Adaptive Beam-forming

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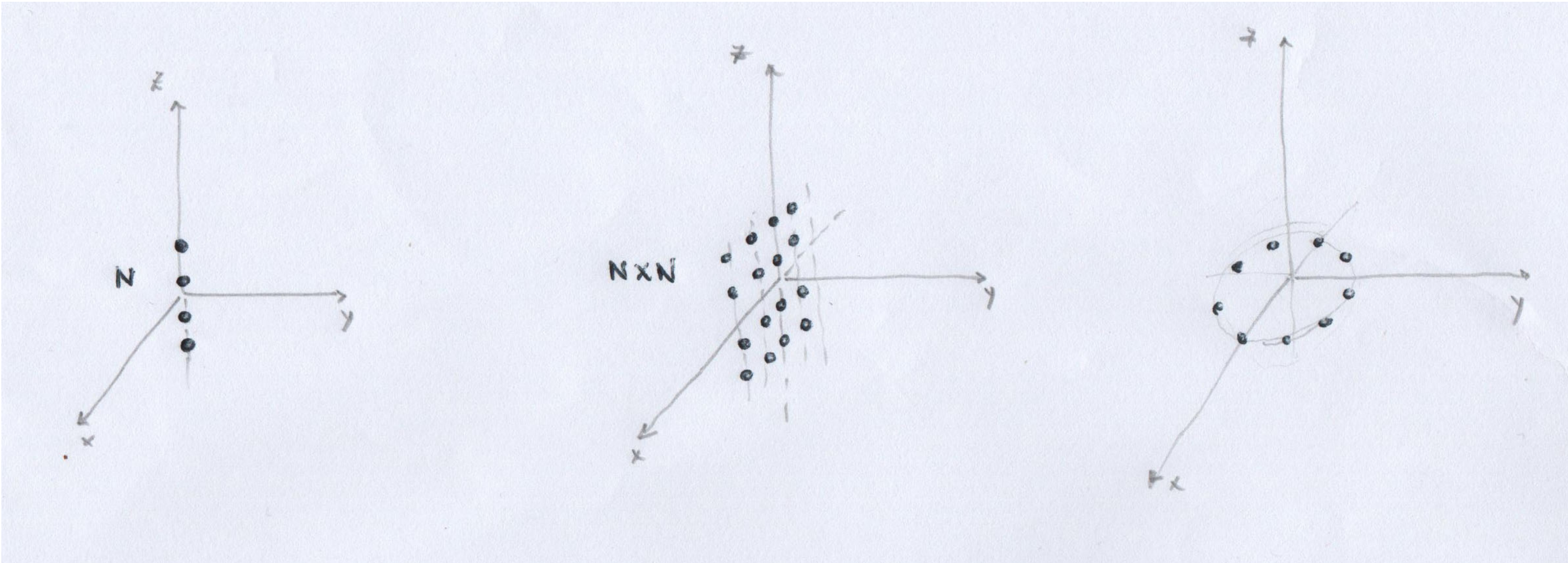
Outline

1. Antenna Arrays
2. Beam-forming
3. Adaptive beam-forming
4. Beam-forming in LTE / NR
5. Project scenario

Antenna arrays

Sets of antennas seen as a single antenna system



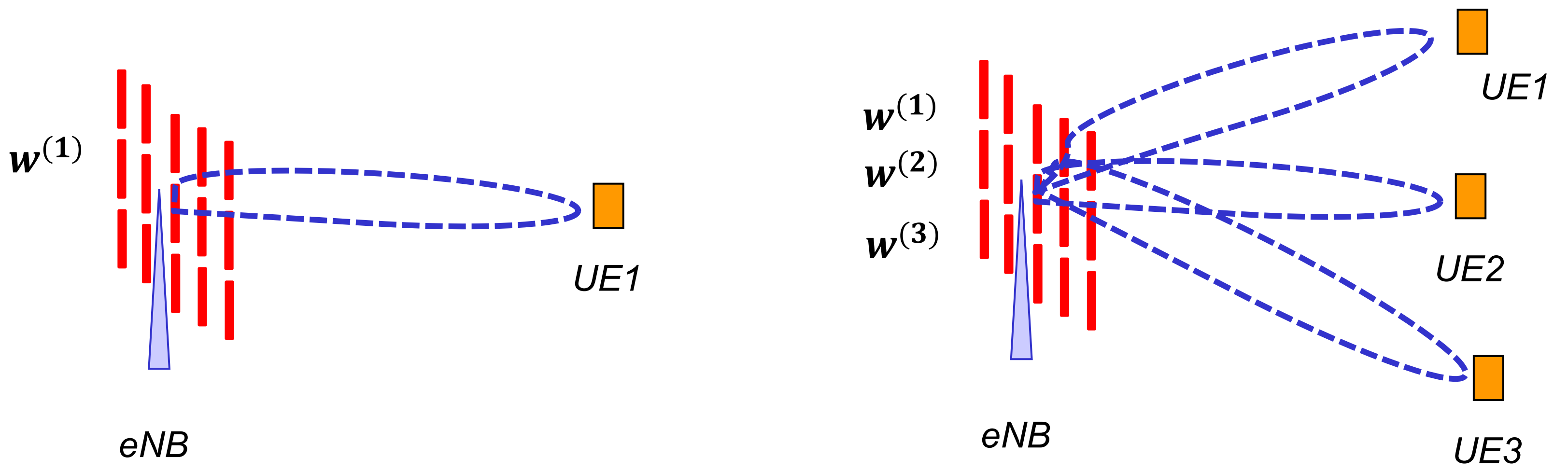
Linear array

Rectangular array

Circular array

Antenna arrays

- **Beamforming:** a method for **controlling the radiation pattern** of the array by means of weights applied to the signals coming from the single array elements.



single user **multi-user**

Advantages

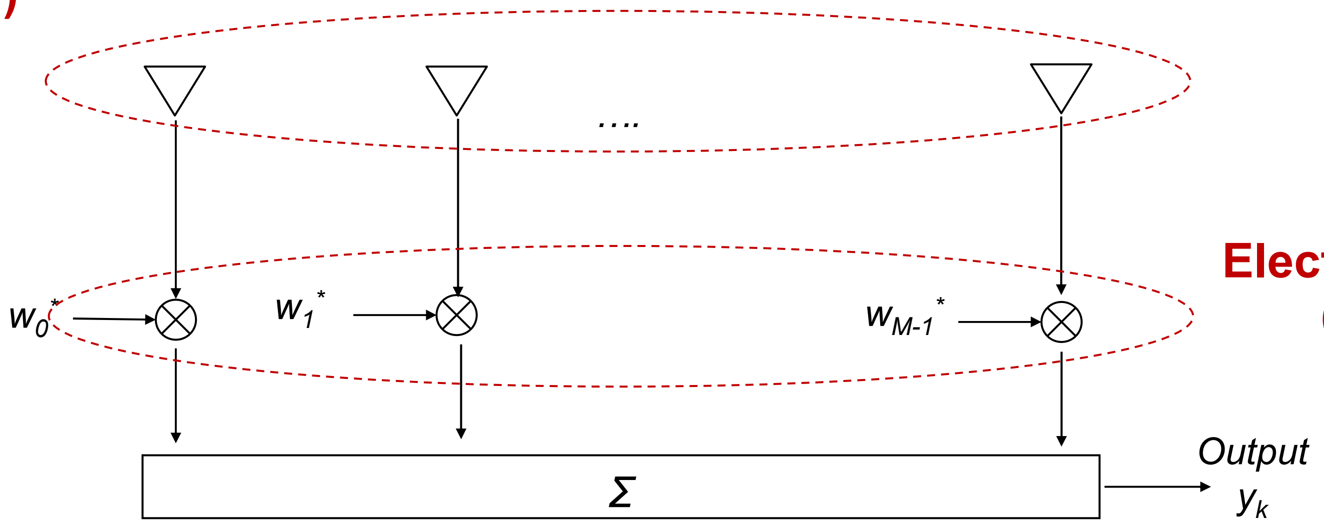
- Increase of **SINR** (impact on link quality / BER, coverage, range, …).
- Increase of the **capacity** (space division multiple access).
- Improvement of user **position** estimation.

Drawbacks

- Hardware **costs**: multiple RF chains are required.
- More **power** consumption.
- Increased **complexity**: signal processing at transmitter and receiver, calibration.
- Practical design problems concerning the space occupied by the array.

Antenna array: structure of the system

Geometric layout of the antenna elements (fixed)



Electronic weights (adaptive)

Spatial filtering

The pattern multiplication principle

For a general array of antennas, the radiation pattern is the product between the element pattern function and the **array pattern function**.

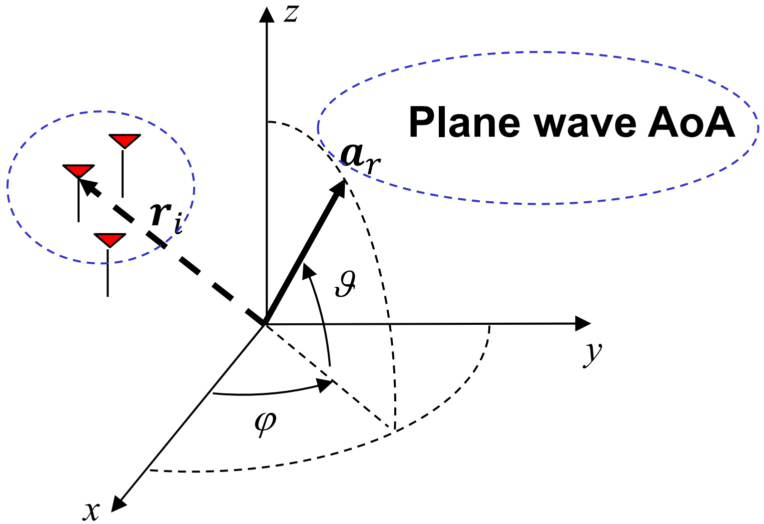
$$\beta = \frac{2\pi}{\lambda}$$

$$|f_{EL}(\theta,\varphi)|^2 \cdot |AF(\theta,\varphi)|^2$$
$$= |f_{EL}(\theta,\varphi)|^2 \cdot \left| \sum_{i=1}^{N} \underbrace{C_i \cdot e^{j(\alpha_i}}_{w_i} {}^{+\beta \boldsymbol{a}_r \cdot \boldsymbol{r}_i)} \right|^2$$

weights

$$AF(\theta,\varphi) = \boldsymbol{w}^H \boldsymbol{s}$$

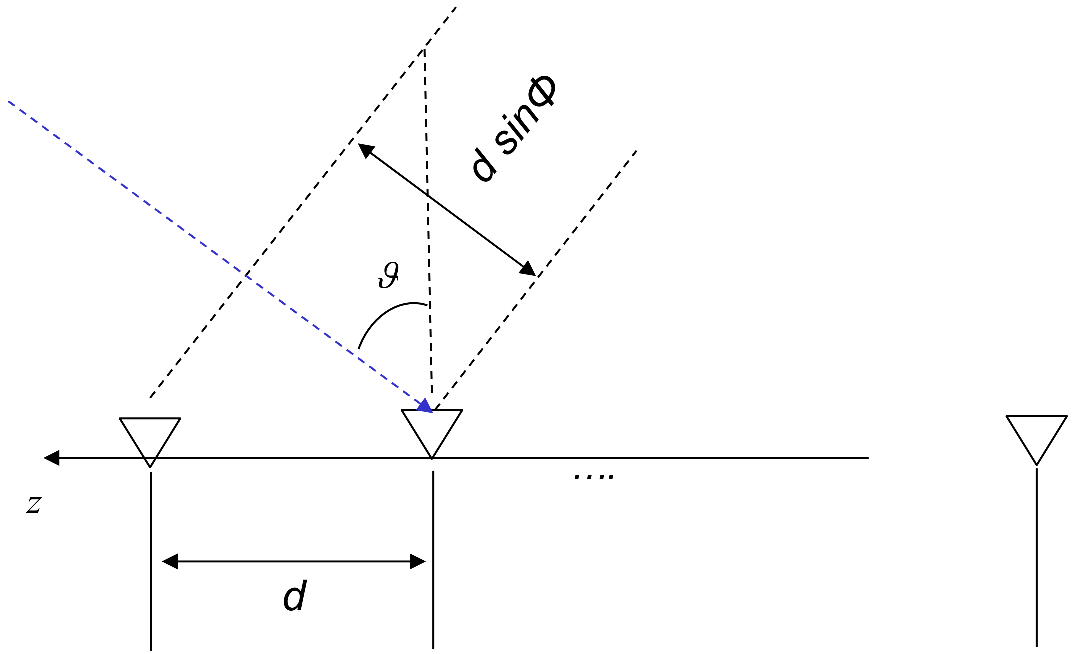
Steering vector



The transmit radiation pattern is equal to the receive on, due to reciprocity.

