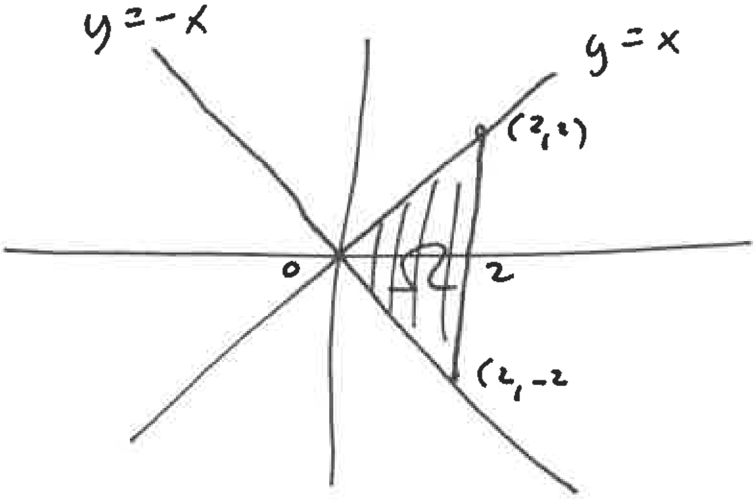
Teste Modelo 3 LEI + BE
AM II

1a) $\Omega = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \wedge x^2 - y^2 \geq 0\}$.



$x^2 - y^2 = 0 \Leftrightarrow$

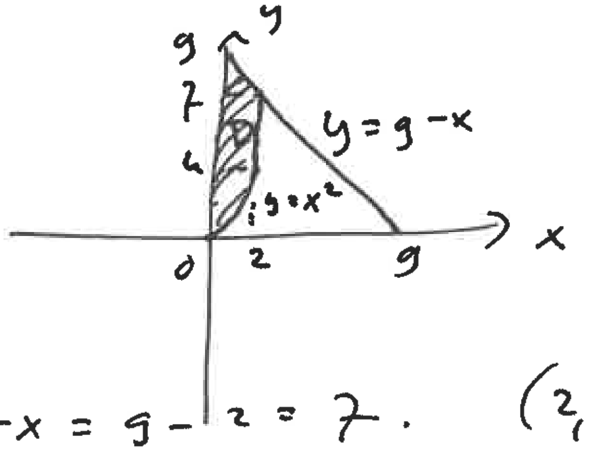
$x^2 = y^2 \Leftrightarrow$

$x = \pm y$

1b) $$\iint_{\Omega} \frac{y}{x^2+16}\, dy\, dx = \int_0^2 \int_{-x}^{x} \frac{y}{x^2+16}\, dy\, dx =$$

$$\frac{1}{2} \int_0^2 \left[\frac{y^2}{x^2+16} \right]_{y=-x}^{y=x} dx = \frac{1}{2} \int_0^2 \frac{x^2}{x^2+16} - \frac{(-x)^2}{x^2+16}\, dx = 0.$$

2) $D = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \wedge x^2 \leq y \leq 9 - x\}$.



$x = 2 : y = 9 - x = 9 - 2 = 7.$ $\quad (2,7)$

$x = 2 : y = x^2 = 4$ $\quad (2,4)$

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Cont. de 2) $y = 9 - x \Leftrightarrow x = 9 - y$

$y = x^2 \Leftrightarrow x = \sqrt{y} \quad (x \geq 0)$

$$D = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq \sqrt{y} \wedge 0 \leq y \leq 2\} \cup$$
$$\{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \wedge 4 \leq y \leq 7\} \cup$$
$$\{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 9-y \wedge 7 \leq y \leq 9\}.$$

$$\int_0^2 \int_{x^2}^{9-x} f(x,y)\, dy\, dx =$$

$$\int_0^4 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy + \int_4^7 \int_0^2 f(x,y)\, dx\, dy + \int_7^9 \int_0^{9-y} f(x,y)\, dx\, dy.$$

Teste Modelo 3 AR II (3)

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3 a) $z = 4 - x - y$ (plano)

