined. For BA, the middle numbers don't match, so that product isn't defined.

The dimensions of AB are given by the outside numbers, which are the rows from the first matrix and the columns from the second matrix.

AB: 3 $\times 3$ 3 $\times$ 4

So the dimensions of AB will be 3×4.

■ 2. Find the product of matrices A and B.

$$A = \begin{bmatrix} 2 & 6 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 \\ 5 & -4 \end{bmatrix}$$

Solution:

Multiply matrix A by matrix B.

$$A \cdot B = \begin{bmatrix} 2 & 6 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 \\ 5 & -4 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2(-2) + 6(5) & 2(0) + 6(-4) \\ -3(-2) + 1(5) & -3(0) + 1(-4) \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 26 & -24 \\ 11 & -4 \end{bmatrix}$$

■ 3. Find the product of matrices A and B.

$$A = \begin{bmatrix} 5 & -1 \\ 0 & 11 \\ 7 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 1 & 8 \\ -3 & 0 & 4 \end{bmatrix}$$

Solution:

Multiply matrix A by matrix B.

$$A \cdot B = \begin{bmatrix} 5 & -1 \\ 0 & 11 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 1 & 8 \\ -3 & 0 & 4 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 5(6) + (-1)(-3) & 5(1) + (-1)(0) & 5(8) + (-1)(4) \\ 0(6) + 11(-3) & 0(1) + 11(0) & 0(8) + 11(4) \\ 7(6) + (-2)(-3) & 7(1) + (-2)(0) & 7(8) + (-2)(4) \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 33 & 5 & 36 \\ -33 & 0 & 44 \\ 48 & 7 & 48 \end{bmatrix}$$

■ 4. Find the product of matrices A and B.

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 8 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 \\ 4 & 8 \end{bmatrix}$$

Solution:

Multiply matrix A by matrix B.

$$A \cdot B = \begin{bmatrix} 3 & -2 \\ 1 & 8 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 4 & 8 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 3(5)+(-2)(4) & 3(2)+(-2)(8) \\ 1(5)+8(4) & 1(2)+8(8) \\ 0(5)+3(4) & 0(2)+3(8) \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 7 & -10 \\ 37 & 66 \\ 12 & 24 \end{bmatrix}$$

■ 5. Use the distributive property to find $A(B+C)$.

$$A = \begin{bmatrix} 2 & 0 \\ 4 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 5 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 1 \\ 3 & -1 \end{bmatrix}$$

Solution:

Applying the distributive property to the initial expression, we get

$$A(B+C) = AB + AC$$

Use matrix multiplication to find $AB + AC$.

$$AB + AC = \begin{bmatrix} 2 & 0 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 3 & -1 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 2(3) + (0)(5) & 2(1) + 0(4) \\ 4(3) + (-2)(5) & 4(1) + (-2)(4) \end{bmatrix}$$

$$+ \begin{bmatrix} 2(6) + 0(3) & 2(1) + 0(-1) \\ 4(6) + (-2)(3) & 4(1) + (-2)(-1) \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 6 & 2 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 2 \\ 18 & 6 \end{bmatrix}$$

Now use matrix addition.

$$AB + AC = \begin{bmatrix} 6 + 12 & 2 + 2 \\ 2 + 18 & -4 + 6 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 18 & 4 \\ 20 & 2 \end{bmatrix}$$

So the value of the original expression is

$$A(B + C) = \begin{bmatrix} 18 & 4 \\ 20 & 2 \end{bmatrix}$$

■ 6. Find the product of matrices A and B.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -2 & 8 & 1 \\ 7 & 3 & 5 & 2 \end{bmatrix}$$

Solution:

Multiply matrix A by matrix B.

$$A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 & 8 & 1 \\ 7 & 3 & 5 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 0(6)+0(7) & 0(-2)+0(3) & 0(8)+0(5) & 0(1)+0(2) \\ 0(6)+0(7) & 0(-2)+0(3) & 0(8)+0(5) & 0(1)+0(2) \\ 0(6)+0(7) & 0(-2)+0(3) & 0(8)+0(5) & 0(1)+0(2) \\ 0(6)+0(7) & 0(-2)+0(3) & 0(8)+0(5) & 0(1)+0(2) \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

IDENTITY MATRICES

■ 1. Write the identity matrix I_4.

Solution:

We always call the identity matrix I, and it's always a square matrix, like 2×2, 3×3, 4×4, etc. For that reason, it's common to abbreviate I_{2x2} as just I_2, or I_{3x3} as just I_3, etc. So, I_4 is the 4×4 identity matrix.

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

■ 2. If we want to find the product IA, where I is the identity matrix and A is a 4×2, then what are the dimensions of I?

Solution:

Start by setting up the equation $I \cdot A = A$, then substitute the dimensions for A into the equation.

$$I \cdot A = A$$

$$I \cdot 4 \times 2 = 4 \times 2$$

Break down the dimensions of the identity matrix as rows × columns, or $R \times C$.

$$R \times C \cdot 4 \times 2 = 4 \times 2$$

For matrix multiplication to be defined, you need the same number of columns in the first matrix as rows in the second matrix.

$$R \times \boxed{4 \cdot 4} \times 2 = 4 \times 2$$

The dimensions of the product come from the rows in the first matrix and the columns in the second matrix, so

$$\boxed{4} \times 4 \cdot 4 \times 2 = \boxed{4} \times 2$$

Therefore, the identity matrix in this case is I_4.

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

■ 3. If we want to find the product IA, where I is the identity matrix and A is a 3×4, then what are the dimensions of I?

Solution:

Start by setting up the equation $I \cdot A = A$, then substitute the dimensions for A into the equation.

$$I \cdot A = A$$

$$I \cdot 3 \times 4 = 3 \times 4$$

Break down the dimensions of the identity matrix as rows $\times$ columns, or $R \times C$.

$$R \times C \cdot 3 \times 4 = 3 \times 4$$

For matrix multiplication to be defined, you need the same number of columns in the first matrix as rows in the second matrix.

$$R \times \boxed{3 \cdot 3} \times 4 = 3 \times 4$$

The dimensions of the product come from the rows in the first matrix and the columns in the second matrix, so

$$\boxed{3} \times 3 \cdot 3 \times 4 = \boxed{3} \times 4$$

Therefore, the identity matrix in this case is I_3.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

■ 4. If we want to find the product IA, where I is the identity matrix and A is given, then what are the dimensions of I? What is the product IA?

$$A = \begin{bmatrix} 2 & 8 \\ -2 & 7 \\ 3 & 5 \end{bmatrix}$$

Solution:

Start by setting up the equation $I \cdot A = A$, then substitute the dimensions for A into the equation.

$$I \cdot A = A$$

$$I \cdot 3 \times 2 = 3 \times 2$$

Break down the dimensions of the identity matrix as rows × columns, or $R \times C$.

$$R \times C \cdot 3 \times 2 = 3 \times 2$$

For matrix multiplication to be defined, you need the same number of columns in the first matrix as rows in the second matrix.

$$R \times \boxed{3 \cdot 3} \times 2 = 3 \times 2$$

The dimensions of the product come from the rows in the first matrix and the columns in the second matrix, so

$$\boxed{3} \times 3 \cdot 3 \times 2 = \boxed{3} \times 2$$

Therefore, the identity matrix in this case is I_3.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The product IA is

$$IA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ -2 & 7 \\ 3 & 5 \end{bmatrix}$$

$$IA = \begin{bmatrix} 1(2) + 0(-2) + 0(3) & 1(8) + 0(7) + 0(5) \\ 0(2) + 1(-2) + 0(3) & 0(8) + 1(7) + 0(5) \\ 0(2) + 0(-2) + 1(3) & 0(8) + 0(7) + 1(5) \end{bmatrix}$$

$$IA = \begin{bmatrix} 2 & 8 \\ -2 & 7 \\ 3 & 5 \end{bmatrix}$$

As we expected, we get back to matrix A after multiplying it by the identity matrix I_3.

