1) $\frac{4}{n^2-4} = \frac{4}{(n-2)(n+2)} \overset{(*)}{=} \frac{1}{n-2} - \frac{1}{n+2}$

(*) $\frac{4}{(n-2)(n+2)} = \frac{a}{n-2} + \frac{b}{n+2} \Leftrightarrow 4 = a(n+2)+b(n-2)$

$\Leftrightarrow 4 = (a+b)n + 2a - 2b \Leftrightarrow \begin{cases} a+b=0 \\ 2a-2b=4 \end{cases} \Leftrightarrow$

$\begin{cases} a=-b \\ 4a=4 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=-1 \end{cases}$

$\sum_{n=3}^{\infty} \frac{1}{n-2} - \frac{1}{n+2}$ é uma série de Mengoli

c/ $b_n = \frac{1}{n-2}$ e $t = 4$.

$\lim_{n\to\infty} b_n = 0$, logo a série de Mengoli converge.

Soma: $S = b_3 + b_4 + b_5 + b_6 - 4 \cdot 0 =$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$

2) $\sqrt{n+2} - \sqrt{n} = \left(\sqrt{n+2} - \sqrt{n}\right)\left(\frac{\sqrt{n+2}+\sqrt{n}}{\sqrt{n+2}+\sqrt{n}}\right)$

$= \frac{n+2-n}{\sqrt{n+2}+\sqrt{n}} = \frac{2}{\sqrt{n+2}+\sqrt{n}}$.

Série dos módulos: $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n+2}+\sqrt{n}}$ é

divergente; Pel. C. d. C. VI:

$\frac{2}{\sqrt{n+2}+\sqrt{n}} \geq \frac{2}{2\sqrt{n+2}} = \frac{1}{\sqrt{n+2}}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}} = \sum_{n=3}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=3}^{\infty} \frac{1}{n^{1/2}}$

é div. pq é uma série de Dirichlet

c/ $s = \frac{1}{2} \leq 1$.

Agora, vamos aplicar o Crit. de Leibniz

à série alternada:

Cont de 2)

i) $\lim_{n\to\infty} (\sqrt{n+2}-\sqrt{n}) = \lim_{n\to\infty} \frac{2}{\sqrt{n+2}+\sqrt{n}} = 0.$ ✓

ii) $\sqrt{n+3} - \sqrt{n+1} < \sqrt{n+2}-\sqrt{n} \quad \forall n \geq 1$

PQ $\frac{2}{\sqrt{n+3}+\sqrt{n+1}} < \frac{2}{\sqrt{n+2}+\sqrt{n}} \quad \forall n \geq 1.$

uma vez que $\sqrt{n+3} > \sqrt{n+2}$ e $\sqrt{n+1} > \sqrt{n}$. ✓

Conclusão: a série alternada é simplesmente convergente.

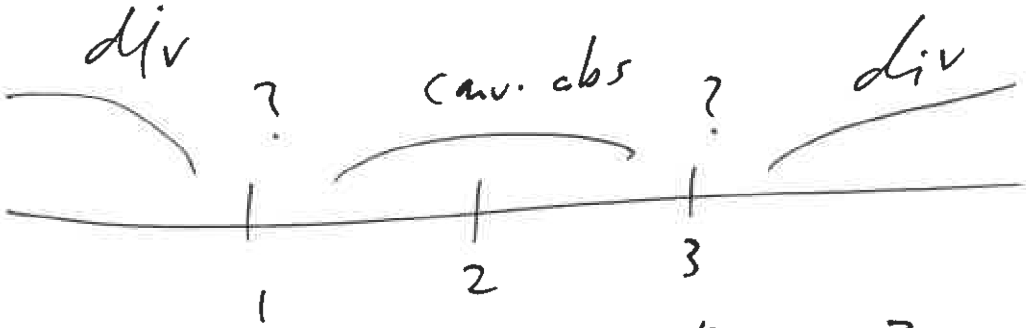
3) Pelo $\underset{3}{\underline{C}}\ \underline{d}\ \underset{n+1}{\underline{Q}}$:

$$\lim_{n\to\infty} \frac{\dfrac{(n+1)^3 |x-2|^{n+1}}{(n+1)^4+1}}{\dfrac{n^3 |x-2|^n}{n^4+1}} = \lim_{n\to\infty} \frac{(n+1)^3(n^4+1)}{n^3((n+1)^4+1)} |x-2|$$

$$= \lim_{n\to\infty} \left(\frac{n^7 + \cdots}{n^7+\cdots}\right) |x-2| = |x-2|.$$

$|x-2| < 1 \iff -1 < x-2 < 1 \iff$

$1 < x < 3$. $(c=2,\ R=1)$



$x=3$: $\displaystyle\sum_{n=0}^{\infty} \frac{n^3(3-2)^4}{n^4+1} = \sum_{n=0}^{\infty} \frac{n^3}{n^4+1}$ diverge.

