1) Trata-se duma série geométrica:

$$\frac{a_{n+1}}{a_n} = \frac{\frac{2^{2n+1}\cdot 4^{n+1}}{3^{3n+1}}}{\frac{2^{2n-1}\, 4^{n}}{3^{3n-2}}} = \frac{2^2\cdot 4}{3^3} = \frac{16}{27}.$$

Como $-1 < \frac{16}{27} < 1$, a série converge.

Soma: $S = \frac{a_1}{1-r} = \frac{\frac{2\cdot 4}{3}}{1-\frac{16}{27}} = \frac{\frac{8}{3}}{\frac{11}{27}} =$

$$\frac{8\cdot 27}{11\cdot 3} = \frac{8\cdot 9}{11} = \frac{72}{11}.$$

2a) Série dos módulos: $\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{n}$.

C.d.C. v1: $\frac{\sqrt{n+3}}{n} \geq \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$

A série $\sum_{n=3}^{\infty} \frac{1}{n^{1/2}}$ diverge (S. de Dirichlet c/ $s=\frac{1}{2}$

Logo, $\sum_{n=3}^{\infty} \frac{\sqrt{n+3}}{n}$ diverge também.

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F1

11/3/24

Cont. de 2a): Vamos aplicar o Crit. de Leibniz à série alternada $\sum_{n=3}^{\infty}(-1)^n \frac{\sqrt{n+3}}{n}$.

i) $\lim_{n\to\infty} \frac{\sqrt{n+3}}{n} = \lim_{n\to\infty} \sqrt{\frac{n+3}{n^2}} = \lim_{n\to\infty} \sqrt{\frac{1}{n}+\frac{3}{n^2}} = 0.$ ✓

ii) Seja $f(x) = \frac{\sqrt{x+3}}{x}$. Então

$$f'(x) = \frac{\frac{1}{2\sqrt{x+3}}\cdot x - \sqrt{x+3}\cdot 1}{x^2} = \frac{x - 2(\sqrt{x+3})^2}{2x^2\sqrt{x+3}} =$$

$$\frac{x-(2x+6)}{2x^2\sqrt{x+3}} = \frac{-x-6}{2x^2\sqrt{x+3}} \leq 0 \quad \forall x \geq -6.$$

Logo $\frac{\sqrt{n+3}}{n}$ é mon. decrescente $\forall n \geq 3$

Segundo o Crit. de Leibniz, a série alternada $\sum_{n=3}^{\infty}(-1)^n \frac{\sqrt{n+3}}{n}$ converge.

Conclusão: $\sum_{n=3}^{\infty}(-1)^n \frac{\sqrt{n+3}}{n}$ é simplesmente convergente.

2b) C. d. Q.: $\dfrac{a_{n+1}}{a_n} = \dfrac{\frac{(n+1)!\,4^{n+1}}{(n+1)^{n+1}}}{\frac{n!\,4^n}{n^n}} =$

$$\frac{(n+1)!\;4^{n+1}\cdot n^n}{n!\;4^n\;(n+1)^{n+1}} = \frac{\cancel{(n+1)}\cdot\cancel{n!}\cdot\cancel{4^n}\cdot 4\cdot n^n}{\cancel{n!}\;\cancel{4^n}\;\cancel{(n+1)}(n+1)^n}$$

$$= \frac{4\cdot n^n}{(n+1)^n} = \frac{4}{\left(\frac{n+1}{n}\right)^n} = \frac{4}{\left(1+\frac{1}{n}\right)^n}.$$

Logo, $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \dfrac{4}{e} > 1.$

Conclusão: a série diverge.

2c) Série dos módulos: $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^n+3^n}$

C.d.C. v1: $\dfrac{2^n}{2^n+3^n} \leq \dfrac{2^n}{3^n} = \left(\dfrac{2}{3}\right)^n.$

A série $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ converge (s. geom. c/ $r=\frac{2}{3}$).

Logo, $\displaystyle\sum \frac{(-2)^n}{2^n+3^n}$ converge absolutamente.