Uniform Linear Arrays (ULA)

A plane wave cumulates at the array elements, a phase contribution that is function of the angle of arrival (AoA) and the antenna spacing.



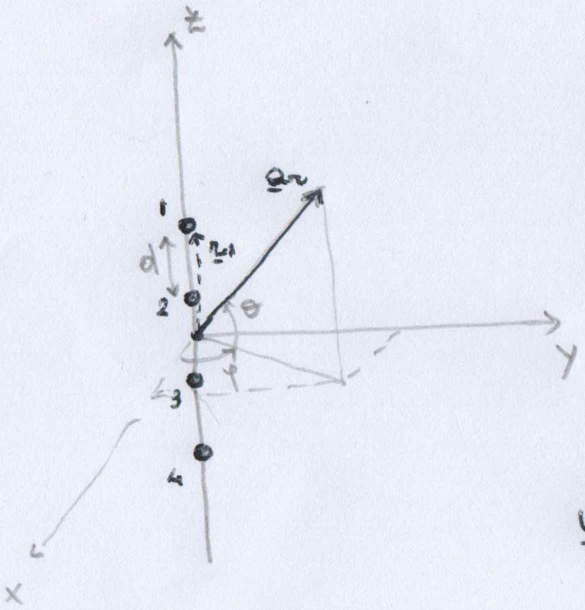
The linear array response depends on a single **Angle of Arrival** (AoA) and it is derived using the **steering vector** (e.g. w.r.t. all the gains equal to *1*):

$$s_n = e^{-j(\beta n \cdot d \sin\vartheta)}$$

$$\beta = \frac{2\pi}{\lambda}$$

The directivity is proportional to *N*, corresponding to the coherent combination of all the incoming signals.

Uniform Linear Arrays (ULA)



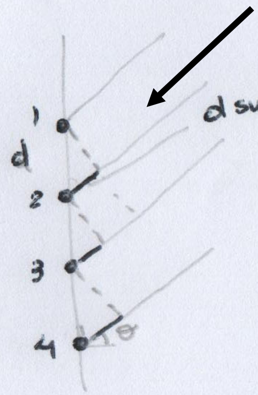
$$\underline{a}_r = \cos\theta\cos\varphi\,\underline{a}_x + \cos\theta\sin\varphi\,\underline{a}_y + \sin\theta\,\underline{a}_z$$

$$\underline{r}_i = \left[(N-i)d - \frac{L}{2}\right]\underline{a}_z$$

$$\underline{a}_r \cdot \underline{r}_i = \left(Nd - \frac{L}{2} - id\right)\sin\theta$$

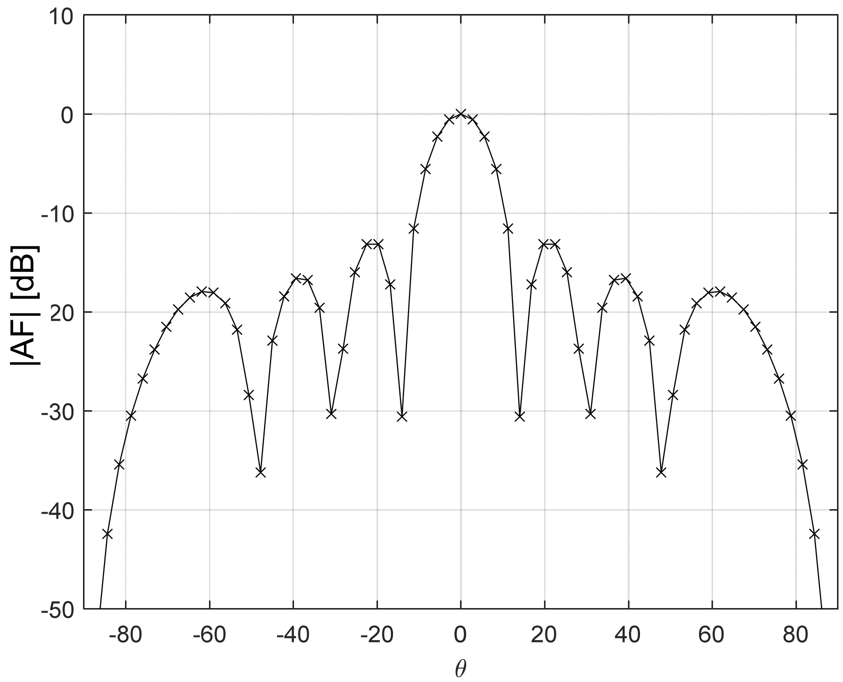
$$\underset{W_i = 1}{\Rightarrow} \quad |AF| = \left|\sum_{i=1}^{N} e^{-j\frac{2\pi}{\lambda} i d \sin\theta}\right|$$

$$\underline{s} = e^{-j2\pi/\lambda\, d\sin\theta}\left[1 \quad e^{-j2\pi/\lambda\, d\sin\theta} \quad e^{-j2\pi/\lambda \cdot 2 \cdot d\sin\theta} \quad e^{-j2\pi/\lambda \cdot 3 \cdot d\sin\theta}\right]$$



$d\sin\theta \rightarrow -\frac{2\pi}{\lambda} d\sin\theta$

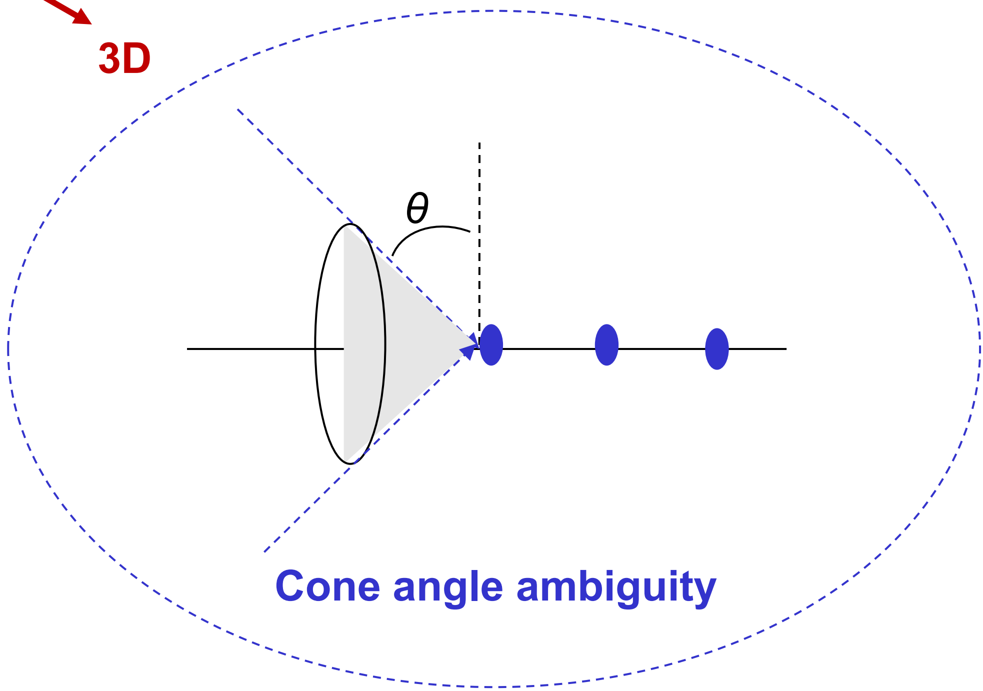
Uniform Linear Arrays (ULA)



Array Factor magnitude

Here N = 8, d = $\lambda/2$ and the AoA θ ….

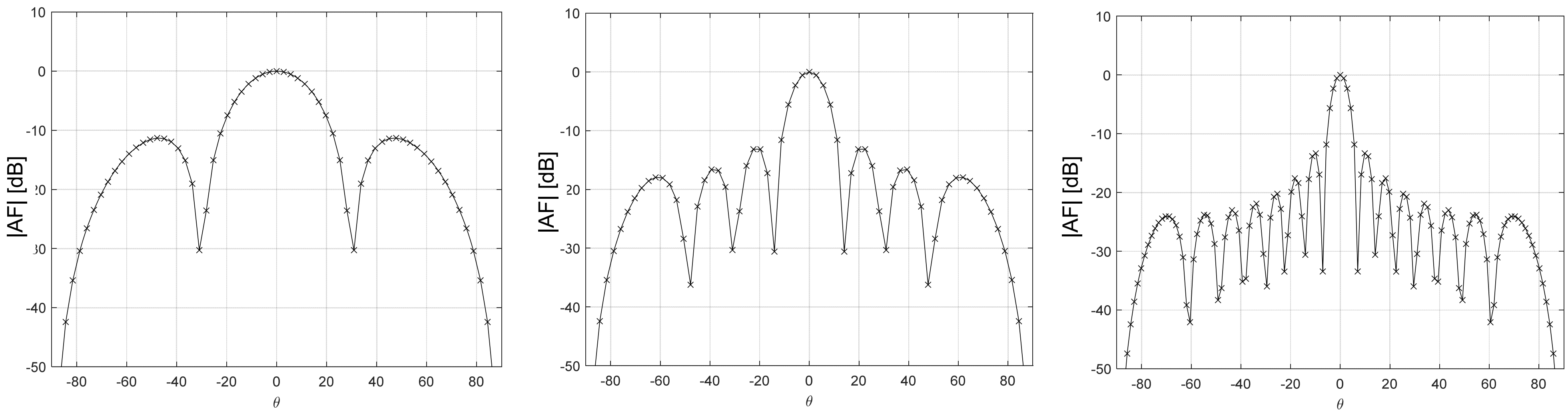
2D → **3D**



Uniform Linear Arrays (ULA)

Beamforming resolution

We can observe that the beam-width is inversely proportional to *N*.

