Outline

1. Antenna Arrays
2. Beam-forming
3. Adaptive beam-forming
4. Beam-forming in LTE / NR
5. Project scenario

The pattern multiplication principle

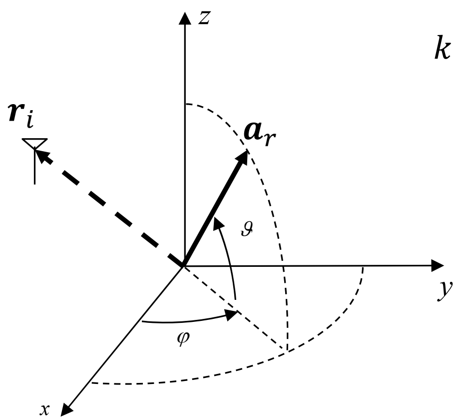
For a general array of antennas, the radiation pattern is the product between the element pattern function and the **array pattern function**.

$$|f_{EL}(\theta,\varphi)|^2 \cdot |AF(\theta,\varphi)|^2$$

$$= |f_{EL}(\theta,\varphi)|^2 \cdot \left| \sum_{i=1}^{N} C_i \cdot e^{j(\alpha_i + k\boldsymbol{a}_r \cdot \boldsymbol{r}_i)} \right|^2$$

weights

$$AF(\theta,\varphi) = \boldsymbol{w}^H \boldsymbol{s}$$



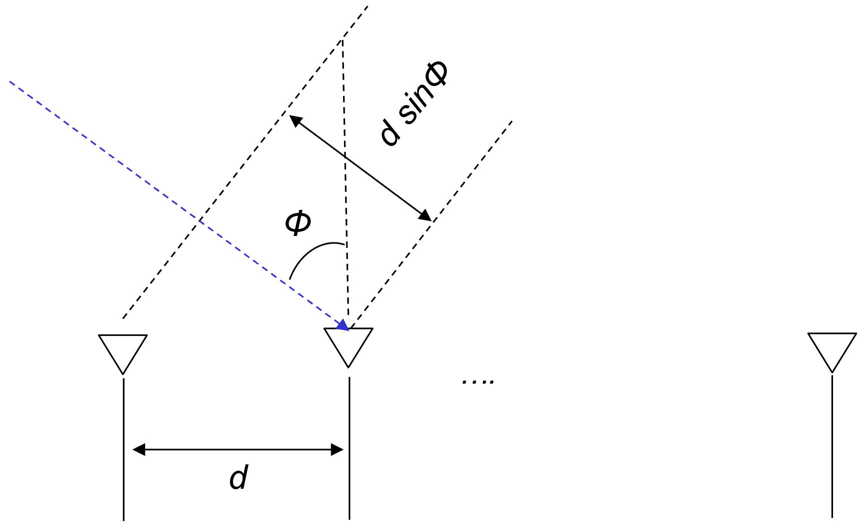
$$k = \frac{2\pi}{\lambda}$$

The transmit radiation pattern is equal to the receive on, due to reciprocity.

Antenna arrays

Uniform Linear Arrays (ULA)

A plane wave cumulates at the array elements, a phase contribution that is function of the angle of arrival (AoA) and the antenna spacing.



Antenna arrays

Uniform Linear Arrays (ULA)

The directivity is proportional to *N*, related to the coherent combination of all the incoming signals.

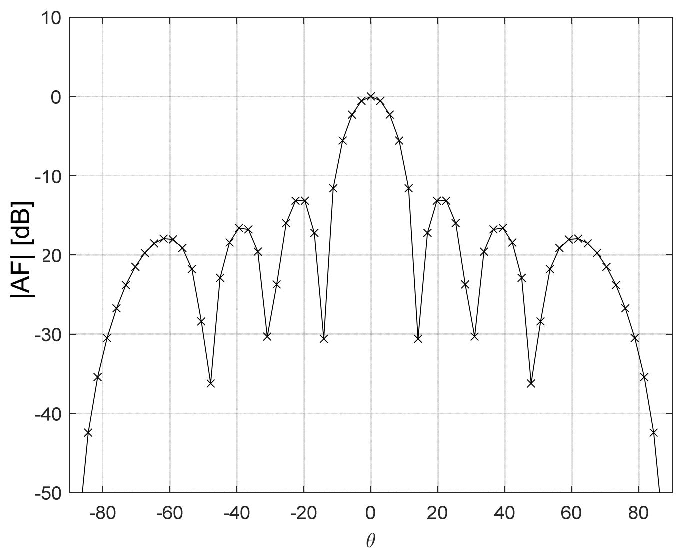
The linear array response depends on a single Angle of Arrival (AoA) and it is derived using the steering vector (e.g. w.r.t. all the gains equal to *1*):