$x = y = 0: \ z = 4 - 0 - 0 = 4 : \ (0,0,4)$

$y = z = 0: \ 0 = 4 - x - 0 \Leftrightarrow x = 4 : \ (4,0,0)$

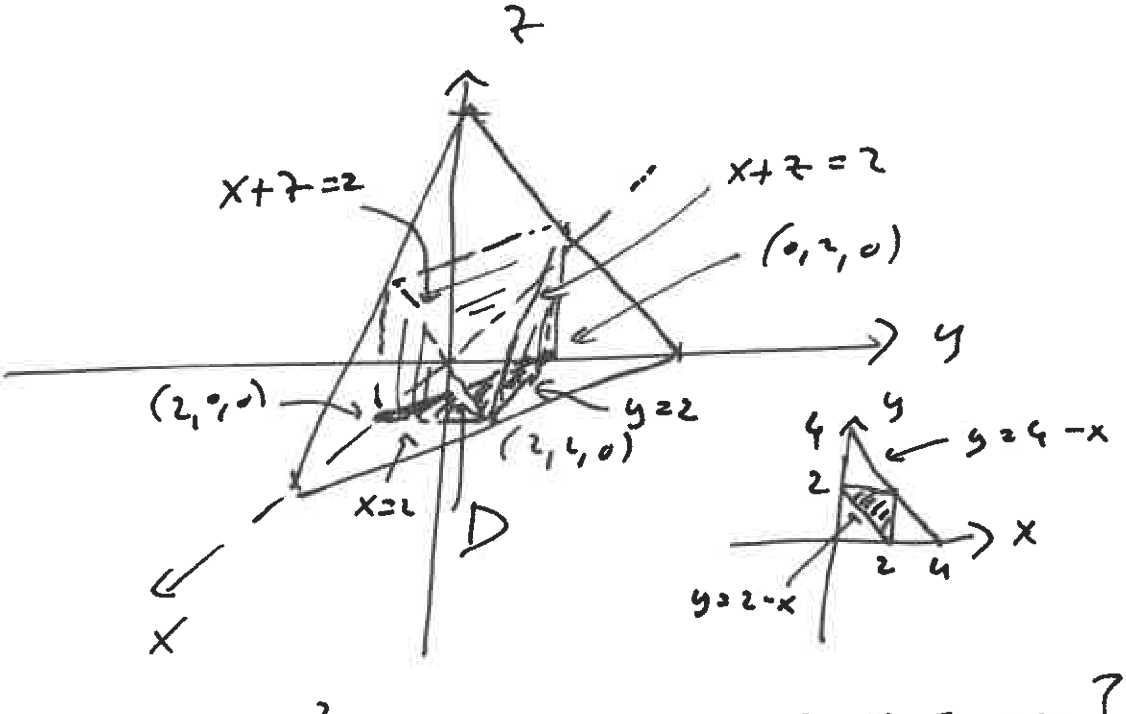
$x = z = 0: \ 0 = 4 - 0 - y \Leftrightarrow y = 4 : \ (0,4,0)$

$z = 0: \ z = 4 - x - y \Leftrightarrow 0 = 4 - x - y \Leftrightarrow x + y = 4$

$x = 2: \ z = 4 - x - y \Leftrightarrow z = 4 - 2 - y \Leftrightarrow y + z = 2$

$y = 2: \ z = 4 - x - y \Leftrightarrow z = 4 - x - 2 \Leftrightarrow x + z = 2$

$x = 2 \wedge x + y = 4 \Leftrightarrow y = 2 : \ (2,2,0)$



$$D = \{(x,y) \in \mathbb{R}^2 : \ 0 \le x \le 2 \ \wedge \ 2 - x \le y \le 2\}.$$

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3 b) $$\mathrm{Vol}(R) = \int_0^2 \int_{2-x}^2 \int_0^{4-x-y} dz\, dy\, dx = \int_0^2 \int_{2-x}^2 [z]_{z=0}^{z=4-x-y}\, dy\, dx$$

$$\int_0^2 \int_{2-x}^2 4-x-y \, dy\, dx = \int_0^2 \left[4y - xy - \frac{y^2}{2}\right]_{y=2-x}^{y=2} dx =$$

$$\int_0^2 -\frac{x^2}{2} + 2x \, dx = \left[-\frac{x^3}{6} + x^2\right]_0^2 = \frac{8}{3}$$