■ 5. If we want to find the product IA, where I is the identity matrix and A is given, then what are the dimensions of I? What is the product IA?

$$A = \begin{bmatrix} 7 & 1 & 3 & -2 \\ 5 & 5 & 2 & 9 \end{bmatrix}$$

Solution:

Start by setting up the equation $I \cdot A = A$, then substitute the dimensions for A into the equation.

$$I \cdot A = A$$

$$I \cdot 2 \times 4 = 2 \times 4$$

Break down the dimensions of the identity matrix as rows $\times$ columns, or $R \times C$.

$$R \times C \cdot 2 \times 4 = 2 \times 4$$

For matrix multiplication to be defined, you need the same number of columns in the first matrix as rows in the second matrix.

$$R \times \boxed{2 \cdot 2} \times 4 = 2 \times 4$$

The dimensions of the product come from the rows in the first matrix and the columns in the second matrix, so

$$\boxed{2} \times 2 \cdot 2 \times 4 = \boxed{2} \times 4$$

Therefore, the identity matrix in this case is I_2.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The product IA is

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 1 & 3 & -2 \\ 5 & 5 & 2 & 9 \end{bmatrix}$$

$$IA = \begin{bmatrix} 1(7) + 0(5) & 1(1) + 0(5) & 1(3) + 0(2) & 1(-2) + 0(9) \\ 0(7) + 1(5) & 0(1) + 1(5) & 0(3) + 1(2) & 0(-2) + 1(9) \end{bmatrix}$$

$$IA = \begin{bmatrix} 7 & 1 & 3 & -2 \\ 5 & 5 & 2 & 9 \end{bmatrix}$$

As we expected, we get back to matrix A after multiplying it by the identity matrix I_2.

■ 6. If A is a 2×4 matrix what are the dimensions of the identity matrix that make the equation true?

$$A \cdot I = A$$

Solution:

Set up the equation $A \cdot I = A$, then substitute the dimensions for A into the equation.

$$A \cdot I = A$$

$$2 \times 4 \cdot I = 2 \times 4$$

Break up the dimensions of I as $R \times C$.

$$2 \times 4 \cdot R \times C = 2 \times 4$$

The number of rows in the second matrix must be equal to the number of columns from the first matrix.

$$2 \times 4 \cdot 4 \times C = 2 \times 4$$

The dimensions of the product come from the rows of the first matrix and the columns of the second matrix, so

$$2 \times 4 \cdot 4 \times 4 = 2 \times 4$$

So the identity matrix is I_4, the 4×4 identity matrix.

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

TRANSFORMATIONS

■ **1.** Find the resulting vector $\vec{b}$ after $\vec{a} = (1,6)$ undergoes a transformation by matrix M.

$$M = \begin{bmatrix} -7 & 1 \\ 0 & -2 \end{bmatrix}$$

Solution:

To apply a transformation matrix to vector $\vec{a}$, we'll multiply the matrix by the vector.

$$\vec{b} = M\vec{a} = \begin{bmatrix} -7 & 1 \\ 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$\vec{b} = M\vec{a} = \begin{bmatrix} -7(1) + 1(6) \\ 0(1) - 2(6) \end{bmatrix}$$

$$\vec{b} = M\vec{a} = \begin{bmatrix} -7 + 6 \\ 0 - 12 \end{bmatrix}$$

$$\vec{b} = M\vec{a} = \begin{bmatrix} -1 \\ -12 \end{bmatrix}$$

■ **2.** Sketch triangle $\triangle ABC$ with vertices $(2,3)$, $(-3, -1)$, and $(1, -4)$, and the transformation of $\triangle ABC$ after it's transformed by matrix L.

$$L = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Solution:

Put the vertices of $\triangle ABC$ into a matrix.

$$\begin{bmatrix} 2 & -3 & 1 \\ 3 & -1 & -4 \end{bmatrix}$$
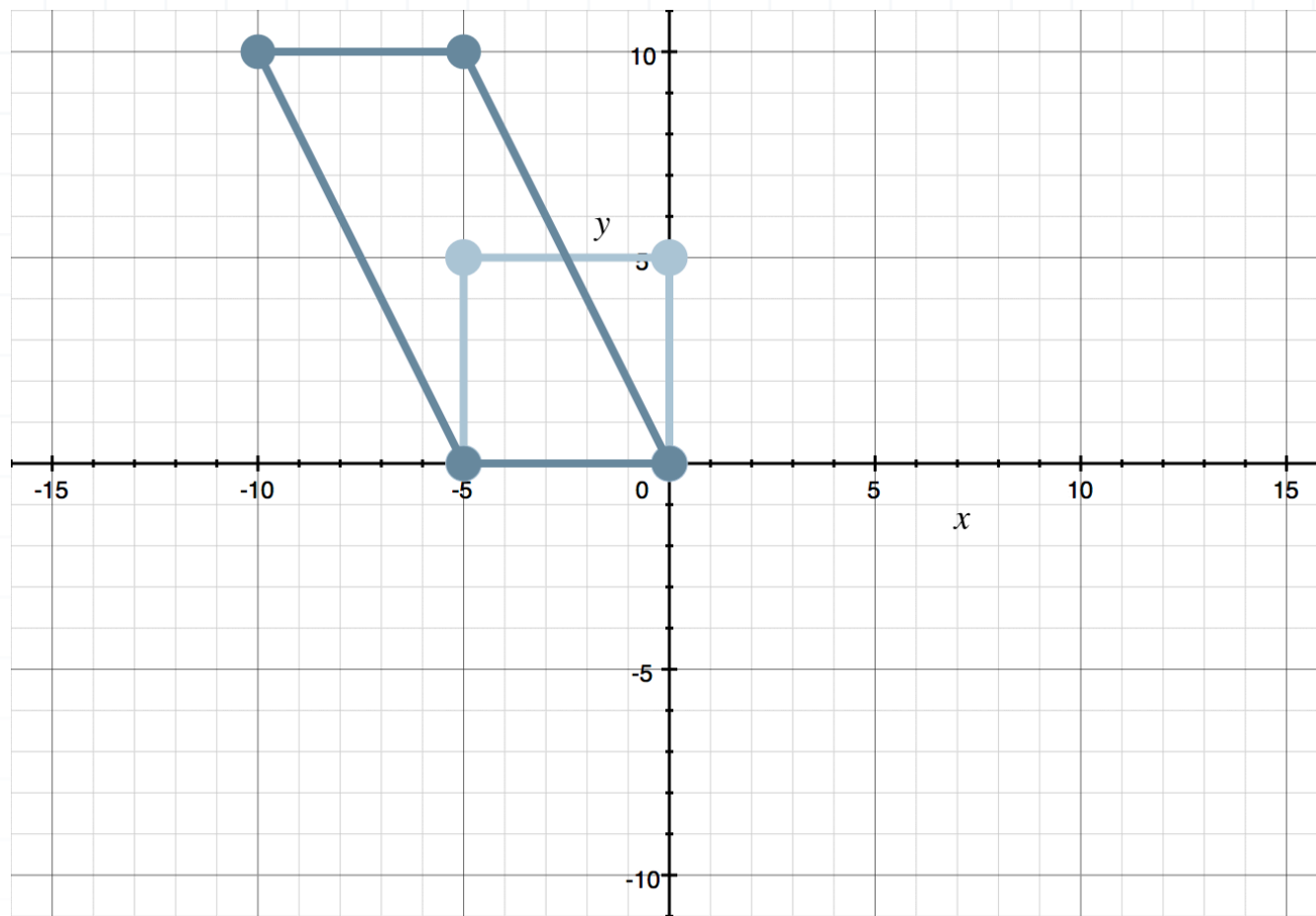
Apply the transformation of L to the vertex matrix.