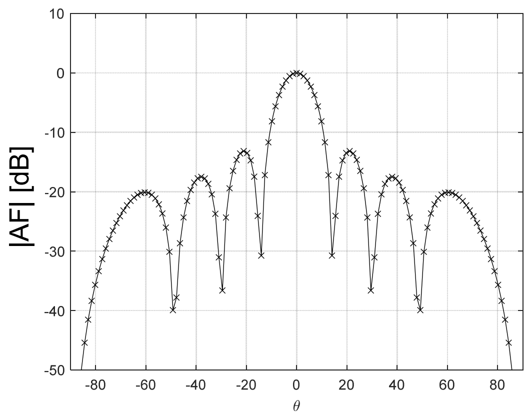


Array Factor magnitude $N = 4,8,16$ and $d = \lambda/2$

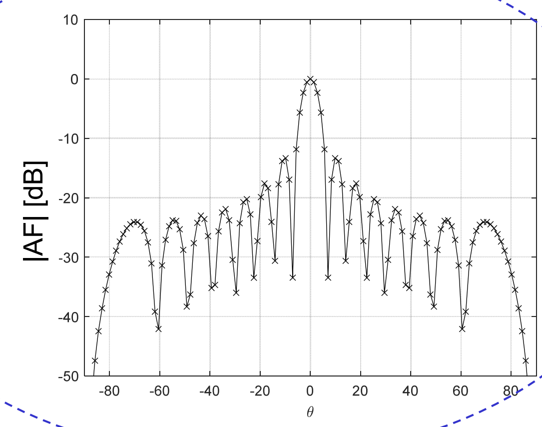
Uniform Linear Arrays (ULA)

Antenna spacing impact

d = λ/4



λ/2



Spatial sampling at

positions = *nd*

$[1 \quad e^{-j2\pi \cdot d \cdot sin\vartheta/\lambda} \quad ...]$

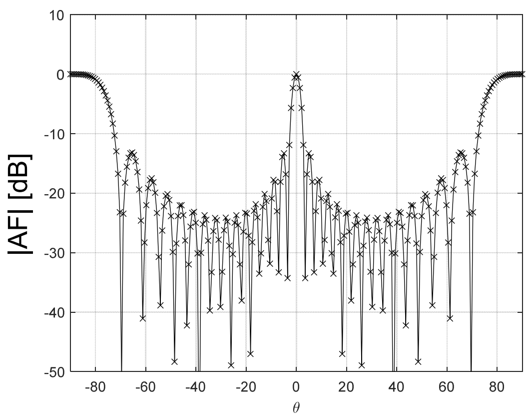
Spatial frequency

$sin\vartheta/\lambda$

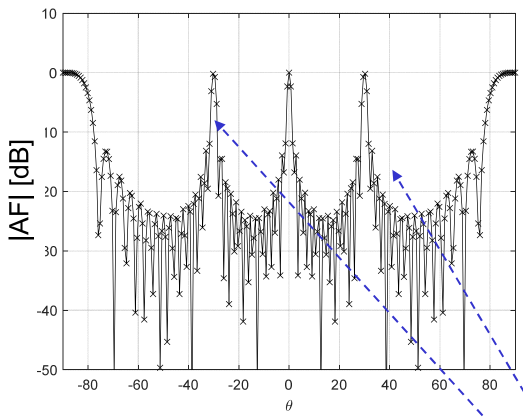
(-1/λ , +1/λ)

$1/d \geq 2/\lambda$

λ



2λ



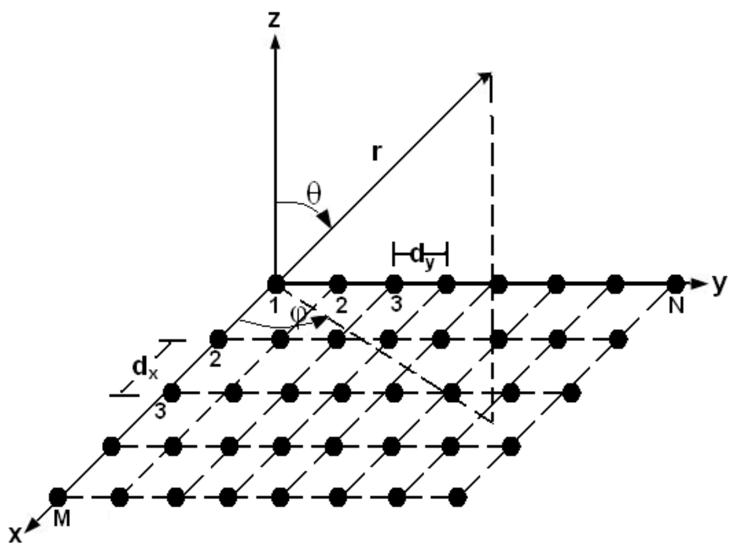
N = 16

Grating lobes (spatial ambiguities)

Uniform 2-D Arrays $N = N_H \times N_V$ (panels)

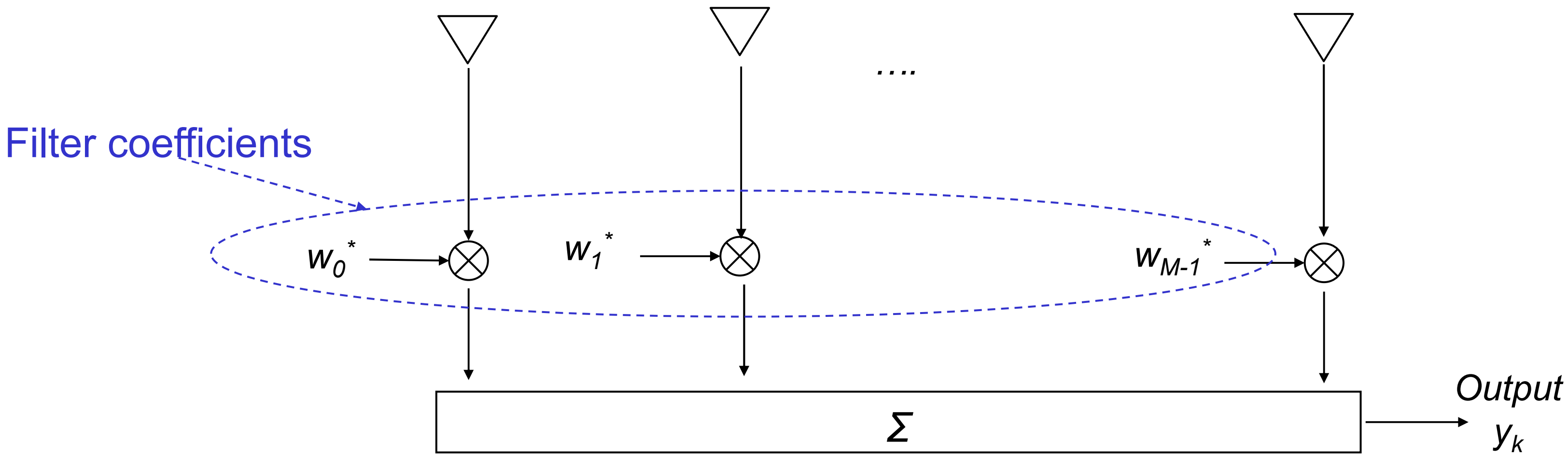
The AF is given by the product of the two linear AFs, corresponding to the **horizontal** and **vertical** directions.

Two AoAs identify the array response in the space.



Beam-forming

- Array operations: the array is a **spatial linear filter**



<u>**Narrow-band** beam-former structure</u>

$$y_n = \sum_k w_k^* \cdot u_k = \boldsymbol{w}^H \cdot \boldsymbol{u}_n$$

Beam-forming

- Array operations: the array is a **spatial linear filter**

Filter vector *N x 1*

$$y_n = \sum_k w_k^* \cdot u_k = \boldsymbol{w}^H \cdot \boldsymbol{u}_n$$

Output signal at time *n*

$$E[y_n y_n^*] = \boldsymbol{w}^H \cdot E[\,\boldsymbol{u}_n \boldsymbol{u}_n^H] \cdot \boldsymbol{w} = \boldsymbol{w}^H \cdot \boldsymbol{R}_{\boldsymbol{u}} \cdot \boldsymbol{w}$$

Power of the output signal

Autocorrelation matrix of the array signals

$$\boldsymbol{R}_{\boldsymbol{u}} = \sum_{i=1}^{M} \sigma_{S,i}^2 \boldsymbol{s}_i \boldsymbol{s}_i^H + \sigma_n^2 \boldsymbol{I}_N = \boldsymbol{S}\boldsymbol{U}\boldsymbol{S}^H + \sigma_n^2 \boldsymbol{I}_N$$

M directional sources with their steering vectors

Steering vector matrix

Autocorrelation matrix of the sources

Beam-forming

- Conventional beamforming
- Null-steering beamforming
- Optimum beam-forming
- Optimum beam-forming with a reference signal (MMSE)

Conventional beamforming

The **phases** <u>are selected to steer the array</u> in a particular direction *(θ_o, φ_o)*

With denoting the steering vector in a direction, the array weights are given by

$$\boldsymbol{w} = \frac{1}{N} \cdot \boldsymbol{s}_o$$

In presence of a single source with amplitude *A* from the direction (θ_o, φ_o), we have

$$y_n = \boldsymbol{w}^H \cdot A\boldsymbol{s}_o = A$$

In an environment consisting of only uncorrelated noise and no directional interferences, this beam former provides the maximum SNR. For uncorrelated noise, the output noise power is given by

$$P_n = \boldsymbol{w}^H \cdot \boldsymbol{R}_n \cdot \boldsymbol{w} = \frac{\sigma_n^2}{N}$$

Array SNR gain (w.r.t. single element)

Null-steering beamforming

A null-steering beam former is used to cancel *K* plane waves arriving from known directions.