Pelo C.d.Q. vz: $\displaystyle\lim_{n\to\infty} \frac{\frac{n^3}{n^4+1}}{\frac{1}{n}} =$

$\displaystyle\lim_{n\to\infty} \frac{n^4}{n^4+1} = 1$. Como $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverge (série harmónica), a série $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{n^4+1}$ também div.

(cont. 3) ⑤

$x = 1$: $\displaystyle\sum_{n=0}^{\infty} \frac{n^3(1-2)^n}{n^4+1} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{n^3}{n^4+1}$.

A série dos módulos diverge, como vimos para $x = 3$.

~~Em~~ Pelo Crit. de Leibniz, a série alternada converge simplesmente, pq

i) $\displaystyle\lim_{n\to\infty} \frac{n^3}{n^4+1} = 0$ ✓

ii) $\displaystyle\frac{(n+1)^3}{(n+1)^4+1} < \frac{n^3}{n^4+1}$ $\forall n \geq 2$

Seja $f(x) = \dfrac{x^3}{x^4+1}$. Então

$$f'(x) = \frac{3x^2(x^4+1) - x^3(4x^3)}{(x^4+1)^2} = \frac{-x^6+3x^2}{(x^4+1)^2}$$

$f'(x) < 0 \iff -x^6 + 3x^2 < 0 \iff$

$x^6 > 3x^2 \iff x^4 > 3 \underset{(x\geq 0)}{\iff} x > \sqrt[4]{3}$

Conclusão: $C = 2$, $R = 1$, $I_0 =]1,3[$, $I = [1,3[$.

4) a) $$\lim_{(x,y)\to(0,0)} \frac{x^4+2x^2y-7y^3}{x^2+y^2} \overset{(*)}{=} 0,$$ logo

f é contínua em (0,0).

(*) Por enquadramento:

$$0 \le \left|\frac{x^4+2x^2y-7y^3}{x^2+y^2}\right| \le \frac{|x|^4+2|x^2y|+7|y|^3}{(x^2+y^2)} =$$

$$\frac{x^4+2x^2|y|+7y^2\cdot|y|}{x^2+y^2} \le \frac{(x^2+y^2)^2+2(x^2+y^2)\cdot|y|+7(x^2+y^2)\cdot|y|}{x^2+y^2}$$

$$= x^2+y^2+2\cdot|y|+7\cdot|y| = x^2+y^2+9\cdot|y|.$$

Como $\lim_{(x,y)\to(0,0)} x^2+y^2+9\cdot|y| = 0$, conclui-se que

$$\lim_{(x,y)\to(0,0)} \frac{x^4+2x^2y-7y^3}{x^2+y^2} = 0$$ também.

4b) Seja $\vec{v} = (v_1, v_2) \in \mathbb{R}^2$ t.q. $v_1^2 + v_2^2 = 1$.

Então $f'_{\vec{v}}(0,0) = \lim_{t \to 0} \frac{f(tv_1, tv_2) - f(0,0)}{t} =$

$$\lim_{t \to 0} \frac{\frac{t^4 v_1^4 + 2t^3 v_1^2 v_2 - 7t^3 v_2^3}{t^2(v_1^2 + v_2^2)} - 0}{t} \underset{(v_1^2+v_2^2=1)}{=}$$

$$\lim_{t \to 0} \frac{t^4 v_1^4 + 2t^3 v_1^2 v_2 - 7t^3 v_2^3}{t^3} =$$

$$\lim_{t \to 0} t v_1^4 + 2v_1^2 v_2 - 7v_2^3 = 2v_1^2 v_2 - 7v_2^3.$$

4c) f não é diferenciável em $(0,0)$, pq

$\nabla f(0,0) \cdot (v_1, v_2) \neq f'_{\vec{v}}(0,0)$, uma

vez que $\nabla f(0,0) \cdot (v_1, v_2) = -v_2$ e

$f'_{\vec{v}}(0,0) = 2v_1^2 v_2 - 7v_2^3$.

5 a) $\nabla f(x,y) = (0,0) \Leftrightarrow (2xy - 4y, x^2 - 4x - 4y) = (0,0) \Leftrightarrow$

$\begin{cases} 2xy - 4y = 0 \\ x^2 - 4x - 4y = 0 \end{cases} \Leftrightarrow \begin{cases} 2y(x-2) = 0 \\ x^2 - 4x - 4y = 0 \end{cases} \Leftrightarrow$

$\begin{cases} x = 2 \vee y = 0 \\ x^2 - 4x - 4y = 0 \end{cases}$

$\underline{x=2}$: $x^2 - 4x - 4y = 0 \Leftrightarrow 4 - 8 - 4y = 0 \Leftrightarrow 4y = -4 \Leftrightarrow y = -1$.

