1) $\dfrac{a_{n+1}}{a_n} = \dfrac{3^{n+1}/5^{n+2}}{3^n/5^{n+1}} = \dfrac{3}{5}.$

A série é geométrica com razão $r = \frac{3}{5}$.

Como $r \in]-1,1[$, a série converge.

A soma: $S = \dfrac{a_2}{1-r} = \dfrac{3^2/5^3}{1-3/5} = \dfrac{3^2/5^3}{2/5} =$

$3^2/2\cdot 5^2 = \dfrac{9}{50}.$

2) A série dos módulos: $\displaystyle\sum_{n=4}^{\infty} \frac{1}{\sqrt{n^3+2n^2+n+5}}$

C.d.C. vi: $\dfrac{1}{\sqrt{n^3+2n^2+n+5}} \leq \dfrac{1}{\sqrt{n^3}} \quad \forall n \geq 1$

$\displaystyle\sum_{n=4}^{\infty} \frac{1}{\sqrt{n^3}} = \sum_{n=4}^{\infty} \frac{1}{n^{3/2}}$ converge (S. de Dirichlet c/ $s = 3/2 > 1$)

Portanto: $\displaystyle\sum_{n=4}^{\infty} \frac{1}{\sqrt{n^3+2n^2+n+5}}$ converge.

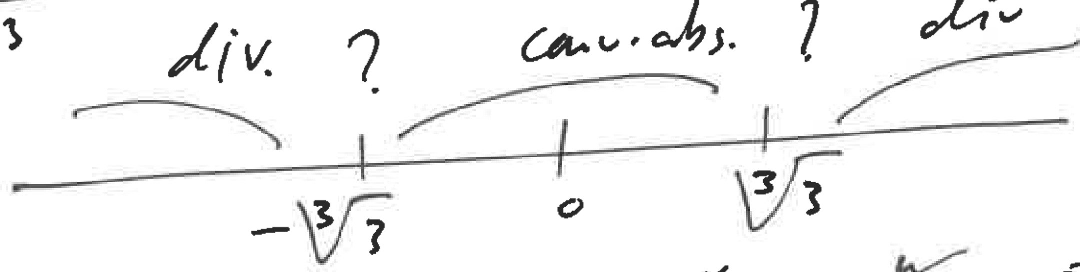
Conclusão: $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^3+2n^2+n+5}}$ conv. abs.

3) $c=0$.

C.d.R: $\lim_{n\to\infty} \sqrt[n]{\dfrac{|x|^{3n}}{n\,3^n}} = \lim_{n\to\infty} \dfrac{|x|^3}{\sqrt[n]{n}\cdot 3} =$

$\dfrac{|x|^3}{3}$.

$\dfrac{|x|^3}{3} = 1 \iff |x|^3 = 3 \iff |x| = \sqrt[3]{3}$.



$x = \sqrt[3]{3}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(\sqrt[3]{3})^{3n}}{n\,3^n} = \sum_{n=1}^{\infty} \frac{\cancel{3^n}}{n\,\cancel{3^n}} = \sum_{n=1}^{\infty} \frac{1}{n}$

diverge (s. harmónica).

$x = -\sqrt[3]{3}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-\sqrt[3]{3})^{3n}}{n\,3^n} = \sum_{n=1}^{\infty} \frac{(-1)^{3n}\,\cancel{3^n}}{n\,\cancel{3^n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

converge simplemente (s. harmónica alternada).

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4) a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^3+5y^3}{x^2+y^2} = 0$, por enquadramento:

$$0 \leq \left| \frac{2x^3+5y^3}{x^2+y^2} \right| \leq \frac{2|x|^3+5|y|^3}{x^2+y^2} = \frac{2x^2|x|+5|y|\cdot y^2}{x^2+y^2} \leq$$

$$\frac{2\cancel{(x^2+y^2)}\cdot|x|+5\cancel{(x^2+y^2)}\cdot|y|}{\cancel{x^2+y^2}} = 2|x|+5|y|.$$

b) $f'_x(0,0) = \lim\limits_{t\to0} \dfrac{f(t,0)-f(0,0)}{t} = \lim\limits_{t\to0} \dfrac{\frac{2t^3+0}{t^2+0}-0}{t} =$

$\lim\limits_{t\to0} \dfrac{2t^3}{t^3} = 2$

$f'_y(0,0) = \lim\limits_{t\to0} \dfrac{f(0,t)-f(0,0)}{t} = \lim\limits_{t\to0} \dfrac{\frac{0+5t^3}{0+t^2}-0}{t} =$

$\lim\limits_{t\to0} \dfrac{5t^3}{t^3} = 5$.

c) f <u>não</u> é dif. em $(0,0)$, pq

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)-\left(f'_x(0,0)x+f'_y(0,0)y\right)}{\sqrt{x^2+y^2}} \neq 0 \quad (*)$$

