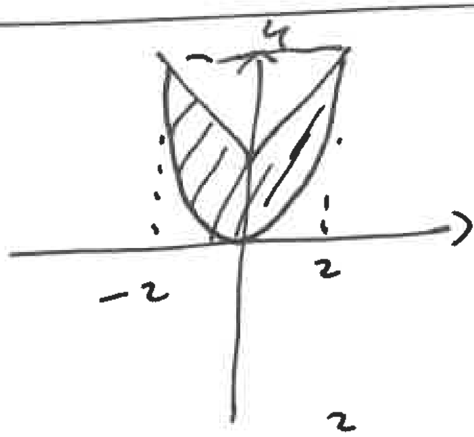
1 a)



$x \geq 0: \; x^2 = 2 + x \iff x^2 - x - 2 = 0 \iff x = 2 \; (y = 4)$

$x \leq 0: \; x^2 = 2 - x \iff x^2 + x - 2 = 0 \iff x = -2 \; (y = 4)$

1 b)
$$\int_{-2}^{0}\int_{0}^{2-x} 2y \, dy \, dx + \int_{0}^{2}\int_{0}^{2+x} 2y \, dy \, dx = 2\int_{0}^{2}\int_{0}^{2+x} 2y \, dy \, dx =$$

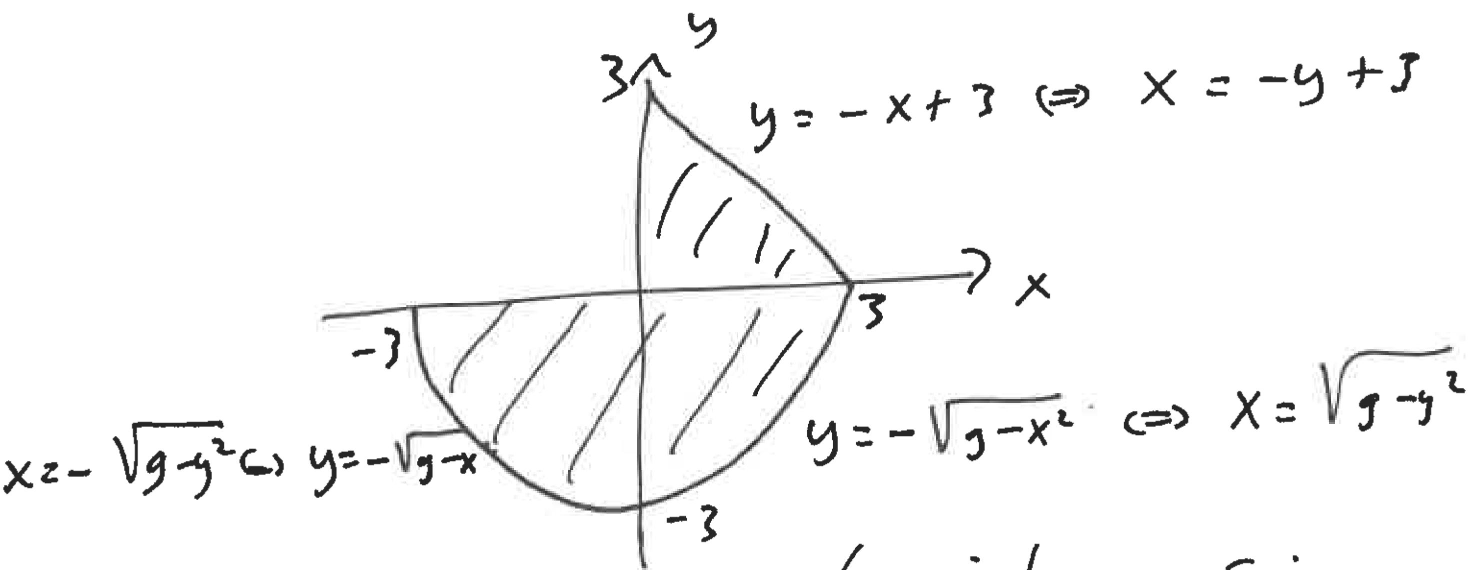
$$2\int_{0}^{2} \left[y^2\right]_{y=0}^{y=2+x} dx = 2\int_{0}^{2} (2+x)^2 - 0 \, dx =$$

$$2\int_{0}^{2} x^2 + 4x + 4 \, dx = 2\left[\frac{x^3}{3} + 2x^2 + 4x\right]_{0}^{2} =$$

$$2\left[\frac{8}{3} + 8 + 8\right] = 2 \cdot \frac{56}{3} = \frac{112}{3}.$$

2) $D = \{(x,y) \in \mathbb{R}^2 : -3 \le x \le 0 \wedge -\sqrt{9-x^2} \le y \le 0\} \cup$
$\{(x,y) \in \mathbb{R}^2 : 0 \le x \le 3 \wedge -\sqrt{9-x^2} \le y \le -x+3\}$

