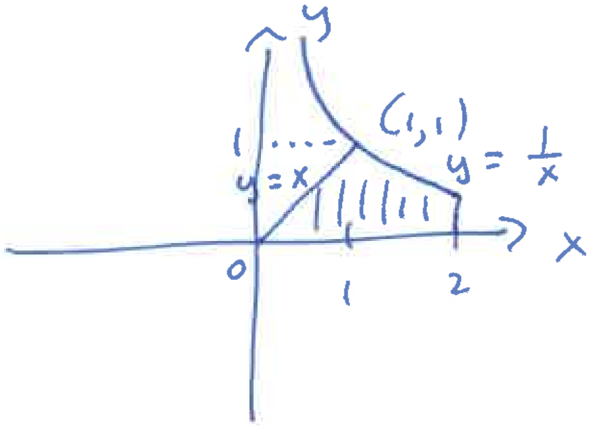
1a) Interseção: $x = \frac{1}{x} \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1.$

$(1,1)$ $\cancel{(-1,-1)}$

D:



1b) $$\iint_D y^2\,dA = \int_0^1\int_0^x y^2\,dy\,dx + \int_1^2\int_0^{1/x} y^2\,dy\,dx =$$

$$\int_0^1 \frac{y^3}{3}\Big|_{y=0}^{y=x} dx + \int_1^2 \frac{y^3}{3}\Big|_{y=0}^{y=\frac{1}{x}} dx =$$

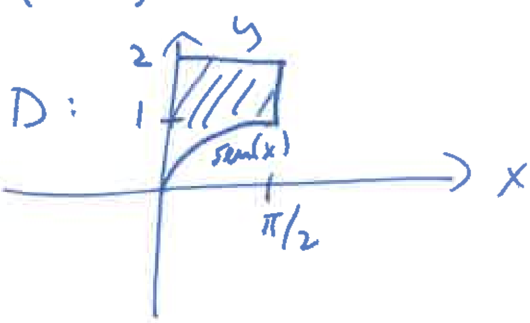
$$\int_0^1 \frac{x^3}{3}\,dx + \int_1^2 \frac{1}{3x^3}\,dx =$$

$$\frac{x^4}{12}\Big|_0^1 - \frac{1}{6x^2}\Big|_1^2 = \frac{1}{12} - 0 - \frac{1}{24} + \frac{1}{6}$$

$$= \frac{5}{24}.$$

2) $D = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le \frac{\pi}{2},\ \operatorname{sen}(x) \le y \le 2\}.$

D:



(Cont. de 2) $y = \operatorname{sen}(x) \Leftrightarrow x = \operatorname{arcsen}(y)$

$$D = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le \operatorname{arcsen} y, \ 0 \le y \le 1\} \cup$$
$$\{(x,y) \in \mathbb{R}^2 : 0 \le x \le \frac{\pi}{2}, \ 1 \le y \le 2\}$$

$$\int_0^{\pi/2}\int_{\operatorname{sen}(x)}^{2} dy\,dx = \int_0^1\int_0^{\operatorname{arcsen} y} dx\,dy + \int_1^2\int_0^{\pi/2} dx\,dy$$

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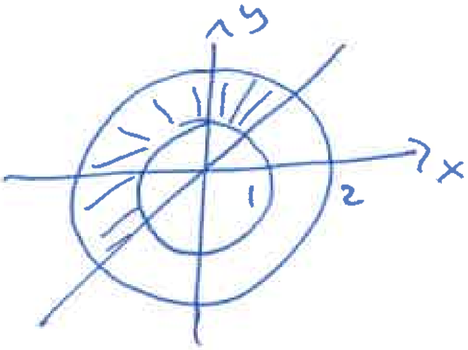
3) $D = \{(x,y) \in \mathbb{R}^2 : 1 \le x^2+y^2 \le 4 \ \wedge \ y \ge x\}$.