steered direction

$$s_n = e^{-j(kn \cdot d\ sin\Phi)}$$

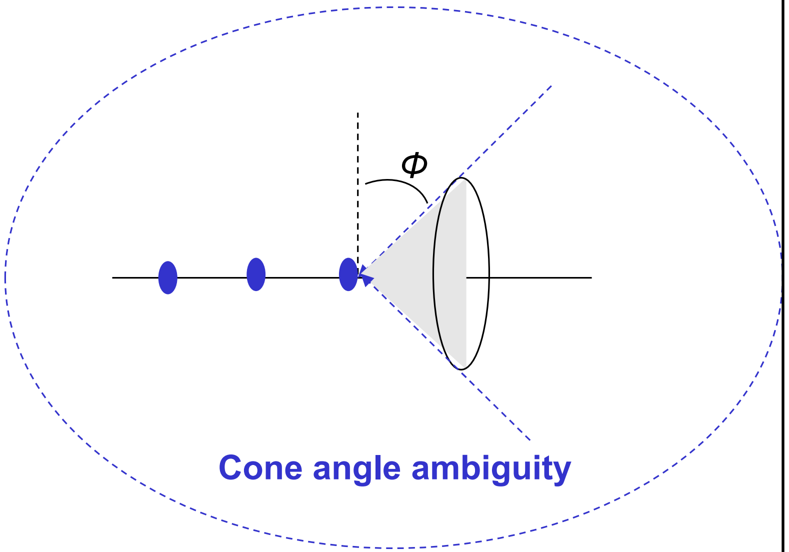
Antenna arrays

Uniform Linear Arrays (ULA)



Array Factor magnitude

Here N = 8, d = $\lambda/2$ and the AoA θ ….



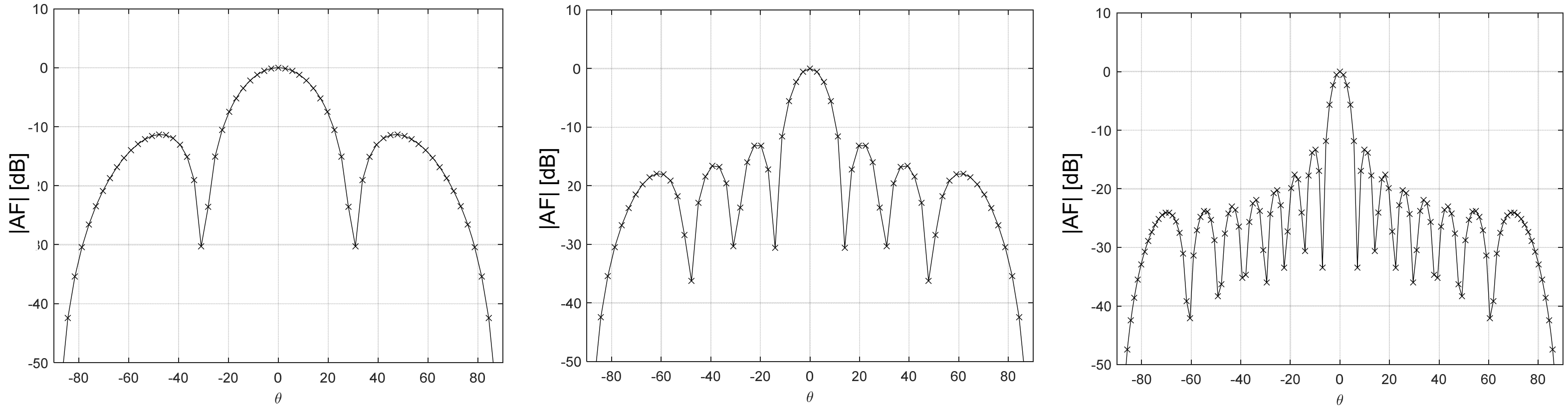
Cone angle ambiguity

Antenna arrays

Uniform Linear Arrays (ULA)

Beamforming resolution

We can observe that the beam-width is inversely proportional to *N*.



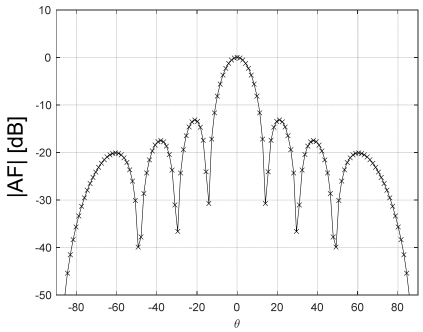
Array Factor magnitude $N = 4,8,16$ and $d = \lambda/2$