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & 1 \\ 3 & -1 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1(2) + 2(3) & 1(-3) + 2(-1) & 1(1) + 2(-4) \\ 2(2) - 1(3) & 2(-3) - 1(-1) & 2(1) - 1(-4) \end{bmatrix}$$

$$\begin{bmatrix} 8 & -5 & -7 \\ 1 & -5 & 6 \end{bmatrix}$$

The original triangle $\triangle ABC$ is sketched in light blue, and its transformation after L is in dark blue.
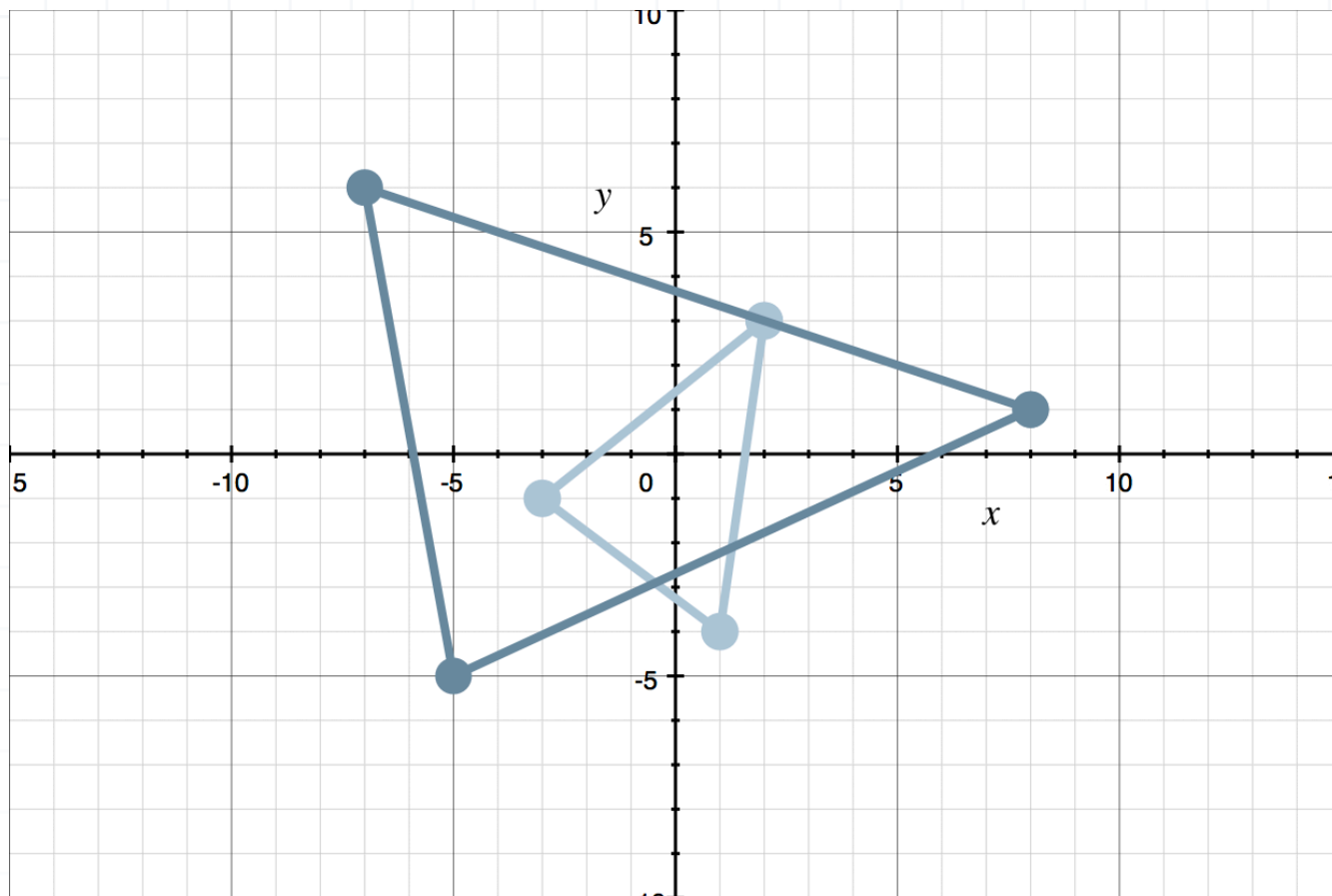
■ 3. Sketch the transformation of the square in the graph after it's transformed by matrix Z.

$$Z = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$
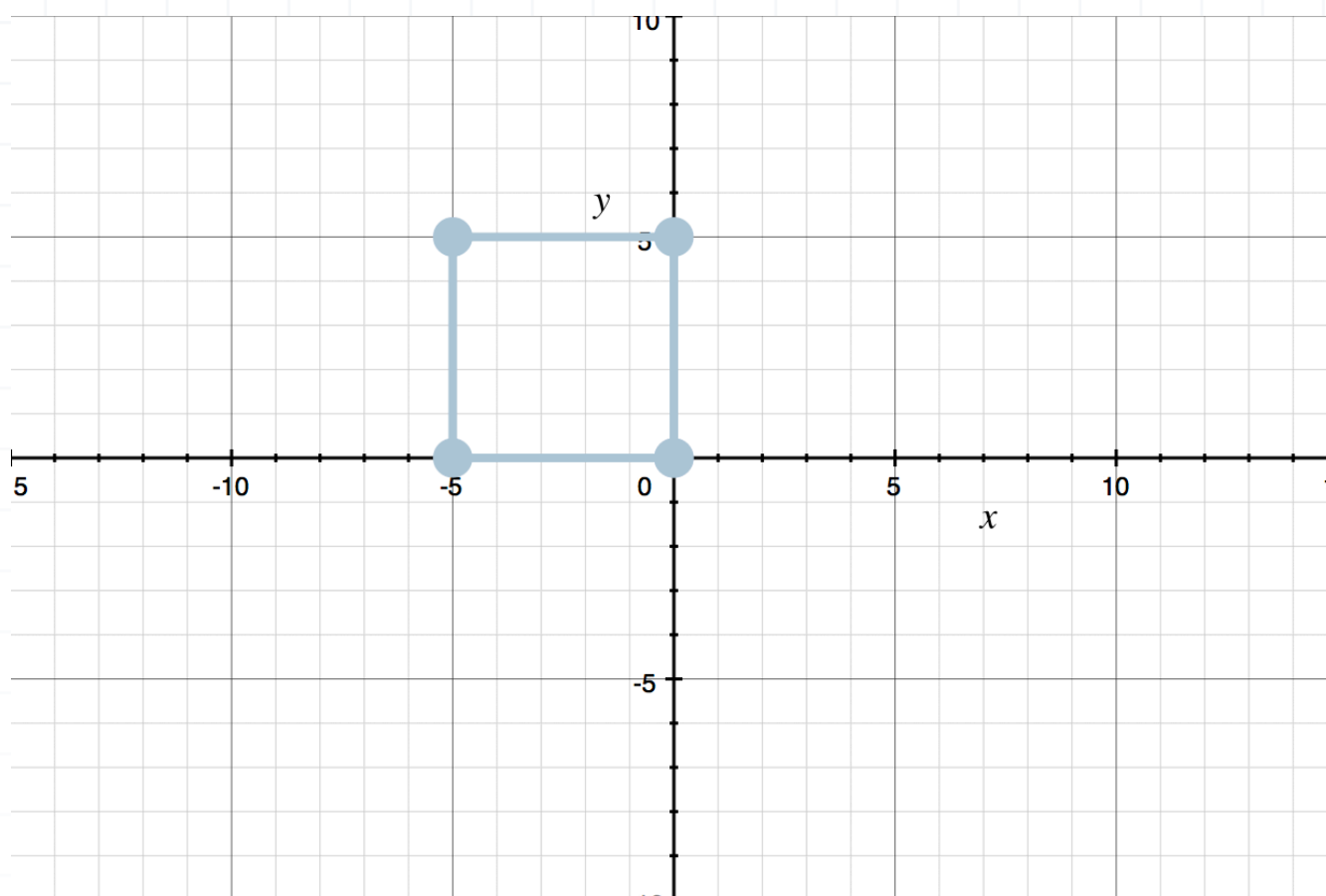
Solution:

Put the vertices of the square into a matrix.

$$\begin{bmatrix} 0 & -5 & -5 & 0 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

Apply the transformation of Z to the vertex matrix.

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & -5 & -5 & 0 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1(0) - 1(0) & 1(-5) - 1(0) & 1(-5) - 1(5) & 1(0) - 1(5) \\ 0(0) + 2(0) & 0(-5) + 2(0) & 0(-5) + 2(5) & 0(0) + 2(5) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -10 & -5 \\ 0 & 0 & 10 & 10 \end{bmatrix}$$

The original square is sketched in light blue, and its transformation after Z is in dark blue.

