Aprendizagem 2021/22

Homework I – Group 117

I. Pen-and-paper

Answer 1

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1- $P(X|C=1) = N\left(\mu_1 = \begin{bmatrix}2\\4\end{bmatrix}, \Sigma_1 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)$ $P(X|C=2) = N\left(\mu_2 = \begin{bmatrix}-1\\-4\end{bmatrix}, \Sigma_2 = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right)$

$\pi_1 = P(C_1=1) = 0{,}7 \quad \pi_2 = P(C_2=1) = 0{,}3$

· E-step

$x_1 = \begin{bmatrix}2\\4\end{bmatrix}$ | Para c=1 |

→ prior : $p(C=1) = 0{,}7$

→ likelihood : $P(x|C=1) = \frac{1}{2\pi}\frac{1}{\det\Sigma^1}\exp\left(-\frac{1}{2}(x_1-\mu_1)^T(\Sigma^1)^{-1}(x_1-\mu_1)\right) =$

$= \frac{1}{2\pi}\frac{1}{\det\Sigma^1}\exp\left(-\frac{1}{2}\left(\begin{bmatrix}2\\4\end{bmatrix}-\begin{bmatrix}2\\4\end{bmatrix}\right)^T \cdot \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}^{-1} \cdot \left(\begin{bmatrix}2\\4\end{bmatrix}-\begin{bmatrix}2\\4\end{bmatrix}\right)\right) = 0{,}1592$

→ joint probability

$P(C=1,x_1) = P(C=1).P(x_1|C=1) = 0{,}7 \times 0{,}1592 = 0{,}1114$

| Para c=2 |

→ prior : $P(C=2) = 0{,}3$

→ likelihood : $P(x|C=2) = \frac{1}{2\pi}\frac{1}{\det\Sigma^2}\exp\left(-\frac{1}{2}(x_1-\mu_2)^T(\Sigma^2)^{-1}(x_1-\mu_2)\right)$

$= \frac{1}{2\pi}\frac{1}{\det\Sigma^2}\exp\left(-\frac{1}{2}\left(\begin{bmatrix}2\\4\end{bmatrix}-\begin{bmatrix}-1\\-4\end{bmatrix}\right)^T\left(\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right)^{-1}\left(\begin{bmatrix}2\\4\end{bmatrix}-\begin{bmatrix}-1\\-4\end{bmatrix}\right)\right) = 9{,}439\times10^{-10}$

→ joint probability

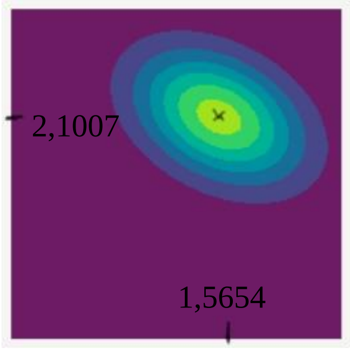
$P(C=2,x_1) = P(C=2).P(x_1|C=2) = 0{,}3\times9{,}439\times10^{-10} =$
$= 2{,}832\times10^{-10}$

NORMALIZAR

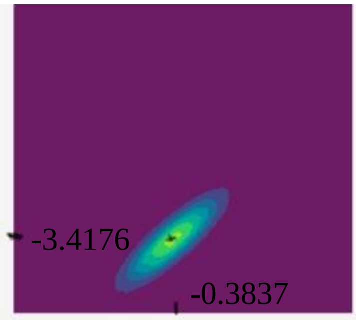
$C=1$: $P(C=1|x_1) = \frac{P(C=1,x_1)}{P(C=1,x_1)+P(C=2,x_1)} = \frac{0{,}1114}{0{,}1114+2{,}832\times10^{-10}} \simeq 1$

$C=2$: $P(C=2|x_1) = \frac{P(C=2,x_1)}{P(C=2,x_1)+P(C=1,x_1)} = \frac{2{,}832\times10^{-10}}{2{,}832\times10^{-10}+0{,}1114} \simeq 0$