The weight vector is the solution of the following problem:

$$\begin{cases} \boldsymbol{w}^H \cdot \boldsymbol{s}_o = 1 \\ \boldsymbol{w}^H \cdot \boldsymbol{s}_i = 0 \qquad\qquad i = 1, \dots, K \end{cases}$$

$$\boldsymbol{w}^H \cdot \boldsymbol{S} = [1 \quad 0 \quad \dots \quad 0] = \boldsymbol{g}_1^T$$

S is generally not square.

$$\boldsymbol{S} = [\boldsymbol{s}_o \quad \boldsymbol{s}_1 \quad \dots \quad \boldsymbol{s}_K]$$

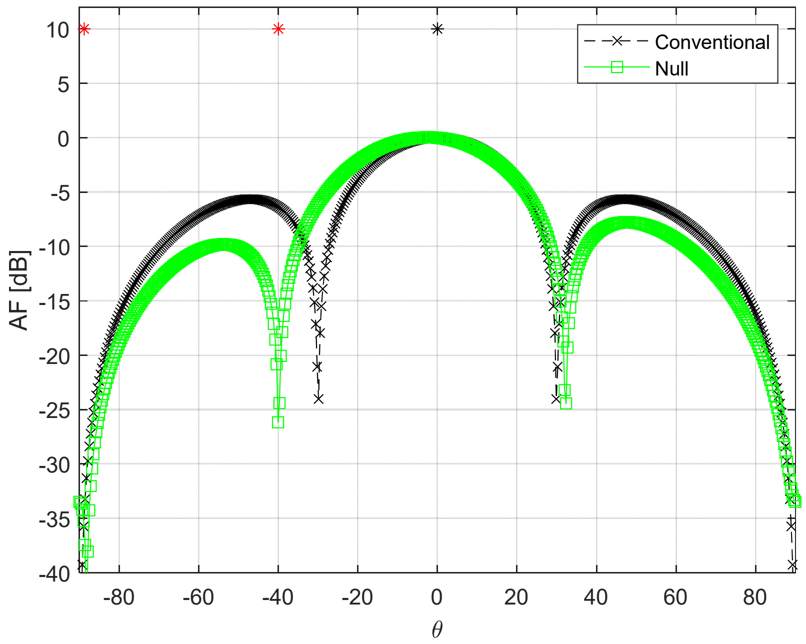
$$\boxed{\mathbf{w}^{\boldsymbol{H}} = \boldsymbol{g}_1^T \cdot \boldsymbol{S}^H \cdot (\boldsymbol{S}\boldsymbol{S}^H)^{-1}}$$

with *K* ≤ *N* - 2

Beam-forming

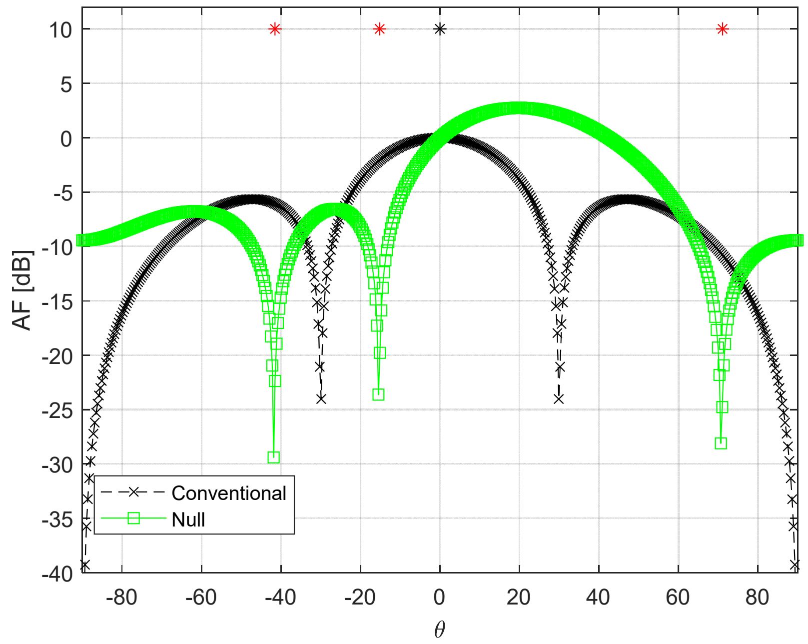
Null-steering beamforming

4 antennas



2 interferers

SINR = 15.6 dB

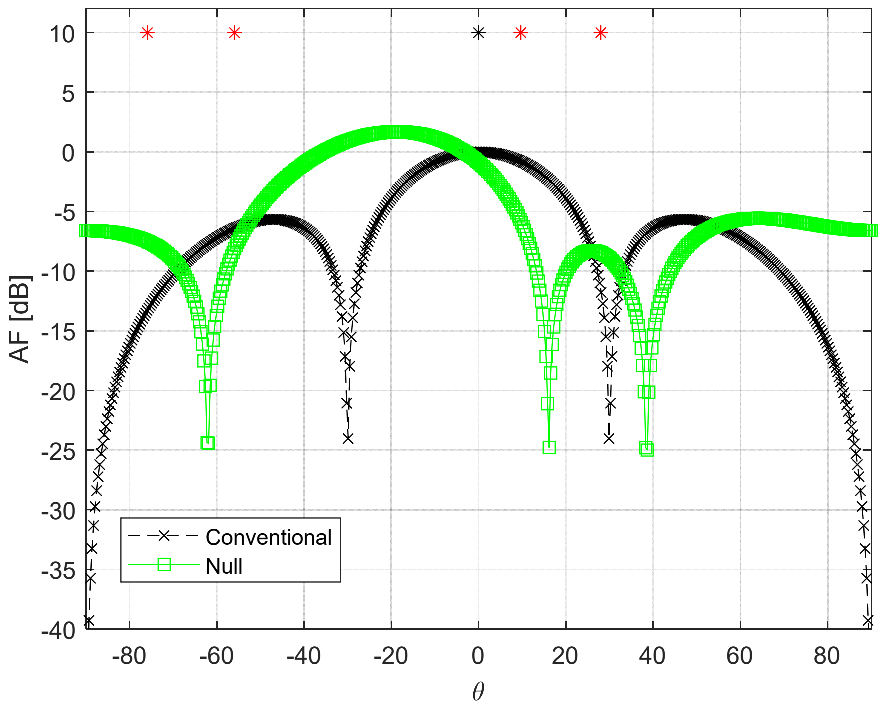


3 interferers

SINR = 14.0 dB

Null-steering beamforming

4 antennas



SINR = 5 dB

4 interferers

Optimum beam-forming

Minimize the interference-plus-noise power at the beamformer output.

The problem is expressed by minimizing the output power keeping a unit response from the signal direction:

$$\text{Minimize} \quad \boldsymbol{w}^H \cdot \boldsymbol{R}_{\boldsymbol{i}+\boldsymbol{n}} \cdot \boldsymbol{w}$$

$$\text{s.t.} \quad \boldsymbol{w}^H \cdot \boldsymbol{s}_o = 1$$

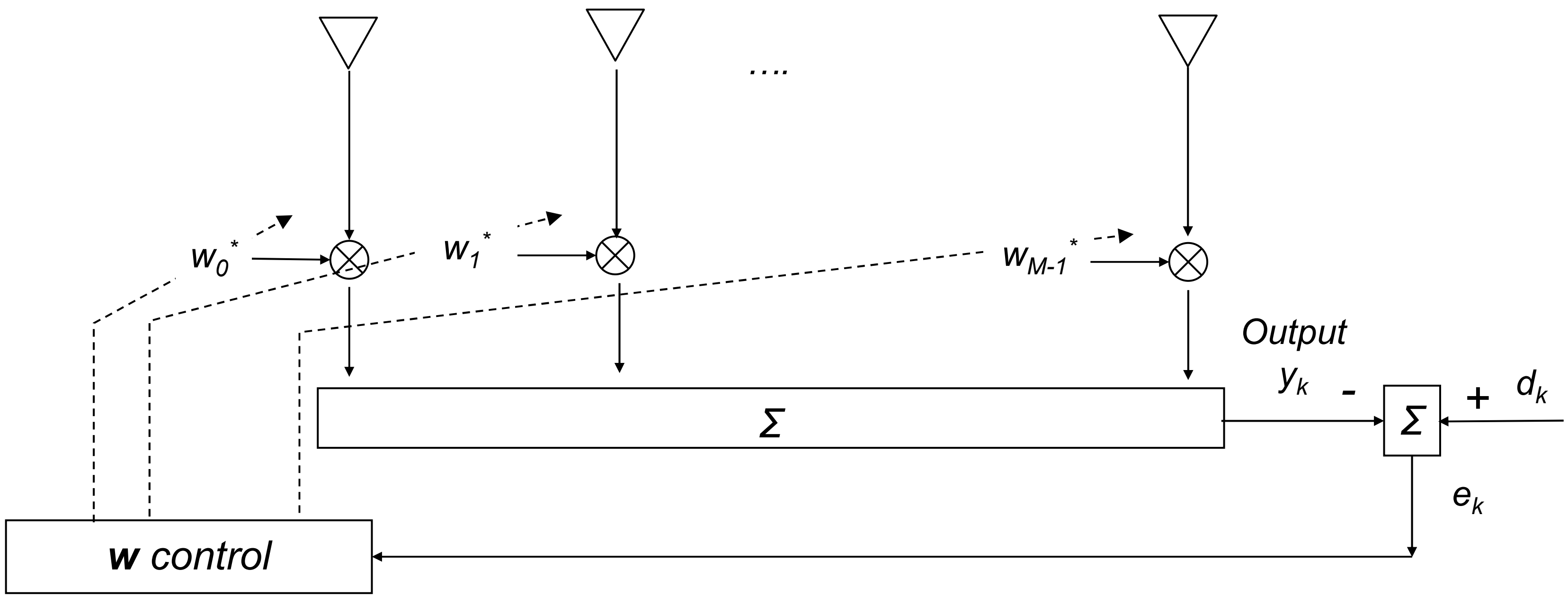
The solution is

$$\boxed{\boldsymbol{w}_{\boldsymbol{o}} = \frac{\boldsymbol{R}_{\boldsymbol{i}+\boldsymbol{n}}^{-1} \cdot \boldsymbol{s}_o}{\boldsymbol{s}_{\boldsymbol{o}}^H \cdot \boldsymbol{R}_{\boldsymbol{i}+\boldsymbol{n}}^{-1} \cdot \boldsymbol{s}_o}}$$

Minimizing the total output noise while keeping the output signal constant is the same as maximizing the output SINR.

This is also called **minimum-variance distortionless response** (MVDR) beamformer.

Optimum beam-forming with a reference signal



Narrow-band beam-former structure with a reference signal

Optimum beam-forming with a reference signal

Weights are adjusted such that the **MSE** between the array output and the reference signal is minimized.

$$\text{MSE} = E[dd^*] + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\,\boldsymbol{w}^H\boldsymbol{p} = \sigma_d^2 + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\,\boldsymbol{w}^H\boldsymbol{p}$$

where we have introduced the cross-correlation between the input and the reference signal

$$\boldsymbol{p} = E[\boldsymbol{u} \cdot d^*]$$

The MSE surface is a quadratic function of and is minimized by setting its gradient with respect to the weights equal to zero.

The **equation** for the optimal weight vector is

$$\boxed{\boldsymbol{w}_{opt} = \boldsymbol{R_u^{-1}} \cdot \boldsymbol{p}}$$

Optimum beam-forming with a reference signal

The **MMSE** is given by

$$MMSE = \sigma_d^2 - \sigma_{\hat{d}}^2 = \sigma_d^2 - \boldsymbol{w}_o^H \cdot \boldsymbol{R}_u \cdot \boldsymbol{w}_o = \sigma_d^2 - \boldsymbol{w}_o^H \cdot \boldsymbol{p} = \sigma_d^2 - \boldsymbol{p}^H \cdot \boldsymbol{R}_{\boldsymbol{u}}^{-\mathbf{1}} \cdot \boldsymbol{p}$$

For a perfect LoS plane wave, the optimal filter and the MVDR are equivalent.

- In digital mobile communications, a synchronization signal may be used for initial weight estimation, followed by the use of the detected signal as a reference signal.

Adaptive beam-forming

Narrowband

Wideband

Beamforming

Switched

Adaptive

Temporal reference

MMSE (training sequence)
LMS
….

Spatial reference (AoA)

Null steering
Conventional
MVDR
….