Cont. de 4 c) (*)

$$\lim_{(x,y)\to(0,0)} \frac{\frac{2x^3+5y^3}{x^2+y^2} - 0 - (2x+5y)}{\sqrt{x^2+y^2}} =$$

$$\lim_{(x,y)\to(0,0)} \frac{2x^3+5y^3-(x^2+y^2)(2x+5y)}{(x^2+y^2)\sqrt{x^2+y^2}} =$$

$$\lim_{(x,y)\to(0,0)} \frac{\cancel{2x^3}+\cancel{5y^3}-\cancel{2x^3}-5x^2y-2xy^2-\cancel{5y^3}}{(x^2+y^2)^{3/2}} =$$

$$\lim_{(x,y)\to(0,0)} \frac{-5x^2y-2xy^2}{(x^2+y^2)^{3/2}}$$ não existe.

$y=kx$: $$\lim_{x\to 0} \frac{-5kx^3-2k^2x^3}{((1+k^2)x^2)^{3/2}} =$$

$$-\lim_{x\to 0} \frac{(5k+2k^2)x^3}{(1+k^2)^{3/2}\cdot|x|^3}$$ não existe, pq

$$\lim_{x\to 0^+} \frac{x^3}{|x|^3} = 1 \neq -1 = \lim_{x\to 0^-} \frac{x^3}{|x|^3}.$$

5 a) $\nabla f(x,y) = (0,0) \Leftrightarrow$

$(y(5x+y-15) + 5xy \,,\, x(5x+y-15) + xy) = (0,0) \Leftrightarrow$

$(y(10x+y-15) \,,\, x(5x+2y-15)) = (0,0) \Leftrightarrow$

$\begin{cases} y(10x+y-15) = 0 \\ x(5x+2y-15) = 0 \end{cases} \Leftrightarrow \begin{cases} y=0 \ \vee \ 10x+y-15 = 0 \\ x=0 \ \vee \ 5x+2y-15 = 0 \end{cases}$

$\underline{x=0}$: $y=0 \ \vee \ 0+y-15 = 0 \Leftrightarrow y=0 \ \vee \ y=15$

$\underline{y=0}$: $x=0 \ \vee \ 5x+0-15 = 0 \Leftrightarrow x=0 \ \vee \ x=3$

~~Pts. críticos: (0,0), (0,15), (3,0).~~

$\begin{cases} 10x+y-15 = 0 \\ 5x+2y-15 = 0 \end{cases} \Leftrightarrow \begin{cases} 10x+y = 15 \\ 5x+2y = 15 \end{cases} \Leftrightarrow \begin{cases} y = 15-10x \\ 5x+2(15-10x)=1 \end{cases}$

$\Leftrightarrow \begin{cases} y = 15-10x \\ -15x = -15 \end{cases} \Leftrightarrow \begin{cases} y = 5 \\ x = 1 \end{cases}$

Pts. críticos $(0,0), (0,15), (3,0)\ (1,5)$.

5 b)

$$H_f(x,y) = \begin{pmatrix} 10y & \cancel{}\ 10x+2y-15 \\ 10x+2y-15 & 2x \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} 0 & -15 \\ -15 & 0 \end{pmatrix} \qquad h_f(0,0) = -(-15)^2 = -225 < 0$$

pt. de sela.

$$H_f(0,15) = \begin{pmatrix} 150 & 15 \\ 15 & 0 \end{pmatrix} \qquad h_f(0,15) = -(15)^2 = -225 < 0$$

pt. de sela

$$H_f(3,0) = \begin{pmatrix} 0 & 15 \\ 15 & 6 \end{pmatrix} \qquad h_f(3,0) = -(15)^2 = -225 < 0$$

pt. de sela.

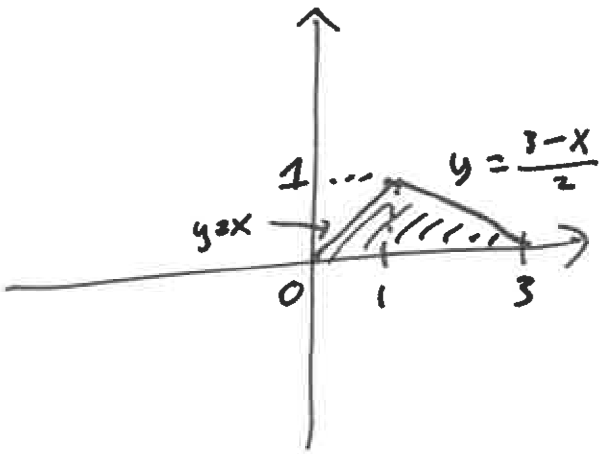
$$H_f(1,5) = \begin{pmatrix} 50 & 5 \\ 5 & 2 \end{pmatrix} \qquad h_f(1,5) = 100 - 25 = 75 > 0$$

$$f''_{xx}(1,5) = 50 > 0$$

mín. local.