1



Cluster 1



Cluster 2

$x_2 = \begin{bmatrix}-1\\-4\end{bmatrix}$ | Para c=1 |

→ prior : $P(C=1) = 0{,}7$

→ likelihood : $P(x|C=1) = 2{,}239\times10^{-17}$

→ joint probability $= 1{,}567\times10^{-17}$

| Para c=2 |

→ prior : $P(C=2) = 0{,}3$

→ likelihood : $P(x|C=2) = 0{,}0796$

→ joint probability $= 0{,}0239$

NORMALIZAR

$C=1$: $P(C=1|x_2) \simeq 0$

$C=2$: $P(C=2|x_2) \simeq 1$

$x_3 = \begin{bmatrix}-1\\2\end{bmatrix}$ | Para c=1 |

→ prior : $P(C=1) = 0{,}7$

→ likelihood : $P(x|C=1) = 0{,}00024$

→ joint probability $= 0{,}00017$

| Para c=2 |

→ prior : $P(C=2) = 0{,}3$

→ likelihood : $P(x|C=2) = 9{,}8206\times10^{-6}$

→ joint probability $= 2{,}9462\times10^{-6}$

NORMALIZAR

$C=1$: $P(C=1|x_3) = 0{,}9827$

$C=2$: $P(C=2|x_3) = 0{,}0173$

$x_4 = \begin{bmatrix}4\\0\end{bmatrix}$ | Para c=1 |

→ prior : $P(C=1) = 0{,}7$

→ likelihood : $P(x|C=1) = 7{,}226\times10^{-6}$

→ joint probability $= 5{,}058\times10^{-6}$

| Para c=2 |

→ prior $P(C=2) = 0{,}3$

→ likelihood : $P(x|C=2) = 2{,}814\times10^{-6}$

→ joint probability $= 8{,}441\times10^{-7}$

NORMALIZAR

$C=1$. $P(C=1|x_4) = 0{,}8570$

$C=2$: $P(C=2|x_4) = 0{,}143$

2

$\mu_1 = \frac{1\times\begin{bmatrix}2\\4\end{bmatrix}+0{,}9827\begin{bmatrix}-1\\2\end{bmatrix}+0{,}857\begin{bmatrix}4\\0\end{bmatrix}}{1+0{,}9827+0{,}857} = \begin{bmatrix}1{,}5654\\2{,}1007\end{bmatrix}$

$\mu_2 = \frac{1\times\begin{bmatrix}-1\\-4\end{bmatrix}+0{,}0173\begin{bmatrix}-1\\2\end{bmatrix}+0{,}143\begin{bmatrix}4\\0\end{bmatrix}}{1+0{,}0173+0{,}143} = \begin{bmatrix}-0{,}3838\\-3{,}4176\end{bmatrix}$

$\Sigma_{11} = \frac{1(2-1{,}5654)^2+0(-1-1{,}5654)^2+0{,}9827(-1-1{,}5654)^2+0{,}857(4-1{,}5654)^2}{1+0+0{,}9827+0{,}857}$

$= 4{,}1328$

$\Sigma_{12} = \Sigma_{21} = \frac{1\times(2-1{,}5654)(4-2{,}1007)+0\times(-1-1{,}5654)(-4-2{,}1007)+0{,}9827(-1-1{,}5654)(2-2{,}1007)+0{,}857(4-1{,}5654)(0-2{,}1007)}{1+0+0{,}9827+0{,}857}$

$= -1{,}1634$

$\Sigma_{22} = \frac{1\times(4-2{,}1007)^2+0(-4-2{,}1007)^2+0{,}9827(2-2{,}1007)^2+0{,}857(0-2{,}1007)^2}{1+0+0{,}9827+0{,}857} =$