Antenna arrays

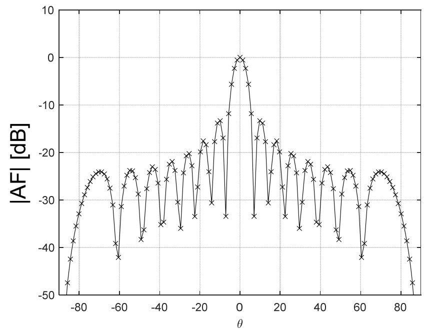
Uniform Linear Arrays (ULA)

Antenna spacing impact

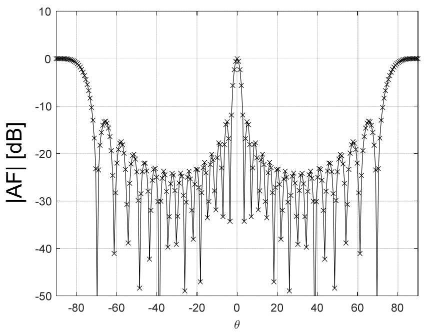
d = λ/4



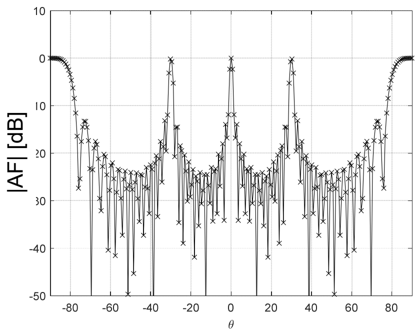
λ/2



λ



2λ



N = 16

Antenna arrays

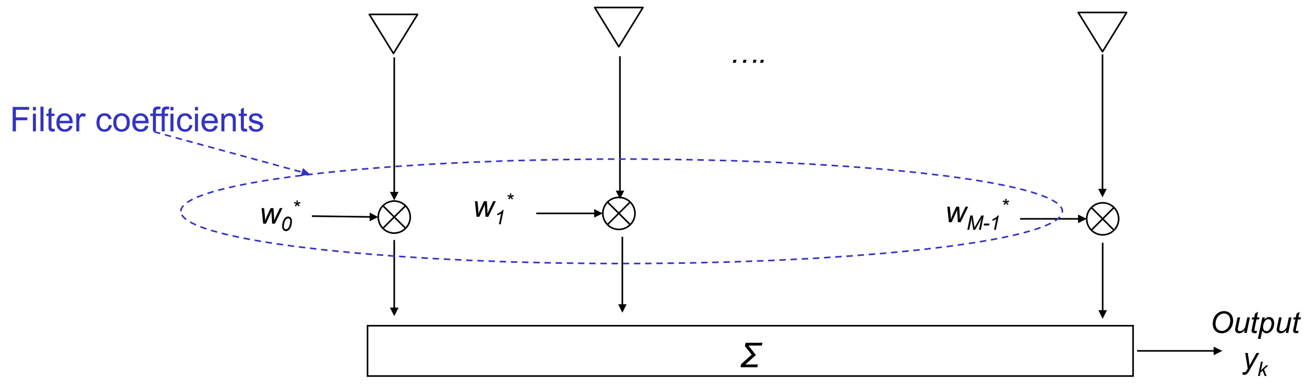
Uniform 2-D Arrays $N = N_H \times N_V$ (panels)

The AF is given by the product of the two linear AFs, corresponding to the horizontal and vertical directions.

Two AoAs identify the array response in the space.

Beam-forming

- Array operations: the array is a **spatial linear filter**



Narrow-band beam-former structure

$$y_n = \sum_k w_k^* \cdot u_k = \boldsymbol{w}^H \cdot \boldsymbol{u}_n$$

Beam-forming

- Array operations: the array is a **spatial linear filter**

Filter vector *N x 1*

$$y_n = \sum_k w_k^* \cdot u_k = \boldsymbol{w}^H \cdot \boldsymbol{u}_n$$

Output signal at time *n*

$$E[y_n y_n^*] = \boldsymbol{w}^H \cdot E[\,\boldsymbol{u}_n \boldsymbol{u}_n^H] \cdot \boldsymbol{w} = \boldsymbol{w}^H \cdot \boldsymbol{R}_{\boldsymbol{u}} \cdot \boldsymbol{w}$$

Power of the output signal

Autocorrelation matrix of the array signals

$$\boldsymbol{R}_{\boldsymbol{u}} = \sum_{i=1}^{M} \sigma_{S,i}^2 \boldsymbol{s}_i \boldsymbol{s}_i^H + \sigma_n^2 \boldsymbol{I}_N = \boldsymbol{S}\boldsymbol{U}\boldsymbol{S}^H + \sigma_n^2 \boldsymbol{I}_N$$

M directional sources with their steering vectors

Steering vector matrix

Autocorrelation matrix of the sources

Beam-forming

Conventional beamforming

The phases are selected to steer the array in a particular direction (θ_o , φ_o)

