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1) $$\frac{1}{(2n-1)(2n+1)} = \frac{1/2}{2n-1} - \frac{1/2}{2n+1}$$

série de Mengoli c/ $b_n = \frac{1/2}{2n-1}$ e $t=1$ (pq $2(n+1)-1 = 2n+2-1 = 2n+1$)

$\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \frac{1/2}{2n-1} = 0$, logo a série converge.

Soma: $b_1 - 1\cdot 0 = \frac{1/2}{2-1} = 1/2$.

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2) $$\left|(-1)^n \frac{(n+1)^2}{3^n}\right| = \frac{(n+1)^2}{3^n}$$

C.d.Q. $$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+2)^2}{3^{n+1}}}{\frac{(n+1)^2}{3^n}} = \frac{(n+2)^2}{(n+1)^2} \cdot \frac{3^n}{3^{n+1}} =$$

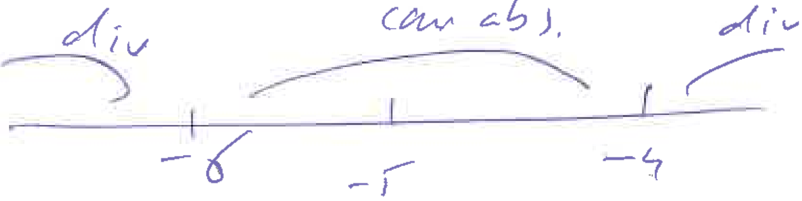
$$\frac{(n+2)^2}{(n+1)^2} \cdot \frac{1}{3}.$$ Logo $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{3} < 1.$

Conclusão: a série é abs. conv.

3) $$\lim_{n\to\infty} \frac{\dfrac{|x+5|^{n+1}}{n+2}}{\dfrac{|x+5|^{n}}{n+1}} = \lim_{n\to\infty} \frac{n+1}{n+2}\cdot|x+5| = |x+5|$$

$|x+5| < 1 \iff -1 < x+5 < 1 \iff -6 < x < -4$

$c = -5$, $R = 1$.

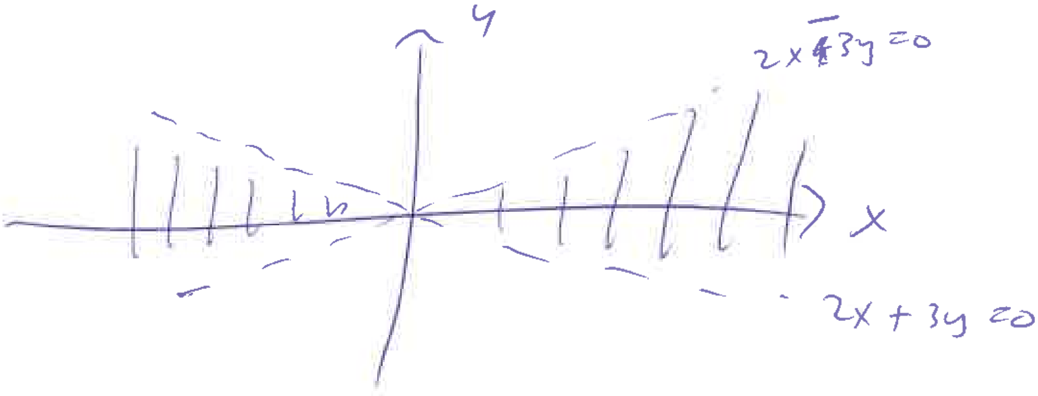


$\underline{x=-4}$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-4+5)^n}{n+1} = \sum_{n=0}^{\infty}\frac{1}{n+1}$ div. (s. harm.)

$\underline{x=-6}$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-6+5)^n}{n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$ simpl. conv. (s. harm. alt.)

4) $D_f = \left\{(x,y)\in\mathbb{R}^2 : \dfrac{2x+3y}{2x-3y} \geq 0 \wedge 2x-3y \neq 0\right\} =$

$\{(x,y)\in\mathbb{R}^2 : 2x+3y \geq 0 \wedge 2x-3y>0\} \cup$
$\{(x,y)\in\mathbb{R}^2 : 2x+3y \leq 0 \wedge 2x-3y<0\}$.