$y = \pm\sqrt{9-x^2} \Leftrightarrow y^2 = 9 - x^2 \Leftrightarrow x^2+y^2 = 9$
(círculo c/ centro em (0,0) e raio 3).



Invertendo a ordem de integração:

$D = \{(x,y) \in \mathbb{R}^2 : -\sqrt{9-y^2} \le x \le \sqrt{9-y^2} \wedge -3 \le y \le 0\} \cup$
$\{(x,y) \in \mathbb{R}^2 : 0 \le x \le -y+3 \wedge 0 \le y \le 3\}$

$$\int_{-3}^{0}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x,y)\,dx\,dy + \int_{0}^{3}\int_{0}^{-y+3} f(x,y)\,dx\,dy$$

F3 AM II

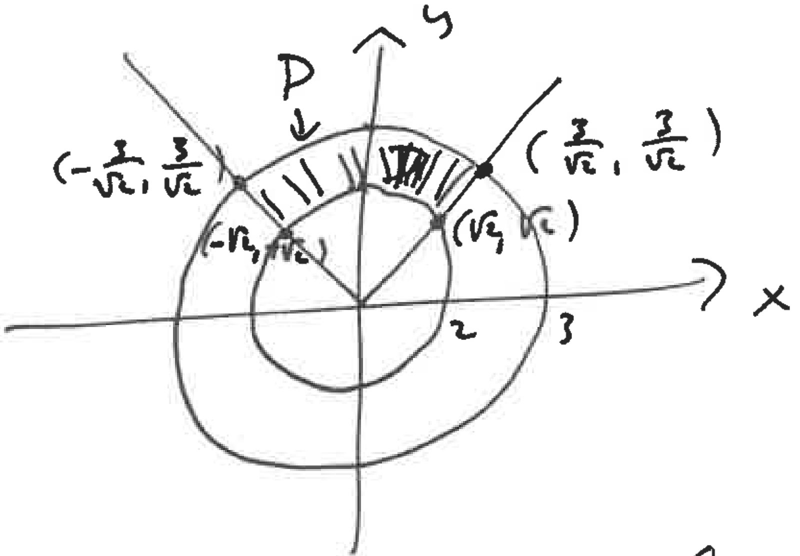
LEI + BE

23/24

3) $D = \{(x,y) \in \mathbb{R}^2 : 4 \leq x^2+y^2 \leq 9 \wedge y \geq |x|\}$.

$y = x \wedge x^2+y^2 = 4 \Leftrightarrow 2x^2 = 4 \Leftrightarrow x^2 = 2 \Leftrightarrow$
$x = \pm\sqrt{2}$

$y = x \wedge x^2+y^2 = 9 \Leftrightarrow 2x^2 = 9 \Leftrightarrow x^2 = \frac{9}{2} \Leftrightarrow$
$x = \pm \frac{3}{\sqrt{2}}$



Em coord. polares: $x = r\cos\theta$, $y = r\,\mathrm{sen}\,\theta$

$2 \leq r \leq 3$, $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$.

$$\iint_D \frac{x-y}{x^2+y^2}\, dA = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_2^3 \frac{r\cos\theta - r\,\mathrm{sen}\,\theta}{r^2}\, r\, dr\, d\theta =$$

Cont de 3) $\displaystyle\int_{\pi/4}^{3\pi/4}\int_{2}^{3} \cos\theta - \operatorname{sen}\theta \, dr \, d\theta =$

$$\int_{\pi/4}^{3\pi/4} \Big[r(\cos\theta - \operatorname{sen}\theta) \Big]_{r=2}^{r=3} d\theta =$$

$$\int_{\pi/4}^{3\pi/4} 3(\cos\theta - \operatorname{sen}\theta) - 2(\cos\theta - \operatorname{sen}\theta) \, d\theta =$$

$$\int_{\pi/4}^{3\pi/4} \cos\theta - \operatorname{sen}\theta \, d\theta = \Big[\operatorname{sen}\theta + \cos\theta \Big]_{\pi/4}^{3\pi/4} =$$

$$\cancel{\frac{\sqrt{2}}{2}} - \cancel{\frac{\sqrt{2}}{2}} - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) = -\sqrt{2}.$$