With denoting the steering vector in a direction, the array weights are given by

$$\boldsymbol{w} = \frac{1}{N} \cdot \boldsymbol{s}_o$$

In presence of a single source with amplitude *A* from the direction (θ_o , φ_o), we have

$$y_n = \boldsymbol{w}^H \cdot A\boldsymbol{s}_o = A$$

In an environment consisting of only uncorrelated noise and no directional interferences, this beam former provides the maximum SNR. For uncorrelated noise, the output noise power is given by

$$P_n = \boldsymbol{w}^H \cdot \boldsymbol{R}_n \cdot \boldsymbol{w} = \frac{\sigma_n^2}{N}$$

Array SNR gain (w.r.t. single element)

Adaptive beamforming

Null-steering beamforming

A null-steering beam former is used to cancel *K* plane waves arriving from known directions.

The weight vector is the solution of the following problem:

$$\begin{cases} \boldsymbol{w}^H \cdot \boldsymbol{s}_o = 1 & \\ \boldsymbol{w}^H \cdot \boldsymbol{s}_i = 0 & i = 1, \dots, K \end{cases}$$

$$\boldsymbol{w}^H \cdot \boldsymbol{S} = [1 \quad 0 \quad \dots \quad 0] = \boldsymbol{g}_1^T$$

S is generally not square.

$$\boldsymbol{S} = [\boldsymbol{s}_o \quad \boldsymbol{s}_1 \quad \dots \quad \boldsymbol{s}_K]$$

$$\boxed{\mathbf{w}^{\boldsymbol{H}} = \boldsymbol{g}_1^T \cdot \boldsymbol{S}^H \cdot (\boldsymbol{S}\boldsymbol{S}^H)^{-1}}$$

with $K \leq N - 2$

Optimum beam-forming

Minimize the interference-plus-noise power at the beamformer output.

The problem is expressed by minimizing the output power keeping a unit response from the signal direction:

$$\text{Minimize} \quad \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w}$$

$$\text{s.t.} \quad \boldsymbol{w}^H \cdot \boldsymbol{s}_o = 1$$

The solution is

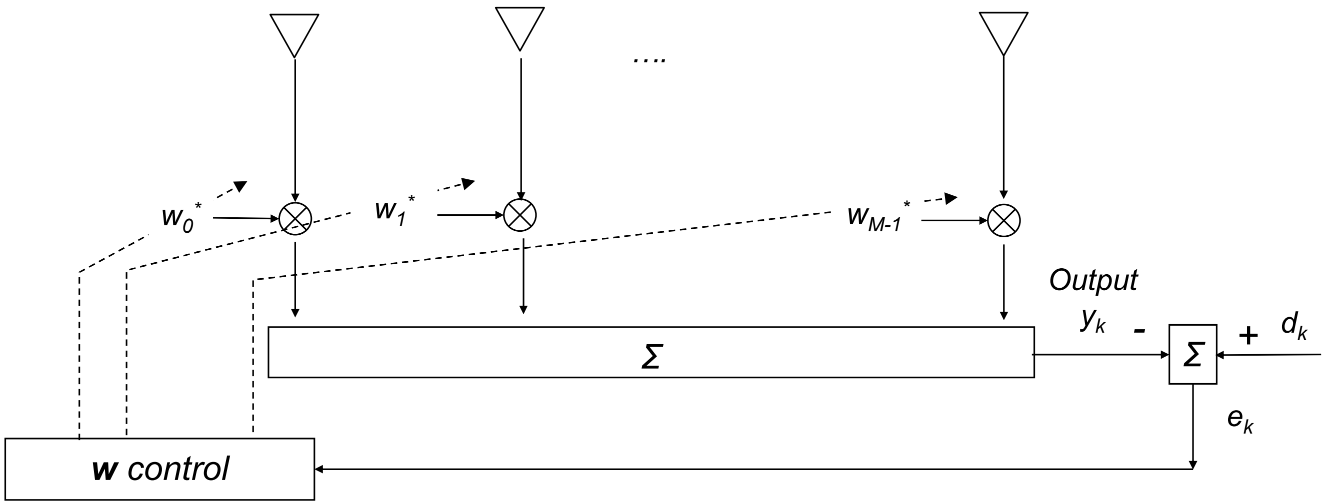
$$\boxed{\boldsymbol{w_o} = \frac{\boldsymbol{R_u^{-1}} \cdot \ \boldsymbol{s}_o}{\boldsymbol{s_o^H} \cdot \boldsymbol{R_u^{-1}} \cdot \ \boldsymbol{s}_o}}$$

Minimizing the total output noise while keeping the output signal constant is the same as maximizing the output SNR, which turns out to be $\boldsymbol{SNR} = \sigma_{S,1}^2 \cdot \boldsymbol{s_o^H} \cdot \boldsymbol{R_u^{-1}} \cdot \ \boldsymbol{s}_o$

This is also called minimum-variance distortionless response (MVDR) beamformer (blind but …).