$= 2{,}6056$

$P(x|C=1) = N\left(\mu_1 = \begin{bmatrix}1{,}5654\\2{,}1007\end{bmatrix}, \Sigma_1 = \begin{bmatrix}4{,}1328 & -1{,}1634\\-1{,}1634 & 2{,}6056\end{bmatrix}\right)$

$\Sigma_{11} = \frac{0(2+0{,}3838)^2+1(-1+0{,}3838)^2+0{,}0173(-1+0{,}3838)^2+0{,}143(4+0{,}3838)^2}{0+1+0{,}0173+0{,}143}$

$= 1{,}1038$

$\Sigma_{12} = \Sigma_{21} = \frac{0\times(2+0{,}3838)(4+3{,}4176)+1(-1+0{,}3838)(-4+3{,}4176)+0{,}0173(-1+0{,}3838)(2+3{,}1476)+0{,}143(4+0{,}3838)(0+3{,}1476)}{0+1+0{,}0173+0{,}143}$

$= 0{,}8605$

$\Sigma_{22} = \frac{0(4+3{,}4176)^2+1\times(-4-3{,}4176)^2+0{,}0173(2+3{,}4176)^2+0{,}143(0+3{,}4176)^2}{0+1+0{,}0173+0{,}143} =$

$= 0{,}8864$

$P(x|C=2) = N\ \mu_2 = \begin{bmatrix}-0{,}3838\\-3{,}4176\end{bmatrix}, \Sigma_2 = \begin{bmatrix}1{,}1038 & 0{,}8605\\0{,}8605 & 0{,}8864\end{bmatrix}$

$P(C=1) = \frac{P(C=1|x_1)+P(C=1|x_2)+P(C=1|x_3)+P(C=1|x_4)}{P(C=1|x_1)+P(C=1|x_2)+P(C=1|x_3)+P(C=1|x_4)+P(C=2|x_1)+P(C=2|x_2)+P(C=2|x_3)+P(C=2|x_4)} =$

$= 0{,}710$

$P(C=0) = 1-P(C=1) = 0{,}290$

3

Answer 2

2 - x_1, x_3 e $x_4 \in$ Cluster 1
$x_2 \in$ Cluster 2

Silhouette para x_1

$a(x_1) = \frac{1}{2}(\|x_1-x_3\|_2 + \|x_1-x_4\|_2) = \frac{1}{2}\left(\sqrt{(2+1)^2+(4-2)^2} + \sqrt{(2-4)^2+(4-0)^2}\right) = 4{,}039$

$b(x_1) = \|x_1-x_2\|_2 = \sqrt{(2+1)^2+(4+4)^2} = 8{,}544$

$s(x_1) = 1 - \frac{a(x_1)}{b(x_1)} = 1 - \frac{4{,}039}{8{,}544} = 0{,}527$

Silhouette para x_2

$a(x_2) = 0$

$b(x_2) = \frac{1}{3}(\|x_2-x_1\|_2 + \|x_2-x_3\|_2 + \|x_2-x_4\|_2) = 6{,}982$

$s(x_2) = 1 - \frac{a(x_2)}{b(x_2)} = 1 - \frac{0}{6{,}982} = 1$

Silhouette para x_3

$a(x_3) = \frac{1}{2}(\|x_3-x_1\|_2 + \|x_3-x_4\|_2) = 4{,}495$

$b(x_3) = \|x_3-x_2\|_2 = 6$

$s(x_3) = 1 - \frac{a(x_3)}{b(x_3)} = 1 - \frac{4{,}495}{6} = 0{,}251$

Silhouette para x_4

$a(x_4) = \frac{1}{2}(\|x_4-x_1\|_2 + \|x_4-x_3\|_2) = 4{,}929$

$b(x_4) = \|x_4-x_2\|_2 = 6{,}403$

$s(x_4) = 1 - \frac{a(x_4)}{b(x_4)} = 0{,}230$

Silhouette de c_1

$s(c_1) = \frac{s(x_1)+s(x_3)+s(x_4)}{3} = \frac{0{,}527+0{,}251+0{,}230}{3} = 0{,}336$