6 a) $D = \{(x,y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \wedge 0 \leq y \leq x\} \cup$

$\{(x,y) \in \mathbb{R}^2 \mid 1 \leq x \leq 3 \wedge 0 \leq y \leq \frac{3-x}{2}\}$



$\frac{3-x}{2} = x \Leftrightarrow$

$\frac{3}{2} - \frac{x}{2} = x \Leftrightarrow$

$\frac{3}{2} x = \frac{3}{2} \Leftrightarrow$

$x = 1$

6 b) $y = x \Leftrightarrow x = y$

$y = \frac{3-x}{2} \Leftrightarrow y = \frac{3}{2} - \frac{x}{2} \Leftrightarrow \frac{x}{2} = \frac{3}{2} - y \Leftrightarrow x = 3-2y$

$$I = \int_0^1 \int_y^{3-2y} xy \, dx \, dy$$

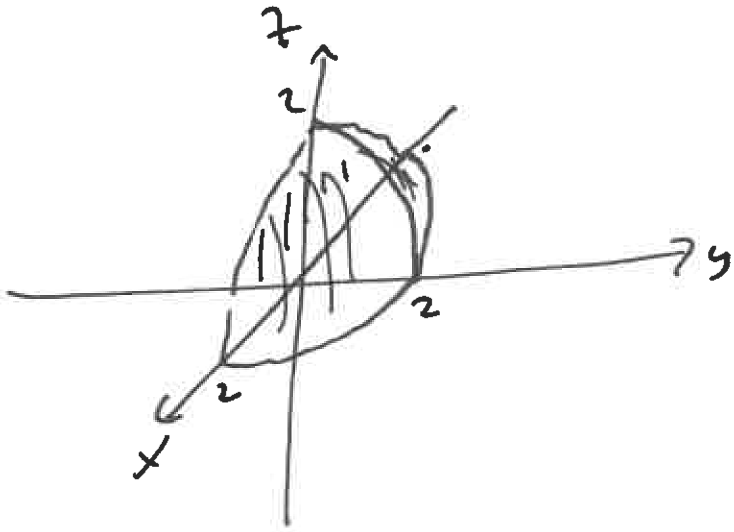
6 c) $$I = \int_0^1 \int_y^{3-2y} xy \, dx \, dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=y}^{x=3-2y} dy$$

$$\int_0^1 \frac{(3-2y)^2 y}{2} - \frac{y^3}{2} dy = \int_0^1 \frac{9y - 12y^2 + 4y^3 - y^3}{2} d$$

$$= \int_0^1 \frac{3y^3 - 12y^2 + 9y}{2} dy = \left[\frac{3y^4}{8} - 2y^3 + \frac{9y^2}{4}\right]_0^1 = (*$$

Cont de 6 c) $(*) = \left[\frac{3}{8} - 2 + \frac{9}{4} - 0\right] = \frac{5}{8}$

7 a) $x^2+y^2+z^2=4$ esfera centrada em $(0,0)$ de raio 2



Coord. esféricas: $x = r\cos\varphi \operatorname{sen}\theta$, $y = r \operatorname{sen}\varphi \operatorname{sen}\theta$, $z = r\cos\theta$

$0 \le r \le 2$, $0 \le \varphi \le \pi$, $0 \le \theta \le \frac{\pi}{2}$.

7 b) $\iiint_R 3z \, dV = \int_0^{\pi}\int_0^{\pi/2}\int_0^{2} 3r\cos\theta \, r^2 \operatorname{sen}\theta \, dr\, d\theta\, d\varphi$

$\int_0^{\pi}\int_0^{\pi/2}\int_0^{2} 3r^3 \operatorname{sen}\theta\cos\theta \, dr\, d\theta\, d\varphi =$

$\int_0^{\pi}\int_0^{\pi/2} \left[\frac{3r^4}{4}\operatorname{sen}\theta\cos\theta\right]_{r=0}^{r=2} d\theta\, d\varphi = (*)$

Cont. di 7 b) : $(*) = \int_0^{\pi}\int_0^{\pi/2} 12 \cdot \operatorname{sen}\theta \cos\theta \, d\theta \, d\varphi =$

$$12\int_0^{\pi} \left[\frac{\operatorname{sen}^2\theta}{2}\right]_{\theta=0}^{\theta=\pi/2} d\varphi = 12\int_0^{\pi} \frac{1}{2} - 0 \, d\varphi =$$

$$6\int_0^{\pi} d\varphi = 6\pi.$$