Coord. pol.: $x = r\cos\theta, \ y = r\operatorname{sen}\theta, \ |J| = r$
$x^2+y^2 = r^2$

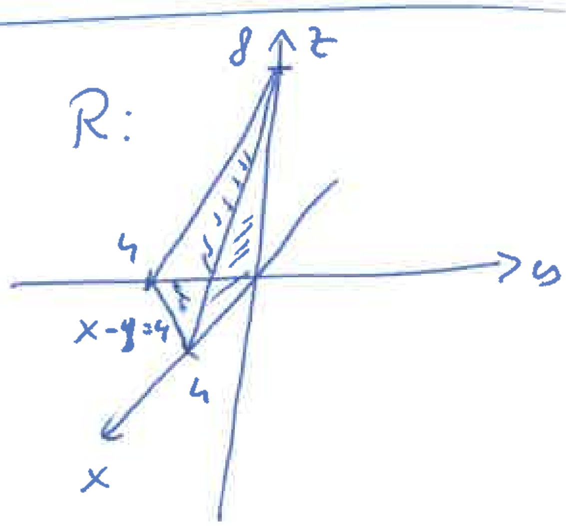
$$\begin{cases} 1 \le r \le 2 \\ \frac{\pi}{4} \le \theta \le \frac{5\pi}{4} \end{cases}$$

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}\,dA = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\int_1^2 \frac{1}{r}\cdot r\,dr\,d\theta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\int_1^2 dr\,d\theta =$$

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} r\Big|_{r=1}^{r=2} d\theta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} 2-1\,d\theta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} d\theta = \theta\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = \frac{5\pi}{4} - \frac{\pi}{4}$$
$$= \pi.$$

4a) $\boxed{z = 8 - 2x + 2y}$

R:



$y = z = 0 : \quad 0 = 8 - 2x \Leftrightarrow x = 4$

$x = z = 0 : \quad 0 = 8 + 2y \Leftrightarrow y = -4$

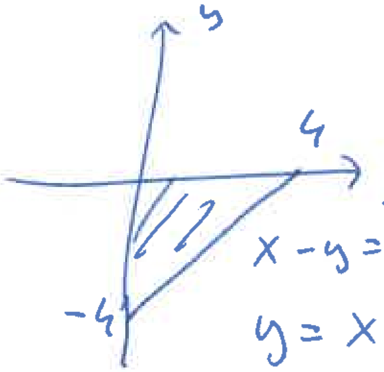
$x = y = 0 : \quad z = 8$

$(4,0,0) \quad (0,-4,0) \quad (0,0,8)$

$z = 0 : \quad 0 = 8 - 2x + 2y \Leftrightarrow$

$2x - 2y = 8 \Leftrightarrow$

$\boxed{x - y = 4}$



$x - y = 4 \Leftrightarrow$

$y = x - 4$

$x \geq 0 \wedge y \leq 0$: 4º quadrante

4b) $$Vol(R) = \int_0^4 \int_{x-4}^{0} 8 - 2x + 2y \, dy \, dx =$$

$$\int_0^4 8y - 2xy + y^2 \Big|_{y=x-4}^{y=0} dx = \int_0^4 0 - \left(8(x-4) - 2x(x-4) + (x-4)\right.$$