Silhouette de c_2

$s(c_2) = \frac{s(x_2)}{1} = \frac{1}{1} = 1$

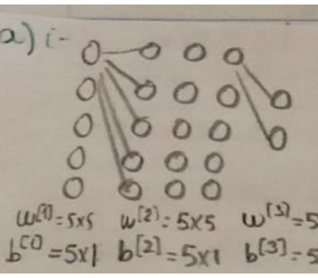
Silhouette de C

$s(C) = \frac{s(c_1)+s(c_2)}{2} = \frac{0{,}336+1}{2} = 0{,}668$

Answer 3-a

i-

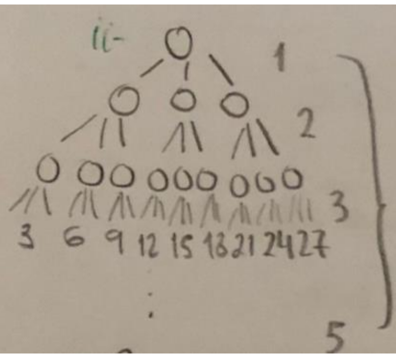
3 - a) i-



Total parâmetros $= 3\times(5\times5+5\times1)+2\times5+2 = 102$

ii-

ii-



$3^5 = 243$ total parâmetros

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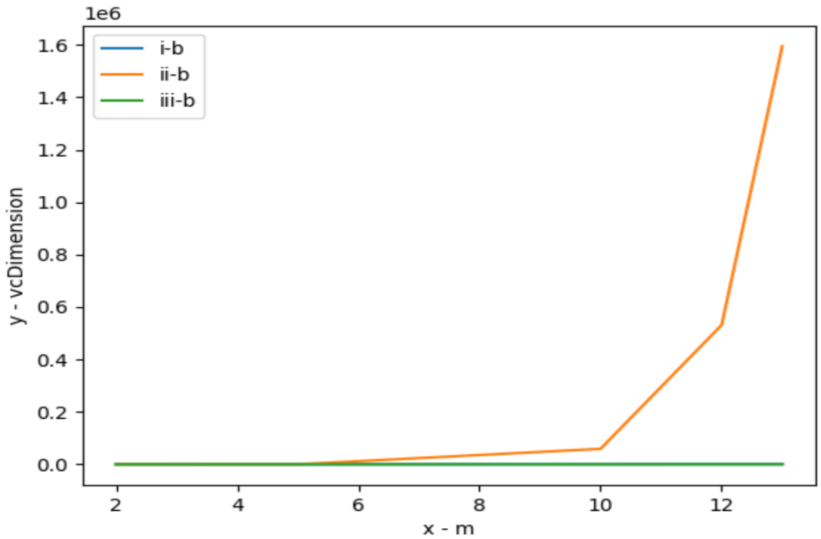
iii-

iii- $P(C=0) = 1 - P(C=1) \longrightarrow$ 1 parametro

$N(\mu, \Sigma)$ $\quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_5 \end{bmatrix} = 5$ parametros $\quad \Sigma = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2 & & \cdot & \cdot & \cdot & \cdot \\ 3 & & & \cdot & \cdot & \cdot \\ 4 & & & & \cdot & \cdot \\ 5 & & & & & \cdot \end{array}$ 15 param.