$\underline{y=0}$: $x^2 - 4x - 4y = 0 \Leftrightarrow x^2 - 4x = 0 \Leftrightarrow x(x-4) = 0 \Leftrightarrow x = 0 \vee x = 4$

Pts. estacionários: $(2,-1)$; $(0,0)$; $(4,0)$

5 b) $H_f(x,y) = \begin{pmatrix} 2y & 2x-4 \\ 2x-4 & -4 \end{pmatrix}$

$(x,y) = (2,-1)$: $H_f(2,-1) = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix}$ $h_f(2,-1) = 8 >$; $f''_{xx}(2,-1) = -2 <$

Logo, f tem um máx. loc. em $(2,-1)$.

$(x,y) = (0,0)$: $H_f(0,0) = \begin{pmatrix} 0 & -4 \\ -4 & -4 \end{pmatrix}$ $h_f(0,0) = -16$

Logo, f tem um pt. de sela em $(0,0)$

Cont. de 5 b)

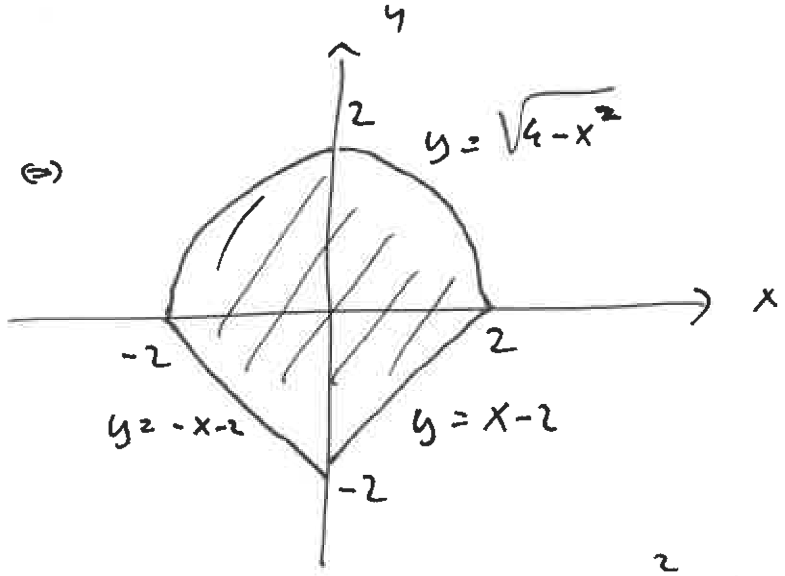
$$H_f(4,0) = \begin{pmatrix} 0 & 4 \\ 4 & -4 \end{pmatrix} \quad h_f(4,0) = -16 < 0$$

Logo, f tem um pt. de sela em $(4,0)$.

6 a)

$$D = \{(x,y) \in \mathbb{R}^2 : -2 \leq x \leq 0,\ -x-2 \leq y \leq \sqrt{4-x^2}\}$$
$$\cup \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2,\ x-2 \leq y \leq \sqrt{4-x^2}\}.$$

$y = \sqrt{4-x^2} \Leftrightarrow$
$y^2 = 4 - x^2 \wedge y \geq 0 \Leftrightarrow$
$x^2+y^2 = 4 \wedge y \geq 0$
semi-círculo positivo.



6 b)

$$\begin{cases} y = \sqrt{4-x^2} \Leftrightarrow x^2+y^2 = 4 \Leftrightarrow x^2 = 4-y^2 \Leftrightarrow x = \pm\sqrt{4-y^2}. \\ y = -x-2 \Leftrightarrow x = -y-2 \\ y = x-2 \Leftrightarrow x = y+2 \end{cases}$$

$$\int_{-2}^{0}\int_{-y-2}^{y+2} y \, dx\, dy + \int_{0}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} y \, dx\, dy$$

6 c) $I = \int_{-2}^{0}\int_{-x-2}^{\sqrt{4-x^2}} y \, dy \, dx + \int_{0}^{2}\int_{x-2}^{\sqrt{4-x^2}} y \, dy \, dx \underset{\text{simetria}}{\overset{\text{usando a}}{=}}$