F3 AM II
LEI + BE
23/24

4 a) $z = 8-2x-4y$ plano em $\mathbb{R}^3$

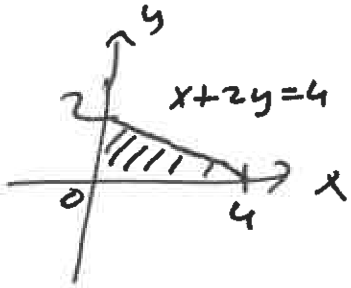
$x=y=0$: $z=8$ $(0,0,8)$

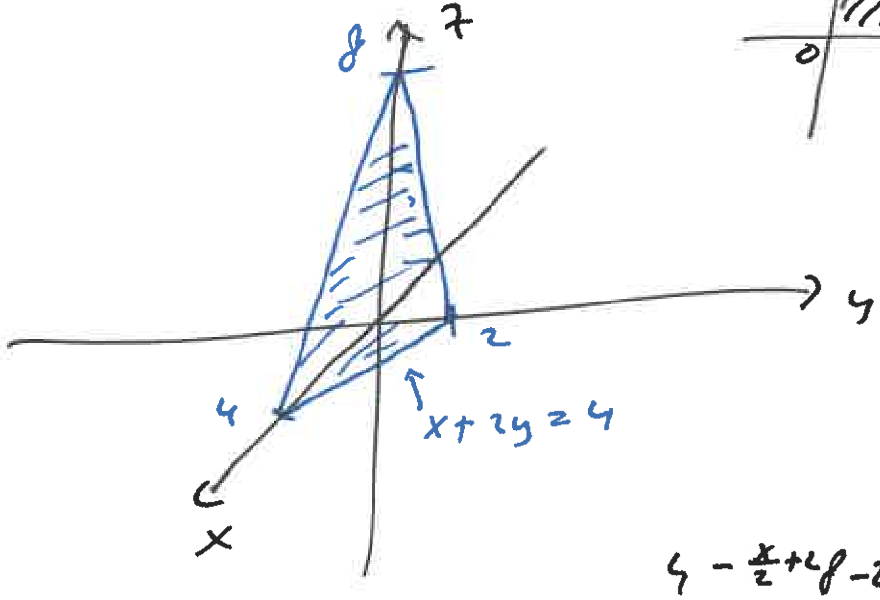
$x=z=0$: $0=8-4y \Leftrightarrow y=2$ $(0,2,0)$

$y=z=0$: $0=8-2x \Leftrightarrow x=4$ $(4,0,0)$

$z=0$: $0=8-2x-4y \Leftrightarrow 2x+4y=8 \Leftrightarrow x+2y=4$

$x \geq 0,\ y \geq 0,\ z \geq 0$: 1° octante.





$x+2y=4 \Leftrightarrow$

$2y = -x+4$

$y = -\frac{x}{2}+2$

b) $\displaystyle Vol(R) = \iiint_R dV = \int_0^4 \int_0^{-\frac{x}{2}+2} \int_0^{8-2x-4y} dz\,dy\,dx$

(cont. de 5^b)

$$\int_0^4\int_0^{-\frac{x}{2}+2} [z]_{z=0}^{z=8-2x-4y} \, dy\,dx =$$

$$\int_0^4\int_0^{-\frac{x}{2}+2} 8-2x-4y \, dy\,dx = \int_0^4 \left[8y-2xy-2y^2\right]_{y=0}^{y=-\frac{x}{2}+2} dx$$