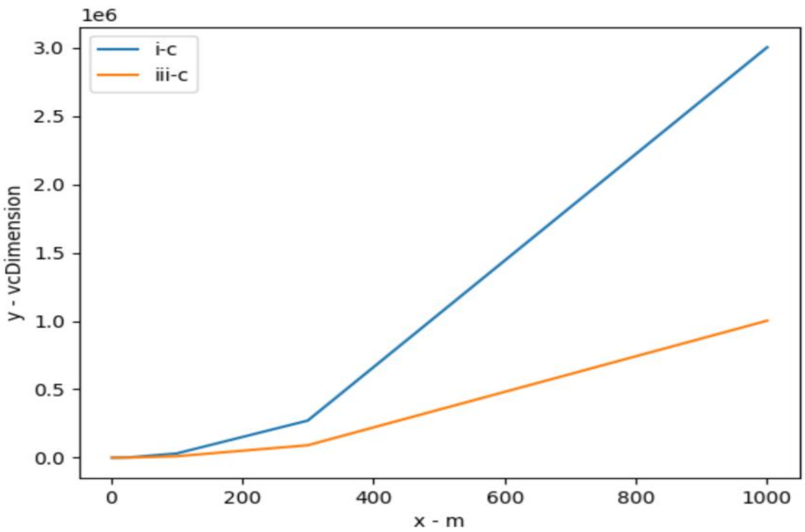
$1 + (15+5) \times 2 = 41$ parâmetros

Answer 3-b



Pela observação do gráfico, verifica-se que a variação da vcDimension da decision tree (ii), aumenta exponencialmente, e verifica-se um aumento abrupto a partir de data dimensionality = 10 face à MLP com 3 hidden layers e ao Bayesian Classifier com uma multivariate Gaussian likelihood.

Answer 3-c



Pela observação do gráfico, verifica-se que um maior aumento da vcDimension no MLP Classifier com 3 hidden layers do que no Bayesian Classifier com uma multivariate Gaussian likelihood a partir da data dimensionality = 100.

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II. Programming and critical analysis

Answer 4

```
ECR K = 2
13.5
ECR k = 3
6.666666666666666
```

```
Silhouette K = 2
0.5967981179111456
Silhouette K = 3
0.5245427800706391
```

a)

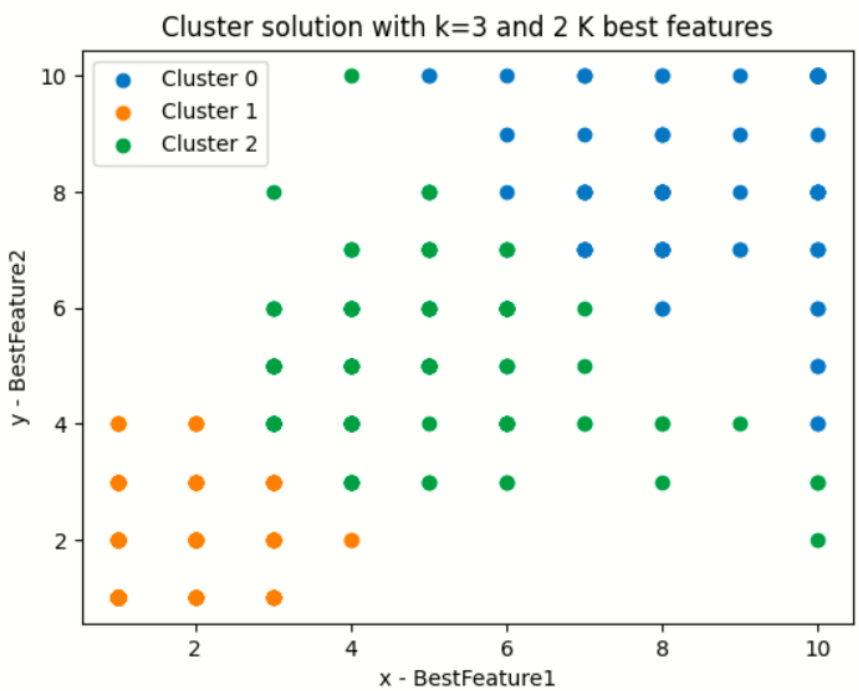
Pelos resultados acima apresentados verifica-se que no algoritmo kMeans, k = 3 apresenta um melhor ECR value que k = 2 para a nossa data.