■ 4. Sketch the transformation of the hexagon after it's transformed by matrix Y.

$$Y = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$$
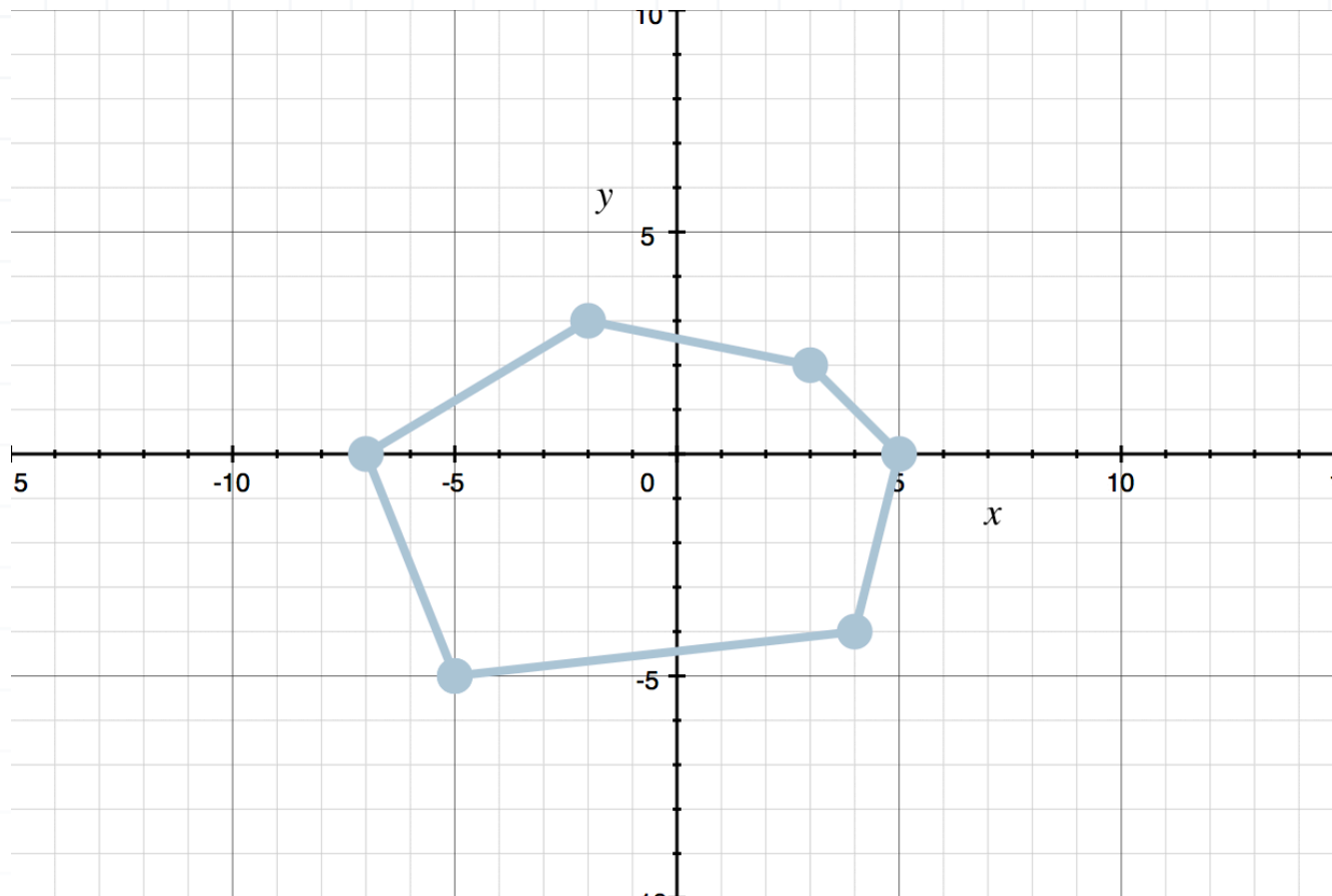
Solution:

Put the vertices of the hexagon into a matrix.

$$\begin{bmatrix} -5 & 4 & 5 & 3 & -2 & -7 & -5 \\ -5 & -4 & 0 & 2 & 3 & 0 & -5 \end{bmatrix}$$
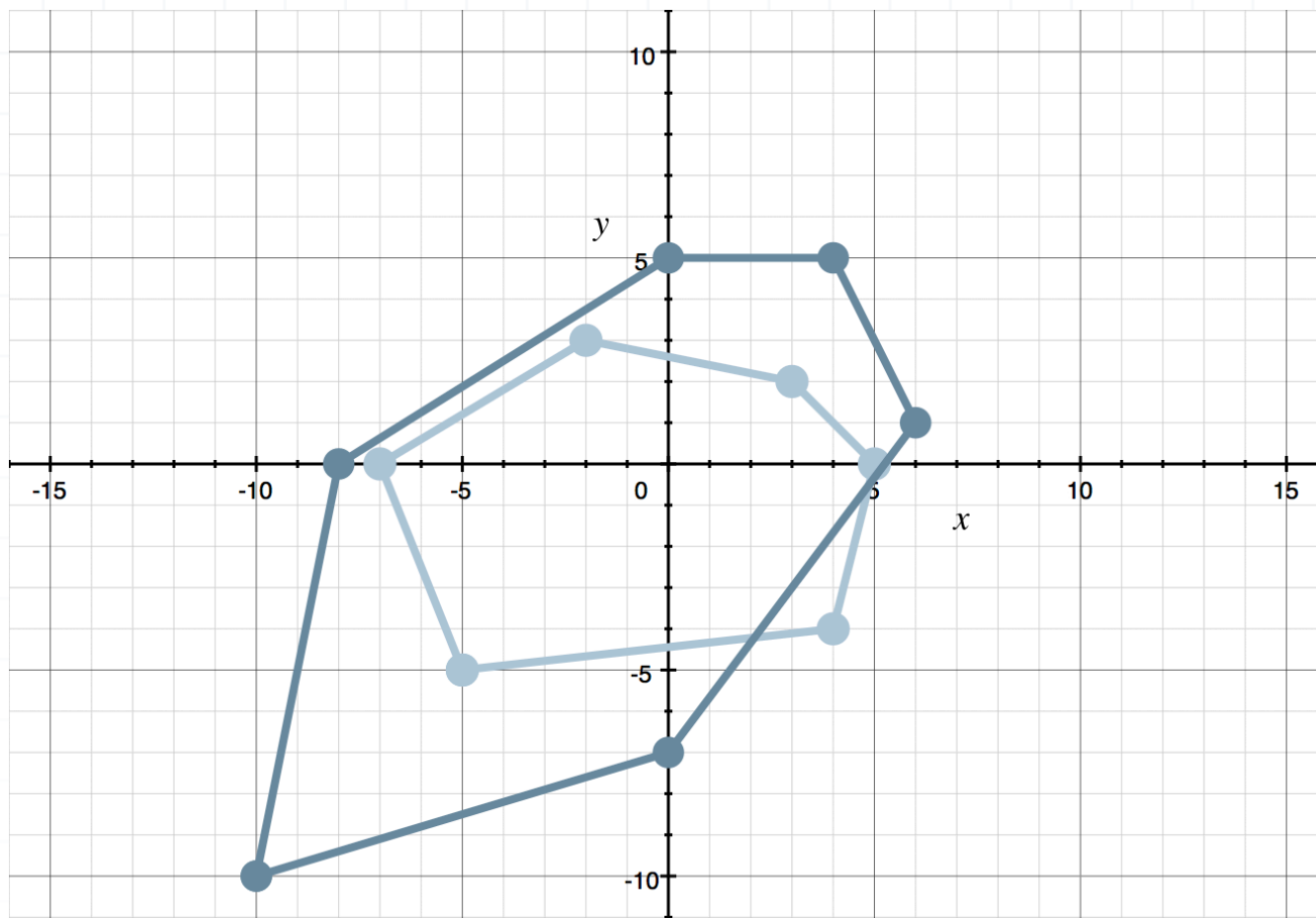
Apply the transformation of Z to the vertex matrix.

$$\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 & 5 & 3 & -2 & -7 \\ -5 & -4 & 0 & 2 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -5(0)-5(2) & 4(0)-4(2) & 5(0)+0(2) & 3(0)+2(2) & -2(0)+3(2) & -7(0)+0(2) \\ -5(1)-5(1) & 4(1)-4(1) & 5(1)+0(1) & 3(1)+2(1) & -2(1)+3(1) & -7(1)+0(1) \end{bmatrix}$$

$$\begin{bmatrix} -10 & -8 & 0 & 4 & 6 & 0 \\ -10 & 0 & 5 & 5 & 1 & -7 \end{bmatrix}$$

The original hexagon is sketched in light blue, and its transformation after Y is in dark blue.

■ 5. What happens to the unit vector $\vec{a} = (1,0)$ after the transformation given by matrix K.

$$K = \begin{bmatrix} 3 & -5 \\ -1 & 0 \end{bmatrix}$$

Solution:

Where $\vec{a} = (1,0)$ lands is given by the first column of the transformation matrix. So $\vec{a}$ will land on $(3, -1)$ after the transformation by K.