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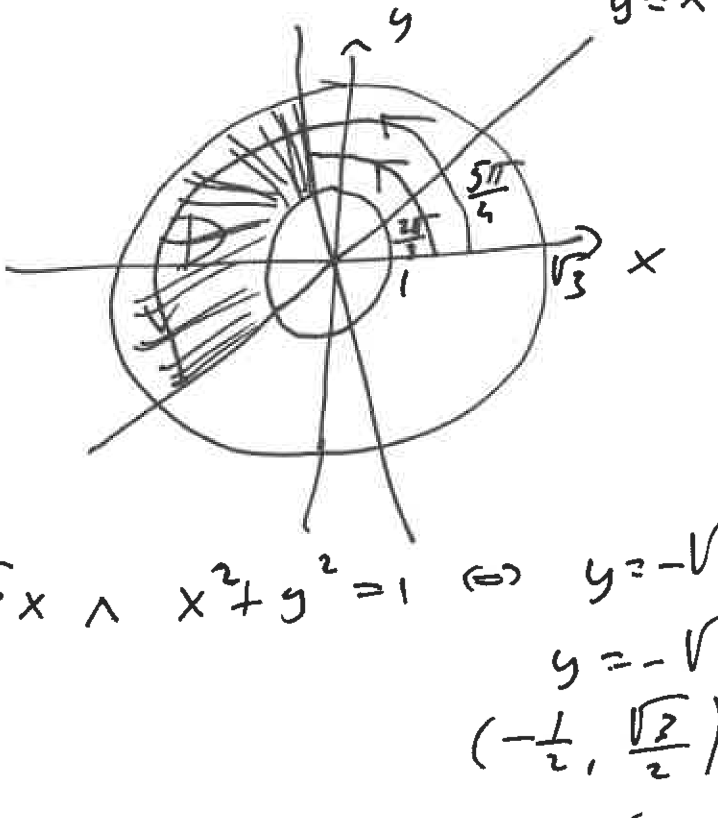
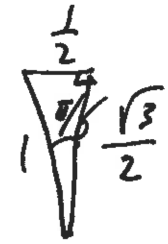
4)





$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$y = -\sqrt{3}x \wedge x^2 + y^2 = 1 \Leftrightarrow y = -\sqrt{3}x \wedge 4x^2 = 1 \Leftrightarrow$

$y = -\sqrt{3}x \wedge x = -\frac{1}{2} \ (x \leq 0)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right).$

$y = -\sqrt{3}x \wedge x^2 + y^2 = 3 \Leftrightarrow (x, y) = \left(-\frac{\sqrt{3}}{2}, \frac{3}{2}\right).$

Em coord. pol: $\{(r, \theta) \in \mathbb{R}^2 : 1 \leq r \leq \sqrt{3}, \frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{4}$

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4 b)

$$\iint_D \frac{x+y}{\sqrt{x^2+y^2}}\, dA = \int_{\frac{2\pi}{3}}^{\frac{5\pi}{4}} \int_1^{\sqrt{3}} \frac{r\cos\theta + r\,\text{sen}\,\theta}{\cancel{r}}\, \cancel{r}\, dr\, d\theta =$$

$$\int_{\frac{2\pi}{3}}^{\frac{5\pi}{4}} \int_1^{\sqrt{3}} r(\cos\theta + \text{sen}\,\theta)\, dr\, d\theta = \int_{\frac{2\pi}{3}}^{\frac{5\pi}{4}} \left[\frac{r^2}{2}(\cos\theta + \text{sen}\,\theta)\right]_{r=1}^{r=\sqrt{3}} d\theta$$

$$\int_{\frac{2\pi}{3}}^{\frac{5\pi}{4}} \frac{3}{2}(\cos\theta + \text{sen}\,\theta) - \frac{1}{2}(\cos\theta + \text{sen}\,\theta)\, d\theta =$$

$$\int_{\frac{2\pi}{3}}^{\frac{5\pi}{4}} \cos\theta + \text{sen}\,\theta\, d\theta = \left[\text{sen}\,\theta - \cos\theta\right]_{\frac{2\pi}{3}}^{\frac{5\pi}{4}} =$$

$$\text{sen}\left(\frac{5\pi}{4}\right) - \cos\left(\frac{5\pi}{4}\right) - \left(\text{sen}\left(\frac{2\pi}{3}\right) - \cos\left(\frac{2\pi}{3}\right)\right) =$$

$$-\cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}} - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = -\left(\frac{1+\sqrt{3}}{2}\right).$$

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(6)

5 a) $x \leq 0, y \geq 0$: 2º quadrante no plano xy

Portanto $\frac{\pi}{2} \leq \varphi \leq \pi$ em coord. esféricas.