$$= \int_0^4 x^2 - 8x + 16 \, dx = \frac{x^3}{3} - 4x^2 + 16x \Big|_0^4 =$$

$$\frac{64}{3} - \cancel{64} + \cancel{64} - 0 = \frac{64}{3}$$

5) $z=\sqrt{x^2+y^2}$ cone, $x^2+y^2+z^2=1$ esfera

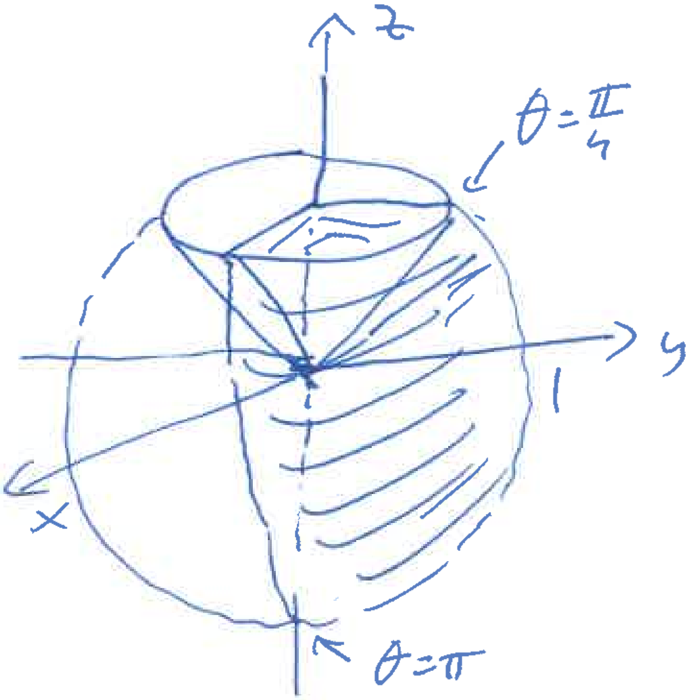
Intersecção: $z=\sqrt{x^2+y^2} \wedge x^2+y^2+z^2=1$

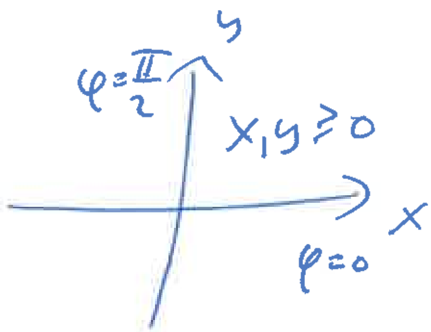
$x^2+y^2+z^2=1 \iff x^2=1-(x^2+y^2).$

$z=\sqrt{x^2+y^2} \iff z^2=x^2+y^2 \wedge z\geq 0.$

$$x^2+y^2=1-(x^2+y^2) \iff$$
$$2(x^2+y^2)=1 \iff$$
$$x^2+y^2=\frac{1}{2}. \qquad \left(z=\sqrt{\tfrac{1}{2}}=\frac{\sqrt{2}}{2}\right)$$

R:





Coord. esféricas: $x^2+y^2+z^2=r^2$, $|J|=r^2\operatorname{sen}\theta$

$$\begin{cases} 0\leq r\leq 1 \\ \frac{\pi}{4}\leq\theta\leq\pi \\ 0\leq\varphi\leq\frac{\pi}{2}\end{cases}$$

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(Cont de 5)

$$Vol(R) = \iiint_R dV =$$

$$\int_0^{\pi/2}\int_{\pi/4}^{\pi}\int_0^1 r^2 \operatorname{sen}\theta \, dr \, d\theta \, d\varphi = \int_0^{\pi/2}\int_{\pi/4}^{\pi} \frac{r^3}{3} \operatorname{sen}\theta \Big|_{r=0}^{r=1} d\theta \, d\varphi$$

$$= \int_0^{\pi/2}\int_{\pi/4}^{\pi} \frac{\operatorname{sen}\theta}{3} \, d\theta \, d\varphi = \int_0^{\pi/2} -\frac{\cos\theta}{3}\Big|_{\theta=\pi/4}^{\theta=\pi} d\varphi =$$

$$\int_0^{\pi/2} -\frac{\cos(\pi)}{3} + \frac{\cos(\pi/4)}{3} \, d\varphi = \int_0^{\pi/2} \frac{1}{3} + \frac{\sqrt{2}}{6} \, d\varphi =$$

$$\left(\frac{1}{3} + \frac{\sqrt{2}}{6}\right)\int_0^{\pi/2} d\varphi = \left(\frac{1}{3} + \frac{\sqrt{2}}{6}\right) \varphi \Big|_0^{\pi/2} = \left(\frac{1}{3} + \frac{\sqrt{2}}{6}\right)\frac{\pi}{2} =$$

$$\frac{\pi}{6} + \frac{\sqrt{2}\pi}{12}.$$