■ 6. What happens to the unit vector $\vec{b} = (0,1)$ after the transformation given by matrix K.

$$K = \begin{bmatrix} 3 & -5 \\ -1 & 0 \end{bmatrix}$$

Solution:

Where $\vec{b} = (0,1)$ lands is given by the second column of the transformation matrix. So $\vec{b}$ will land on $(-5,0)$ after the transformation by K.

MATRIX INVERSES, AND INVERTIBLE AND SINGULAR MATRICES

■ 1. Find the determinant of the matrix.

$$B = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

Solution:

The determinant is

$$B = \begin{vmatrix} -3 & 8 \\ 0 & -2 \end{vmatrix}$$

$$B = (-3)(-2) - (8)(0)$$

$$B = 6 - 0$$

$$B = 6$$

■ 2. Find the determinant of the matrix.

$$B = \begin{bmatrix} 1 & -6 \\ 5 & 5 \end{bmatrix}$$

Solution:

The determinant is

$$B = \begin{vmatrix} 1 & -6 \\ 5 & 5 \end{vmatrix}$$

$$B = (1)(5) - (-6)(5)$$

$$B = 5 + 30$$

$$B = 35$$

■ 3. Find the inverse of matrix G.

$$G = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

Solution:

Plug into the formula for the inverse matrix.

$$G^{-1} = \frac{1}{|G|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$G^{-1} = \frac{1}{\begin{vmatrix} -3 & 8 \\ 0 & -2 \end{vmatrix}} \begin{bmatrix} -2 & -8 \\ 0 & -3 \end{bmatrix}$$

$$G^{-1} = \frac{1}{(-3)(-2) - (8)(0)} \begin{bmatrix} -2 & -8 \\ 0 & -3 \end{bmatrix}$$

$$G^{-1} = \frac{1}{6} \begin{bmatrix} -2 & -8 \\ 0 & -3 \end{bmatrix}$$

$$G^{-1} = \begin{bmatrix} -\frac{1}{3} & -\frac{4}{3} \\ 0 & -\frac{1}{2} \end{bmatrix}$$

■ 4. Find the inverse of matrix N.

$$N = \begin{bmatrix} 11 & -4 \\ 5 & -3 \end{bmatrix}$$

Solution:

Plug into the formula for the inverse matrix.

$$N^{-1} = \frac{1}{|N|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$N^{-1} = \frac{1}{\begin{vmatrix} 11 & -4 \\ 5 & -3 \end{vmatrix}} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = \frac{1}{(11)(-3) - (-4)(5)} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = \frac{1}{-33 + 20} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = -\frac{1}{13}\begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = \begin{bmatrix} \dfrac{3}{13} & -\dfrac{4}{13} \\ \dfrac{5}{13} & -\dfrac{11}{13} \end{bmatrix}$$

■ 5. Is the matrix invertible or singular?

$$Z = \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

Solution:

Find the determinant of the matrix.

$$\begin{vmatrix} 4 & 2 \\ -2 & -1 \end{vmatrix}$$

$$(4)(-1) - (2)(-2)$$

$$-4 + 4$$

$$0$$

Because the determinant is 0, Z is a singular matrix that has no inverse.

■ 6. Is the matrix invertible or singular?

$$Y = \begin{bmatrix} 0 & 6 \\ 2 & -1 \end{bmatrix}$$

Solution:

Find the determinant of the matrix.

$$\begin{vmatrix} 0 & 6 \\ 2 & -1 \end{vmatrix}$$

$$(0)(-1) - (6)(2)$$

$$0 - 12$$

$$-12$$

Because the determinant is non-zero, Y is an invertible matrix with a defined inverse.

SOLVING SYSTEMS WITH INVERSE MATRICES

■ 1. Use an inverse matrix to solve the system.

$$-4x + 3y = -14$$

$$7x - 4y = 32$$

Solution:

Transfer the system into a matrix equation.

$$\begin{bmatrix} -4 & 3 \\ 7 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -14 \\ 32 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M^{-1} = \frac{1}{(-4)(-4) - (3)(7)} \begin{bmatrix} -4 & -3 \\ -7 & -4 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{5} \begin{bmatrix} -4 & -3 \\ -7 & -4 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{7}{5} & \frac{4}{5} \end{bmatrix}$$

The solution to the system is

$$\vec{a} = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{7}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} -14 \\ 32 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{4}{5}(-14) + \frac{3}{5}(32) \\ \frac{7}{5}(-14) + \frac{4}{5}(32) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\frac{56}{5} + \frac{96}{5} \\ -\frac{98}{5} + \frac{128}{5} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{40}{5} \\ \frac{30}{5} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

■ 2. Use an inverse matrix to solve the system.

$$6x - 11y = 2$$

$$-10x + 7y = -26$$

Solution:

Transfer the system into a matrix equation.

$$\begin{bmatrix} 6 & -11 \\ -10 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -26 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M^{-1} = \frac{1}{(6)(7) - (-11)(-10)} \begin{bmatrix} 7 & 11 \\ 10 & 6 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{68} \begin{bmatrix} 7 & 11 \\ 10 & 6 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\dfrac{7}{68} & -\dfrac{11}{68} \\ -\dfrac{10}{68} & -\dfrac{6}{68} \end{bmatrix}$$

The solution to the system is

$$\vec{a} = \begin{bmatrix} -\dfrac{7}{68} & -\dfrac{11}{8} \\ -\dfrac{10}{68} & -\dfrac{6}{68} \end{bmatrix} \begin{bmatrix} 2 \\ -26 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\dfrac{7}{68}(2) - \dfrac{11}{68}(-26) \\ -\dfrac{10}{68}(2) - \dfrac{6}{68}(-26) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\dfrac{14}{68} + \dfrac{286}{68} \\ -\dfrac{20}{68} + \dfrac{156}{68} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \dfrac{272}{68} \\ \dfrac{136}{68} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

■ 3. Use an inverse matrix to solve the system.

$$13y - 6x = -81$$

$$7x + 17 = -22y$$

Solution:

Transfer the system into a matrix equation.

$$\begin{bmatrix} -6 & 13 \\ 7 & 22 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -81 \\ -17 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M^{-1} = \frac{1}{(-6)(22) - (13)(7)} \begin{bmatrix} 22 & -13 \\ -7 & -6 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{223} \begin{bmatrix} 22 & -13 \\ -7 & -6 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\dfrac{22}{223} & \dfrac{13}{223} \\ \dfrac{7}{223} & \dfrac{6}{223} \end{bmatrix}$$

The solution to the system is

$$\vec{a} = \begin{bmatrix} -\dfrac{22}{223} & \dfrac{13}{223} \\ \dfrac{7}{223} & \dfrac{6}{223} \end{bmatrix} \begin{bmatrix} -81 \\ -17 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\dfrac{22}{223}(-81) + \dfrac{13}{223}(-17) \\ \dfrac{7}{223}(-81) + \dfrac{6}{223}(-17) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \dfrac{1,782}{223} - \dfrac{221}{223} \\ -\dfrac{567}{223} - \dfrac{102}{223} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \dfrac{1,561}{223} \\ -\dfrac{669}{223} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

■ 4. Sketch a graph of vectors to visually find the solution to the system.

$$3x = 3$$

$$x - y = -2$$

Solution:

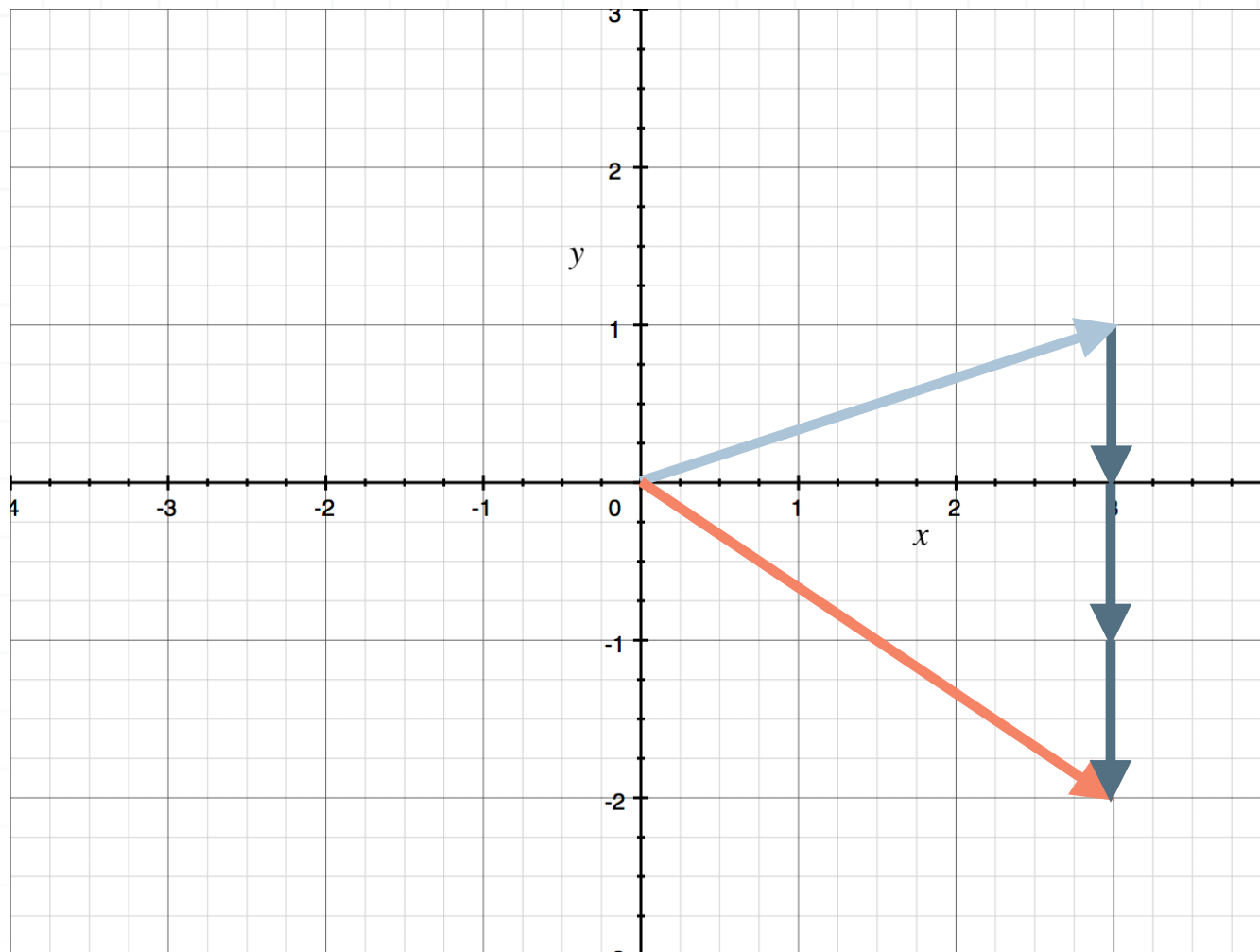Put the system into a matrix equation.

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} y = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

From the vector equation, we can sketch the vectors $\vec{s} = (3,1)$ for x, $\vec{t} = (0, -1)$ for y, and the resulting vector $\vec{u} = (3, -2)$.
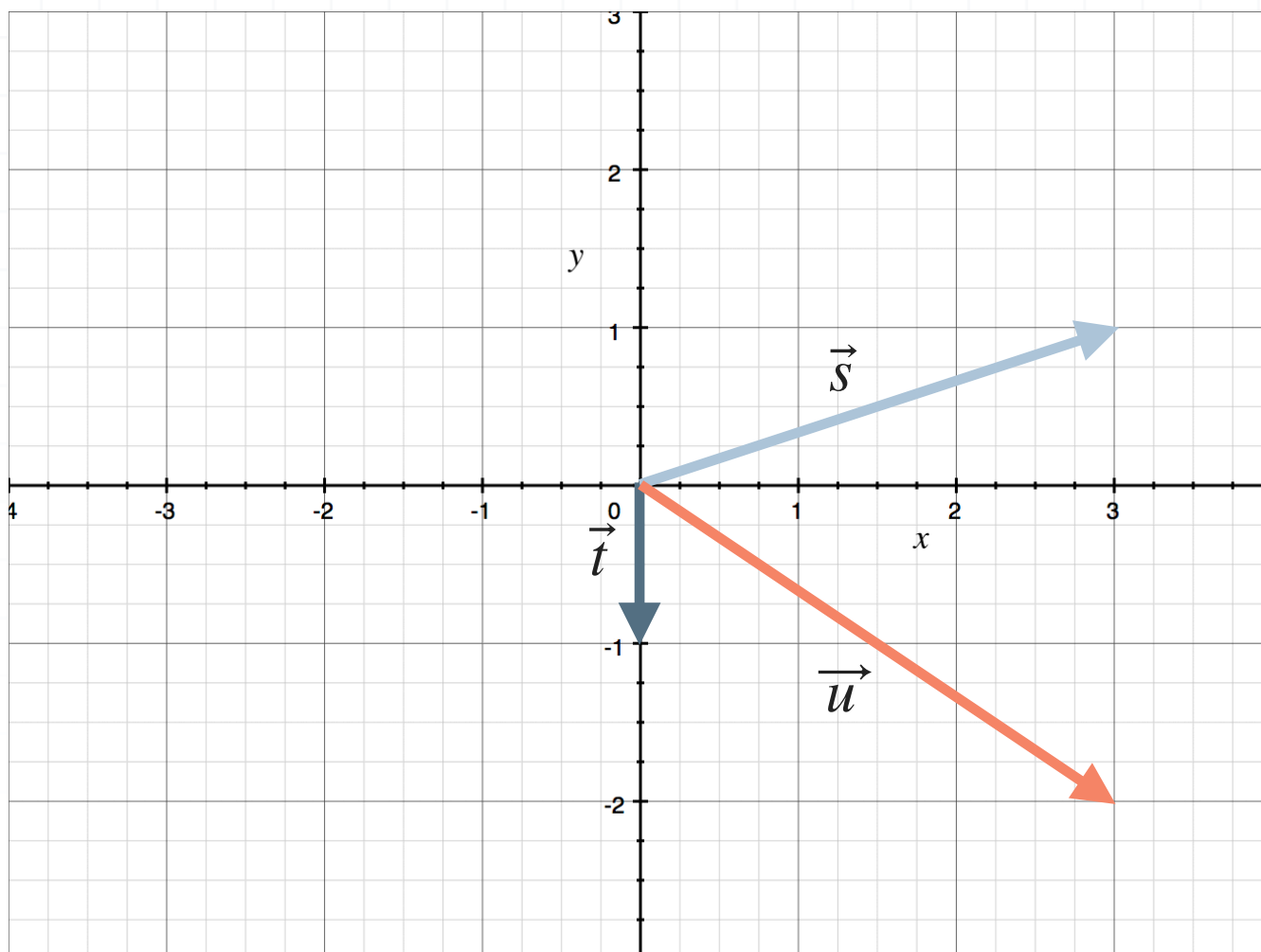
If we play around a little bit with the vectors in the graph, we can see that putting one $\vec{s}$ and three $\vec{t}$s together will get us back to the terminal point of $\vec{u}$, so $x = 1$ and $y = 3$.