Beam-forming

Optimum beam-forming with a reference signal



Narrow-band beam-former structure with a reference signal

Optimum beam-forming with a reference signal

Weights are adjusted such that the MSE between the array output and the reference signal is minimized.

$$\mathrm{MSE} = E[d_n d_n^*] + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\ \boldsymbol{w}^H \boldsymbol{p} = \sigma_d^2 + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\ \boldsymbol{w}^H \boldsymbol{p}$$

where we have introduced the cross-correlation between the input and the reference signal

$$\boldsymbol{p} = E[\boldsymbol{u}_n \cdot d_n^*]$$

The MSE surface is a quadratic function of and is minimized by setting its gradient with respect to the weights equal to zero.

The **Wiener–Hopf equation** for the optimal weight vector is

$$\boldsymbol{w}_{opt} = \boldsymbol{R}_{\boldsymbol{u}}^{-\mathbf{1}} \cdot \boldsymbol{p}$$

Optimum beam-forming with a reference signal

The **MMSE** is given by

$$MMSE = \sigma_d^2 - \sigma_{\hat{d}}^2 = \sigma_d^2 - \boldsymbol{w}_o^H \cdot \boldsymbol{R}_u \cdot \boldsymbol{w}_o = \sigma_d^2 - \boldsymbol{w}_o^H \cdot \boldsymbol{p} = \sigma_d^2 - \boldsymbol{p}^H \cdot \boldsymbol{R}_{\boldsymbol{u}}^{-1} \cdot \boldsymbol{p}$$

For a perfect LoS plane wave, the Wiener filter and the MVDR are equivalent.

- In digital mobile communications, a synchronization signal may be used for initial weight estimation, followed by the use of the detected signal as a reference signal.

Adaptive beam-forming

Without the perfect knowledge of the second-order statistics, the use of adaptive methods allows their direct estimation from the collected data and the contemporary update of the BF weights.

The LMS algorithm

Constrained LMS algorithm

The weights are subject to constraints at each iteration.

Unconstrained LMS algorithm

The weights are not constrained at each iteration.

For example this is applicable when weights are updated using a reference signal and no knowledge of AoA is used.

Adaptive beam-forming

The LMS algorithm

The algorithm updates the weights at each iteration by estimating the **gradient** of the **MSE** and moving them in the **negative direction** of the gradient.

A real-time unconstrained LMS algorithm for determining optimal weight is

step size

$$\boxed{\boldsymbol{w_{n+1}} = \boldsymbol{w_n} - \frac{1}{2}\boldsymbol{\mu} \cdot \widehat{\nabla}_{\boldsymbol{w}} \boldsymbol{MSE}(\boldsymbol{w})|_{\boldsymbol{w}=\boldsymbol{w_n}}}$$

$$\mathrm{MSE} = E[d_n d_n^*] + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\ \boldsymbol{w}^H \boldsymbol{p} = \sigma_d^2 + \boldsymbol{w}^H \cdot \boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\ \boldsymbol{w}^H \boldsymbol{p}$$

$$\boldsymbol{\nabla_w MSE}(\boldsymbol{w}) = 2\boldsymbol{R_u} \cdot \boldsymbol{w} - \mathbf{2}\ \boldsymbol{p}$$

$$\downarrow$$

$$\widehat{\boldsymbol{\nabla}}_{\boldsymbol{w}} \boldsymbol{MSE}(\boldsymbol{w}) = 2\boldsymbol{u}_n \cdot \boldsymbol{u}_n^H \cdot \boldsymbol{w}_n - 2\boldsymbol{u}_n \cdot d_n^* = 2\boldsymbol{u}_n \cdot e_n^*$$

The LMS algorithm

$$\boldsymbol{w_{n+1}} = \boldsymbol{w_n} - \frac{1}{2}\boldsymbol{\mu} \cdot 2\boldsymbol{u}_n \cdot e_n^* = \boldsymbol{w_n} - \boldsymbol{\mu} \cdot \boldsymbol{u}_n \cdot\ e_n^*$$

$$0 < \mu < \frac{2}{\lambda_{max}}$$

As the sum of all eigenvalues of $\boldsymbol{R_u}$ equals its trace, one may select the gradient step size

$$\mu = \frac{1}{Tr(\boldsymbol{R_u})}$$

Also considering that each diagonal element of $\boldsymbol{R_u}$ is equal to the average power measured on the corresponding element of the array.

