AM II LEI + BE

F2 (V2)

6/5/2024

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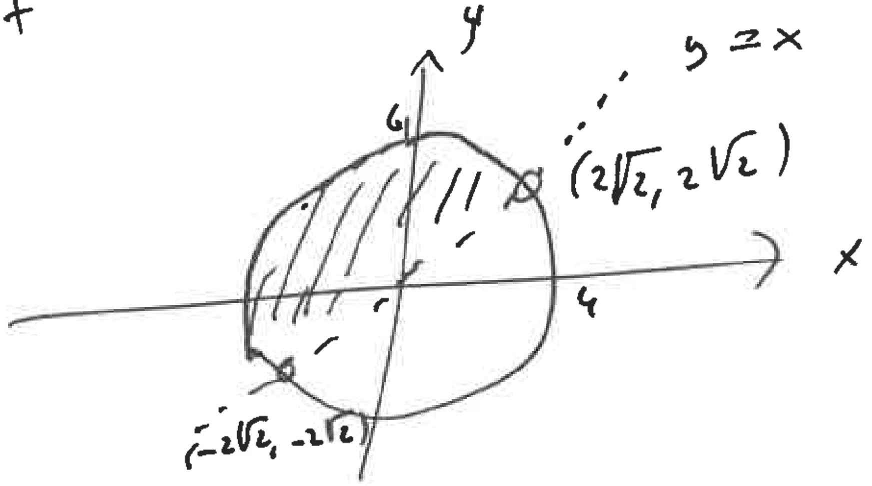
1) $D_f = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \le 16 \wedge y > x\}$



$x=y \wedge x^2+y^2=16 \Leftrightarrow x=y \wedge 2x^2=16 \Leftrightarrow$

$x=y \wedge x=\pm\sqrt{8}=\pm 2\sqrt{2}$

2) a) f cont. em $(0,0) \Leftrightarrow \lim\limits_{(x,y)\to(0,0)} \dfrac{2x^3+5xy-3y^3}{\sqrt{x^2+y^2}} = 0$

$$0 \le \left|\frac{2x^3+5xy-3y^3}{\sqrt{x^2+y^2}}\right| \le \frac{2|x|^3+5|x||y|+3|y|^3}{\sqrt{x^2+y^2}}$$

$$\le \frac{5\left(\sqrt{x^2+y^2}\right)^3+5\left(\sqrt{x^2+y^2}\right)^2}{\sqrt{x^2+y^2}} = 5\left(\sqrt{x^2+y^2}\right)^2+5\sqrt{x^2+y^2}$$

Como $\lim\limits_{(x,y)\to(0,0)} \sqrt{x^2+y^2} = 0$, $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^3+5xy-3y^3}{\sqrt{x^2+y^2}} = 0$

Conclusão: f é cont. em $(0,0)$.

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2 b) $f'_x(0,0) = \lim_{t\to 0} \dfrac{f(t,0)-f(0,0)}{t} = \lim_{t\to 0} \dfrac{\frac{2t^3}{\sqrt{t^2}} - 0}{t} =$

$\lim_{t\to 0} \dfrac{2t^3}{|t|\cdot t} = \lim_{t\to 0} \dfrac{2t^2}{|t|} = \lim_{t\to 0} \dfrac{2|t|^2}{|t|} =$

$\lim_{t\to 0} 2|t| = 0.$

$f'_y(0,0) = \lim_{t\to 0} \dfrac{f(0,t)-f(0,0)}{t} = \lim_{t\to 0} \dfrac{\frac{-3t^3}{\sqrt{t^2}} - 0}{t}$

$= 0$

c) f não é dif. em $(0,0)$, pq

$\lim_{(x,y)\to(0,0)} \dfrac{f(x,y) - f(0,0) - (f'_x(0,0)x + f'_y(0,0)y)}{\sqrt{x^2+y^2}}$ $f($

Prova: $\lim_{(x,y)\to(0,0)} \dfrac{f(x,y) - f(0,0) - (f'_x(0,0)x + f'_y(0,0)y)}{\sqrt{x^2+y^2}} =$

$\lim_{(x,y)\to(0,0)} \dfrac{\frac{2x^3+5xy-3y^3}{\sqrt{x^2+y^2}} - 0}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \dfrac{2x^3+5xy-}{x^2+y^2}$

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(3)

Cont. de 2 c): Limite trajectorial:

$y=kx$: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=kx}} \frac{2x^3+5xy-3y^3}{x^2+y^2} =$

$$\lim_{x\to 0} \frac{2x^3+5kx^2-3k^3x^3}{x^2+k^2x^2} =$$

$$\lim_{x\to 0} \frac{2x^3+5kx^2-3k^3x^3}{(1+k^2)x^2} =$$

$$\lim_{x\to 0} \frac{2x+5k-3k^3x}{(1+k^2)} = \frac{5k}{1+k^2}$$ depende de k!

Logo $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^3+5xy-3y^3}{x^2+y^2}$ não existe.

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3) Pela Regra da Cadeia:

$$g'_r(\sqrt{2}, \pi/4) = f'_x(1,1) \cdot (r\cos(\theta))'_r \Big|_{(\sqrt{2}, \pi/4)} +$$

$$f'_y(1,1)(r \operatorname{sen}\theta)'_r \Big|_{(\sqrt{2}, \pi/4)} = 1 \cdot \cos(\pi/4) = \frac{\sqrt{2}}{2}.$$

$$g'_\theta(\sqrt{2}, \pi/4) = f'_x(1,1)(r\cos\theta)'_\theta \Big|_{(\sqrt{2}, \pi/4)} +$$

$$f'_y(1,1)(r \operatorname{sen}\theta)'_\theta \Big|_{(\sqrt{2}, \pi/4)} = 1 \cdot (-\sqrt{2} \operatorname{sen}(\pi/4)) = -1.$$

Note-se que $x(\sqrt{2}, \pi/4) = y(\sqrt{2}, \pi/4) = 1$.

4 a) $\nabla f(x,y) = (0,0) \Leftrightarrow$

$(-2y + x^2 - 3, 2y - 2x) = (0,0) \Leftrightarrow$

$$\begin{cases} -2y + x^2 - 3 = 0 \\ x = y \end{cases} \Leftrightarrow \begin{cases} x^2 - 2x - 3 = 0 \\ x = y \end{cases} \Leftrightarrow \begin{cases} x = -1 \vee x = 3 \\ x = y \end{cases}$$

Pts. críticos: $(-1,-1)$, $(3,3)$.

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4 b) $H_f(x,y) = \begin{pmatrix} 2x & -2 \\ -2 & 2 \end{pmatrix}$

$H_f(-1,-1) = \begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$.

$\det(H_f(-1,-1)) = -2 \cdot 2 - (-2)^2 = -8 < 0$

f tem um pt. de sela em $(-1,-1)$.

$H_f(3,3) = \begin{pmatrix} 6 & -2 \\ -2 & 2 \end{pmatrix}$.

$\det(H_f(3,3)) = 6 \cdot 2 - (-2)^2 = 8 > 0$

$f''_{xx}(3,3) = 6 > 0$

f tem um mínimo local em $(3,3)$.