◼ 5. Sketch a graph of vectors to visually find the solution to the system.
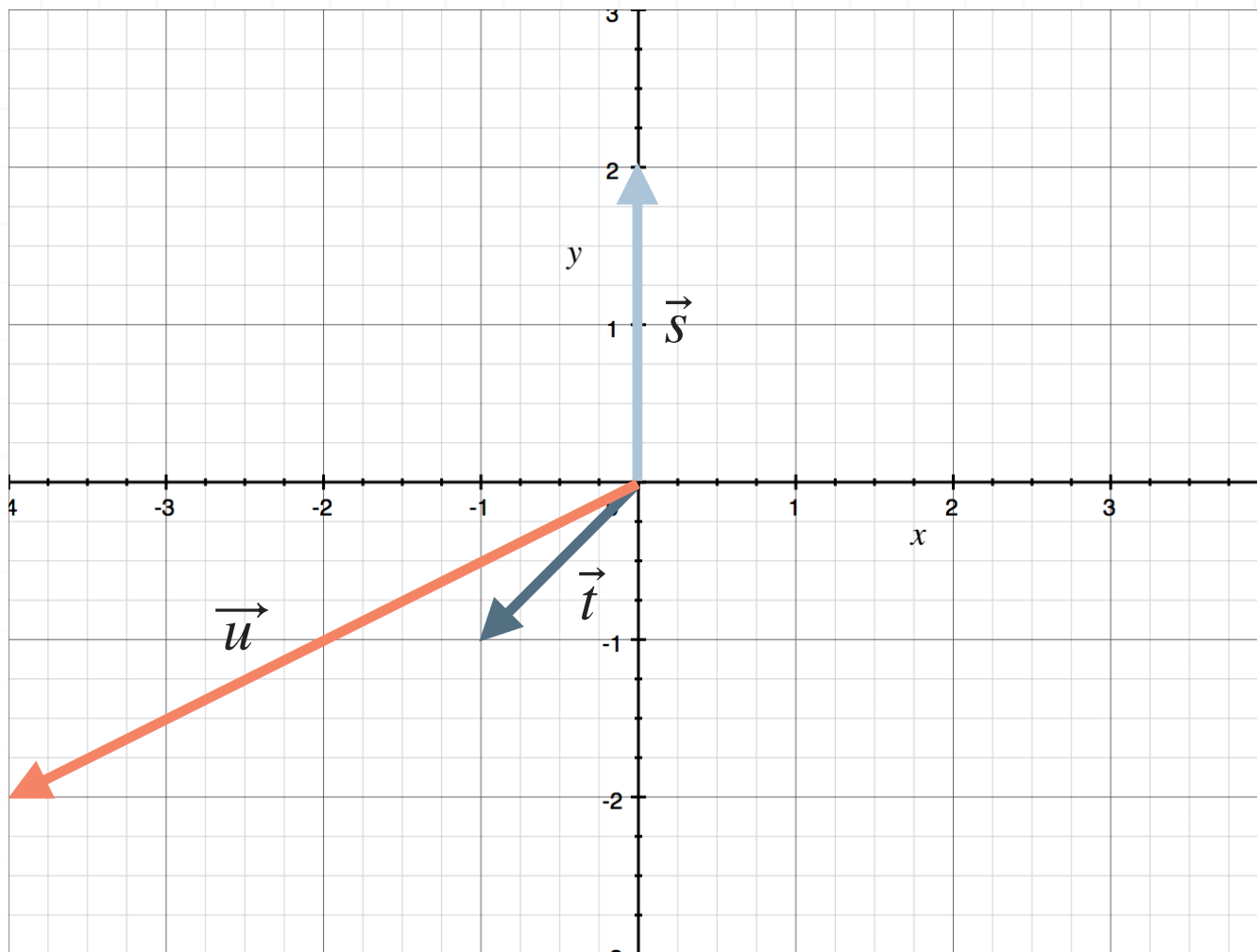
$$-y = -4$$

$$2x - y = -2$$

Solution:

Put the system into a matrix equation.

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} x + \begin{bmatrix} -1 \\ -1 \end{bmatrix} y = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$
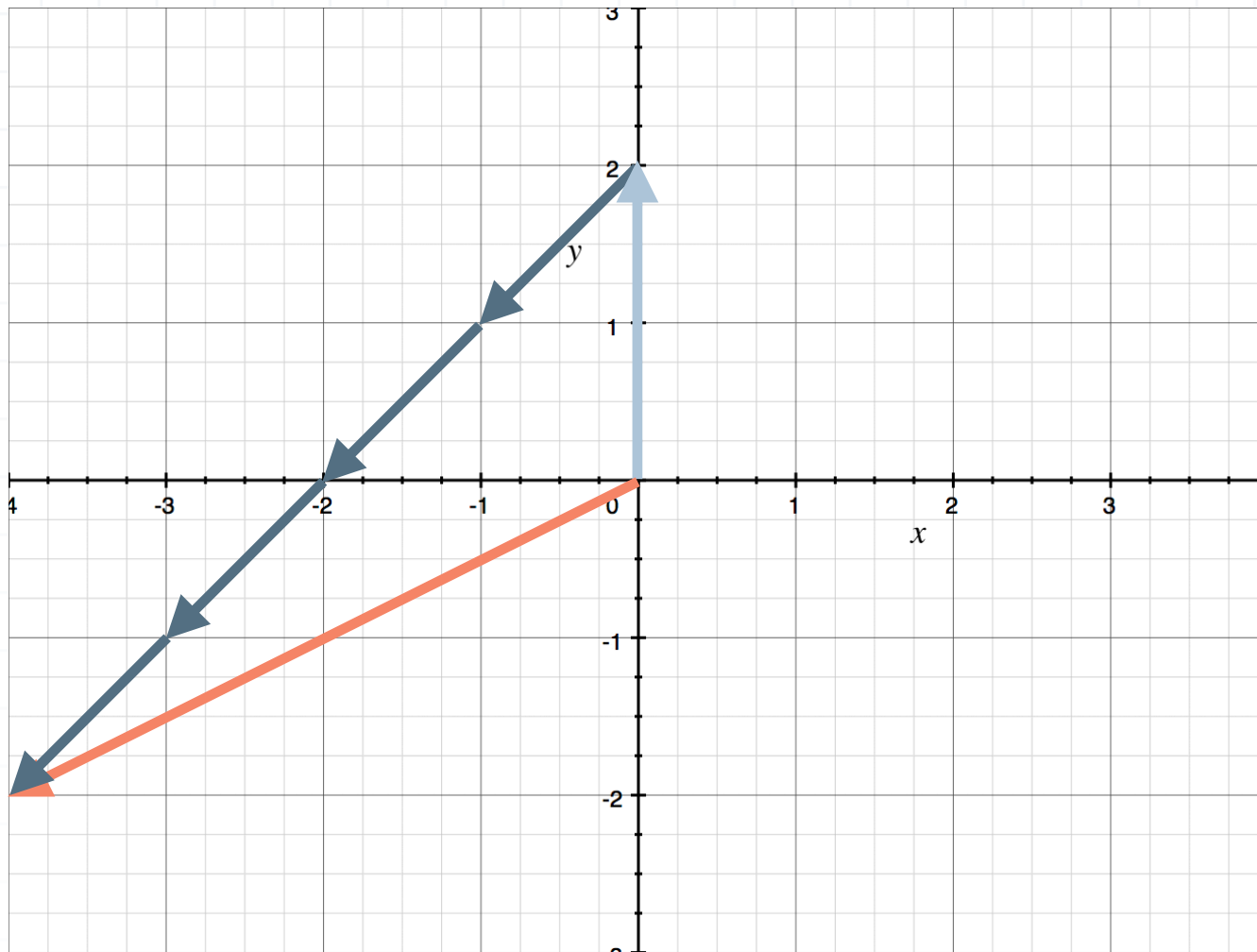
From the vector equation, we can sketch the vectors $\vec{s} = (0,2)$ for x, $\vec{t} = (-1, -1)$ for y, and the resulting vector $\vec{u} = (-4, -2)$.

If we play around a little bit with the vectors in the graph, we can see that putting one $\vec{s}$ and four $\vec{t}$s together will get us back to the terminal point of $\vec{u}$, so $x = 1$ and $y = 4$.