Adaptive beam-forming

The excess MSE or the misadjustment noise

The misadjustment is caused by the use of the noisy estimate of the gradient.

Even when the mean of the estimated weights converges to the optimal weights, they have finite covariance.

Therefore, the average value of the excess MSE does not approach zero.

Increasing the step size the misadjustment noise.

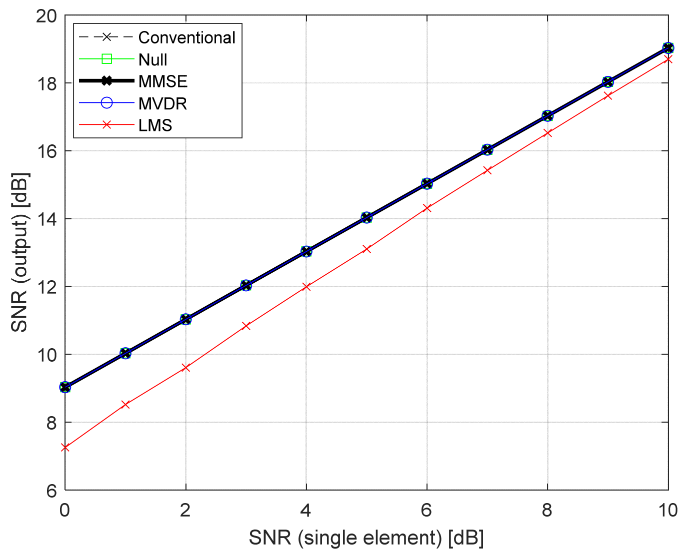
On the other hand, an increase of the step size makes the convergence faster.

Many schemes have variable step size to overcome the problem.