Sendo o ECR a média dos pontos mal classificados concluimos que ao adicionarmos um novo cluster vai existir uma maior margem para classificação de pontos, e portanto menos pontos mal classificados, assim é natural que o ECR seja mais pequeno para k = 3.

b)

Pelos resultados acima apresentados verifica-se que no algoritmo kMeans, k = 2 apresenta uma melhor silhouette que k = 3 para a nossa data. Isto deve-se ao facto de para k = 2 os clusters serem mais compactos e estarem mais separados entre si.

Answer 5



Answer 6

```
ECR Ex5
11.666666666666666
Silhouette Ex5
0.7074226869204926
```

No exercício 5 verifica-se que a silhouette do algoritmo kMeans com k = 2 e apenas selecionando as 2 melhores features da nossa data segundo a mutual information é bastante boa, isto significa que os cluster são compactos e estão afastados entre si, algo que se pode verificar pelo gráfico apresentado na resposta 5.

Quanto ao ECR value verifica-se que este é melhor do que para k = 2 usando toda a data, mas pior do que para k = 3 usando toda a data.

Sendo o ECR a média dos pontos mal classificados concluimos que ao adicionarmos um novo cluster vai existir uma maior margem para classificação de pontos, e portanto menos pontos mal classificados, assim é natural que o ECR seja mais pequeno do que para k = 2. Dado que no exercício 5 apenas se selecionam as duas melhores features, a nossa data torna-se mais imprecisa o que leva a um maior grupo de pontos mal classificados, portanto é normal que o ECR value do exercício 5 seja maior que o de k = 3.

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III. APPENDIX

Paste your programming code here using Consolas 9pt or 10pt.