Blind

Switched BF

Finite number of fixed beams with high sensitivity in particular directions.

The beams are selected according to predetermined fixed beam patterns.

A simple approach …

The SNR is measured for each beam and the maximum is determined.

The beam associated with the largest SNR is selected.

The gradient descent algorithm

The algorithm updates the weights at each iteration by estimating the **gradient** of the **MSE** and moving them in the **negative direction** of the gradient.

A real-time algorithm for determining the weight vector

step size

$$\boxed{\boldsymbol{w}_{n+1} = \boldsymbol{w}_n - \frac{1}{2}\boldsymbol{\mu} \cdot \widehat{\nabla}_{\boldsymbol{w}} \boldsymbol{MSE}(\boldsymbol{w})|_{\boldsymbol{w}=\boldsymbol{w}_n}}$$

n -> time

$$\mathrm{MSE} = E[d_n d_n^*] + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \boldsymbol{2}\ \boldsymbol{w}^H \boldsymbol{p} = \sigma_d^2 + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \boldsymbol{2}\ \boldsymbol{w}^H \boldsymbol{p}$$

$$\nabla_{\boldsymbol{w}} \boldsymbol{MSE}(\boldsymbol{w}) = 2\boldsymbol{R_u} \cdot \boldsymbol{w} - \boldsymbol{2}\ \boldsymbol{p}$$

$$\downarrow$$

$$\widehat{\nabla}_{\boldsymbol{w}} \boldsymbol{MSE}(\boldsymbol{w}) = 2\boldsymbol{u}_n \cdot \boldsymbol{u}_n^H \cdot \boldsymbol{w}_n - 2\boldsymbol{u}_n \cdot d_n^* = 2\boldsymbol{u}_n \cdot e_n^*$$

The LMS algorithm

$$\boxed{\boldsymbol{w}_{\boldsymbol{n}+\mathbf{1}} = \boldsymbol{w}_{\boldsymbol{n}} - \frac{1}{2}\boldsymbol{\mu} \cdot 2\boldsymbol{u}_n \cdot e_n^* = \boldsymbol{w}_{\boldsymbol{n}} - \boldsymbol{\mu} \cdot \boldsymbol{u}_n \cdot e_n^*}$$

$$0 < \mu < \frac{2}{\lambda_{max}}$$

As the sum of all eigenvalues of $\boldsymbol{R_u}$ equals its trace, one may select the gradient step size

$$\mu = \frac{1}{Tr(\boldsymbol{R_u})}$$

Also considering that each diagonal element of $\boldsymbol{R_u}$ is equal to the average power measured on the corresponding element of the array.

The excess MSE

The excess MSE w.r.t. optimal MMSE is caused by the use of the noisy estimate of the gradient.

Increasing the step size, we increase the misadjustment noise.

On the other hand, an increase of the step size makes the convergence faster.

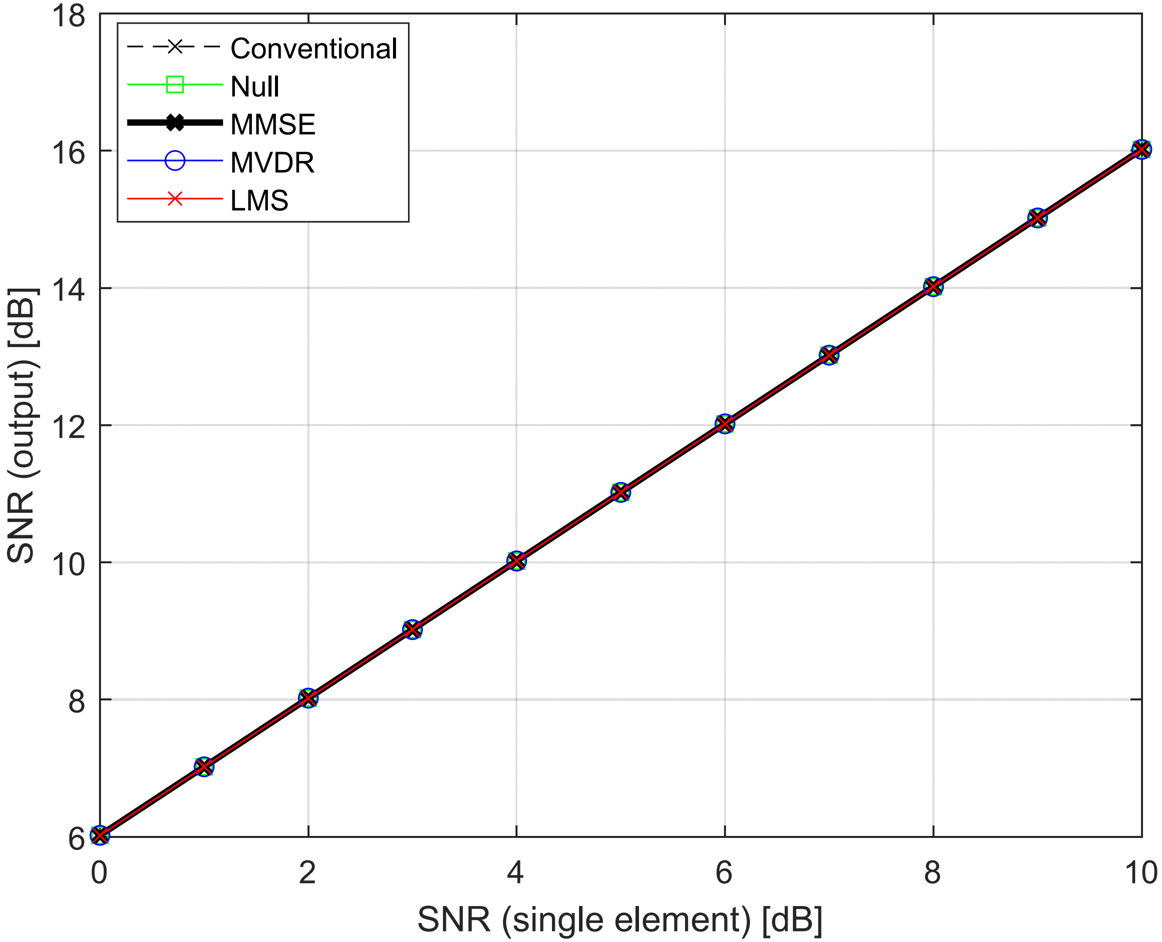
Many schemes have variable step size to overcome the problem.