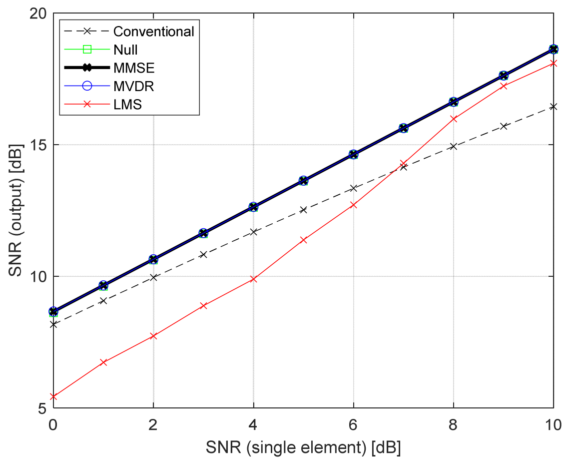
Adaptive beam-forming

- Example: no interferers

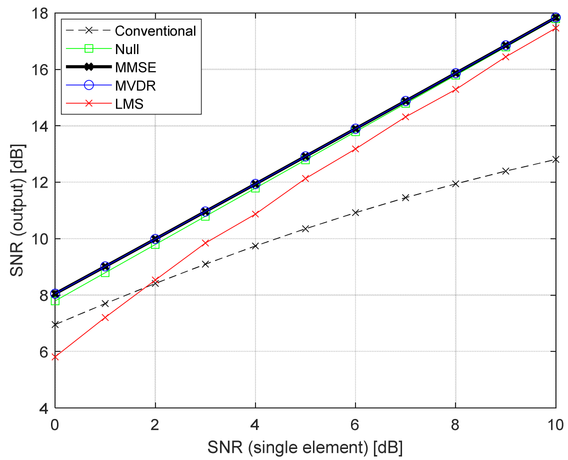
N = 8 and *d* = *λ*/2



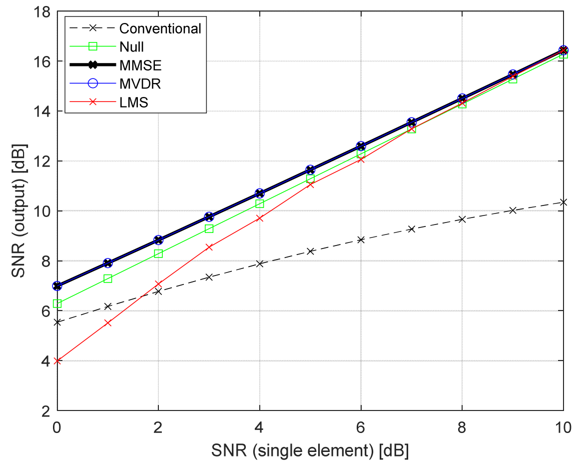
Adaptive beam-forming



1 interferer



3 interferers

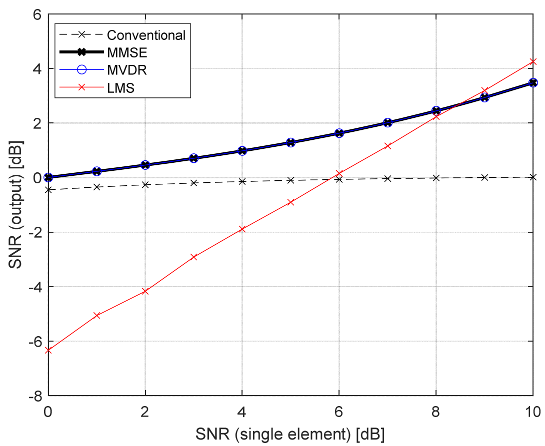


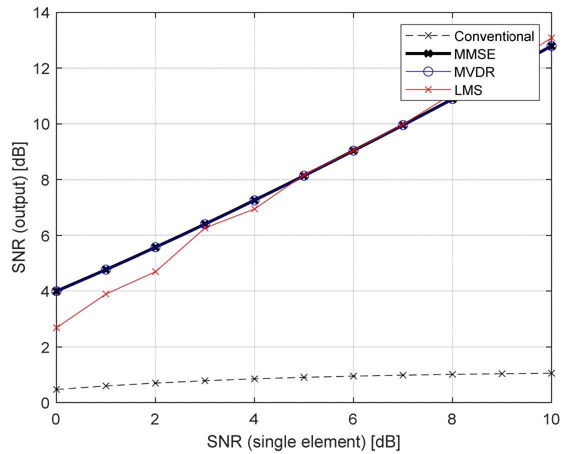
5 interferers

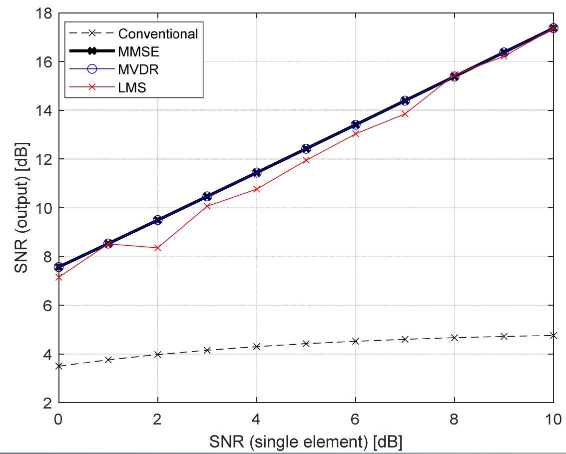
N = *8* and *d* = *λ/2*

Adaptive beamforming

Adaptive beam-forming







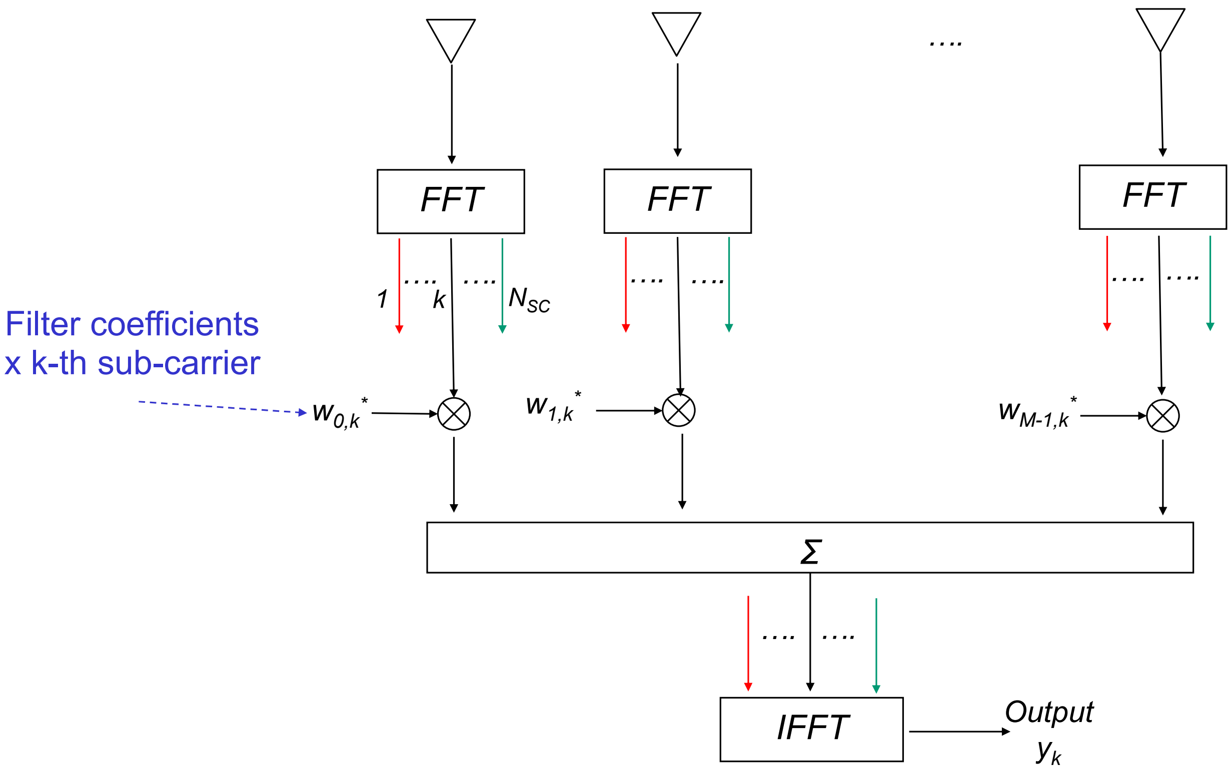
Example:

1 interferer (1˚, 4˚, 8˚ angular sep.)

$N = 8$ and $d = \lambda/2$

Adaptive beam-forming

- Wideband signals → *Frequency-Domain Beam Forming*



Adaptive beamforming

Adaptive beam-forming

- Frequency-Domain Beam Forming

The weights required for each frequency bin are selected independently, and this selection may be performed in parallel, leading to a faster weight update.

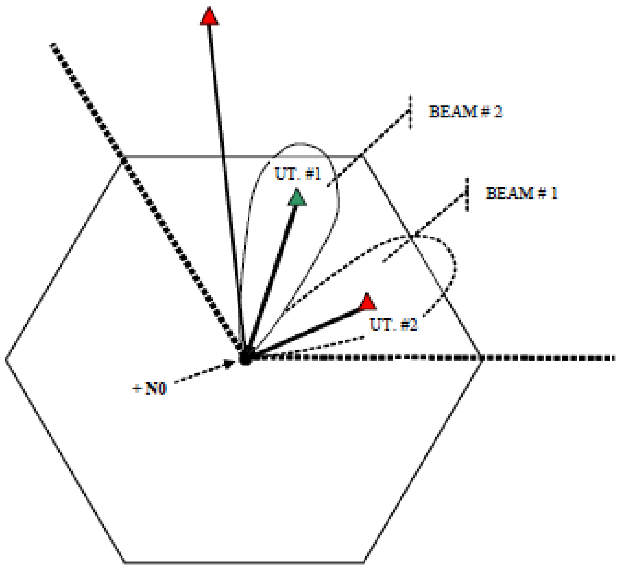
When adaptive algorithms such as the LMS algorithm is used for weight update, a different step size may be used for each bin, leading to faster convergence.

Beam-Forming in LTE-A

Beamforming

1 or 2 layers x SU-MIMO or MU-MIMO

Possibility of adaptive BF



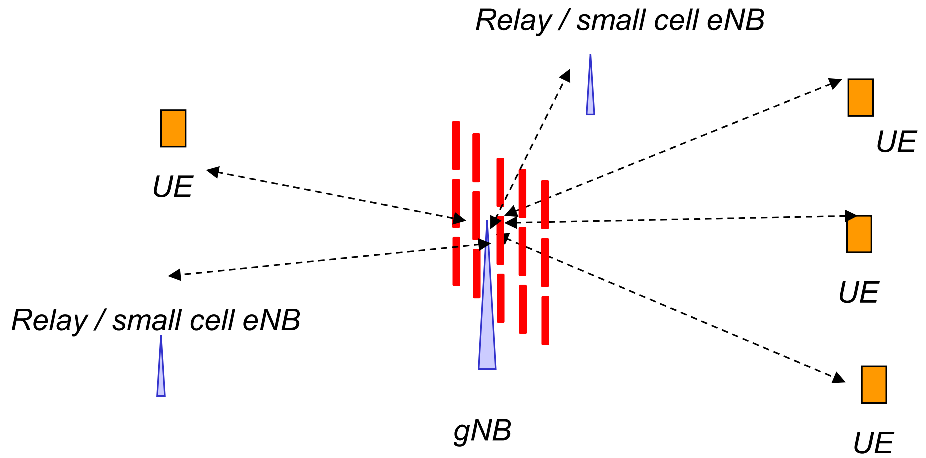
LTE does not specify BF methods.

Also the number of antennas and the antenna architecture are left to implementation.

Adaptive beamforming

Beam-Forming in NR

- **Massive MIMO**



3D beamforming with up to 256 antenna elements

5G NR co-located with LTE macro sites

Needed accurate and timely channel est.

- **3D beam-forming at mmWaves: beam tracking, path / angle diversity …**

mmWave mobile communications is possible

[T. Rappport et al., "*Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!*", IEEE Access]

Adaptive beamforming