F2 AM II LEI + BE (1)

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1) $f(x) = 7x^2 - 6x + 1 \qquad c = 2$

$$T_f(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2}(x-2)^2$$

$$f(2) = 28 - 12 + 1 = 17$$

$$f'(2) = 14x - 6\Big|_{x=2} = 28 - 6 = 22$$

$$f''(2) = 14.$$

$$\boxed{T_f(x) = 17 + 22(x-2) + 7(x-2)^2}$$

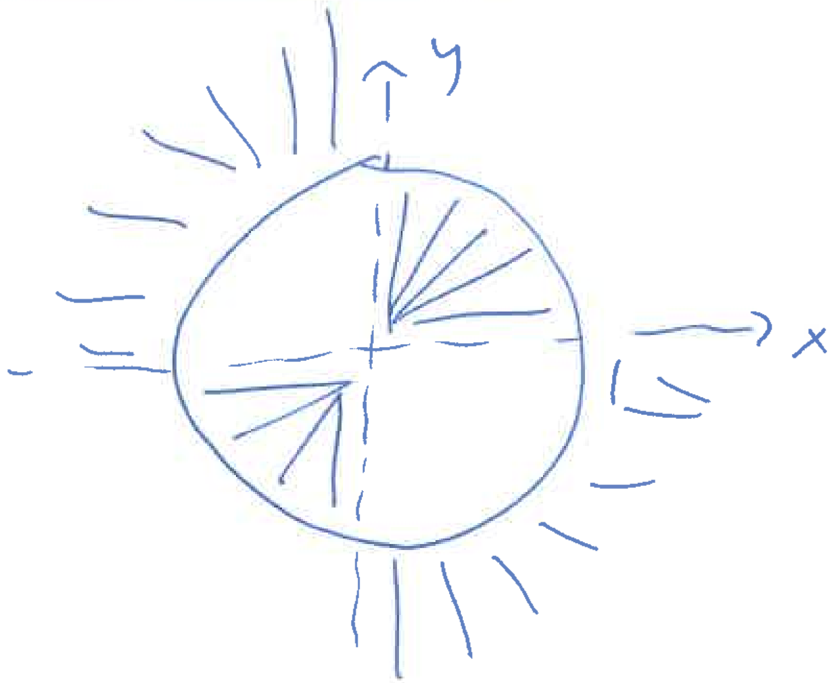
2) $D_f = \left\{ (x,y) \in \mathbb{R}^2 : \dfrac{1-x^2-y^2}{xy} \geq 0 \wedge xy \neq 0 \right\}$

$= \{ (x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1 \wedge xy > 0 \} \cup$
$\{ (x,y) \in \mathbb{R}^2 : x^2+y^2 \geq 1 \wedge xy < 0 \}.$

$= \{ (x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1 \wedge x > 0 \wedge y > 0 \} \cup$
$\{ (x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1 \wedge x < 0 \wedge y < 0 \} \cup$
$\{ (x,y) \in \mathbb{R}^2 : x^2+y^2 \geq 1 \wedge x > 0 \wedge y < 0 \} \cup$
$\{ (x,y) \in \mathbb{R}^2 : x^2+y^2 \geq 1 \wedge x < 0 \wedge y > 0 \}.$

Cont. de 2)

D_f:



3ª) f não é cont. em $(0,0)$, pq

$$\lim_{(x,y)\to(0,0)} \frac{x^5-3xy+5y^4}{x^2+y^2} \neq 0. \quad (*)$$

$$(*) \lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{x^5-3xy+5y^4}{x^2+y^2} = \lim_{x\to 0} \frac{x^5-3x^2+5x^4}{2x^2} =$$

$$\lim_{x\to 0} \frac{x^3}{2} - \frac{3}{2} + \frac{5x^2}{2} = 0 - \frac{3}{2} + 0 = -\frac{3}{2} \neq 0.$$

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3 b) $f'_x(0,0) = \lim_{t \to 0} \dfrac{f(t,0) - f(0,0)}{t} =$

$\lim_{t\to 0} \dfrac{\dfrac{t^5 - 0 + 0}{t^2 + 0} - 0}{t} = \lim_{t \to 0} t^2 = 0$

$f'_y(0,0) = \lim_{\cancel{t} \to 0} \dfrac{f(0,t) - f(0,0)}{t} =$

$\lim_{t\to 0} \dfrac{\dfrac{0 - 0 + 5t^4}{0 + t^2} - 0}{t} = \lim_{t \to 0} 5t = 0.$

$$\boxed{f'_x(0,0) = f'_y(0,0) = 0}$$

4 a) $F(x,y,z) = 3x^2 - 2xz^2 + 4x^2y - 5y^3z.$

$S: \ F(x,y,z) = 0,$

<u>Teoria</u>: $\nabla F(1,1,1) \perp T_S(1,1,1).$

$\nabla F(1,1,1) = \left(6x - 2z^2 + 8xy,\ 4x^2 - 15y^2z,\ -4xz - 5y\right.$ ... $\Big|_{(1,1}$ ...

$= (12, -11, -9)$

Cont. de 4 a) Podemos tomar $\vec{v} = (12, -11, -9)$.

4 b) $\vec{w} \parallel T_S(1,1,1) \Leftrightarrow \vec{w} \perp \vec{v} \Leftrightarrow \vec{w} \cdot \vec{v} = 0$

$\vec{w} = (w_1, w_2, w_3)$.

$(w_1, w_2, w_3) \cdot (12, -11, -9) = 0 \Leftrightarrow$

$12w_1 - 11w_2 - 9w_3 = 0$

Podemos tomar $\vec{w} = (11, 12, 0)$.

5 a) $\nabla f(x,y) = (0,0) \Leftrightarrow$

$(-3x^2 + 2y, -2y + 2x) = (0,0) \Leftrightarrow$

$\begin{cases} -3x^2 + 2y = 0 \\ -2y + 2x = 0 \end{cases} \Leftrightarrow \begin{cases} -3x^2 + 2x = 0 \\ x = y \end{cases} \Leftrightarrow \begin{cases} x(-3x+2) = \\ x = y \end{cases}$

$\Leftrightarrow \begin{cases} x = 0 \lor x = \frac{2}{3} \\ x = y \end{cases}$ Pts. estac.: $(0,0)$, $(\frac{2}{3}, \frac{2}{3})$

5 b) $$H_f(x,y) = \begin{pmatrix} -6x & 2 \\ 2 & -2 \end{pmatrix}$$

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$$H_f(0,0) = \begin{pmatrix} 0 & 2 \\ 2 & -2 \end{pmatrix} \qquad h_f(0,0) = -4 < 0$$

pt. de sela

$$H_f\left(\frac{2}{3}, \frac{2}{3}\right) = \begin{pmatrix} -4 & 2 \\ 2 & -2 \end{pmatrix} \qquad h_f\left(\frac{2}{3}, \frac{2}{3}\right) = 8-4 = 4 > 0$$

$$f''_{xx}\left(\frac{2}{3}, \frac{2}{3}\right) = -4 < 0$$

f tem um máx. loc. em $\left(\frac{2}{3}, \frac{2}{3}\right)$.