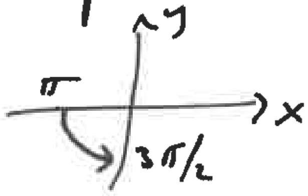
$$\int_0^4 8\left(-\frac{x}{2}+2\right) - 2x\left(-\frac{x}{2}+2\right) - 2\left(-\frac{x}{2}+2\right)^2 - 0\,dx =$$

$$\int_0^4 \frac{x^2}{2} - 4x + 8\,dx = \left[\frac{x^3}{6} - 2x^2 + 8x\right]_0^4 =$$

$$\frac{64}{6} - \cancel{32} + \cancel{32} - 0 = \frac{64}{6} = \frac{32}{3}$$

5) a) $x \le 0,\ y \le 0$: 3° quadrante no plano-xy

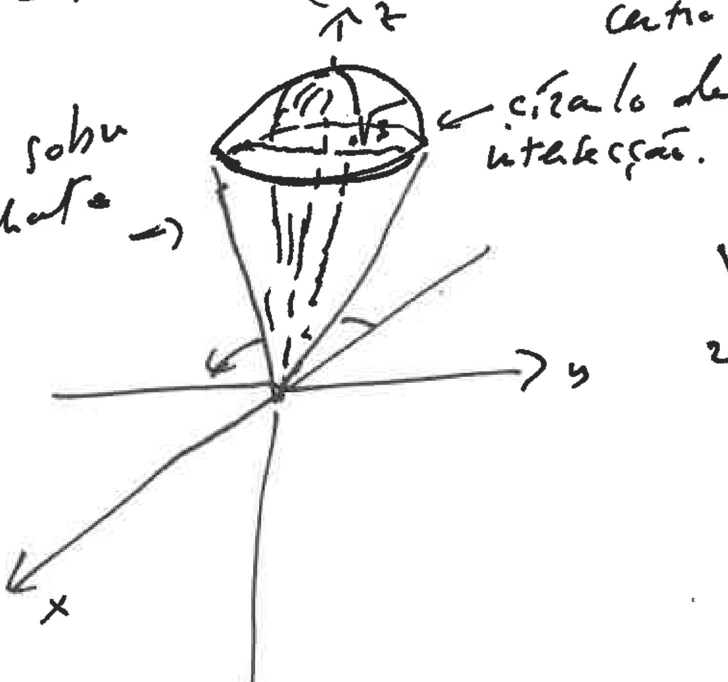
Portanto, $\pi \le \varphi \le \frac{3\pi}{2}$.

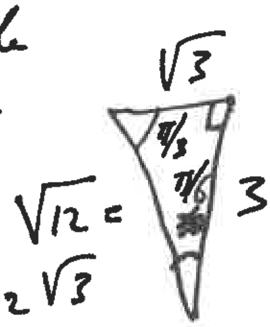


$\sqrt{3(x^2+y^2)} = \sqrt{12-x^2-y^2} \Leftrightarrow 3(x^2+y^2) = 12-(x^2+y^2) \Leftrightarrow$

$4(x^2+y^2) = 12 \Leftrightarrow (x^2+y^2) = 3$ círculo c/ centro em $(0,0)$ e $r=\sqrt{3}$.

A parte sobre o 3° quadrante ⇒





$z = \sqrt{3(x^2+y^2)}$ semi-cone positivo

$z = \sqrt{12-x^2-y^2}$ semi-esfera positiva c/ $r = \sqrt{12} = 2\sqrt{3}$.

b) Em coordenadas esféricas:

$0 \le r \le 2\sqrt{3}$, $\pi \le \varphi \le \frac{3\pi}{2}$, $0 \le \theta \le \frac{\pi}{6}$.

$$Vol(R) = \iiint\limits_R dV = \int_{\pi}^{3\pi/2}\int_{0}^{\pi/6}\int_{0}^{2\sqrt{3}} r^2 \,\text{sen}(\theta)\, dr\, d\theta$$

$$= \int_{\pi}^{3\pi/2}\int_{0}^{\pi/6} \left[\frac{r^3}{3}\,\text{sen}(\theta)\right]_{0}^{r=2\sqrt{3}} d\theta\, d\varphi =$$

$$\int_{\pi}^{3\pi/2}\int_{0}^{\pi/6} 8\sqrt{3}\,\text{sen}\,\theta\, d\theta\, d\varphi = \int_{\pi}^{3\pi/2}\left[-8\sqrt{3}\cos\theta\right]_{\theta=0}^{\theta=\pi/6} d$$

$$= \int_{\pi}^{3\pi/2} -8\sqrt{3}\left(\frac{\sqrt{3}}{2}-1\right) d\varphi = 8\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)\int_{\pi}^{3\pi/2} d\varphi =$$

$$8\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)\cdot\left[3\pi/2 - \pi\right] = 4\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)\pi$$