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5) a) $f'_x(0,0) = \lim_{t\to 0} \frac{f(t,0)-f(0,0)}{t} = \lim_{t\to 0} \frac{0-0}{t} = 0$

$f'_y(0,0) = \lim_{t\to 0} \frac{f(0,t)-f(0,0)}{t} = \lim_{t\to 0} \frac{\frac{5t^4}{t^2} - 0}{t} =$

$\lim_{t\to 0} \frac{5t^4}{t^3} = \lim_{t\to 0} 5t = 0.$

b) f <u>não</u> é dif. em $(0,0)$, pq

$f'_{\vec{v}}(0,0) = \lim_{t\to 0} \frac{f(tv_1, tv_2) - f(0,0)}{t} =$

$\lim_{t\to 0} \frac{\frac{3t^3 v_1 v_2^2 + 5t^4 v_2^4}{t^2(v_1^2+v^2)} - 0}{t} \underset{(v_1^2+v_2^2=1)}{=}$

$\lim_{t\to 0} \frac{3t^3 v_1 v_2^2 + 5t^4 v_2^4}{t^3} = \lim_{t\to 0} 3v_1v_2^2 + 5t v_2^4 =$

$3v_1v_2^2$. Portanto, $f'_{\vec{v}}(0,0) \neq f'_x(0,0)v_1 + f'_y(0,0)v_2$

pq $f'_x(0,0)v_1 + f'_y(0,0)v_2 = 0.$

6 a) $\nabla f(x,y) = (0,0) \Leftrightarrow (-6y+6x,\ 6y^2 - 6x) = (0,0)$

$\Leftrightarrow \begin{cases} x = y \\ 6x^2 - 6x = 0 \end{cases} \Leftrightarrow \begin{cases} x = y \\ 6x(x-1) = 0 \end{cases} \Leftrightarrow \begin{cases} x = y \\ x = 0 \vee x = 1 \end{cases}$

Pts estac. $(0,0), (1,1)$

6 b) $H_f(x,y) = \begin{pmatrix} 6 & -6 \\ -6 & 12y \end{pmatrix}$

$H_f(0,0) = \begin{pmatrix} 6 & -6 \\ -6 & 0 \end{pmatrix}$ $h_f(0,0) = -36 < 0$
pt. de sela

$H_f(1,1) = \begin{pmatrix} 6 & -6 \\ -6 & 12 \end{pmatrix}$ $h_f(1,1) = 36 > 0$
$f''_{xx}(1,1) = 6 > 0$
mín. local.

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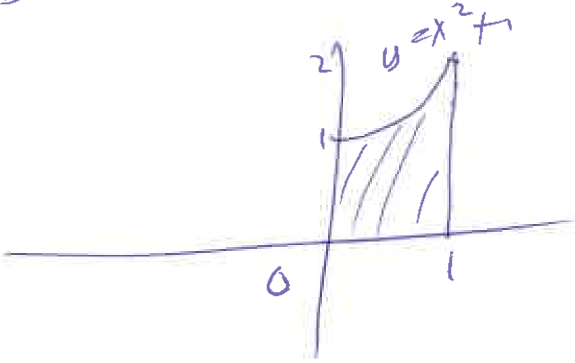
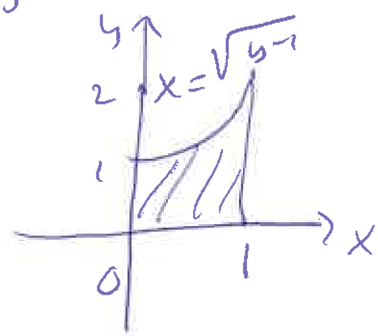
7 a) $D = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1 \wedge 0 \leq y \leq x^2+1\}$.



7 b) $y = x^2+1 \Leftrightarrow x^2 = y-1 \Leftrightarrow x = \pm\sqrt{y-1}$



$D = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1\} \cup$
$\{(x,y) \in \mathbb{R}^2 : \sqrt{y-1} \leq x \leq 1, 1 \leq y \leq 2\}$

$$I = \int_0^1 \int_0^1 f(x,y)\,dx\,dy + \int_1^2 \int_{\sqrt{y-1}}^1 f(x,y)\,dx\,dy$$

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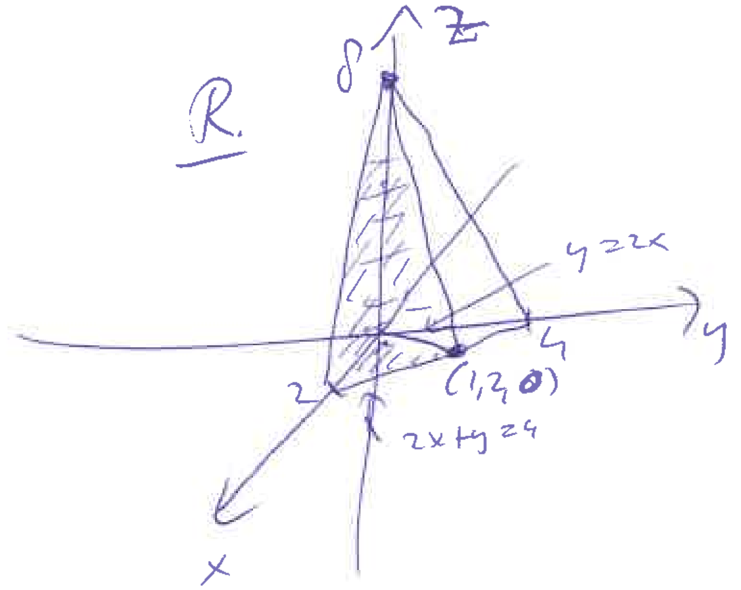
8) a) $z = 8 - 4x - 2y$

