Precalculus Workbook

Polar curves

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POLAR COORDINATES

■ 1. Find the rectangular point that's equivalent to the polar point.

$$(r, \theta) = \left(-14, \frac{5\pi}{6}\right)$$

■ 2. Convert the rectangular point $(x, y) = (5\sqrt{2}, 5\sqrt{2})$ to polar coordinates.

■ 3. Convert the rectangular point to polar coordinates.

$$(x, y) = \left(\cos\left(\frac{2\pi}{7}\right), \sin\left(\frac{2\pi}{7}\right)\right)$$

■ 4. Convert the polar point into rectangular coordinates.

$$(r, \theta) = \left(6, \frac{\pi}{6}\right)$$

■ 5. Convert the rectangular point $(x, y) = (2\sqrt{3}, -6)$ to polar coordinates.

■ 6. Convert the polar point $(r, \theta) = (8, 2\pi)$ into rectangular coordinates.

MULTIPLE WAYS TO EXPRESS POLAR POINTS

■ 1. Find five polar points that are equivalent to $(6, -\pi/4)$, where the angle θ lies within $(-3\pi,3\pi)$.

■ 2. Find a point that's equivalent to $(3,101\pi/2)$, with a positive radius r and a polar angle θ that lies within $(0,2\pi]$.

■ 3. How many points are equivalent to the point $(1,\pi/7)$ and have a polar angle in the interval $0 < \theta < 5\pi$?

■ 4. Find four polar points that are equivalent to the point $(-10,16\pi/7)$ and have a polar angle in the interval $(-2\pi,2\pi)$.

■ 5. How many points in $0 < \theta < 3\pi/2$ are equivalent to the point $(2, -2\pi/3)$?

■ 6. Find a point equivalent to $(-1,53\pi/6)$ with positive radius r and a polar angle θ within $(0,2\pi]$.

CONVERTING EQUATIONS

■ 1. Convert the rectangular equation into polar coordinates.

$$(x+9)^2+(y-13)^2=64$$

■ 2. Convert the rectangular equation into polar coordinates.

$$\frac{(x+5)^2}{9}+\frac{(y-7)^2}{4}=1$$

■ 3. Convert the rectangular equation into polar coordinates.

$$x=\frac{1}{y+3}$$

■ 4. Convert the polar equation to rectangular coordinates.

$$r^2=4\sin^2\theta$$

■ 5. Convert the polar equation to rectangular coordinates.

$$\theta=r^2+1$$

6. Convert the polar equation to rectangular coordinates.

$$r = e^{\theta}$$

GRAPHING POLAR CURVES IN A RECTANGULAR SYSTEM

■ 1. Sketch the graph of $r = 3\sin\theta - 3$ in a rectangular coordinate system.

■ 2. Convert $r = 4\cos\theta$ to rectangular coordinates and then sketch the graph of the resulting equation.

■ 3. Convert $r = 6\sin\theta - 2\cos\theta$ into rectangular coordinates and then sketch the graph of the resulting equation.

■ 4. Convert $r^2\sin(2\theta) = 1$ into rectangular coordinates and then sketch the graph of the resulting equation. Hint: Apply the trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$.

■ 5. Convert $r\cos\theta - \tan\theta + 2 = 0$ into rectangular coordinates and then sketch the graph of the resulting equation.

■ 6. Convert the polar equation into rectangular coordinates and the sketch the graph of the resulting equation.

$$r^2 = \frac{\sin\theta}{\cos^3\theta}$$

GRAPHING CIRCLES

- 1. Sketch the graph of $r = -1$.

- 2. Sketch the graph of $r = 8 \sin \theta$.

- 3. Sketch the graph of $r = 5 \cos \theta$.

- 4. Sketch the graph of $r = -3 \cos \theta$.

- 5. Sketch the graph of $r = -\sin \theta$.

- 6. Sketch the graph of $r^2 = 4 \sin^2 \theta$.

GRAPHING ROSES

- 1. Sketch the graph of the rose $r = 2\sin(5\theta)$.

- 2. Sketch the graph of the rose $r = -4\cos(3\theta)$.

- 3. Sketch the graph of the rose $r = -3\sin(2\theta)$.

- 4. Sketch the graph of the rose $r = 7\cos(4\theta)$.

- 5. How many petals will the rose have?

$$r = 5\cos(12\theta)$$

- 6. Write an equation of a rose that has 9 petals and passes through the point $(6, 3\pi/2)$.

GRAPHING CARDIOIDS

■ 1. Create a table of values for $r = 5 - 5\cos\theta$ over the interval $0 \leq \theta \leq 2\pi$, then use it to sketch the graph.

■ 2. Create a table of values for $r = 3 - 3\sin\theta$ over the interval $0 \leq \theta \leq 2\pi$, then use it to sketch the graph.

■ 3. Create a table of values for $r = -2 - 2\sin\theta$ over the interval $0 \leq \theta \leq 2\pi$, then use it to sketch the graph.

■ 4. Create a table of values for $r = -3 - 3\cos\theta$ over the interval $0 \leq \theta \leq 2\pi$, then use it to sketch the graph.

■ 5. Sketch the graph of the cardioid $r = 2 + 2\cos\theta$.

■ 6. Find the equation of the cardioid that extends out a distance of 2 from the pole, is symmetric across the vertical axis, and mostly sits above of the horizontal axis. Then sketch its graph.

GRAPHING LIMACONS

- 1. Sketch the graph of $r = 2 + 5\cos\theta$.

- 2. Sketch the graph of $r = 4 - \sin\theta$.

- 3. Sketch the graph of $r = 7 - 6\cos\theta$.

- 4. Sketch the graph of $r = 3 + 2\sin\theta$.

