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(1)

1) $D_f = \{ (x,y) \in \mathbb{R}^2 : \frac{x-y-1}{x+y} > 0 \wedge x+y \neq 0 \}$

$= \{ (x,y) \in \mathbb{R}^2 : x-y-1 > 0 \wedge x+y > 0 \} \cup$
$\{ (x,y) \in \mathbb{R}^2 : x-y-1 < 0 \wedge x+y < 0 \}.$

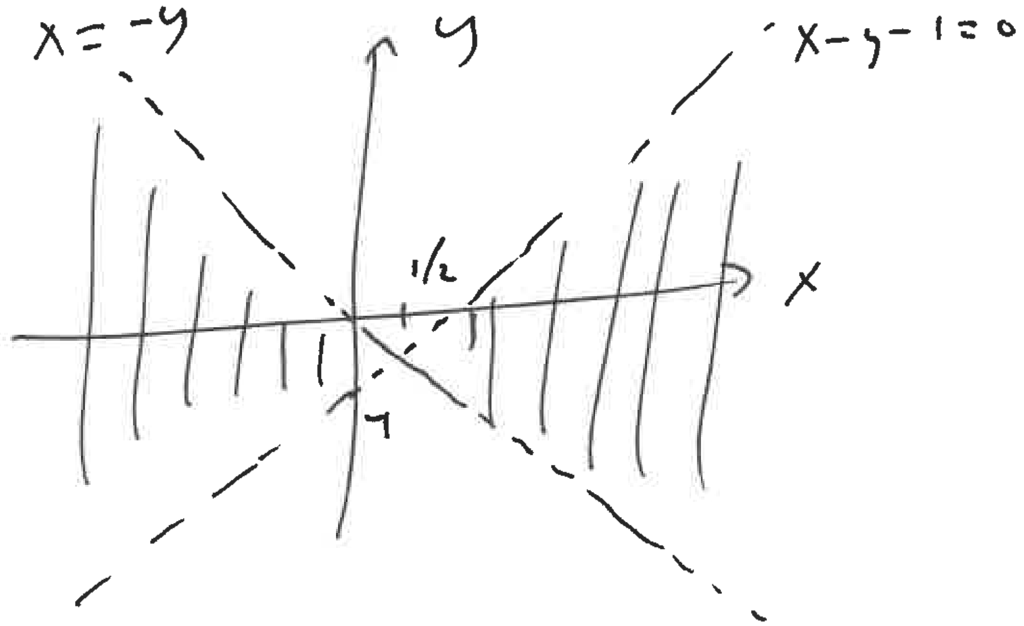
Pts. de interseção:

$x-y-1 = 0 \wedge x+y = 0 \Leftrightarrow \begin{cases} y = -x \\ -x = x-1 \end{cases} \Leftrightarrow \begin{cases} y = -x \\ 2x = 1 \end{cases} \Leftrightarrow$

$(x,y) = \left(\frac{1}{2}, -\frac{1}{2}\right).$

$x-y-1 = 0 \wedge x = 0 \Leftrightarrow x = 0 \wedge y = -1 \quad (0,-1)$

$x-y-1 = 0 \wedge y = 0 \Leftrightarrow x = 1 \wedge y = 0 \quad (1,0)$



2 a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4+3x^2y^2-5y^4}{x^2+y^2} = 0$, pq

$$0 \leq \left| \frac{x^4+3x^2y^2-5y^4}{x^2+y^2} \right| \leq \frac{x^4+3x^2y^2+5y^4}{x^2+y^2}$$

$$\leq \frac{x^4}{x^2} + \frac{3x^2y^2}{x^2} + \frac{5y^4}{y^2} = x^2+3y^2+5y^2 =$$

$x^2+8y^2$.

Como $\lim\limits_{(x,y)\to(0,0)} x^2+8y^2 = 0$, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4+3x^2y^2-5y^4}{x^2+y^2} = 0$

pa enquadramento.

Conclusão: $f$ é cont. em $(0,0)$.