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F1

11/3/24

2 d) $\dfrac{\sqrt{n+1}-\sqrt{n}}{n} = \left(\dfrac{\sqrt{n+1}-\sqrt{n}}{n}\right)\left(\dfrac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}\right)$

$= \dfrac{n+1-n}{(\sqrt{n+1}+\sqrt{n})n} = \dfrac{1}{n(\sqrt{n+1}+\sqrt{n})}. \leq \dfrac{1}{n\sqrt{n}} = \dfrac{1}{n^{3/2}}$

Como $\sum_{n=1}^{\infty} \dfrac{1}{n^{3/2}}$ conv., $\sum_{n=1}^{\infty} \dfrac{\sqrt{n+1}-\sqrt{n}}{n}$

também converge pelo C.d.C. V1.

3) $c = \frac{1}{4}$ pq $4x-1=0 \Leftrightarrow x = \frac{1}{4}$.

C.d.R. $\lim_{n\to\infty} \sqrt[n]{\dfrac{6^n |4x-1|^{n-1}}{n}} =$

$\lim_{n\to\infty} \dfrac{6^{\frac{n}{n}}|4x-1|^{\frac{n-1}{n}}}{\sqrt[n]{n}} = 6|4x-1|.$

$6|4x-1| = 1 \Leftrightarrow |4x-1| = \frac{1}{6} \Leftrightarrow 4x-1 = -\frac{1}{6} \vee$

$4x-1 = \frac{1}{6} \Leftrightarrow x = +\frac{5}{24} \vee x = \frac{7}{24}.$

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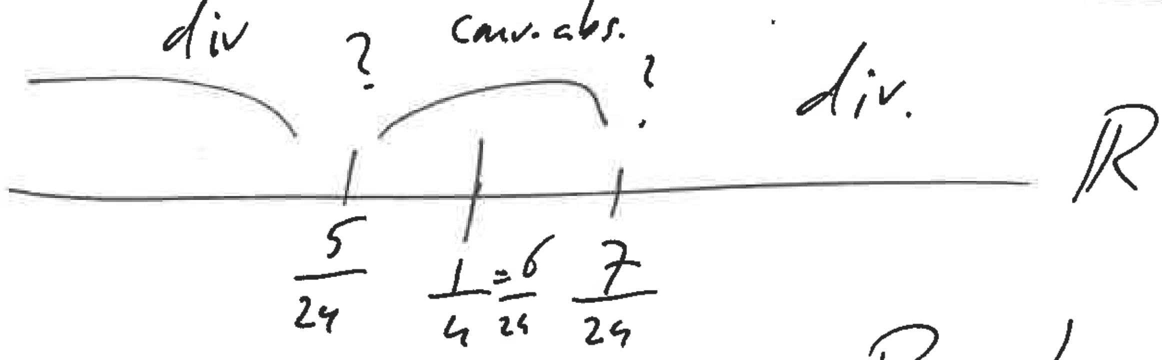
F1

11/3/24



$R = \frac{1}{24}$.

$$x = \frac{7}{24}: \sum_{n=1}^{\infty} \frac{6^n\left(\frac{7}{6}-1\right)^{n-1}}{n} = \sum_{n=1}^{\infty} \frac{6^n \cdot \left(\frac{1}{6}\right)^{n-1}}{n} =$$

$$\sum_{n=1}^{\infty} \frac{6 \cdot \cancel{6^{n-1}} \cdot \cancel{\left(\frac{1}{6}\right)^{n-1}}}{n} = \sum_{n=1}^{\infty} \frac{6}{n} = 6 \sum_{n=1}^{\infty} \frac{1}{n}$$

Esta série diverge (6 × série harm.)

$$x = \frac{5}{24}: \sum_{n=1}^{\infty} \frac{6^n\left(\frac{5}{6}-1\right)^{n-1}}{n} = \sum_{n=1}^{\infty} \frac{6^n\left(-\frac{1}{6}\right)^{n-1}}{n} =$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 6}{n} = 6 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}.$$

Esta série é simplesmente convergente (6 × série harmónica alternada).