- 5. Sketch the graph of $r = 1 - 20\sin\theta$.

- 6. Sketch the graph of $r = 9 + 4\cos\theta$.

GRAPHING LEMNISCATES

■ 1. Sketch the graph of $r^2 = -4\cos(2\theta)$.

■ 2. Sketch the graph of $r^2 = 7\sin(2\theta)$.

■ 3. Sketch the graph of $r^2 = 4\cos(2\theta)$.

■ 4. Sketch the graph of $r^2 = -16\sin(2\theta)$.

■ 5. Find the equation of the lemniscate that has loops that sit along the horizontal axis and extend out to a distance of 5, then sketch its graph.

■ 6. Find the equation of the lemniscate that has loops that sit in the first and third quadrants and extend out to a distance of $\sqrt{5}$, then sketch its graph.

INTERSECTION OF POLAR CURVES

■ 1. Find the points of intersection of $r = 4 - 4\cos\theta$ and $r = 4 + 4\sin\theta$.

■ 2. Find the points of intersection of $r = 3 - 4\sin\theta$ and $r = -\sin\theta$.

■ 3. Find the points of intersection of $r = -3 - 3\sin\theta$ and $r = 3 - \sin\theta$.

■ 4. Find the points of intersection of $r = 1 + 2\sin\theta$ and $r = 1 + 2\cos\theta$.

■ 5. How many points of intersection exist for the graphs of the polar equations? Hint: Think just about the shape of each curve, without trying to solve algebraically for the points of intersection.

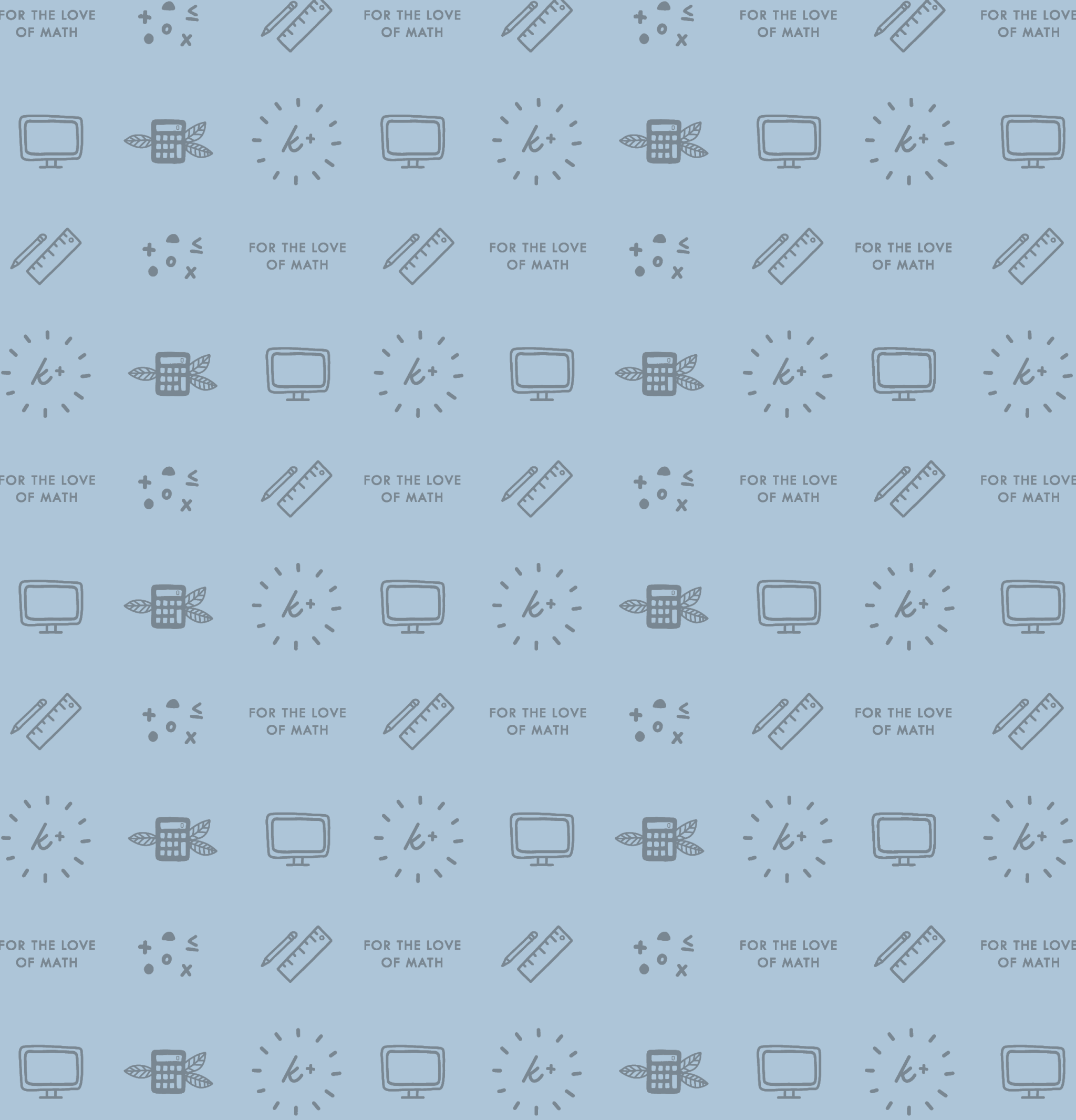
$$r = 5\sin(3\theta)$$

$$r = 4$$

■ 6. How many points of intersection exist for the graphs of the polar equations? Hint: Think just about the shape of each curve, without trying to solve algebraically for the points of intersection.

$r = 2\sin(2\theta)$

$r^2 = 9\sin(2\theta)$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