Adaptive beam-forming

- Example: no interferers

N = *8* and *d* = *λ/2*

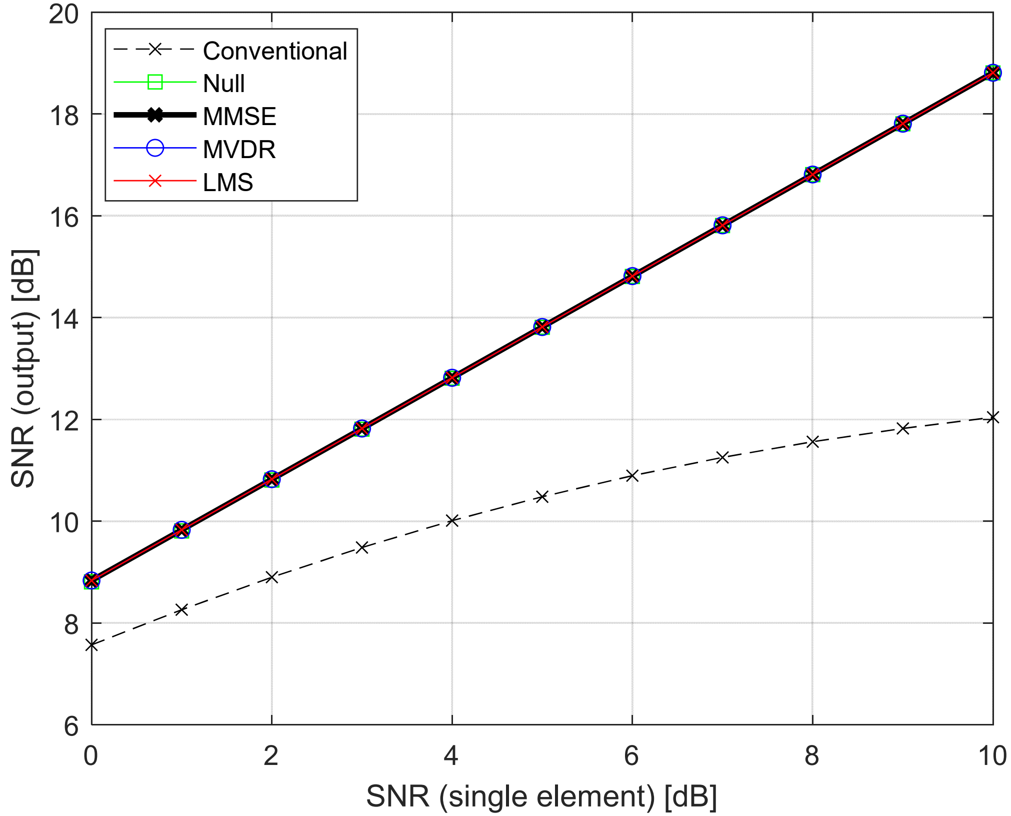
10*log10(N) ≈ 9

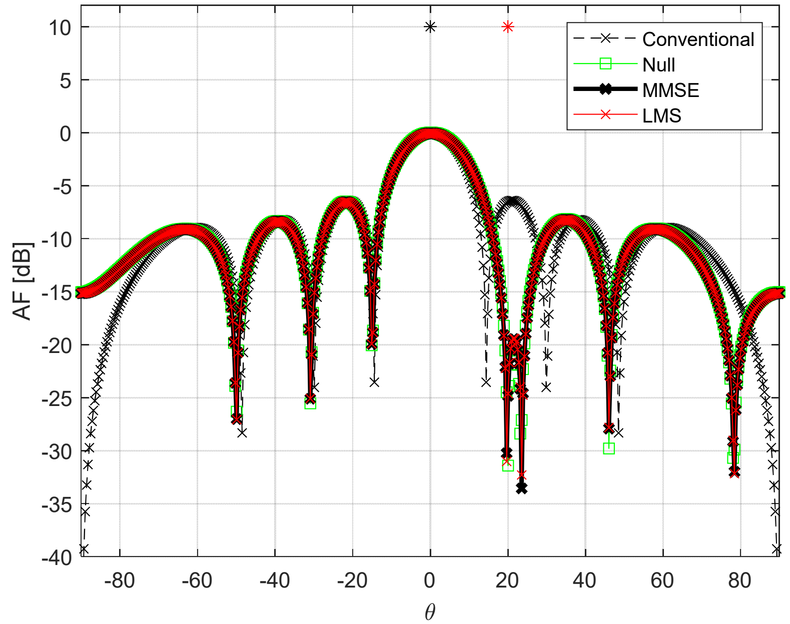


- Example: 1 interferer

N = 8 and *d* = *λ*/2

Angular separation = 20˚



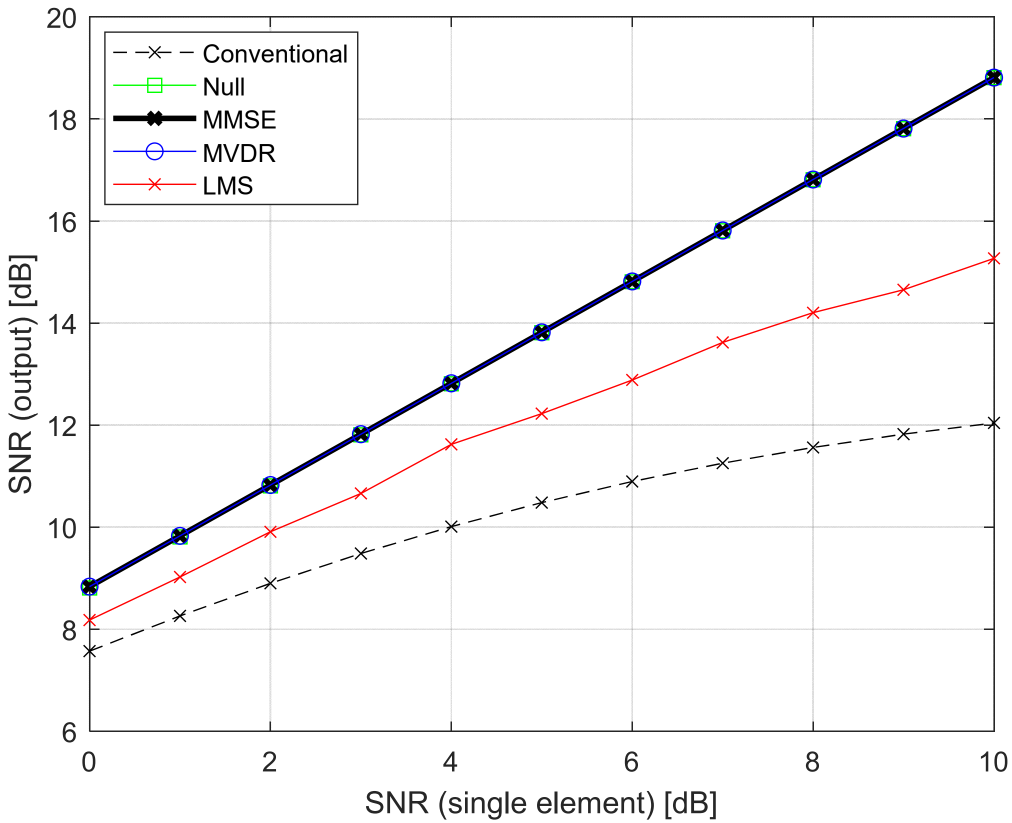


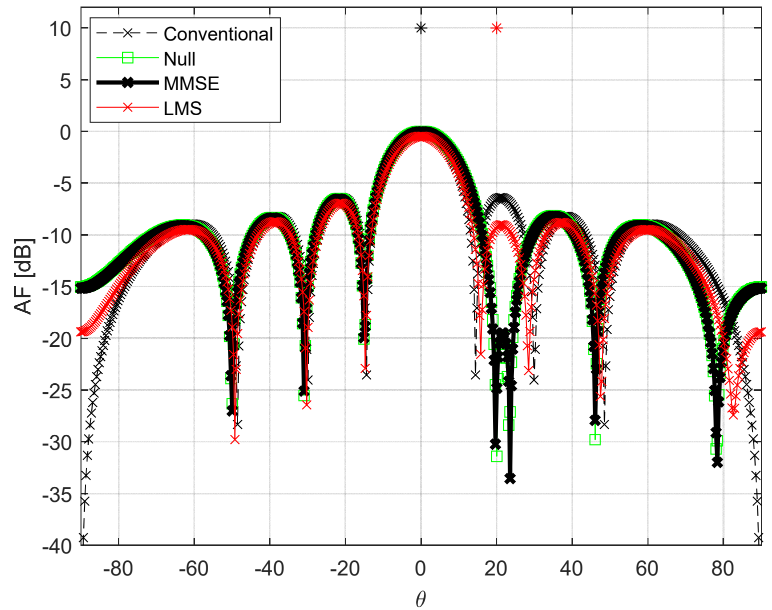
LMS: «good» initial state …

- Example: 1 interferer

N = 8 and *d* = $\lambda/2$

Angular separation = 20˚



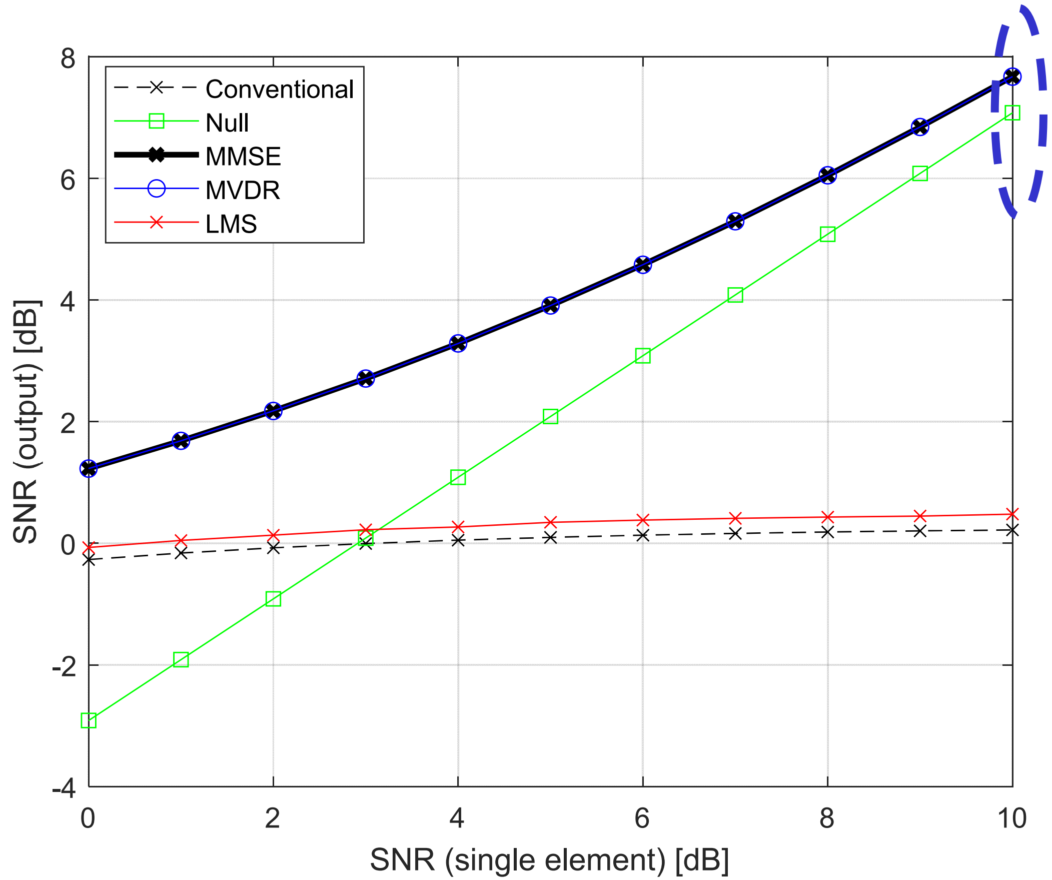


LMS: initial state based on conventional BF

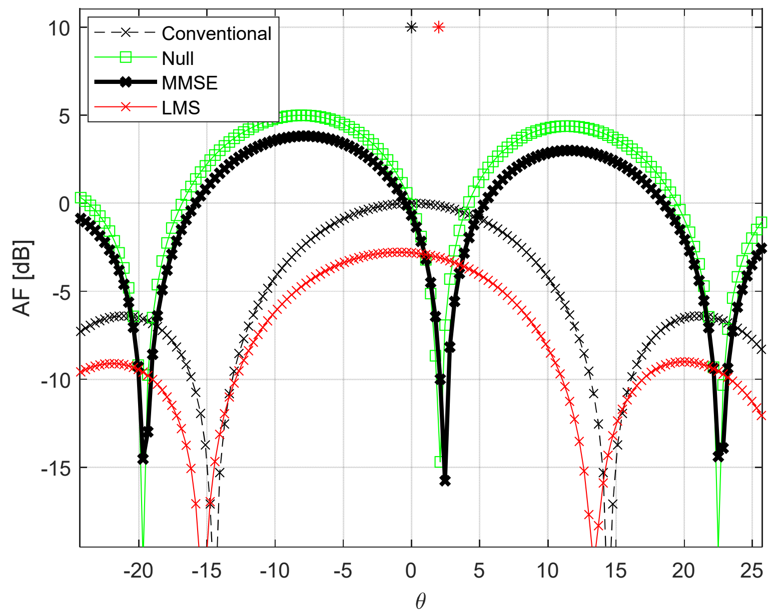
- Example: 1 interferer

N = *8* and *d* = *λ/2*

Angular separation = 2°



SNR = 10 dB



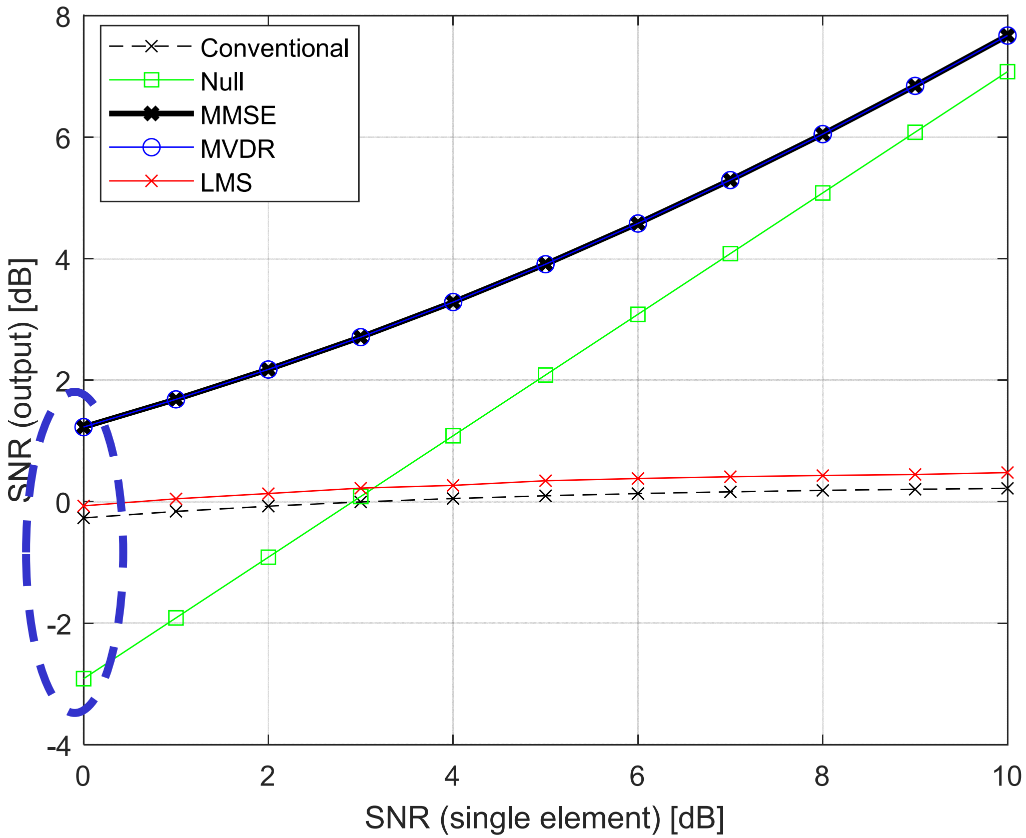
LMS: initial state based on conventional BF