$z = \sqrt{\frac{x^2+y^2}{3}}$ meio cone

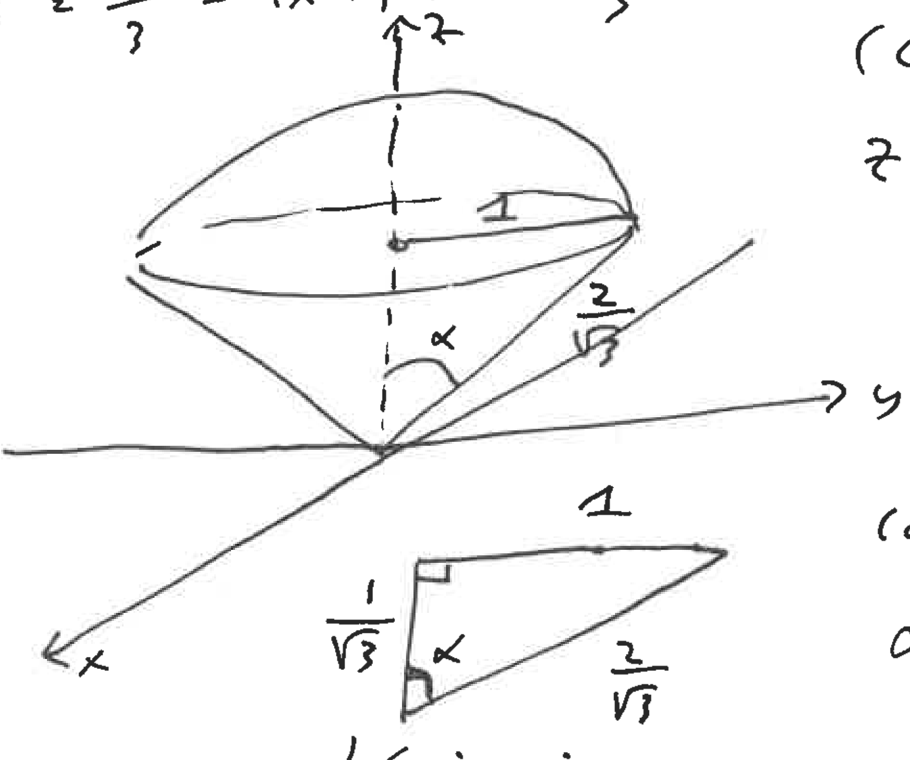
$z = \sqrt{\frac{4}{3} - x^2 - y^2}$ meia esfera de raio $\sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

Interseção: $\sqrt{\frac{x^2+y^2}{3}} = \sqrt{\frac{4}{3} - (x^2+y^2)} \Leftrightarrow$

$\frac{x^2+y^2}{3} = \frac{4}{3} - (x^2+y^2) \Leftrightarrow \frac{4}{3}(x^2+y^2) = \frac{4}{3} \Leftrightarrow (x^2+y^2) = 1$

(círculo de raio 1)

$z = \sqrt{\frac{1}{3}}$ altura desse círculo.



$\cos(\alpha) = \frac{1}{2} \Leftrightarrow$

$\alpha = \frac{\pi}{3}$.

Em coord. esféricas:

$\{(r, \varphi, \theta) \in \mathbb{R}^3 : 0 \leq r \leq \frac{2}{\sqrt{3}}, \frac{\pi}{2} \leq \varphi \leq \pi, 0 \leq \theta \leq \frac{\pi}{3}\}$

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5 b) $\mathrm{Vol}(R) = \iiint_R dV = \int_{\frac{\pi}{2}}^{\pi} \int_0^{\pi/3} \int_0^{\frac{2}{\sqrt{3}}} r^2 \operatorname{sen}\theta \, dr \, d\theta \, d\varphi =$

$$\int_{\frac{\pi}{2}}^{\pi} \int_0^{\frac{\pi}{3}} \left[\frac{r^3}{3} \operatorname{sen}\theta\right]_{r=0}^{r=\frac{2}{\sqrt{3}}} d\theta \, d\varphi =$$

$$\int_{\frac{\pi}{2}}^{\pi} \int_0^{\frac{\pi}{3}} \frac{8}{9\sqrt{3}} \operatorname{sen}\theta \, d\theta \, d\varphi = \frac{8}{9\sqrt{3}} \int_{\frac{\pi}{2}}^{\pi} \left[-\cos\theta\right]_{\theta=0}^{\theta=\frac{\pi}{3}} d\varphi$$

$$= \frac{8}{9\sqrt{3}} \int_{\frac{\pi}{2}}^{\pi} -\cos\left(\frac{\pi}{3}\right) + \cos(0) \, d\varphi = \frac{8}{9\sqrt{3}} \int_{\frac{\pi}{2}}^{\pi} -\frac{1}{2} + 1 \, d\varphi$$

$$= \frac{8}{9\sqrt{3}} \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} \, d\varphi = \frac{4}{9\sqrt{3}} \int_{\frac{\pi}{2}}^{\pi} d\varphi = \frac{4}{9\sqrt{3}} \left[\varphi\right]_{\frac{\pi}{2}}^{\pi} =$$

$$\frac{4}{9\sqrt{3}} \left(\pi - \frac{\pi}{2}\right) = \frac{4}{9\sqrt{3}} \cdot \frac{\pi}{2} = \frac{2\pi}{9\sqrt{3}}.$$