2 b) $f'_x(0,0) = \lim\limits_{t\to0} \dfrac{f(t,0)-f(0,0)}{t} =$

$$\lim_{t\to0} \frac{\frac{t^4}{t^2}-0}{t} = \lim_{t\to0} \frac{t^4}{t^3} = \lim_{t\to0} t = 0.$$

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(3)

(cont. de 2 b))

$$f'_y(0,0) = \lim_{t\to 0} \frac{f(0,t) - f(0,0)}{t} =$$

$$\lim_{t\to 0} \frac{\frac{-5t^4}{t^2} - 0}{t} = \lim_{t\to 0} -5\frac{t^4}{t^3} =$$

$$\lim_{t\to 0} -5t = 0.$$

2 c) $f$ é dif. em $(0,0)$, pq

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - f(0,0) - (f'_x(0,0)x + f'_y(0,0)y)}{\sqrt{x^2+y^2}} = 0$$

Prova: $$\frac{f(x,y) - f(0,0) - (f'_x(0,0)x + f'_y(0,0)y)}{\sqrt{x^2+y^2}} =$$

$$\frac{\frac{x^4 - 3x^2y^2 - 5y^4}{x^2+y^2} - 0}{\sqrt{x^2+y^2}} = \frac{x^4 - 3x^2y^2 - 5y^4}{(x^2+y^2)^{3/2}}.$$

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Cont. de 2c) Enquadramento:

$$0 \leq \left| \frac{x^4 - 3x^2y^2 - 5y^4}{(x^2+y^2)^{3/2}} \right| \leq \frac{x^4 + 3x^2y^2 + 5y^4}{(x^2+y^2)^{3/2}} \leq$$

$$\frac{(x^2+y^2)^2 + 3(x^2+y^2)^2 + 5(x^2+y^2)^2}{(x^2+y^2)^{3/2}} =$$

$$9\,(x^2+y^2)^{3/2}$$

Como $\lim\limits_{(x,y)} (x^2+y^2)^{3/2} = 0$, vemos que

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 3x^2y^2 - 5y^4}{(x^2+y^2)^{3/2}} = 0.$$

3) $S=\{(x,y,z)\in\mathbb{R}^3 \mid F(x,y,z)=0\}$, onde

$$F(x,y,z)=x\sqrt{x^2+y^2}+y^3-z.$$

A equação do plano tangente a $S$ em $(-4,3,7)$:

$$F'_x(-4,3,7)(x+4)+F'_y(-4,3,7)(y-3)+F'_z(-4,3,7)(z-7)=0$$

$$F'_x(-4,3,7)=\sqrt{x^2+y^2}+\frac{x^2}{\sqrt{x^2+y^2}}\Big|_{(-4,3,7)}=5+\frac{16}{5}=\frac{4}{5}$$

$$F'_y(-4,3,7)=\frac{xy}{\sqrt{x^2+y^2}}+3y^2\Big|_{(-4,3,7)}=\frac{-12}{5}+27=$$

$$\frac{123}{5}$$

$$F'_z(-4,3,7)=-1.$$

Portanto, a equação de $T_S(-4,3,7)$ é

$$\frac{41}{5}(x+4)+\frac{123}{5}(y-3)-(z-7)=0$$

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4 a) $\nabla f(x,y) = (0,0) \Leftrightarrow$

$(7 + 2y - 2x, \; -8 + 2x + 3y^2) = (0,0) \Leftrightarrow$

$\begin{cases} 7 + 2y - 2x = 0 \\ -8 + 2x + 3y^2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x = 2y + 7 \\ -8 + 2y + 7 + 3y^2 = 0 \end{cases} \Leftrightarrow$

$\begin{cases} 2x = 2y + 7 \\ 3y^2 + 2y - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x = 2y + 7 \\ y = -1 \vee y = \frac{1}{3} \end{cases}$

Pts. estac.: $\left(\frac{5}{2}, -1\right), \left(\frac{23}{6}, \frac{1}{3}\right)$

4 b) $H_f(x,y) = \begin{pmatrix} -2 & 2 \\ 2 & 6y \end{pmatrix}$

$H_f\left(\frac{5}{2}, -1\right) = \begin{pmatrix} -2 & 2 \\ 2 & -6 \end{pmatrix}$ $\quad \left.\begin{matrix} h_f\left(\frac{5}{2}, -1\right) = 12 - 4 = 8 > 0 \\ f''_{xx}\left(\frac{5}{2}, -1\right) = -2 < 0 \end{matrix}\right\}$ máx. local.

$H_f\left(\frac{23}{6}, \frac{1}{3}\right) = \begin{pmatrix} -2 & 2 \\ 2 & 2 \end{pmatrix}$ $\quad h_f\left(\frac{23}{6}, \frac{1}{3}\right) = -8 < 0$

pt. de sela.

$f$ tem um máximo local em $\left(\frac{5}{2}, -1\right)$ e um pt. de sela em $\left(\frac{23}{6}, \frac{1}{3}\right)$.