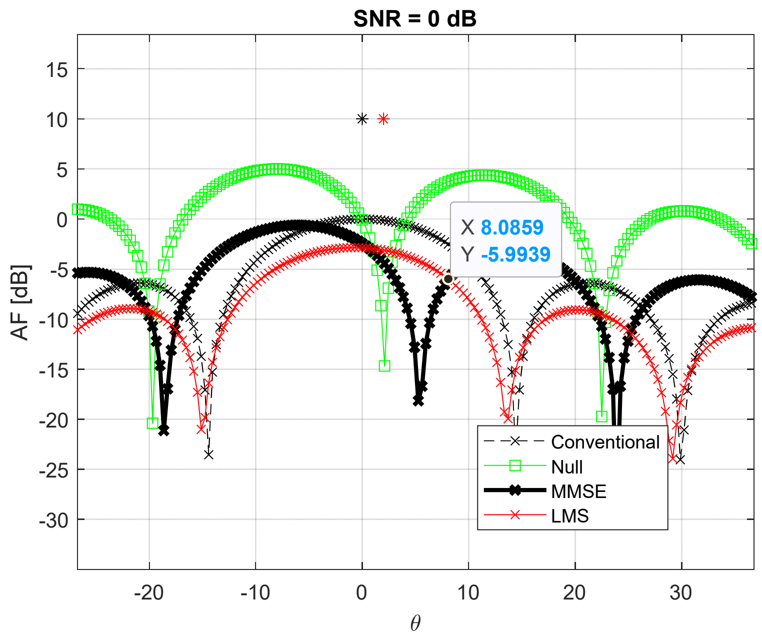
Adaptive beam-forming

- Example: 1 interferer

N = 8 and *d* = λ/2

Angular separation = 2°

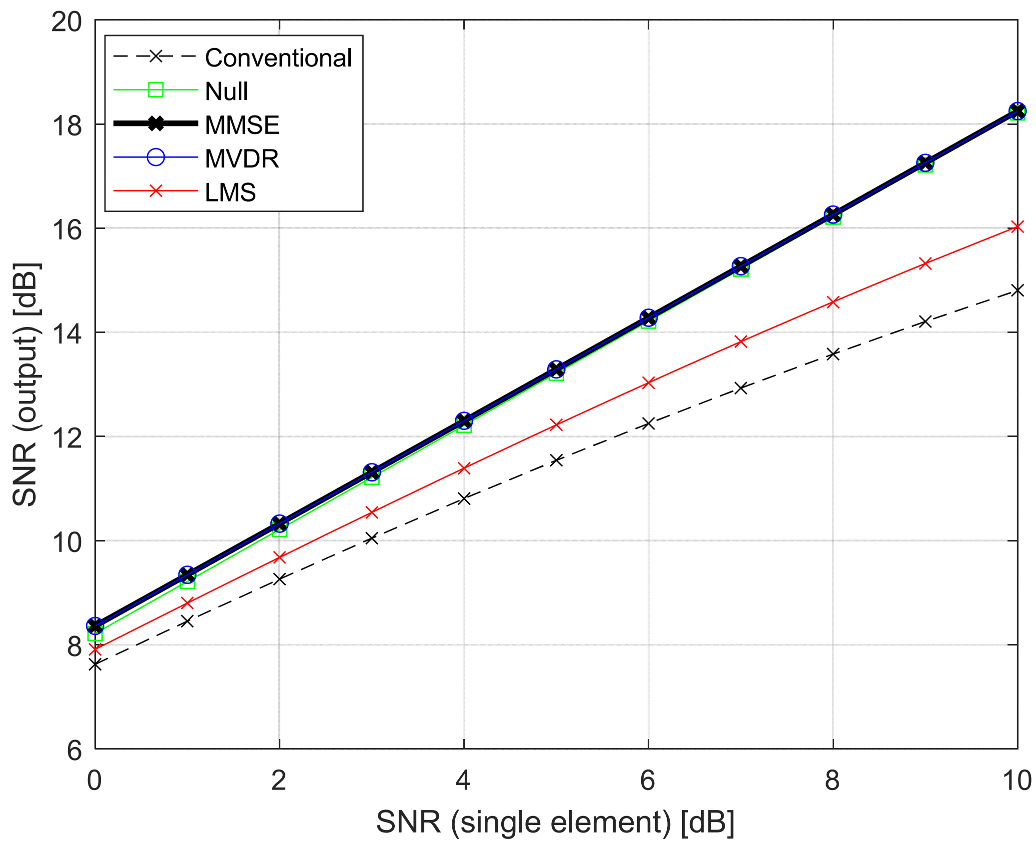




LMS: initial state based on conventional BF

- Example: 2 interferers (random positions)

N = 8 and *d* = $\lambda/2$

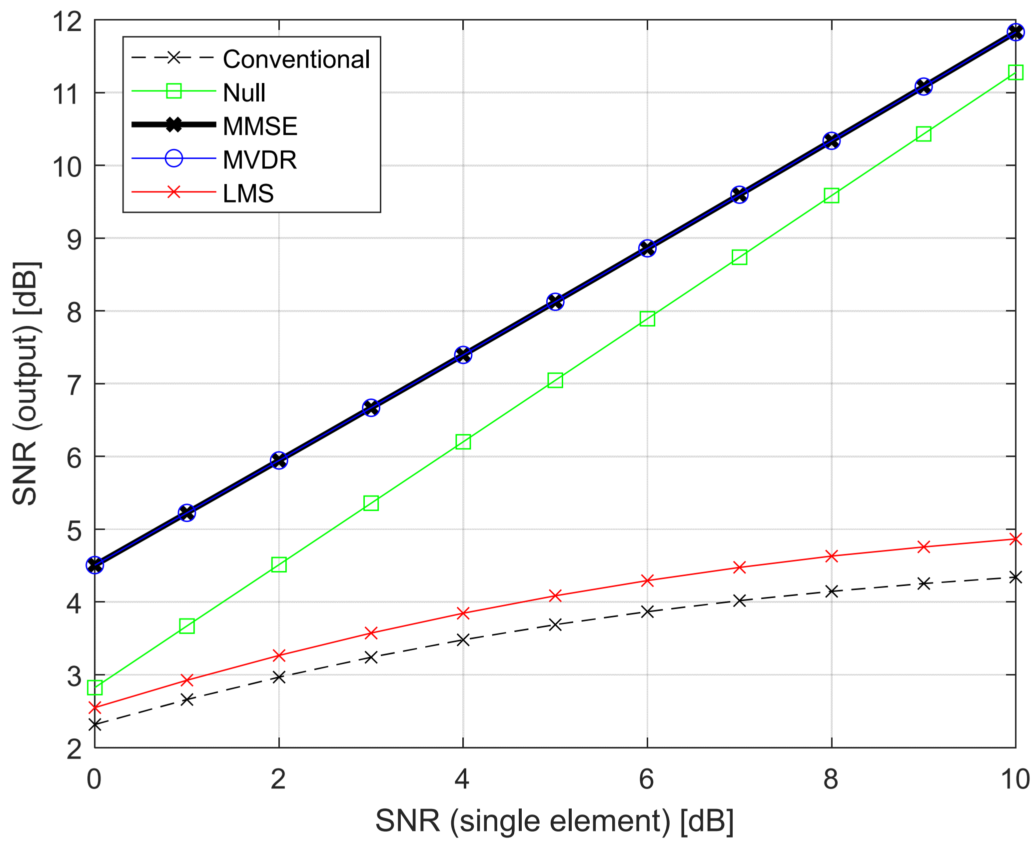


LMS: initial state based on conventional BF

Adaptive beam-forming

- Example: 12 interferers (random positions)

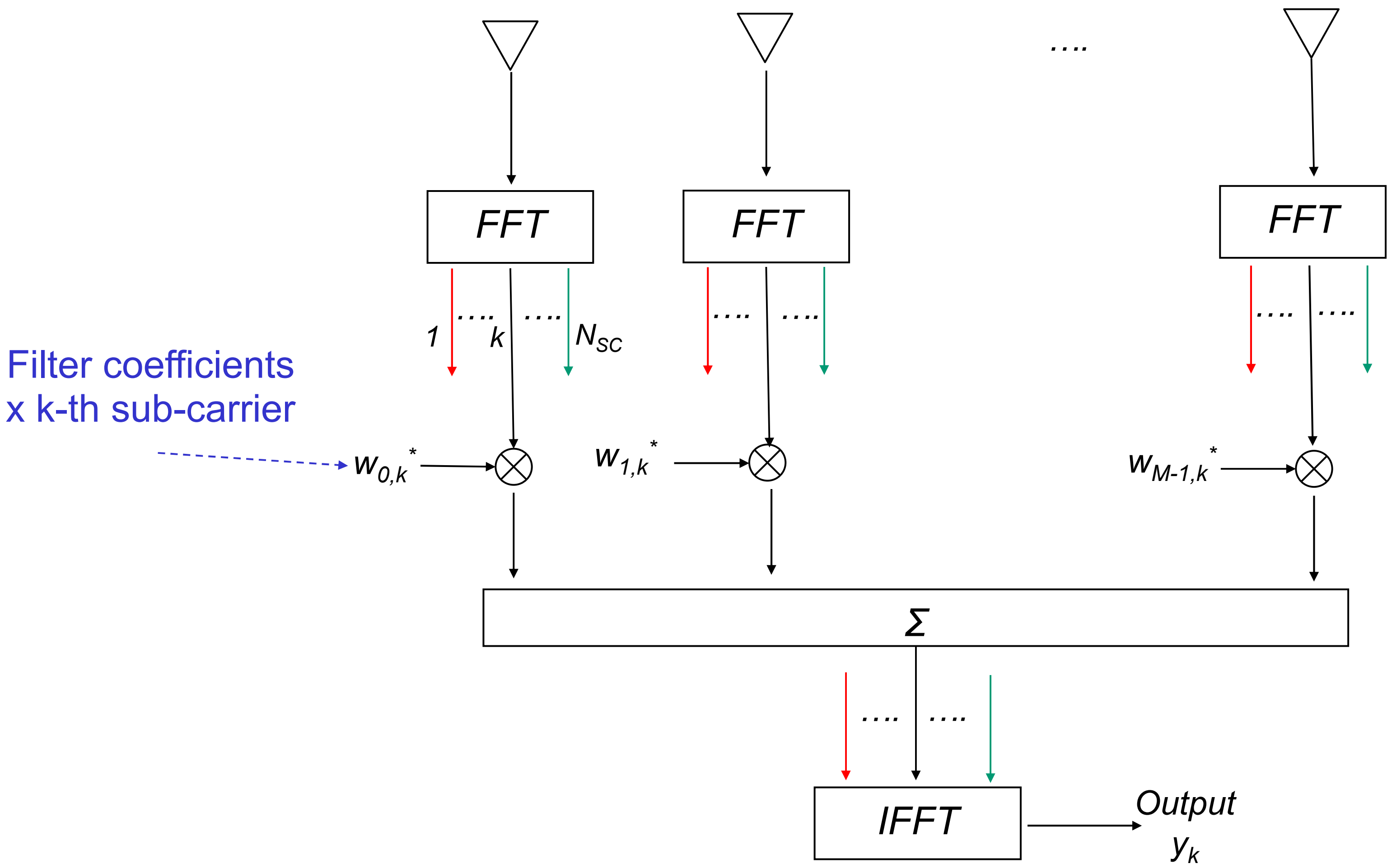
N = 8 and *d* = λ/2



LMS: initial state based on conventional BF

Adaptive beam-forming

- **Wideband signals** → *Frequency-Domain Beam Forming*



Adaptive beam-forming

Frequency-Domain Beam Forming

The weights required for each frequency bin are selected independently, and this selection may be performed in parallel, leading to a faster weight update.