$x = y = 0 : \ z = 8$

$x = z = 0 : \ y = 4$

$y = z = 0 : \ x = 2$

$(2,0,0), (0,4,0), (0,0,8)$



$z = 0 : \ 8 - 4x - 2y = 0 \iff$

$4x + 2y = 8 \iff$

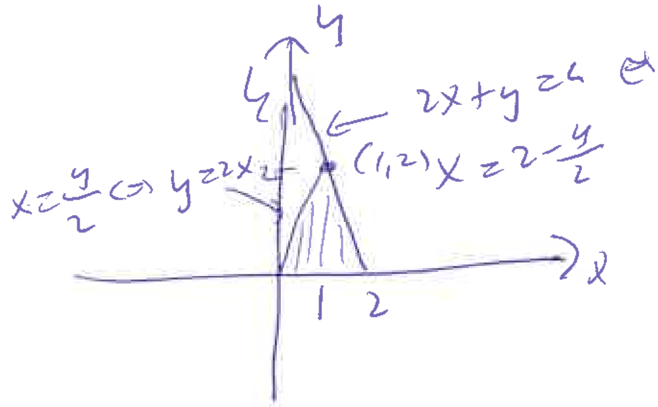
$\boxed{2x + y = 4}$

$y = 2x \ \wedge \ 2x + y = 4 \iff$

$4x = 4 \ \wedge \ y = 2x \iff$

$x = 1 \ \wedge \ y = 2x$

$(1,2).$



8b) $\text{Vol}(R) = \int_0^2 \int_{\frac{y}{2}}^{2-\frac{y}{2}} 8 - 4x - 2y \, dx \, dy =$

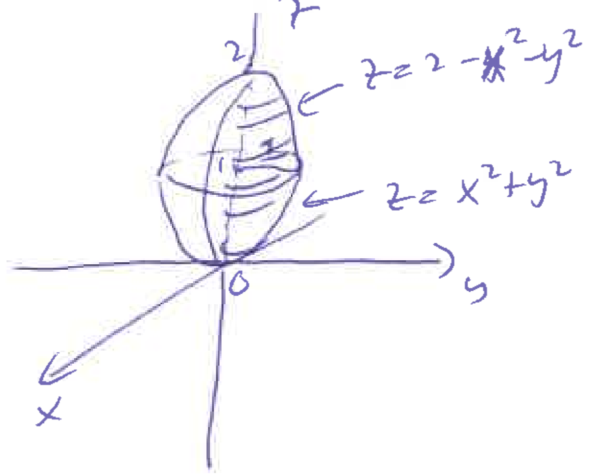
$\int_0^2 8x - 2x^2 - 2xy \Big|_{x=\frac{y}{2}}^{x=2-\frac{y}{2}} dy = \int_0^2 8\left(2-\frac{y}{2}\right) - 2\left(2-\frac{y}{2}\right)^2 - 2\left(2-\frac{y}{2}\right)y$

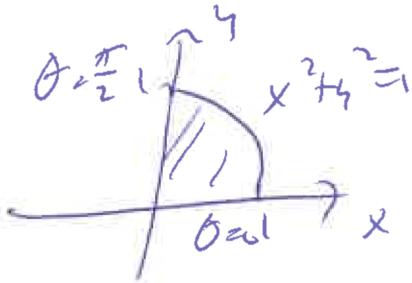
$-\left(4y - \frac{y^2}{2} - y^2\right) dy =$

Cont. de 8b)

$$\int_0^2 2y^2 - 4y + 8\,dy = \frac{2y^3}{3} - 2y^2 + 8y\Big|_0^2 = \frac{16}{3} - 8 + 16 = \frac{40}{3}$$

9a) $x^2+y^2 = 2 - x^2 - y^2 \iff 2(x^2+y^2) = 2 \iff x^2+y^2 = 1$





9b) Coord. cilíndricas: $x = r\cos\theta,\ y = r\,\mathrm{sen}\,\theta,\ z = z$

Jacobiano $= r$, $\quad x^2+y^2 = r^2$

$$\iiint_R (x^2+y^2)^{\frac{3}{2}}\,dV = \int_0^{\pi/2}\int_0^1\int_{r^2}^{2-r^2} (r^2)^{\frac{3}{2}}\cdot r\,dz\,dr\,d\theta =$$

$$\int_0^{\pi/2}\int_0^1\int_{r^2}^{2-r^2} r^4\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^1 r^4 z\Big|_{z=r^2}^{z=2-r^2} dr\,d\theta =$$

$$\int_0^{\pi/2}\int_0^1 r^4(2-r^2) - r^6\,dr\,d\theta = \int_0^{\pi/2}\int_0^1 2r^4 - 2r^6\,dr\,d\theta$$