Use **highlighting** or colored text to facilitate the analysis by your faculty hosts.

```
# Grupo 117 Aprendizagem HomeWork 4
# Bernardo Castico ist196845
# Hugo Rita ist196870

import pandas as pd
import sklearn
from scipy.io import arff
from sklearn.cluster import KMeans
import numpy as np
from sklearn.metrics import silhouette_score
from sklearn.feature_selection import SelectKBest
from sklearn.feature_selection import mutual_info_classif
import matplotlib.pyplot as plt

def getDataToMatrix(lines):
    realLines = []
    data = []
    toDelete = []
    for i in range(len(lines)):
        if i > 11:
            realLines += [lines[i]]
    for i in range(len(realLines)):
        for j in range(len(realLines[i])):
            if realLines[i][j] == "benign\n":
                realLines[i][j] = "benign"
            elif realLines[i][j] == "malignant\n":
                realLines[i][j] = "malignant"
            elif realLines[i][j] == '?':
                toDelete += [i]
            else:
                realLines[i][j] = int(realLines[i][j])
    for i in range(len(realLines)):
        if i not in toDelete:
            data += [realLines[i]]
    return data

def splitData(list):
    a = []
    b = []
    for i in list:
        a.append(i[:-1])
```

```
        b.append(i[-1])
    return [a,b]

def main():
    data, res2 = [],[]
    cluster02, cluster12, cluster03, cluster13, cluster23, cluster05, cluster15, cluster25 =
0,0,0,0,0,0,0,0
    Benign02, malignant02, Benign12, malignant12, Benign03, malignant03, Benign13, malignant13,
Benign23, malignant23 = 0,0,0,0,0,0,0,0,0,0
    Benign05, malignant05, Benign15, malignant15, Benign25, malignant25 = 0,0,0,0,0,0
    xCluster0, yCluster0, xCluster1, yCluster1, xCluster2, yCluster2 = [],[],[],[],[],[]
    with open("HW3-breast.txt") as f:
        lines = f.readlines()
    for line in lines:
        tmp = line.split(',')
        res2.append(tmp)
    data = getDataToMatrix(res2)

    trainDataSplit = splitData(data)

    kMeans2 = KMeans(n_clusters=2, random_state=0).fit(trainDataSplit[0])
    kMeans3 = KMeans(n_clusters=3, random_state=0).fit(trainDataSplit[0])

    kLabels2 = kMeans2.labels_
    kLabels3 = kMeans3.labels_

    for i in range(len(kLabels2)):
        if kLabels2[i] == 0:
            cluster02 += 1
            if trainDataSplit[1][i] == 'malignant':
                malignant02 += 1
            else:
                Benign02 += 1
        elif kLabels2[i] == 1:
            cluster12 += 1
            if trainDataSplit[1][i] == 'malignant':
                malignant12 += 1
            else:
                Benign12 += 1
        if kLabels3[i] == 0:
            cluster03 += 1
            if trainDataSplit[1][i] == 'malignant':
                malignant03 += 1
            else:
                Benign03 += 1
        elif kLabels3[i] == 1:
            cluster13 += 1
```

```
            if trainDataSplit[1][i] == 'malignant':
                malignant13 += 1
            else:
                Benign13 += 1
        elif kLabels3[i] == 2:
            cluster23 += 1
            if trainDataSplit[1][i] == 'malignant':
                malignant23 += 1
            else:
                Benign23 += 1

    ECR2 = 0.5*((cluster02-max(Benign02,malignant02)) + (cluster12-max(Benign12, malignant12)))
    ECR3 = (1/3)*((cluster03-max(Benign03,malignant03)) + (cluster13-max(Benign13, malignant13))+
(cluster23-max(Benign23, malignant23)))

    print("ECR K = 2")
    print(ECR2)
    print("ECR k = 3")
    print(ECR3)
    print("Silhouette K = 2")
    print(silhouette_score(trainDataSplit[0], kLabels2))
    print("Silhouette K = 3")
    print(silhouette_score(trainDataSplit[0], kLabels3))

    #EX5

    decision = SelectKBest(mutual_info_classif, k=2).fit(trainDataSplit[0], trainDataSplit[1])
    decisionTrainData = decision.transform(trainDataSplit[0])

    kMeans3Ex5 = KMeans(n_clusters=3, random_state=0).fit(decisionTrainData)
    kLabelsEx5 = kMeans3Ex5.labels_

    for i in range(len(kLabelsEx5)):
        if kLabelsEx5[i] == 0:
            cluster05 += 1
            if trainDataSplit[1][i] == 'malignant':
                malignant05 += 1
            else:
                Benign05 += 1
        elif kLabelsEx5[i] == 1:
            cluster15 += 1
            if trainDataSplit[1][i] == 'malignant':
                malignant15 += 1
            else:
                Benign15 += 1
        elif kLabelsEx5[i] == 2:
            cluster25 += 1
```

```
            if trainDataSplit[1][i] == 'malignant':
                malignant25 += 1
            else:
                Benign25 += 1

    ECR5 = (1/3)*((cluster05-max(Benign05,malignant05)) + (cluster15-max(Benign15, malignant15))+
(cluster25-max(Benign25, malignant25)))
    print("ECR Ex5")
    print(ECR5)
    print("Silhouette Ex5")
    print(silhouette_score(decisionTrainData, kLabelsEx5))

    for i in range(len(kLabelsEx5)):
        if kLabelsEx5[i] == 0:
            xCluster0 += [decisionTrainData[i][0]]
            yCluster0 += [decisionTrainData[i][1]]
        elif kLabelsEx5[i] == 1:
            xCluster1 += [decisionTrainData[i][0]]
            yCluster1 += [decisionTrainData[i][1]]
        else:
            xCluster2 += [decisionTrainData[i][0]]
            yCluster2 += [decisionTrainData[i][1]]

    plt.scatter(xCluster0, yCluster0, label="Cluster 0")
    plt.scatter(xCluster1, yCluster1, label="Cluster 1")
    plt.scatter(xCluster2, yCluster2, label="Cluster 2")

    plt.xlabel('x - BestFeature1')
    plt.ylabel('y - BestFeature2')
    plt.title('Cluster solution with k=3 and 2 K best features')

    # show a legend on the plot
    plt.legend()
    plt.show()

main()
```

END