■ 6. Sketch a graph of vectors to visually find the solution to the system.
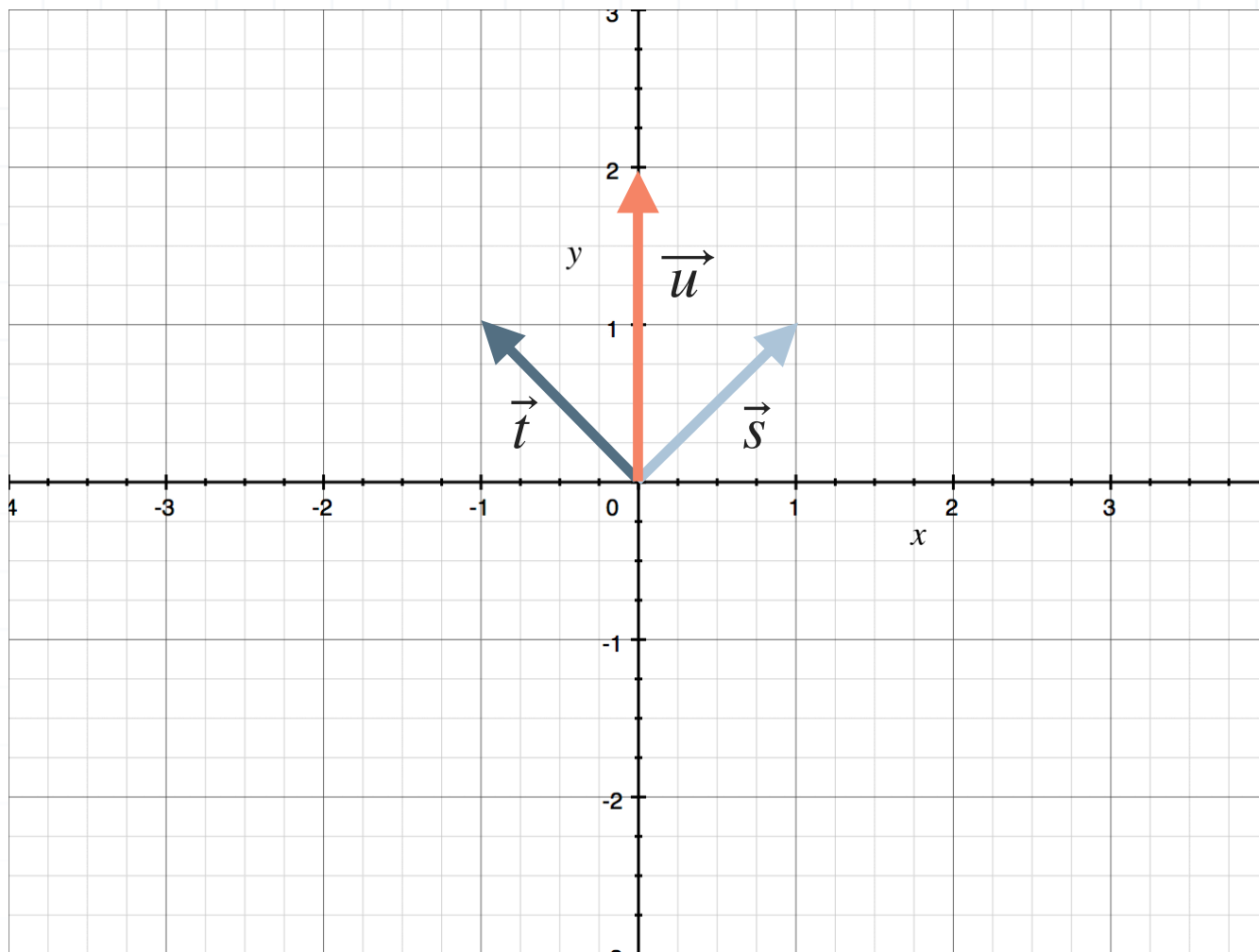
$$x - y = 0$$

$$x + y = 2$$

Solution:

Put the system into a matrix equation.

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$
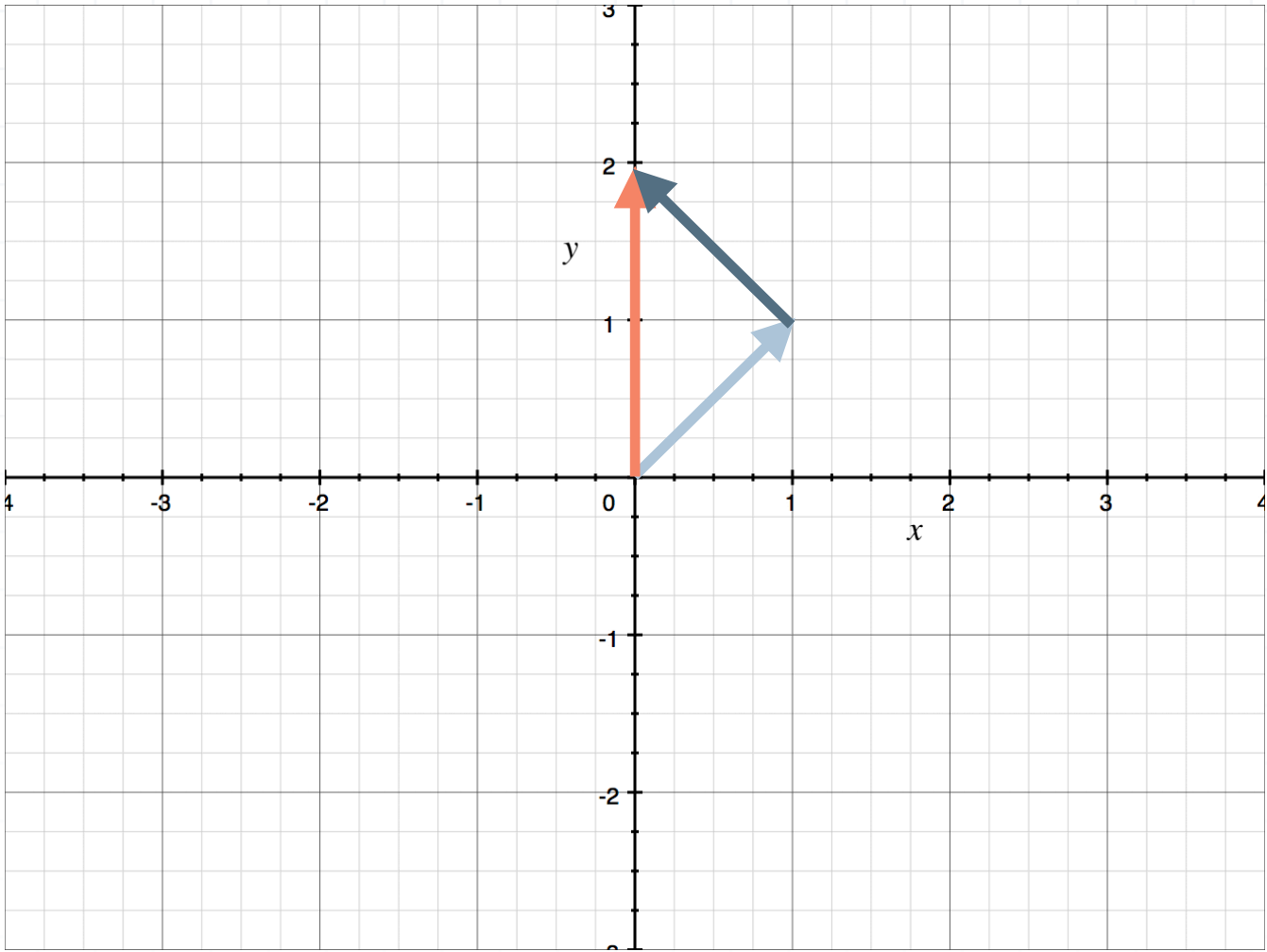
From the vector equation, we can sketch the vectors $\vec{s} = (1,1)$ for x, $\vec{t} = (-1,1)$ for y, and the resulting vector $\vec{u} = (0,2)$.

If we play around a little bit with the vectors in the graph, we can see that putting one $\vec{s}$ and one $\vec{t}$ together will get us back to the terminal point of $\vec{u}$, so $x = 1$ and $y = 1$.

SOLVING SYSTEMS WITH CRAMER'S RULE

■ 1. Use Cramer's Rule to find the expression that would give the solution for x. You do not need to solve the system.

$$2x - y = 5$$

$$x + 3y = 15$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} 5 & -1 \\ 15 & 3 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$\frac{\begin{vmatrix} 5 & -1 \\ 15 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}}$$

■ 2. Use Cramer's Rule to find the expression that would give the solution for x. You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$\frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

■ 3. Use Cramer's Rule to find the expression that would give the solution for y. You do not need to solve the system.

$$3x + 4y = 11$$

$$2x - 3y = -4$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix}$$

To find D_y, replace the second column of the coefficient matrix with the answer column.

$$D_y = \begin{vmatrix} 3 & 11 \\ 2 & -4 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_y/D.

$$\frac{\begin{vmatrix} 3 & 11 \\ 2 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix}}$$

■ 4. Use Cramer's Rule to find the expression that would give the solution for y. You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

To find D_y, replace the second column of the coefficient matrix with the answer column.

$$D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_y/D.

$$\frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

■ 5. Use Cramer's Rule to solve for x.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$\frac{\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}}$$

Calculate the value of x.

$$x = \frac{1(5) - 2(4)}{3(5) - 2(6)}$$

$$x = \frac{5 - 8}{15 - 12}$$

$$x = \frac{-3}{3}$$

$$x = -1$$

■ 6. Use Cramer's Rule to solve for y.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

To find D_y, replace the second column of the coefficient matrix with the answer column.

$$D_y = \begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_y/D.

$$\frac{\begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}}$$

Calculate the value of y.

$$y = \frac{3(4) - 1(6)}{3(5) - 2(6)}$$

$$y = \frac{12 - 6}{15 - 12}$$

$$y = \frac{6}{3}$$

$$y = 2$$