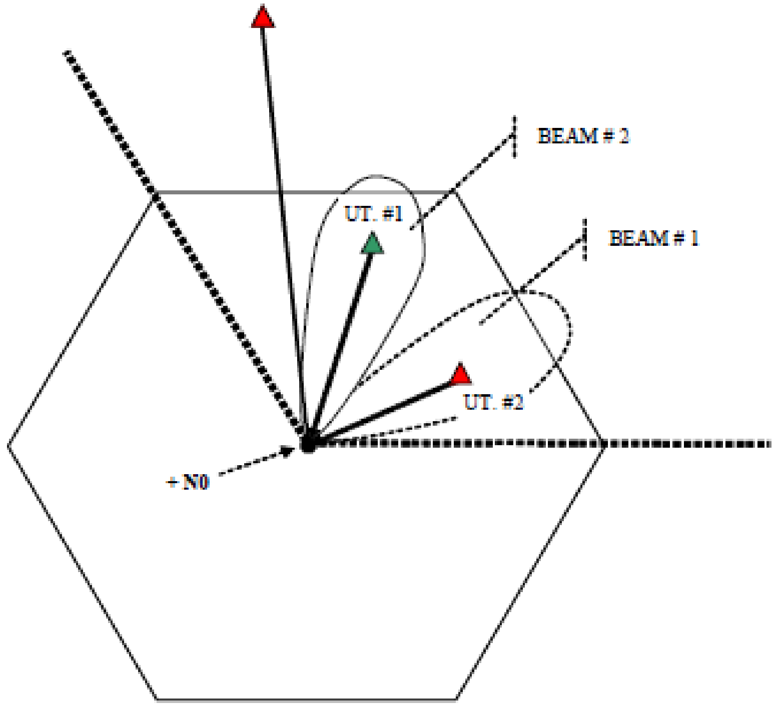
When adaptive algorithms such as the LMS algorithm is used for weight update, a different step size may be used for each bin, leading to faster convergence.

Beam-Forming in LTE-A

Beamforming

1 or 2 layers x SU-MIMO or MU-MIMO

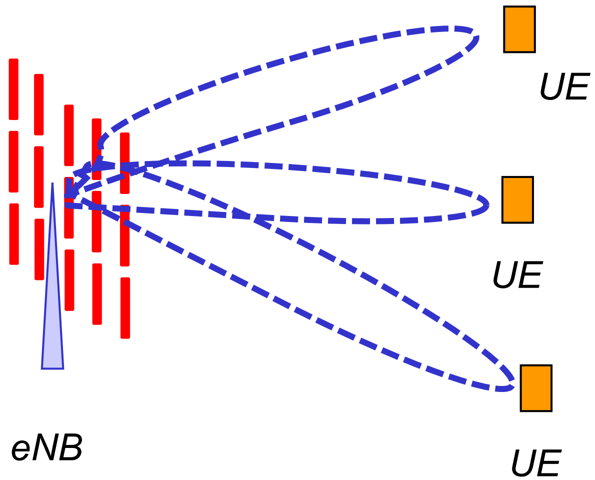
Possibility of adaptive BF



LTE does not specify BF methods.

Also the number of antennas and the antenna architecture are left to implementation.

- **Massive MIMO – Beamforming with high spatial resolution**



Especially at mm waves (e.g. in LoS conditions), really compact antenna arrays (at least 8 x 8) will be able to steer and track simultaneous multiple beams towards the devices to be served with benefits on spectral and energy efficiency.

- New Radio (NR - 5G) bandwidth

Frequency range designation	Corresponding frequency range
FR1	450 MHz – 6000 MHz
FR2	24250 MHz – 52600 MHz

Δf [kHz]	5 MHz	10 MHz	15 MHz	20 MHz	25 MHz	30 MHz	40 MHz	50 MHz	60 MHz	70 MHz	80 MHz	90 MHz	100 MHz
	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}	N_{RB}
15	25	52	79	106	133	160	216	270	N.A	N.A	N.A	N.A	N.A
30	11	24	38	51	65	78	106	133	162	189	217	245	273
60	N.A	11	18	24	31	38	51	65	79	93	107	121	135

Δf [kHz]	50 MHz	100 MHz	200 MHz	400 MHz
	N_{RB}	N_{RB}	N_{RB}	N_{RB}
60	66	132	264	N.A
120	32	66	132	264

[3GPP 38.104, BS radio transmission and reception (Release 15)]

- **mmWave mobile communications is possible**

[T. Rappport et al., "*Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!*", IEEE Access]

- **What do we expect in terms of coverage ?**

Cell radius decrease from typical 2 - 4 km between 1.8 and 3 GHz till to 2 - 300 m at 26 GHz.

The coverage at mmWaves can be between 40 % and 60 % of the cell.

[5G PPP Architecture Working Group, "View on 5G Architecture ", V3.0, 2019]

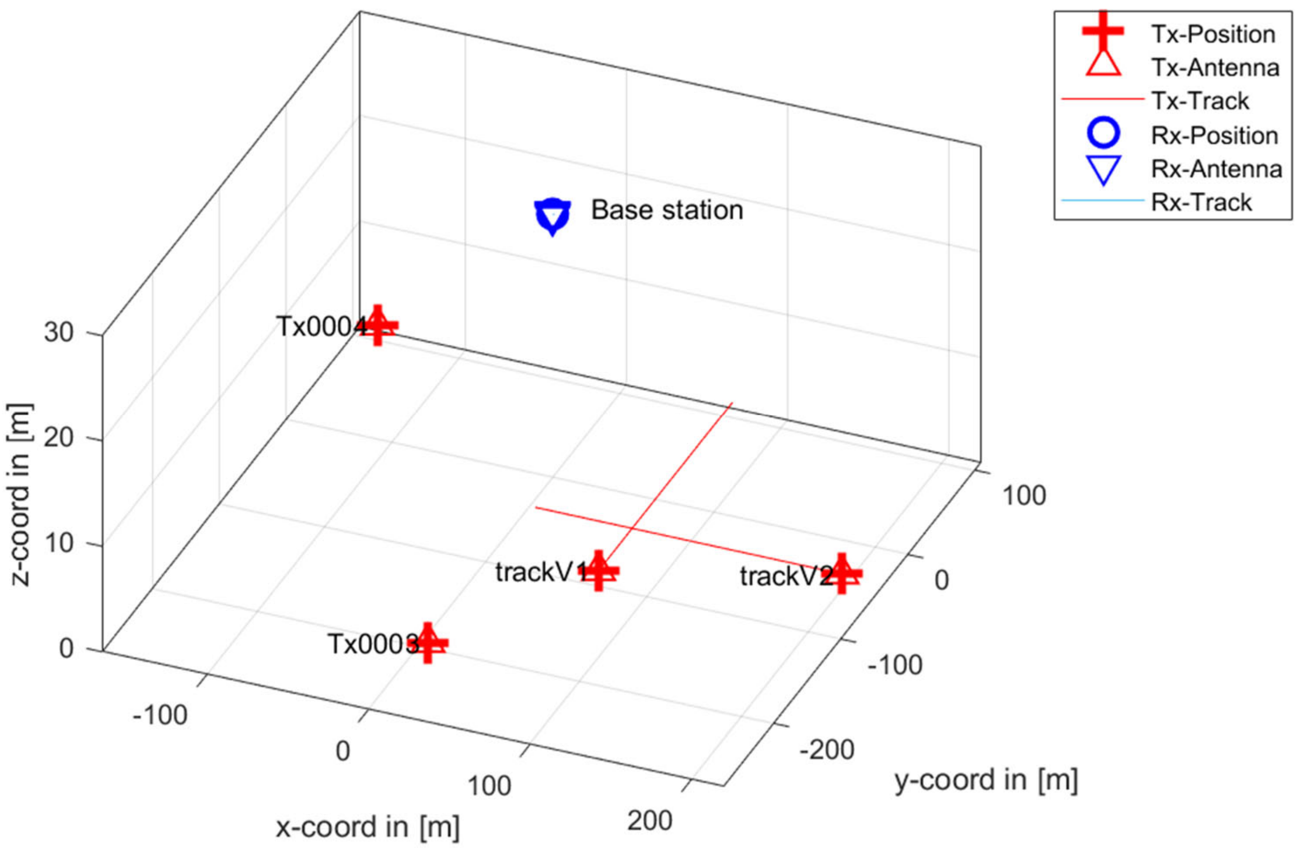
References

[1] S. Haykin, ”Adaptive Filter Theory”, Prentice Hall.

[2] D.G. Manolakis, “Statistical and Adaptive Signal Processing: Spectral Estimation, Signal Modeling, Adaptive Filtering and Array Processing”, Artech House.

[3] L. C. Godara, «Application of Antenna Arrays to Mobile Communications, Part II: Beam-Forming and Direction-of-Arrival Considerations», Proc. of the IEEE, Vol. 85, No. 8, Aug. 1997.

An environment with a BS equipped with an antenna array (rectangular)



The idea is to track, with adaptive BF, two vehicles in presence of one or more fixed interferers.

The project scenario

Project blocks		Main options
System model	Geometric scenario and parameters	2D / 3D
	Channel model	LoS / 3GPP / …
	Signals	NB / OFDM / 3GPP
	Adaptive BF	DoA based / LMS / …
		Use of a preamble / training sequence / OFDM pilots

Project results and levels (D is the minimum level to be achieved)

Levels	Geometry	Channel	Signal	BF
D (2 pnt)	2D	LoS, free space	Narrow-band	DoA based
C (3 pnt)	3D	LoS, free space	OFDM	DoA based
B (5/6 pnt)	3D	LoS + simplified multipath	OFDM	LMS (time or frequency domain)
A (6 pnt)	3D	3GPP	OFDM / 3GPP / NR (> 6 GHz)	LMS (time and frequency domain)

- Any feature in the code that is parameterized (e.g. the code works for any number of interferers, terminals, etc …) constitutes an added value for the project evaluation.
- The project can be done in a team (till to 3 persons) but the exam discussion of the project is always personal.
- We expect a final brief report or presentation + commented (Matlab or Simulink) code.
- The type of numerical results in the report and the related comments are important for the project evaluation.
- If a team divides the tasks, please specify how you have organized and divided the activity in the final report.
- This table is just a guideline, please do not take it as a rigid homework. We really appreciate your own ideas !