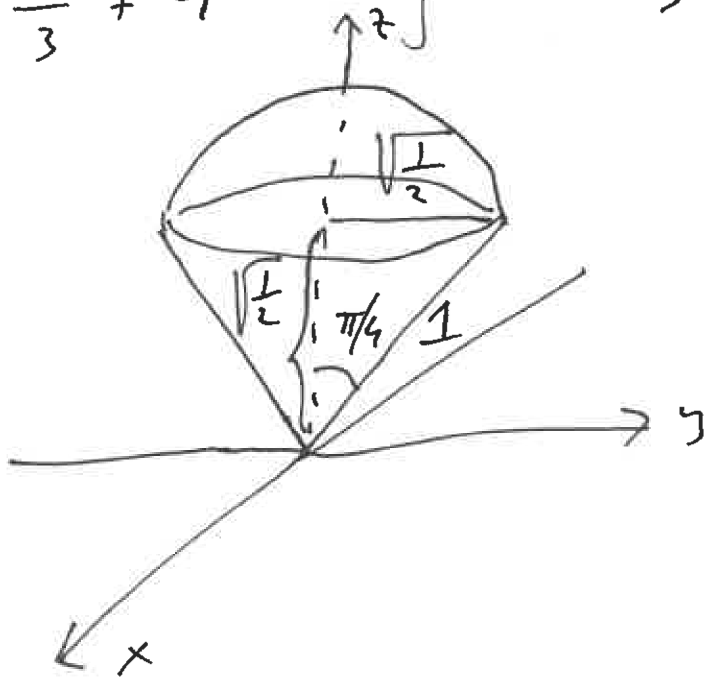
$$2\int_0^2\int_{x-2}^{\sqrt{4-x^2}} y\,dy\,dx = 2\int_0^2 \left[\frac{y^2}{2}\right]_{y=x-2}^{y=\sqrt{4-x^2}} dx =$$

$$2\int_0^2 \frac{4-x^2}{2} - \frac{(x-2)^2}{2}\,dx = 2\int_0^2 \cancel{2} - \frac{x^2}{2} - \frac{x^2}{2} + 2x - \cancel{2}\,dx.$$

$$2\int_0^2 -x^2+2x\,dx = 2\left[-\frac{x^3}{3}+x^2\right]_0^2 =$$

$$2\left[-\frac{8}{3}+4-0\right] = 2\cdot\frac{4}{3} = \frac{8}{3}.$$

7 a)



$$\sqrt{x^2+y^2} = \sqrt{1-x^2-y^2} \Leftrightarrow$$
$$x^2+y^2 = 1-(x^2+y^2) \Leftrightarrow$$
$$2(x^2+y^2) = 1 \Leftrightarrow$$
$$x^2+y^2 = \frac{1}{2}.$$

$$\begin{cases} z = \sqrt{1-x^2-y^2} \Leftrightarrow z^2 = 1-x^2-y^2 \wedge z \geq 0 \Leftrightarrow x^2+y^2+z^2 = 1 \wedge z \geq 0 \\ tg\left(\frac{\pi}{4}\right) = \frac{\sqrt{\frac{1}{2}}}{\sqrt{\frac{1}{2}}} = 1. \end{cases}$$

esfera de raio 1.

(cont de $7^{a)}$) Em coord. esféricas:

$x = r\cos\varphi \,\mathrm{sen}\,\theta$, $y = r\,\mathrm{sen}\,\varphi\,\mathrm{sen}\,\theta$, $z = r\cos\theta$

$0 \le r \le 1$, $0 \le \varphi \le 2\pi$, $0 \le \theta \le \frac{\pi}{4}$.

$7^{b)}$ $$\iiint_S 2z\,dV = \int_0^{\pi/4}\int_0^{2\pi}\int_0^1 2r\cos\theta \cdot r^2 \,\mathrm{sen}\,\theta \,dr\,d\varphi\,d\theta$$

$$= 2\int_0^{\pi/4}\int_0^{2\pi}\int_0^1 r^3 \cos\theta\,\mathrm{sen}\,\theta\,dr\,d\varphi\,d\theta =$$

$$2\int_0^{\pi/4}\int_0^{2\pi}\left[\frac{r^4}{4}\cos\theta\,\mathrm{sen}\,\theta\right]_{r=0}^{r=1} d\varphi\,d\theta =$$

$$2\int_0^{\pi/4}\int_0^{2\pi}\frac{1}{4}\cos\theta\,\mathrm{sen}\,\theta - 0\,d\varphi\,d\theta =$$

$$\frac{1}{2}\int_0^{\pi/4}\left[(\cos\theta\,\mathrm{sen}\,\theta)\cdot\varphi\right]_{\varphi=0}^{\varphi=2\pi} d\theta =$$

$$\frac{1}{2}\int_0^{\pi/4}(\cos\theta\,\mathrm{sen}\,\theta)\cdot 2\pi\,d\theta =$$

$$\pi\int_0^{\pi/4}\cos\theta\,\mathrm{sen}\,\theta\,d\theta =$$

$$\pi\left[\frac{\operatorname{sen}^2\theta}{2}\right]_0^{\pi/4} = \pi\left[\frac{\left(\sqrt{\frac{1}{2}}\right)^2}{2} - \frac{0^2}{2}\right] =$$

$$= \frac{\pi}{4}.$$